Computer Algebra
Independent Integration Tests

Summer 2023 edition

4-Trig-functions/4.4-Cotangent/112-4.4.2.1-a+b-cot-^m-c+d-cot-^n

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Contents

CHAPTER 1

INTRODUCTION

This report gives the result of running the computer algebra independent integration test.The download section in on the main webpage contains links to download the problems in plain text format used for all CAS systems. The number of integrals in this report is [106]. This is test number [112].

1.1 Listing of CAS systems tested

The following are the CAS systems tested:

1. Mathematica 13.3.1 (August 16, 2023) on windows 10.
2. Rubi 4.16.1 (Dec 19, 2018) on Mathematica 13.3 on windows 10
3. Maple 2023.1 (July, 12, 2023) on windows 10.
4. Maxima 5.47 (June 1, 2023) using Lisp SBCL 2.3.0 on Linux via sagemath 10.1 (Aug 20, 2023).
5. FriCAS 1.3.9 (July 8, 2023) based on sbcl 2.3.0 on Linux via sagemath 10.1 (Aug 20, 2023).
6. Giac/Xcas 1.9.0-57 (June 26, 2023) on Linux via sagemath 10.1 (Aug 20, 2023).
7. Sympy 1.12 (May 10, 2023) Using Python 3.11.3 on Linux.
8. Mupad using Matlab 2021a with Symbolic Math Toolbox Version 8.7 on windows 10.

Maxima and Fricas and Giac are called using Sagemath. This was done using Sagemath `integrate` command by changing the name of the algorithm to use the different CAS systems.

Sympy was run directly in Python not via sagemath.

1.2 Results

Important note: A number of problems in this test suite have no antiderivative in closed form. This means the antiderivative of these integrals can not be expressed in terms of elementary, special functions or `Hypergeometric2F1` functions. `RootSum` and `RootOf` are not allowed. If a CAS returns the above integral unevaluated within the time limit, then the result is counted as passed and assigned an A grade.

However, if CAS times out, then it is assigned an F grade even if the integral is not integrable, as this implies CAS could not determine that the integral is not integrable in the time limit.

If a CAS returns an antiderivative to such an integral, it is assigned an A grade automatically and this special result is listed in the introduction section of each individual test report to make it easy to identify as this can be important result to investigate.

The results given in in the table below reflects the above.

System	% solved	% Failed
Rubi	100.00 (106)	0.00 (0)
Mathematica	99.06 (105)	0.94 (1)
Maple	97.17 (103)	2.83 (3)
Fricas	97.17 (103)	2.83 (3)
Mupad	97.17 (103)	2.83 (3)
Giac	2.83 (3)	97.17 (103)
Maxima	2.83 (3)	97.17 (103)
Sympy	1.89 (2)	98.11 (104)

Table 1.1: Percentage solved for each CAS

The table below gives additional break down of the grading of quality of the antiderivatives generated by each CAS. The grading is given using the letters A,B,C and F with A being the best quality. The grading is accomplished by comparing the antiderivative generated with the optimal antiderivatives included in the test suite. The following table describes the meaning of these grades.

grade	description
A	Integral was solved and antiderivative is optimal in quality and leaf size.
B	Integral was solved and antiderivative is optimal in quality but leaf size is larger than twice the optimal antiderivatives leaf size.
C	Integral was solved and antiderivative is non-optimal in quality. This can be due to one or more of the following reasons 1. antiderivative contains a hypergeometric function and the optimal antiderivative does not. 2. antiderivative contains a special function and the optimal antiderivative does not. 3. antiderivative contains the imaginary unit and the optimal antiderivative does not.
F	Integral was not solved. Either the integral was returned unevaluated within the time limit, or it timed out, or CAS hanged or crashed or an exception was raised.

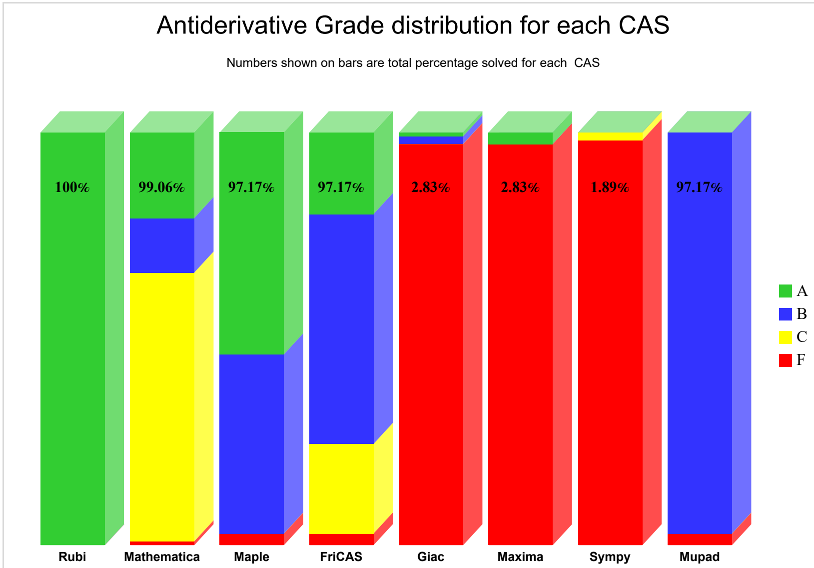
Table 1.2: Description of grading applied to integration result

Grading is implemented for all CAS systems. Based on the above, the following table summarizes the grading for this test suite.

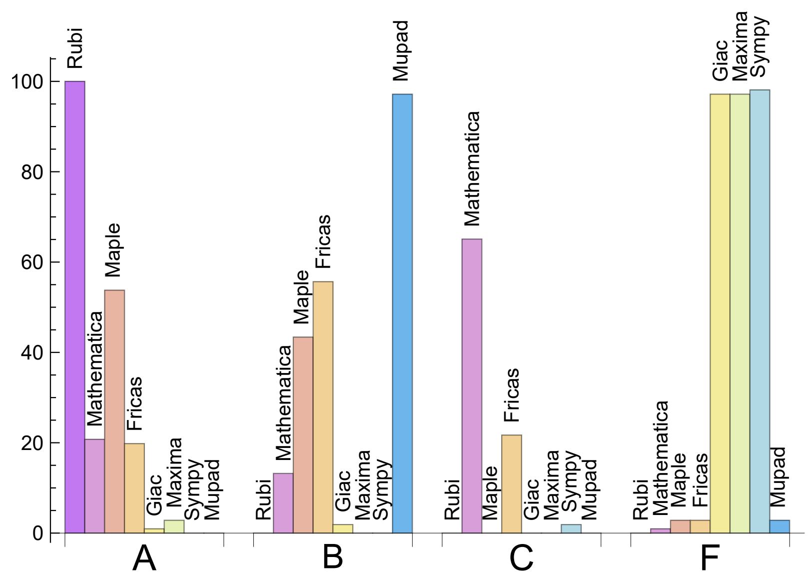
System	% A grade	% B grade	% C grade	% F grade
Rubi	100.000	0.000	0.000	0.000
Maple	53.774	43.396	0.000	2.830
Mathematica	20.755	13.208	65.094	0.943
Fricas	19.811	55.660	21.698	2.830
Maxima	2.830	0.000	0.000	97.170
Giac	0.943	1.887	0.000	97.170
Mupad	0.000	97.170	0.000	2.830
Sympy	0.000	0.000	1.887	98.113

Table 1.3: Antiderivative Grade distribution of each CAS

The following is a Bar chart illustration of the data in the above table.



The figure below compares the grades of the CAS systems.



The following table shows the distribution of the different types of failures for each CAS. There are 3 types failures. The first is when CAS returns the input within the time limit, which means it could not solve it. This is the typical failure and given as **F**.

The second failure is due to time out. CAS could not solve the integral within the 3 minutes time limit which is assigned. This is assigned **F(-1)**.

The third is due to an exception generated, indicated as **F(-2)**. This most likely indicates an interface problem between sagemath and the CAS (applicable only to FriCAS, Maxima and Giac) or it could be an indication of an internal error in the CAS itself. This type of error requires more investigation to determine the cause.

System	Number failed	Percentage normal failure	Percentage timeout failure	Percentage exception failure
Rubi	0	0.00	0.00	0.00
Mathematica	1	100.00	0.00	0.00
Fricas	3	100.00	0.00	0.00
Maple	3	100.00	0.00	0.00
Mupad	3	0.00	100.00	0.00
Giac	103	95.15	4.85	0.00
Maxima	103	24.27	0.97	74.76
Sympy	104	94.23	4.81	0.96

Table 1.4: Failure statistics for each CAS

1.3 Time and leaf size Performance

The table below summarizes the performance of each CAS system in terms of time used and leaf size of results.

Mean size is the average leaf size produced by the CAS (before any normalization). The Normalized mean is relative to the mean size of the optimal anti-derivative given in the input files.

For example, if CAS has **Normalized mean** of 3, then the mean size of its leaf size is 3 times as large as the mean size of the optimal leaf size.

Median size is value of leaf size where half the values are larger than this and half are smaller (before any normalization). i.e. The Middle value.

Similarly the **Normalized median** is relative to the median leaf size of the optimal.

For example, if a CAS has Normalized median of 1.2, then its median is 1.2 as large as the median leaf size of the optimal.

System	Mean time (sec)
Maple	0.12
Giac	0.36
Maxima	0.42
Rubi	0.52
Sympy	1.07
Fricas	1.42
Mathematica	1.99
Mupad	15.48

Table 1.5: Time performance for each CAS

System	Mean size	Normalized mean	Median size	Normalized median
Maxima	203.67	1.70	185.00	1.67
Rubi	232.98	1.00	221.50	1.00
Mathematica	236.03	1.26	203.00	0.83
Giac	249.33	2.05	241.00	2.17
Maple	530.62	3.30	333.00	1.29
Fricas	1996.96	7.20	849.00	3.87
Sympy	2244.00	22.30	2244.00	22.30
Mupad	3189.42	10.43	366.00	1.69

Table 1.6: Leaf size performance for each CAS

1.4 Performance based on number of rules Rubi used

This section shows how each CAS performed based on the number of rules Rubi needed to solve the same integral. One diagram is given for each CAS.

On the y axis is the percentage solved which Rubi itself needed the number of rules given the x axis. These plots show that as more rules are needed then most CAS system percentage of solving decreases which indicates the integral is becoming more complicated to solve.

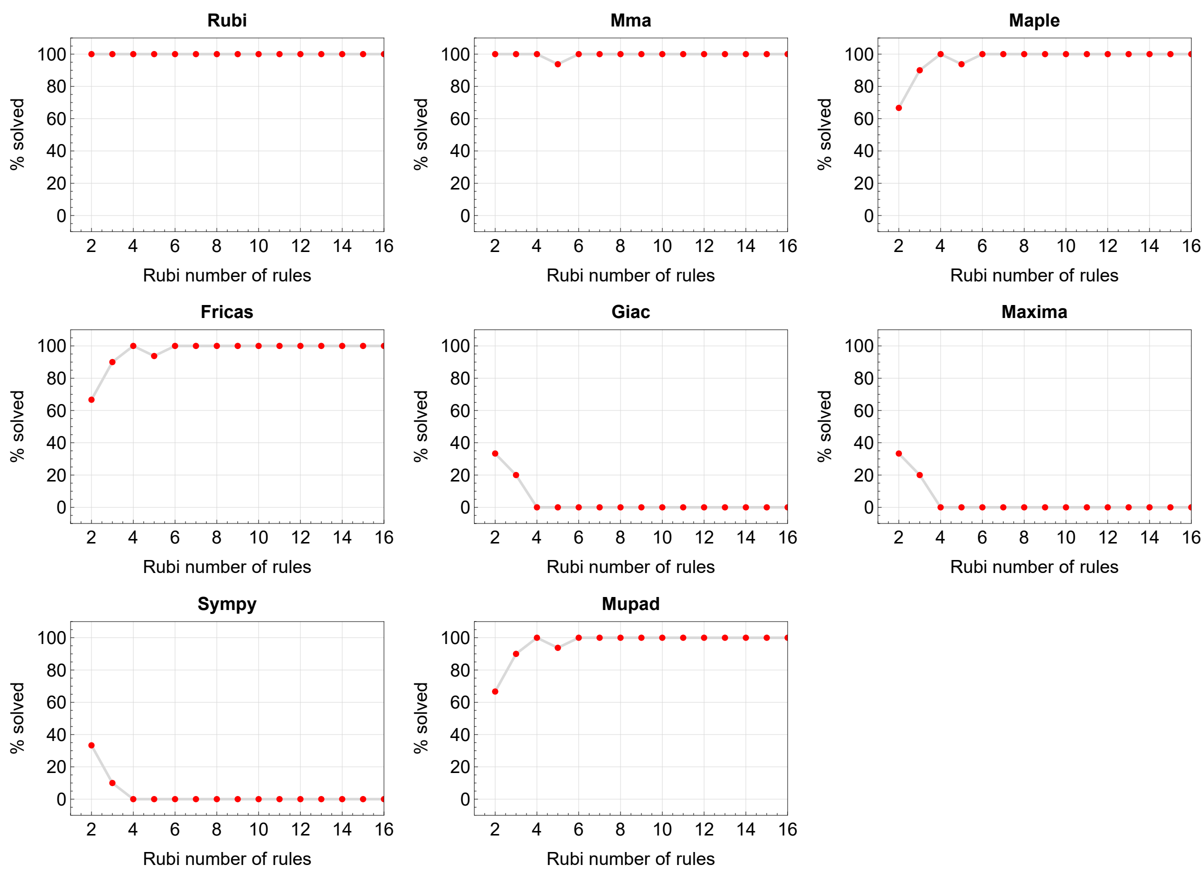


Figure 1.1: Solving statistics per number of Rubi rules used

1.5 Performance based on number of steps Rubi used

This section shows how each CAS performed based on the number of steps Rubi needed to solve the same integral. Note that the number of steps Rubi needed can be much higher than the number of rules, as the same rule could be used more than once.

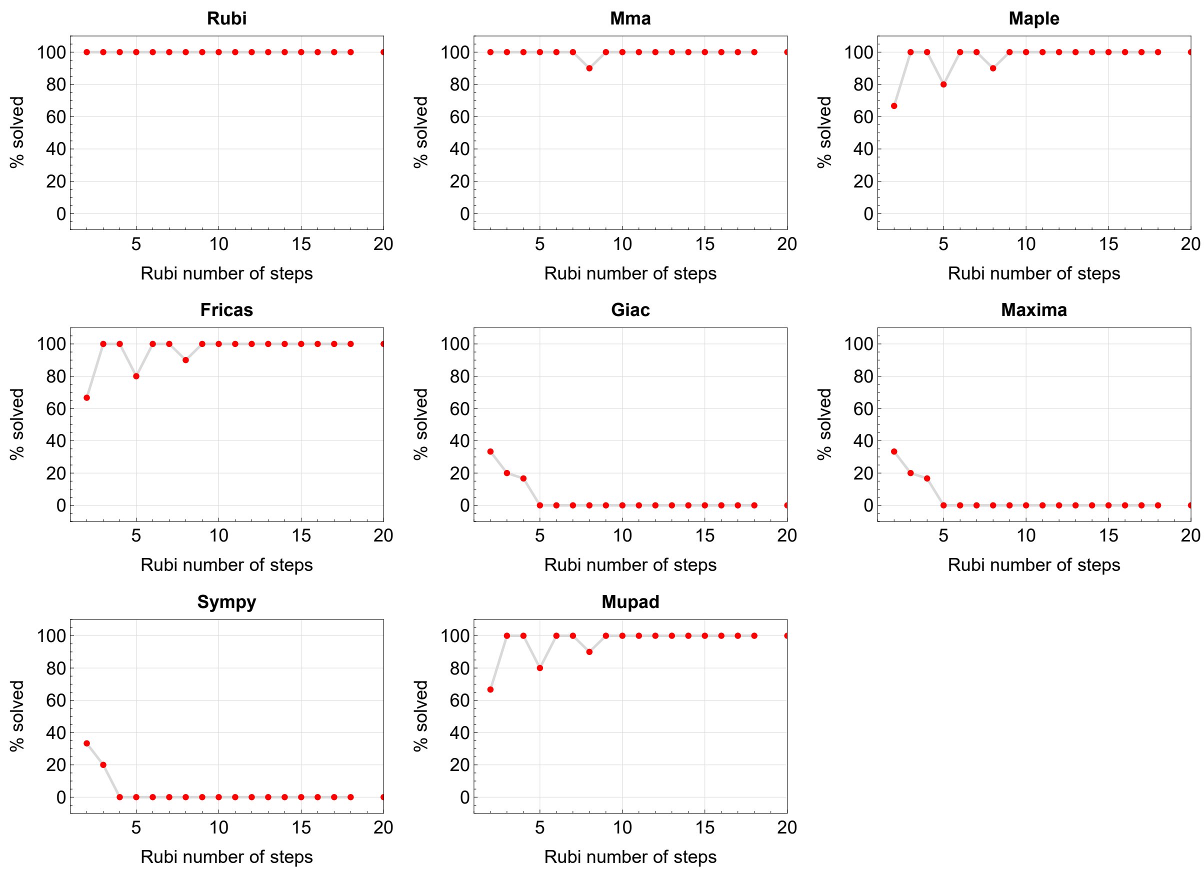


Figure 1.2: Solving statistics per number of Rubi steps used

The above diagram show that the precentage of solved intergals decreases for most CAS systems as the number of steps increases. As expected, for integrals that required less steps by Rubi, CAS systems had more success which indicates the integral was not as hard to solve. As Rubi needed more steps to solve the integral, the solved percentage decreased for most CAS systems which indicates the integral is becoming harder to solve.

1.6 Solved integrals histogram based on leaf size of result

The following shows the distribution of solved integrals for each CAS system based on leaf size of the antiderivatives produced by each CAS. It shows that most integrals solved produced leaf size less than about 100 to 150. The bin size used is 40.

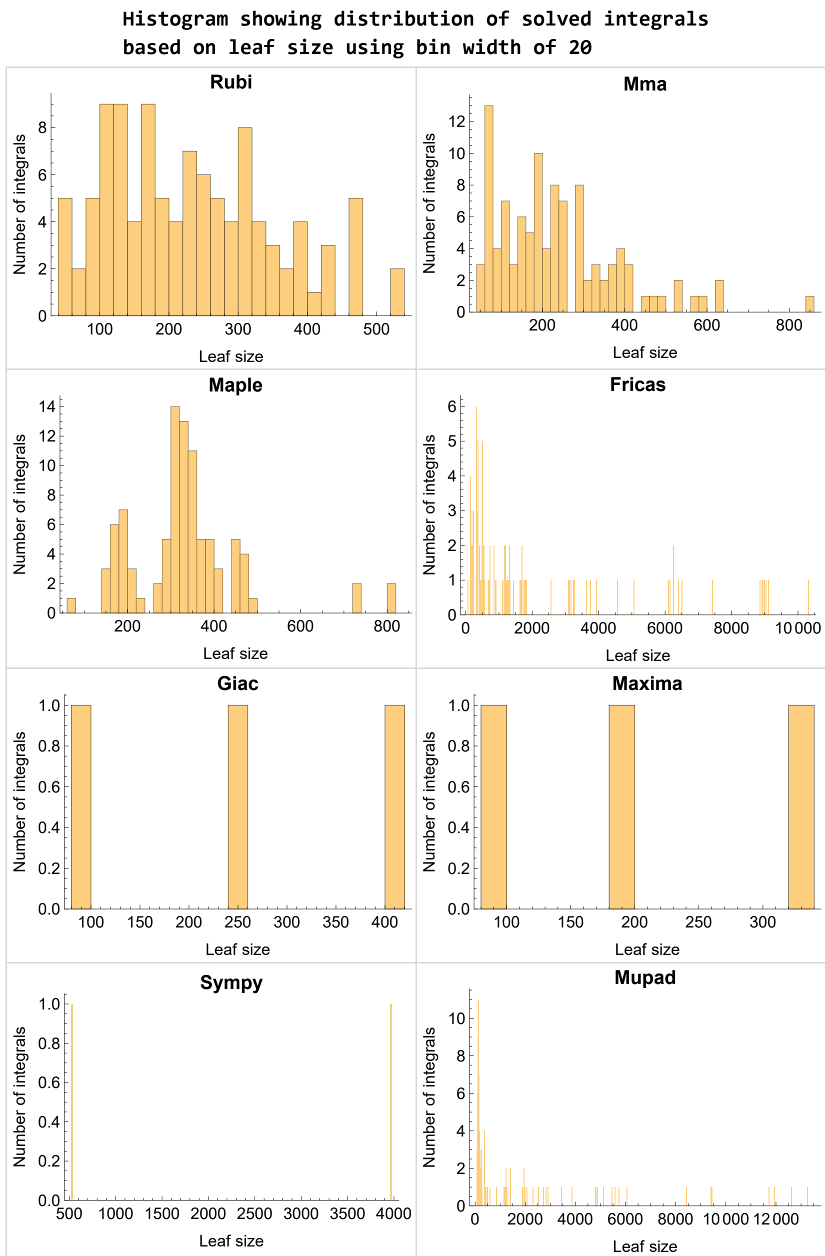


Figure 1.3: Solved integrals based on leaf size distribution

1.7 Solved integrals histogram based on CPU time used

The following shows the distribution of solved integrals for each CAS system based on CPU time used in seconds. The bin size used is 0.1 second.

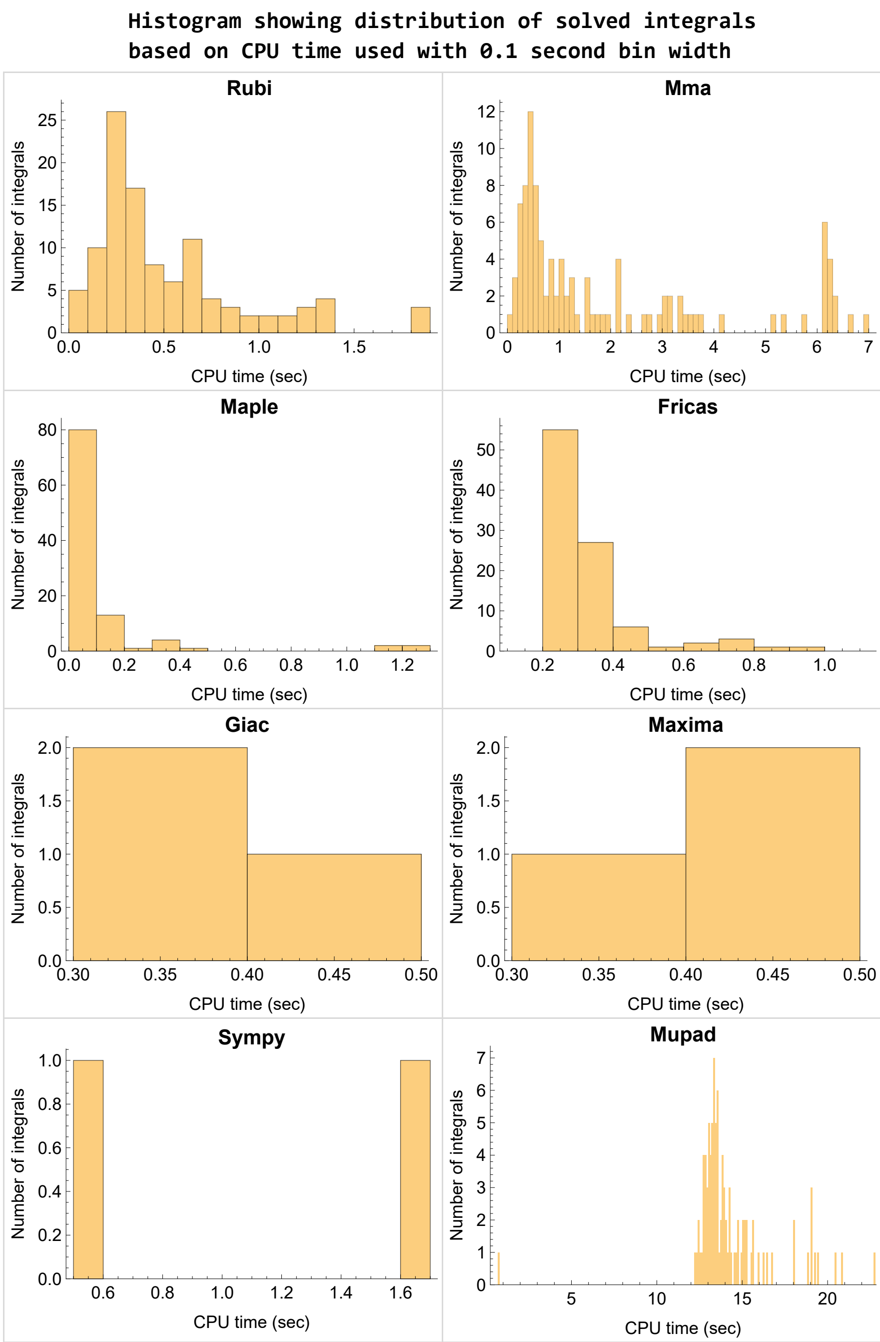


Figure 1.4: Solved integrals histogram based on CPU time used

1.8 Leaf size vs. CPU time used

The following shows the relation between the CPU time used to solve an integral and the leaf size of the antiderivative.

The result for Fricas, Maxima and Giac is shifted more to the right than the other CAS system due to the use of sagemath to call them, which causes an initial slight delay in the timing to start the integration due to overhead of starting a new process each time. This should also be taken into account when looking at the timing of these three CAS systems. Direct calls not using sagemath would result in faster timings, but current implementation uses sagemath as this makes testing much easier to do.

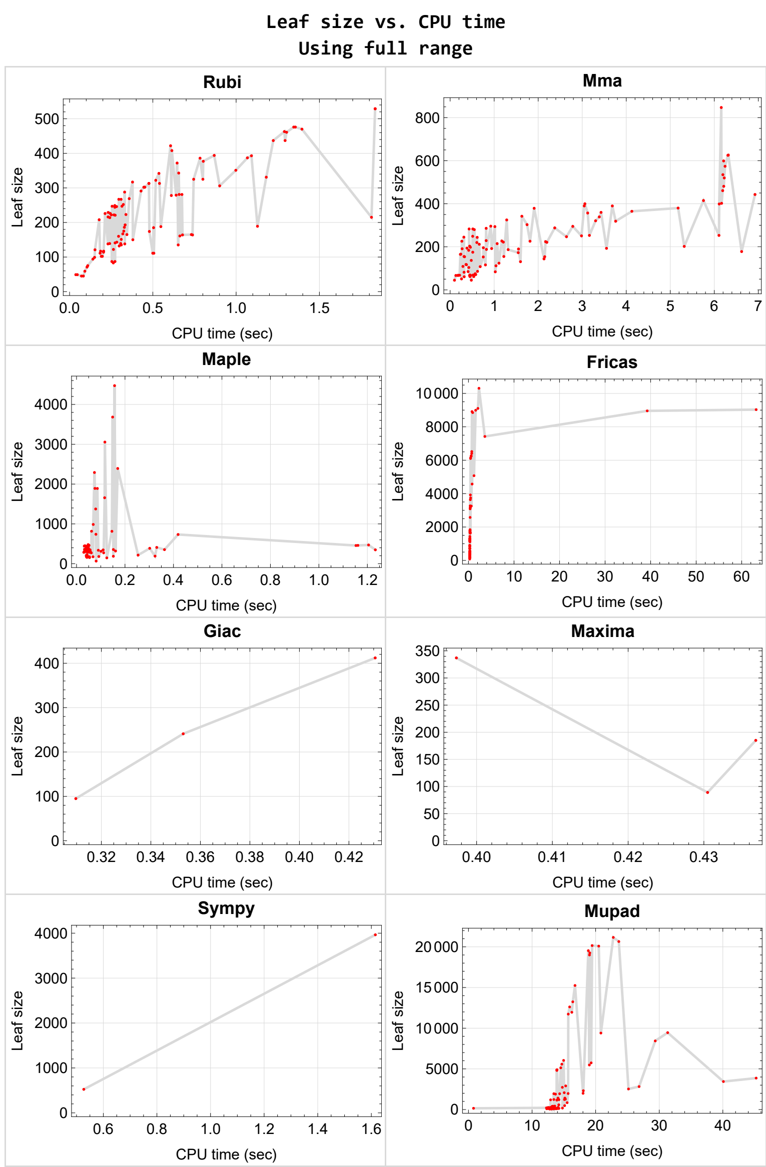


Figure 1.5: Leaf size vs. CPU time. Full range

1.9 list of integrals with no known antiderivative

{}

1.10 List of integrals solved by CAS but has no known antiderivative

Rubi {}

Mathematica {}

Maple {}

Maxima {}

Fricas {}

Sympy {}

Giac {}

Mupad {}

1.11 list of integrals solved by CAS but failed verification

The following are integrals solved by CAS but the verification phase failed to verify the anti-derivative produced is correct. This does not necessarily mean that the anti-derivative is wrong as additional methods of verification might be needed, or more time is needed (3 minutes time limit was used). These integrals are listed here to make it possible to do further investigation to determine why the result could not be verified.

Rubi {}

Mathematica $\{13, 16, 17, 19, 21\}$

Maple {}

Maxima Verification phase not currently implemented.

Fricas Verification phase not currently implemented.

Sympy Verification phase not currently implemented.

Giac Verification phase not currently implemented.

Mupad Verification phase not currently implemented.

1.12 Timing

The command `AbsoluteTiming[]` was used in Mathematica to obtain the elapsed time for each integrate call. In Maple, the command `Usage` was used as in the following example

```
cpu_time := Usage(assign ('result_of_int',int(expr,x)),output='realtime'
```

For all other CAS systems, the elapsed time to complete each integral was found by taking the difference between the time after the call completed from the time before the call was made. This was done using Python's `time.time()` call.

All elapsed times shown are in seconds. A time limit of 3 CPU minutes was used for each integral. If the integrate command did not complete within this time limit, the integral was aborted and considered to have failed and assigned an F grade. The time used by failed integrals due to time out was not counted in the final statistics.

1.13 Verification

A verification phase was applied on the result of integration for `Rubi` and `Mathematica`.

Future version of this report will implement verification for the other CAS systems. For the integrals whose result was not run through a verification phase, it is assumed that the antiderivative was correct.

Verification phase also had 3 minutes time out. An integral whose result was not verified could still be correct, but further investigation is needed on those integrals. These integrals were marked in the summary table below and also in each integral separate section so they are easy to identify and locate.

1.14 Important notes about some of the results

Important note about Maxima results

Since tests were run in a batch mode, and using an automated script, then any integral where Maxima needed an interactive response from the user to answer a question during the evaluation of the integral will fail.

The exception raised is `ValueError`. Therefore Maxima results is lower than what would result if Maxima was run directly and each question was answered correctly.

The percentage of such failures were not counted for each test file, but for an example, for the `Timofeev` test file, there were about 14 such integrals out of total 705, or about 2 percent. This percentage can be higher or lower depending on the specific input test file.

Such integrals can be identified by looking at the output of the integration in each section for Maxima. The exception message will indicate the cause of error.

Maxima integrate was run using SageMath with the following settings set by default

```
'besselexpand : true'
'display2d : false'
'domain : complex'
'keepfloat : true'
'load(to_poly_solve)'
'load(simplify_sum)'
'load(abs_integrate)' 'load(diag)'
```

SageMath automatic loading of Maxima `abs_integrate` was found to cause some problems. So the following code was added to disable this effect.

```
from sage.interfaces.maxima_lib import maxima_lib
maxima_lib.set('extra_definite_integration_methods', '[]')
maxima_lib.set('extra_integration_methods', '[]')
```

See `https://ask.sagemath.org/question/43088/integrate-results-that-are-different-from-using-maxima/` for reference.

Important note about FriCAS result

There were few integrals which failed due to SageMath interface and not because FriCAS system could not do the integration.

These will fail With error `Exception raised: NotImplementedError`.

The number of such cases seems to be very small. About 1 or 2 percent of all integrals. These can be identified by looking at the exception message given in the result.

Important note about finding leaf size of antiderivative

For Mathematica, Rubi, and Maple, the builtin system function `LeafSize` was used to find the leaf size of each antiderivative.

The other CAS systems (SageMath and Sympy) do not have special builtin function for this purpose at this time. Therefore the leaf size for Fricas and Sympy antiderivative was determined using the following function, thanks to user `slelievre` at `https://ask.sagemath.org/question/57123/could-we-have-a-leaf_count-function-in-base-sagemath/`

```
def tree_size(expr):
    r"""
    Return the tree size of this expression.
    """
    if expr not in SR:
        # deal with lists, tuples, vectors
        return 1 + sum(tree_size(a) for a in expr)
    expr = SR(expr)
```

```
    x, aa = expr.operator(), expr.operands()
    if x is None:
        return 1
    else:
        return 1 + sum(tree_size(a) for a in aa)
```

For Sympy, which was called directly from Python, the following code was used to obtain the leafsize of its result

```
try:
  # 1.7 is a fudge factor since it is low side from actual leaf count
  leafCount = round(1.7*count_ops(anti))

  except Exception as ee:
       leafCount =1
```

Important note about Mupad results

Matlab's symbolic toolbox does not have a leaf count function to measure the size of the antiderivative. Maple was used to determine the leaf size of Mupad output by post processing Mupad result.

Currently no grading of the antiderivative for Mupad is implemented. If it can integrate the problem, it was assigned a B grade automatically as a placeholder. In the future, when grading function is implemented for Mupad, the tests will be rerun again.

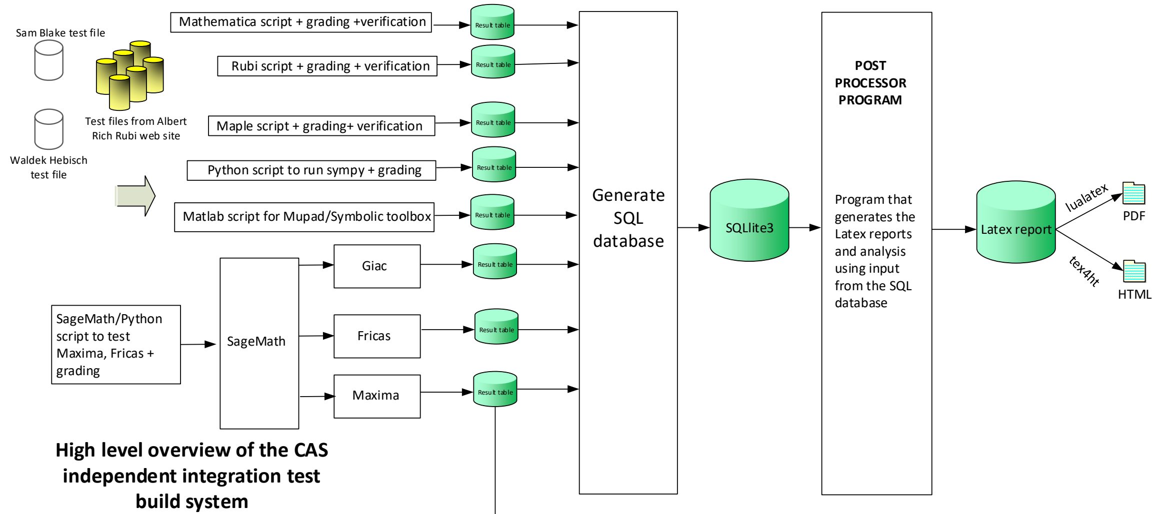
The following is an example of using Matlab's symbolic toolbox (Mupad) to solve an integral

```
integrand = evalin(symengine,'cos(x)*sin(x)')
the_variable = evalin(symengine,'x')
anti = int(integrand,the_variable)
```

Which gives `sin(x)^2/2`

1.15 Design of the test system

The following diagram gives a high level view of the current test build system.



One record (line) per one integral result. The line is CSV comma separated. This is description of each record

1. integer, the problem number.
2. integer. 0 for failed, 1 for passed, -1 for timeout, -2 for CAS specific exception. (this is not the grade field)
3. integer. Leaf size of result.
4. integer. Leaf size of the optimal antiderivative.
5. number. CPU time used to solve this integral. 0 if failed.
6. string. The integral in Latex format
7. string. The input used in CAS own syntax.
8. string. The result (antiderivative) produced by CAS in Latex format
9. string. The optimal antiderivative in Latex format.
10. integer. 0 or 1. Indicates if problem has known antiderivative or not
11. String. The result (antiderivative) in CAS own syntax.
12. String. The grade of the antiderivative. Can be "A", "B", "C", or "F"
13. String. Small string description of why the grade was given.
14. integer. 1 if result was verified or 0 if not verified. (For mma, rubi and maple only)

*The following fields are present only in **Rubi** Table file*

15. integer. Number of steps used.
16. integer. Number of rules used.
17. integer. Integrand leaf size.
18. real number. Ratio. Field 16 over field 17
19. String of form "{n,n,..}" which is list of the rules used by Rubi
20. String. The optimal antiderivative in Mathematica syntx

Nasser M. Abbasi
June 27, 2023
Design.vsdx

CHAPTER 2

DETAILED SUMMARY TABLES OF RESULTS

2.1 List of integrals sorted by grade for each CAS

Rubi

A grade { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106 }

B grade { }

C grade { }

F normal fail { }

F(-1) timedout fail { }

F(-2) exception fail { }

Mma

A grade { 8, 9, 10, 11, 12, 13, 14, 31, 32, 90, 91, 92, 96, 97, 98, 100, 101, 102, 103, 104, 105, 106 }

B grade { 1, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 95 }

C grade { 2, 3, 4, 5, 6, 7, 27, 28, 29, 30, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 93, 94, 99 }

F normal fail { 89 }

F(-1) timedout fail { }

F(-2) exception fail { }

Maple

A grade { 8, 9, 10, 11, 12, 13, 14, 29, 30, 31, 32, 33, 34, 42, 43, 47, 49, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 92, 93, 94 }

B grade { 2, 3, 4, 5, 6, 7, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 35, 36, 37, 38, 39, 40, 41, 44, 45, 46, 48, 50, 90, 91, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106 }

C grade { }

F normal fail { 1, 88, 89 }

F(-1) timedout fail { }

F(-2) exception fail { }

Fricas

A grade { 2, 4, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 28, 35, 36, 37, 38, 40, 92 }

B grade { 3, 5, 6, 7, 27, 39, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 90, 91, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106 }

C grade { 8, 9, 10, 11, 12, 13, 14, 29, 30, 31, 32, 33, 34, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50 }

F normal fail { 1, 88, 89 }

F(-1) timedout fail { }

F(-2) exception fail { }

Maxima

A grade { 92, 93, 94 }

B grade { }

C grade { }

F normal fail { 1, 41, 42, 44, 45, 46, 47, 48, 49, 50, 88, 90, 91, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106 }

F(-1) timedout fail { 43 }

F(-2) exception fail { 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 89 }

Giac

A grade { 92 }

B grade { 93, 94 }

C grade { }

F normal fail { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106 }

F(-1) timedout fail { 21, 22, 67, 68, 80 }

F(-2) exception fail { }

Mupad

A grade { }

B grade { 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106 }

C grade { }

F normal fail { }

F(-1) timedout fail { 1, 88, 89 }

F(-2) exception fail { }

Sympy

A grade { }

B grade { }

C grade { 92, 93 }

F normal fail { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 77, 78, 79, 80, 84, 85, 86, 87, 88, 89, 90, 91, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106 }

F(-1) timedout fail { 75, 76, 81, 82, 83 }

F(-2) exception fail { 94 }

2.2 Detailed conclusion table per each integral for all CAS systems

Detailed conclusion table per each integral is given by the table below. The elapsed time is in seconds. For failed result it is given as **F(-1)** if the failure was due to timeout. It is given as **F(-2)** if the failure was due to an exception being raised, which could indicate a bug in the system. If the failure was due to integral not being evaluated within the time limit, then it is given as **F**.

In this table, the column `N.S.` means **normalized size** and is defined as $\frac{\text{antiderivative leaf size}}{\text{optimal antiderivative leaf size}}$. To make the table fit the page, the name `Mathematica` was abbreviated to `MMA`.

Problem 1	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	49	49	117	0	0	0	0	0	0
N.S.	1	1.00	2.39	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.037	0.370	0.000	0.000	0.000	0.000	0.000	0.000

Problem 2	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	F(-2)	A	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	116	116	68	319	0	377	0	0	144
N.S.	1	1.00	0.59	2.75	0.00	3.25	0.00	0.00	1.24
time (sec)	N/A	0.204	0.215	0.161	0.000	0.286	0.000	0.000	13.778

Problem 3	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	F(-2)	B	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	94	94	67	303	0	334	0	0	98
N.S.	1	1.00	0.71	3.22	0.00	3.55	0.00	0.00	1.04
time (sec)	N/A	0.140	0.130	0.045	0.000	0.278	0.000	0.000	13.505

Problem 4	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	F(-2)	A	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	71	71	154	287	0	236	0	0	128
N.S.	1	1.00	2.17	4.04	0.00	3.32	0.00	0.00	1.80
time (sec)	N/A	0.105	0.327	0.053	0.000	0.269	0.000	0.000	13.278

Problem 5	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	F(-2)	B	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	49	49	165	273	0	172	0	0	65
N.S.	1	1.00	3.37	5.57	0.00	3.51	0.00	0.00	1.33
time (sec)	N/A	0.046	0.233	0.112	0.000	0.277	0.000	0.000	13.062

Problem 6	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	F(-2)	B	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	75	75	191	294	0	321	0	0	84
N.S.	1	1.00	2.55	3.92	0.00	4.28	0.00	0.00	1.12
time (sec)	N/A	0.109	0.272	0.046	0.000	0.274	0.000	0.000	13.330

Problem 7	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	F(-2)	B	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	99	99	203	309	0	358	0	0	103
N.S.	1	1.00	2.05	3.12	0.00	3.62	0.00	0.00	1.04
time (sec)	N/A	0.150	0.444	0.049	0.000	0.280	0.000	0.000	13.898

Problem 8	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	C	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	269	269	187	187	0	507	0	0	125
N.S.	1	1.00	0.70	0.70	0.00	1.88	0.00	0.00	0.46
time (sec)	N/A	0.358	1.318	0.152	0.000	0.289	0.000	0.000	13.912

Problem 9	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	C	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	246	246	112	172	0	423	0	0	104
N.S.	1	1.00	0.46	0.70	0.00	1.72	0.00	0.00	0.42
time (sec)	N/A	0.282	1.041	0.041	0.000	0.279	0.000	0.000	13.366

Problem 10	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	C	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	244	244	175	170	0	372	0	0	104
N.S.	1	1.00	0.72	0.70	0.00	1.52	0.00	0.00	0.43
time (sec)	N/A	0.272	0.514	0.049	0.000	0.303	0.000	0.000	13.070

Problem 11	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	C	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	222	222	253	155	0	323	0	0	86
N.S.	1	1.00	1.14	0.70	0.00	1.45	0.00	0.00	0.39
time (sec)	N/A	0.250	6.107	0.054	0.000	0.299	0.000	0.000	12.740

Problem 12	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	C	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	222	222	184	159	0	413	0	0	86
N.S.	1	1.00	0.83	0.72	0.00	1.86	0.00	0.00	0.39
time (sec)	N/A	0.267	0.422	0.043	0.000	0.282	0.000	0.000	12.877

Problem 13	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	C	F	F	B
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD
size	247	247	400	174	0	446	0	0	99
N.S.	1	1.00	1.62	0.70	0.00	1.81	0.00	0.00	0.40
time (sec)	N/A	0.314	3.060	0.053	0.000	0.287	0.000	0.000	13.090

Problem 14	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	C	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	249	249	173	174	0	542	0	0	99
N.S.	1	1.00	0.69	0.70	0.00	2.18	0.00	0.00	0.40
time (sec)	N/A	0.272	1.547	0.068	0.000	0.285	0.000	0.000	13.749

Problem 15	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	B	F(-2)	A	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	186	186	847	354	0	535	0	0	177
N.S.	1	1.00	4.55	1.90	0.00	2.88	0.00	0.00	0.95
time (sec)	N/A	0.329	6.159	0.363	0.000	0.282	0.000	0.000	14.778

Problem 16	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	B	F(-2)	A	F	F	B
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD
size	160	160	380	339	0	487	0	0	143
N.S.	1	1.00	2.38	2.12	0.00	3.04	0.00	0.00	0.89
time (sec)	N/A	0.294	5.179	0.043	0.000	0.282	0.000	0.000	14.193

Problem 17	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	B	F(-2)	A	F	F	B
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD
size	138	138	360	323	0	366	0	0	136
N.S.	1	1.00	2.61	2.34	0.00	2.65	0.00	0.00	0.99
time (sec)	N/A	0.230	3.428	0.054	0.000	0.274	0.000	0.000	13.523

Problem 18	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	B	F(-2)	A	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	117	117	342	309	0	349	0	0	100
N.S.	1	1.00	2.92	2.64	0.00	2.98	0.00	0.00	0.85
time (sec)	N/A	0.187	1.629	0.057	0.000	0.277	0.000	0.000	13.031

Problem 19	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	B	F(-2)	A	F	F	B
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD
size	114	114	357	305	0	372	0	0	119
N.S.	1	1.00	3.13	2.68	0.00	3.26	0.00	0.00	1.04
time (sec)	N/A	0.204	3.125	0.043	0.000	0.277	0.000	0.000	12.453

Problem 20	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	B	F(-2)	A	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	117	117	599	303	0	378	0	0	101
N.S.	1	1.00	5.12	2.59	0.00	3.23	0.00	0.00	0.86
time (sec)	N/A	0.205	6.212	0.043	0.000	0.323	0.000	0.000	12.308

Problem 21	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	B	F(-2)	A	F	F(-1)	B
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD
size	141	141	415	323	0	485	0	0	126
N.S.	1	1.00	2.94	2.29	0.00	3.44	0.00	0.00	0.89
time (sec)	N/A	0.273	5.755	0.047	0.000	0.285	0.000	0.000	13.545

Problem 22	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	B	F(-2)	A	F	F(-1)	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	165	165	625	338	0	514	0	0	129
N.S.	1	1.00	3.79	2.05	0.00	3.12	0.00	0.00	0.78
time (sec)	N/A	0.344	6.309	0.090	0.000	0.293	0.000	0.000	14.208

Problem 23	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	B	F(-2)	A	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	111	111	296	312	0	400	0	0	123
N.S.	1	1.00	2.67	2.81	0.00	3.60	0.00	0.00	1.11
time (sec)	N/A	0.500	0.923	0.101	0.000	0.293	0.000	0.000	12.929

Problem 24	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	B	F(-2)	A	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	87	87	280	298	0	333	0	0	79
N.S.	1	1.00	3.22	3.43	0.00	3.83	0.00	0.00	0.91
time (sec)	N/A	0.273	0.549	0.046	0.000	0.275	0.000	0.000	12.784

Problem 25	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	B	F(-2)	A	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	87	87	283	304	0	331	0	0	102
N.S.	1	1.00	3.25	3.49	0.00	3.80	0.00	0.00	1.17
time (sec)	N/A	0.254	0.431	0.046	0.000	0.290	0.000	0.000	12.728

Problem 26	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	B	F(-2)	A	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	83	83	283	304	0	321	0	0	79
N.S.	1	1.00	3.41	3.66	0.00	3.87	0.00	0.00	0.95
time (sec)	N/A	0.263	0.516	0.042	0.000	0.282	0.000	0.000	12.775

Problem 27	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	F(-2)	B	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	111	111	229	319	0	472	0	0	123
N.S.	1	1.00	2.06	2.87	0.00	4.25	0.00	0.00	1.11
time (sec)	N/A	0.506	0.819	0.041	0.000	0.282	0.000	0.000	12.671

Problem 28	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	F(-2)	A	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	135	135	86	333	0	500	0	0	132
N.S.	1	1.00	0.64	2.47	0.00	3.70	0.00	0.00	0.98
time (sec)	N/A	0.652	0.622	0.042	0.000	0.279	0.000	0.000	13.312

Problem 29	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F(-2)	C	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	281	281	288	191	0	1173	0	0	375
N.S.	1	1.00	1.02	0.68	0.00	4.17	0.00	0.00	1.33
time (sec)	N/A	0.658	2.376	0.324	0.000	0.297	0.000	0.000	13.298

Problem 30	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F(-2)	C	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	279	279	155	197	0	1172	0	0	376
N.S.	1	1.00	0.56	0.71	0.00	4.20	0.00	0.00	1.35
time (sec)	N/A	0.640	1.220	0.040	0.000	0.296	0.000	0.000	13.435

Problem 31	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	C	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	278	278	221	200	0	1114	0	0	366
N.S.	1	1.00	0.79	0.72	0.00	4.01	0.00	0.00	1.32
time (sec)	N/A	0.611	2.197	0.041	0.000	0.330	0.000	0.000	13.129

Problem 32	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	C	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	281	281	131	197	0	1181	0	0	366
N.S.	1	1.00	0.47	0.70	0.00	4.20	0.00	0.00	1.30
time (sec)	N/A	0.675	1.598	0.045	0.000	0.301	0.000	0.000	13.137

Problem 33	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F(-2)	C	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	306	306	227	212	0	1257	0	0	414
N.S.	1	1.00	0.74	0.69	0.00	4.11	0.00	0.00	1.35
time (sec)	N/A	0.901	1.174	0.042	0.000	0.315	0.000	0.000	13.241

Problem 34	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F(-2)	C	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	331	331	84	227	0	1751	0	0	425
N.S.	1	1.00	0.25	0.69	0.00	5.29	0.00	0.00	1.28
time (sec)	N/A	1.181	1.025	0.043	0.000	0.326	0.000	0.000	13.570

Problem 35	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	F(-2)	A	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	164	164	390	349	0	567	0	0	154
N.S.	1	1.00	2.38	2.13	0.00	3.46	0.00	0.00	0.94
time (sec)	N/A	0.678	3.684	1.233	0.000	0.294	0.000	0.000	13.529

Problem 36	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	F(-2)	A	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	164	164	365	349	0	533	0	0	178
N.S.	1	1.00	2.23	2.13	0.00	3.25	0.00	0.00	1.09
time (sec)	N/A	0.738	4.128	0.042	0.000	0.295	0.000	0.000	13.267

Problem 37	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	F(-2)	A	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	161	161	443	349	0	518	0	0	151
N.S.	1	1.00	2.75	2.17	0.00	3.22	0.00	0.00	0.94
time (sec)	N/A	0.662	6.925	0.048	0.000	0.283	0.000	0.000	13.339

Problem 38	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	F(-2)	A	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	165	165	339	349	0	504	0	0	173
N.S.	1	1.00	2.05	2.12	0.00	3.05	0.00	0.00	1.05
time (sec)	N/A	0.731	3.388	0.056	0.000	0.286	0.000	0.000	13.412

Problem 39	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	F(-2)	B	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	189	189	325	364	0	697	0	0	175
N.S.	1	1.00	1.72	1.93	0.00	3.69	0.00	0.00	0.93
time (sec)	N/A	1.128	1.285	0.046	0.000	0.287	0.000	0.000	13.337

Problem 40	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	F(-2)	A	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	215	215	124	379	0	718	0	0	193
N.S.	1	1.00	0.58	1.76	0.00	3.34	0.00	0.00	0.90
time (sec)	N/A	1.813	1.109	0.050	0.000	0.303	0.000	0.000	13.533

Problem 41	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	F	C	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	223	223	69	356	0	180	0	0	119
N.S.	1	1.00	0.31	1.60	0.00	0.81	0.00	0.00	0.53
time (sec)	N/A	0.338	0.522	0.152	0.000	0.269	0.000	0.000	12.981

Problem 42	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	C	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	135	135	61	174	0	142	0	0	210
N.S.	1	1.00	0.45	1.29	0.00	1.05	0.00	0.00	1.56
time (sec)	N/A	0.311	0.319	0.043	0.000	0.265	0.000	0.000	12.260

Problem 43	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F(-1)	C	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	139	139	75	197	0	202	0	0	254
N.S.	1	1.00	0.54	1.42	0.00	1.45	0.00	0.00	1.83
time (sec)	N/A	0.307	0.569	0.042	0.000	0.268	0.000	0.000	12.865

Problem 44	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	F	C	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	221	221	67	452	0	190	0	0	254
N.S.	1	1.00	0.30	2.05	0.00	0.86	0.00	0.00	1.15
time (sec)	N/A	0.274	0.440	0.035	0.000	0.262	0.000	0.000	12.595

Problem 45	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	F	C	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	214	214	67	442	0	178	0	0	238
N.S.	1	1.00	0.31	2.07	0.00	0.83	0.00	0.00	1.11
time (sec)	N/A	0.242	0.466	0.043	0.000	0.288	0.000	0.000	12.460

Problem 46	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	F	C	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	121	121	51	181	0	157	0	0	230
N.S.	1	1.00	0.42	1.50	0.00	1.30	0.00	0.00	1.90
time (sec)	N/A	0.153	0.258	0.092	0.000	0.253	0.000	0.000	12.872

Problem 47	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	C	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	139	139	62	173	0	198	0	0	208
N.S.	1	1.00	0.45	1.24	0.00	1.42	0.00	0.00	1.50
time (sec)	N/A	0.241	0.450	0.043	0.000	0.259	0.000	0.000	13.077

Problem 48	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	F	C	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	226	226	71	356	0	202	0	0	121
N.S.	1	1.00	0.31	1.58	0.00	0.89	0.00	0.00	0.54
time (sec)	N/A	0.244	0.581	0.039	0.000	0.259	0.000	0.000	12.836

Problem 49	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	C	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	143	143	62	195	0	238	0	0	242
N.S.	1	1.00	0.43	1.36	0.00	1.66	0.00	0.00	1.69
time (sec)	N/A	0.284	0.537	0.043	0.000	0.265	0.000	0.000	13.178

Problem 50	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	F	C	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	216	216	69	444	0	230	0	0	238
N.S.	1	1.00	0.32	2.06	0.00	1.06	0.00	0.00	1.10
time (sec)	N/A	0.231	0.452	0.032	0.000	0.275	0.000	0.000	13.103

Problem 51	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F(-2)	B	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	247	247	68	303	0	843	0	0	153
N.S.	1	1.00	0.28	1.23	0.00	3.41	0.00	0.00	0.62
time (sec)	N/A	0.255	0.181	0.042	0.000	0.276	0.000	0.000	13.860

Problem 52	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F(-2)	B	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	226	226	155	287	0	730	0	0	128
N.S.	1	1.00	0.69	1.27	0.00	3.23	0.00	0.00	0.57
time (sec)	N/A	0.213	0.315	0.030	0.000	0.274	0.000	0.000	13.492

Problem 53	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F(-2)	B	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	208	208	166	273	0	721	0	0	118
N.S.	1	1.00	0.80	1.31	0.00	3.47	0.00	0.00	0.57
time (sec)	N/A	0.178	0.242	0.061	0.000	0.273	0.000	0.000	13.200

Problem 54	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F(-2)	B	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	229	229	194	295	0	890	0	0	137
N.S.	1	1.00	0.85	1.29	0.00	3.89	0.00	0.00	0.60
time (sec)	N/A	0.232	0.382	0.032	0.000	0.289	0.000	0.000	13.314

Problem 55	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F(-2)	B	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	252	252	196	311	0	905	0	0	158
N.S.	1	1.00	0.78	1.23	0.00	3.59	0.00	0.00	0.63
time (sec)	N/A	0.323	0.754	0.034	0.000	0.274	0.000	0.000	13.333

Problem 56	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F(-2)	B	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	317	317	224	360	0	1313	0	0	1274
N.S.	1	1.00	0.71	1.14	0.00	4.14	0.00	0.00	4.02
time (sec)	N/A	0.378	2.168	0.033	0.000	0.298	0.000	0.000	15.244

Problem 57	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F(-2)	B	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	288	288	220	321	0	1230	0	0	1157
N.S.	1	1.00	0.76	1.11	0.00	4.27	0.00	0.00	4.02
time (sec)	N/A	0.331	0.606	0.035	0.000	0.293	0.000	0.000	13.899

Problem 58	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F(-2)	B	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	267	267	192	306	0	1183	0	0	1234
N.S.	1	1.00	0.72	1.15	0.00	4.43	0.00	0.00	4.62
time (sec)	N/A	0.296	0.943	0.048	0.000	0.286	0.000	0.000	13.425

Problem 59	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F(-2)	B	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	267	267	188	301	0	1298	0	0	1196
N.S.	1	1.00	0.70	1.13	0.00	4.86	0.00	0.00	4.48
time (sec)	N/A	0.309	0.825	0.042	0.000	0.309	0.000	0.000	12.918

Problem 60	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F(-2)	B	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	291	291	82	326	0	1318	0	0	1214
N.S.	1	1.00	0.28	1.12	0.00	4.53	0.00	0.00	4.17
time (sec)	N/A	0.429	0.316	0.042	0.000	0.315	0.000	0.000	14.232

Problem 61	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F(-2)	B	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	322	322	85	347	0	1436	0	0	1227
N.S.	1	1.00	0.26	1.08	0.00	4.46	0.00	0.00	3.81
time (sec)	N/A	0.517	0.408	0.037	0.000	0.318	0.000	0.000	15.093

Problem 62	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F(-2)	B	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	372	372	251	410	0	1795	0	0	2317
N.S.	1	1.00	0.67	1.10	0.00	4.83	0.00	0.00	6.23
time (sec)	N/A	0.646	2.984	0.041	0.000	0.344	0.000	0.000	18.064

Problem 63	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F(-2)	B	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	342	342	247	371	0	1643	0	0	2071
N.S.	1	1.00	0.72	1.08	0.00	4.80	0.00	0.00	6.06
time (sec)	N/A	0.536	2.643	0.037	0.000	0.343	0.000	0.000	15.059

Problem 64	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F(-2)	B	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	313	313	215	337	0	1633	0	0	1896
N.S.	1	1.00	0.69	1.08	0.00	5.22	0.00	0.00	6.06
time (sec)	N/A	0.476	1.068	0.039	0.000	0.414	0.000	0.000	13.685

Problem 65	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F(-2)	B	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	313	313	193	332	0	1679	0	0	1951
N.S.	1	1.00	0.62	1.06	0.00	5.36	0.00	0.00	6.23
time (sec)	N/A	0.476	3.552	0.036	0.000	0.376	0.000	0.000	13.428

Problem 66	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F(-2)	B	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	313	313	104	331	0	1692	0	0	1946
N.S.	1	1.00	0.33	1.06	0.00	5.41	0.00	0.00	6.22
time (sec)	N/A	0.540	0.425	0.034	0.000	0.370	0.000	0.000	14.342

Problem 67	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F(-2)	B	F	F(-1)	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	343	343	108	359	0	1804	0	0	1969
N.S.	1	1.00	0.31	1.05	0.00	5.26	0.00	0.00	5.74
time (sec)	N/A	0.655	0.683	0.037	0.000	0.363	0.000	0.000	15.672

Problem 68	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F(-2)	B	F	F(-1)	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	377	377	116	388	0	1839	0	0	1992
N.S.	1	1.00	0.31	1.03	0.00	4.88	0.00	0.00	5.28
time (sec)	N/A	0.803	0.802	0.043	0.000	0.341	0.000	0.000	18.021

Problem 69	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F(-2)	B	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	325	325	286	347	0	3267	0	0	5579
N.S.	1	1.00	0.88	1.07	0.00	10.05	0.00	0.00	17.17
time (sec)	N/A	0.800	0.818	0.110	0.000	0.373	0.000	0.000	14.684

Problem 70	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F(-2)	B	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	302	302	249	326	0	3137	0	0	5129
N.S.	1	1.00	0.82	1.08	0.00	10.39	0.00	0.00	16.98
time (sec)	N/A	0.448	0.512	0.043	0.000	0.384	0.000	0.000	14.514

Problem 71	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F(-2)	B	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	302	302	226	332	0	3088	0	0	4808
N.S.	1	1.00	0.75	1.10	0.00	10.23	0.00	0.00	15.92
time (sec)	N/A	0.452	0.266	0.042	0.000	0.360	0.000	0.000	13.900

Problem 72	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F(-2)	B	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	302	302	245	332	0	3160	0	0	4871
N.S.	1	1.00	0.81	1.10	0.00	10.46	0.00	0.00	16.13
time (sec)	N/A	0.446	0.312	0.048	0.000	0.381	0.000	0.000	13.881

Problem 73	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F(-2)	B	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	325	325	198	354	0	3637	0	0	4899
N.S.	1	1.00	0.61	1.09	0.00	11.19	0.00	0.00	15.07
time (sec)	N/A	0.745	0.460	0.044	0.000	0.413	0.000	0.000	13.929

Problem 74	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F(-2)	B	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	351	351	109	371	0	3742	0	0	6042
N.S.	1	1.00	0.31	1.06	0.00	10.66	0.00	0.00	17.21
time (sec)	N/A	0.998	0.291	0.042	0.000	0.475	0.000	0.000	14.960

Problem 75	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F(-2)	B	F(-1)	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	437	437	461	409	0	6403	0	0	13244
N.S.	1	1.00	1.05	0.94	0.00	14.65	0.00	0.00	30.31
time (sec)	N/A	1.223	6.191	0.331	0.000	0.712	0.000	0.000	16.405

Problem 76	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F(-2)	B	F(-1)	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	393	393	390	387	0	6258	0	0	12617
N.S.	1	1.00	0.99	0.98	0.00	15.92	0.00	0.00	32.10
time (sec)	N/A	1.092	3.046	0.302	0.000	0.586	0.000	0.000	15.910

Problem 77	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F(-2)	B	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	387	387	321	391	0	6150	0	0	11953
N.S.	1	1.00	0.83	1.01	0.00	15.89	0.00	0.00	30.89
time (sec)	N/A	1.068	3.304	0.040	0.000	0.473	0.000	0.000	16.253

Problem 78	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F(-2)	B	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	386	386	399	391	0	6104	0	0	11731
N.S.	1	1.00	1.03	1.01	0.00	15.81	0.00	0.00	30.39
time (sec)	N/A	0.783	6.112	0.044	0.000	0.452	0.000	0.000	15.694

Problem 79	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F(-2)	B	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	394	394	295	396	0	6248	0	0	9400
N.S.	1	1.00	0.75	1.01	0.00	15.86	0.00	0.00	23.86
time (sec)	N/A	0.869	2.786	0.045	0.000	0.607	0.000	0.000	20.823

Problem 80	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F(-2)	B	F	F(-1)	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	437	437	244	414	0	6519	0	0	15251
N.S.	1	1.00	0.56	0.95	0.00	14.92	0.00	0.00	34.90
time (sec)	N/A	1.294	0.615	0.048	0.000	0.723	0.000	0.000	16.753

Problem 81	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F(-2)	B	F(-1)	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	529	529	626	471	0	9101	0	0	20651
N.S.	1	1.00	1.18	0.89	0.00	17.20	0.00	0.00	39.04
time (sec)	N/A	1.835	6.319	1.205	0.000	2.082	0.000	0.000	23.643

Problem 82	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F(-2)	B	F(-1)	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	476	476	574	460	0	9029	0	0	20089
N.S.	1	1.00	1.21	0.97	0.00	18.97	0.00	0.00	42.20
time (sec)	N/A	1.356	6.245	1.161	0.000	63.174	0.000	0.000	20.486

Problem 83	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F(-2)	B	F(-1)	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	470	470	520	457	0	8955	0	0	19256
N.S.	1	1.00	1.11	0.97	0.00	19.05	0.00	0.00	40.97
time (sec)	N/A	1.395	6.227	1.153	0.000	39.249	0.000	0.000	19.079

Problem 84	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F(-2)	B	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	461	461	535	456	0	8913	0	0	19000
N.S.	1	1.00	1.16	0.99	0.00	19.33	0.00	0.00	41.21
time (sec)	N/A	1.303	6.195	0.051	0.000	0.776	0.000	0.000	19.029

Problem 85	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F(-2)	B	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	463	463	481	460	0	8853	0	0	19534
N.S.	1	1.00	1.04	0.99	0.00	19.12	0.00	0.00	42.19
time (sec)	N/A	1.290	6.215	0.051	0.000	0.931	0.000	0.000	18.849

Problem 86	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F(-2)	B	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	476	476	402	465	0	8991	0	0	20155
N.S.	1	1.00	0.84	0.98	0.00	18.89	0.00	0.00	42.34
time (sec)	N/A	1.346	6.166	0.051	0.000	1.580	0.000	0.000	19.447

Problem 87	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F(-2)	B	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	529	529	303	480	0	10308	0	0	21158
N.S.	1	1.00	0.57	0.91	0.00	19.49	0.00	0.00	40.00
time (sec)	N/A	1.835	1.747	0.048	0.000	2.307	0.000	0.000	22.776

Problem 88	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	167	167	118	0	0	0	0	0	0
N.S.	1	1.00	0.71	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.320	0.360	0.000	0.000	0.000	0.000	0.000	0.000

Problem 89	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	F	F	F(-2)	F	F	F	F(-1)
verified	N/A	Yes	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	193	193	0	0	0	0	0	0	0
N.S.	1	1.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.334	0.000	0.000	0.000	0.000	0.000	0.000	0.000

Problem 90	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	B	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	45	45	45	734	0	159	0	0	1410
N.S.	1	1.00	1.00	16.31	0.00	3.53	0.00	0.00	31.33
time (sec)	N/A	0.083	0.099	0.419	0.000	0.299	0.000	0.000	15.159

Problem 91	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	B	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	45	45	45	739	0	159	0	0	1410
N.S.	1	1.00	1.00	16.42	0.00	3.53	0.00	0.00	31.33
time (sec)	N/A	0.072	0.481	0.080	0.000	0.285	0.000	0.000	14.073

Problem 92	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	C	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	59	59	67	66	89	79	524	95	155
N.S.	1	1.00	1.14	1.12	1.51	1.34	8.88	1.61	2.63
time (sec)	N/A	0.092	0.177	0.081	0.430	0.273	0.527	0.310	0.797

Problem 93	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	A	B	C	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	111	111	144	147	185	340	3964	241	268
N.S.	1	1.00	1.30	1.32	1.67	3.06	35.71	2.17	2.41
time (sec)	N/A	0.183	2.132	0.125	0.437	0.296	1.615	0.353	14.088

Problem 94	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	A	B	F(-2)	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	175	175	202	216	337	549	0	412	481
N.S.	1	1.00	1.15	1.23	1.93	3.14	0.00	2.35	2.75
time (sec)	N/A	0.325	5.318	0.254	0.397	0.340	0.000	0.431	15.121

Problem 95	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	B	F	B	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	188	188	379	2392	0	5073	0	0	3864
N.S.	1	1.00	2.02	12.72	0.00	26.98	0.00	0.00	20.55
time (sec)	N/A	0.548	1.908	0.170	0.000	1.217	0.000	0.000	45.238

Problem 96	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	B	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	150	150	294	1657	0	3252	0	0	2823
N.S.	1	1.00	1.96	11.05	0.00	21.68	0.00	0.00	18.82
time (sec)	N/A	0.380	1.016	0.116	0.000	0.628	0.000	0.000	26.830

Problem 97	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	B	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	122	122	212	814	0	1329	0	0	843
N.S.	1	1.00	1.74	6.67	0.00	10.89	0.00	0.00	6.91
time (sec)	N/A	0.263	0.662	0.146	0.000	0.326	0.000	0.000	15.521

Problem 98	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	B	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	151	151	222	1375	0	1684	0	0	3442
N.S.	1	1.00	1.47	9.11	0.00	11.15	0.00	0.00	22.79
time (sec)	N/A	0.311	1.208	0.079	0.000	0.312	0.000	0.000	40.088

Problem 99	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	F	B	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	408	408	178	986	0	1148	0	0	2529
N.S.	1	1.00	0.44	2.42	0.00	2.81	0.00	0.00	6.20
time (sec)	N/A	0.614	6.621	0.069	0.000	0.290	0.000	0.000	25.166

Problem 100	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	B	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	422	422	153	816	0	849	0	0	583
N.S.	1	1.00	0.36	1.93	0.00	2.01	0.00	0.00	1.38
time (sec)	N/A	0.606	0.753	0.062	0.000	0.305	0.000	0.000	14.202

Problem 101	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	B	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	102	102	154	1890	0	1773	0	0	2909
N.S.	1	1.00	1.51	18.53	0.00	17.38	0.00	0.00	28.52
time (sec)	N/A	0.190	2.148	0.086	0.000	0.324	0.000	0.000	15.288

Problem 102	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	B	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	138	138	226	3683	0	4572	0	0	5737
N.S.	1	1.00	1.64	26.69	0.00	33.13	0.00	0.00	41.57
time (sec)	N/A	0.331	1.815	0.149	0.000	0.804	0.000	0.000	19.294

Problem 103	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	B	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	185	185	319	4472	0	7422	0	0	9453
N.S.	1	1.00	1.72	24.17	0.00	40.12	0.00	0.00	51.10
time (sec)	N/A	0.504	3.760	0.157	0.000	3.599	0.000	0.000	31.324

Problem 104	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	B	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	102	102	137	1891	0	1219	0	0	2731
N.S.	1	1.00	1.34	18.54	0.00	11.95	0.00	0.00	26.77
time (sec)	N/A	0.197	0.480	0.076	0.000	0.313	0.000	0.000	14.765

Problem 105	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	B	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	132	132	190	2291	0	2574	0	0	5475
N.S.	1	1.00	1.44	17.36	0.00	19.50	0.00	0.00	41.48
time (sec)	N/A	0.298	1.553	0.074	0.000	0.358	0.000	0.000	19.009

Problem 106	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	B	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	174	174	253	3055	0	3922	0	0	8438
N.S.	1	1.00	1.45	17.56	0.00	22.54	0.00	0.00	48.49
time (sec)	N/A	0.479	3.163	0.117	0.000	0.419	0.000	0.000	29.371

2.3 Detailed conclusion table specific for Rubi results

The following table is specific to Rubi only. It gives additional statistics for each integral. the column **steps** is the number of steps used by Rubi to obtain the antiderivative. The **rules** column is the number of unique rules used. The **integrand size** column is the leaf size of the integrand. Finally the ratio $\frac{\text{number of rules}}{\text{integrand size}}$ is also given. The larger this ratio is, the harder the integral is to solve. In this test file, problem number [48] had the largest ratio of [.9091000000000000019]

Table 2.1: Rubi specific breakdown of results for each integral

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
1	A	2	2	1.00	15	0.133
2	A	5	3	1.00	23	0.130
3	A	4	3	1.00	23	0.130
4	A	3	3	1.00	23	0.130
5	A	2	2	1.00	23	0.087
6	A	3	3	1.00	23	0.130
7	A	4	3	1.00	23	0.130
8	A	16	12	1.00	25	0.480
9	A	15	12	1.00	25	0.480
10	A	15	12	1.00	25	0.480
11	A	14	11	1.00	25	0.440
12	A	13	10	1.00	25	0.400
13	A	14	11	1.00	25	0.440
14	A	14	11	1.00	25	0.440
15	A	7	5	1.00	25	0.200
16	A	6	5	1.00	25	0.200
17	A	5	5	1.00	25	0.200
18	A	4	4	1.00	25	0.160
19	A	4	4	1.00	25	0.160
20	A	4	4	1.00	25	0.160
21	A	5	5	1.00	25	0.200
22	A	6	5	1.00	25	0.200
23	A	7	6	1.00	25	0.240
24	A	6	6	1.00	25	0.240

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
25	A	6	5	1.00	25	0.200
26	A	6	6	1.00	25	0.240
27	A	7	6	1.00	25	0.240
28	A	10	10	1.00	25	0.400
29	A	17	14	1.00	25	0.560
30	A	18	15	1.00	25	0.600
31	A	17	14	1.00	25	0.560
32	A	18	15	1.00	25	0.600
33	A	18	15	1.00	25	0.600
34	A	20	16	1.00	25	0.640
35	A	8	8	1.00	25	0.320
36	A	8	7	1.00	25	0.280
37	A	8	8	1.00	25	0.320
38	A	8	7	1.00	25	0.280
39	A	9	8	1.00	25	0.320
40	A	10	8	1.00	25	0.320
41	A	12	9	1.00	13	0.692
42	A	6	5	1.00	11	0.454
43	A	8	7	1.00	13	0.538
44	A	14	9	1.00	11	0.818
45	A	12	8	1.00	13	0.615
46	A	5	4	1.00	11	0.364
47	A	6	5	1.00	13	0.385
48	A	13	10	1.00	11	0.909
49	A	8	7	1.00	13	0.538
50	A	13	9	1.00	11	0.818
51	A	12	8	1.00	23	0.348
52	A	11	8	1.00	23	0.348
53	A	10	7	1.00	23	0.304
54	A	11	8	1.00	23	0.348
55	A	12	8	1.00	23	0.348
56	A	13	9	1.00	25	0.360
57	A	12	9	1.00	25	0.360
58	A	11	8	1.00	25	0.320
59	A	11	8	1.00	25	0.320

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
60	A	12	9	1.00	25	0.360
61	A	13	9	1.00	25	0.360
62	A	14	10	1.00	25	0.400
63	A	13	10	1.00	25	0.400
64	A	12	9	1.00	25	0.360
65	A	12	9	1.00	25	0.360
66	A	12	9	1.00	25	0.360
67	A	13	10	1.00	25	0.400
68	A	14	10	1.00	25	0.400
69	A	15	12	1.00	25	0.480
70	A	14	11	1.00	25	0.440
71	A	14	11	1.00	25	0.440
72	A	14	11	1.00	25	0.440
73	A	15	12	1.00	25	0.480
74	A	16	13	1.00	25	0.520
75	A	16	13	1.00	25	0.520
76	A	15	12	1.00	25	0.480
77	A	15	12	1.00	25	0.480
78	A	15	12	1.00	25	0.480
79	A	15	12	1.00	25	0.480
80	A	16	13	1.00	25	0.520
81	A	17	14	1.00	25	0.560
82	A	16	13	1.00	25	0.520
83	A	16	13	1.00	25	0.520
84	A	16	13	1.00	25	0.520
85	A	16	13	1.00	25	0.520
86	A	16	13	1.00	25	0.520
87	A	17	13	1.00	25	0.520
88	A	5	3	1.00	12	0.250
89	A	8	5	1.00	23	0.217
90	A	3	3	1.00	27	0.111
91	A	3	3	1.00	27	0.111
92	A	2	2	1.00	23	0.087
93	A	3	3	1.00	23	0.130
94	A	4	3	1.00	23	0.130

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
95	A	10	5	1.00	25	0.200
96	A	9	5	1.00	25	0.200
97	A	8	5	1.00	25	0.200
98	A	10	7	1.00	27	0.259
99	A	13	9	1.00	27	0.333
100	A	13	9	1.00	27	0.333
101	A	7	4	1.00	25	0.160
102	A	8	5	1.00	25	0.200
103	A	9	5	1.00	25	0.200
104	A	7	4	1.00	27	0.148
105	A	8	5	1.00	27	0.185
106	A	9	5	1.00	27	0.185

CHAPTER 3

LISTING OF INTEGRALS

3.1 $\int (a + ia\cot(c+dx))^n \, dx$

Optimal result

Integrand size = 15, antiderivative size = 49

$$\int (a+ia\cot(c+dx))^n \, dx = \frac{i(a+ia\cot(c+dx))^n \operatorname{Hypergeometric2F1}\left(1,n,1+n,\frac{1}{2}(1+i\cot(c+dx))\right)}{2dn}$$

```
[Out] 1/2*I*(a+I*a*cot(d*x+c))^n*hypergeom([1, n],[1+n],1/2+1/2*I*cot(d*x+c))/d/n
```

Rubi [A] (verified)

Time = 0.04 (sec) , antiderivative size = 49, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {3562, 70}

$$\int (a+ia\cot(c+dx))^n \, dx = \frac{i(a+ia\cot(c+dx))^n \operatorname{Hypergeometric2F1}\left(1,n,n+1,\frac{1}{2}(i\cot(c+dx)+1)\right)}{2dn}$$

```
[In] Int[(a + I*a*Cot[c + d*x])^n,x]
```

```
[Out] ((I/2)*(a + I*a*Cot[c + d*x])^n*Hypergeometric2F1[1, n, 1 + n, (1 + I*Cot[c
 + d*x])/2])/(d*n)
```

Rule 70

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[(b
*c - a*d)^n*((a + b*x)^(m + 1)/(b^(n + 1)*(m + 1)))*Hypergeometric2F1[-n, m
 + 1, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m}, x]
```

```
&& NeQ[b*c - a*d, 0] &&  !IntegerQ[m] && IntegerQ[n]
```

Rule 3562

```
Int[((a_) + (b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[-b/d, Su
bst[Int[(a + x)^(n - 1)/(a - x), x], x, b*Tan[c + d*x]], x] /; FreeQ[{a, b,
 c, d, n}, x] && EqQ[a^2 + b^2, 0]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \frac{(ia)\text{Subst}\left(\int \frac{(a+x)^{-1+n}}{a-x}\,dx, x, ia\cot(c+dx)\right)}{d}\\ &= \frac{i(a+ia\cot(c+dx))^n\,\text{Hypergeometric2F1}\left(1,n,1+n,\frac{1}{2}(1+i\cot(c+dx))\right)}{2dn}\end{aligned}$$

Mathematica [B] (verified)

Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 117 vs. 2(49) = 98.

Time = 0.37 (sec) , antiderivative size = 117, normalized size of antiderivative = 2.39

$$\int (a+ia\cot(c+dx))^n\,dx$$
$$= \frac{i(a+ia\cot(c+dx))^n\,\left(2(1+n)\,\text{Hypergeometric2F1}(1,n,1+n,1+i\cot(c+dx)) + (n+in\cot(c+dx))\,(\right.}{}$$

```
[In] Integrate[(a + I*a*Cot[c + d*x])^n,x]
```

```
[Out] ((I/4)*(a + I*a*Cot[c + d*x])^n*(2*(1 + n)*Hypergeometric2F1[1, n, 1 + n, 1
 + I*Cot[c + d*x]] + (n + I*n*Cot[c + d*x])*(Hypergeometric2F1[1, 1 + n, 2
+ n, (1 + I*Cot[c + d*x])/2] - 2*Hypergeometric2F1[1, 1 + n, 2 + n, 1 + I*C
ot[c + d*x]])))/(d*n*(1 + n))
```

Maple [F]

$$\int (a+ia\cot(dx+c))^n\,dx$$

```
[In] int((a+I*a*cot(d*x+c))^n,x)
```

```
[Out] int((a+I*a*cot(d*x+c))^n,x)
```

Fricas [F]

$$\int (a+ia\cot(c+dx))^n\,dx = \int (i\,a\cot(dx+c)+a)^n\,dx$$

```
[In] integrate((a+I*a*cot(d*x+c))^n,x, algorithm="fricas")
[Out] integral((-2*a/(e^(2*I*d*x + 2*I*c) - 1))^n, x)
```

Sympy [F]

$$\int (a+ia\cot(c+dx))^n\,dx = \int (ia\cot(c+dx)+a)^n\,dx$$

```
[In] integrate((a+I*a*cot(d*x+c))**n,x)
[Out] Integral((I*a*cot(c + d*x) + a)**n, x)
```

Maxima [F]

$$\int (a+ia\cot(c+dx))^n\,dx = \int (i\,a\cot(dx+c)+a)^n\,dx$$

```
[In] integrate((a+I*a*cot(d*x+c))^n,x, algorithm="maxima")
[Out] integrate((I*a*cot(d*x + c) + a)^n, x)
```

Giac [F]

$$\int (a+ia\cot(c+dx))^n\,dx = \int (i\,a\cot(dx+c)+a)^n\,dx$$

```
[In] integrate((a+I*a*cot(d*x+c))^n,x, algorithm="giac")
[Out] integrate((I*a*cot(d*x + c) + a)^n, x)
```

Mupad [F(-1)]

Timed out.

$$\int (a+ia\cot(c+dx))^n\,dx=\int (a+a\cot(c+d\,x)\;1\mathrm{i})^n\,dx$$

```
[In] int((a + a*cot(c + d*x)*1i)^n,x)
```

```
[Out] int((a + a*cot(c + d*x)*1i)^n, x)
```

3.2 $\int (e\cot(c+dx))^{5/2}(a+a\cot(c+dx))\,dx$

Optimal result

Integrand size = 23, antiderivative size = 116

$$\int (e\cot(c+dx))^{5/2}(a+a\cot(c+dx))\,dx=-\frac{\sqrt{2}ae^{5/2}\text{arctanh}\left(\frac{\sqrt{e}+\sqrt{e}\cot(c+dx)}{\sqrt{2}\sqrt{e\cot(c+dx)}}\right)}{d}+\frac{2ae^2\sqrt{e\cot(c+dx)}}{d}-\frac{2ae(e\cot(c+dx))^{3/2}}{3d}-\frac{2a(e\cot(c+dx))^{5/2}}{5d}$$

```
[Out] -2/3*a*e*(e*cot(d*x+c))^(3/2)/d-2/5*a*(e*cot(d*x+c))^(5/2)/d-a*e^(5/2)*arct
anh(1/2*(e^(1/2)+cot(d*x+c)*e^(1/2))*2^(1/2)/(e*cot(d*x+c))^(1/2))*2^(1/2)/
d+2*a*e^2*(e*cot(d*x+c))^(1/2)/d
```

Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 116, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.130, Rules used = {3609, 3613, 214}

$$\int (e\cot(c+dx))^{5/2}(a+a\cot(c+dx))\,dx=-\frac{\sqrt{2}ae^{5/2}\text{arctanh}\left(\frac{\sqrt{e}\cot(c+dx)+\sqrt{e}}{\sqrt{2}\sqrt{e\cot(c+dx)}}\right)}{d}+\frac{2ae^2\sqrt{e\cot(c+dx)}}{d}-\frac{2ae(e\cot(c+dx))^{3/2}}{3d}-\frac{2a(e\cot(c+dx))^{5/2}}{5d}$$

```
[In] Int[(e*Cot[c + d*x])^(5/2)*(a + a*Cot[c + d*x]),x]
```

```
[Out] -((Sqrt[2]*a*e^(5/2)*ArcTanh[(Sqrt[e] + Sqrt[e]*Cot[c + d*x])/(Sqrt[2]*Sqrt
[e*Cot[c + d*x]])])/d) + (2*a*e^2*Sqrt[e*Cot[c + d*x]])/d - (2*a*e*(e*Cot[c
 + d*x])^(3/2))/(3*d) - (2*a*(e*Cot[c + d*x])^(5/2))/(5*d)
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 3609

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)]), x_Symbol] :> Simp[d*((a + b*Tan[e + f*x])^m/(f*m)), x] + Int
[(a + b*Tan[e + f*x])^(m - 1)*Simp[a*c - b*d + (b*c + a*d)*Tan[e + f*x], x]
, x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2,
 0] && GtQ[m, 0]
```

Rule 3613

```
Int[((c_) + (d_.)*tan[(e_.) + (f_.)*(x_)])/Sqrt[(b_.)*tan[(e_.) + (f_.)*(x_
)]], x_Symbol] :> Dist[-2*(d^2/f), Subst[Int[1/(2*c*d + b*x^2), x], x, (c -
 d*Tan[e + f*x])/Sqrt[b*Tan[e + f*x]]], x] /; FreeQ[{b, c, d, e, f}, x] &&
EqQ[c^2 - d^2, 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\frac{2a(e\cot(c+dx))^{5/2}}{5d} + \int (e\cot(c+dx))^{3/2}(-ae+ae\cot(c+dx))\,dx \\
&= -\frac{2ae(e\cot(c+dx))^{3/2}}{3d} - \frac{2a(e\cot(c+dx))^{5/2}}{5d} \\
&\quad + \int \sqrt{e\cot(c+dx)}\left(-ae^2-ae^2\cot(c+dx)\right)\,dx \\
&= \frac{2ae^2\sqrt{e\cot(c+dx)}}{d} - \frac{2ae(e\cot(c+dx))^{3/2}}{3d} \\
&\quad - \frac{2a(e\cot(c+dx))^{5/2}}{5d} + \int \frac{ae^3-ae^3\cot(c+dx)}{\sqrt{e\cot(c+dx)}}\,dx \\
&= \frac{2ae^2\sqrt{e\cot(c+dx)}}{d} - \frac{2ae(e\cot(c+dx))^{3/2}}{3d} - \frac{2a(e\cot(c+dx))^{5/2}}{5d} \\
&\quad - \frac{(2a^2e^6)\,\text{Subst}\left(\int \frac{1}{2a^2e^6-ex^2}\,dx, x, \frac{ae^3+ae^3\cot(c+dx)}{\sqrt{e\cot(c+dx)}}\right)}{d} \\
&= -\frac{\sqrt{2}ae^{5/2}\text{arctanh}\left(\frac{\sqrt{e}+\sqrt{e}\cot(c+dx)}{\sqrt{2}\sqrt{e\cot(c+dx)}}\right)}{d} + \frac{2ae^2\sqrt{e\cot(c+dx)}}{d} \\
&\quad - \frac{2ae(e\cot(c+dx))^{3/2}}{3d} - \frac{2a(e\cot(c+dx))^{5/2}}{5d}
\end{aligned}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 0.22 (sec) , antiderivative size = 68, normalized size of antiderivative = 0.59

$$\int (e\cot(c+dx))^{5/2}(a+a\cot(c+dx))\,dx =$$
$$-\frac{2ae(e\cot(c+dx))^{3/2}\,\big(3\cot(c+dx)\,\text{Hypergeometric2F1}\big(-\tfrac{5}{4},1,-\tfrac{1}{4},-\tan^2(c+dx)\big)+5\,\text{Hypergeometric}\cdots}{15d}$$

```
[In] Integrate[(e*Cot[c + d*x])^(5/2)*(a + a*Cot[c + d*x]),x]
```

```
[Out] (-2*a*e*(e*Cot[c + d*x])^(3/2)*(3*Cot[c + d*x]*Hypergeometric2F1[-5/4, 1, -
1/4, -Tan[c + d*x]^2] + 5*Hypergeometric2F1[-3/4, 1, 1/4, -Tan[c + d*x]^2])
)/(15*d)
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 318 vs. 2(95) = 190.

Time = 0.16 (sec) , antiderivative size = 319, normalized size of antiderivative = 2.75

method	result
derivativedivides	$-\frac{a\Big(\frac{2(e\cot(dx+c))^{\frac{5}{2}}}{5}+\frac{2e(e\cot(dx+c))^{\frac{3}{2}}}{3}-2\sqrt{e\cot(dx+c)}\,e^2+2e^3\Big(\frac{(e^2)^{\frac14}\sqrt{2}\Big(\ln\Big(\frac{e\cot(dx+c)+(e^2)^{\frac14}\sqrt{e\cot(dx+c)}\sqrt{2}+\sqrt{\cdots}}{e\cot(dx+c)-(e^2)^{\frac14}\sqrt{e\cot(dx+c)}\sqrt{2}+\sqrt{\cdots}}\cdots}{\cdots}\cdots}{\cdots}$
default	$-\frac{a\Big(\frac{2(e\cot(dx+c))^{\frac{5}{2}}}{5}+\frac{2e(e\cot(dx+c))^{\frac{3}{2}}}{3}-2\sqrt{e\cot(dx+c)}\,e^2+2e^3\Big(\frac{(e^2)^{\frac14}\sqrt{2}\Big(\ln\Big(\frac{e\cot(dx+c)+(e^2)^{\frac14}\sqrt{e\cot(dx+c)}\sqrt{2}+\sqrt{\cdots}}{e\cot(dx+c)-(e^2)^{\frac14}\sqrt{e\cot(dx+c)}\sqrt{2}+\sqrt{\cdots}}\cdots}{\cdots}\cdots}{\cdots}$
parts	$-\frac{2ae\Big(\frac{(e\cot(dx+c))^{\frac{3}{2}}}{3}-\frac{e^2\sqrt{2}\Big(\ln\Big(\frac{e\cot(dx+c)-(e^2)^{\frac14}\sqrt{e\cot(dx+c)}\sqrt{2}+\sqrt{e^2}}{e\cot(dx+c)+(e^2)^{\frac14}\sqrt{e\cot(dx+c)}\sqrt{2}+\sqrt{e^2}}\Big)+2\arctan\Big(\frac{\sqrt{2}\sqrt{e\cot(dx+c)}}{(e^2)^{\frac14}}+1\Big)-2\arctan\Big(-\cdots}{8(e^2)^{\frac14}}\cdots}{d}$

```
[In] int((e*cot(d*x+c))^(5/2)*(a+a*cot(d*x+c)),x,method=_RETURNVERBOSE)
```

```
[Out] -1/d*a*(2/5*(e*cot(d*x+c))^(5/2)+2/3*e*(e*cot(d*x+c))^(3/2)-2*(e*cot(d*x+c)
)^(1/2)*e^2+2*e^3*(1/8/e*(e^2)^(1/4)*2^(1/2)*(ln((e*cot(d*x+c)+(e^2)^(1/4)*
(e*cot(d*x+c))^(1/2)*2^(1/2)+(e^2)^(1/2))/(e*cot(d*x+c)-(e^2)^(1/4)*(e*cot(
d*x+c))^(1/2)*2^(1/2)+(e^2)^(1/2)))+2*arctan(2^(1/2)/(e^2)^(1/4)*(e*cot(d*x
```

```
+c))^(1/2)+1)-2*arctan(-2^(1/2)/(e^2)^(1/4)*(e*cot(d*x+c))^(1/2)+1))-1/8/(e
^2)^(1/4)*2^(1/2)*(ln((e*cot(d*x+c)-(e^2)^(1/4)*(e*cot(d*x+c))^(1/2)*2^(1/2
)+(e^2)^(1/2))/(e*cot(d*x+c)+(e^2)^(1/4)*(e*cot(d*x+c))^(1/2)*2^(1/2)+(e^2)
^(1/2)))+2*arctan(2^(1/2)/(e^2)^(1/4)*(e*cot(d*x+c))^(1/2)+1)-2*arctan(-2^(
1/2)/(e^2)^(1/4)*(e*cot(d*x+c))^(1/2)+1))))
```

Fricas [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 377, normalized size of antiderivative = 3.25

$$\int (e\cot(c+dx))^{5/2}(a+a\cot(c+dx))\,dx = \left[\frac{15\sqrt{2}(ae^2\cos(2\,dx+2\,c)-ae^2)\sqrt{e}\log\left(\sqrt{2}\sqrt{e}\sqrt{\frac{e\cos(2\,dx+2\,c)+e}{\sin(2\,dx+2\,c)}}(\cos(2\,dx+2\,c)-\sin(2\right.}{}\right.$$

```
[In] integrate((e*cot(d*x+c))^(5/2)*(a+a*cot(d*x+c)),x, algorithm="fricas")
```

```
[Out] [1/30*(15*sqrt(2)*(a*e^2*cos(2*d*x + 2*c) - a*e^2)*sqrt(e)*log(sqrt(2)*sqrt
(e)*sqrt((e*cos(2*d*x + 2*c) + e)/sin(2*d*x + 2*c))*(cos(2*d*x + 2*c) - sin
(2*d*x + 2*c) - 1) + 2*e*sin(2*d*x + 2*c) + e) + 4*(18*a*e^2*cos(2*d*x + 2*
c) + 5*a*e^2*sin(2*d*x + 2*c) - 12*a*e^2)*sqrt((e*cos(2*d*x + 2*c) + e)/sin
(2*d*x + 2*c)))/(d*cos(2*d*x + 2*c) - d), 1/15*(15*sqrt(2)*(a*e^2*cos(2*d*x
 + 2*c) - a*e^2)*sqrt(-e)*arctan(1/2*sqrt(2)*sqrt(-e)*sqrt((e*cos(2*d*x + 2
*c) + e)/sin(2*d*x + 2*c))*(cos(2*d*x + 2*c) + sin(2*d*x + 2*c) + 1)/(e*cos
(2*d*x + 2*c) + e)) + 2*(18*a*e^2*cos(2*d*x + 2*c) + 5*a*e^2*sin(2*d*x + 2*
c) - 12*a*e^2)*sqrt((e*cos(2*d*x + 2*c) + e)/sin(2*d*x + 2*c)))/(d*cos(2*d*
x + 2*c) - d)]
```

Sympy [F]

$$\int (e\cot(c+dx))^{5/2}(a+a\cot(c+dx))\,dx = a\left(\int (e\cot(c+dx))^{\frac{5}{2}}\,dx + \int (e\cot(c+dx))^{\frac{5}{2}}\cot(c+dx)\,dx\right)$$

```
[In] integrate((e*cot(d*x+c))**(5/2)*(a+a*cot(d*x+c)),x)
```

```
[Out] a*(Integral((e*cot(c + d*x))**(5/2), x) + Integral((e*cot(c + d*x))**(5/2)*
cot(c + d*x), x))
```

Maxima [F(-2)]

Exception generated.

$$\int (e\cot(c+dx))^{5/2}(a+a\cot(c+dx))\,dx = \text{Exception raised: ValueError}$$

```
[In] integrate((e*cot(d*x+c))^(5/2)*(a+a*cot(d*x+c)),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(e>0)', see `assume?` for more detai
ls)Is e
```

Giac [F]

$$\int (e\cot(c+dx))^{5/2}(a+a\cot(c+dx))\,dx = \int (a\cot(dx+c)+a)(e\cot(dx+c))^{\frac{5}{2}}\,dx$$

```
[In] integrate((e*cot(d*x+c))^(5/2)*(a+a*cot(d*x+c)),x, algorithm="giac")
```

```
[Out] integrate((a*cot(d*x + c) + a)*(e*cot(d*x + c))^(5/2), x)
```

Mupad [B] (verification not implemented)

Time = 13.78 (sec) , antiderivative size = 144, normalized size of antiderivative = 1.24

$$\int (e\cot(c+dx))^{5/2}(a+a\cot(c+dx))\,dx = \frac{2\,a\,e^2\,\sqrt{e\cot(c+d\,x)}}{d} - \frac{2\,a\,e\,(e\cot(c+d\,x))^{3/2}}{3\,d} - \frac{2\,a\,(e\cot(c+d\,x))^{5/2}}{5\,d} + \frac{(-1)^{1/4}\,a\,e^{5/2}\,\mathrm{atan}\left(\frac{(-1)^{1/4}\,\sqrt{e\cot(c+d\,x)}\,1\mathrm{i}}{\sqrt{e}}\right)}{d} - \frac{(-1)^{1/4}\,a\,e^{5/2}\,\mathrm{atanh}\left(\frac{(-1)^{1/4}\,\sqrt{e\cot(c+d\,x)}}{\sqrt{e}}\right)}{d} + \frac{(-1)^{1/4}\,a\,e^{5/2}\,\mathrm{atan}\left(\frac{(-1)^{1/4}\,\sqrt{e\cot(c+d\,x)}}{\sqrt{e}}\right)\,(1+1\mathrm{i})}{d}$$

```
[In] int((e*cot(c + d*x))^(5/2)*(a + a*cot(c + d*x)),x)
```

```
[Out] (2*a*e^2*(e*cot(c + d*x))^(1/2))/d - (2*a*e*(e*cot(c + d*x))^(3/2))/(3*d) -
 (2*a*(e*cot(c + d*x))^(5/2))/(5*d) + ((-1)^(1/4)*a*e^(5/2)*atan(((-1)^(1/4
)*(e*cot(c + d*x))^(1/2))/e^(1/2))*(1 + 1i))/d + ((-1)^(1/4)*a*e^(5/2)*atan
(((-1)^(1/4)*(e*cot(c + d*x))^(1/2)*1i)/e^(1/2)))/d - ((-1)^(1/4)*a*e^(5/2)
*atanh(((-1)^(1/4)*(e*cot(c + d*x))^(1/2))/e^(1/2)))/d
```

3.3 $\int (e\cot(c+dx))^{3/2}(a+a\cot(c+dx))\,dx$

Optimal result

Integrand size = 23, antiderivative size = 94

$$\int (e\cot(c+dx))^{3/2}(a+a\cot(c+dx))\,dx = -\frac{\sqrt{2}ae^{3/2}\arctan\left(\frac{\sqrt{e}-\sqrt{e}\cot(c+dx)}{\sqrt{2}\sqrt{e\cot(c+dx)}}\right)}{d} - \frac{2ae\sqrt{e\cot(c+dx)}}{d} - \frac{2a(e\cot(c+dx))^{3/2}}{3d}$$

```
[Out] -2/3*a*(e*cot(d*x+c))^(3/2)/d-a*e^(3/2)*arctan(1/2*(e^(1/2)-cot(d*x+c)*e^(1
/2))*2^(1/2)/(e*cot(d*x+c))^(1/2))*2^(1/2)/d-2*a*e*(e*cot(d*x+c))^(1/2)/d
```

Rubi [A] (verified)

Time = 0.14 (sec) , antiderivative size = 94, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.130, Rules used = {3609, 3613, 211}

$$\int (e\cot(c+dx))^{3/2}(a+a\cot(c+dx))\,dx = -\frac{\sqrt{2}ae^{3/2}\arctan\left(\frac{\sqrt{e}-\sqrt{e}\cot(c+dx)}{\sqrt{2}\sqrt{e\cot(c+dx)}}\right)}{d} - \frac{2ae\sqrt{e\cot(c+dx)}}{d} - \frac{2a(e\cot(c+dx))^{3/2}}{3d}$$

```
[In] Int[(e*Cot[c + d*x])^(3/2)*(a + a*Cot[c + d*x]),x]
```

```
[Out] -((Sqrt[2]*a*e^(3/2)*ArcTan[(Sqrt[e] - Sqrt[e]*Cot[c + d*x])/(Sqrt[2]*Sqrt[
e*Cot[c + d*x]])])/d) - (2*a*e*Sqrt[e*Cot[c + d*x]])/d - (2*a*(e*Cot[c + d*
x])^(3/2))/(3*d)
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 3609

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)]), x_Symbol] :> Simp[d*((a + b*Tan[e + f*x])^m/(f*m)), x] + Int
[(a + b*Tan[e + f*x])^(m - 1)*Simp[a*c - b*d + (b*c + a*d)*Tan[e + f*x], x]
, x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2,
 0] && GtQ[m, 0]
```

Rule 3613

```
Int[((c_) + (d_.)*tan[(e_.) + (f_.)*(x_)])/Sqrt[(b_.)*tan[(e_.) + (f_.)*(x_
)]], x_Symbol] :> Dist[-2*(d^2/f), Subst[Int[1/(2*c*d + b*x^2), x], x, (c -
 d*Tan[e + f*x])/Sqrt[b*Tan[e + f*x]]], x] /; FreeQ[{b, c, d, e, f}, x] &&
EqQ[c^2 - d^2, 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\frac{2a(e\cot(c+dx))^{3/2}}{3d} + \int \sqrt{e\cot(c+dx)}(-ae+ae\cot(c+dx))\,dx \\
&= -\frac{2ae\sqrt{e\cot(c+dx)}}{d} - \frac{2a(e\cot(c+dx))^{3/2}}{3d} + \int \frac{-ae^2-ae^2\cot(c+dx)}{\sqrt{e\cot(c+dx)}}\,dx \\
&= -\frac{2ae\sqrt{e\cot(c+dx)}}{d} - \frac{2a(e\cot(c+dx))^{3/2}}{3d} \\
&\quad - \frac{(2a^2e^4)\,\text{Subst}\left(\int \frac{1}{-2a^2e^4-ex^2}\,dx, x, \frac{-ae^2+ae^2\cot(c+dx)}{\sqrt{e\cot(c+dx)}}\right)}{d} \\
&= -\frac{\sqrt{2}ae^{3/2}\arctan\left(\frac{\sqrt{e}-\sqrt{e}\cot(c+dx)}{\sqrt{2}\sqrt{e\cot(c+dx)}}\right)}{d} - \frac{2ae\sqrt{e\cot(c+dx)}}{d} - \frac{2a(e\cot(c+dx))^{3/2}}{3d}
\end{aligned}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 0.13 (sec) , antiderivative size = 67, normalized size of antiderivative = 0.71

$$\int (e\cot(c+dx))^{3/2}(a+a\cot(c+dx))\,dx =$$

$$-\frac{2ae\sqrt{e\cot(c+dx)}\left(\cot(c+dx)\,\text{Hypergeometric2F1}\left(-\frac{3}{4},1,\frac{1}{4},-\tan^2(c+dx)\right)+3\,\text{Hypergeometric2F1}\left(-\right.\right.}{3d}$$

```
[In] Integrate[(e*Cot[c + d*x])^(3/2)*(a + a*Cot[c + d*x]),x]
```

[Out] (-2*a*e*Sqrt[e*Cot[c + d*x]]*(Cot[c + d*x]*Hypergeometric2F1[-3/4, 1, 1/4, -Tan[c + d*x]^2] + 3*Hypergeometric2F1[-1/4, 1, 3/4, -Tan[c + d*x]^2]))/(3*d)

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 302 vs. $2(77) = 154$.

Time = 0.04 (sec) , antiderivative size = 303, normalized size of antiderivative = 3.22

method	result
parts	$-\frac{2ae\left(\sqrt{e\cot(dx+c)}-\frac{(e^2)^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\frac{e\cot(dx+c)+(e^2)^{\frac{1}{4}}\sqrt{e\cot(dx+c)}\,\sqrt{2}+\sqrt{e^2}}{e\cot(dx+c)-(e^2)^{\frac{1}{4}}\sqrt{e\cot(dx+c)}\,\sqrt{2}+\sqrt{e^2}}\right)+2\arctan\left(\frac{\sqrt{2}\,\sqrt{e\cot(dx+c)}}{(e^2)^{\frac{1}{4}}}+1\right)-2\arctan\left(\cdots\right.\right.}{8}\cdots\right.}{d}$
derivativedivides	$-\frac{a\left(\frac{2(e\cot(dx+c))^{\frac{3}{2}}}{3}+2e\sqrt{e\cot(dx+c)}-2e^2\left(\frac{(e^2)^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\frac{e\cot(dx+c)+(e^2)^{\frac{1}{4}}\sqrt{e\cot(dx+c)}\,\sqrt{2}+\sqrt{e^2}}{e\cot(dx+c)-(e^2)^{\frac{1}{4}}\sqrt{e\cot(dx+c)}\,\sqrt{2}+\sqrt{e^2}}\right)+2\arctan\left(\frac{\sqrt{2}\,\sqrt{e\cdots}}{(e^2\cdots}\right.\right.}{8e}\cdots\right.\right.}{\cdots}$
default	$-\frac{a\left(\frac{2(e\cot(dx+c))^{\frac{3}{2}}}{3}+2e\sqrt{e\cot(dx+c)}-2e^2\left(\frac{(e^2)^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\frac{e\cot(dx+c)+(e^2)^{\frac{1}{4}}\sqrt{e\cot(dx+c)}\,\sqrt{2}+\sqrt{e^2}}{e\cot(dx+c)-(e^2)^{\frac{1}{4}}\sqrt{e\cot(dx+c)}\,\sqrt{2}+\sqrt{e^2}}\right)+2\arctan\left(\frac{\sqrt{2}\,\sqrt{e\cdots}}{(e^2\cdots}\right.\right.}{8e}\cdots\right.\right.}{\cdots}$

[In] int((e*cot(d*x+c))^(3/2)*(a+a*cot(d*x+c)),x,method=_RETURNVERBOSE)

[Out] -2*a/d*e*((e*cot(d*x+c))^(1/2)-1/8*(e^2)^(1/4)*2^(1/2)*(ln((e*cot(d*x+c)+(e^2)^(1/4)*(e*cot(d*x+c))^(1/2)*2^(1/2)+(e^2)^(1/2))/(e*cot(d*x+c)-(e^2)^(1/4)*(e*cot(d*x+c))^(1/2)*2^(1/2)+(e^2)^(1/2)))+2*arctan(2^(1/2)/(e^2)^(1/4)*(e*cot(d*x+c))^(1/2)+1)-2*arctan(-2^(1/2)/(e^2)^(1/4)*(e*cot(d*x+c))^(1/2)+1)))+a/d*(-2/3*(e*cot(d*x+c))^(3/2)+1/4*e^2/(e^2)^(1/4)*2^(1/2)*(ln((e*cot(d*x+c)-(e^2)^(1/4)*(e*cot(d*x+c))^(1/2)*2^(1/2)+(e^2)^(1/2))/(e*cot(d*x+c)+(e^2)^(1/4)*(e*cot(d*x+c))^(1/2)*2^(1/2)+(e^2)^(1/2)))+2*arctan(2^(1/2)/(e^2)^(1/4)*(e*cot(d*x+c))^(1/2)+1)-2*arctan(-2^(1/2)/(e^2)^(1/4)*(e*cot(d*x+c))^(1/2)+1)))

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 166 vs. 2(78) = 156.

Time = 0.28 (sec) , antiderivative size = 334, normalized size of antiderivative = 3.55

$$\int (e\cot(c+dx))^{3/2}(a+a\cot(c+dx))\,dx = \left[\frac{3\sqrt{2}a\sqrt{-e}e\log\left(\sqrt{2}\sqrt{-e}\sqrt{\frac{e\cos(2\,dx+2\,c)+e}{\sin(2\,dx+2\,c)}}(\cos(2\,dx+2\,c)+\sin(2\,dx+2\,c)-1)-2\,e\right.}{}\right.$$

$$-\frac{3\sqrt{2}ae^{\frac{3}{2}}\arctan\left(-\frac{\sqrt{2}\sqrt{e}\sqrt{\frac{e\cos(2\,dx+2\,c)+e}{\sin(2\,dx+2\,c)}}(\cos(2\,dx+2\,c)-\sin(2\,dx+2\,c)+1)}{2\,(e\cos(2\,dx+2\,c)+e)}\right)\sin(2\,dx+2\,c)+2\,(ae\cos(2\,dx+2\,c)+}{3\,d\sin(2\,dx+2\,c)}$$

```
[In] integrate((e*cot(d*x+c))^(3/2)*(a+a*cot(d*x+c)),x, algorithm="fricas")
```

```
[Out] [1/6*(3*sqrt(2)*a*sqrt(-e)*e*log(sqrt(2)*sqrt(-e)*sqrt((e*cos(2*d*x + 2*c)
+ e)/sin(2*d*x + 2*c))*(cos(2*d*x + 2*c) + sin(2*d*x + 2*c) - 1) - 2*e*sin(
2*d*x + 2*c) + e)*sin(2*d*x + 2*c) - 4*(a*e*cos(2*d*x + 2*c) + 3*a*e*sin(2*
d*x + 2*c) + a*e)*sqrt((e*cos(2*d*x + 2*c) + e)/sin(2*d*x + 2*c)))/(d*sin(2
*d*x + 2*c)), -1/3*(3*sqrt(2)*a*e^(3/2)*arctan(-1/2*sqrt(2)*sqrt(e)*sqrt((e
*cos(2*d*x + 2*c) + e)/sin(2*d*x + 2*c))*(cos(2*d*x + 2*c) - sin(2*d*x + 2*
c) + 1)/(e*cos(2*d*x + 2*c) + e))*sin(2*d*x + 2*c) + 2*(a*e*cos(2*d*x + 2*c
) + 3*a*e*sin(2*d*x + 2*c) + a*e)*sqrt((e*cos(2*d*x + 2*c) + e)/sin(2*d*x +
 2*c)))/(d*sin(2*d*x + 2*c))]
```

Sympy [F]

$$\int (e\cot(c+dx))^{3/2}(a+a\cot(c+dx))\,dx = a\left(\int (e\cot(c+dx))^{\frac{3}{2}}\,dx + \int (e\cot(c+dx))^{\frac{3}{2}}\cot(c+dx)\,dx\right)$$

```
[In] integrate((e*cot(d*x+c))**(3/2)*(a+a*cot(d*x+c)),x)
```

```
[Out] a*(Integral((e*cot(c + d*x))**(3/2), x) + Integral((e*cot(c + d*x))**(3/2)*
cot(c + d*x), x))
```

Maxima [F(-2)]

Exception generated.

$$\int (e\cot(c+dx))^{3/2}(a+a\cot(c+dx))\,dx = \text{Exception raised: ValueError}$$

```
[In] integrate((e*cot(d*x+c))^(3/2)*(a+a*cot(d*x+c)),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(e>0)', see `assume?` for more detai
ls)Is e
```

Giac [F]

$$\int (e\cot(c+dx))^{3/2}(a+a\cot(c+dx))\,dx = \int (a\cot(dx+c)+a)(e\cot(dx+c))^{\frac{3}{2}}\,dx$$

```
[In] integrate((e*cot(d*x+c))^(3/2)*(a+a*cot(d*x+c)),x, algorithm="giac")
```

```
[Out] integrate((a*cot(d*x + c) + a)*(e*cot(d*x + c))^(3/2), x)
```

Mupad [B] (verification not implemented)

Time = 13.51 (sec) , antiderivative size = 98, normalized size of antiderivative = 1.04

$$\int (e\cot(c+dx))^{3/2}(a+a\cot(c+dx))\,dx = -\frac{2a\,(e\cot(c+d\,x))^{3/2}}{3\,d} - \frac{2\,a\,e\,\sqrt{e\cot(c+d\,x)}}{d} + \frac{(-1)^{1/4}\,a\,e^{3/2}\,\mathrm{atan}\left(\frac{(-1)^{1/4}\,\sqrt{e\cot(c+d\,x)}}{\sqrt{e}}\right)\,(1-\mathrm{i})}{d} + \frac{(-1)^{1/4}\,a\,e^{3/2}\,\mathrm{atanh}\left(\frac{(-1)^{1/4}\,\sqrt{e\cot(c+d\,x)}}{\sqrt{e}}\right)\,(-1-\mathrm{i})}{d}$$

```
[In] int((e*cot(c + d*x))^(3/2)*(a + a*cot(c + d*x)),x)
```

```
[Out] ((-1)^(1/4)*a*e^(3/2)*atan(((-1)^(1/4)*(e*cot(c + d*x))^(1/2))/e^(1/2))*(1
- 1i))/d - (2*a*e*(e*cot(c + d*x))^(1/2))/d - (2*a*(e*cot(c + d*x))^(3/2))/
(3*d) - ((-1)^(1/4)*a*e^(3/2)*atanh(((-1)^(1/4)*(e*cot(c + d*x))^(1/2))/e^(
1/2))*(1 + 1i))/d
```

3.4 $\int \sqrt{e\cot(c+dx)}(a+a\cot(c+dx))\,dx$

Optimal result

Integrand size = 23, antiderivative size = 71

$$\int \sqrt{e\cot(c+dx)}(a+a\cot(c+dx))\,dx=\frac{\sqrt{2}a\sqrt{e}\operatorname{arctanh}\left(\frac{\sqrt{e}+\sqrt{e}\cot(c+dx)}{\sqrt{2}\sqrt{e\cot(c+dx)}}\right)}{d}-\frac{2a\sqrt{e\cot(c+dx)}}{d}$$

```
[Out] a*arctanh(1/2*(e^(1/2)+cot(d*x+c)*e^(1/2))*2^(1/2)/(e*cot(d*x+c))^(1/2))*2^
(1/2)*e^(1/2)/d-2*a*(e*cot(d*x+c))^(1/2)/d
```

Rubi [A] (verified)

Time = 0.10 (sec) , antiderivative size = 71, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.130, Rules used = {3609, 3613, 214}

$$\int \sqrt{e\cot(c+dx)}(a+a\cot(c+dx))\,dx=\frac{\sqrt{2}a\sqrt{e}\operatorname{arctanh}\left(\frac{\sqrt{e}\cot(c+dx)+\sqrt{e}}{\sqrt{2}\sqrt{e\cot(c+dx)}}\right)}{d}-\frac{2a\sqrt{e\cot(c+dx)}}{d}$$

```
[In] Int[Sqrt[e*Cot[c + d*x]]*(a + a*Cot[c + d*x]),x]
```

```
[Out] (Sqrt[2]*a*Sqrt[e]*ArcTanh[(Sqrt[e] + Sqrt[e]*Cot[c + d*x])/(Sqrt[2]*Sqrt[e
*Cot[c + d*x]])])/d - (2*a*Sqrt[e*Cot[c + d*x]])/d
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 3609

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)]), x_Symbol] :> Simp[d*((a + b*Tan[e + f*x])^m/(f*m)), x] + Int
[(a + b*Tan[e + f*x])^(m - 1)*Simp[a*c - b*d + (b*c + a*d)*Tan[e + f*x], x]
, x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2,
 0] && GtQ[m, 0]
```

Rule 3613

```
Int[((c_) + (d_.)*tan[(e_.) + (f_.)*(x_)])/Sqrt[(b_.)*tan[(e_.) + (f_.)*(x_
)]], x_Symbol] :> Dist[-2*(d^2/f), Subst[Int[1/(2*c*d + b*x^2), x], x, (c -
 d*Tan[e + f*x])/Sqrt[b*Tan[e + f*x]]], x] /; FreeQ[{b, c, d, e, f}, x] &&
EqQ[c^2 - d^2, 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\frac{2a\sqrt{e\cot(c+dx)}}{d} + \int \frac{-ae+ae\cot(c+dx)}{\sqrt{e\cot(c+dx)}}\,dx \\
&= -\frac{2a\sqrt{e\cot(c+dx)}}{d} - \frac{(2a^2e^2)\,\text{Subst}\left(\int \frac{1}{2a^2e^2-ex^2}\,dx, x, \frac{-ae-ae\cot(c+dx)}{\sqrt{e\cot(c+dx)}}\right)}{d} \\
&= \frac{\sqrt{2}a\sqrt{e}\operatorname{arctanh}\left(\frac{\sqrt{e}+\sqrt{e}\cot(c+dx)}{\sqrt{2}\sqrt{e\cot(c+dx)}}\right)}{d} - \frac{2a\sqrt{e\cot(c+dx)}}{d}
\end{aligned}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 0.33 (sec) , antiderivative size = 154, normalized size of antiderivative = 2.17

$$\int \sqrt{e\cot(c+dx)}(a+a\cot(c+dx))\,dx =$$
$$-\frac{a\sqrt{e\cot(c+dx)}\left(8\,\text{Hypergeometric2F1}\left(-\frac{1}{4},1,\frac{3}{4},-\tan^2(c+dx)\right)+\sqrt{2}\left(2\arctan\left(1-\sqrt{2}\sqrt{\tan(c+dx}\right.\right.\right.}{}$$

```
[In] Integrate[Sqrt[e*Cot[c + d*x]]*(a + a*Cot[c + d*x]),x]
```

```
[Out] -1/4*(a*Sqrt[e*Cot[c + d*x]]*(8*Hypergeometric2F1[-1/4, 1, 3/4, -Tan[c + d*
x]^2] + Sqrt[2]*(2*ArcTan[1 - Sqrt[2]*Sqrt[Tan[c + d*x]]] - 2*ArcTan[1 + Sq
rt[2]*Sqrt[Tan[c + d*x]]] + Log[1 - Sqrt[2]*Sqrt[Tan[c + d*x]] + Tan[c + d*
x]] - Log[1 + Sqrt[2]*Sqrt[Tan[c + d*x]] + Tan[c + d*x]])*Sqrt[Tan[c + d*x]
]))/d
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 286 vs. 2(58) = 116.

Time = 0.05 (sec) , antiderivative size = 287, normalized size of antiderivative = 4.04

method	result
parts	$-\frac{ae\sqrt{2}\left(\ln\left(\frac{e\cot(dx+c)-\left(e^{2}\right)^{\frac{1}{4}}\sqrt{e\cot(dx+c)}\,\sqrt{2}+\sqrt{e^{2}}}{e\cot(dx+c)+\left(e^{2}\right)^{\frac{1}{4}}\sqrt{e\cot(dx+c)}\,\sqrt{2}+\sqrt{e^{2}}}\right)+2\arctan\left(\frac{\sqrt{2}\,\sqrt{e\cot(dx+c)}}{\left(e^{2}\right)^{\frac{1}{4}}}+1\right)-2\arctan\left(-\frac{\sqrt{2}\,\sqrt{e\cot(dx+c)}}{\left(e^{2}\right)^{\frac{1}{4}}}\right.\right.}{4d\left(e^{2}\right)^{\frac{1}{4}}}$
derivativedivides	$-\frac{a\left(2\sqrt{e\cot(dx+c)}-2e\left(\frac{\left(e^{2}\right)^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\frac{e\cot(dx+c)+\left(e^{2}\right)^{\frac{1}{4}}\sqrt{e\cot(dx+c)}\,\sqrt{2}+\sqrt{e^{2}}}{e\cot(dx+c)-\left(e^{2}\right)^{\frac{1}{4}}\sqrt{e\cot(dx+c)}\,\sqrt{2}+\sqrt{e^{2}}}\right)+2\arctan\left(\frac{\sqrt{2}\,\sqrt{e\cot(dx+c)}}{\left(e^{2}\right)^{\frac{1}{4}}}+1\right)-2\,\mathrm{arc}\right.}{8e}\right.\right.}{}$
default	$-\frac{a\left(2\sqrt{e\cot(dx+c)}-2e\left(\frac{\left(e^{2}\right)^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\frac{e\cot(dx+c)+\left(e^{2}\right)^{\frac{1}{4}}\sqrt{e\cot(dx+c)}\,\sqrt{2}+\sqrt{e^{2}}}{e\cot(dx+c)-\left(e^{2}\right)^{\frac{1}{4}}\sqrt{e\cot(dx+c)}\,\sqrt{2}+\sqrt{e^{2}}}\right)+2\arctan\left(\frac{\sqrt{2}\,\sqrt{e\cot(dx+c)}}{\left(e^{2}\right)^{\frac{1}{4}}}+1\right)-2\,\mathrm{arc}\right.}{8e}\right.\right.}{}$

```
[In] int((e*cot(d*x+c))^(1/2)*(a+a*cot(d*x+c)),x,method=_RETURNVERBOSE)
```

```
[Out] -1/4*a/d*e/(e^2)^(1/4)*2^(1/2)*(ln((e*cot(d*x+c)-(e^2)^(1/4)*(e*cot(d*x+c))
^(1/2)*2^(1/2)+(e^2)^(1/2))/(e*cot(d*x+c)+(e^2)^(1/4)*(e*cot(d*x+c))^(1/2)*
2^(1/2)+(e^2)^(1/2)))+2*arctan(2^(1/2)/(e^2)^(1/4)*(e*cot(d*x+c))^(1/2)+1)-
2*arctan(-2^(1/2)/(e^2)^(1/4)*(e*cot(d*x+c))^(1/2)+1))+a/d*(-2*(e*cot(d*x+c
))^(1/2)+1/4*(e^2)^(1/4)*2^(1/2)*(ln((e*cot(d*x+c)+(e^2)^(1/4)*(e*cot(d*x+c
))^(1/2)*2^(1/2)+(e^2)^(1/2))/(e*cot(d*x+c)-(e^2)^(1/4)*(e*cot(d*x+c))^(1/2
)*2^(1/2)+(e^2)^(1/2)))+2*arctan(2^(1/2)/(e^2)^(1/4)*(e*cot(d*x+c))^(1/2)+1
)-2*arctan(-2^(1/2)/(e^2)^(1/4)*(e*cot(d*x+c))^(1/2)+1)))
```

Fricas [A] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 119 vs. $2(58) = 116$.

Time = 0.27 (sec) , antiderivative size = 236, normalized size of antiderivative = 3.32

$$\int \sqrt{e\cot(c+dx)}(a+a\cot(c+dx))\,dx$$
$$= \left[\frac{\sqrt{2}a\sqrt{e}\log\left(-\sqrt{2}\sqrt{e}\sqrt{\frac{e\cos(2\,dx+2\,c)+e}{\sin(2\,dx+2\,c)}}(\cos(2\,dx+2\,c)-\sin(2\,dx+2\,c)-1)+2\,e\sin(2\,dx+2\,c)+e\right)-}{2\,d}\right.$$
$$\left.-\frac{\sqrt{2}a\sqrt{-e}\arctan\left(\frac{\sqrt{2}\sqrt{-e}\sqrt{\frac{e\cos(2\,dx+2\,c)+e}{\sin(2\,dx+2\,c)}}(\cos(2\,dx+2\,c)+\sin(2\,dx+2\,c)+1)}{2\,(e\cos(2\,dx+2\,c)+e)}\right)+2\,a\sqrt{\frac{e\cos(2\,dx+2\,c)+e}{\sin(2\,dx+2\,c)}}}{d}\right]$$

```
[In] integrate((e*cot(d*x+c))^(1/2)*(a+a*cot(d*x+c)),x, algorithm="fricas")
```

```
[Out] [1/2*(sqrt(2)*a*sqrt(e)*log(-sqrt(2)*sqrt(e)*sqrt((e*cos(2*d*x + 2*c) + e)/
sin(2*d*x + 2*c))*(cos(2*d*x + 2*c) - sin(2*d*x + 2*c) - 1) + 2*e*sin(2*d*x
 + 2*c) + e) - 4*a*sqrt((e*cos(2*d*x + 2*c) + e)/sin(2*d*x + 2*c)))/d, -(sq
rt(2)*a*sqrt(-e)*arctan(1/2*sqrt(2)*sqrt(-e)*sqrt((e*cos(2*d*x + 2*c) + e)/
sin(2*d*x + 2*c))*(cos(2*d*x + 2*c) + sin(2*d*x + 2*c) + 1)/(e*cos(2*d*x +
2*c) + e)) + 2*a*sqrt((e*cos(2*d*x + 2*c) + e)/sin(2*d*x + 2*c)))/d]
```

Sympy [F]

$$\int \sqrt{e\cot(c+dx)}(a+a\cot(c+dx))\,dx = a\left(\int \sqrt{e\cot(c+dx)}\,dx + \int \sqrt{e\cot(c+dx)}\cot(c+dx)\,dx\right)$$

```
[In] integrate((e*cot(d*x+c))**(1/2)*(a+a*cot(d*x+c)),x)
```

```
[Out] a*(Integral(sqrt(e*cot(c + d*x)), x) + Integral(sqrt(e*cot(c + d*x))*cot(c
+ d*x), x))
```

Maxima [F(-2)]

Exception generated.

$$\int \sqrt{e\cot(c+dx)}(a+a\cot(c+dx))\,dx = \text{Exception raised: ValueError}$$

```
[In] integrate((e*cot(d*x+c))^(1/2)*(a+a*cot(d*x+c)),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(e>0)', see `assume?` for more detai
ls)Is e
```

Giac [F]

$$\int \sqrt{e\cot(c+dx)}(a+a\cot(c+dx))\,dx = \int (a\cot(dx+c)+a)\sqrt{e\cot(dx+c)}\,dx$$

```
[In] integrate((e*cot(d*x+c))^(1/2)*(a+a*cot(d*x+c)),x, algorithm="giac")
```

```
[Out] integrate((a*cot(d*x + c) + a)*sqrt(e*cot(d*x + c)), x)
```

Mupad [B] (verification not implemented)

Time = 13.28 (sec) , antiderivative size = 128, normalized size of antiderivative = 1.80

$$\begin{aligned}\int \sqrt{e\cot(c+dx)}(a+a\cot(c+dx))\,dx &= -\frac{2a\sqrt{e\cot(c+dx)}}{d}\\&\quad-\frac{(-1)^{1/4}a\sqrt{e}\left(\operatorname{atan}\left(\frac{(-1)^{1/4}\sqrt{e\cot(c+dx)}}{\sqrt{e}}\right)-\operatorname{atanh}\left(\frac{(-1)^{1/4}\sqrt{e\cot(c+dx)}}{\sqrt{e}}\right)\right)}{d}\\&\quad-\frac{(-1)^{1/4}a\sqrt{e}\operatorname{atan}\left(\frac{(-1)^{1/4}\sqrt{e\cot(c+dx)}}{\sqrt{e}}\right)1\mathrm{i}}{d}-\frac{(-1)^{1/4}a\sqrt{e}\operatorname{atanh}\left(\frac{(-1)^{1/4}\sqrt{e\cot(c+dx)}}{\sqrt{e}}\right)1\mathrm{i}}{d}\end{aligned}$$

```
[In] int((e*cot(c + d*x))^(1/2)*(a + a*cot(c + d*x)),x)
```

```
[Out] - (2*a*(e*cot(c + d*x))^(1/2))/d - ((-1)^(1/4)*a*e^(1/2)*atan(((-1)^(1/4)*(
e*cot(c + d*x))^(1/2))/e^(1/2))*1i)/d - ((-1)^(1/4)*a*e^(1/2)*atanh(((-1)^(
1/4)*(e*cot(c + d*x))^(1/2))/e^(1/2))*1i)/d - ((-1)^(1/4)*a*e^(1/2)*(atan((
(-1)^(1/4)*(e*cot(c + d*x))^(1/2))/e^(1/2)) - atanh(((-1)^(1/4)*(e*cot(c +
d*x))^(1/2))/e^(1/2))))/d
```

3.5 $\int \frac{a+a\cot(c+dx)}{\sqrt{e\cot(c+dx)}}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 49

$$\int \frac{a+a\cot(c+dx)}{\sqrt{e\cot(c+dx)}}\,dx = \frac{\sqrt{2}a\arctan\left(\frac{\sqrt{e}(1-\cot(c+dx))}{\sqrt{2}\sqrt{e\cot(c+dx)}}\right)}{d\sqrt{e}}$$

```
[Out] a*arctan(1/2*(1-cot(d*x+c))*e^(1/2)*2^(1/2)/(e*cot(d*x+c))^(1/2))*2^(1/2)/d
/e^(1/2)
```

Rubi [A] (verified)

Time = 0.05 (sec) , antiderivative size = 49, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.087, Rules used = {3613, 211}

$$\int \frac{a+a\cot(c+dx)}{\sqrt{e\cot(c+dx)}}\,dx = \frac{\sqrt{2}a\arctan\left(\frac{\sqrt{e}(1-\cot(c+dx))}{\sqrt{2}\sqrt{e\cot(c+dx)}}\right)}{d\sqrt{e}}$$

```
[In] Int[(a + a*Cot[c + d*x])/Sqrt[e*Cot[c + d*x]],x]
```

```
[Out] (Sqrt[2]*a*ArcTan[(Sqrt[e]*(1 - Cot[c + d*x]))/(Sqrt[2]*Sqrt[e*Cot[c + d*x]
])])/(d*Sqrt[e])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 3613

```
Int[((c_) + (d_.)*tan[(e_.) + (f_.)*(x_)])/Sqrt[(b_.)*tan[(e_.) + (f_.)*(x_
)]], x_Symbol] :> Dist[-2*(d^2/f), Subst[Int[1/(2*c*d + b*x^2), x], x, (c -
 d*Tan[e + f*x])/Sqrt[b*Tan[e + f*x]]], x] /; FreeQ[{b, c, d, e, f}, x] &&
EqQ[c^2 - d^2, 0]
```

Rubi steps

$$\text{integral} = -\frac{(2a^2)\,\text{Subst}\left(\int \frac{1}{-2a^2-ex^2}\,dx, x, \frac{a-a\cot(c+dx)}{\sqrt{e\cot(c+dx)}}\right)}{d}$$
$$= \frac{\sqrt{2}a\arctan\left(\frac{\sqrt{e}(1-\cot(c+dx))}{\sqrt{2}\sqrt{e\cot(c+dx)}}\right)}{d\sqrt{e}}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 0.23 (sec) , antiderivative size = 165, normalized size of antiderivative = 3.37

$$\int \frac{a+a\cot(c+dx)}{\sqrt{e\cot(c+dx)}}\,dx$$
$$= \frac{a\Big(3\sqrt{2}\Big(-2\arctan\left(1-\sqrt{2}\sqrt{\tan(c+dx)}\right)+2\arctan\left(1+\sqrt{2}\sqrt{\tan(c+dx)}\right)-\log\left(1-\sqrt{2}\sqrt{\tan(c+\ldots}{12d\ldots}$$

```
[In] Integrate[(a + a*Cot[c + d*x])/Sqrt[e*Cot[c + d*x]],x]
```

```
[Out] (a*(3*Sqrt[2]*(-2*ArcTan[1 - Sqrt[2]*Sqrt[Tan[c + d*x]]] + 2*ArcTan[1 + Sqr
t[2]*Sqrt[Tan[c + d*x]]] - Log[1 - Sqrt[2]*Sqrt[Tan[c + d*x]] + Tan[c + d*x
]] + Log[1 + Sqrt[2]*Sqrt[Tan[c + d*x]] + Tan[c + d*x]]) + 8*Hypergeometric
2F1[3/4, 1, 7/4, -Tan[c + d*x]^2]*Tan[c + d*x]^(3/2)))/(12*d*Sqrt[e*Cot[c +
 d*x]]*Sqrt[Tan[c + d*x]])
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 272 vs. 2(40) = 80.

Time = 0.11 (sec) , antiderivative size = 273, normalized size of antiderivative = 5.57

method	result
derivativedivides	$-\frac{a\left(\frac{(e^2)^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\frac{e\cot(dx+c)+(e^2)^{\frac{1}{4}}\sqrt{e\cot(dx+c)}\sqrt{2}+\sqrt{e^2}}{e\cot(dx+c)-(e^2)^{\frac{1}{4}}\sqrt{e\cot(dx+c)}\sqrt{2}+\sqrt{e^2}}\right)+2\arctan\left(\frac{\sqrt{2}\sqrt{e\cot(dx+c)}}{(e^2)^{\frac{1}{4}}}+1\right)-2\arctan\left(-\frac{\sqrt{2}\sqrt{e\cot(dx+c)}}{(e^2)^{\frac{1}{4}}}+\right.\right.}{4e}\right.}{}$
default	$-\frac{a\left(\frac{(e^2)^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\frac{e\cot(dx+c)+(e^2)^{\frac{1}{4}}\sqrt{e\cot(dx+c)}\sqrt{2}+\sqrt{e^2}}{e\cot(dx+c)-(e^2)^{\frac{1}{4}}\sqrt{e\cot(dx+c)}\sqrt{2}+\sqrt{e^2}}\right)+2\arctan\left(\frac{\sqrt{2}\sqrt{e\cot(dx+c)}}{(e^2)^{\frac{1}{4}}}+1\right)-2\arctan\left(-\frac{\sqrt{2}\sqrt{e\cot(dx+c)}}{(e^2)^{\frac{1}{4}}}+\right.\right.}{4e}\right.}{}$
parts	$-\frac{a(e^2)^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\frac{e\cot(dx+c)+(e^2)^{\frac{1}{4}}\sqrt{e\cot(dx+c)}\sqrt{2}+\sqrt{e^2}}{e\cot(dx+c)-(e^2)^{\frac{1}{4}}\sqrt{e\cot(dx+c)}\sqrt{2}+\sqrt{e^2}}\right)+2\arctan\left(\frac{\sqrt{2}\sqrt{e\cot(dx+c)}}{(e^2)^{\frac{1}{4}}}+1\right)-2\arctan\left(-\frac{\sqrt{2}\sqrt{e\cot(dx}}{(e^2)^{\frac{1}{4}}}\right.\right.}{4de}$

```
[In] int((a+a*cot(d*x+c))/(e*cot(d*x+c))^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/d*a*(1/4/e*(e^2)^(1/4)*2^(1/2)*(ln((e*cot(d*x+c)+(e^2)^(1/4)*(e*cot(d*x+
c))^(1/2)*2^(1/2)+(e^2)^(1/2))/(e*cot(d*x+c)-(e^2)^(1/4)*(e*cot(d*x+c))^(1/
2)*2^(1/2)+(e^2)^(1/2)))+2*arctan(2^(1/2)/(e^2)^(1/4)*(e*cot(d*x+c))^(1/2)+
1)-2*arctan(-2^(1/2)/(e^2)^(1/4)*(e*cot(d*x+c))^(1/2)+1))+1/4/(e^2)^(1/4)*2
^(1/2)*(ln((e*cot(d*x+c)-(e^2)^(1/4)*(e*cot(d*x+c))^(1/2)*2^(1/2)+(e^2)^(1/
2))/(e*cot(d*x+c)+(e^2)^(1/4)*(e*cot(d*x+c))^(1/2)*2^(1/2)+(e^2)^(1/2)))+2*
arctan(2^(1/2)/(e^2)^(1/4)*(e*cot(d*x+c))^(1/2)+1)-2*arctan(-2^(1/2)/(e^2)^
(1/4)*(e*cot(d*x+c))^(1/2)+1)))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 82 vs. $2(38) = 76$.

Time = 0.28 (sec) , antiderivative size = 172, normalized size of antiderivative = 3.51

$$\int \frac{a + a\cot(c + dx)}{\sqrt{e\cot(c + dx)}}\,dx$$

$$= \left[\frac{\sqrt{2}a\sqrt{-\frac{1}{e}}\log\left(-\sqrt{2}\sqrt{\frac{e\cos(2\,dx+2\,c)+e}{\sin(2\,dx+2\,c)}}\sqrt{-\frac{1}{e}}(\cos(2\,dx+2\,c)+\sin(2\,dx+2\,c)-1)-2\,\sin(2\,dx+2\,c)+1\right.}{2\,d}\right.$$

```
[In] integrate((a+a*cot(d*x+c))/(e*cot(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] [1/2*sqrt(2)*a*sqrt(-1/e)*log(-sqrt(2)*sqrt((e*cos(2*d*x + 2*c) + e)/sin(2*
d*x + 2*c))*sqrt(-1/e)*(cos(2*d*x + 2*c) + sin(2*d*x + 2*c) - 1) - 2*sin(2*
d*x + 2*c) + 1)/d, sqrt(2)*a*arctan(-1/2*sqrt(2)*sqrt((e*cos(2*d*x + 2*c) +
```

```
 e)/sin(2*d*x + 2*c))*(cos(2*d*x + 2*c) - sin(2*d*x + 2*c) + 1)/(sqrt(e)*(c
os(2*d*x + 2*c) + 1)))/(d*sqrt(e))]
```

Sympy [F]

$$\int \frac{a + a\cot(c + dx)}{\sqrt{e\cot(c + dx)}}\,dx = a\left(\int \frac{1}{\sqrt{e\cot{(c + dx)}}}\,dx + \int \frac{\cot{(c + dx)}}{\sqrt{e\cot{(c + dx)}}}\,dx\right)$$

```
[In] integrate((a+a*cot(d*x+c))/(e*cot(d*x+c))**(1/2),x)
```

```
[Out] a*(Integral(1/sqrt(e*cot(c + d*x)), x) + Integral(cot(c + d*x)/sqrt(e*cot(c
 + d*x)), x))
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{a + a\cot(c + dx)}{\sqrt{e\cot(c + dx)}}\,dx = \text{Exception raised: ValueError}$$

```
[In] integrate((a+a*cot(d*x+c))/(e*cot(d*x+c))^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(e>0)', see `assume?` for more detai
ls)Is e
```

Giac [F]

$$\int \frac{a + a\cot(c + dx)}{\sqrt{e\cot(c + dx)}}\,dx = \int \frac{a\cot{(dx + c)} + a}{\sqrt{e\cot{(dx + c)}}}\,dx$$

```
[In] integrate((a+a*cot(d*x+c))/(e*cot(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] integrate((a*cot(d*x + c) + a)/sqrt(e*cot(d*x + c)), x)
```

Mupad [B] (verification not implemented)

Time = 13.06 (sec) , antiderivative size = 65, normalized size of antiderivative = 1.33

$$\int \frac{a + a\cot(c + dx)}{\sqrt{e\cot(c + dx)}}\,dx = \frac{(-1)^{1/4}\,a\,\mathrm{atan}\left(\frac{(-1)^{1/4}\,\sqrt{e\cot(c+d\,x)}}{\sqrt{e}}\right)\,(-1 + 1\mathrm{i})}{d\,\sqrt{e}} + \frac{(-1)^{1/4}\,a\,\mathrm{atanh}\left(\frac{(-1)^{1/4}\,\sqrt{e\cot(c+d\,x)}}{\sqrt{e}}\right)\,(1 + 1\mathrm{i})}{d\,\sqrt{e}}$$

```
[In] int((a + a*cot(c + d*x))/(e*cot(c + d*x))^(1/2),x)
```

```
[Out] ((-1)^(1/4)*a*atanh(((-1)^(1/4)*(e*cot(c + d*x))^(1/2))/e^(1/2))*(1 + 1i))/
(d*e^(1/2)) - ((-1)^(1/4)*a*atan(((-1)^(1/4)*(e*cot(c + d*x))^(1/2))/e^(1/2
))*(1 - 1i))/(d*e^(1/2))
```

3.6 $\int \frac{a+a\cot(c+dx)}{(e\cot(c+dx))^{3/2}}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 75

$$\int \frac{a+a\cot(c+dx)}{(e\cot(c+dx))^{3/2}}\,dx = -\frac{\sqrt{2}a\operatorname{arctanh}\left(\frac{\sqrt{e}+\sqrt{e}\cot(c+dx)}{\sqrt{2}\sqrt{e\cot(c+dx)}}\right)}{de^{3/2}}+\frac{2a}{de\sqrt{e\cot(c+dx)}}$$

```
[Out] -a*arctanh(1/2*(e^(1/2)+cot(d*x+c)*e^(1/2))*2^(1/2)/(e*cot(d*x+c))^(1/2))*2
^(1/2)/d/e^(3/2)+2*a/d/e/(e*cot(d*x+c))^(1/2)
```

Rubi [A] (verified)

Time = 0.11 (sec) , antiderivative size = 75, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.130, Rules used = {3610, 3613, 214}

$$\int \frac{a+a\cot(c+dx)}{(e\cot(c+dx))^{3/2}}\,dx = \frac{2a}{de\sqrt{e\cot(c+dx)}}-\frac{\sqrt{2}a\operatorname{arctanh}\left(\frac{\sqrt{e}\cot(c+dx)+\sqrt{e}}{\sqrt{2}\sqrt{e\cot(c+dx)}}\right)}{de^{3/2}}$$

```
[In] Int[(a + a*Cot[c + d*x])/(e*Cot[c + d*x])^(3/2),x]
```

```
[Out] -((Sqrt[2]*a*ArcTanh[(Sqrt[e] + Sqrt[e]*Cot[c + d*x])/(Sqrt[2]*Sqrt[e*Cot[c
 + d*x]])])/(d*e^(3/2))) + (2*a)/(d*e*Sqrt[e*Cot[c + d*x]])
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 3610

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)]), x_Symbol] :> Simp[(b*c - a*d)*((a + b*Tan[e + f*x])^(m + 1)/
(f*(m + 1)*(a^2 + b^2))), x] + Dist[1/(a^2 + b^2), Int[(a + b*Tan[e + f*x])
^(m + 1)*Simp[a*c + b*d - (b*c - a*d)*Tan[e + f*x], x], x], x] /; FreeQ[{a,
 b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && LtQ[m, -1]
```

Rule 3613

```
Int[((c_) + (d_.)*tan[(e_.) + (f_.)*(x_)])/Sqrt[(b_.)*tan[(e_.) + (f_.)*(x_
)]], x_Symbol] :> Dist[-2*(d^2/f), Subst[Int[1/(2*c*d + b*x^2), x], x, (c -
 d*Tan[e + f*x])/Sqrt[b*Tan[e + f*x]]], x] /; FreeQ[{b, c, d, e, f}, x] &&
EqQ[c^2 - d^2, 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{2a}{de\sqrt{e\cot(c+dx)}} + \frac{\int \frac{ae-ae\cot(c+dx)}{\sqrt{e\cot(c+dx)}}\,dx}{e^2} \\
&= \frac{2a}{de\sqrt{e\cot(c+dx)}} - \frac{(2a^2)\,\text{Subst}\left(\int \frac{1}{2a^2e^2-ex^2}\,dx, x, \frac{ae+ae\cot(c+dx)}{\sqrt{e\cot(c+dx)}}\right)}{d} \\
&= -\frac{\sqrt{2}a\text{arctanh}\left(\frac{\sqrt{e}+\sqrt{e}\cot(c+dx)}{\sqrt{2}\sqrt{e\cot(c+dx)}}\right)}{de^{3/2}} + \frac{2a}{de\sqrt{e\cot(c+dx)}}
\end{aligned}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 0.27 (sec) , antiderivative size = 191, normalized size of antiderivative = 2.55

$$\int \frac{a+a\cot(c+dx)}{(e\cot(c+dx))^{3/2}}\,dx = \frac{a\left(6\sqrt{2}\arctan\left(1-\sqrt{2}\sqrt{\tan(c+dx)}\right)-6\sqrt{2}\arctan\left(1+\sqrt{2}\sqrt{\tan(c+dx)}\right)+3\cdots\right.}{}$$

```
[In] Integrate[(a + a*Cot[c + d*x])/(e*Cot[c + d*x])^(3/2),x]
```

```
[Out] (a*(6*Sqrt[2]*ArcTan[1 - Sqrt[2]*Sqrt[Tan[c + d*x]]] - 6*Sqrt[2]*ArcTan[1 +
 Sqrt[2]*Sqrt[Tan[c + d*x]]] + 3*Sqrt[2]*Log[1 - Sqrt[2]*Sqrt[Tan[c + d*x]]
 + Tan[c + d*x]] - 3*Sqrt[2]*Log[1 + Sqrt[2]*Sqrt[Tan[c + d*x]] + Tan[c + d
*x]] + 24*Sqrt[Tan[c + d*x]] + 8*Hypergeometric2F1[3/4, 1, 7/4, -Tan[c + d*
x]^2]*Tan[c + d*x]^(3/2)))/(12*d*(e*Cot[c + d*x])^(3/2)*Tan[c + d*x]^(3/2))
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 293 vs. $2(62) = 124$.

Time = 0.05 (sec) , antiderivative size = 294, normalized size of antiderivative = 3.92

method	result
derivativedivides	$-\frac{a\left(\frac{\left(e^{2}\right)^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\frac{e\cot(dx+c)+\left(e^{2}\right)^{\frac{1}{4}}\sqrt{e\cot(dx+c)}\,\sqrt{2}+\sqrt{e^{2}}}{e\cot(dx+c)-\left(e^{2}\right)^{\frac{1}{4}}\sqrt{e\cot(dx+c)}\,\sqrt{2}+\sqrt{e^{2}}}\right)+2\arctan\left(\frac{\sqrt{2}\,\sqrt{e\cot(dx+c)}}{\left(e^{2}\right)^{\frac{1}{4}}}+1\right)-2\arctan\left(-\frac{\sqrt{2}\,\sqrt{e\cot(dx+c)}}{\left(e^{2}\right)^{\frac{1}{4}}}\right.\right.}{4e}}{}}{}$
default	$-\frac{a\left(\frac{\left(e^{2}\right)^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\frac{e\cot(dx+c)+\left(e^{2}\right)^{\frac{1}{4}}\sqrt{e\cot(dx+c)}\,\sqrt{2}+\sqrt{e^{2}}}{e\cot(dx+c)-\left(e^{2}\right)^{\frac{1}{4}}\sqrt{e\cot(dx+c)}\,\sqrt{2}+\sqrt{e^{2}}}\right)+2\arctan\left(\frac{\sqrt{2}\,\sqrt{e\cot(dx+c)}}{\left(e^{2}\right)^{\frac{1}{4}}}+1\right)-2\arctan\left(-\frac{\sqrt{2}\,\sqrt{e\cot(dx+c)}}{\left(e^{2}\right)^{\frac{1}{4}}}\right.\right.}{4e}}{}}{}$
parts	$-\frac{2ae\left(-\frac{\sqrt{2}\left(\ln\left(\frac{e\cot(dx+c)-\left(e^{2}\right)^{\frac{1}{4}}\sqrt{e\cot(dx+c)}\,\sqrt{2}+\sqrt{e^{2}}}{e\cot(dx+c)+\left(e^{2}\right)^{\frac{1}{4}}\sqrt{e\cot(dx+c)}\,\sqrt{2}+\sqrt{e^{2}}}\right)+2\arctan\left(\frac{\sqrt{2}\,\sqrt{e\cot(dx+c)}}{\left(e^{2}\right)^{\frac{1}{4}}}+1\right)-2\arctan\left(-\frac{\sqrt{2}\,\sqrt{e\cot(dx+c)}}{\left(e^{2}\right)^{\frac{1}{4}}}+\right.\right.}{8e^{2}\left(e^{2}\right)^{\frac{1}{4}}}\right.}{d}$

```
[In] int((a+a*cot(d*x+c))/(e*cot(d*x+c))^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/d*a*(2/e*(1/8/e*(e^2)^(1/4)*2^(1/2)*(ln((e*cot(d*x+c)+(e^2)^(1/4)*(e*cot
(d*x+c))^(1/2)*2^(1/2)+(e^2)^(1/2))/(e*cot(d*x+c)-(e^2)^(1/4)*(e*cot(d*x+c)
)^(1/2)*2^(1/2)+(e^2)^(1/2)))+2*arctan(2^(1/2)/(e^2)^(1/4)*(e*cot(d*x+c))^(
1/2)+1)-2*arctan(-2^(1/2)/(e^2)^(1/4)*(e*cot(d*x+c))^(1/2)+1))-1/8/(e^2)^(1
/4)*2^(1/2)*(ln((e*cot(d*x+c)-(e^2)^(1/4)*(e*cot(d*x+c))^(1/2)*2^(1/2)+(e^2
)^(1/2))/(e*cot(d*x+c)+(e^2)^(1/4)*(e*cot(d*x+c))^(1/2)*2^(1/2)+(e^2)^(1/2)
))+2*arctan(2^(1/2)/(e^2)^(1/4)*(e*cot(d*x+c))^(1/2)+1)-2*arctan(-2^(1/2)/(
e^2)^(1/4)*(e*cot(d*x+c))^(1/2)+1)))-2/e/(e*cot(d*x+c))^(1/2))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 163 vs. $2(62) = 124$.

Time = 0.27 (sec) , antiderivative size = 321, normalized size of antiderivative = 4.28

$$\int \frac{a + a\cot(c+dx)}{(e\cot(c+dx))^{3/2}}\,dx = \left[\frac{4\,a\sqrt{\frac{e\cos(2\,dx+2\,c)+e}{\sin(2\,dx+2\,c)}}\sin(2\,dx+2\,c) + \frac{\sqrt{2}(ae\cos(2\,dx+2\,c)+ae)\log\left(\frac{\sqrt{2}\sqrt{\frac{e\cos(2\,dx+2\,c)+e}{\sin(2\,dx+2\,c)}}(\cos(2\,dx}{\sqrt{e}}\right.}{\sqrt{e}}}{2\left(de^2\cos(2\,dx+2\,c)+de^2\right)}\right.$$

```
[In] integrate((a+a*cot(d*x+c))/(e*cot(d*x+c))^(3/2),x, algorithm="fricas")
```

```
[Out] [1/2*(4*a*sqrt((e*cos(2*d*x + 2*c) + e)/sin(2*d*x + 2*c))*sin(2*d*x + 2*c)
+ sqrt(2)*(a*e*cos(2*d*x + 2*c) + a*e)*log(sqrt(2)*sqrt((e*cos(2*d*x + 2*c)
 + e)/sin(2*d*x + 2*c))*(cos(2*d*x + 2*c) - sin(2*d*x + 2*c) - 1)/sqrt(e) +
 2*sin(2*d*x + 2*c) + 1)/sqrt(e))/(d*e^2*cos(2*d*x + 2*c) + d*e^2), (sqrt(2
)*(a*e*cos(2*d*x + 2*c) + a*e)*sqrt(-1/e)*arctan(1/2*sqrt(2)*sqrt((e*cos(2*
d*x + 2*c) + e)/sin(2*d*x + 2*c))*sqrt(-1/e)*(cos(2*d*x + 2*c) + sin(2*d*x
+ 2*c) + 1)/(cos(2*d*x + 2*c) + 1)) + 2*a*sqrt((e*cos(2*d*x + 2*c) + e)/sin
(2*d*x + 2*c))*sin(2*d*x + 2*c))/(d*e^2*cos(2*d*x + 2*c) + d*e^2)]
```

Sympy [F]

$$\int \frac{a + a\cot(c+dx)}{(e\cot(c+dx))^{3/2}}\,dx = a\left(\int \frac{1}{(e\cot(c+dx))^{\frac{3}{2}}}\,dx + \int \frac{\cot(c+dx)}{(e\cot(c+dx))^{\frac{3}{2}}}\,dx\right)$$

```
[In] integrate((a+a*cot(d*x+c))/(e*cot(d*x+c))**(3/2),x)
```

```
[Out] a*(Integral((e*cot(c + d*x))**(-3/2), x) + Integral(cot(c + d*x)/(e*cot(c +
 d*x))**(3/2), x))
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{a + a\cot(c+dx)}{(e\cot(c+dx))^{3/2}}\,dx = \text{Exception raised: ValueError}$$

```
[In] integrate((a+a*cot(d*x+c))/(e*cot(d*x+c))^(3/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(e>0)', see `assume?` for more detai
ls)Is e
```

Giac [F]

$$\int \frac{a+a\cot(c+dx)}{(e\cot(c+dx))^{3/2}}\,dx=\int \frac{a\cot{(dx+c)}+a}{(e\cot{(dx+c)})^{\frac{3}{2}}}\,dx$$

```
[In] integrate((a+a*cot(d*x+c))/(e*cot(d*x+c))^(3/2),x, algorithm="giac")
```

```
[Out] integrate((a*cot(d*x + c) + a)/(e*cot(d*x + c))^(3/2), x)
```

Mupad [B] (verification not implemented)

Time = 13.33 (sec) , antiderivative size = 84, normalized size of antiderivative = 1.12

$$\int \frac{a+a\cot(c+dx)}{(e\cot(c+dx))^{3/2}}\,dx=\frac{2\,a}{d\,e\,\sqrt{e\cot{(c+d\,x)}}}+\frac{{(-1)}^{1/4}\,a\,\mathrm{atan}\left(\frac{{(-1)}^{1/4}\,\sqrt{e\cot(c+d\,x)}}{\sqrt{e}}\right)\,(1+1\mathrm{i})}{d\,e^{3/2}}+\frac{{(-1)}^{1/4}\,a\,\mathrm{atanh}\left(\frac{{(-1)}^{1/4}\,\sqrt{e\cot(c+d\,x)}}{\sqrt{e}}\right)\,(-1+1\mathrm{i})}{d\,e^{3/2}}$$

```
[In] int((a + a*cot(c + d*x))/(e*cot(c + d*x))^(3/2),x)
```

```
[Out] (2*a)/(d*e*(e*cot(c + d*x))^(1/2)) + ((-1)^(1/4)*a*atan(((-1)^(1/4)*(e*cot(
c + d*x))^(1/2))/e^(1/2))*(1 + 1i))/(d*e^(3/2)) - ((-1)^(1/4)*a*atanh(((-1)
^(1/4)*(e*cot(c + d*x))^(1/2))/e^(1/2))*(1 - 1i))/(d*e^(3/2))
```

3.7 $\int \frac{a+a\cot(c+dx)}{(e\cot(c+dx))^{5/2}}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 99

$$\int \frac{a+a\cot(c+dx)}{(e\cot(c+dx))^{5/2}}\,dx = -\frac{\sqrt{2}a\arctan\left(\frac{\sqrt{e}-\sqrt{e}\cot(c+dx)}{\sqrt{2}\sqrt{e\cot(c+dx)}}\right)}{de^{5/2}} + \frac{2a}{3de(e\cot(c+dx))^{3/2}} + \frac{2a}{de^2\sqrt{e\cot(c+dx)}}$$

```
[Out] 2/3*a/d/e/(e*cot(d*x+c))^(3/2)-a*arctan(1/2*(e^(1/2)-cot(d*x+c)*e^(1/2))*2^
(1/2)/(e*cot(d*x+c))^(1/2))*2^(1/2)/d/e^(5/2)+2*a/d/e^2/(e*cot(d*x+c))^(1/2
)
```

Rubi [A] (verified)

Time = 0.15 (sec) , antiderivative size = 99, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.130$, Rules used = {3610, 3613, 211}

$$\int \frac{a+a\cot(c+dx)}{(e\cot(c+dx))^{5/2}}\,dx = -\frac{\sqrt{2}a\arctan\left(\frac{\sqrt{e}-\sqrt{e}\cot(c+dx)}{\sqrt{2}\sqrt{e\cot(c+dx)}}\right)}{de^{5/2}} + \frac{2a}{de^2\sqrt{e\cot(c+dx)}} + \frac{2a}{3de(e\cot(c+dx))^{3/2}}$$

```
[In] Int[(a + a*Cot[c + d*x])/(e*Cot[c + d*x])^(5/2),x]
```

```
[Out] -((Sqrt[2]*a*ArcTan[(Sqrt[e] - Sqrt[e]*Cot[c + d*x])/(Sqrt[2]*Sqrt[e*Cot[c
+ d*x]])])/(d*e^(5/2))) + (2*a)/(3*d*e*(e*Cot[c + d*x])^(3/2)) + (2*a)/(d*e
^2*Sqrt[e*Cot[c + d*x]])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 3610

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)]), x_Symbol] :> Simp[(b*c - a*d)*((a + b*Tan[e + f*x])^(m + 1)/
(f*(m + 1)*(a^2 + b^2))), x] + Dist[1/(a^2 + b^2), Int[(a + b*Tan[e + f*x])
^(m + 1)*Simp[a*c + b*d - (b*c - a*d)*Tan[e + f*x], x], x], x] /; FreeQ[{a,
 b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && LtQ[m, -1]
```

Rule 3613

```
Int[((c_) + (d_.)*tan[(e_.) + (f_.)*(x_)])/Sqrt[(b_.)*tan[(e_.) + (f_.)*(x_
)]], x_Symbol] :> Dist[-2*(d^2/f), Subst[Int[1/(2*c*d + b*x^2), x], x, (c -
 d*Tan[e + f*x])/Sqrt[b*Tan[e + f*x]]], x] /; FreeQ[{b, c, d, e, f}, x] &&
EqQ[c^2 - d^2, 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{2a}{3de(e\cot(c+dx))^{3/2}} + \frac{\int \frac{ae-ae\cot(c+dx)}{(e\cot(c+dx))^{3/2}}\,dx}{e^2} \\
&= \frac{2a}{3de(e\cot(c+dx))^{3/2}} + \frac{2a}{de^2\sqrt{e\cot(c+dx)}} + \frac{\int \frac{-ae^2-ae^2\cot(c+dx)}{\sqrt{e\cot(c+dx)}}\,dx}{e^4} \\
&= \frac{2a}{3de(e\cot(c+dx))^{3/2}} + \frac{2a}{de^2\sqrt{e\cot(c+dx)}} \\
&\quad - \frac{(2a^2)\,\text{Subst}\left(\int \frac{1}{-2a^2e^4-ex^2}\,dx, x, \frac{-ae^2+ae^2\cot(c+dx)}{\sqrt{e\cot(c+dx)}}\right)}{d} \\
&= -\frac{\sqrt{2}a\arctan\left(\frac{\sqrt{e}-\sqrt{e}\cot(c+dx)}{\sqrt{2}\sqrt{e\cot(c+dx)}}\right)}{de^{5/2}} + \frac{2a}{3de(e\cot(c+dx))^{3/2}} + \frac{2a}{de^2\sqrt{e\cot(c+dx)}}
\end{aligned}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 0.44 (sec) , antiderivative size = 203, normalized size of antiderivative = 2.05

$$\int \frac{a+a\cot(c+dx)}{(e\cot(c+dx))^{5/2}}\,dx = \frac{a\left(6\sqrt{2}\arctan\left(1-\sqrt{2}\sqrt{\tan(c+dx)}\right) - 6\sqrt{2}\arctan\left(1+\sqrt{2}\sqrt{\tan(c+dx)}\right) + \right.}{}$$

```
[In] Integrate[(a + a*Cot[c + d*x])/(e*Cot[c + d*x])^(5/2),x]
```

```
[Out] (a*(6*Sqrt[2]*ArcTan[1 - Sqrt[2]*Sqrt[Tan[c + d*x]]] - 6*Sqrt[2]*ArcTan[1 +
 Sqrt[2]*Sqrt[Tan[c + d*x]]] + 3*Sqrt[2]*Log[1 - Sqrt[2]*Sqrt[Tan[c + d*x]]
 + Tan[c + d*x]] - 3*Sqrt[2]*Log[1 + Sqrt[2]*Sqrt[Tan[c + d*x]] + Tan[c + d
*x]] + 24*Sqrt[Tan[c + d*x]] + 8*Tan[c + d*x]^(3/2) - 8*Hypergeometric2F1[3
/4, 1, 7/4, -Tan[c + d*x]^2]*Tan[c + d*x]^(3/2)))/(12*d*(e*Cot[c + d*x])^(5
/2)*Tan[c + d*x]^(5/2))
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 308 vs. $2(82) = 164$.

Time = 0.05 (sec) , antiderivative size = 309, normalized size of antiderivative = 3.12

method	result
derivativedivides	$-\frac{a\left(-\frac{(e^2)^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\frac{e\cot(dx+c)+(e^2)^{\frac{1}{4}}\sqrt{e\cot(dx+c)}\sqrt{2}+\sqrt{e^2}}{e\cot(dx+c)-(e^2)^{\frac{1}{4}}\sqrt{e\cot(dx+c)}\sqrt{2}+\sqrt{e^2}}\right)+2\arctan\left(\frac{\sqrt{2}\sqrt{e\cot(dx+c)}}{(e^2)^{\frac{1}{4}}}+1\right)-2\arctan\left(-\frac{\sqrt{2}\sqrt{e\cot(dx+c}}{(e^2)^{\frac{1}{4}}}\right.\right.}{4e}\right.}{}$
default	$-\frac{a\left(-\frac{(e^2)^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\frac{e\cot(dx+c)+(e^2)^{\frac{1}{4}}\sqrt{e\cot(dx+c)}\sqrt{2}+\sqrt{e^2}}{e\cot(dx+c)-(e^2)^{\frac{1}{4}}\sqrt{e\cot(dx+c)}\sqrt{2}+\sqrt{e^2}}\right)+2\arctan\left(\frac{\sqrt{2}\sqrt{e\cot(dx+c)}}{(e^2)^{\frac{1}{4}}}+1\right)-2\arctan\left(-\frac{\sqrt{2}\sqrt{e\cot(dx+c}}{(e^2)^{\frac{1}{4}}}\right.\right.}{4e}\right.}{}$
parts	$-\frac{2ae\left(-\frac{(e^2)^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\frac{e\cot(dx+c)+(e^2)^{\frac{1}{4}}\sqrt{e\cot(dx+c)}\sqrt{2}+\sqrt{e^2}}{e\cot(dx+c)-(e^2)^{\frac{1}{4}}\sqrt{e\cot(dx+c)}\sqrt{2}+\sqrt{e^2}}\right)+2\arctan\left(\frac{\sqrt{2}\sqrt{e\cot(dx+c)}}{(e^2)^{\frac{1}{4}}}+1\right)-2\arctan\left(-\frac{\sqrt{2}\sqrt{e\cot(dx}}{(e^2)^{\frac{1}{4}}}\right.\right.}{8e^4}\right.}{d}$

```
[In] int((a+a*cot(d*x+c))/(e*cot(d*x+c))^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/d*a*(2/e^2*(-1/8/e*(e^2)^(1/4)*2^(1/2)*(ln((e*cot(d*x+c)+(e^2)^(1/4)*(e*
cot(d*x+c))^(1/2)*2^(1/2)+(e^2)^(1/2))/(e*cot(d*x+c)-(e^2)^(1/4)*(e*cot(d*x
+c))^(1/2)*2^(1/2)+(e^2)^(1/2)))+2*arctan(2^(1/2)/(e^2)^(1/4)*(e*cot(d*x+c)
)^(1/2)+1)-2*arctan(-2^(1/2)/(e^2)^(1/4)*(e*cot(d*x+c))^(1/2)+1))-1/8/(e^2)
^(1/4)*2^(1/2)*(ln((e*cot(d*x+c)-(e^2)^(1/4)*(e*cot(d*x+c))^(1/2)*2^(1/2)+(
e^2)^(1/2))/(e*cot(d*x+c)+(e^2)^(1/4)*(e*cot(d*x+c))^(1/2)*2^(1/2)+(e^2)^(1
/2)))+2*arctan(2^(1/2)/(e^2)^(1/4)*(e*cot(d*x+c))^(1/2)+1)-2*arctan(-2^(1/2
)/(e^2)^(1/4)*(e*cot(d*x+c))^(1/2)+1)))-2/e^2/(e*cot(d*x+c))^(1/2)-2/3/e/(e
*cot(d*x+c))^(3/2))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 176 vs. 2(83) = 166.

Time = 0.28 (sec) , antiderivative size = 358, normalized size of antiderivative = 3.62

$$\int \frac{a+a\cot(c+dx)}{(e\cot(c+dx))^{5/2}}\,dx = \left[\frac{3\sqrt{2}(ae\cos(2\,dx+2\,c)+ae)\sqrt{-\frac{1}{e}}\log\left(\sqrt{2}\sqrt{\frac{e\cos(2\,dx+2\,c)+e}{\sin(2\,dx+2\,c)}}\sqrt{-\frac{1}{e}}(\cos(2\,dx+2\right.}{}\right.$$

$$-\frac{\frac{3\sqrt{2}(ae\cos(2\,dx+2\,c)+ae)\arctan\left(-\frac{\sqrt{2}\sqrt{\frac{e\cos(2\,dx+2\,c)+e}{\sin(2\,dx+2\,c)}}(\cos(2\,dx+2\,c)-\sin(2\,dx+2\,c)+1)}{2\sqrt{e}(\cos(2\,dx+2\,c)+1)}\right)}{\sqrt{e}}+2\,(a\cos(2\,dx+2\,c)-3\,a\sin(2\,dx+}{3\,(de^3\cos(2\,dx+2\,c)+de^3)}$$

```
[In] integrate((a+a*cot(d*x+c))/(e*cot(d*x+c))^(5/2),x, algorithm="fricas")
```

```
[Out] [1/6*(3*sqrt(2)*(a*e*cos(2*d*x + 2*c) + a*e)*sqrt(-1/e)*log(sqrt(2)*sqrt((e
*cos(2*d*x + 2*c) + e)/sin(2*d*x + 2*c))*sqrt(-1/e)*(cos(2*d*x + 2*c) + sin
(2*d*x + 2*c) - 1) - 2*sin(2*d*x + 2*c) + 1) - 4*(a*cos(2*d*x + 2*c) - 3*a*
sin(2*d*x + 2*c) - a)*sqrt((e*cos(2*d*x + 2*c) + e)/sin(2*d*x + 2*c)))/(d*e
^3*cos(2*d*x + 2*c) + d*e^3), -1/3*(3*sqrt(2)*(a*e*cos(2*d*x + 2*c) + a*e)*
arctan(-1/2*sqrt(2)*sqrt((e*cos(2*d*x + 2*c) + e)/sin(2*d*x + 2*c))*(cos(2*
d*x + 2*c) - sin(2*d*x + 2*c) + 1)/(sqrt(e)*(cos(2*d*x + 2*c) + 1)))/sqrt(e
) + 2*(a*cos(2*d*x + 2*c) - 3*a*sin(2*d*x + 2*c) - a)*sqrt((e*cos(2*d*x + 2
*c) + e)/sin(2*d*x + 2*c)))/(d*e^3*cos(2*d*x + 2*c) + d*e^3)]
```

Sympy [F]

$$\int \frac{a+a\cot(c+dx)}{(e\cot(c+dx))^{5/2}}\,dx = a\left(\int \frac{1}{(e\cot(c+dx))^{\frac{5}{2}}}\,dx+\int \frac{\cot(c+dx)}{(e\cot(c+dx))^{\frac{5}{2}}}\,dx\right)$$

```
[In] integrate((a+a*cot(d*x+c))/(e*cot(d*x+c))**(5/2),x)
```

```
[Out] a*(Integral((e*cot(c + d*x))**(-5/2), x) + Integral(cot(c + d*x)/(e*cot(c +
 d*x))**(5/2), x))
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{a + a\cot(c + dx)}{(e\cot(c + dx))^{5/2}}\, dx = \text{Exception raised: ValueError}$$

```
[In] integrate((a+a*cot(d*x+c))/(e*cot(d*x+c))^(5/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(e>0)', see `assume?` for more detai
ls)Is e
```

Giac [F]

$$\int \frac{a + a\cot(c + dx)}{(e\cot(c + dx))^{5/2}}\, dx = \int \frac{a\cot(dx + c) + a}{(e\cot(dx + c))^{\frac{5}{2}}}\, dx$$

```
[In] integrate((a+a*cot(d*x+c))/(e*cot(d*x+c))^(5/2),x, algorithm="giac")
```

```
[Out] integrate((a*cot(d*x + c) + a)/(e*cot(d*x + c))^(5/2), x)
```

Mupad [B] (verification not implemented)

Time = 13.90 (sec) , antiderivative size = 103, normalized size of antiderivative = 1.04

$$\int \frac{a + a\cot(c + dx)}{(e\cot(c + dx))^{5/2}}\, dx = \frac{2a}{d\,e^2\sqrt{e\cot(c + d\,x)}} + \frac{2a}{3\,d\,e\,(e\cot(c + d\,x))^{3/2}} + \frac{(-1)^{1/4}\,a\,\mathrm{atan}\left(\frac{(-1)^{1/4}\sqrt{e\cot(c+d\,x)}}{\sqrt{e}}\right)\,(1 - \mathrm{i})}{d\,e^{5/2}} + \frac{(-1)^{1/4}\,a\,\mathrm{atanh}\left(\frac{(-1)^{1/4}\sqrt{e\cot(c+d\,x)}}{\sqrt{e}}\right)\,(-1 - \mathrm{i})}{d\,e^{5/2}}$$

```
[In] int((a + a*cot(c + d*x))/(e*cot(c + d*x))^(5/2),x)
```

```
[Out] (2*a)/(d*e^2*(e*cot(c + d*x))^(1/2)) + (2*a)/(3*d*e*(e*cot(c + d*x))^(3/2))
 + ((-1)^(1/4)*a*atan(((-1)^(1/4)*(e*cot(c + d*x))^(1/2))/e^(1/2))*(1 - 1i)
)/(d*e^(5/2)) - ((-1)^(1/4)*a*atanh(((-1)^(1/4)*(e*cot(c + d*x))^(1/2))/e^(
1/2))*(1 + 1i))/(d*e^(5/2))
```

3.8 $\int (e\cot(c+dx))^{5/2}(a+a\cot(c+dx))^2\,dx$

Optimal result

Integrand size = 25, antiderivative size = 269

$$\int (e\cot(c+dx))^{5/2}(a+a\cot(c+dx))^2\,dx=\frac{\sqrt{2}a^2e^{5/2}\arctan\left(1-\frac{\sqrt{2}\sqrt{e\cot(c+dx)}}{\sqrt{e}}\right)}{d}-\frac{\sqrt{2}a^2e^{5/2}\arctan\left(1+\frac{\sqrt{2}\sqrt{e\cot(c+dx)}}{\sqrt{e}}\right)}{d}+\frac{4a^2e^2\sqrt{e\cot(c+dx)}}{d}-\frac{4a^2(e\cot(c+dx))^{5/2}}{5d}-\frac{2a^2(e\cot(c+dx))^{7/2}}{7de}+\frac{a^2e^{5/2}\log\left(\sqrt{e}+\sqrt{e}\cot(c+dx)-\sqrt{2}\sqrt{e\cot(c+dx)}\right)}{\sqrt{2}d}-\frac{a^2e^{5/2}\log\left(\sqrt{e}+\sqrt{e}\cot(c+dx)+\sqrt{2}\sqrt{e\cot(c+dx)}\right)}{\sqrt{2}d}$$

```
[Out] -4/5*a^2*(e*cot(d*x+c))^(5/2)/d-2/7*a^2*(e*cot(d*x+c))^(7/2)/d/e+1/2*a^2*e^
(5/2)*ln(e^(1/2)+cot(d*x+c)*e^(1/2)-2^(1/2)*(e*cot(d*x+c))^(1/2))/d*2^(1/2)
-1/2*a^2*e^(5/2)*ln(e^(1/2)+cot(d*x+c)*e^(1/2)+2^(1/2)*(e*cot(d*x+c))^(1/2)
)/d*2^(1/2)+a^2*e^(5/2)*arctan(1-2^(1/2)*(e*cot(d*x+c))^(1/2)/e^(1/2))*2^(1
/2)/d-a^2*e^(5/2)*arctan(1+2^(1/2)*(e*cot(d*x+c))^(1/2)/e^(1/2))*2^(1/2)/d+
4*a^2*e^2*(e*cot(d*x+c))^(1/2)/d
```

Rubi [A] (verified)

Time = 0.36 (sec) , antiderivative size = 269, normalized size of antiderivative = 1.00, number of steps used = 16, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.480, Rules used = {3624, 12, 16, 3554, 3557, 335, 217, 1179, 642, 1176, 631, 210}

$$\int (e\cot(c+dx))^{5/2}(a+a\cot(c+dx))^2\,dx = \frac{\sqrt{2}a^2e^{5/2}\arctan\left(1-\frac{\sqrt{2}\sqrt{e\cot(c+dx)}}{\sqrt{e}}\right)}{d} - \frac{\sqrt{2}a^2e^{5/2}\arctan\left(\frac{\sqrt{2}\sqrt{e\cot(c+dx)}}{\sqrt{e}}+1\right)}{d} + \frac{a^2e^{5/2}\log\left(\sqrt{e}\cot(c+dx)-\sqrt{2}\sqrt{e\cot(c+dx)}+\sqrt{e}\right)}{\sqrt{2}d} - \frac{a^2e^{5/2}\log\left(\sqrt{e}\cot(c+dx)+\sqrt{2}\sqrt{e\cot(c+dx)}+\sqrt{e}\right)}{\sqrt{2}d} + \frac{4a^2e^2\sqrt{e\cot(c+dx)}}{d} - \frac{2a^2(e\cot(c+dx))^{7/2}}{7de} - \frac{4a^2(e\cot(c+dx))^{5/2}}{5d}$$

```
[In] Int[(e*Cot[c + d*x])^(5/2)*(a + a*Cot[c + d*x])^2,x]
```

```
[Out] (Sqrt[2]*a^2*e^(5/2)*ArcTan[1 - (Sqrt[2]*Sqrt[e*Cot[c + d*x]])/Sqrt[e]])/d
- (Sqrt[2]*a^2*e^(5/2)*ArcTan[1 + (Sqrt[2]*Sqrt[e*Cot[c + d*x]])/Sqrt[e]])/
d + (4*a^2*e^2*Sqrt[e*Cot[c + d*x]])/d - (4*a^2*(e*Cot[c + d*x])^(5/2))/(5*
d) - (2*a^2*(e*Cot[c + d*x])^(7/2))/(7*d*e) + (a^2*e^(5/2)*Log[Sqrt[e] + Sq
rt[e]*Cot[c + d*x] - Sqrt[2]*Sqrt[e*Cot[c + d*x]]])/(Sqrt[2]*d) - (a^2*e^(5
/2)*Log[Sqrt[e] + Sqrt[e]*Cot[c + d*x] + Sqrt[2]*Sqrt[e*Cot[c + d*x]]])/(Sq
rt[2]*d)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 16

```
Int[(u_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Dist[1/b^m, Int[u*(b*v)
^(m + n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4),
 x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b
}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &&
AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1179

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 3554

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[b*((b*Tan[c + d
*x])^(n - 1)/(d*(n - 1))), x] - Dist[b^2, Int[(b*Tan[c + d*x])^(n - 2), x],
 x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
```

Rule 3557

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[b/d, Subst[Int[
x^n/(b^2 + x^2), x], x, b*Tan[c + d*x]], x] /; FreeQ[{b, c, d, n}, x] &&  !
IntegerQ[n]
```

Rule 3624

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)])^2, x_Symbol] :> Simp[d^2*((a + b*Tan[e + f*x])^(m + 1)/(b*f*(
m + 1))), x] + Int[(a + b*Tan[e + f*x])^m*Simp[c^2 - d^2 + 2*c*d*Tan[e + f*
x], x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b*c - a*d, 0] &&  !LeQ
[m, -1] &&  !(EqQ[m, 2] && EqQ[a, 0])
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= -\frac{2a^2(e\cot(c+dx))^{7/2}}{7de} + \int 2a^2\cot(c+dx)(e\cot(c+dx))^{5/2}\,dx \\
&= -\frac{2a^2(e\cot(c+dx))^{7/2}}{7de} + \left(2a^2\right)\int \cot(c+dx)(e\cot(c+dx))^{5/2}\,dx \\
&= -\frac{2a^2(e\cot(c+dx))^{7/2}}{7de} + \frac{\left(2a^2\right)\int (e\cot(c+dx))^{7/2}\,dx}{e} \\
&= -\frac{4a^2(e\cot(c+dx))^{5/2}}{5d} - \frac{2a^2(e\cot(c+dx))^{7/2}}{7de} - \left(2a^2e\right)\int (e\cot(c+dx))^{3/2}\,dx \\
&= \frac{4a^2e^2\sqrt{e\cot(c+dx)}}{d} - \frac{4a^2(e\cot(c+dx))^{5/2}}{5d} \\
&\quad - \frac{2a^2(e\cot(c+dx))^{7/2}}{7de} + \left(2a^2e^3\right)\int \frac{1}{\sqrt{e\cot(c+dx)}}\,dx \\
&= \frac{4a^2e^2\sqrt{e\cot(c+dx)}}{d} - \frac{4a^2(e\cot(c+dx))^{5/2}}{5d} - \frac{2a^2(e\cot(c+dx))^{7/2}}{7de} \\
&\quad - \frac{\left(2a^2e^4\right)\operatorname{Subst}\left(\int \frac{1}{\sqrt{x}(e^2+x^2)}\,dx, x, e\cot(c+dx)\right)}{d} \\
&= \frac{4a^2e^2\sqrt{e\cot(c+dx)}}{d} - \frac{4a^2(e\cot(c+dx))^{5/2}}{5d} - \frac{2a^2(e\cot(c+dx))^{7/2}}{7de} \\
&\quad - \frac{\left(4a^2e^4\right)\operatorname{Subst}\left(\int \frac{1}{e^2+x^4}\,dx, x, \sqrt{e\cot(c+dx)}\right)}{d}
\end{aligned}
$$

$$= \frac{4a^2e^2\sqrt{e\cot(c+dx)}}{d} - \frac{4a^2(e\cot(c+dx))^{5/2}}{5d} - \frac{2a^2(e\cot(c+dx))^{7/2}}{7de} - \frac{(2a^2e^3)\,\mathrm{Subst}\left(\int \frac{e-x^2}{e^2+x^4}\,dx, x, \sqrt{e\cot(c+dx)}\right)}{d} - \frac{(2a^2e^3)\,\mathrm{Subst}\left(\int \frac{e+x^2}{e^2+x^4}\,dx, x, \sqrt{e\cot(c+dx)}\right)}{d}$$

$$= \frac{4a^2e^2\sqrt{e\cot(c+dx)}}{d} - \frac{4a^2(e\cot(c+dx))^{5/2}}{5d} - \frac{2a^2(e\cot(c+dx))^{7/2}}{7de} + \frac{(a^2e^{5/2})\,\mathrm{Subst}\left(\int \frac{\sqrt{2}\sqrt{e}+2x}{-e-\sqrt{2}\sqrt{e}x-x^2}\,dx, x, \sqrt{e\cot(c+dx)}\right)}{\sqrt{2}d} + \frac{(a^2e^{5/2})\,\mathrm{Subst}\left(\int \frac{\sqrt{2}\sqrt{e}-2x}{-e+\sqrt{2}\sqrt{e}x-x^2}\,dx, x, \sqrt{e\cot(c+dx)}\right)}{\sqrt{2}d} - \frac{(a^2e^3)\,\mathrm{Subst}\left(\int \frac{1}{e-\sqrt{2}\sqrt{e}x+x^2}\,dx, x, \sqrt{e\cot(c+dx)}\right)}{d} - \frac{(a^2e^3)\,\mathrm{Subst}\left(\int \frac{1}{e+\sqrt{2}\sqrt{e}x+x^2}\,dx, x, \sqrt{e\cot(c+dx)}\right)}{d}$$

$$= \frac{4a^2e^2\sqrt{e\cot(c+dx)}}{d} - \frac{4a^2(e\cot(c+dx))^{5/2}}{5d} - \frac{2a^2(e\cot(c+dx))^{7/2}}{7de} + \frac{a^2e^{5/2}\log\left(\sqrt{e}+\sqrt{e}\cot(c+dx)-\sqrt{2}\sqrt{e\cot(c+dx)}\right)}{\sqrt{2}d} - \frac{a^2e^{5/2}\log\left(\sqrt{e}+\sqrt{e}\cot(c+dx)+\sqrt{2}\sqrt{e\cot(c+dx)}\right)}{\sqrt{2}d} - \frac{(\sqrt{2}a^2e^{5/2})\,\mathrm{Subst}\left(\int \frac{1}{-1-x^2}\,dx, x, 1-\frac{\sqrt{2}\sqrt{e\cot(c+dx)}}{\sqrt{e}}\right)}{d} + \frac{(\sqrt{2}a^2e^{5/2})\,\mathrm{Subst}\left(\int \frac{1}{-1-x^2}\,dx, x, 1+\frac{\sqrt{2}\sqrt{e\cot(c+dx)}}{\sqrt{e}}\right)}{d}$$

$$= \frac{\sqrt{2}a^2e^{5/2}\arctan\left(1-\frac{\sqrt{2}\sqrt{e\cot(c+dx)}}{\sqrt{e}}\right)}{d} - \frac{\sqrt{2}a^2e^{5/2}\arctan\left(1+\frac{\sqrt{2}\sqrt{e\cot(c+dx)}}{\sqrt{e}}\right)}{d} + \frac{4a^2e^2\sqrt{e\cot(c+dx)}}{d} - \frac{4a^2(e\cot(c+dx))^{5/2}}{5d} - \frac{2a^2(e\cot(c+dx))^{7/2}}{7de} + \frac{a^2e^{5/2}\log\left(\sqrt{e}+\sqrt{e}\cot(c+dx)-\sqrt{2}\sqrt{e\cot(c+dx)}\right)}{\sqrt{2}d} - \frac{a^2e^{5/2}\log\left(\sqrt{e}+\sqrt{e}\cot(c+dx)+\sqrt{2}\sqrt{e\cot(c+dx)}\right)}{\sqrt{2}d}$$

Mathematica [A] (verified)

Time = 1.32 (sec) , antiderivative size = 187, normalized size of antiderivative = 0.70

$$\int (e\cot(c+dx))^{5/2}(a+a\cot(c+dx))^2\,dx =$$
$$-\frac{a^2(e\cot(c+dx))^{5/2}\left(-70\sqrt{2}\arctan\left(1-\sqrt{2}\sqrt{\cot(c+dx)}\right)+70\sqrt{2}\arctan\left(1+\sqrt{2}\sqrt{\cot(c+dx)}\right)-280\cdots\right.}{\cdots}$$

```
[In] Integrate[(e*Cot[c + d*x])^(5/2)*(a + a*Cot[c + d*x])^2,x]
```

```
[Out] -1/70*(a^2*(e*Cot[c + d*x])^(5/2)*(-70*Sqrt[2]*ArcTan[1 - Sqrt[2]*Sqrt[Cot[
c + d*x]]] + 70*Sqrt[2]*ArcTan[1 + Sqrt[2]*Sqrt[Cot[c + d*x]]] - 280*Sqrt[C
ot[c + d*x]] + 56*Cot[c + d*x]^(5/2) + 20*Cot[c + d*x]^(7/2) - 35*Sqrt[2]*L
og[1 - Sqrt[2]*Sqrt[Cot[c + d*x]] + Cot[c + d*x]] + 35*Sqrt[2]*Log[1 + Sqrt
[2]*Sqrt[Cot[c + d*x]] + Cot[c + d*x]]))/(d*Cot[c + d*x]^(5/2))
```

Maple [A] (verified)

Time = 0.15 (sec) , antiderivative size = 187, normalized size of antiderivative = 0.70

method	result
derivativedivides	$-\frac{2a^2\left(\frac{(e\cot(dx+c))^{\frac{7}{2}}}{7}+\frac{2e(e\cot(dx+c))^{\frac{5}{2}}}{5}-2\sqrt{e\cot(dx+c)}\,e^3+\frac{e^3\left(e^2\right)^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\frac{e\cot(dx+c)+\left(e^2\right)^{\frac{1}{4}}\sqrt{e\cot(dx+c)}\,\sqrt{2}+\sqrt{e^2}}{e\cot(dx+c)-\left(e^2\right)^{\frac{1}{4}}\sqrt{e\cot(dx+c)}\,\sqrt{2}+\sqrt{e^2}}\right)\cdots\right.}{\cdots}\right)}{de}$
default	$-\frac{2a^2\left(\frac{(e\cot(dx+c))^{\frac{7}{2}}}{7}+\frac{2e(e\cot(dx+c))^{\frac{5}{2}}}{5}-2\sqrt{e\cot(dx+c)}\,e^3+\frac{e^3\left(e^2\right)^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\frac{e\cot(dx+c)+\left(e^2\right)^{\frac{1}{4}}\sqrt{e\cot(dx+c)}\,\sqrt{2}+\sqrt{e^2}}{e\cot(dx+c)-\left(e^2\right)^{\frac{1}{4}}\sqrt{e\cot(dx+c)}\,\sqrt{2}+\sqrt{e^2}}\right)\cdots\right.}{\cdots}\right)}{de}$
parts	$-\frac{2a^2e\left(\frac{(e\cot(dx+c))^{\frac{3}{2}}}{3}-\frac{e^2\sqrt{2}\left(\ln\left(\frac{e\cot(dx+c)-\left(e^2\right)^{\frac{1}{4}}\sqrt{e\cot(dx+c)}\,\sqrt{2}+\sqrt{e^2}}{e\cot(dx+c)+\left(e^2\right)^{\frac{1}{4}}\sqrt{e\cot(dx+c)}\,\sqrt{2}+\sqrt{e^2}}\right)+2\arctan\left(\frac{\sqrt{2}\,\sqrt{e\cot(dx+c)}}{\left(e^2\right)^{\frac{1}{4}}}+1\right)-2\arctan\left(-\cdots\right)\right)}{8\left(e^2\right)^{\frac{1}{4}}}\cdots\right)}{d}$

```
[In] int((e*cot(d*x+c))^(5/2)*(a+a*cot(d*x+c))^2,x,method=_RETURNVERBOSE)
```

```
[Out] -2/d*a^2/e*(1/7*(e*cot(d*x+c))^(7/2)+2/5*e*(e*cot(d*x+c))^(5/2)-2*(e*cot(d*
x+c))^(1/2)*e^3+1/4*e^3*(e^2)^(1/4)*2^(1/2)*(ln((e*cot(d*x+c)+(e^2)^(1/4)*(
e*cot(d*x+c))^(1/2)*2^(1/2)+(e^2)^(1/2))/(e*cot(d*x+c)-(e^2)^(1/4)*(e*cot(d
*x+c))^(1/2)*2^(1/2)+(e^2)^(1/2)))+2*arctan(2^(1/2)/(e^2)^(1/4)*(e*cot(d*x+
c))^(1/2)+1)-2*arctan(-2^(1/2)/(e^2)^(1/4)*(e*cot(d*x+c))^(1/2)+1)))
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.29 (sec) , antiderivative size = 507, normalized size of antiderivative = 1.88

$$\int (e\cot(c+dx))^{5/2}(a+a\cot(c+dx))^2\,dx =$$

$$-\frac{35\left(-\frac{a^8e^{10}}{d^4}\right)^{\frac{1}{4}}(d\cos(2\,dx+2\,c)-d)\log\left(a^2e^2\sqrt{\frac{e\cos(2\,dx+2\,c)+e}{\sin(2\,dx+2\,c)}}+\left(-\frac{a^8e^{10}}{d^4}\right)^{\frac{1}{4}}d\right)\sin(2\,dx+2\,c)+35\left(-\frac{a^8}{c}\right.}{}$$

[In] integrate((e*cot(d*x+c))^(5/2)*(a+a*cot(d*x+c))^2,x, algorithm="fricas")

[Out] -1/35*(35*(-a^8*e^10/d^4)^(1/4)*(d*cos(2*d*x + 2*c) - d)*log(a^2*e^2*sqrt((e*cos(2*d*x + 2*c) + e)/sin(2*d*x + 2*c)) + (-a^8*e^10/d^4)^(1/4)*d)*sin(2*d*x + 2*c) + 35*(-a^8*e^10/d^4)^(1/4)*(I*d*cos(2*d*x + 2*c) - I*d)*log(a^2*e^2*sqrt((e*cos(2*d*x + 2*c) + e)/sin(2*d*x + 2*c)) + I*(-a^8*e^10/d^4)^(1/4)*d)*sin(2*d*x + 2*c) + 35*(-a^8*e^10/d^4)^(1/4)*(-I*d*cos(2*d*x + 2*c) + I*d)*log(a^2*e^2*sqrt((e*cos(2*d*x + 2*c) + e)/sin(2*d*x + 2*c)) - I*(-a^8*e^10/d^4)^(1/4)*d)*sin(2*d*x + 2*c) - 35*(-a^8*e^10/d^4)^(1/4)*(d*cos(2*d*x + 2*c) - d)*log(a^2*e^2*sqrt((e*cos(2*d*x + 2*c) + e)/sin(2*d*x + 2*c)) - (-a^8*e^10/d^4)^(1/4)*d)*sin(2*d*x + 2*c) - 2*(5*a^2*e^2*cos(2*d*x + 2*c)^2 + 10*a^2*e^2*cos(2*d*x + 2*c) + 5*a^2*e^2 + 28*(3*a^2*e^2*cos(2*d*x + 2*c) - 2*a^2*e^2)*sin(2*d*x + 2*c))*sqrt((e*cos(2*d*x + 2*c) + e)/sin(2*d*x + 2*c)))/((d*cos(2*d*x + 2*c) - d)*sin(2*d*x + 2*c))

Sympy [F]

$$\int (e\cot(c+dx))^{5/2}(a+a\cot(c+dx))^2\,dx = a^2\left(\int (e\cot(c+dx))^{\frac{5}{2}}\,dx + \int 2(e\cot(c+dx))^{\frac{5}{2}}\cot(c+dx)\,dx + \int (e\cot(c+dx))^{\frac{5}{2}}\cot^2(c+dx)\,dx\right)$$

[In] integrate((e*cot(d*x+c))**(5/2)*(a+a*cot(d*x+c))**2,x)

[Out] a**2*(Integral((e*cot(c + d*x))**(5/2), x) + Integral(2*(e*cot(c + d*x))**(5/2)*cot(c + d*x), x) + Integral((e*cot(c + d*x))**(5/2)*cot(c + d*x)**2, x))

Maxima [F(-2)]

Exception generated.

$$\int (e\cot(c+dx))^{5/2}(a+a\cot(c+dx))^2\,dx = \text{Exception raised: ValueError}$$

```
[In] integrate((e*cot(d*x+c))^(5/2)*(a+a*cot(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(e>0)', see `assume?` for more detai
ls)Is e
```

Giac [F]

$$\int (e\cot(c+dx))^{5/2}(a+a\cot(c+dx))^2\,dx = \int (a\cot(dx+c)+a)^2(e\cot(dx+c))^{\frac{5}{2}}\,dx$$

```
[In] integrate((e*cot(d*x+c))^(5/2)*(a+a*cot(d*x+c))^2,x, algorithm="giac")
```

```
[Out] integrate((a*cot(d*x + c) + a)^2*(e*cot(d*x + c))^(5/2), x)
```

Mupad [B] (verification not implemented)

Time = 13.91 (sec) , antiderivative size = 125, normalized size of antiderivative = 0.46

$$\begin{aligned}\int (e\cot(c+dx))^{5/2}(a+a\cot(c+dx))^2\,dx &= \frac{4a^2e^2\sqrt{e\cot(c+dx)}}{d} \\ &- \frac{4a^2(e\cot(c+dx))^{5/2}}{5d} - \frac{2a^2(e\cot(c+dx))^{7/2}}{7de} \\ &+ \frac{(-1)^{1/4}a^2e^{5/2}\,\mathrm{atan}\left(\frac{(-1)^{1/4}\sqrt{e\cot(c+dx)}}{\sqrt{e}}\right)2\mathrm{i}}{d} \\ &+ \frac{2(-1)^{1/4}a^2e^{5/2}\,\mathrm{atan}\left(\frac{(-1)^{1/4}\sqrt{e\cot(c+dx)}\,1\mathrm{i}}{\sqrt{e}}\right)}{d}\end{aligned}$$

```
[In] int((e*cot(c + d*x))^(5/2)*(a + a*cot(c + d*x))^2,x)
```

```
[Out] (4*a^2*e^2*(e*cot(c + d*x))^(1/2))/d - (4*a^2*(e*cot(c + d*x))^(5/2))/(5*d)
 - (2*a^2*(e*cot(c + d*x))^(7/2))/(7*d*e) + ((-1)^(1/4)*a^2*e^(5/2)*atan(((
-1)^(1/4)*(e*cot(c + d*x))^(1/2))/e^(1/2))*2i)/d + (2*(-1)^(1/4)*a^2*e^(5/2
)*atan(((-1)^(1/4)*(e*cot(c + d*x))^(1/2)*1i)/e^(1/2)))/d
```

3.9 $\int (e\cot(c+dx))^{3/2}(a+a\cot(c+dx))^2\,dx$

Optimal result

Integrand size = 25, antiderivative size = 246

$$\int (e\cot(c+dx))^{3/2}(a+a\cot(c+dx))^2\,dx = -\frac{\sqrt{2}a^2e^{3/2}\arctan\left(1-\frac{\sqrt{2}\sqrt{e\cot(c+dx)}}{\sqrt{e}}\right)}{d} + \frac{\sqrt{2}a^2e^{3/2}\arctan\left(1+\frac{\sqrt{2}\sqrt{e\cot(c+dx)}}{\sqrt{e}}\right)}{d} - \frac{4a^2(e\cot(c+dx))^{3/2}}{3d} - \frac{2a^2(e\cot(c+dx))^{5/2}}{5de} + \frac{a^2e^{3/2}\log\left(\sqrt{e}+\sqrt{e}\cot(c+dx)-\sqrt{2}\sqrt{e\cot(c+dx)}\right)}{\sqrt{2}d} - \frac{a^2e^{3/2}\log\left(\sqrt{e}+\sqrt{e}\cot(c+dx)+\sqrt{2}\sqrt{e\cot(c+dx)}\right)}{\sqrt{2}d}$$

```
[Out] -4/3*a^2*(e*cot(d*x+c))^(3/2)/d-2/5*a^2*(e*cot(d*x+c))^(5/2)/d/e+1/2*a^2*e^
(3/2)*ln(e^(1/2)+cot(d*x+c)*e^(1/2)-2^(1/2)*(e*cot(d*x+c))^(1/2))/d*2^(1/2)
-1/2*a^2*e^(3/2)*ln(e^(1/2)+cot(d*x+c)*e^(1/2)+2^(1/2)*(e*cot(d*x+c))^(1/2)
)/d*2^(1/2)-a^2*e^(3/2)*arctan(1-2^(1/2)*(e*cot(d*x+c))^(1/2)/e^(1/2))*2^(1
/2)/d+a^2*e^(3/2)*arctan(1+2^(1/2)*(e*cot(d*x+c))^(1/2)/e^(1/2))*2^(1/2)/d
```

Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 246, normalized size of antiderivative = 1.00, number of steps used = 15, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.480, Rules

used = {3624, 12, 16, 3554, 3557, 335, 303, 1176, 631, 210, 1179, 642}

$$\int (e\cot(c+dx))^{3/2}(a+a\cot(c+dx))^2\,dx = -\frac{\sqrt{2}a^2e^{3/2}\arctan\left(1-\frac{\sqrt{2}\sqrt{e\cot(c+dx)}}{\sqrt{e}}\right)}{d}+\frac{\sqrt{2}a^2e^{3/2}\arctan\left(\frac{\sqrt{2}\sqrt{e\cot(c+dx)}}{\sqrt{e}}+1\right)}{d} + \frac{a^2e^{3/2}\log\left(\sqrt{e}\cot(c+dx)-\sqrt{2}\sqrt{e\cot(c+dx)}+\sqrt{e}\right)}{\sqrt{2}d} - \frac{a^2e^{3/2}\log\left(\sqrt{e}\cot(c+dx)+\sqrt{2}\sqrt{e\cot(c+dx)}+\sqrt{e}\right)}{\sqrt{2}d} - \frac{2a^2(e\cot(c+dx))^{5/2}}{5de} - \frac{4a^2(e\cot(c+dx))^{3/2}}{3d}$$

```
[In] Int[(e*Cot[c + d*x])^(3/2)*(a + a*Cot[c + d*x])^2,x]
```

```
[Out] -((Sqrt[2]*a^2*e^(3/2)*ArcTan[1 - (Sqrt[2]*Sqrt[e*Cot[c + d*x]])/Sqrt[e]])/
d) + (Sqrt[2]*a^2*e^(3/2)*ArcTan[1 + (Sqrt[2]*Sqrt[e*Cot[c + d*x]])/Sqrt[e]
])/d - (4*a^2*(e*Cot[c + d*x])^(3/2))/(3*d) - (2*a^2*(e*Cot[c + d*x])^(5/2)
)/(5*d*e) + (a^2*e^(3/2)*Log[Sqrt[e] + Sqrt[e]*Cot[c + d*x] - Sqrt[2]*Sqrt[
e*Cot[c + d*x]]])/(Sqrt[2]*d) - (a^2*e^(3/2)*Log[Sqrt[e] + Sqrt[e]*Cot[c +
d*x] + Sqrt[2]*Sqrt[e*Cot[c + d*x]]])/(Sqrt[2]*d)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 16

```
Int[(u_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Dist[1/b^m, Int[u*(b*v)
^(m + n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 303

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*s), Int[(r + s*x^2)/(a + b*x^4
), x], x] - Dist[1/(2*s), Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
```

```
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1179

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 3554

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[b*((b*Tan[c + d
*x])^(n - 1)/(d*(n - 1))), x] - Dist[b^2, Int[(b*Tan[c + d*x])^(n - 2), x],
 x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
```

Rule 3557

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[b/d, Subst[Int[
x^n/(b^2 + x^2), x], x, b*Tan[c + d*x]], x] /; FreeQ[{b, c, d, n}, x] &&  !
```

```
IntegerQ[n]
```

Rule 3624

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)])^2, x_Symbol] :> Simp[d^2*((a + b*Tan[e + f*x])^(m + 1)/(b*f*(
m + 1))), x] + Int[(a + b*Tan[e + f*x])^m*Simp[c^2 - d^2 + 2*c*d*Tan[e + f*
x], x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b*c - a*d, 0] &&  !LeQ
[m, -1] &&  !(EqQ[m, 2] && EqQ[a, 0])
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= -\frac{2a^2(e\cot(c+dx))^{5/2}}{5de}+\int 2a^2\cot(c+dx)(e\cot(c+dx))^{3/2}\,dx\\
&= -\frac{2a^2(e\cot(c+dx))^{5/2}}{5de}+\left(2a^2\right)\int \cot(c+dx)(e\cot(c+dx))^{3/2}\,dx\\
&= -\frac{2a^2(e\cot(c+dx))^{5/2}}{5de}+\frac{\left(2a^2\right)\int (e\cot(c+dx))^{5/2}\,dx}{e}\\
&= -\frac{4a^2(e\cot(c+dx))^{3/2}}{3d}-\frac{2a^2(e\cot(c+dx))^{5/2}}{5de}-\left(2a^2e\right)\int \sqrt{e\cot(c+dx)}\,dx\\
&= -\frac{4a^2(e\cot(c+dx))^{3/2}}{3d}-\frac{2a^2(e\cot(c+dx))^{5/2}}{5de}+\frac{\left(2a^2e^2\right)\text{Subst}\left(\int \frac{\sqrt{x}}{e^2+x^2}\,dx,x,e\cot(c+dx)\right)}{d}\\
&= -\frac{4a^2(e\cot(c+dx))^{3/2}}{3d}-\frac{2a^2(e\cot(c+dx))^{5/2}}{5de}\\
&\quad+\frac{\left(4a^2e^2\right)\text{Subst}\left(\int \frac{x^2}{e^2+x^4}\,dx,x,\sqrt{e\cot(c+dx)}\right)}{d}\\
&= -\frac{4a^2(e\cot(c+dx))^{3/2}}{3d}-\frac{2a^2(e\cot(c+dx))^{5/2}}{5de}\\
&\quad-\frac{\left(2a^2e^2\right)\text{Subst}\left(\int \frac{e-x^2}{e^2+x^4}\,dx,x,\sqrt{e\cot(c+dx)}\right)}{d}\\
&\quad+\frac{\left(2a^2e^2\right)\text{Subst}\left(\int \frac{e+x^2}{e^2+x^4}\,dx,x,\sqrt{e\cot(c+dx)}\right)}{d}
\end{aligned}
$$

$$=-\frac{4a^2(e\cot(c+dx))^{3/2}}{3d}-\frac{2a^2(e\cot(c+dx))^{5/2}}{5de}$$
$$+\frac{\left(a^2e^{3/2}\right)\text{Subst}\left(\int\frac{\sqrt{2}\sqrt{e}+2x}{-e-\sqrt{2}\sqrt{e}x-x^2}\,dx,x,\sqrt{e\cot(c+dx)}\right)}{\sqrt{2}d}$$
$$+\frac{\left(a^2e^{3/2}\right)\text{Subst}\left(\int\frac{\sqrt{2}\sqrt{e}-2x}{-e+\sqrt{2}\sqrt{e}x-x^2}\,dx,x,\sqrt{e\cot(c+dx)}\right)}{\sqrt{2}d}$$
$$+\frac{\left(a^2e^2\right)\text{Subst}\left(\int\frac{1}{e-\sqrt{2}\sqrt{e}x+x^2}\,dx,x,\sqrt{e\cot(c+dx)}\right)}{d}$$
$$+\frac{\left(a^2e^2\right)\text{Subst}\left(\int\frac{1}{e+\sqrt{2}\sqrt{e}x+x^2}\,dx,x,\sqrt{e\cot(c+dx)}\right)}{d}$$
$$=-\frac{4a^2(e\cot(c+dx))^{3/2}}{3d}-\frac{2a^2(e\cot(c+dx))^{5/2}}{5de}$$
$$+\frac{a^2e^{3/2}\log\left(\sqrt{e}+\sqrt{e}\cot(c+dx)-\sqrt{2}\sqrt{e\cot(c+dx)}\right)}{\sqrt{2}d}$$
$$-\frac{a^2e^{3/2}\log\left(\sqrt{e}+\sqrt{e}\cot(c+dx)+\sqrt{2}\sqrt{e\cot(c+dx)}\right)}{\sqrt{2}d}$$
$$+\frac{\left(\sqrt{2}a^2e^{3/2}\right)\text{Subst}\left(\int\frac{1}{-1-x^2}\,dx,x,1-\frac{\sqrt{2}\sqrt{e\cot(c+dx)}}{\sqrt{e}}\right)}{d}$$
$$-\frac{\left(\sqrt{2}a^2e^{3/2}\right)\text{Subst}\left(\int\frac{1}{-1-x^2}\,dx,x,1+\frac{\sqrt{2}\sqrt{e\cot(c+dx)}}{\sqrt{e}}\right)}{d}$$
$$=-\frac{\sqrt{2}a^2e^{3/2}\arctan\left(1-\frac{\sqrt{2}\sqrt{e\cot(c+dx)}}{\sqrt{e}}\right)}{d}+\frac{\sqrt{2}a^2e^{3/2}\arctan\left(1+\frac{\sqrt{2}\sqrt{e\cot(c+dx)}}{\sqrt{e}}\right)}{d}$$
$$-\frac{4a^2(e\cot(c+dx))^{3/2}}{3d}-\frac{2a^2(e\cot(c+dx))^{5/2}}{5de}$$
$$+\frac{a^2e^{3/2}\log\left(\sqrt{e}+\sqrt{e}\cot(c+dx)-\sqrt{2}\sqrt{e\cot(c+dx)}\right)}{\sqrt{2}d}$$
$$-\frac{a^2e^{3/2}\log\left(\sqrt{e}+\sqrt{e}\cot(c+dx)+\sqrt{2}\sqrt{e\cot(c+dx)}\right)}{\sqrt{2}d}$$

Mathematica [A] (verified)

Time = 1.04 (sec) , antiderivative size = 112, normalized size of antiderivative = 0.46

$$\int (e\cot(c+dx))^{3/2}(a+a\cot(c+dx))^2\,dx = -\frac{2a^2(e\cot(c+dx))^{3/2}\left(-15\arctan\left(\sqrt[4]{-\cot^2(c+dx)}\right)\sqrt[4]{-\cot(c+dx)}+15\text{arctanh}\left(\sqrt[4]{-\cot^2(c+dx)}\right)\cdots\right.}{15d\cot^{\frac{7}{4}}(c+dx)}$$

```
[In] Integrate[(e*Cot[c + d*x])^(3/2)*(a + a*Cot[c + d*x])^2,x]
```

```
[Out] (-2*a^2*(e*Cot[c + d*x])^(3/2)*(-15*ArcTan[(-Cot[c + d*x]^2)^(1/4)]*(-Cot[c
 + d*x])^(1/4) + 15*ArcTanh[(-Cot[c + d*x]^2)^(1/4)]*(-Cot[c + d*x])^(1/4)
+ Cot[c + d*x]^(7/4)*(10 + 3*Cot[c + d*x])))/(15*d*Cot[c + d*x]^(7/4))
```

Maple [A] (verified)

Time = 0.04 (sec) , antiderivative size = 172, normalized size of antiderivative = 0.70

method	result
derivativedivides	$-\frac{2a^2\left(\frac{(e\cot(dx+c))^{\frac{5}{2}}}{5}+\frac{2e(e\cot(dx+c))^{\frac{3}{2}}}{3}-\frac{e^3\sqrt{2}\left(\ln\left(\frac{e\cot(dx+c)-(e^2)^{\frac{1}{4}}\sqrt{e\cot(dx+c)}\sqrt{2}+\sqrt{e^2}}{e\cot(dx+c)+(e^2)^{\frac{1}{4}}\sqrt{e\cot(dx+c)}\sqrt{2}+\sqrt{e^2}}\right)+2\arctan\left(\frac{\sqrt{2}\sqrt{e\cot(dx+c)}}{(e^2)^{\frac{1}{4}}}\cdots\right.\right.}{4(e^2)^{\frac{1}{4}}}\right.}{de}$
default	$-\frac{2a^2\left(\frac{(e\cot(dx+c))^{\frac{5}{2}}}{5}+\frac{2e(e\cot(dx+c))^{\frac{3}{2}}}{3}-\frac{e^3\sqrt{2}\left(\ln\left(\frac{e\cot(dx+c)-(e^2)^{\frac{1}{4}}\sqrt{e\cot(dx+c)}\sqrt{2}+\sqrt{e^2}}{e\cot(dx+c)+(e^2)^{\frac{1}{4}}\sqrt{e\cot(dx+c)}\sqrt{2}+\sqrt{e^2}}\right)+2\arctan\left(\frac{\sqrt{2}\sqrt{e\cot(dx+c)}}{(e^2)^{\frac{1}{4}}}\cdots\right.\right.}{4(e^2)^{\frac{1}{4}}}\right.}{de}$
parts	$-\frac{2a^2e\left(\sqrt{e\cot(dx+c)}-\frac{(e^2)^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\frac{e\cot(dx+c)+(e^2)^{\frac{1}{4}}\sqrt{e\cot(dx+c)}\sqrt{2}+\sqrt{e^2}}{e\cot(dx+c)-(e^2)^{\frac{1}{4}}\sqrt{e\cot(dx+c)}\sqrt{2}+\sqrt{e^2}}\right)+2\arctan\left(\frac{\sqrt{2}\sqrt{e\cot(dx+c)}}{(e^2)^{\frac{1}{4}}}+1\right)-2\arctan\cdots\right.}{8}\right.}{d}$

```
[In] int((e*cot(d*x+c))^(3/2)*(a+a*cot(d*x+c))^2,x,method=_RETURNVERBOSE)
```

```
[Out] -2/d*a^2/e*(1/5*(e*cot(d*x+c))^(5/2)+2/3*e*(e*cot(d*x+c))^(3/2)-1/4*e^3/(e^
2)^(1/4)*2^(1/2)*(ln((e*cot(d*x+c)-(e^2)^(1/4)*(e*cot(d*x+c))^(1/2)*2^(1/2)
+(e^2)^(1/2))/(e*cot(d*x+c)+(e^2)^(1/4)*(e*cot(d*x+c))^(1/2)*2^(1/2)+(e^2)^
(1/2)))+2*arctan(2^(1/2)/(e^2)^(1/4)*(e*cot(d*x+c))^(1/2)+1)-2*arctan(-2^(1
/2)/(e^2)^(1/4)*(e*cot(d*x+c))^(1/2)+1)))
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.28 (sec) , antiderivative size = 423, normalized size of antiderivative = 1.72

$$\int (e\cot(c+dx))^{3/2}(a+a\cot(c+dx))^2\,dx = \frac{15\left(-\frac{a^8e^6}{d^4}\right)^{\frac{1}{4}}(d\cos(2\,dx+2\,c)-d)\log\left(a^6e^4\sqrt{\frac{e\cos(2\,dx+2\,c)+e}{\sin(2\,dx+2\,c)}}+\left(-\frac{a^8e^6}{d^4}\right)^{\frac{3}{4}}d^3\right)-15\left(\right.$$

```
[In] integrate((e*cot(d*x+c))^(3/2)*(a+a*cot(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] 1/15*(15*(-a^8*e^6/d^4)^(1/4)*(d*cos(2*d*x + 2*c) - d)*log(a^6*e^4*sqrt((e*
cos(2*d*x + 2*c) + e)/sin(2*d*x + 2*c)) + (-a^8*e^6/d^4)^(3/4)*d^3) - 15*(-
a^8*e^6/d^4)^(1/4)*(I*d*cos(2*d*x + 2*c) - I*d)*log(a^6*e^4*sqrt((e*cos(2*d
*x + 2*c) + e)/sin(2*d*x + 2*c)) + I*(-a^8*e^6/d^4)^(3/4)*d^3) - 15*(-a^8*e
^6/d^4)^(1/4)*(-I*d*cos(2*d*x + 2*c) + I*d)*log(a^6*e^4*sqrt((e*cos(2*d*x +
 2*c) + e)/sin(2*d*x + 2*c)) - I*(-a^8*e^6/d^4)^(3/4)*d^3) - 15*(-a^8*e^6/d
^4)^(1/4)*(d*cos(2*d*x + 2*c) - d)*log(a^6*e^4*sqrt((e*cos(2*d*x + 2*c) + e
)/sin(2*d*x + 2*c)) - (-a^8*e^6/d^4)^(3/4)*d^3) + 2*(3*a^2*e*cos(2*d*x + 2*
c) + 10*a^2*e*sin(2*d*x + 2*c) + 3*a^2*e)*sqrt((e*cos(2*d*x + 2*c) + e)/sin
(2*d*x + 2*c)))/(d*cos(2*d*x + 2*c) - d)
```

Sympy [F]

$$\int (e\cot(c+dx))^{3/2}(a+a\cot(c+dx))^2\,dx = a^2\left(\int (e\cot(c+dx))^{\frac{3}{2}}\,dx + \int 2(e\cot(c+dx))^{\frac{3}{2}}\cot(c+dx)\,dx + \int (e\cot(c+dx))^{\frac{3}{2}}\cot^2(c+dx)\,dx\right)$$

```
[In] integrate((e*cot(d*x+c))**(3/2)*(a+a*cot(d*x+c))**2,x)
```

```
[Out] a**2*(Integral((e*cot(c + d*x))**(3/2), x) + Integral(2*(e*cot(c + d*x))**(
3/2)*cot(c + d*x), x) + Integral((e*cot(c + d*x))**(3/2)*cot(c + d*x)**2, x
))
```

Maxima [F(-2)]

Exception generated.

$$\int (e\cot(c+dx))^{3/2}(a+a\cot(c+dx))^2\,dx = \text{Exception raised: ValueError}$$

```
[In] integrate((e*cot(d*x+c))^(3/2)*(a+a*cot(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(e>0)', see `assume?` for more detai
ls)Is e
```

Giac [F]

$$\int (e\cot(c+dx))^{3/2}(a+a\cot(c+dx))^2\,dx = \int (a\cot(dx+c)+a)^2(e\cot(dx+c))^{\frac{3}{2}}\,dx$$

```
[In] integrate((e*cot(d*x+c))^(3/2)*(a+a*cot(d*x+c))^2,x, algorithm="giac")
```

```
[Out] integrate((a*cot(d*x + c) + a)^2*(e*cot(d*x + c))^(3/2), x)
```

Mupad [B] (verification not implemented)

Time = 13.37 (sec) , antiderivative size = 104, normalized size of antiderivative = 0.42

$$\int (e\cot(c+dx))^{3/2}(a+a\cot(c+dx))^2\,dx = \frac{2(-1)^{1/4}a^2e^{3/2}\operatorname{atan}\left(\frac{(-1)^{1/4}\sqrt{e\cot(c+dx)}}{\sqrt{e}}\right)}{d} - \frac{2a^2(e\cot(c+dx))^{5/2}}{5de} - \frac{4a^2(e\cot(c+dx))^{3/2}}{3d} + \frac{(-1)^{1/4}a^2e^{3/2}\operatorname{atan}\left(\frac{(-1)^{1/4}\sqrt{e\cot(c+dx)}\,1\mathrm{i}}{\sqrt{e}}\right)2\mathrm{i}}{d}$$

```
[In] int((e*cot(c + d*x))^(3/2)*(a + a*cot(c + d*x))^2,x)
```

```
[Out] (2*(-1)^(1/4)*a^2*e^(3/2)*atan(((-1)^(1/4)*(e*cot(c + d*x))^(1/2))/e^(1/2))
)/d - (2*a^2*(e*cot(c + d*x))^(5/2))/(5*d*e) - (4*a^2*(e*cot(c + d*x))^(3/2
))/(3*d) + ((-1)^(1/4)*a^2*e^(3/2)*atan(((-1)^(1/4)*(e*cot(c + d*x))^(1/2)*
1i)/e^(1/2))*2i)/d
```

3.10 $\int \sqrt{e\cot(c+dx)}(a+a\cot(c+dx))^2\,dx$

Optimal result

Integrand size = 25, antiderivative size = 244

$$
\begin{aligned}
\int \sqrt{e\cot(c+dx)}(a+a\cot(c+dx))^2\,dx
&= -\frac{\sqrt{2}a^2\sqrt{e}\arctan\left(1-\frac{\sqrt{2}\sqrt{e\cot(c+dx)}}{\sqrt{e}}\right)}{d}+\frac{\sqrt{2}a^2\sqrt{e}\arctan\left(1+\frac{\sqrt{2}\sqrt{e\cot(c+dx)}}{\sqrt{e}}\right)}{d} \\
&\quad -\frac{4a^2\sqrt{e\cot(c+dx)}}{d}-\frac{2a^2(e\cot(c+dx))^{3/2}}{3de} \\
&\quad -\frac{a^2\sqrt{e}\log\left(\sqrt{e}+\sqrt{e}\cot(c+dx)-\sqrt{2}\sqrt{e\cot(c+dx)}\right)}{\sqrt{2}d} \\
&\quad +\frac{a^2\sqrt{e}\log\left(\sqrt{e}+\sqrt{e}\cot(c+dx)+\sqrt{2}\sqrt{e\cot(c+dx)}\right)}{\sqrt{2}d}
\end{aligned}
$$

```
[Out] -2/3*a^2*(e*cot(d*x+c))^(3/2)/d/e-1/2*a^2*ln(e^(1/2)+cot(d*x+c)*e^(1/2)-2^(
1/2)*(e*cot(d*x+c))^(1/2))*e^(1/2)/d*2^(1/2)+1/2*a^2*ln(e^(1/2)+cot(d*x+c)*
e^(1/2)+2^(1/2)*(e*cot(d*x+c))^(1/2))*e^(1/2)/d*2^(1/2)-a^2*arctan(1-2^(1/2
)*(e*cot(d*x+c))^(1/2)/e^(1/2))*2^(1/2)*e^(1/2)/d+a^2*arctan(1+2^(1/2)*(e*c
ot(d*x+c))^(1/2)/e^(1/2))*2^(1/2)*e^(1/2)/d-4*a^2*(e*cot(d*x+c))^(1/2)/d
```

Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 244, normalized size of antiderivative = 1.00, number of steps used = 15, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.480, Rules used = {3624, 12, 16, 3554, 3557, 335, 217, 1179, 642, 1176, 631, 210}

$$\int \sqrt{e\cot(c+dx)}(a+a\cot(c+dx))^2\,dx$$
$$= -\frac{\sqrt{2}a^2\sqrt{e}\arctan\left(1-\frac{\sqrt{2}\sqrt{e\cot(c+dx)}}{\sqrt{e}}\right)}{d}+\frac{\sqrt{2}a^2\sqrt{e}\arctan\left(\frac{\sqrt{2}\sqrt{e\cot(c+dx)}}{\sqrt{e}}+1\right)}{d}$$
$$-\frac{2a^2(e\cot(c+dx))^{3/2}}{3de}-\frac{4a^2\sqrt{e\cot(c+dx)}}{d}$$
$$-\frac{a^2\sqrt{e}\log\left(\sqrt{e}\cot(c+dx)-\sqrt{2}\sqrt{e\cot(c+dx)}+\sqrt{e}\right)}{\sqrt{2}d}$$
$$+\frac{a^2\sqrt{e}\log\left(\sqrt{e}\cot(c+dx)+\sqrt{2}\sqrt{e\cot(c+dx)}+\sqrt{e}\right)}{\sqrt{2}d}$$

```
[In] Int[Sqrt[e*Cot[c + d*x]]*(a + a*Cot[c + d*x])^2,x]
```

```
[Out] -((Sqrt[2]*a^2*Sqrt[e]*ArcTan[1 - (Sqrt[2]*Sqrt[e*Cot[c + d*x]])/Sqrt[e]])/
d) + (Sqrt[2]*a^2*Sqrt[e]*ArcTan[1 + (Sqrt[2]*Sqrt[e*Cot[c + d*x]])/Sqrt[e]
])/d - (4*a^2*Sqrt[e*Cot[c + d*x]])/d - (2*a^2*(e*Cot[c + d*x])^(3/2))/(3*d
*e) - (a^2*Sqrt[e]*Log[Sqrt[e] + Sqrt[e]*Cot[c + d*x] - Sqrt[2]*Sqrt[e*Cot[
c + d*x]]])/(Sqrt[2]*d) + (a^2*Sqrt[e]*Log[Sqrt[e] + Sqrt[e]*Cot[c + d*x] +
 Sqrt[2]*Sqrt[e*Cot[c + d*x]]])/(Sqrt[2]*d)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 16

```
Int[(u_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Dist[1/b^m, Int[u*(b*v)
^(m + n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4),
```

```
 x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b
}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &&
AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1179

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 3554

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[b*((b*Tan[c + d
*x])^(n - 1)/(d*(n - 1))), x] - Dist[b^2, Int[(b*Tan[c + d*x])^(n - 2), x],
 x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
```

Rule 3557

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[b/d, Subst[Int[
x^n/(b^2 + x^2), x], x, b*Tan[c + d*x]], x] /; FreeQ[{b, c, d, n}, x] &&  !
IntegerQ[n]
```

Rule 3624

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)])^2, x_Symbol] :> Simp[d^2*((a + b*Tan[e + f*x])^(m + 1)/(b*f*(
m + 1))), x] + Int[(a + b*Tan[e + f*x])^m*Simp[c^2 - d^2 + 2*c*d*Tan[e + f*
x], x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b*c - a*d, 0] &&  !LeQ
[m, -1] &&  !(EqQ[m, 2] && EqQ[a, 0])
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= -\frac{2a^2(e\cot(c+dx))^{3/2}}{3de} + \int 2a^2\cot(c+dx)\sqrt{e\cot(c+dx)}\,dx \\
&= -\frac{2a^2(e\cot(c+dx))^{3/2}}{3de} + \left(2a^2\right)\int \cot(c+dx)\sqrt{e\cot(c+dx)}\,dx \\
&= -\frac{2a^2(e\cot(c+dx))^{3/2}}{3de} + \frac{\left(2a^2\right)\int (e\cot(c+dx))^{3/2}\,dx}{e} \\
&= -\frac{4a^2\sqrt{e\cot(c+dx)}}{d} - \frac{2a^2(e\cot(c+dx))^{3/2}}{3de} - \left(2a^2e\right)\int \frac{1}{\sqrt{e\cot(c+dx)}}\,dx \\
&= -\frac{4a^2\sqrt{e\cot(c+dx)}}{d} - \frac{2a^2(e\cot(c+dx))^{3/2}}{3de} \\
&\quad + \frac{\left(2a^2e^2\right)\text{Subst}\left(\int \frac{1}{\sqrt{x}(e^2+x^2)}\,dx, x, e\cot(c+dx)\right)}{d} \\
&= -\frac{4a^2\sqrt{e\cot(c+dx)}}{d} - \frac{2a^2(e\cot(c+dx))^{3/2}}{3de} + \frac{\left(4a^2e^2\right)\text{Subst}\left(\int \frac{1}{e^2+x^4}\,dx, x, \sqrt{e\cot(c+dx)}\right)}{d} \\
&= -\frac{4a^2\sqrt{e\cot(c+dx)}}{d} - \frac{2a^2(e\cot(c+dx))^{3/2}}{3de} \\
&\quad + \frac{\left(2a^2e\right)\text{Subst}\left(\int \frac{e-x^2}{e^2+x^4}\,dx, x, \sqrt{e\cot(c+dx)}\right)}{d} \\
&\quad + \frac{\left(2a^2e\right)\text{Subst}\left(\int \frac{e+x^2}{e^2+x^4}\,dx, x, \sqrt{e\cot(c+dx)}\right)}{d}
\end{aligned}
$$

$$= -\frac{4a^2\sqrt{e\cot(c+dx)}}{d} - \frac{2a^2(e\cot(c+dx))^{3/2}}{3de} - \frac{(a^2\sqrt{e})\,\text{Subst}\left(\int \frac{\sqrt{2}\sqrt{e}+2x}{-e-\sqrt{2}\sqrt{e}x-x^2}\,dx, x, \sqrt{e\cot(c+dx)}\right)}{\sqrt{2}d} - \frac{(a^2\sqrt{e})\,\text{Subst}\left(\int \frac{\sqrt{2}\sqrt{e}-2x}{-e+\sqrt{2}\sqrt{e}x-x^2}\,dx, x, \sqrt{e\cot(c+dx)}\right)}{\sqrt{2}d} + \frac{(a^2e)\,\text{Subst}\left(\int \frac{1}{e-\sqrt{2}\sqrt{e}x+x^2}\,dx, x, \sqrt{e\cot(c+dx)}\right)}{d} + \frac{(a^2e)\,\text{Subst}\left(\int \frac{1}{e+\sqrt{2}\sqrt{e}x+x^2}\,dx, x, \sqrt{e\cot(c+dx)}\right)}{d}$$

$$= -\frac{4a^2\sqrt{e\cot(c+dx)}}{d} - \frac{2a^2(e\cot(c+dx))^{3/2}}{3de} - \frac{a^2\sqrt{e}\log\left(\sqrt{e}+\sqrt{e}\cot(c+dx)-\sqrt{2}\sqrt{e\cot(c+dx)}\right)}{\sqrt{2}d} + \frac{a^2\sqrt{e}\log\left(\sqrt{e}+\sqrt{e}\cot(c+dx)+\sqrt{2}\sqrt{e\cot(c+dx)}\right)}{\sqrt{2}d} + \frac{(\sqrt{2}a^2\sqrt{e})\,\text{Subst}\left(\int \frac{1}{-1-x^2}\,dx, x, 1-\frac{\sqrt{2}\sqrt{e\cot(c+dx)}}{\sqrt{e}}\right)}{d} - \frac{(\sqrt{2}a^2\sqrt{e})\,\text{Subst}\left(\int \frac{1}{-1-x^2}\,dx, x, 1+\frac{\sqrt{2}\sqrt{e\cot(c+dx)}}{\sqrt{e}}\right)}{d}$$

$$= -\frac{\sqrt{2}a^2\sqrt{e}\arctan\left(1-\frac{\sqrt{2}\sqrt{e\cot(c+dx)}}{\sqrt{e}}\right)}{d} + \frac{\sqrt{2}a^2\sqrt{e}\arctan\left(1+\frac{\sqrt{2}\sqrt{e\cot(c+dx)}}{\sqrt{e}}\right)}{d} - \frac{4a^2\sqrt{e\cot(c+dx)}}{d} - \frac{2a^2(e\cot(c+dx))^{3/2}}{3de} - \frac{a^2\sqrt{e}\log\left(\sqrt{e}+\sqrt{e}\cot(c+dx)-\sqrt{2}\sqrt{e\cot(c+dx)}\right)}{\sqrt{2}d} + \frac{a^2\sqrt{e}\log\left(\sqrt{e}+\sqrt{e}\cot(c+dx)+\sqrt{2}\sqrt{e\cot(c+dx)}\right)}{\sqrt{2}d}$$

Mathematica [A] (verified)

Time = 0.51 (sec) , antiderivative size = 175, normalized size of antiderivative = 0.72

$$\int \sqrt{e\cot(c+dx)}(a+a\cot(c+dx))^2\,dx =$$
$$-\frac{a^2\sqrt{e\cot(c+dx)}\left(6\sqrt{2}\arctan\left(1-\sqrt{2}\sqrt{\cot(c+dx)}\right)-6\sqrt{2}\arctan\left(1+\sqrt{2}\sqrt{\cot(c+dx)}\right)+24\sqrt{\cot}\right.}{}$$

```
[In] Integrate[Sqrt[e*Cot[c + d*x]]*(a + a*Cot[c + d*x])^2,x]
```

```
[Out] -1/6*(a^2*Sqrt[e*Cot[c + d*x]]*(6*Sqrt[2]*ArcTan[1 - Sqrt[2]*Sqrt[Cot[c + d
*x]]] - 6*Sqrt[2]*ArcTan[1 + Sqrt[2]*Sqrt[Cot[c + d*x]]] + 24*Sqrt[Cot[c +
d*x]] + 4*Cot[c + d*x]^(3/2) + 3*Sqrt[2]*Log[1 - Sqrt[2]*Sqrt[Cot[c + d*x]]
 + Cot[c + d*x]] - 3*Sqrt[2]*Log[1 + Sqrt[2]*Sqrt[Cot[c + d*x]] + Cot[c + d
*x]]))/(d*Sqrt[Cot[c + d*x]])
```

Maple [A] (verified)

Time = 0.05 (sec) , antiderivative size = 170, normalized size of antiderivative = 0.70

method	result
derivativedivides	$-\frac{2a^2\left(\frac{(e\cot(dx+c))^{\frac{3}{2}}}{3}+2e\sqrt{e\cot(dx+c)}-\frac{e\left(e^2\right)^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\frac{e\cot(dx+c)+\left(e^2\right)^{\frac{1}{4}}\sqrt{e\cot(dx+c)}\sqrt{2}+\sqrt{e^2}}{e\cot(dx+c)-\left(e^2\right)^{\frac{1}{4}}\sqrt{e\cot(dx+c)}\sqrt{2}+\sqrt{e^2}}\right)+2\arctan\left(\frac{\sqrt{2}\sqrt{e\cot(d}}{\left(e^2\right)^{\frac{1}{4}}}\right.\right.}{4}\right.}{de}$
default	$-\frac{2a^2\left(\frac{(e\cot(dx+c))^{\frac{3}{2}}}{3}+2e\sqrt{e\cot(dx+c)}-\frac{e\left(e^2\right)^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\frac{e\cot(dx+c)+\left(e^2\right)^{\frac{1}{4}}\sqrt{e\cot(dx+c)}\sqrt{2}+\sqrt{e^2}}{e\cot(dx+c)-\left(e^2\right)^{\frac{1}{4}}\sqrt{e\cot(dx+c)}\sqrt{2}+\sqrt{e^2}}\right)+2\arctan\left(\frac{\sqrt{2}\sqrt{e\cot(d}}{\left(e^2\right)^{\frac{1}{4}}}\right.\right.}{4}\right.}{de}$
parts	$-\frac{a^2e\sqrt{2}\left(\ln\left(\frac{e\cot(dx+c)-\left(e^2\right)^{\frac{1}{4}}\sqrt{e\cot(dx+c)}\sqrt{2}+\sqrt{e^2}}{e\cot(dx+c)+\left(e^2\right)^{\frac{1}{4}}\sqrt{e\cot(dx+c)}\sqrt{2}+\sqrt{e^2}}\right)+2\arctan\left(\frac{\sqrt{2}\sqrt{e\cot(dx+c)}}{\left(e^2\right)^{\frac{1}{4}}}+1\right)-2\arctan\left(-\frac{\sqrt{2}\sqrt{e\cot(dx+c)}}{\left(e^2\right)^{\frac{1}{4}}}\right.\right.}{4d\left(e^2\right)^{\frac{1}{4}}}$

```
[In] int((e*cot(d*x+c))^(1/2)*(a+a*cot(d*x+c))^2,x,method=_RETURNVERBOSE)
```

```
[Out] -2/d*a^2/e*(1/3*(e*cot(d*x+c))^(3/2)+2*e*(e*cot(d*x+c))^(1/2)-1/4*e*(e^2)^(
1/4)*2^(1/2)*(ln((e*cot(d*x+c)+(e^2)^(1/4)*(e*cot(d*x+c))^(1/2)*2^(1/2)+(e^
2)^(1/2))/(e*cot(d*x+c)-(e^2)^(1/4)*(e*cot(d*x+c))^(1/2)*2^(1/2)+(e^2)^(1/2
```

```
)))+2*arctan(2^(1/2)/(e^2)^(1/4)*(e*cot(d*x+c))^(1/2)+1)-2*arctan(-2^(1/2)/
(e^2)^(1/4)*(e*cot(d*x+c))^(1/2)+1)))
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.30 (sec) , antiderivative size = 372, normalized size of antiderivative = 1.52

$$\int \sqrt{e\cot(c+dx)}(a+a\cot(c+dx))^2\,dx$$
$$= \frac{3\left(-\frac{a^8e^2}{d^4}\right)^{\frac{1}{4}} d\log\left(a^2\sqrt{\frac{e\cos(2\,dx+2\,c)+e}{\sin(2\,dx+2\,c)}}+\left(-\frac{a^8e^2}{d^4}\right)^{\frac{1}{4}} d\right)\sin\left(2\,dx+2\,c\right)+3i\left(-\frac{a^8e^2}{d^4}\right)^{\frac{1}{4}} d\log\left(a^2\sqrt{\frac{e\cos(2\,dx+2}{\sin(2\,dx+2}}\right.}{}$$

```
[In] integrate((e*cot(d*x+c))^(1/2)*(a+a*cot(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] 1/3*(3*(-a^8*e^2/d^4)^(1/4)*d*log(a^2*sqrt((e*cos(2*d*x + 2*c) + e)/sin(2*d
*x + 2*c)) + (-a^8*e^2/d^4)^(1/4)*d)*sin(2*d*x + 2*c) + 3*I*(-a^8*e^2/d^4)^
(1/4)*d*log(a^2*sqrt((e*cos(2*d*x + 2*c) + e)/sin(2*d*x + 2*c)) + I*(-a^8*e
^2/d^4)^(1/4)*d)*sin(2*d*x + 2*c) - 3*I*(-a^8*e^2/d^4)^(1/4)*d*log(a^2*sqrt
((e*cos(2*d*x + 2*c) + e)/sin(2*d*x + 2*c)) - I*(-a^8*e^2/d^4)^(1/4)*d)*sin
(2*d*x + 2*c) - 3*(-a^8*e^2/d^4)^(1/4)*d*log(a^2*sqrt((e*cos(2*d*x + 2*c) +
 e)/sin(2*d*x + 2*c)) - (-a^8*e^2/d^4)^(1/4)*d)*sin(2*d*x + 2*c) - 2*(a^2*c
os(2*d*x + 2*c) + 6*a^2*sin(2*d*x + 2*c) + a^2)*sqrt((e*cos(2*d*x + 2*c) +
e)/sin(2*d*x + 2*c)))/(d*sin(2*d*x + 2*c))
```

Sympy [F]

$$\int \sqrt{e\cot(c+dx)}(a+a\cot(c+dx))^2\,dx = a^2\left(\int \sqrt{e\cot\left(c+dx\right)}\,dx + \int 2\sqrt{e\cot\left(c+dx\right)}\cot\left(c+dx\right)dx + \int \sqrt{e\cot\left(c+dx\right)}\cot^2\left(c+dx\right)dx\right)$$

```
[In] integrate((e*cot(d*x+c))**(1/2)*(a+a*cot(d*x+c))**2,x)
```

```
[Out] a**2*(Integral(sqrt(e*cot(c + d*x)), x) + Integral(2*sqrt(e*cot(c + d*x))*c
ot(c + d*x), x) + Integral(sqrt(e*cot(c + d*x))*cot(c + d*x)**2, x))
```

Maxima [F(-2)]

Exception generated.

$$\int \sqrt{e\cot(c+dx)}(a+a\cot(c+dx))^2\,dx = \text{Exception raised: ValueError}$$

```
[In] integrate((e*cot(d*x+c))^(1/2)*(a+a*cot(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(e>0)', see `assume?` for more detai
ls)Is e
```

Giac [F]

$$\int \sqrt{e\cot(c+dx)}(a+a\cot(c+dx))^2\,dx = \int (a\cot(dx+c)+a)^2\sqrt{e\cot(dx+c)}\,dx$$

```
[In] integrate((e*cot(d*x+c))^(1/2)*(a+a*cot(d*x+c))^2,x, algorithm="giac")
```

```
[Out] integrate((a*cot(d*x + c) + a)^2*sqrt(e*cot(d*x + c)), x)
```

Mupad [B] (verification not implemented)

Time = 13.07 (sec) , antiderivative size = 104, normalized size of antiderivative = 0.43

$$\int \sqrt{e\cot(c+dx)}(a+a\cot(c+dx))^2\,dx = -\frac{4a^2\sqrt{e\cot(c+dx)}}{d} - \frac{2a^2(e\cot(c+dx))^{3/2}}{3de} - \frac{(-1)^{1/4}a^2\sqrt{e}\,\mathrm{atan}\left(\frac{(-1)^{1/4}\sqrt{e\cot(c+dx)}}{\sqrt{e}}\right)2i}{d} - \frac{2(-1)^{1/4}a^2\sqrt{e}\,\mathrm{atan}\left(\frac{(-1)^{1/4}\sqrt{e\cot(c+dx)}\,1i}{\sqrt{e}}\right)}{d}$$

```
[In] int((e*cot(c + d*x))^(1/2)*(a + a*cot(c + d*x))^2,x)
```

```
[Out] - (4*a^2*(e*cot(c + d*x))^(1/2))/d - (2*a^2*(e*cot(c + d*x))^(3/2))/(3*d*e)
 - ((-1)^(1/4)*a^2*e^(1/2)*atan(((-1)^(1/4)*(e*cot(c + d*x))^(1/2))/e^(1/2)
)*2i)/d - (2*(-1)^(1/4)*a^2*e^(1/2)*atan(((-1)^(1/4)*(e*cot(c + d*x))^(1/2)
*1i)/e^(1/2)))/d
```

3.11 $\int \frac{(a+a\cot(c+dx))^2}{\sqrt{e\cot(c+dx)}}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 222

$$\int \frac{(a+a\cot(c+dx))^2}{\sqrt{e\cot(c+dx)}}\,dx = \frac{\sqrt{2}a^2\arctan\left(1-\frac{\sqrt{2}\sqrt{e\cot(c+dx)}}{\sqrt{e}}\right)}{d\sqrt{e}} - \frac{\sqrt{2}a^2\arctan\left(1+\frac{\sqrt{2}\sqrt{e\cot(c+dx)}}{\sqrt{e}}\right)}{d\sqrt{e}} - \frac{2a^2\sqrt{e\cot(c+dx)}}{de} - \frac{a^2\log\left(\sqrt{e}+\sqrt{e}\cot(c+dx)-\sqrt{2}\sqrt{e\cot(c+dx)}\right)}{\sqrt{2}d\sqrt{e}} + \frac{a^2\log\left(\sqrt{e}+\sqrt{e}\cot(c+dx)+\sqrt{2}\sqrt{e\cot(c+dx)}\right)}{\sqrt{2}d\sqrt{e}}$$

```
[Out] -1/2*a^2*ln(e^(1/2)+cot(d*x+c)*e^(1/2)-2^(1/2)*(e*cot(d*x+c))^(1/2))/d*2^(1
/2)/e^(1/2)+1/2*a^2*ln(e^(1/2)+cot(d*x+c)*e^(1/2)+2^(1/2)*(e*cot(d*x+c))^(1
/2))/d*2^(1/2)/e^(1/2)+a^2*arctan(1-2^(1/2)*(e*cot(d*x+c))^(1/2)/e^(1/2))*2
^(1/2)/d/e^(1/2)-a^2*arctan(1+2^(1/2)*(e*cot(d*x+c))^(1/2)/e^(1/2))*2^(1/2)
/d/e^(1/2)-2*a^2*(e*cot(d*x+c))^(1/2)/d/e
```

Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 222, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.440$, Rules used = {3624, 12, 16, 3557, 335, 303, 1176, 631, 210, 1179, 642}

$$\int \frac{(a+a\cot(c+dx))^2}{\sqrt{e\cot(c+dx)}}\,dx = \frac{\sqrt{2}a^2\arctan\left(1-\frac{\sqrt{2}\sqrt{e\cot(c+dx)}}{\sqrt{e}}\right)}{d\sqrt{e}} - \frac{\sqrt{2}a^2\arctan\left(\frac{\sqrt{2}\sqrt{e\cot(c+dx)}}{\sqrt{e}}+1\right)}{d\sqrt{e}} - \frac{2a^2\sqrt{e\cot(c+dx)}}{de} - \frac{a^2\log\left(\sqrt{e}\cot(c+dx)-\sqrt{2}\sqrt{e\cot(c+dx)}+\sqrt{e}\right)}{\sqrt{2}d\sqrt{e}} + \frac{a^2\log\left(\sqrt{e}\cot(c+dx)+\sqrt{2}\sqrt{e\cot(c+dx)}+\sqrt{e}\right)}{\sqrt{2}d\sqrt{e}}$$

```
[In] Int[(a + a*Cot[c + d*x])^2/Sqrt[e*Cot[c + d*x]],x]
```

```
[Out] (Sqrt[2]*a^2*ArcTan[1 - (Sqrt[2]*Sqrt[e*Cot[c + d*x]])/Sqrt[e]])/(d*Sqrt[e]
) - (Sqrt[2]*a^2*ArcTan[1 + (Sqrt[2]*Sqrt[e*Cot[c + d*x]])/Sqrt[e]])/(d*Sqr
t[e]) - (2*a^2*Sqrt[e*Cot[c + d*x]])/(d*e) - (a^2*Log[Sqrt[e] + Sqrt[e]*Cot
[c + d*x] - Sqrt[2]*Sqrt[e*Cot[c + d*x]]])/(Sqrt[2]*d*Sqrt[e]) + (a^2*Log[S
qrt[e] + Sqrt[e]*Cot[c + d*x] + Sqrt[2]*Sqrt[e*Cot[c + d*x]]])/(Sqrt[2]*d*S
qrt[e])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 16

```
Int[(u_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Dist[1/b^m, Int[u*(b*v)
^(m + n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 303

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*s), Int[(r + s*x^2)/(a + b*x^4
```

```
), x], x] - Dist[1/(2*s), Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1179

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 3557

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[b/d, Subst[Int[
x^n/(b^2 + x^2), x], x, b*Tan[c + d*x]], x] /; FreeQ[{b, c, d, n}, x] &&  !
IntegerQ[n]
```

Rule 3624

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)])^2, x_Symbol] :> Simp[d^2*((a + b*Tan[e + f*x])^(m + 1)/(b*f*(
m + 1))), x] + Int[(a + b*Tan[e + f*x])^m*Simp[c^2 - d^2 + 2*c*d*Tan[e + f*
x], x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b*c - a*d, 0] &&  !LeQ
[m, -1] &&  !(EqQ[m, 2] && EqQ[a, 0])
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\frac{2a^2\sqrt{e\cot(c+dx)}}{de}+\int \frac{2a^2\cot(c+dx)}{\sqrt{e\cot(c+dx)}}\,dx\\
&= -\frac{2a^2\sqrt{e\cot(c+dx)}}{de}+\left(2a^2\right)\int \frac{\cot(c+dx)}{\sqrt{e\cot(c+dx)}}\,dx\\
&= -\frac{2a^2\sqrt{e\cot(c+dx)}}{de}+\frac{\left(2a^2\right)\int \sqrt{e\cot(c+dx)}\,dx}{e}\\
&= -\frac{2a^2\sqrt{e\cot(c+dx)}}{de}-\frac{\left(2a^2\right)\text{Subst}\left(\int \frac{\sqrt{x}}{e^2+x^2}\,dx,x,e\cot(c+dx)\right)}{d}\\
&= -\frac{2a^2\sqrt{e\cot(c+dx)}}{de}-\frac{\left(4a^2\right)\text{Subst}\left(\int \frac{x^2}{e^2+x^4}\,dx,x,\sqrt{e\cot(c+dx)}\right)}{d}\\
&= -\frac{2a^2\sqrt{e\cot(c+dx)}}{de}+\frac{\left(2a^2\right)\text{Subst}\left(\int \frac{e-x^2}{e^2+x^4}\,dx,x,\sqrt{e\cot(c+dx)}\right)}{d}\\
&\quad -\frac{\left(2a^2\right)\text{Subst}\left(\int \frac{e+x^2}{e^2+x^4}\,dx,x,\sqrt{e\cot(c+dx)}\right)}{d}\\
&= -\frac{2a^2\sqrt{e\cot(c+dx)}}{de}-\frac{a^2\text{Subst}\left(\int \frac{1}{e-\sqrt{2}\sqrt{e}x+x^2}\,dx,x,\sqrt{e\cot(c+dx)}\right)}{d}\\
&\quad -\frac{a^2\text{Subst}\left(\int \frac{1}{e+\sqrt{2}\sqrt{e}x+x^2}\,dx,x,\sqrt{e\cot(c+dx)}\right)}{d}\\
&\quad -\frac{a^2\text{Subst}\left(\int \frac{\sqrt{2}\sqrt{e}+2x}{-e-\sqrt{2}\sqrt{e}x-x^2}\,dx,x,\sqrt{e\cot(c+dx)}\right)}{\sqrt{2}d\sqrt{e}}\\
&\quad -\frac{a^2\text{Subst}\left(\int \frac{\sqrt{2}\sqrt{e}-2x}{-e+\sqrt{2}\sqrt{e}x-x^2}\,dx,x,\sqrt{e\cot(c+dx)}\right)}{\sqrt{2}d\sqrt{e}}
\end{aligned}$$

$$
\begin{aligned}
&= -\frac{2a^2\sqrt{e\cot(c+dx)}}{de} - \frac{a^2\log\left(\sqrt{e}+\sqrt{e}\cot(c+dx)-\sqrt{2}\sqrt{e\cot(c+dx)}\right)}{\sqrt{2}d\sqrt{e}} \\
&\quad + \frac{a^2\log\left(\sqrt{e}+\sqrt{e}\cot(c+dx)+\sqrt{2}\sqrt{e\cot(c+dx)}\right)}{\sqrt{2}d\sqrt{e}} \\
&\quad - \frac{\left(\sqrt{2}a^2\right)\text{Subst}\left(\int\frac{1}{-1-x^2}\,dx, x, 1-\frac{\sqrt{2}\sqrt{e\cot(c+dx)}}{\sqrt{e}}\right)}{d\sqrt{e}} \\
&\quad + \frac{\left(\sqrt{2}a^2\right)\text{Subst}\left(\int\frac{1}{-1-x^2}\,dx, x, 1+\frac{\sqrt{2}\sqrt{e\cot(c+dx)}}{\sqrt{e}}\right)}{d\sqrt{e}} \\
&= \frac{\sqrt{2}a^2\arctan\left(1-\frac{\sqrt{2}\sqrt{e\cot(c+dx)}}{\sqrt{e}}\right)}{d\sqrt{e}} - \frac{\sqrt{2}a^2\arctan\left(1+\frac{\sqrt{2}\sqrt{e\cot(c+dx)}}{\sqrt{e}}\right)}{d\sqrt{e}} \\
&\quad - \frac{2a^2\sqrt{e\cot(c+dx)}}{de} - \frac{a^2\log\left(\sqrt{e}+\sqrt{e}\cot(c+dx)-\sqrt{2}\sqrt{e\cot(c+dx)}\right)}{\sqrt{2}d\sqrt{e}} \\
&\quad + \frac{a^2\log\left(\sqrt{e}+\sqrt{e}\cot(c+dx)+\sqrt{2}\sqrt{e\cot(c+dx)}\right)}{\sqrt{2}d\sqrt{e}}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 6.11 (sec) , antiderivative size = 253, normalized size of antiderivative = 1.14

$$
\begin{aligned}
\int\frac{(a+a\cot(c+dx))^2}{\sqrt{e\cot(c+dx)}}\,dx &= -\frac{2\cos(c+dx)(a+a\cot(c+dx))^2\sin(c+dx)}{d\sqrt{e\cot(c+dx)}(\cos(c+dx)+\sin(c+dx))^2} \\
&\quad - \frac{2\arctan\left(\sqrt[4]{-\cot(c+dx)}\sqrt[4]{\cot(c+dx)}\right)\sqrt[4]{-\cot(c+dx)}\sqrt[4]{\cot(c+dx)}(a+a\cot(c+dx))^2\sin^2(c+d}{d\sqrt{e\cot(c+dx)}(\cos(c+dx)+\sin(c+dx))^2} \\
&\quad + \frac{2\text{arctanh}\left(\sqrt[4]{-\cot(c+dx)}\sqrt[4]{\cot(c+dx)}\right)\sqrt[4]{-\cot(c+dx)}\sqrt[4]{\cot(c+dx)}(a+a\cot(c+dx))^2\sin^2(c+d}{d\sqrt{e\cot(c+dx)}(\cos(c+dx)+\sin(c+dx))^2}
\end{aligned}
$$

```
[In] Integrate[(a + a*Cot[c + d*x])^2/Sqrt[e*Cot[c + d*x]],x]
```

```
[Out] (-2*Cos[c + d*x]*(a + a*Cot[c + d*x])^2*Sin[c + d*x])/(d*Sqrt[e*Cot[c + d*x
]]*(Cos[c + d*x] + Sin[c + d*x])^2) - (2*ArcTan[(-Cot[c + d*x])^(1/4)*Cot[c
 + d*x]^(1/4)]*(-Cot[c + d*x])^(1/4)*Cot[c + d*x]^(1/4)*(a + a*Cot[c + d*x]
)^2*Sin[c + d*x]^2)/(d*Sqrt[e*Cot[c + d*x]]*(Cos[c + d*x] + Sin[c + d*x])^2
) + (2*ArcTanh[(-Cot[c + d*x])^(1/4)*Cot[c + d*x]^(1/4)]*(-Cot[c + d*x])^(1
/4)*Cot[c + d*x]^(1/4)*(a + a*Cot[c + d*x])^2*Sin[c + d*x]^2)/(d*Sqrt[e*Cot
[c + d*x]]*(Cos[c + d*x] + Sin[c + d*x])^2)
```

Maple [A] (verified)

Time = 0.05 (sec) , antiderivative size = 155, normalized size of antiderivative = 0.70

method	result
derivativedivides	$-\frac{2a^2\left(\sqrt{e\cot(dx+c)}+\frac{e\sqrt{2}\left(\ln\left(\frac{e\cot(dx+c)-\left(e^2\right)^{\frac{1}{4}}\sqrt{e\cot(dx+c)}\sqrt{2}+\sqrt{e^2}}{e\cot(dx+c)+\left(e^2\right)^{\frac{1}{4}}\sqrt{e\cot(dx+c)}\sqrt{2}+\sqrt{e^2}}\right)+2\arctan\left(\frac{\sqrt{2}\sqrt{e\cot(dx+c)}}{\left(e^2\right)^{\frac{1}{4}}}+1\right)-2\arctan\left(-\frac{\sqrt{2}}{}\right.\right.}{4\left(e^2\right)^{\frac{1}{4}}}\right.}{de}$
default	$-\frac{2a^2\left(\sqrt{e\cot(dx+c)}+\frac{e\sqrt{2}\left(\ln\left(\frac{e\cot(dx+c)-\left(e^2\right)^{\frac{1}{4}}\sqrt{e\cot(dx+c)}\sqrt{2}+\sqrt{e^2}}{e\cot(dx+c)+\left(e^2\right)^{\frac{1}{4}}\sqrt{e\cot(dx+c)}\sqrt{2}+\sqrt{e^2}}\right)+2\arctan\left(\frac{\sqrt{2}\sqrt{e\cot(dx+c)}}{\left(e^2\right)^{\frac{1}{4}}}+1\right)-2\arctan\left(-\frac{\sqrt{2}}{}\right.\right.}{4\left(e^2\right)^{\frac{1}{4}}}\right.}{de}$
parts	$-\frac{a^2\left(e^2\right)^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\frac{e\cot(dx+c)+\left(e^2\right)^{\frac{1}{4}}\sqrt{e\cot(dx+c)}\sqrt{2}+\sqrt{e^2}}{e\cot(dx+c)-\left(e^2\right)^{\frac{1}{4}}\sqrt{e\cot(dx+c)}\sqrt{2}+\sqrt{e^2}}\right)+2\arctan\left(\frac{\sqrt{2}\sqrt{e\cot(dx+c)}}{\left(e^2\right)^{\frac{1}{4}}}+1\right)-2\arctan\left(-\frac{\sqrt{2}\sqrt{e\cot(d}}{\left(e^2\right)^{\frac{1}{4}}}\right.\right.}{4de}$

```
[In] int((a+a*cot(d*x+c))^2/(e*cot(d*x+c))^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -2/d*a^2/e*((e*cot(d*x+c))^(1/2)+1/4*e/(e^2)^(1/4)*2^(1/2)*(ln((e*cot(d*x+c
)-(e^2)^(1/4)*(e*cot(d*x+c))^(1/2)*2^(1/2)+(e^2)^(1/2))/(e*cot(d*x+c)+(e^2)
^(1/4)*(e*cot(d*x+c))^(1/2)*2^(1/2)+(e^2)^(1/2)))+2*arctan(2^(1/2)/(e^2)^(1
/4)*(e*cot(d*x+c))^(1/2)+1)-2*arctan(-2^(1/2)/(e^2)^(1/4)*(e*cot(d*x+c))^(1
/2)+1)))
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.30 (sec) , antiderivative size = 323, normalized size of antiderivative = 1.45

$$\int \frac{(a+a\cot(c+dx))^2}{\sqrt{e\cot(c+dx)}}\,dx =$$
$$-\frac{\left(-\frac{a^8}{d^4e^2}\right)^{\frac{1}{4}}de\log\left(a^6\sqrt{\frac{e\cos(2\,dx+2\,c)+e}{\sin(2\,dx+2\,c)}}+\left(-\frac{a^8}{d^4e^2}\right)^{\frac{3}{4}}d^3e^2\right)-i\left(-\frac{a^8}{d^4e^2}\right)^{\frac{1}{4}}de\log\left(a^6\sqrt{\frac{e\cos(2\,dx+2\,c)+e}{\sin(2\,dx+2\,c)}}+i\left(-\right.\right.}{}$$

```
[In] integrate((a+a*cot(d*x+c))^2/(e*cot(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] -((-a^8/(d^4*e^2))^(1/4)*d*e*log(a^6*sqrt((e*cos(2*d*x + 2*c) + e)/sin(2*d*
x + 2*c)) + (-a^8/(d^4*e^2))^(3/4)*d^3*e^2) - I*(-a^8/(d^4*e^2))^(1/4)*d*e*
```

```
log(a^6*sqrt((e*cos(2*d*x + 2*c) + e)/sin(2*d*x + 2*c)) + I*(-a^8/(d^4*e^2)
)^(3/4)*d^3*e^2) + I*(-a^8/(d^4*e^2))^(1/4)*d*e*log(a^6*sqrt((e*cos(2*d*x +
 2*c) + e)/sin(2*d*x + 2*c)) - I*(-a^8/(d^4*e^2))^(3/4)*d^3*e^2) - (-a^8/(d
^4*e^2))^(1/4)*d*e*log(a^6*sqrt((e*cos(2*d*x + 2*c) + e)/sin(2*d*x + 2*c))
- (-a^8/(d^4*e^2))^(3/4)*d^3*e^2) + 2*a^2*sqrt((e*cos(2*d*x + 2*c) + e)/sin
(2*d*x + 2*c)))/(d*e)
```

Sympy [F]

$$\int \frac{(a+a\cot(c+dx))^2}{\sqrt{e\cot(c+dx)}}\,dx = a^2\left(\int \frac{1}{\sqrt{e\cot(c+dx)}}\,dx + \int \frac{2\cot(c+dx)}{\sqrt{e\cot(c+dx)}}\,dx + \int \frac{\cot^2(c+dx)}{\sqrt{e\cot(c+dx)}}\,dx\right)$$

```
[In] integrate((a+a*cot(d*x+c))**2/(e*cot(d*x+c))**(1/2),x)
```

```
[Out] a**2*(Integral(1/sqrt(e*cot(c + d*x)), x) + Integral(2*cot(c + d*x)/sqrt(e*
cot(c + d*x)), x) + Integral(cot(c + d*x)**2/sqrt(e*cot(c + d*x)), x))
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{(a+a\cot(c+dx))^2}{\sqrt{e\cot(c+dx)}}\,dx = \text{Exception raised: ValueError}$$

```
[In] integrate((a+a*cot(d*x+c))^2/(e*cot(d*x+c))^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(e>0)', see `assume?` for more detai
ls)Is e
```

Giac [F]

$$\int \frac{(a+a\cot(c+dx))^2}{\sqrt{e\cot(c+dx)}}\,dx = \int \frac{(a\cot(dx+c)+a)^2}{\sqrt{e\cot(dx+c)}}\,dx$$

```
[In] integrate((a+a*cot(d*x+c))^2/(e*cot(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] integrate((a*cot(d*x + c) + a)^2/sqrt(e*cot(d*x + c)), x)
```

Mupad [B] (verification not implemented)

Time = 12.74 (sec) , antiderivative size = 86, normalized size of antiderivative = 0.39

$$\int \frac{(a+a\cot(c+dx))^2}{\sqrt{e\cot(c+dx)}}\,dx = \frac{2\,(-1)^{1/4}\,a^2\,\mathrm{atanh}\left(\frac{(-1)^{1/4}\,\sqrt{e\cot(c+d\,x)}}{\sqrt{e}}\right)}{d\,\sqrt{e}} - \frac{2\,(-1)^{1/4}\,a^2\,\mathrm{atan}\left(\frac{(-1)^{1/4}\,\sqrt{e\cot(c+d\,x)}}{\sqrt{e}}\right)}{d\,\sqrt{e}} - \frac{2\,a^2\,\sqrt{e\cot\left(c+d\,x\right)}}{d\,e}$$

```
[In] int((a + a*cot(c + d*x))^2/(e*cot(c + d*x))^(1/2),x)
```

```
[Out] (2*(-1)^(1/4)*a^2*atanh(((-1)^(1/4)*(e*cot(c + d*x))^(1/2))/e^(1/2)))/(d*e^
(1/2)) - (2*(-1)^(1/4)*a^2*atan(((-1)^(1/4)*(e*cot(c + d*x))^(1/2))/e^(1/2)
))/(d*e^(1/2)) - (2*a^2*(e*cot(c + d*x))^(1/2))/(d*e)
```

3.12 $\int \frac{(a+a\cot(c+dx))^2}{(e\cot(c+dx))^{3/2}}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 222

$$\int \frac{(a+a\cot(c+dx))^2}{(e\cot(c+dx))^{3/2}}\,dx = \frac{\sqrt{2}a^2\arctan\left(1-\frac{\sqrt{2}\sqrt{e\cot(c+dx)}}{\sqrt{e}}\right)}{de^{3/2}} - \frac{\sqrt{2}a^2\arctan\left(1+\frac{\sqrt{2}\sqrt{e\cot(c+dx)}}{\sqrt{e}}\right)}{de^{3/2}} + \frac{2a^2}{de\sqrt{e\cot(c+dx)}} + \frac{a^2\log\left(\sqrt{e}+\sqrt{e}\cot(c+dx)-\sqrt{2}\sqrt{e\cot(c+dx)}\right)}{\sqrt{2}de^{3/2}} - \frac{a^2\log\left(\sqrt{e}+\sqrt{e}\cot(c+dx)+\sqrt{2}\sqrt{e\cot(c+dx)}\right)}{\sqrt{2}de^{3/2}}$$

```
[Out] 1/2*a^2*ln(e^(1/2)+cot(d*x+c)*e^(1/2)-2^(1/2)*(e*cot(d*x+c))^(1/2))/d/e^(3/
2)*2^(1/2)-1/2*a^2*ln(e^(1/2)+cot(d*x+c)*e^(1/2)+2^(1/2)*(e*cot(d*x+c))^(1/
2))/d/e^(3/2)*2^(1/2)+a^2*arctan(1-2^(1/2)*(e*cot(d*x+c))^(1/2)/e^(1/2))*2^
(1/2)/d/e^(3/2)-a^2*arctan(1+2^(1/2)*(e*cot(d*x+c))^(1/2)/e^(1/2))*2^(1/2)/
d/e^(3/2)+2*a^2/d/e/(e*cot(d*x+c))^(1/2)
```

Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 222, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.400, Rules used = {3623, 12, 3557, 335, 217, 1179, 642, 1176, 631, 210}

$$\int \frac{(a+a\cot(c+dx))^2}{(e\cot(c+dx))^{3/2}}\,dx = \frac{\sqrt{2}a^2\arctan\left(1-\frac{\sqrt{2}\sqrt{e\cot(c+dx)}}{\sqrt{e}}\right)}{de^{3/2}} - \frac{\sqrt{2}a^2\arctan\left(\frac{\sqrt{2}\sqrt{e\cot(c+dx)}}{\sqrt{e}}+1\right)}{de^{3/2}} + \frac{a^2\log\left(\sqrt{e}\cot(c+dx)-\sqrt{2}\sqrt{e\cot(c+dx)}+\sqrt{e}\right)}{\sqrt{2}de^{3/2}} - \frac{a^2\log\left(\sqrt{e}\cot(c+dx)+\sqrt{2}\sqrt{e\cot(c+dx)}+\sqrt{e}\right)}{\sqrt{2}de^{3/2}} + \frac{2a^2}{de\sqrt{e\cot(c+dx)}}$$

```
[In] Int[(a + a*Cot[c + d*x])^2/(e*Cot[c + d*x])^(3/2),x]
```

```
[Out] (Sqrt[2]*a^2*ArcTan[1 - (Sqrt[2]*Sqrt[e*Cot[c + d*x]])/Sqrt[e]])/(d*e^(3/2)
) - (Sqrt[2]*a^2*ArcTan[1 + (Sqrt[2]*Sqrt[e*Cot[c + d*x]])/Sqrt[e]])/(d*e^(
3/2)) + (2*a^2)/(d*e*Sqrt[e*Cot[c + d*x]]) + (a^2*Log[Sqrt[e] + Sqrt[e]*Cot
[c + d*x] - Sqrt[2]*Sqrt[e*Cot[c + d*x]]])/(Sqrt[2]*d*e^(3/2)) - (a^2*Log[S
qrt[e] + Sqrt[e]*Cot[c + d*x] + Sqrt[2]*Sqrt[e*Cot[c + d*x]]])/(Sqrt[2]*d*e
^(3/2))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4),
 x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b
}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &&
AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1179

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 3557

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[b/d, Subst[Int[
x^n/(b^2 + x^2), x], x, b*Tan[c + d*x]], x] /; FreeQ[{b, c, d, n}, x] &&  !
IntegerQ[n]
```

Rule 3623

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)])^2, x_Symbol] :> Simp[(b*c - a*d)^2*((a + b*Tan[e + f*x])^(m +
 1)/(b*f*(m + 1)*(a^2 + b^2))), x] + Dist[1/(a^2 + b^2), Int[(a + b*Tan[e +
 f*x])^(m + 1)*Simp[a*c^2 + 2*b*c*d - a*d^2 - (b*c^2 - 2*a*c*d - b*d^2)*Tan
[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
&& LtQ[m, -1] && NeQ[a^2 + b^2, 0]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{2a^2}{de\sqrt{e\cot(c+dx)}} + \frac{\int \frac{2a^2e}{\sqrt{e\cot(c+dx)}}\,dx}{e^2}\\
&= \frac{2a^2}{de\sqrt{e\cot(c+dx)}} + \frac{(2a^2)\int \frac{1}{\sqrt{e\cot(c+dx)}}\,dx}{e}\\
&= \frac{2a^2}{de\sqrt{e\cot(c+dx)}} - \frac{(2a^2)\,\text{Subst}\left(\int \frac{1}{\sqrt{x}(e^2+x^2)}\,dx, x, e\cot(c+dx)\right)}{d}\\
&= \frac{2a^2}{de\sqrt{e\cot(c+dx)}} - \frac{(4a^2)\,\text{Subst}\left(\int \frac{1}{e^2+x^4}\,dx, x, \sqrt{e\cot(c+dx)}\right)}{d}\\
&= \frac{2a^2}{de\sqrt{e\cot(c+dx)}} - \frac{(2a^2)\,\text{Subst}\left(\int \frac{e-x^2}{e^2+x^4}\,dx, x, \sqrt{e\cot(c+dx)}\right)}{de}\\
&\quad - \frac{(2a^2)\,\text{Subst}\left(\int \frac{e+x^2}{e^2+x^4}\,dx, x, \sqrt{e\cot(c+dx)}\right)}{de}\\
&= \frac{2a^2}{de\sqrt{e\cot(c+dx)}} + \frac{a^2\text{Subst}\left(\int \frac{\sqrt{2}\sqrt{e}+2x}{-e-\sqrt{2}\sqrt{e}x-x^2}\,dx, x, \sqrt{e\cot(c+dx)}\right)}{\sqrt{2}de^{3/2}}\\
&\quad + \frac{a^2\text{Subst}\left(\int \frac{\sqrt{2}\sqrt{e}-2x}{-e+\sqrt{2}\sqrt{e}x-x^2}\,dx, x, \sqrt{e\cot(c+dx)}\right)}{\sqrt{2}de^{3/2}}\\
&\quad - \frac{a^2\text{Subst}\left(\int \frac{1}{e-\sqrt{2}\sqrt{e}x+x^2}\,dx, x, \sqrt{e\cot(c+dx)}\right)}{de}\\
&\quad - \frac{a^2\text{Subst}\left(\int \frac{1}{e+\sqrt{2}\sqrt{e}x+x^2}\,dx, x, \sqrt{e\cot(c+dx)}\right)}{de}\\
&= \frac{2a^2}{de\sqrt{e\cot(c+dx)}} + \frac{a^2\log\left(\sqrt{e}+\sqrt{e}\cot(c+dx)-\sqrt{2}\sqrt{e\cot(c+dx)}\right)}{\sqrt{2}de^{3/2}}\\
&\quad - \frac{a^2\log\left(\sqrt{e}+\sqrt{e}\cot(c+dx)+\sqrt{2}\sqrt{e\cot(c+dx)}\right)}{\sqrt{2}de^{3/2}}\\
&\quad - \frac{(\sqrt{2}a^2)\,\text{Subst}\left(\int \frac{1}{-1-x^2}\,dx, x, 1-\frac{\sqrt{2}\sqrt{e\cot(c+dx)}}{\sqrt{e}}\right)}{de^{3/2}}\\
&\quad + \frac{(\sqrt{2}a^2)\,\text{Subst}\left(\int \frac{1}{-1-x^2}\,dx, x, 1+\frac{\sqrt{2}\sqrt{e\cot(c+dx)}}{\sqrt{e}}\right)}{de^{3/2}}
\end{aligned}
$$

$$= \frac{\sqrt{2}a^2 \arctan\left(1 - \frac{\sqrt{2}\sqrt{e\cot(c+dx)}}{\sqrt{e}}\right)}{de^{3/2}} - \frac{\sqrt{2}a^2 \arctan\left(1 + \frac{\sqrt{2}\sqrt{e\cot(c+dx)}}{\sqrt{e}}\right)}{de^{3/2}} + \frac{2a^2}{de\sqrt{e\cot(c+dx)}} + \frac{a^2 \log\left(\sqrt{e} + \sqrt{e}\cot(c+dx) - \sqrt{2}\sqrt{e\cot(c+dx)}\right)}{\sqrt{2}de^{3/2}} - \frac{a^2 \log\left(\sqrt{e} + \sqrt{e}\cot(c+dx) + \sqrt{2}\sqrt{e\cot(c+dx)}\right)}{\sqrt{2}de^{3/2}}$$

Mathematica [A] (verified)

Time = 0.42 (sec) , antiderivative size = 184, normalized size of antiderivative = 0.83

$$\int \frac{(a + a\cot(c+dx))^2}{(e\cot(c+dx))^{3/2}}\,dx = \frac{a^2\left(4 + 2\sqrt{2}\arctan\left(1 - \sqrt{2}\sqrt{\cot(c+dx)}\right)\sqrt{\cot(c+dx)} - 2\sqrt{2}\arctan\left(1 + \right.\right.}{}$$

```
[In] Integrate[(a + a*Cot[c + d*x])^2/(e*Cot[c + d*x])^(3/2),x]
```

```
[Out] (a^2*(4 + 2*Sqrt[2]*ArcTan[1 - Sqrt[2]*Sqrt[Cot[c + d*x]]]*Sqrt[Cot[c + d*x
]] - 2*Sqrt[2]*ArcTan[1 + Sqrt[2]*Sqrt[Cot[c + d*x]]]*Sqrt[Cot[c + d*x]] +
Sqrt[2]*Sqrt[Cot[c + d*x]]*Log[1 - Sqrt[2]*Sqrt[Cot[c + d*x]] + Cot[c + d*x
]] - Sqrt[2]*Sqrt[Cot[c + d*x]]*Log[1 + Sqrt[2]*Sqrt[Cot[c + d*x]] + Cot[c
+ d*x]]))/(2*d*e*Sqrt[e*Cot[c + d*x]])
```

Maple [A] (verified)

Time = 0.04 (sec) , antiderivative size = 159, normalized size of antiderivative = 0.72

method	result
derivativedivides	$-\frac{2a^2\left(\frac{(e^2)^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\frac{e\cot(dx+c)+(e^2)^{\frac{1}{4}}\sqrt{e\cot(dx+c)}\sqrt{2}+\sqrt{e^2}}{e\cot(dx+c)-(e^2)^{\frac{1}{4}}\sqrt{e\cot(dx+c)}\sqrt{2}+\sqrt{e^2}}\right)+2\arctan\left(\frac{\sqrt{2}\sqrt{e\cot(dx+c)}}{(e^2)^{\frac{1}{4}}}+1\right)-2\arctan\left(-\frac{\sqrt{2}\sqrt{e\cot(dx+c}}{(e^2)^{\frac{1}{4}}}\right.\right.}{4e}\right.}{de}$
default	$-\frac{2a^2\left(\frac{(e^2)^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\frac{e\cot(dx+c)+(e^2)^{\frac{1}{4}}\sqrt{e\cot(dx+c)}\sqrt{2}+\sqrt{e^2}}{e\cot(dx+c)-(e^2)^{\frac{1}{4}}\sqrt{e\cot(dx+c)}\sqrt{2}+\sqrt{e^2}}\right)+2\arctan\left(\frac{\sqrt{2}\sqrt{e\cot(dx+c)}}{(e^2)^{\frac{1}{4}}}+1\right)-2\arctan\left(-\frac{\sqrt{2}\sqrt{e\cot(dx+c}}{(e^2)^{\frac{1}{4}}}\right.\right.}{4e}\right.}{de}$
parts	$-\frac{2a^2e\left(-\frac{\sqrt{2}\left(\ln\left(\frac{e\cot(dx+c)-(e^2)^{\frac{1}{4}}\sqrt{e\cot(dx+c)}\sqrt{2}+\sqrt{e^2}}{e\cot(dx+c)+(e^2)^{\frac{1}{4}}\sqrt{e\cot(dx+c)}\sqrt{2}+\sqrt{e^2}}\right)+2\arctan\left(\frac{\sqrt{2}\sqrt{e\cot(dx+c)}}{(e^2)^{\frac{1}{4}}}+1\right)-2\arctan\left(-\frac{\sqrt{2}\sqrt{e\cot(dx+c)}}{(e^2)^{\frac{1}{4}}}+\right.\right.}{8e^2(e^2)^{\frac{1}{4}}}\right.}{d}$

```
[In] int((a+a*cot(d*x+c))^2/(e*cot(d*x+c))^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] -2/d*a^2/e*(1/4/e*(e^2)^(1/4)*2^(1/2)*(ln((e*cot(d*x+c)+(e^2)^(1/4)*(e*cot(
d*x+c))^(1/2)*2^(1/2)+(e^2)^(1/2))/(e*cot(d*x+c)-(e^2)^(1/4)*(e*cot(d*x+c))
^(1/2)*2^(1/2)+(e^2)^(1/2)))+2*arctan(2^(1/2)/(e^2)^(1/4)*(e*cot(d*x+c))^(1
/2)+1)-2*arctan(-2^(1/2)/(e^2)^(1/4)*(e*cot(d*x+c))^(1/2)+1))-1/(e*cot(d*x+
c))^(1/2))
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.28 (sec) , antiderivative size = 413, normalized size of antiderivative = 1.86

$$\int \frac{(a+a\cot(c+dx))^2}{(e\cot(c+dx))^{3/2}}\,dx = \frac{2\,a^2\sqrt{\frac{e\cos(2\,dx+2\,c)+e}{\sin(2\,dx+2\,c)}}\sin\left(2\,dx+2\,c\right)-\left(de^2\cos\left(2\,dx+2\,c\right)+de^2\right)\left(-\frac{a^8}{d^4e^6}\right)^{\frac{1}{4}}\log\left(\right.}{}$$

```
[In] integrate((a+a*cot(d*x+c))^2/(e*cot(d*x+c))^(3/2),x, algorithm="fricas")
```

```
[Out] (2*a^2*sqrt((e*cos(2*d*x + 2*c) + e)/sin(2*d*x + 2*c))*sin(2*d*x + 2*c) - (
d*e^2*cos(2*d*x + 2*c) + d*e^2)*(-a^8/(d^4*e^6))^(1/4)*log(d*e^2*(-a^8/(d^4
*e^6))^(1/4) + a^2*sqrt((e*cos(2*d*x + 2*c) + e)/sin(2*d*x + 2*c))) + (-I*d
*e^2*cos(2*d*x + 2*c) - I*d*e^2)*(-a^8/(d^4*e^6))^(1/4)*log(I*d*e^2*(-a^8/(
d^4*e^6))^(1/4) + a^2*sqrt((e*cos(2*d*x + 2*c) + e)/sin(2*d*x + 2*c))) + (I
*d*e^2*cos(2*d*x + 2*c) + I*d*e^2)*(-a^8/(d^4*e^6))^(1/4)*log(-I*d*e^2*(-a^
8/(d^4*e^6))^(1/4) + a^2*sqrt((e*cos(2*d*x + 2*c) + e)/sin(2*d*x + 2*c))) +
 (d*e^2*cos(2*d*x + 2*c) + d*e^2)*(-a^8/(d^4*e^6))^(1/4)*log(-d*e^2*(-a^8/(
```

```
d^4*e^6))^(1/4) + a^2*sqrt((e*cos(2*d*x + 2*c) + e)/sin(2*d*x + 2*c))))/(d*
e^2*cos(2*d*x + 2*c) + d*e^2)
```

Sympy [F]

$$\int \frac{(a+a\cot(c+dx))^2}{(e\cot(c+dx))^{3/2}}\,dx = a^2\left(\int \frac{1}{(e\cot{(c+dx)})^{\frac{3}{2}}}\,dx + \int \frac{2\cot{(c+dx)}}{(e\cot{(c+dx)})^{\frac{3}{2}}}\,dx + \int \frac{\cot^2{(c+dx)}}{(e\cot{(c+dx)})^{\frac{3}{2}}}\,dx\right)$$

```
[In] integrate((a+a*cot(d*x+c))**2/(e*cot(d*x+c))**(3/2),x)
```

```
[Out] a**2*(Integral((e*cot(c + d*x))**(-3/2), x) + Integral(2*cot(c + d*x)/(e*co
t(c + d*x))**(3/2), x) + Integral(cot(c + d*x)**2/(e*cot(c + d*x))**(3/2),
x))
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{(a+a\cot(c+dx))^2}{(e\cot(c+dx))^{3/2}}\,dx = \text{Exception raised: ValueError}$$

```
[In] integrate((a+a*cot(d*x+c))^2/(e*cot(d*x+c))^(3/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(e>0)', see `assume?` for more detai
ls)Is e
```

Giac [F]

$$\int \frac{(a+a\cot(c+dx))^2}{(e\cot(c+dx))^{3/2}}\,dx = \int \frac{(a\cot{(dx+c)}+a)^2}{(e\cot{(dx+c)})^{\frac{3}{2}}}\,dx$$

```
[In] integrate((a+a*cot(d*x+c))^2/(e*cot(d*x+c))^(3/2),x, algorithm="giac")
```

```
[Out] integrate((a*cot(d*x + c) + a)^2/(e*cot(d*x + c))^(3/2), x)
```

Mupad [B] (verification not implemented)

Time = 12.88 (sec) , antiderivative size = 86, normalized size of antiderivative = 0.39

$$\int \frac{(a+a\cot(c+dx))^2}{(e\cot(c+dx))^{3/2}}\,dx=\frac{2\,a^2}{d\,e\,\sqrt{e\cot\left(c+d\,x\right)}}+\frac{{\left(-1\right)}^{1/4}\,a^2\,\mathrm{atan}\left(\frac{{\left(-1\right)}^{1/4}\,\sqrt{e\cot\left(c+d\,x\right)}}{\sqrt{e}}\right)\,2{}\mathrm{i}}{d\,e^{3/2}}+\frac{{\left(-1\right)}^{1/4}\,a^2\,\mathrm{atanh}\left(\frac{{\left(-1\right)}^{1/4}\,\sqrt{e\cot\left(c+d\,x\right)}}{\sqrt{e}}\right)\,2{}\mathrm{i}}{d\,e^{3/2}}$$

```
[In] int((a + a*cot(c + d*x))^2/(e*cot(c + d*x))^(3/2),x)
```

```
[Out] (2*a^2)/(d*e*(e*cot(c + d*x))^(1/2)) + ((-1)^(1/4)*a^2*atan(((-1)^(1/4)*(e*
cot(c + d*x))^(1/2))/e^(1/2))*2i)/(d*e^(3/2)) + ((-1)^(1/4)*a^2*atanh(((-1)
^(1/4)*(e*cot(c + d*x))^(1/2))/e^(1/2))*2i)/(d*e^(3/2))
```

3.13 $\int \frac{(a+a\cot(c+dx))^2}{(e\cot(c+dx))^{5/2}}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 247

$$\begin{aligned}\int \frac{(a+a\cot(c+dx))^2}{(e\cot(c+dx))^{5/2}}\,dx &= -\frac{\sqrt{2}a^2\arctan\left(1-\frac{\sqrt{2}\sqrt{e\cot(c+dx)}}{\sqrt{e}}\right)}{de^{5/2}}\\ &+\frac{\sqrt{2}a^2\arctan\left(1+\frac{\sqrt{2}\sqrt{e\cot(c+dx)}}{\sqrt{e}}\right)}{de^{5/2}}+\frac{2a^2}{3de(e\cot(c+dx))^{3/2}}\\ &+\frac{4a^2}{de^2\sqrt{e\cot(c+dx)}}+\frac{a^2\log\left(\sqrt{e}+\sqrt{e}\cot(c+dx)-\sqrt{2}\sqrt{e\cot(c+dx)}\right)}{\sqrt{2}de^{5/2}}\\ &-\frac{a^2\log\left(\sqrt{e}+\sqrt{e}\cot(c+dx)+\sqrt{2}\sqrt{e\cot(c+dx)}\right)}{\sqrt{2}de^{5/2}}\end{aligned}$$

```
[Out] 2/3*a^2/d/e/(e*cot(d*x+c))^(3/2)+1/2*a^2*ln(e^(1/2)+cot(d*x+c)*e^(1/2)-2^(1
/2)*(e*cot(d*x+c))^(1/2))/d/e^(5/2)*2^(1/2)-1/2*a^2*ln(e^(1/2)+cot(d*x+c)*e
^(1/2)+2^(1/2)*(e*cot(d*x+c))^(1/2))/d/e^(5/2)*2^(1/2)-a^2*arctan(1-2^(1/2)
*(e*cot(d*x+c))^(1/2)/e^(1/2))*2^(1/2)/d/e^(5/2)+a^2*arctan(1+2^(1/2)*(e*co
t(d*x+c))^(1/2)/e^(1/2))*2^(1/2)/d/e^(5/2)+4*a^2/d/e^2/(e*cot(d*x+c))^(1/2)
```

Rubi [A] (verified)

Time = 0.31 (sec) , antiderivative size = 247, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.440, Rules used = {3623, 12, 3555, 3557, 335, 303, 1176, 631, 210, 1179, 642}

$$\int \frac{(a+a\cot(c+dx))^2}{(e\cot(c+dx))^{5/2}}\,dx = -\frac{\sqrt{2}a^2\arctan\left(1-\frac{\sqrt{2}\sqrt{e\cot(c+dx)}}{\sqrt{e}}\right)}{de^{5/2}} + \frac{\sqrt{2}a^2\arctan\left(\frac{\sqrt{2}\sqrt{e\cot(c+dx)}}{\sqrt{e}}+1\right)}{de^{5/2}} + \frac{a^2\log\left(\sqrt{e}\cot(c+dx)-\sqrt{2}\sqrt{e\cot(c+dx)}+\sqrt{e}\right)}{\sqrt{2}de^{5/2}} - \frac{a^2\log\left(\sqrt{e}\cot(c+dx)+\sqrt{2}\sqrt{e\cot(c+dx)}+\sqrt{e}\right)}{\sqrt{2}de^{5/2}} + \frac{4a^2}{de^2\sqrt{e\cot(c+dx)}} + \frac{2a^2}{3de(e\cot(c+dx))^{3/2}}$$

```
[In] Int[(a + a*Cot[c + d*x])^2/(e*Cot[c + d*x])^(5/2),x]
```

```
[Out] -((Sqrt[2]*a^2*ArcTan[1 - (Sqrt[2]*Sqrt[e*Cot[c + d*x]])/Sqrt[e]])/(d*e^(5/
2))) + (Sqrt[2]*a^2*ArcTan[1 + (Sqrt[2]*Sqrt[e*Cot[c + d*x]])/Sqrt[e]])/(d*
e^(5/2)) + (2*a^2)/(3*d*e*(e*Cot[c + d*x])^(3/2)) + (4*a^2)/(d*e^2*Sqrt[e*C
ot[c + d*x]]) + (a^2*Log[Sqrt[e] + Sqrt[e]*Cot[c + d*x] - Sqrt[2]*Sqrt[e*Co
t[c + d*x]]])/(Sqrt[2]*d*e^(5/2)) - (a^2*Log[Sqrt[e] + Sqrt[e]*Cot[c + d*x]
 + Sqrt[2]*Sqrt[e*Cot[c + d*x]]])/(Sqrt[2]*d*e^(5/2))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 303

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*s), Int[(r + s*x^2)/(a + b*x^4
), x], x] - Dist[1/(2*s), Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
```

```
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1179

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 3555

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*Tan[c + d*x]
)^(n + 1)/(b*d*(n + 1)), x] - Dist[1/b^2, Int[(b*Tan[c + d*x])^(n + 2), x],
 x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1]
```

Rule 3557

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[b/d, Subst[Int[
x^n/(b^2 + x^2), x], x, b*Tan[c + d*x]], x] /; FreeQ[{b, c, d, n}, x] &&  !
```

```
IntegerQ[n]
```

Rule 3623

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)])^2, x_Symbol] :> Simp[(b*c - a*d)^2*((a + b*Tan[e + f*x])^(m +
 1)/(b*f*(m + 1)*(a^2 + b^2))), x] + Dist[1/(a^2 + b^2), Int[(a + b*Tan[e +
 f*x])^(m + 1)*Simp[a*c^2 + 2*b*c*d - a*d^2 - (b*c^2 - 2*a*c*d - b*d^2)*Tan
[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
&& LtQ[m, -1] && NeQ[a^2 + b^2, 0]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{2a^2}{3de(e\cot(c+dx))^{3/2}} + \frac{\int \frac{2a^2e}{(e\cot(c+dx))^{3/2}}\,dx}{e^2} \\
&= \frac{2a^2}{3de(e\cot(c+dx))^{3/2}} + \frac{(2a^2)\int \frac{1}{(e\cot(c+dx))^{3/2}}\,dx}{e} \\
&= \frac{2a^2}{3de(e\cot(c+dx))^{3/2}} + \frac{4a^2}{de^2\sqrt{e\cot(c+dx)}} - \frac{(2a^2)\int \sqrt{e\cot(c+dx)}\,dx}{e^3} \\
&= \frac{2a^2}{3de(e\cot(c+dx))^{3/2}} + \frac{4a^2}{de^2\sqrt{e\cot(c+dx)}} + \frac{(2a^2)\,\text{Subst}\left(\int \frac{\sqrt{x}}{e^2+x^2}\,dx, x, e\cot(c+dx)\right)}{de^2} \\
&= \frac{2a^2}{3de(e\cot(c+dx))^{3/2}} + \frac{4a^2}{de^2\sqrt{e\cot(c+dx)}} + \frac{(4a^2)\,\text{Subst}\left(\int \frac{x^2}{e^2+x^4}\,dx, x, \sqrt{e\cot(c+dx)}\right)}{de^2} \\
&= \frac{2a^2}{3de(e\cot(c+dx))^{3/2}} + \frac{4a^2}{de^2\sqrt{e\cot(c+dx)}} \\
&\quad - \frac{(2a^2)\,\text{Subst}\left(\int \frac{e-x^2}{e^2+x^4}\,dx, x, \sqrt{e\cot(c+dx)}\right)}{de^2} \\
&\quad + \frac{(2a^2)\,\text{Subst}\left(\int \frac{e+x^2}{e^2+x^4}\,dx, x, \sqrt{e\cot(c+dx)}\right)}{de^2}
\end{aligned}
$$

$$= \frac{2a^2}{3de(e\cot(c+dx))^{3/2}} + \frac{4a^2}{de^2\sqrt{e\cot(c+dx)}} + \frac{a^2\text{Subst}\left(\int \frac{\sqrt{2}\sqrt{e}+2x}{-e-\sqrt{2}\sqrt{e}x-x^2}\,dx, x, \sqrt{e\cot(c+dx)}\right)}{\sqrt{2}de^{5/2}} + \frac{a^2\text{Subst}\left(\int \frac{\sqrt{2}\sqrt{e}-2x}{-e+\sqrt{2}\sqrt{e}x-x^2}\,dx, x, \sqrt{e\cot(c+dx)}\right)}{\sqrt{2}de^{5/2}} + \frac{a^2\text{Subst}\left(\int \frac{1}{e-\sqrt{2}\sqrt{e}x+x^2}\,dx, x, \sqrt{e\cot(c+dx)}\right)}{de^2} + \frac{a^2\text{Subst}\left(\int \frac{1}{e+\sqrt{2}\sqrt{e}x+x^2}\,dx, x, \sqrt{e\cot(c+dx)}\right)}{de^2}$$

$$= \frac{2a^2}{3de(e\cot(c+dx))^{3/2}} + \frac{4a^2}{de^2\sqrt{e\cot(c+dx)}} + \frac{a^2\log\left(\sqrt{e}+\sqrt{e}\cot(c+dx)-\sqrt{2}\sqrt{e\cot(c+dx)}\right)}{\sqrt{2}de^{5/2}} - \frac{a^2\log\left(\sqrt{e}+\sqrt{e}\cot(c+dx)+\sqrt{2}\sqrt{e\cot(c+dx)}\right)}{\sqrt{2}de^{5/2}} + \frac{\left(\sqrt{2}a^2\right)\text{Subst}\left(\int \frac{1}{-1-x^2}\,dx, x, 1-\frac{\sqrt{2}\sqrt{e\cot(c+dx)}}{\sqrt{e}}\right)}{de^{5/2}} - \frac{\left(\sqrt{2}a^2\right)\text{Subst}\left(\int \frac{1}{-1-x^2}\,dx, x, 1+\frac{\sqrt{2}\sqrt{e\cot(c+dx)}}{\sqrt{e}}\right)}{de^{5/2}}$$

$$= -\frac{\sqrt{2}a^2\arctan\left(1-\frac{\sqrt{2}\sqrt{e\cot(c+dx)}}{\sqrt{e}}\right)}{de^{5/2}} + \frac{\sqrt{2}a^2\arctan\left(1+\frac{\sqrt{2}\sqrt{e\cot(c+dx)}}{\sqrt{e}}\right)}{de^{5/2}} + \frac{2a^2}{3de(e\cot(c+dx))^{3/2}} + \frac{4a^2}{de^2\sqrt{e\cot(c+dx)}} + \frac{a^2\log\left(\sqrt{e}+\sqrt{e}\cot(c+dx)-\sqrt{2}\sqrt{e\cot(c+dx)}\right)}{\sqrt{2}de^{5/2}} - \frac{a^2\log\left(\sqrt{e}+\sqrt{e}\cot(c+dx)+\sqrt{2}\sqrt{e\cot(c+dx)}\right)}{\sqrt{2}de^{5/2}}$$

Mathematica [A] (warning: unable to verify)

Time = 3.06 (sec) , antiderivative size = 400, normalized size of antiderivative = 1.62

$$\int \frac{(a+a\cot(c+dx))^2}{(e\cot(c+dx))^{5/2}}\,dx = \frac{a^2\left(48\cos^2(c+dx)+6\sqrt{2}\arctan\left(1-\sqrt{2}\sqrt{\cot(c+dx)}\right)\cot^{\frac{5}{2}}(c+dx)\sin^2(c+\right.}{}$$

[In] Integrate[(a + a*Cot[c + d*x])^2/(e*Cot[c + d*x])^(5/2),x]

[Out] (a^2*(48*Cos[c + d*x]^2 + 6*Sqrt[2]*ArcTan[1 - Sqrt[2]*Sqrt[Cot[c + d*x]]]*Cot[c + d*x]^(5/2)*Sin[c + d*x]^2 - 6*Sqrt[2]*ArcTan[1 + Sqrt[2]*Sqrt[Cot[c + d*x]]]*Cot[c + d*x]^(5/2)*Sin[c + d*x]^2 + 3*Sqrt[2]*Cot[c + d*x]^(5/2)*Log[1 - Sqrt[2]*Sqrt[Cot[c + d*x]] + Cot[c + d*x]]*Sin[c + d*x]^2 - 3*Sqrt[2]*Cot[c + d*x]^(5/2)*Log[1 + Sqrt[2]*Sqrt[Cot[c + d*x]] + Cot[c + d*x]]*Sin[c + d*x]^2 + 4*Sin[2*(c + d*x)] + 6*ArcTan[(-Cot[c + d*x]^2)^(1/4)]*(4*(-Cot[c + d*x])^(1/4)*Cot[c + d*x]^(9/4)*Sin[c + d*x]^2 - (-Cot[c + d*x]^2)^(3/4)*Sin[2*(c + d*x)]) - 6*ArcTanh[(-Cot[c + d*x]^2)^(1/4)]*(4*Cos[c + d*x]^2*(-Cot[c + d*x]^2)^(1/4) + (-Cot[c + d*x]^2)^(3/4)*Sin[2*(c + d*x)]))*(1 + Tan[c + d*x])^2)/(12*d*e^2*Sqrt[e*Cot[c + d*x]]*(Cos[c + d*x] + Sin[c + d*x])^2)

Maple [A] (verified)

Time = 0.05 (sec) , antiderivative size = 174, normalized size of antiderivative = 0.70

method	result
derivativedivides	$-\frac{2a^2\left(-\frac{\sqrt{2}\left(\ln\left(\frac{e\cot(dx+c)-(e^2)^{\frac{1}{4}}\sqrt{e\cot(dx+c)}\sqrt{2}+\sqrt{e^2}}{e\cot(dx+c)+(e^2)^{\frac{1}{4}}\sqrt{e\cot(dx+c)}\sqrt{2}+\sqrt{e^2}}\right)+2\arctan\left(\frac{\sqrt{2}\sqrt{e\cot(dx+c)}}{(e^2)^{\frac{1}{4}}}+1\right)-2\arctan\left(-\frac{\sqrt{2}\sqrt{e\cot(dx+c)}}{(e^2)^{\frac{1}{4}}}+1\right.\right.}{4e(e^2)^{\frac{1}{4}}}\right.}{de}$
default	$-\frac{2a^2\left(-\frac{\sqrt{2}\left(\ln\left(\frac{e\cot(dx+c)-(e^2)^{\frac{1}{4}}\sqrt{e\cot(dx+c)}\sqrt{2}+\sqrt{e^2}}{e\cot(dx+c)+(e^2)^{\frac{1}{4}}\sqrt{e\cot(dx+c)}\sqrt{2}+\sqrt{e^2}}\right)+2\arctan\left(\frac{\sqrt{2}\sqrt{e\cot(dx+c)}}{(e^2)^{\frac{1}{4}}}+1\right)-2\arctan\left(-\frac{\sqrt{2}\sqrt{e\cot(dx+c)}}{(e^2)^{\frac{1}{4}}}+1\right.\right.}{4e(e^2)^{\frac{1}{4}}}\right.}{de}$
parts	$-\frac{2a^2e\left(-\frac{(e^2)^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\frac{e\cot(dx+c)+(e^2)^{\frac{1}{4}}\sqrt{e\cot(dx+c)}\sqrt{2}+\sqrt{e^2}}{e\cot(dx+c)-(e^2)^{\frac{1}{4}}\sqrt{e\cot(dx+c)}\sqrt{2}+\sqrt{e^2}}\right)+2\arctan\left(\frac{\sqrt{2}\sqrt{e\cot(dx+c)}}{(e^2)^{\frac{1}{4}}}+1\right)-2\arctan\left(-\frac{\sqrt{2}\sqrt{e\cot(d}}{(e^2)^{\frac{1}{4}}}\right.\right.}{8e^4}\right.}{d}$

[In] int((a+a*cot(d*x+c))^2/(e*cot(d*x+c))^(5/2),x,method=_RETURNVERBOSE)

```
[Out] -2/d*a^2/e*(-1/4/e/(e^2)^(1/4)*2^(1/2)*(ln((e*cot(d*x+c)-(e^2)^(1/4)*(e*cot
(d*x+c))^(1/2)*2^(1/2)+(e^2)^(1/2))/(e*cot(d*x+c)+(e^2)^(1/4)*(e*cot(d*x+c)
)^(1/2)*2^(1/2)+(e^2)^(1/2)))+2*arctan(2^(1/2)/(e^2)^(1/4)*(e*cot(d*x+c))^(
1/2)+1)-2*arctan(-2^(1/2)/(e^2)^(1/4)*(e*cot(d*x+c))^(1/2)+1))-1/3/(e*cot(d
*x+c))^(3/2)-2/e/(e*cot(d*x+c))^(1/2))
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.29 (sec) , antiderivative size = 446, normalized size of antiderivative = 1.81

$$\int \frac{(a+a\cot(c+dx))^2}{(e\cot(c+dx))^{5/2}}\,dx = \frac{3\left(de^3\cos\left(2\,dx+2\,c\right)+de^3\right)\left(-\frac{a^8}{d^4e^{10}}\right)^{\frac{1}{4}}\log\left(d^3e^8\left(-\frac{a^8}{d^4e^{10}}\right)^{\frac{3}{4}}+a^6\sqrt{\frac{e\cos(2\,dx+2\,c)}{\sin(2\,dx+2\,c)}}\right.}{}$$

```
[In] integrate((a+a*cot(d*x+c))^2/(e*cot(d*x+c))^(5/2),x, algorithm="fricas")
```

```
[Out] 1/3*(3*(d*e^3*cos(2*d*x + 2*c) + d*e^3)*(-a^8/(d^4*e^10))^(1/4)*log(d^3*e^8
*(-a^8/(d^4*e^10))^(3/4) + a^6*sqrt((e*cos(2*d*x + 2*c) + e)/sin(2*d*x + 2*
c))) - 3*(I*d*e^3*cos(2*d*x + 2*c) + I*d*e^3)*(-a^8/(d^4*e^10))^(1/4)*log(I
*d^3*e^8*(-a^8/(d^4*e^10))^(3/4) + a^6*sqrt((e*cos(2*d*x + 2*c) + e)/sin(2*
d*x + 2*c))) - 3*(-I*d*e^3*cos(2*d*x + 2*c) - I*d*e^3)*(-a^8/(d^4*e^10))^(1
/4)*log(-I*d^3*e^8*(-a^8/(d^4*e^10))^(3/4) + a^6*sqrt((e*cos(2*d*x + 2*c) +
 e)/sin(2*d*x + 2*c))) - 3*(d*e^3*cos(2*d*x + 2*c) + d*e^3)*(-a^8/(d^4*e^10
))^(1/4)*log(-d^3*e^8*(-a^8/(d^4*e^10))^(3/4) + a^6*sqrt((e*cos(2*d*x + 2*c
) + e)/sin(2*d*x + 2*c))) - 2*(a^2*cos(2*d*x + 2*c) - 6*a^2*sin(2*d*x + 2*c
) - a^2)*sqrt((e*cos(2*d*x + 2*c) + e)/sin(2*d*x + 2*c)))/(d*e^3*cos(2*d*x
+ 2*c) + d*e^3)
```

Sympy [F]

$$\int \frac{(a+a\cot(c+dx))^2}{(e\cot(c+dx))^{5/2}}\,dx = a^2\left(\int \frac{1}{\left(e\cot\left(c+dx\right)\right)^{\frac{5}{2}}}\,dx + \int \frac{2\cot\left(c+dx\right)}{\left(e\cot\left(c+dx\right)\right)^{\frac{5}{2}}}\,dx + \int \frac{\cot^2\left(c+dx\right)}{\left(e\cot\left(c+dx\right)\right)^{\frac{5}{2}}}\,dx\right)$$

```
[In] integrate((a+a*cot(d*x+c))**2/(e*cot(d*x+c))**(5/2),x)
```

```
[Out] a**2*(Integral((e*cot(c + d*x))**(-5/2), x) + Integral(2*cot(c + d*x)/(e*co
t(c + d*x))**(5/2), x) + Integral(cot(c + d*x)**2/(e*cot(c + d*x))**(5/2),
x))
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{(a + a\cot(c + dx))^2}{(e\cot(c + dx))^{5/2}}\,dx = \text{Exception raised: ValueError}$$

```
[In] integrate((a+a*cot(d*x+c))^2/(e*cot(d*x+c))^(5/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(e>0)', see `assume?` for more detai
ls)Is e
```

Giac [F]

$$\int \frac{(a + a\cot(c + dx))^2}{(e\cot(c + dx))^{5/2}}\,dx = \int \frac{(a\cot(dx + c) + a)^2}{(e\cot(dx + c))^{\frac{5}{2}}}\,dx$$

```
[In] integrate((a+a*cot(d*x+c))^2/(e*cot(d*x+c))^(5/2),x, algorithm="giac")
```

```
[Out] integrate((a*cot(d*x + c) + a)^2/(e*cot(d*x + c))^(5/2), x)
```

Mupad [B] (verification not implemented)

Time = 13.09 (sec) , antiderivative size = 99, normalized size of antiderivative = 0.40

$$\int \frac{(a + a\cot(c + dx))^2}{(e\cot(c + dx))^{5/2}}\,dx = \frac{4\,a^2\cot(c + d\,x) + \frac{2\,a^2}{3}}{d\,e\,(e\cot(c + d\,x))^{3/2}} + \frac{2\,(-1)^{1/4}\,a^2\,\mathrm{atan}\left(\frac{(-1)^{1/4}\,\sqrt{e\cot(c+d\,x)}}{\sqrt{e}}\right)}{d\,e^{5/2}} - \frac{2\,(-1)^{1/4}\,a^2\,\mathrm{atanh}\left(\frac{(-1)^{1/4}\,\sqrt{e\cot(c+d\,x)}}{\sqrt{e}}\right)}{d\,e^{5/2}}$$

```
[In] int((a + a*cot(c + d*x))^2/(e*cot(c + d*x))^(5/2),x)
```

```
[Out] (4*a^2*cot(c + d*x) + (2*a^2)/3)/(d*e*(e*cot(c + d*x))^(3/2)) + (2*(-1)^(1/
4)*a^2*atan(((-1)^(1/4)*(e*cot(c + d*x))^(1/2))/e^(1/2)))/(d*e^(5/2)) - (2*
(-1)^(1/4)*a^2*atanh(((-1)^(1/4)*(e*cot(c + d*x))^(1/2))/e^(1/2)))/(d*e^(5/
2))
```

3.14 $\int \frac{(a+a\cot(c+dx))^2}{(e\cot(c+dx))^{7/2}}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 249

$$\begin{aligned}\int \frac{(a+a\cot(c+dx))^2}{(e\cot(c+dx))^{7/2}}\,dx = &-\frac{\sqrt{2}a^2\arctan\left(1-\frac{\sqrt{2}\sqrt{e\cot(c+dx)}}{\sqrt{e}}\right)}{de^{7/2}} \\ &+\frac{\sqrt{2}a^2\arctan\left(1+\frac{\sqrt{2}\sqrt{e\cot(c+dx)}}{\sqrt{e}}\right)}{de^{7/2}}+\frac{2a^2}{5de(e\cot(c+dx))^{5/2}} \\ &+\frac{4a^2}{3de^2(e\cot(c+dx))^{3/2}}-\frac{a^2\log\left(\sqrt{e}+\sqrt{e}\cot(c+dx)-\sqrt{2}\sqrt{e\cot(c+dx)}\right)}{\sqrt{2}de^{7/2}} \\ &+\frac{a^2\log\left(\sqrt{e}+\sqrt{e}\cot(c+dx)+\sqrt{2}\sqrt{e\cot(c+dx)}\right)}{\sqrt{2}de^{7/2}}\end{aligned}$$

```
[Out] 2/5*a^2/d/e/(e*cot(d*x+c))^(5/2)+4/3*a^2/d/e^2/(e*cot(d*x+c))^(3/2)-1/2*a^2
*ln(e^(1/2)+cot(d*x+c)*e^(1/2)-2^(1/2)*(e*cot(d*x+c))^(1/2))/d/e^(7/2)*2^(1
/2)+1/2*a^2*ln(e^(1/2)+cot(d*x+c)*e^(1/2)+2^(1/2)*(e*cot(d*x+c))^(1/2))/d/e
^(7/2)*2^(1/2)-a^2*arctan(1-2^(1/2)*(e*cot(d*x+c))^(1/2)/e^(1/2))*2^(1/2)/d
/e^(7/2)+a^2*arctan(1+2^(1/2)*(e*cot(d*x+c))^(1/2)/e^(1/2))*2^(1/2)/d/e^(7/
2)
```

Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 249, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.440, Rules used = {3623, 12, 3555, 3557, 335, 217, 1179, 642, 1176, 631, 210}

$$\begin{aligned}\int \frac{(a+a\cot(c+dx))^2}{(e\cot(c+dx))^{7/2}}\,dx=&-\frac{\sqrt{2}a^2\arctan\left(1-\frac{\sqrt{2}\sqrt{e\cot(c+dx)}}{\sqrt{e}}\right)}{de^{7/2}}\\&+\frac{\sqrt{2}a^2\arctan\left(\frac{\sqrt{2}\sqrt{e\cot(c+dx)}}{\sqrt{e}}+1\right)}{de^{7/2}}\\&-\frac{a^2\log\left(\sqrt{e}\cot(c+dx)-\sqrt{2}\sqrt{e\cot(c+dx)}+\sqrt{e}\right)}{\sqrt{2}de^{7/2}}\\&+\frac{a^2\log\left(\sqrt{e}\cot(c+dx)+\sqrt{2}\sqrt{e\cot(c+dx)}+\sqrt{e}\right)}{\sqrt{2}de^{7/2}}\\&+\frac{4a^2}{3de^2(e\cot(c+dx))^{3/2}}+\frac{2a^2}{5de(e\cot(c+dx))^{5/2}}\end{aligned}$$

```
[In] Int[(a + a*Cot[c + d*x])^2/(e*Cot[c + d*x])^(7/2),x]
```

```
[Out] -((Sqrt[2]*a^2*ArcTan[1 - (Sqrt[2]*Sqrt[e*Cot[c + d*x]])/Sqrt[e]])/(d*e^(7/
2))) + (Sqrt[2]*a^2*ArcTan[1 + (Sqrt[2]*Sqrt[e*Cot[c + d*x]])/Sqrt[e]])/(d*
e^(7/2)) + (2*a^2)/(5*d*e*(e*Cot[c + d*x])^(5/2)) + (4*a^2)/(3*d*e^2*(e*Cot
[c + d*x])^(3/2)) - (a^2*Log[Sqrt[e] + Sqrt[e]*Cot[c + d*x] - Sqrt[2]*Sqrt[
e*Cot[c + d*x]]])/(Sqrt[2]*d*e^(7/2)) + (a^2*Log[Sqrt[e] + Sqrt[e]*Cot[c +
d*x] + Sqrt[2]*Sqrt[e*Cot[c + d*x]]])/(Sqrt[2]*d*e^(7/2))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4),
 x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b
}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &&
```

```
AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1179

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 3555

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*Tan[c + d*x]
)^(n + 1)/(b*d*(n + 1)), x] - Dist[1/b^2, Int[(b*Tan[c + d*x])^(n + 2), x],
 x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1]
```

Rule 3557

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[b/d, Subst[Int[
x^n/(b^2 + x^2), x], x, b*Tan[c + d*x]], x] /; FreeQ[{b, c, d, n}, x] &&  !
```

```
IntegerQ[n]
```

Rule 3623

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)])^2, x_Symbol] :> Simp[(b*c - a*d)^2*((a + b*Tan[e + f*x])^(m +
 1)/(b*f*(m + 1)*(a^2 + b^2))), x] + Dist[1/(a^2 + b^2), Int[(a + b*Tan[e +
 f*x])^(m + 1)*Simp[a*c^2 + 2*b*c*d - a*d^2 - (b*c^2 - 2*a*c*d - b*d^2)*Tan
[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
&& LtQ[m, -1] && NeQ[a^2 + b^2, 0]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{2a^2}{5de(e\cot(c+dx))^{5/2}} + \frac{\int \frac{2a^2e}{(e\cot(c+dx))^{5/2}}\,dx}{e^2} \\
&= \frac{2a^2}{5de(e\cot(c+dx))^{5/2}} + \frac{(2a^2)\int \frac{1}{(e\cot(c+dx))^{5/2}}\,dx}{e} \\
&= \frac{2a^2}{5de(e\cot(c+dx))^{5/2}} + \frac{4a^2}{3de^2(e\cot(c+dx))^{3/2}} - \frac{(2a^2)\int \frac{1}{\sqrt{e\cot(c+dx)}}\,dx}{e^3} \\
&= \frac{2a^2}{5de(e\cot(c+dx))^{5/2}} + \frac{4a^2}{3de^2(e\cot(c+dx))^{3/2}} + \frac{(2a^2)\,\text{Subst}\left(\int \frac{1}{\sqrt{x}(e^2+x^2)}\,dx, x, e\cot(c+dx)\right)}{de^2} \\
&= \frac{2a^2}{5de(e\cot(c+dx))^{5/2}} + \frac{4a^2}{3de^2(e\cot(c+dx))^{3/2}} + \frac{(4a^2)\,\text{Subst}\left(\int \frac{1}{e^2+x^4}\,dx, x, \sqrt{e\cot(c+dx)}\right)}{de^2} \\
&= \frac{2a^2}{5de(e\cot(c+dx))^{5/2}} + \frac{4a^2}{3de^2(e\cot(c+dx))^{3/2}} \\
&\quad + \frac{(2a^2)\,\text{Subst}\left(\int \frac{e-x^2}{e^2+x^4}\,dx, x, \sqrt{e\cot(c+dx)}\right)}{de^3} \\
&\quad + \frac{(2a^2)\,\text{Subst}\left(\int \frac{e+x^2}{e^2+x^4}\,dx, x, \sqrt{e\cot(c+dx)}\right)}{de^3}
\end{aligned}
$$

$$= \frac{2a^2}{5de(e\cot(c+dx))^{5/2}} + \frac{4a^2}{3de^2(e\cot(c+dx))^{3/2}} - \frac{a^2\text{Subst}\left(\int \frac{\sqrt{2}\sqrt{e}+2x}{-e-\sqrt{2}\sqrt{e}x-x^2}\,dx, x, \sqrt{e\cot(c+dx)}\right)}{\sqrt{2}de^{7/2}} - \frac{a^2\text{Subst}\left(\int \frac{\sqrt{2}\sqrt{e}-2x}{-e+\sqrt{2}\sqrt{e}x-x^2}\,dx, x, \sqrt{e\cot(c+dx)}\right)}{\sqrt{2}de^{7/2}} + \frac{a^2\text{Subst}\left(\int \frac{1}{e-\sqrt{2}\sqrt{e}x+x^2}\,dx, x, \sqrt{e\cot(c+dx)}\right)}{de^3} + \frac{a^2\text{Subst}\left(\int \frac{1}{e+\sqrt{2}\sqrt{e}x+x^2}\,dx, x, \sqrt{e\cot(c+dx)}\right)}{de^3}$$

$$= \frac{2a^2}{5de(e\cot(c+dx))^{5/2}} + \frac{4a^2}{3de^2(e\cot(c+dx))^{3/2}} - \frac{a^2\log\left(\sqrt{e}+\sqrt{e}\cot(c+dx)-\sqrt{2}\sqrt{e\cot(c+dx)}\right)}{\sqrt{2}de^{7/2}} + \frac{a^2\log\left(\sqrt{e}+\sqrt{e}\cot(c+dx)+\sqrt{2}\sqrt{e\cot(c+dx)}\right)}{\sqrt{2}de^{7/2}} + \frac{\left(\sqrt{2}a^2\right)\text{Subst}\left(\int \frac{1}{-1-x^2}\,dx, x, 1-\frac{\sqrt{2}\sqrt{e\cot(c+dx)}}{\sqrt{e}}\right)}{de^{7/2}} - \frac{\left(\sqrt{2}a^2\right)\text{Subst}\left(\int \frac{1}{-1-x^2}\,dx, x, 1+\frac{\sqrt{2}\sqrt{e\cot(c+dx)}}{\sqrt{e}}\right)}{de^{7/2}}$$

$$= -\frac{\sqrt{2}a^2\arctan\left(1-\frac{\sqrt{2}\sqrt{e\cot(c+dx)}}{\sqrt{e}}\right)}{de^{7/2}} + \frac{\sqrt{2}a^2\arctan\left(1+\frac{\sqrt{2}\sqrt{e\cot(c+dx)}}{\sqrt{e}}\right)}{de^{7/2}} + \frac{2a^2}{5de(e\cot(c+dx))^{5/2}} + \frac{4a^2}{3de^2(e\cot(c+dx))^{3/2}} - \frac{a^2\log\left(\sqrt{e}+\sqrt{e}\cot(c+dx)-\sqrt{2}\sqrt{e\cot(c+dx)}\right)}{\sqrt{2}de^{7/2}} + \frac{a^2\log\left(\sqrt{e}+\sqrt{e}\cot(c+dx)+\sqrt{2}\sqrt{e\cot(c+dx)}\right)}{\sqrt{2}de^{7/2}}$$

Mathematica [A] (verified)

Time = 1.55 (sec) , antiderivative size = 173, normalized size of antiderivative = 0.69

$$\int \frac{(a+a\cot(c+dx))^2}{(e\cot(c+dx))^{7/2}}\,dx =$$

$$-\frac{a^2\sqrt{e\cot(c+dx)}(1+\cot(c+dx))^2\left(-20\cos^2(c+dx)+30\arctan\left(\sqrt[4]{-\cot^2(c+dx)}\right)(-\cot(c+dx))^{3/4}\right.}{1}$$

[In] Integrate[(a + a*Cot[c + d*x])^2/(e*Cot[c + d*x])^(7/2),x]

[Out] -1/15*(a^2*Sqrt[e*Cot[c + d*x]]*(1 + Cot[c + d*x])^2*(-20*Cos[c + d*x]^2 + 30*ArcTan[(-Cot[c + d*x]^2)^(1/4)]*(-Cot[c + d*x])^(3/4)*Cot[c + d*x]^(11/4)*Sin[c + d*x]^2 + 30*ArcTanh[(-Cot[c + d*x]^2)^(1/4)]*(-Cot[c + d*x])^(3/4)*Cot[c + d*x]^(11/4)*Sin[c + d*x]^2 - 3*Sin[2*(c + d*x)])*Tan[c + d*x]^4)/(d*e^4*(Cos[c + d*x] + Sin[c + d*x])^2)

Maple [A] (verified)

Time = 0.07 (sec) , antiderivative size = 174, normalized size of antiderivative = 0.70

method	result
derivativedivides	$-\frac{2a^2\left(-\frac{(e^2)^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\frac{e\cot(dx+c)+(e^2)^{\frac{1}{4}}\sqrt{e\cot(dx+c)}\sqrt{2}+\sqrt{e^2}}{e\cot(dx+c)-(e^2)^{\frac{1}{4}}\sqrt{e\cot(dx+c)}\sqrt{2}+\sqrt{e^2}}\right)+2\arctan\left(\frac{\sqrt{2}\sqrt{e\cot(dx+c)}}{(e^2)^{\frac{1}{4}}}+1\right)-2\arctan\left(-\frac{\sqrt{2}\sqrt{e\cot(dx}}{(e^2)^{\frac{1}{4}}}\right.\right.}{4e^3}\right.}{de}$
default	$-\frac{2a^2\left(-\frac{(e^2)^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\frac{e\cot(dx+c)+(e^2)^{\frac{1}{4}}\sqrt{e\cot(dx+c)}\sqrt{2}+\sqrt{e^2}}{e\cot(dx+c)-(e^2)^{\frac{1}{4}}\sqrt{e\cot(dx+c)}\sqrt{2}+\sqrt{e^2}}\right)+2\arctan\left(\frac{\sqrt{2}\sqrt{e\cot(dx+c)}}{(e^2)^{\frac{1}{4}}}+1\right)-2\arctan\left(-\frac{\sqrt{2}\sqrt{e\cot(dx}}{(e^2)^{\frac{1}{4}}}\right.\right.}{4e^3}\right.}{de}$
parts	$-\frac{2a^2e\left(-\frac{1}{5e^2(e\cot(dx+c))^{\frac{5}{2}}}+\frac{1}{e^4\sqrt{e\cot(dx+c)}}+\frac{\sqrt{2}\left(\ln\left(\frac{e\cot(dx+c)-(e^2)^{\frac{1}{4}}\sqrt{e\cot(dx+c)}\sqrt{2}+\sqrt{e^2}}{e\cot(dx+c)+(e^2)^{\frac{1}{4}}\sqrt{e\cot(dx+c)}\sqrt{2}+\sqrt{e^2}}\right)+2\arctan\left(\frac{\sqrt{2}\sqrt{e\cot(dx+}}{(e^2)^{\frac{1}{4}}}\right.\right.}{8e^4(e^2)^{\frac{1}{4}}}\right.}{d}$

[In] int((a+a*cot(d*x+c))^2/(e*cot(d*x+c))^(7/2),x,method=_RETURNVERBOSE)

[Out] -2/d*a^2/e*(-1/4/e^3*(e^2)^(1/4)*2^(1/2)*(ln((e*cot(d*x+c)+(e^2)^(1/4)*(e*cot(d*x+c))^(1/2)*2^(1/2)+(e^2)^(1/2))/(e*cot(d*x+c)-(e^2)^(1/4)*(e*cot(d*x+c))^(1/2)*2^(1/2)+(e^2)^(1/2)))+2*arctan(2^(1/2)/(e^2)^(1/4)*(e*cot(d*x+c))

```
^(1/2)+1)-2*arctan(-2^(1/2)/(e^2)^(1/4)*(e*cot(d*x+c))^(1/2)+1))-1/5/(e*cot
(d*x+c))^(5/2)-2/3/e/(e*cot(d*x+c))^(3/2))
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.29 (sec) , antiderivative size = 542, normalized size of antiderivative = 2.18

$$\int \frac{(a+a\cot(c+dx))^2}{(e\cot(c+dx))^{7/2}}\,dx = \frac{15\left(de^4\cos\left(2\,dx+2\,c\right)^2+2\,de^4\cos\left(2\,dx+2\,c\right)+de^4\right)\left(-\frac{a^8}{d^4e^{14}}\right)^{\frac{1}{4}}\log\left(de^4\left(-\frac{a^8}{d^4e^{14}}\right)^{\frac{1}{4}}\right.}{}$$

```
[In] integrate((a+a*cot(d*x+c))^2/(e*cot(d*x+c))^(7/2),x, algorithm="fricas")
```

```
[Out] 1/15*(15*(d*e^4*cos(2*d*x + 2*c)^2 + 2*d*e^4*cos(2*d*x + 2*c) + d*e^4)*(-a^
8/(d^4*e^14))^(1/4)*log(d*e^4*(-a^8/(d^4*e^14))^(1/4) + a^2*sqrt((e*cos(2*d
*x + 2*c) + e)/sin(2*d*x + 2*c))) - 15*(-I*d*e^4*cos(2*d*x + 2*c)^2 - 2*I*d
*e^4*cos(2*d*x + 2*c) - I*d*e^4)*(-a^8/(d^4*e^14))^(1/4)*log(I*d*e^4*(-a^8/
(d^4*e^14))^(1/4) + a^2*sqrt((e*cos(2*d*x + 2*c) + e)/sin(2*d*x + 2*c))) -
15*(I*d*e^4*cos(2*d*x + 2*c)^2 + 2*I*d*e^4*cos(2*d*x + 2*c) + I*d*e^4)*(-a^
8/(d^4*e^14))^(1/4)*log(-I*d*e^4*(-a^8/(d^4*e^14))^(1/4) + a^2*sqrt((e*cos(
2*d*x + 2*c) + e)/sin(2*d*x + 2*c))) - 15*(d*e^4*cos(2*d*x + 2*c)^2 + 2*d*e
^4*cos(2*d*x + 2*c) + d*e^4)*(-a^8/(d^4*e^14))^(1/4)*log(-d*e^4*(-a^8/(d^4*
e^14))^(1/4) + a^2*sqrt((e*cos(2*d*x + 2*c) + e)/sin(2*d*x + 2*c))) - 2*(10
*a^2*cos(2*d*x + 2*c)^2 - 10*a^2 + 3*(a^2*cos(2*d*x + 2*c) - a^2)*sin(2*d*x
 + 2*c))*sqrt((e*cos(2*d*x + 2*c) + e)/sin(2*d*x + 2*c)))/(d*e^4*cos(2*d*x
+ 2*c)^2 + 2*d*e^4*cos(2*d*x + 2*c) + d*e^4)
```

Sympy [F]

$$\int \frac{(a+a\cot(c+dx))^2}{(e\cot(c+dx))^{7/2}}\,dx = a^2\left(\int \frac{1}{\left(e\cot\left(c+dx\right)\right)^{\frac{7}{2}}}\,dx + \int \frac{2\cot\left(c+dx\right)}{\left(e\cot\left(c+dx\right)\right)^{\frac{7}{2}}}\,dx + \int \frac{\cot^2\left(c+dx\right)}{\left(e\cot\left(c+dx\right)\right)^{\frac{7}{2}}}\,dx\right)$$

```
[In] integrate((a+a*cot(d*x+c))**2/(e*cot(d*x+c))**(7/2),x)
```

```
[Out] a**2*(Integral((e*cot(c + d*x))**(-7/2), x) + Integral(2*cot(c + d*x)/(e*co
t(c + d*x))**(7/2), x) + Integral(cot(c + d*x)**2/(e*cot(c + d*x))**(7/2),
x))
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{(a+a\cot(c+dx))^2}{(e\cot(c+dx))^{7/2}}\,dx = \text{Exception raised: ValueError}$$

```
[In] integrate((a+a*cot(d*x+c))^2/(e*cot(d*x+c))^(7/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(e>0)', see `assume?` for more detai
ls)Is e
```

Giac [F]

$$\int \frac{(a+a\cot(c+dx))^2}{(e\cot(c+dx))^{7/2}}\,dx = \int \frac{(a\cot(dx+c)+a)^2}{(e\cot(dx+c))^{\frac{7}{2}}}\,dx$$

```
[In] integrate((a+a*cot(d*x+c))^2/(e*cot(d*x+c))^(7/2),x, algorithm="giac")
```

```
[Out] integrate((a*cot(d*x + c) + a)^2/(e*cot(d*x + c))^(7/2), x)
```

Mupad [B] (verification not implemented)

Time = 13.75 (sec) , antiderivative size = 99, normalized size of antiderivative = 0.40

$$\int \frac{(a+a\cot(c+dx))^2}{(e\cot(c+dx))^{7/2}}\,dx = \frac{\frac{4a^2\cot(c+dx)}{3}+\frac{2a^2}{5}}{d\,e\,(e\cot(c+d\,x))^{5/2}} - \frac{(-1)^{1/4}\,a^2\,\mathrm{atan}\left(\frac{(-1)^{1/4}\sqrt{e\cot(c+d\,x)}}{\sqrt{e}}\right)\,2i}{d\,e^{7/2}} - \frac{(-1)^{1/4}\,a^2\,\mathrm{atanh}\left(\frac{(-1)^{1/4}\sqrt{e\cot(c+d\,x)}}{\sqrt{e}}\right)\,2i}{d\,e^{7/2}}$$

```
[In] int((a + a*cot(c + d*x))^2/(e*cot(c + d*x))^(7/2),x)
```

```
[Out] ((4*a^2*cot(c + d*x))/3 + (2*a^2)/5)/(d*e*(e*cot(c + d*x))^(5/2)) - ((-1)^(
1/4)*a^2*atan(((-1)^(1/4)*(e*cot(c + d*x))^(1/2))/e^(1/2))*2i)/(d*e^(7/2))
 - ((-1)^(1/4)*a^2*atanh(((-1)^(1/4)*(e*cot(c + d*x))^(1/2))/e^(1/2))*2i)/(d
*e^(7/2))
```

3.15 $\int (e\cot(c+dx))^{5/2}(a+a\cot(c+dx))^3\,dx$

Optimal result

Integrand size = 25, antiderivative size = 186

$$\int (e\cot(c+dx))^{5/2}(a+a\cot(c+dx))^3\,dx = \frac{2\sqrt{2}a^3e^{5/2}\arctan\left(\frac{\sqrt{e}-\sqrt{e}\cot(c+dx)}{\sqrt{2}\sqrt{e\cot(c+dx)}}\right)}{d} + \frac{4a^3e^2\sqrt{e\cot(c+dx)}}{d} + \frac{4a^3e(e\cot(c+dx))^{3/2}}{3d} - \frac{4a^3(e\cot(c+dx))^{5/2}}{5d} - \frac{40a^3(e\cot(c+dx))^{7/2}}{63de} - \frac{2(e\cot(c+dx))^{7/2}\left(a^3+a^3\cot(c+dx)\right)}{9de}$$

```
[Out] 4/3*a^3*e*(e*cot(d*x+c))^(3/2)/d-4/5*a^3*(e*cot(d*x+c))^(5/2)/d-40/63*a^3*(
e*cot(d*x+c))^(7/2)/d/e-2/9*(e*cot(d*x+c))^(7/2)*(a^3+a^3*cot(d*x+c))/d/e+2
*a^3*e^(5/2)*arctan(1/2*(e^(1/2)-cot(d*x+c)*e^(1/2))*2^(1/2)/(e*cot(d*x+c))
^(1/2))*2^(1/2)/d+4*a^3*e^2*(e*cot(d*x+c))^(1/2)/d
```

Rubi [A] (verified)

Time = 0.33 (sec) , antiderivative size = 186, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {3647, 3711, 3609, 3613, 211}

$$\int (e\cot(c+dx))^{5/2}(a+a\cot(c+dx))^3\,dx = \frac{2\sqrt{2}a^3e^{5/2}\arctan\left(\frac{\sqrt{e}-\sqrt{e}\cot(c+dx)}{\sqrt{2}\sqrt{e\cot(c+dx)}}\right)}{d} + \frac{4a^3e^2\sqrt{e\cot(c+dx)}}{d} - \frac{2\left(a^3\cot(c+dx)+a^3\right)(e\cot(c+dx))^{7/2}}{9de} - \frac{40a^3(e\cot(c+dx))^{7/2}}{63de} - \frac{4a^3(e\cot(c+dx))^{5/2}}{5d} + \frac{4a^3e(e\cot(c+dx))^{3/2}}{3d}$$

```
[In] Int[(e*Cot[c + d*x])^(5/2)*(a + a*Cot[c + d*x])^3,x]
```

```
[Out] (2*Sqrt[2]*a^3*e^(5/2)*ArcTan[(Sqrt[e] - Sqrt[e]*Cot[c + d*x])/(Sqrt[2]*Sqr
t[e*Cot[c + d*x]])])/d + (4*a^3*e^2*Sqrt[e*Cot[c + d*x]])/d + (4*a^3*e*(e*C
ot[c + d*x])^(3/2))/(3*d) - (4*a^3*(e*Cot[c + d*x])^(5/2))/(5*d) - (40*a^3*
(e*Cot[c + d*x])^(7/2))/(63*d*e) - (2*(e*Cot[c + d*x])^(7/2)*(a^3 + a^3*Cot
[c + d*x]))/(9*d*e)
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 3609

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)]), x_Symbol] :> Simp[d*((a + b*Tan[e + f*x])^m/(f*m)), x] + Int
[(a + b*Tan[e + f*x])^(m - 1)*Simp[a*c - b*d + (b*c + a*d)*Tan[e + f*x], x]
, x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2,
 0] && GtQ[m, 0]
```

Rule 3613

```
Int[((c_) + (d_.)*tan[(e_.) + (f_.)*(x_)])/Sqrt[(b_.)*tan[(e_.) + (f_.)*(x_
)]], x_Symbol] :> Dist[-2*(d^2/f), Subst[Int[1/(2*c*d + b*x^2), x], x, (c -
 d*Tan[e + f*x])/Sqrt[b*Tan[e + f*x]]], x] /; FreeQ[{b, c, d, e, f}, x] &&
EqQ[c^2 - d^2, 0]
```

Rule 3647

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)])^(n_), x_Symbol] :> Simp[b^2*(a + b*Tan[e + f*x])^(m - 2)*((c
+ d*Tan[e + f*x])^(n + 1)/(d*f*(m + n - 1))), x] + Dist[1/(d*(m + n - 1)),
Int[(a + b*Tan[e + f*x])^(m - 3)*(c + d*Tan[e + f*x])^n*Simp[a^3*d*(m + n -
 1) - b^2*(b*c*(m - 2) + a*d*(1 + n)) + b*d*(m + n - 1)*(3*a^2 - b^2)*Tan[e
 + f*x] - b^2*(b*c*(m - 2) - a*d*(3*m + 2*n - 4))*Tan[e + f*x]^2, x], x], x
] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2,
 0] && NeQ[c^2 + d^2, 0] && IntegerQ[2*m] && GtQ[m, 2] && (GeQ[n, -1] || In
tegerQ[m]) &&  !(IGtQ[n, 2] && ( !IntegerQ[m] || (EqQ[c, 0] && NeQ[a, 0])))
```

Rule 3711

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*tan[(e_.)
+ (f_.)*(x_)] + (C_.)*tan[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[C*((a +
 b*Tan[e + f*x])^(m + 1)/(b*f*(m + 1))), x] + Int[(a + b*Tan[e + f*x])^m*Si
mp[A - C + B*Tan[e + f*x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] &&
NeQ[A*b^2 - a*b*B + a^2*C, 0] &&  !LeQ[m, -1]
```

Rubi steps

$$\text{integral} = -\frac{2(e\cot(c+dx))^{7/2}\left(a^3+a^3\cot(c+dx)\right)}{9de} - \frac{2\int (e\cot(c+dx))^{5/2}\left(-a^3e-9a^3e\cot(c+dx)-10a^3e\cot^2(c+dx)\right)\,dx}{9e}$$

$$= -\frac{40a^3(e\cot(c+dx))^{7/2}}{63de} - \frac{2(e\cot(c+dx))^{7/2}\left(a^3+a^3\cot(c+dx)\right)}{9de} - \frac{2\int (e\cot(c+dx))^{5/2}\left(9a^3e-9a^3e\cot(c+dx)\right)\,dx}{9e}$$

$$= -\frac{4a^3(e\cot(c+dx))^{5/2}}{5d} - \frac{40a^3(e\cot(c+dx))^{7/2}}{63de} - \frac{2(e\cot(c+dx))^{7/2}\left(a^3+a^3\cot(c+dx)\right)}{9de} - \frac{2\int (e\cot(c+dx))^{3/2}\left(9a^3e^2+9a^3e^2\cot(c+dx)\right)\,dx}{9e}$$

$$= \frac{4a^3e(e\cot(c+dx))^{3/2}}{3d} - \frac{4a^3(e\cot(c+dx))^{5/2}}{5d} - \frac{40a^3(e\cot(c+dx))^{7/2}}{63de} - \frac{2(e\cot(c+dx))^{7/2}\left(a^3+a^3\cot(c+dx)\right)}{9de} - \frac{2\int \sqrt{e\cot(c+dx)}\left(-9a^3e^3+9a^3e^3\cot(c+dx)\right)\,dx}{9e}$$

$$= \frac{4a^3e^2\sqrt{e\cot(c+dx)}}{d} + \frac{4a^3e(e\cot(c+dx))^{3/2}}{3d} - \frac{4a^3(e\cot(c+dx))^{5/2}}{5d} - \frac{40a^3(e\cot(c+dx))^{7/2}}{63de} - \frac{2(e\cot(c+dx))^{7/2}\left(a^3+a^3\cot(c+dx)\right)}{9de} - \frac{2\int \frac{-9a^3e^4-9a^3e^4\cot(c+dx)}{\sqrt{e\cot(c+dx)}}\,dx}{9e}$$

$$= \frac{4a^3e^2\sqrt{e\cot(c+dx)}}{d} + \frac{4a^3e(e\cot(c+dx))^{3/2}}{3d} - \frac{4a^3(e\cot(c+dx))^{5/2}}{5d} - \frac{40a^3(e\cot(c+dx))^{7/2}}{63de} - \frac{2(e\cot(c+dx))^{7/2}\left(a^3+a^3\cot(c+dx)\right)}{9de} + \frac{(36a^6e^7)\,\text{Subst}\left(\int \frac{1}{-162a^6e^8-ex^2}\,dx, x, \frac{-9a^3e^4+9a^3e^4\cot(c+dx)}{\sqrt{e\cot(c+dx)}}\right)}{d}$$

$$= \frac{2\sqrt{2}a^3e^{5/2}\arctan\left(\frac{\sqrt{e}-\sqrt{e}\cot(c+dx)}{\sqrt{2}\sqrt{e\cot(c+dx)}}\right)}{d} + \frac{4a^3e^2\sqrt{e\cot(c+dx)}}{d} + \frac{4a^3e(e\cot(c+dx))^{3/2}}{3d} - \frac{4a^3(e\cot(c+dx))^{5/2}}{5d} - \frac{40a^3(e\cot(c+dx))^{7/2}}{63de} - \frac{2(e\cot(c+dx))^{7/2}\left(a^3+a^3\cot(c+dx)\right)}{9de}$$

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 847 vs. 2(186) = 372.

Time = 6.16 (sec) , antiderivative size = 847, normalized size of antiderivative = 4.55

$$\int (e\cot(c+dx))^{5/2}(a+a\cot(c+dx))^3\,dx =$$
$$-\frac{2\cos^2(c+dx)(e\cot(c+dx))^{5/2}(a+a\cot(c+dx))^3\sin(c+dx)}{9d(\cos(c+dx)+\sin(c+dx))^3}$$
$$-\frac{6\cos(c+dx)(e\cot(c+dx))^{5/2}(a+a\cot(c+dx))^3\sin^2(c+dx)}{7d(\cos(c+dx)+\sin(c+dx))^3}$$
$$-\frac{4(e\cot(c+dx))^{5/2}(a+a\cot(c+dx))^3\sin^3(c+dx)}{5d(\cos(c+dx)+\sin(c+dx))^3}$$
$$-\frac{2\arctan\left(\sqrt[4]{-\cot(c+dx)}\sqrt[4]{\cot(c+dx)}\right)\sqrt[4]{-\cot(c+dx)}(e\cot(c+dx))^{5/2}(a+a\cot(c+dx))^3\sin^3(c+dx)}{d\cot^{\frac{11}{4}}(c+dx)(\cos(c+dx)+\sin(c+dx))^3}$$
$$+\frac{2\text{arctanh}\left(\sqrt[4]{-\cot(c+dx)}\sqrt[4]{\cot(c+dx)}\right)\sqrt[4]{-\cot(c+dx)}(e\cot(c+dx))^{5/2}(a+a\cot(c+dx))^3\sin^3(c+dx)}{d\cot^{\frac{11}{4}}(c+dx)(\cos(c+dx)+\sin(c+dx))^3}$$
$$+\frac{\sqrt{2}\arctan\left(1-\sqrt{2}\sqrt{\cot(c+dx)}\right)(e\cot(c+dx))^{5/2}(a+a\cot(c+dx))^3\sin^3(c+dx)}{d\cot^{\frac{5}{2}}(c+dx)(\cos(c+dx)+\sin(c+dx))^3}$$
$$-\frac{\sqrt{2}\arctan\left(1+\sqrt{2}\sqrt{\cot(c+dx)}\right)(e\cot(c+dx))^{5/2}(a+a\cot(c+dx))^3\sin^3(c+dx)}{d\cot^{\frac{5}{2}}(c+dx)(\cos(c+dx)+\sin(c+dx))^3}$$
$$+\frac{(e\cot(c+dx))^{5/2}(a+a\cot(c+dx))^3\log\left(1-\sqrt{2}\sqrt{\cot(c+dx)}+\cot(c+dx)\right)\sin^3(c+dx)}{\sqrt{2}d\cot^{\frac{5}{2}}(c+dx)(\cos(c+dx)+\sin(c+dx))^3}$$
$$-\frac{(e\cot(c+dx))^{5/2}(a+a\cot(c+dx))^3\log\left(1+\sqrt{2}\sqrt{\cot(c+dx)}+\cot(c+dx)\right)\sin^3(c+dx)}{\sqrt{2}d\cot^{\frac{5}{2}}(c+dx)(\cos(c+dx)+\sin(c+dx))^3}$$
$$+\frac{4(e\cot(c+dx))^{5/2}(a+a\cot(c+dx))^3\sin^3(c+dx)\tan(c+dx)}{3d(\cos(c+dx)+\sin(c+dx))^3}$$
$$+\frac{4(e\cot(c+dx))^{5/2}(a+a\cot(c+dx))^3\sin^3(c+dx)\tan^2(c+dx)}{d(\cos(c+dx)+\sin(c+dx))^3}$$

```
[In] Integrate[(e*Cot[c + d*x])^(5/2)*(a + a*Cot[c + d*x])^3,x]
```

```
[Out] (-2*Cos[c + d*x]^2*(e*Cot[c + d*x])^(5/2)*(a + a*Cot[c + d*x])^3*Sin[c + d*
x])/(9*d*(Cos[c + d*x] + Sin[c + d*x])^3) - (6*Cos[c + d*x]*(e*Cot[c + d*x]
)^(5/2)*(a + a*Cot[c + d*x])^3*Sin[c + d*x]^2)/(7*d*(Cos[c + d*x] + Sin[c +
 d*x])^3) - (4*(e*Cot[c + d*x])^(5/2)*(a + a*Cot[c + d*x])^3*Sin[c + d*x]^3
)/(5*d*(Cos[c + d*x] + Sin[c + d*x])^3) - (2*ArcTan[(-Cot[c + d*x])^(1/4)*C
ot[c + d*x]^(1/4)]*(-Cot[c + d*x])^(1/4)*(e*Cot[c + d*x])^(5/2)*(a + a*Cot[
```

```
c + d*x])^3*Sin[c + d*x]^3)/(d*Cot[c + d*x]^(11/4)*(Cos[c + d*x] + Sin[c +
d*x])^3) + (2*ArcTanh[(-Cot[c + d*x])^(1/4)*Cot[c + d*x]^(1/4)]*(-Cot[c + d
*x])^(1/4)*(e*Cot[c + d*x])^(5/2)*(a + a*Cot[c + d*x])^3*Sin[c + d*x]^3)/(d
*Cot[c + d*x]^(11/4)*(Cos[c + d*x] + Sin[c + d*x])^3) + (Sqrt[2]*ArcTan[1 -
 Sqrt[2]*Sqrt[Cot[c + d*x]]]*(e*Cot[c + d*x])^(5/2)*(a + a*Cot[c + d*x])^3*
Sin[c + d*x]^3)/(d*Cot[c + d*x]^(5/2)*(Cos[c + d*x] + Sin[c + d*x])^3) - (S
qrt[2]*ArcTan[1 + Sqrt[2]*Sqrt[Cot[c + d*x]]]*(e*Cot[c + d*x])^(5/2)*(a + a
*Cot[c + d*x])^3*Sin[c + d*x]^3)/(d*Cot[c + d*x]^(5/2)*(Cos[c + d*x] + Sin[
c + d*x])^3) + ((e*Cot[c + d*x])^(5/2)*(a + a*Cot[c + d*x])^3*Log[1 - Sqrt[
2]*Sqrt[Cot[c + d*x]] + Cot[c + d*x]]*Sin[c + d*x]^3)/(Sqrt[2]*d*Cot[c + d*
x]^(5/2)*(Cos[c + d*x] + Sin[c + d*x])^3) - ((e*Cot[c + d*x])^(5/2)*(a + a*
Cot[c + d*x])^3*Log[1 + Sqrt[2]*Sqrt[Cot[c + d*x]] + Cot[c + d*x]]*Sin[c +
d*x]^3)/(Sqrt[2]*d*Cot[c + d*x]^(5/2)*(Cos[c + d*x] + Sin[c + d*x])^3) + (4
*(e*Cot[c + d*x])^(5/2)*(a + a*Cot[c + d*x])^3*Sin[c + d*x]^3*Tan[c + d*x])
/(3*d*(Cos[c + d*x] + Sin[c + d*x])^3) + (4*(e*Cot[c + d*x])^(5/2)*(a + a*C
ot[c + d*x])^3*Sin[c + d*x]^3*Tan[c + d*x]^2)/(d*(Cos[c + d*x] + Sin[c + d*
x])^3)
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 353 vs. $2(157) = 314$.

Time = 0.36 (sec) , antiderivative size = 354, normalized size of antiderivative = 1.90

method	result
derivativedivides	$-\frac{2a^3\left(\frac{(e\cot(dx+c))^{\frac{9}{2}}}{9}+\frac{3e(e\cot(dx+c))^{\frac{7}{2}}}{7}+\frac{2e^2(e\cot(dx+c))^{\frac{5}{2}}}{5}-\frac{2e^3(e\cot(dx+c))^{\frac{3}{2}}}{3}-2\sqrt{e\cot(dx+c)}\,e^4+2e^5\left(\frac{(e^2)^{\frac{1}{4}}\sqrt{2}\left(\ln \cdots\right.}{\cdots}\right.\right.}{\cdots}$
default	$-\frac{2a^3\left(\frac{(e\cot(dx+c))^{\frac{9}{2}}}{9}+\frac{3e(e\cot(dx+c))^{\frac{7}{2}}}{7}+\frac{2e^2(e\cot(dx+c))^{\frac{5}{2}}}{5}-\frac{2e^3(e\cot(dx+c))^{\frac{3}{2}}}{3}-2\sqrt{e\cot(dx+c)}\,e^4+2e^5\left(\frac{(e^2)^{\frac{1}{4}}\sqrt{2}\left(\ln \cdots\right.}{\cdots}\right.\right.}{\cdots}$
parts	$-\frac{2a^3e\left(\frac{(e\cot(dx+c))^{\frac{3}{2}}}{3}-\frac{e^2\sqrt{2}\left(\ln\left(\frac{e\cot(dx+c)-(e^2)^{\frac{1}{4}}\sqrt{e\cot(dx+c)}\sqrt{2}+\sqrt{e^2}}{e\cot(dx+c)+(e^2)^{\frac{1}{4}}\sqrt{e\cot(dx+c)}\sqrt{2}+\sqrt{e^2}}\right)+2\arctan\left(\frac{\sqrt{2}\sqrt{e\cot(dx+c)}}{(e^2)^{\frac{1}{4}}}+1\right)-2\arctan\left(-\cdots\right.\right.}{8(e^2)^{\frac{1}{4}}}\cdots\right.}{d}$

```
[In] int((e*cot(d*x+c))^(5/2)*(a+a*cot(d*x+c))^3,x,method=_RETURNVERBOSE)
```

```
[Out] -2/d*a^3/e^2*(1/9*(e*cot(d*x+c))^(9/2)+3/7*e*(e*cot(d*x+c))^(7/2)+2/5*e^2*(
e*cot(d*x+c))^(5/2)-2/3*e^3*(e*cot(d*x+c))^(3/2)-2*(e*cot(d*x+c))^(1/2)*e^4
+2*e^5*(1/8/e*(e^2)^(1/4)*2^(1/2)*(ln((e*cot(d*x+c)+(e^2)^(1/4)*(e*cot(d*x+
```

```
c))^(1/2)*2^(1/2)+(e^2)^(1/2))/(e*cot(d*x+c)-(e^2)^(1/4)*(e*cot(d*x+c))^(1/
2)*2^(1/2)+(e^2)^(1/2)))+2*arctan(2^(1/2)/(e^2)^(1/4)*(e*cot(d*x+c))^(1/2)+
1)-2*arctan(-2^(1/2)/(e^2)^(1/4)*(e*cot(d*x+c))^(1/2)+1))+1/8/(e^2)^(1/4)*2
^(1/2)*(ln((e*cot(d*x+c)-(e^2)^(1/4)*(e*cot(d*x+c))^(1/2)*2^(1/2)+(e^2)^(1/
2))/(e*cot(d*x+c)+(e^2)^(1/4)*(e*cot(d*x+c))^(1/2)*2^(1/2)+(e^2)^(1/2)))+2*
arctan(2^(1/2)/(e^2)^(1/4)*(e*cot(d*x+c))^(1/2)+1)-2*arctan(-2^(1/2)/(e^2)^
(1/4)*(e*cot(d*x+c))^(1/2)+1))))
```

Fricas [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 535, normalized size of antiderivative = 2.88

$$\int (e\cot(c+dx))^{5/2}(a+a\cot(c+dx))^3\,dx = \left[\frac{315\sqrt{2}\left(a^3e^2\cos(2\,dx+2\,c)^2-2\,a^3e^2\cos(2\,dx+2\,c)+a^3e^2\right)\sqrt{-e}\log\left(-\sqrt{2}\sqrt{-e}\sqrt{\frac{ec}{}}\right.}{}\right.$$

```
[In] integrate((e*cot(d*x+c))^(5/2)*(a+a*cot(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] [1/315*(315*sqrt(2)*(a^3*e^2*cos(2*d*x + 2*c)^2 - 2*a^3*e^2*cos(2*d*x + 2*c
) + a^3*e^2)*sqrt(-e)*log(-sqrt(2)*sqrt(-e)*sqrt((e*cos(2*d*x + 2*c) + e)/s
in(2*d*x + 2*c))*(cos(2*d*x + 2*c) + sin(2*d*x + 2*c) - 1) - 2*e*sin(2*d*x
+ 2*c) + e) + 2*(721*a^3*e^2*cos(2*d*x + 2*c)^2 - 1330*a^3*e^2*cos(2*d*x +
2*c) + 469*a^3*e^2 - 15*(23*a^3*e^2*cos(2*d*x + 2*c) - 5*a^3*e^2)*sin(2*d*x
 + 2*c))*sqrt((e*cos(2*d*x + 2*c) + e)/sin(2*d*x + 2*c)))/(d*cos(2*d*x + 2*
c)^2 - 2*d*cos(2*d*x + 2*c) + d), 2/315*(315*sqrt(2)*(a^3*e^2*cos(2*d*x + 2
*c)^2 - 2*a^3*e^2*cos(2*d*x + 2*c) + a^3*e^2)*sqrt(e)*arctan(-1/2*sqrt(2)*s
qrt(e)*sqrt((e*cos(2*d*x + 2*c) + e)/sin(2*d*x + 2*c))*(cos(2*d*x + 2*c) -
sin(2*d*x + 2*c) + 1)/(e*cos(2*d*x + 2*c) + e)) + (721*a^3*e^2*cos(2*d*x +
2*c)^2 - 1330*a^3*e^2*cos(2*d*x + 2*c) + 469*a^3*e^2 - 15*(23*a^3*e^2*cos(2
*d*x + 2*c) - 5*a^3*e^2)*sin(2*d*x + 2*c))*sqrt((e*cos(2*d*x + 2*c) + e)/si
n(2*d*x + 2*c)))/(d*cos(2*d*x + 2*c)^2 - 2*d*cos(2*d*x + 2*c) + d)]
```

Sympy [F]

$$\int (e\cot(c+dx))^{5/2}(a+a\cot(c+dx))^3\,dx = a^3\left(\int (e\cot{(c+dx)})^{\frac{5}{2}}\,dx + \int 3(e\cot{(c+dx)})^{\frac{5}{2}}\cot{(c+dx)}\,dx + \int 3(e\cot{(c+dx)})^{\frac{5}{2}}\cot^2{(c+dx)}\,dx + \int (e\cot{(c+dx)})^{\frac{5}{2}}\cot^3{(c+dx)}\,dx\right)$$

```
[In] integrate((e*cot(d*x+c))**(5/2)*(a+a*cot(d*x+c))**3,x)
```

```
[Out] a**3*(Integral((e*cot(c + d*x))**(5/2), x) + Integral(3*(e*cot(c + d*x))**(
5/2)*cot(c + d*x), x) + Integral(3*(e*cot(c + d*x))**(5/2)*cot(c + d*x)**2,
 x) + Integral((e*cot(c + d*x))**(5/2)*cot(c + d*x)**3, x))
```

Maxima [F(-2)]

Exception generated.

$$\int (e\cot(c+dx))^{5/2}(a+a\cot(c+dx))^3\,dx = \text{Exception raised: ValueError}$$

```
[In] integrate((e*cot(d*x+c))^(5/2)*(a+a*cot(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(e>0)', see `assume?` for more detai
ls)Is e
```

Giac [F]

$$\int (e\cot(c+dx))^{5/2}(a+a\cot(c+dx))^3\,dx = \int (a\cot{(dx+c)}+a)^3(e\cot{(dx+c)})^{\frac{5}{2}}\,dx$$

```
[In] integrate((e*cot(d*x+c))^(5/2)*(a+a*cot(d*x+c))^3,x, algorithm="giac")
```

```
[Out] integrate((a*cot(d*x + c) + a)^3*(e*cot(d*x + c))^(5/2), x)
```

Mupad [B] (verification not implemented)

Time = 14.78 (sec) , antiderivative size = 177, normalized size of antiderivative = 0.95

$$\int (e\cot(c+dx))^{5/2}(a+a\cot(c+dx))^3\,dx = \frac{4\,a^3\,e^2\,\sqrt{e\cot\left(c+d\,x\right)}}{d} - \frac{4\,a^3\,\left(e\cot(c+d\,x)\right)^{5/2}}{5\,d} - \frac{6\,a^3\,\left(e\cot(c+d\,x)\right)^{7/2}}{7\,d\,e} - \frac{2\,a^3\,\left(e\cot(c+d\,x)\right)^{9/2}}{9\,d\,e^2} + \frac{4\,a^3\,e\,\left(e\cot(c+d\,x)\right)^{3/2}}{3\,d} - \frac{\sqrt{2}\,a^3\,e^{5/2}\,\left(2\,\mathrm{atan}\left(\frac{\sqrt{2}\,\sqrt{e\cot(c+d\,x)}}{2\,\sqrt{e}}\right)+2\,\mathrm{atan}\left(\frac{\sqrt{2}\,\sqrt{e\cot(c+d\,x)}}{2\,\sqrt{e}}+\frac{\sqrt{2}\,\left(e\cot(c+d\,x)\right)^{3/2}}{2\,e^{3/2}}\right)\right)}{d}$$

```
[In] int((e*cot(c + d*x))^(5/2)*(a + a*cot(c + d*x))^3,x)
```

```
[Out] (4*a^3*e^2*(e*cot(c + d*x))^(1/2))/d - (4*a^3*(e*cot(c + d*x))^(5/2))/(5*d)
 - (6*a^3*(e*cot(c + d*x))^(7/2))/(7*d*e) - (2*a^3*(e*cot(c + d*x))^(9/2))/
(9*d*e^2) + (4*a^3*e*(e*cot(c + d*x))^(3/2))/(3*d) - (2^(1/2)*a^3*e^(5/2)*(
2*atan((2^(1/2)*(e*cot(c + d*x))^(1/2))/(2*e^(1/2))) + 2*atan((2^(1/2)*(e*c
ot(c + d*x))^(1/2))/(2*e^(1/2)) + (2^(1/2)*(e*cot(c + d*x))^(3/2))/(2*e^(3/
2)))))/d
```

3.16 $\int (e\cot(c+dx))^{3/2}(a+a\cot(c+dx))^3\,dx$

Optimal result

Integrand size = 25, antiderivative size = 160

$$\int (e\cot(c+dx))^{3/2}(a+a\cot(c+dx))^3\,dx = -\frac{2\sqrt{2}a^3e^{3/2}\text{arctanh}\left(\frac{\sqrt{e}+\sqrt{e}\cot(c+dx)}{\sqrt{2}\sqrt{e\cot(c+dx)}}\right)}{d} + \frac{4a^3e\sqrt{e\cot(c+dx)}}{d} - \frac{4a^3(e\cot(c+dx))^{3/2}}{3d} - \frac{32a^3(e\cot(c+dx))^{5/2}}{35de} - \frac{2(e\cot(c+dx))^{5/2}\left(a^3+a^3\cot(c+dx)\right)}{7de}$$

```
[Out] -4/3*a^3*(e*cot(d*x+c))^(3/2)/d-32/35*a^3*(e*cot(d*x+c))^(5/2)/d/e-2/7*(e*c
ot(d*x+c))^(5/2)*(a^3+a^3*cot(d*x+c))/d/e-2*a^3*e^(3/2)*arctanh(1/2*(e^(1/2
)+cot(d*x+c)*e^(1/2))*2^(1/2)/(e*cot(d*x+c))^(1/2))*2^(1/2)/d+4*a^3*e*(e*co
t(d*x+c))^(1/2)/d
```

Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 160, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {3647, 3711, 3609, 3613, 214}

$$\int (e\cot(c+dx))^{3/2}(a+a\cot(c+dx))^3\,dx = -\frac{2\sqrt{2}a^3e^{3/2}\text{arctanh}\left(\frac{\sqrt{e}\cot(c+dx)+\sqrt{e}}{\sqrt{2}\sqrt{e\cot(c+dx)}}\right)}{d} - \frac{32a^3(e\cot(c+dx))^{5/2}}{35de} - \frac{4a^3(e\cot(c+dx))^{3/2}}{3d} + \frac{4a^3e\sqrt{e\cot(c+dx)}}{d} - \frac{2\left(a^3\cot(c+dx)+a^3\right)(e\cot(c+dx))^{5/2}}{7de}$$

```
[In] Int[(e*Cot[c + d*x])^(3/2)*(a + a*Cot[c + d*x])^3,x]
```

```
[Out] (-2*Sqrt[2]*a^3*e^(3/2)*ArcTanh[(Sqrt[e] + Sqrt[e]*Cot[c + d*x])/(Sqrt[2]*S
qrt[e*Cot[c + d*x]])])/d + (4*a^3*e*Sqrt[e*Cot[c + d*x]])/d - (4*a^3*(e*Cot
[c + d*x])^(3/2))/(3*d) - (32*a^3*(e*Cot[c + d*x])^(5/2))/(35*d*e) - (2*(e*
Cot[c + d*x])^(5/2)*(a^3 + a^3*Cot[c + d*x]))/(7*d*e)
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 3609

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)]), x_Symbol] :> Simp[d*((a + b*Tan[e + f*x])^m/(f*m)), x] + Int
[(a + b*Tan[e + f*x])^(m - 1)*Simp[a*c - b*d + (b*c + a*d)*Tan[e + f*x], x]
, x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2,
 0] && GtQ[m, 0]
```

Rule 3613

```
Int[((c_) + (d_.)*tan[(e_.) + (f_.)*(x_)])/Sqrt[(b_.)*tan[(e_.) + (f_.)*(x_
)]], x_Symbol] :> Dist[-2*(d^2/f), Subst[Int[1/(2*c*d + b*x^2), x], x, (c -
 d*Tan[e + f*x])/Sqrt[b*Tan[e + f*x]]], x] /; FreeQ[{b, c, d, e, f}, x] &&
EqQ[c^2 - d^2, 0]
```

Rule 3647

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)])^(n_), x_Symbol] :> Simp[b^2*(a + b*Tan[e + f*x])^(m - 2)*((c
+ d*Tan[e + f*x])^(n + 1)/(d*f*(m + n - 1))), x] + Dist[1/(d*(m + n - 1)),
Int[(a + b*Tan[e + f*x])^(m - 3)*(c + d*Tan[e + f*x])^n*Simp[a^3*d*(m + n -
 1) - b^2*(b*c*(m - 2) + a*d*(1 + n)) + b*d*(m + n - 1)*(3*a^2 - b^2)*Tan[e
 + f*x] - b^2*(b*c*(m - 2) - a*d*(3*m + 2*n - 4))*Tan[e + f*x]^2, x], x], x
] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2,
 0] && NeQ[c^2 + d^2, 0] && IntegerQ[2*m] && GtQ[m, 2] && (GeQ[n, -1] || In
tegerQ[m]) &&  !(IGtQ[n, 2] && ( !IntegerQ[m] || (EqQ[c, 0] && NeQ[a, 0])))
```

Rule 3711

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*tan[(e_.)
+ (f_.)*(x_)] + (C_.)*tan[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[C*((a +
 b*Tan[e + f*x])^(m + 1)/(b*f*(m + 1))), x] + Int[(a + b*Tan[e + f*x])^m*Si
mp[A - C + B*Tan[e + f*x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] &&
NeQ[A*b^2 - a*b*B + a^2*C, 0] &&  !LeQ[m, -1]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= -\frac{2(e\cot(c+dx))^{5/2}\left(a^3+a^3\cot(c+dx)\right)}{7de} - \frac{2\int(e\cot(c+dx))^{3/2}\left(-a^3e-7a^3e\cot(c+dx)-8a^3e\cot^2(c+dx)\right)\,dx}{7e}\\
&= -\frac{32a^3(e\cot(c+dx))^{5/2}}{35de} - \frac{2(e\cot(c+dx))^{5/2}\left(a^3+a^3\cot(c+dx)\right)}{7de} - \frac{2\int(e\cot(c+dx))^{3/2}\left(7a^3e-7a^3e\cot(c+dx)\right)\,dx}{7e}\\
&= -\frac{4a^3(e\cot(c+dx))^{3/2}}{3d} - \frac{32a^3(e\cot(c+dx))^{5/2}}{35de} - \frac{2(e\cot(c+dx))^{5/2}\left(a^3+a^3\cot(c+dx)\right)}{7de} - \frac{2\int\sqrt{e\cot(c+dx)}\left(7a^3e^2+7a^3e^2\cot(c+dx)\right)\,dx}{7e}\\
&= \frac{4a^3e\sqrt{e\cot(c+dx)}}{d} - \frac{4a^3(e\cot(c+dx))^{3/2}}{3d} - \frac{32a^3(e\cot(c+dx))^{5/2}}{35de} - \frac{2(e\cot(c+dx))^{5/2}\left(a^3+a^3\cot(c+dx)\right)}{7de} - \frac{2\int\frac{-7a^3e^3+7a^3e^3\cot(c+dx)}{\sqrt{e\cot(c+dx)}}\,dx}{7e}\\
&= \frac{4a^3e\sqrt{e\cot(c+dx)}}{d} - \frac{4a^3(e\cot(c+dx))^{3/2}}{3d} - \frac{32a^3(e\cot(c+dx))^{5/2}}{35de} - \frac{2(e\cot(c+dx))^{5/2}\left(a^3+a^3\cot(c+dx)\right)}{7de} + \frac{(28a^6e^5)\operatorname{Subst}\left(\int\frac{1}{98a^6e^6-ex^2}\,dx,x,\frac{-7a^3e^3-7a^3e^3\cot(c+dx)}{\sqrt{e\cot(c+dx)}}\right)}{d}\\
&= -\frac{2\sqrt{2}a^3e^{3/2}\operatorname{arctanh}\left(\frac{\sqrt{e}+\sqrt{e}\cot(c+dx)}{\sqrt{2}\sqrt{e\cot(c+dx)}}\right)}{d} + \frac{4a^3e\sqrt{e\cot(c+dx)}}{d} - \frac{4a^3(e\cot(c+dx))^{3/2}}{3d} - \frac{32a^3(e\cot(c+dx))^{5/2}}{35de} - \frac{2(e\cot(c+dx))^{5/2}\left(a^3+a^3\cot(c+dx)\right)}{7de}
\end{aligned}
$$

Mathematica [B] (warning: unable to verify)

Leaf count is larger than twice the leaf count of optimal. 380 vs. $2(160) = 320$.

Time = 5.18 (sec) , antiderivative size = 380, normalized size of antiderivative = 2.38

$$\int (e\cot(c+dx))^{3/2}(a+a\cot(c+dx))^3\,dx =$$

$$-\frac{a^3(e\cot(c+dx))^{3/2}(1+\cot(c+dx))^3\left(-420\arctan\left(\sqrt[4]{-\cot^2(c+dx)}\right)\sqrt[4]{-\cot(c+dx)}\sin^3(c+dx)+4\right.}{}$$

```
[In] Integrate[(e*Cot[c + d*x])^(3/2)*(a + a*Cot[c + d*x])^3,x]
```

```
[Out] -1/210*(a^3*(e*Cot[c + d*x])^(3/2)*(1 + Cot[c + d*x])^3*(-420*ArcTan[(-Cot[
c + d*x]^2)^(1/4)]*(-Cot[c + d*x])^(1/4)*Sin[c + d*x]^3 + 420*ArcTanh[(-Cot
[c + d*x]^2)^(1/4)]*(-Cot[c + d*x])^(1/4)*Sin[c + d*x]^3 + Cot[c + d*x]^(1/
4)*Sin[c + d*x]*(60*Cos[c + d*x]^2*Cot[c + d*x]^(3/2) - 210*Sqrt[2]*ArcTan[
1 - Sqrt[2]*Sqrt[Cot[c + d*x]]]*Sin[c + d*x]^2 + 210*Sqrt[2]*ArcTan[1 + Sqr
t[2]*Sqrt[Cot[c + d*x]]]*Sin[c + d*x]^2 - 840*Sqrt[Cot[c + d*x]]*Sin[c + d*
x]^2 + 280*Cot[c + d*x]^(3/2)*Sin[c + d*x]^2 - 105*Sqrt[2]*Log[1 - Sqrt[2]*
Sqrt[Cot[c + d*x]] + Cot[c + d*x]]*Sin[c + d*x]^2 + 105*Sqrt[2]*Log[1 + Sqr
t[2]*Sqrt[Cot[c + d*x]] + Cot[c + d*x]]*Sin[c + d*x]^2 + 126*Cot[c + d*x]^(
3/2)*Sin[2*(c + d*x)])))/(d*Cot[c + d*x]^(7/4)*(Cos[c + d*x] + Sin[c + d*x]
)^3)
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 338 vs. $2(135) = 270$.

Time = 0.04 (sec) , antiderivative size = 339, normalized size of antiderivative = 2.12

method	result
derivativedivides	$-\frac{2a^3\left(\frac{(e\cot(dx+c))^{\frac{7}{2}}}{7}+\frac{3e(e\cot(dx+c))^{\frac{5}{2}}}{5}+\frac{2e^2(e\cot(dx+c))^{\frac{3}{2}}}{3}-2\sqrt{e\cot(dx+c)}\,e^3+2e^4\left(\frac{\left(e^2\right)^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\frac{e\cot(dx+c)+\left(e^2\right)}{e\cot(dx+c)-\left(e^2\right)}\right.\right.}{}\right.\right.}{}$
default	$-\frac{2a^3\left(\frac{(e\cot(dx+c))^{\frac{7}{2}}}{7}+\frac{3e(e\cot(dx+c))^{\frac{5}{2}}}{5}+\frac{2e^2(e\cot(dx+c))^{\frac{3}{2}}}{3}-2\sqrt{e\cot(dx+c)}\,e^3+2e^4\left(\frac{\left(e^2\right)^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\frac{e\cot(dx+c)+\left(e^2\right)}{e\cot(dx+c)-\left(e^2\right)}\right.\right.}{}\right.\right.}{}$
parts	$-\frac{2a^3e\left(\sqrt{e\cot(dx+c)}-\frac{\left(e^2\right)^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\frac{e\cot(dx+c)+\left(e^2\right)^{\frac{1}{4}}\sqrt{e\cot(dx+c)}\sqrt{2}+\sqrt{e^2}}{e\cot(dx+c)-\left(e^2\right)^{\frac{1}{4}}\sqrt{e\cot(dx+c)}\sqrt{2}+\sqrt{e^2}}\right)+2\arctan\left(\frac{\sqrt{2}\sqrt{e\cot(dx+c)}}{\left(e^2\right)^{\frac{1}{4}}}+1\right)-2\,\mathrm{arcta}\right.}{8}\right.}{d}$

```
[In] int((e*cot(d*x+c))^(3/2)*(a+a*cot(d*x+c))^3,x,method=_RETURNVERBOSE)
```

```
[Out] -2/d*a^3/e^2*(1/7*(e*cot(d*x+c))^(7/2)+3/5*e*(e*cot(d*x+c))^(5/2)+2/3*e^2*(
e*cot(d*x+c))^(3/2)-2*(e*cot(d*x+c))^(1/2)*e^3+2*e^4*(1/8/e*(e^2)^(1/4)*2^(
1/2)*(ln((e*cot(d*x+c)+(e^2)^(1/4)*(e*cot(d*x+c))^(1/2)*2^(1/2)+(e^2)^(1/2)
)/(e*cot(d*x+c)-(e^2)^(1/4)*(e*cot(d*x+c))^(1/2)*2^(1/2)+(e^2)^(1/2)))+2*ar
ctan(2^(1/2)/(e^2)^(1/4)*(e*cot(d*x+c))^(1/2)+1)-2*arctan(-2^(1/2)/(e^2)^(1
/4)*(e*cot(d*x+c))^(1/2)+1))-1/8/(e^2)^(1/4)*2^(1/2)*(ln((e*cot(d*x+c)-(e^2
)^(1/4)*(e*cot(d*x+c))^(1/2)*2^(1/2)+(e^2)^(1/2))/(e*cot(d*x+c)+(e^2)^(1/4)
*(e*cot(d*x+c))^(1/2)*2^(1/2)+(e^2)^(1/2)))+2*arctan(2^(1/2)/(e^2)^(1/4)*(e
*cot(d*x+c))^(1/2)+1)-2*arctan(-2^(1/2)/(e^2)^(1/4)*(e*cot(d*x+c))^(1/2)+1)
)))
```

Fricas [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 487, normalized size of antiderivative = 3.04

$$\int (e\cot(c+dx))^{3/2}(a+a\cot(c+dx))^3\,dx = \left[\frac{105\sqrt{2}(a^3e\cos(2\,dx+2\,c)-a^3e)\sqrt{e}\log\left(\sqrt{2}\sqrt{e}\sqrt{\frac{e\cos(2\,dx+2\,c)+e}{\sin(2\,dx+2\,c)}}(\cos(2\,dx+2\,c)-\mathrm{s}\right.}{}\right.$$

```
[In] integrate((e*cot(d*x+c))^(3/2)*(a+a*cot(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] [1/105*(105*sqrt(2)*(a^3*e*cos(2*d*x + 2*c) - a^3*e)*sqrt(e)*log(sqrt(2)*sq
rt(e)*sqrt((e*cos(2*d*x + 2*c) + e)/sin(2*d*x + 2*c))*(cos(2*d*x + 2*c) - s
in(2*d*x + 2*c) - 1) + 2*e*sin(2*d*x + 2*c) + e)*sin(2*d*x + 2*c) - 2*(55*a
^3*e*cos(2*d*x + 2*c)^2 - 30*a^3*e*cos(2*d*x + 2*c) - 85*a^3*e - 21*(13*a^3
*e*cos(2*d*x + 2*c) - 7*a^3*e)*sin(2*d*x + 2*c))*sqrt((e*cos(2*d*x + 2*c) +
 e)/sin(2*d*x + 2*c)))/((d*cos(2*d*x + 2*c) - d)*sin(2*d*x + 2*c)), 2/105*(
105*sqrt(2)*(a^3*e*cos(2*d*x + 2*c) - a^3*e)*sqrt(-e)*arctan(1/2*sqrt(2)*sq
rt(-e)*sqrt((e*cos(2*d*x + 2*c) + e)/sin(2*d*x + 2*c))*(cos(2*d*x + 2*c) +
sin(2*d*x + 2*c) + 1)/(e*cos(2*d*x + 2*c) + e))*sin(2*d*x + 2*c) - (55*a^3*
e*cos(2*d*x + 2*c)^2 - 30*a^3*e*cos(2*d*x + 2*c) - 85*a^3*e - 21*(13*a^3*e*
cos(2*d*x + 2*c) - 7*a^3*e)*sin(2*d*x + 2*c))*sqrt((e*cos(2*d*x + 2*c) + e)
/sin(2*d*x + 2*c)))/((d*cos(2*d*x + 2*c) - d)*sin(2*d*x + 2*c))]
```

Sympy [F]

$$\int (e\cot(c+dx))^{3/2}(a+a\cot(c+dx))^3\,dx = a^3\left(\int (e\cot(c+dx))^{\frac{3}{2}}\,dx + \int 3(e\cot(c+dx))^{\frac{3}{2}}\cot(c+dx)\,dx + \int 3(e\cot(c+dx))^{\frac{3}{2}}\cot^2(c+dx)\,dx + \int (e\cot(c+dx))^{\frac{3}{2}}\cot^3(c+dx)\,dx\right)$$

```
[In] integrate((e*cot(d*x+c))**(3/2)*(a+a*cot(d*x+c))**3,x)
```

```
[Out] a**3*(Integral((e*cot(c + d*x))**(3/2), x) + Integral(3*(e*cot(c + d*x))**(
3/2)*cot(c + d*x), x) + Integral(3*(e*cot(c + d*x))**(3/2)*cot(c + d*x)**2,
 x) + Integral((e*cot(c + d*x))**(3/2)*cot(c + d*x)**3, x))
```

Maxima [F(-2)]

Exception generated.

$$\int (e\cot(c+dx))^{3/2}(a+a\cot(c+dx))^3\,dx = \text{Exception raised: ValueError}$$

```
[In] integrate((e*cot(d*x+c))^(3/2)*(a+a*cot(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(e>0)', see `assume?` for more detai
ls)Is e
```

Giac [F]

$$\int (e\cot(c+dx))^{3/2}(a+a\cot(c+dx))^3\,dx = \int (a\cot(dx+c)+a)^3(e\cot(dx+c))^{\frac{3}{2}}\,dx$$

```
[In] integrate((e*cot(d*x+c))^(3/2)*(a+a*cot(d*x+c))^3,x, algorithm="giac")
[Out] integrate((a*cot(d*x + c) + a)^3*(e*cot(d*x + c))^(3/2), x)
```

Mupad [B] (verification not implemented)

Time = 14.19 (sec) , antiderivative size = 143, normalized size of antiderivative = 0.89

$$\int (e\cot(c+dx))^{3/2}(a+a\cot(c+dx))^3\,dx = \frac{4\,a^3\,e\,\sqrt{e\cot(c+d\,x)}}{d} - \frac{6\,a^3\,(e\cot(c+d\,x))^{5/2}}{5\,d\,e} - \frac{2\,a^3\,(e\cot(c+d\,x))^{7/2}}{7\,d\,e^2} - \frac{4\,a^3\,(e\cot(c+d\,x))^{3/2}}{3\,d} + \frac{\sqrt{2}\,a^3\,e^{3/2}\,\mathrm{atan}\left(\frac{\sqrt{2}\,a^6\,e^{9/2}\,\sqrt{e\cot(c+d\,x)}\,32\mathrm{i}}{32\,a^6\,e^5+32\,a^6\,e^5\cot(c+d\,x)}\right)\,2\mathrm{i}}{d}$$

```
[In] int((e*cot(c + d*x))^(3/2)*(a + a*cot(c + d*x))^3,x)
[Out] (4*a^3*e*(e*cot(c + d*x))^(1/2))/d - (6*a^3*(e*cot(c + d*x))^(5/2))/(5*d*e)
 - (2*a^3*(e*cot(c + d*x))^(7/2))/(7*d*e^2) - (4*a^3*(e*cot(c + d*x))^(3/2)
)/(3*d) + (2^(1/2)*a^3*e^(3/2)*atan((2^(1/2)*a^6*e^(9/2)*(e*cot(c + d*x))^(
1/2)*32i)/(32*a^6*e^5 + 32*a^6*e^5*cot(c + d*x)))*2i)/d
```

3.17 $\int \sqrt{e \cot(c+dx)}(a+a\cot(c+dx))^3\,dx$

Optimal result

Integrand size = 25, antiderivative size = 138

$$\int \sqrt{e\cot(c+dx)}(a+a\cot(c+dx))^3\,dx = -\frac{2\sqrt{2}a^3\sqrt{e}\arctan\left(\frac{\sqrt{e}-\sqrt{e}\cot(c+dx)}{\sqrt{2}\sqrt{e\cot(c+dx)}}\right)}{d} - \frac{4a^3\sqrt{e\cot(c+dx)}}{d} - \frac{8a^3(e\cot(c+dx))^{3/2}}{5de} - \frac{2(e\cot(c+dx))^{3/2}\left(a^3+a^3\cot(c+dx)\right)}{5de}$$

```
[Out] -8/5*a^3*(e*cot(d*x+c))^(3/2)/d/e-2/5*(e*cot(d*x+c))^(3/2)*(a^3+a^3*cot(d*x
+c))/d/e-2*a^3*arctan(1/2*(e^(1/2)-cot(d*x+c)*e^(1/2))*2^(1/2)/(e*cot(d*x+c
))^(1/2))*2^(1/2)*e^(1/2)/d-4*a^3*(e*cot(d*x+c))^(1/2)/d
```

Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 138, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {3647, 3711, 3609, 3613, 211}

$$\int \sqrt{e\cot(c+dx)}(a+a\cot(c+dx))^3\,dx = -\frac{2\sqrt{2}a^3\sqrt{e}\arctan\left(\frac{\sqrt{e}-\sqrt{e}\cot(c+dx)}{\sqrt{2}\sqrt{e\cot(c+dx)}}\right)}{d} - \frac{8a^3(e\cot(c+dx))^{3/2}}{5de} - \frac{4a^3\sqrt{e\cot(c+dx)}}{d} - \frac{2\left(a^3\cot(c+dx)+a^3\right)(e\cot(c+dx))^{3/2}}{5de}$$

```
[In] Int[Sqrt[e*Cot[c + d*x]]*(a + a*Cot[c + d*x])^3,x]
```

```
[Out] (-2*Sqrt[2]*a^3*Sqrt[e]*ArcTan[(Sqrt[e] - Sqrt[e]*Cot[c + d*x])/(Sqrt[2]*Sq
rt[e*Cot[c + d*x]])])/d - (4*a^3*Sqrt[e*Cot[c + d*x]])/d - (8*a^3*(e*Cot[c
+ d*x])^(3/2))/(5*d*e) - (2*(e*Cot[c + d*x])^(3/2)*(a^3 + a^3*Cot[c + d*x])
)/(5*d*e)
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 3609

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)]), x_Symbol] :> Simp[d*((a + b*Tan[e + f*x])^m/(f*m)), x] + Int
[(a + b*Tan[e + f*x])^(m - 1)*Simp[a*c - b*d + (b*c + a*d)*Tan[e + f*x], x]
, x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2,
 0] && GtQ[m, 0]
```

Rule 3613

```
Int[((c_) + (d_.)*tan[(e_.) + (f_.)*(x_)])/Sqrt[(b_.)*tan[(e_.) + (f_.)*(x_
)]], x_Symbol] :> Dist[-2*(d^2/f), Subst[Int[1/(2*c*d + b*x^2), x], x, (c -
 d*Tan[e + f*x])/Sqrt[b*Tan[e + f*x]]], x] /; FreeQ[{b, c, d, e, f}, x] &&
EqQ[c^2 - d^2, 0]
```

Rule 3647

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)])^(n_), x_Symbol] :> Simp[b^2*(a + b*Tan[e + f*x])^(m - 2)*((c
+ d*Tan[e + f*x])^(n + 1)/(d*f*(m + n - 1))), x] + Dist[1/(d*(m + n - 1)),
Int[(a + b*Tan[e + f*x])^(m - 3)*(c + d*Tan[e + f*x])^n*Simp[a^3*d*(m + n -
 1) - b^2*(b*c*(m - 2) + a*d*(1 + n)) + b*d*(m + n - 1)*(3*a^2 - b^2)*Tan[e
 + f*x] - b^2*(b*c*(m - 2) - a*d*(3*m + 2*n - 4))*Tan[e + f*x]^2, x], x], x
] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2,
 0] && NeQ[c^2 + d^2, 0] && IntegerQ[2*m] && GtQ[m, 2] && (GeQ[n, -1] || In
tegerQ[m]) &&  !(IGtQ[n, 2] && ( !IntegerQ[m] || (EqQ[c, 0] && NeQ[a, 0])))
```

Rule 3711

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*tan[(e_.)
+ (f_.)*(x_)] + (C_.)*tan[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[C*((a +
 b*Tan[e + f*x])^(m + 1)/(b*f*(m + 1))), x] + Int[(a + b*Tan[e + f*x])^m*Si
mp[A - C + B*Tan[e + f*x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] &&
NeQ[A*b^2 - a*b*B + a^2*C, 0] &&  !LeQ[m, -1]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\frac{2(e\cot(c+dx))^{3/2}\left(a^3+a^3\cot(c+dx)\right)}{5de} \\
&\quad -\frac{2\int\sqrt{e\cot(c+dx)}\left(-a^3e-5a^3e\cot(c+dx)-6a^3e\cot^2(c+dx)\right)\,dx}{5e} \\
&= -\frac{8a^3(e\cot(c+dx))^{3/2}}{5de}-\frac{2(e\cot(c+dx))^{3/2}\left(a^3+a^3\cot(c+dx)\right)}{5de} \\
&\quad -\frac{2\int\sqrt{e\cot(c+dx)}\left(5a^3e-5a^3e\cot(c+dx)\right)\,dx}{5e} \\
&= -\frac{4a^3\sqrt{e\cot(c+dx)}}{d}-\frac{8a^3(e\cot(c+dx))^{3/2}}{5de} \\
&\quad -\frac{2(e\cot(c+dx))^{3/2}\left(a^3+a^3\cot(c+dx)\right)}{5de}-\frac{2\int\frac{5a^3e^2+5a^3e^2\cot(c+dx)}{\sqrt{e\cot(c+dx)}}\,dx}{5e} \\
&= -\frac{4a^3\sqrt{e\cot(c+dx)}}{d}-\frac{8a^3(e\cot(c+dx))^{3/2}}{5de} \\
&\quad -\frac{2(e\cot(c+dx))^{3/2}\left(a^3+a^3\cot(c+dx)\right)}{5de} \\
&\quad +\frac{(20a^6e^3)\,\text{Subst}\left(\int\frac{1}{-50a^6e^4-ex^2}\,dx,x,\frac{5a^3e^2-5a^3e^2\cot(c+dx)}{\sqrt{e\cot(c+dx)}}\right)}{d} \\
&= -\frac{2\sqrt{2}a^3\sqrt{e}\arctan\left(\frac{\sqrt{e}-\sqrt{e}\cot(c+dx)}{\sqrt{2}\sqrt{e\cot(c+dx)}}\right)}{d}-\frac{4a^3\sqrt{e\cot(c+dx)}}{d} \\
&\quad -\frac{8a^3(e\cot(c+dx))^{3/2}}{5de}-\frac{2(e\cot(c+dx))^{3/2}\left(a^3+a^3\cot(c+dx)\right)}{5de}
\end{aligned}$$

Mathematica [B] (warning: unable to verify)

Leaf count is larger than twice the leaf count of optimal. 360 vs. $2(138) = 276$.

Time = 3.43 (sec) , antiderivative size = 360, normalized size of antiderivative = 2.61

$$\int\sqrt{e\cot(c+dx)}(a+a\cot(c+dx))^3\,dx =$$

$$-\frac{a^3\sqrt{e\cot(c+dx)}(1+\cot(c+dx))^3\left(-20\arctan\left(\sqrt[4]{-\cot^2(c+dx)}\right)\sqrt[4]{-\cot(c+dx)}\sin^3(c+dx)+2\right.}{}$$

```
[In] Integrate[Sqrt[e*Cot[c + d*x]]*(a + a*Cot[c + d*x])^3,x]
```

```
[Out] -1/10*(a^3*Sqrt[e*Cot[c + d*x]]*(1 + Cot[c + d*x])^3*(-20*ArcTan[(-Cot[c + d*x]^2)^(1/4)]*(-Cot[c + d*x])^(1/4)*Sin[c + d*x]^3 + 20*ArcTanh[(-Cot[c +
```

```
d*x]^2)^(1/4)]*(-Cot[c + d*x])^(1/4)*Sin[c + d*x]^3 + Cot[c + d*x]^(1/4)*Si
n[c + d*x]*(4*Cos[c + d*x]^2*Sqrt[Cot[c + d*x]] + 10*Sqrt[2]*ArcTan[1 - Sqr
t[2]*Sqrt[Cot[c + d*x]]]*Sin[c + d*x]^2 - 10*Sqrt[2]*ArcTan[1 + Sqrt[2]*Sqr
t[Cot[c + d*x]]]*Sin[c + d*x]^2 + 40*Sqrt[Cot[c + d*x]]*Sin[c + d*x]^2 + 5*
Sqrt[2]*Log[1 - Sqrt[2]*Sqrt[Cot[c + d*x]] + Cot[c + d*x]]*Sin[c + d*x]^2 -
 5*Sqrt[2]*Log[1 + Sqrt[2]*Sqrt[Cot[c + d*x]] + Cot[c + d*x]]*Sin[c + d*x]^
2 + 10*Sqrt[Cot[c + d*x]]*Sin[2*(c + d*x)])))/(d*Cot[c + d*x]^(3/4)*(Cos[c
+ d*x] + Sin[c + d*x])^3)
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 322 vs. $2(117) = 234$.

Time = 0.05 (sec) , antiderivative size = 323, normalized size of antiderivative = 2.34

method	result
derivativedivides	$-\frac{2a^3\left(\frac{(e\cot(dx+c))^{\frac{5}{2}}}{5}+e(e\cot(dx+c))^{\frac{3}{2}}+2\sqrt{e\cot(dx+c)}\,e^2-2e^3\left(\frac{(e^2)^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\frac{e\cot(dx+c)+(e^2)^{\frac{1}{4}}\sqrt{e\cot(dx+c)}\,\sqrt{2}+\cdots}{e\cot(dx+c)-(e^2)^{\frac{1}{4}}\sqrt{e\cot(dx+c)}\,\sqrt{2}+\cdots}\right)\cdots\right.}{\cdots}\right.\right.}{\cdots}$
default	$-\frac{2a^3\left(\frac{(e\cot(dx+c))^{\frac{5}{2}}}{5}+e(e\cot(dx+c))^{\frac{3}{2}}+2\sqrt{e\cot(dx+c)}\,e^2-2e^3\left(\frac{(e^2)^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\frac{e\cot(dx+c)+(e^2)^{\frac{1}{4}}\sqrt{e\cot(dx+c)}\,\sqrt{2}+\cdots}{e\cot(dx+c)-(e^2)^{\frac{1}{4}}\sqrt{e\cot(dx+c)}\,\sqrt{2}+\cdots}\right)\cdots\right.}{\cdots}\right.\right.}{\cdots}$
parts	$-\frac{a^3e\sqrt{2}\left(\ln\left(\frac{e\cot(dx+c)-(e^2)^{\frac{1}{4}}\sqrt{e\cot(dx+c)}\,\sqrt{2}+\sqrt{e^2}}{e\cot(dx+c)+(e^2)^{\frac{1}{4}}\sqrt{e\cot(dx+c)}\,\sqrt{2}+\sqrt{e^2}}\right)+2\arctan\left(\frac{\sqrt{2}\,\sqrt{e\cot(dx+c)}}{(e^2)^{\frac{1}{4}}}+1\right)-2\arctan\left(-\frac{\sqrt{2}\,\sqrt{e\cot(dx+c)}}{(e^2)^{\frac{1}{4}}}\cdots\right.\right.}{4d(e^2)^{\frac{1}{4}}}$

```
[In] int((e*cot(d*x+c))^(1/2)*(a+a*cot(d*x+c))^3,x,method=_RETURNVERBOSE)
```

```
[Out] -2/d*a^3/e^2*(1/5*(e*cot(d*x+c))^(5/2)+e*(e*cot(d*x+c))^(3/2)+2*(e*cot(d*x+
c))^(1/2)*e^2-2*e^3*(1/8/e*(e^2)^(1/4)*2^(1/2)*(ln((e*cot(d*x+c)+(e^2)^(1/4
)*(e*cot(d*x+c))^(1/2)*2^(1/2)+(e^2)^(1/2))/(e*cot(d*x+c)-(e^2)^(1/4)*(e*co
t(d*x+c))^(1/2)*2^(1/2)+(e^2)^(1/2)))+2*arctan(2^(1/2)/(e^2)^(1/4)*(e*cot(d
*x+c))^(1/2)+1)-2*arctan(-2^(1/2)/(e^2)^(1/4)*(e*cot(d*x+c))^(1/2)+1))+1/8/
(e^2)^(1/4)*2^(1/2)*(ln((e*cot(d*x+c)-(e^2)^(1/4)*(e*cot(d*x+c))^(1/2)*2^(1
/2)+(e^2)^(1/2))/(e*cot(d*x+c)+(e^2)^(1/4)*(e*cot(d*x+c))^(1/2)*2^(1/2)+(e^
2)^(1/2)))+2*arctan(2^(1/2)/(e^2)^(1/4)*(e*cot(d*x+c))^(1/2)+1)-2*arctan(-2
^(1/2)/(e^2)^(1/4)*(e*cot(d*x+c))^(1/2)+1))))
```

Fricas [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 366, normalized size of antiderivative = 2.65

$$\int \sqrt{e\cot(c+dx)}(a+a\cot(c+dx))^3\,dx$$
$$= \left[\frac{5\sqrt{2}(a^3\cos(2\,dx+2\,c)-a^3)\sqrt{-e}\log\left(\sqrt{2}\sqrt{-e}\sqrt{\frac{e\cos(2\,dx+2\,c)+e}{\sin(2\,dx+2\,c)}}(\cos(2\,dx+2\,c)+\sin(2\,dx+2\,c)-1)-\right.}{5\,(d\cos(2\,dx+2}\right.$$
$$-\frac{2\left(5\sqrt{2}(a^3\cos(2\,dx+2\,c)-a^3)\sqrt{e}\arctan\left(-\frac{\sqrt{2}\sqrt{e}\sqrt{\frac{e\cos(2\,dx+2\,c)+e}{\sin(2\,dx+2\,c)}}(\cos(2\,dx+2\,c)-\sin(2\,dx+2\,c)+1)}{2\,(e\cos(2\,dx+2\,c)+e)}\right)+(9\,a^3\cos(\right.}{5\,(d\cos(2\,dx+2\,c)-d)}$$

```
[In] integrate((e*cot(d*x+c))^(1/2)*(a+a*cot(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] [1/5*(5*sqrt(2)*(a^3*cos(2*d*x + 2*c) - a^3)*sqrt(-e)*log(sqrt(2)*sqrt(-e)*
sqrt((e*cos(2*d*x + 2*c) + e)/sin(2*d*x + 2*c))*(cos(2*d*x + 2*c) + sin(2*d
*x + 2*c) - 1) - 2*e*sin(2*d*x + 2*c) + e) - 2*(9*a^3*cos(2*d*x + 2*c) - 5*
a^3*sin(2*d*x + 2*c) - 11*a^3)*sqrt((e*cos(2*d*x + 2*c) + e)/sin(2*d*x + 2*
c)))/(d*cos(2*d*x + 2*c) - d), -2/5*(5*sqrt(2)*(a^3*cos(2*d*x + 2*c) - a^3)
*sqrt(e)*arctan(-1/2*sqrt(2)*sqrt(e)*sqrt((e*cos(2*d*x + 2*c) + e)/sin(2*d*
x + 2*c))*(cos(2*d*x + 2*c) - sin(2*d*x + 2*c) + 1)/(e*cos(2*d*x + 2*c) + e
)) + (9*a^3*cos(2*d*x + 2*c) - 5*a^3*sin(2*d*x + 2*c) - 11*a^3)*sqrt((e*cos
(2*d*x + 2*c) + e)/sin(2*d*x + 2*c)))/(d*cos(2*d*x + 2*c) - d)]
```

Sympy [F]

$$\int \sqrt{e\cot(c+dx)}(a+a\cot(c+dx))^3\,dx = a^3\left(\int \sqrt{e\cot(c+dx)}\,dx + \int 3\sqrt{e\cot(c+dx)}\cot(c+dx)\,dx + \int 3\sqrt{e\cot(c+dx)}\cot^2(c+dx)\,dx + \int \sqrt{e\cot(c+dx)}\cot^3(c+dx)\,dx\right)$$

```
[In] integrate((e*cot(d*x+c))**(1/2)*(a+a*cot(d*x+c))**3,x)
```

```
[Out] a**3*(Integral(sqrt(e*cot(c + d*x)), x) + Integral(3*sqrt(e*cot(c + d*x))*c
ot(c + d*x), x) + Integral(3*sqrt(e*cot(c + d*x))*cot(c + d*x)**2, x) + Int
egral(sqrt(e*cot(c + d*x))*cot(c + d*x)**3, x))
```

Maxima [F(-2)]

Exception generated.

$$\int \sqrt{e\cot(c+dx)}(a+a\cot(c+dx))^3\,dx = \text{Exception raised: ValueError}$$

```
[In] integrate((e*cot(d*x+c))^(1/2)*(a+a*cot(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(e>0)', see `assume?` for more detai
ls)Is e
```

Giac [F]

$$\int \sqrt{e\cot(c+dx)}(a+a\cot(c+dx))^3\,dx = \int (a\cot(dx+c)+a)^3\sqrt{e\cot(dx+c)}\,dx$$

```
[In] integrate((e*cot(d*x+c))^(1/2)*(a+a*cot(d*x+c))^3,x, algorithm="giac")
```

```
[Out] integrate((a*cot(d*x + c) + a)^3*sqrt(e*cot(d*x + c)), x)
```

Mupad [B] (verification not implemented)

Time = 13.52 (sec) , antiderivative size = 136, normalized size of antiderivative = 0.99

$$\int \sqrt{e\cot(c+dx)}(a+a\cot(c+dx))^3\,dx = \frac{\sqrt{2}\,a^3\sqrt{e}\left(2\,\mathrm{atan}\left(\frac{\sqrt{2}\sqrt{e\cot(c+dx)}}{2\sqrt{e}}\right)+2\,\mathrm{atan}\left(\frac{\sqrt{2}\sqrt{e\cot(c+dx)}}{2\sqrt{e}}+\frac{\sqrt{2}\,(e\cot(c+dx))^{3/2}}{2\,e^{3/2}}\right)\right)}{d} - \frac{2\,a^3\,(e\cot(c+d\,x))^{3/2}}{d\,e} - \frac{2\,a^3\,(e\cot(c+d\,x))^{5/2}}{5\,d\,e^2} - \frac{4\,a^3\sqrt{e\cot(c+d\,x)}}{d}$$

```
[In] int((e*cot(c + d*x))^(1/2)*(a + a*cot(c + d*x))^3,x)
```

```
[Out] (2^(1/2)*a^3*e^(1/2)*(2*atan((2^(1/2)*(e*cot(c + d*x))^(1/2))/(2*e^(1/2)))
+ 2*atan((2^(1/2)*(e*cot(c + d*x))^(1/2))/(2*e^(1/2)) + (2^(1/2)*(e*cot(c +
 d*x))^(3/2))/(2*e^(3/2)))))/d - (2*a^3*(e*cot(c + d*x))^(3/2))/(d*e) - (2*
a^3*(e*cot(c + d*x))^(5/2))/(5*d*e^2) - (4*a^3*(e*cot(c + d*x))^(1/2))/d
```

3.18 $\int \frac{(a+a\cot(c+dx))^3}{\sqrt{e\cot(c+dx)}}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 117

$$\int \frac{(a+a\cot(c+dx))^3}{\sqrt{e\cot(c+dx)}}\,dx = \frac{2\sqrt{2}a^3\text{arctanh}\left(\frac{\sqrt{e}+\sqrt{e}\cot(c+dx)}{\sqrt{2}\sqrt{e\cot(c+dx)}}\right)}{d\sqrt{e}} - \frac{16a^3\sqrt{e\cot(c+dx)}}{3de} - \frac{2\sqrt{e\cot(c+dx)}(a^3+a^3\cot(c+dx))}{3de}$$

```
[Out] 2*a^3*arctanh(1/2*(e^(1/2)+cot(d*x+c)*e^(1/2))*2^(1/2)/(e*cot(d*x+c))^(1/2)
)*2^(1/2)/d/e^(1/2)-16/3*a^3*(e*cot(d*x+c))^(1/2)/d/e-2/3*(a^3+a^3*cot(d*x+
c))*(e*cot(d*x+c))^(1/2)/d/e
```

Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 117, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.160, Rules used = {3647, 3711, 3613, 214}

$$\int \frac{(a+a\cot(c+dx))^3}{\sqrt{e\cot(c+dx)}}\,dx = \frac{2\sqrt{2}a^3\text{arctanh}\left(\frac{\sqrt{e}\cot(c+dx)+\sqrt{e}}{\sqrt{2}\sqrt{e\cot(c+dx)}}\right)}{d\sqrt{e}} - \frac{16a^3\sqrt{e\cot(c+dx)}}{3de} - \frac{2(a^3\cot(c+dx)+a^3)\sqrt{e\cot(c+dx)}}{3de}$$

```
[In] Int[(a + a*Cot[c + d*x])^3/Sqrt[e*Cot[c + d*x]],x]
```

```
[Out] (2*Sqrt[2]*a^3*ArcTanh[(Sqrt[e] + Sqrt[e]*Cot[c + d*x])/(Sqrt[2]*Sqrt[e*Cot
[c + d*x]])])/(d*Sqrt[e]) - (16*a^3*Sqrt[e*Cot[c + d*x]])/(3*d*e) - (2*Sqrt
[e*Cot[c + d*x]]*(a^3 + a^3*Cot[c + d*x]))/(3*d*e)
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 3613

```
Int[((c_) + (d_.)*tan[(e_.) + (f_.)*(x_)])/Sqrt[(b_.)*tan[(e_.) + (f_.)*(x_
)]], x_Symbol] :> Dist[-2*(d^2/f), Subst[Int[1/(2*c*d + b*x^2), x], x, (c -
 d*Tan[e + f*x])/Sqrt[b*Tan[e + f*x]]], x] /; FreeQ[{b, c, d, e, f}, x] &&
EqQ[c^2 - d^2, 0]
```

Rule 3647

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)])^(n_), x_Symbol] :> Simp[b^2*(a + b*Tan[e + f*x])^(m - 2)*((c
+ d*Tan[e + f*x])^(n + 1)/(d*f*(m + n - 1))), x] + Dist[1/(d*(m + n - 1)),
Int[(a + b*Tan[e + f*x])^(m - 3)*(c + d*Tan[e + f*x])^n*Simp[a^3*d*(m + n -
 1) - b^2*(b*c*(m - 2) + a*d*(1 + n)) + b*d*(m + n - 1)*(3*a^2 - b^2)*Tan[e
 + f*x] - b^2*(b*c*(m - 2) - a*d*(3*m + 2*n - 4))*Tan[e + f*x]^2, x], x], x
] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2,
 0] && NeQ[c^2 + d^2, 0] && IntegerQ[2*m] && GtQ[m, 2] && (GeQ[n, -1] || In
tegerQ[m]) &&  !(IGtQ[n, 2] && ( !IntegerQ[m] || (EqQ[c, 0] && NeQ[a, 0])))
```

Rule 3711

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*tan[(e_.)
+ (f_.)*(x_)] + (C_.)*tan[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[C*((a +
 b*Tan[e + f*x])^(m + 1)/(b*f*(m + 1))), x] + Int[(a + b*Tan[e + f*x])^m*Si
mp[A - C + B*Tan[e + f*x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] &&
NeQ[A*b^2 - a*b*B + a^2*C, 0] &&  !LeQ[m, -1]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\frac{2\sqrt{e\cot(c+dx)}(a^3+a^3\cot(c+dx))}{3de}-\frac{2\int\frac{-a^3e-3a^3e\cot(c+dx)-4a^3e\cot^2(c+dx)}{\sqrt{e\cot(c+dx)}}\,dx}{3e}\\
&= -\frac{16a^3\sqrt{e\cot(c+dx)}}{3de}-\frac{2\sqrt{e\cot(c+dx)}(a^3+a^3\cot(c+dx))}{3de}-\frac{2\int\frac{3a^3e-3a^3e\cot(c+dx)}{\sqrt{e\cot(c+dx)}}\,dx}{3e}\\
&= -\frac{16a^3\sqrt{e\cot(c+dx)}}{3de}-\frac{2\sqrt{e\cot(c+dx)}(a^3+a^3\cot(c+dx))}{3de}\\
&\quad+\frac{(12a^6e)\,\text{Subst}\left(\int\frac{1}{18a^6e^2-ex^2}\,dx,x,\frac{3a^3e+3a^3e\cot(c+dx)}{\sqrt{e\cot(c+dx)}}\right)}{d}
\end{aligned}$$

$$= \frac{2\sqrt{2}a^3\text{arctanh}\left(\frac{\sqrt{e}+\sqrt{e}\cot(c+dx)}{\sqrt{2}\sqrt{e\cot(c+dx)}}\right)}{d\sqrt{e}} - \frac{16a^3\sqrt{e\cot(c+dx)}}{3de} - \frac{2\sqrt{e\cot(c+dx)}(a^3+a^3\cot(c+dx))}{3de}$$

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 342 vs. 2(117) = 234.

Time = 1.63 (sec) , antiderivative size = 342, normalized size of antiderivative = 2.92

$$\int \frac{(a+a\cot(c+dx))^3}{\sqrt{e\cot(c+dx)}}\,dx = -\frac{a^3(1+\cot(c+dx))^3\sin(c+dx)\left(4\cos^2(c+dx)+6\sqrt{2}\arctan\left(1-\sqrt{2}\sqrt{\cot(c+dx)}\right)\sqrt{\cot(c+dx)}\sin\right.}{}$$

```
[In] Integrate[(a + a*Cot[c + d*x])^3/Sqrt[e*Cot[c + d*x]],x]
```

```
[Out] -1/6*(a^3*(1 + Cot[c + d*x])^3*Sin[c + d*x]*(4*Cos[c + d*x]^2 + 6*Sqrt[2]*A
rcTan[1 - Sqrt[2]*Sqrt[Cot[c + d*x]]]*Sqrt[Cot[c + d*x]]*Sin[c + d*x]^2 - 6
*Sqrt[2]*ArcTan[1 + Sqrt[2]*Sqrt[Cot[c + d*x]]]*Sqrt[Cot[c + d*x]]*Sin[c +
d*x]^2 + 12*ArcTan[(-Cot[c + d*x]^2)^(1/4)]*(-Cot[c + d*x]^2)^(1/4)*Sin[c +
 d*x]^2 - 12*ArcTanh[(-Cot[c + d*x]^2)^(1/4)]*(-Cot[c + d*x]^2)^(1/4)*Sin[c
 + d*x]^2 + 3*Sqrt[2]*Sqrt[Cot[c + d*x]]*Log[1 - Sqrt[2]*Sqrt[Cot[c + d*x]]
 + Cot[c + d*x]]*Sin[c + d*x]^2 - 3*Sqrt[2]*Sqrt[Cot[c + d*x]]*Log[1 + Sqrt
[2]*Sqrt[Cot[c + d*x]] + Cot[c + d*x]]*Sin[c + d*x]^2 + 18*Sin[2*(c + d*x)]
))/(d*Sqrt[e*Cot[c + d*x]]*(Cos[c + d*x] + Sin[c + d*x])^3)
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 308 vs. 2(98) = 196.

Time = 0.06 (sec) , antiderivative size = 309, normalized size of antiderivative = 2.64

method	result
derivativedivides	$-\frac{2a^3\left(\frac{(e\cot(dx+c))^{\frac{3}{2}}}{3}+3e\sqrt{e\cot(dx+c)}-2e^2\left(\frac{(e^2)^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\frac{e\cot(dx+c)+(e^2)^{\frac{1}{4}}\sqrt{e\cot(dx+c)}\sqrt{2}+\sqrt{e^2}}{e\cot(dx+c)-(e^2)^{\frac{1}{4}}\sqrt{e\cot(dx+c)}\sqrt{2}+\sqrt{e^2}}\right)+2\arctan\left(\frac{\sqrt{2}\sqrt{}}{}\right.\right.}{8e}\right.\right.}{}$
default	$-\frac{2a^3\left(\frac{(e\cot(dx+c))^{\frac{3}{2}}}{3}+3e\sqrt{e\cot(dx+c)}-2e^2\left(\frac{(e^2)^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\frac{e\cot(dx+c)+(e^2)^{\frac{1}{4}}\sqrt{e\cot(dx+c)}\sqrt{2}+\sqrt{e^2}}{e\cot(dx+c)-(e^2)^{\frac{1}{4}}\sqrt{e\cot(dx+c)}\sqrt{2}+\sqrt{e^2}}\right)+2\arctan\left(\frac{\sqrt{2}\sqrt{}}{}\right.\right.}{8e}\right.\right.}{}$
parts	$-\frac{a^3(e^2)^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\frac{e\cot(dx+c)+(e^2)^{\frac{1}{4}}\sqrt{e\cot(dx+c)}\sqrt{2}+\sqrt{e^2}}{e\cot(dx+c)-(e^2)^{\frac{1}{4}}\sqrt{e\cot(dx+c)}\sqrt{2}+\sqrt{e^2}}\right)+2\arctan\left(\frac{\sqrt{2}\sqrt{e\cot(dx+c)}}{(e^2)^{\frac{1}{4}}}+1\right)-2\arctan\left(-\frac{\sqrt{2}\sqrt{e\cot}}{(e^2)}\right.\right.}{4de}$

```
[In] int((a+a*cot(d*x+c))^3/(e*cot(d*x+c))^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -2/d*a^3/e^2*(1/3*(e*cot(d*x+c))^(3/2)+3*e*(e*cot(d*x+c))^(1/2)-2*e^2*(1/8/
e*(e^2)^(1/4)*2^(1/2)*(ln((e*cot(d*x+c)+(e^2)^(1/4)*(e*cot(d*x+c))^(1/2)*2^
(1/2)+(e^2)^(1/2))/(e*cot(d*x+c)-(e^2)^(1/4)*(e*cot(d*x+c))^(1/2)*2^(1/2)+(
e^2)^(1/2)))+2*arctan(2^(1/2)/(e^2)^(1/4)*(e*cot(d*x+c))^(1/2)+1)-2*arctan(
-2^(1/2)/(e^2)^(1/4)*(e*cot(d*x+c))^(1/2)+1))-1/8/(e^2)^(1/4)*2^(1/2)*(ln((
e*cot(d*x+c)-(e^2)^(1/4)*(e*cot(d*x+c))^(1/2)*2^(1/2)+(e^2)^(1/2))/(e*cot(d
*x+c)+(e^2)^(1/4)*(e*cot(d*x+c))^(1/2)*2^(1/2)+(e^2)^(1/2)))+2*arctan(2^(1/
2)/(e^2)^(1/4)*(e*cot(d*x+c))^(1/2)+1)-2*arctan(-2^(1/2)/(e^2)^(1/4)*(e*cot
(d*x+c))^(1/2)+1))))
```

Fricas [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 349, normalized size of antiderivative = 2.98

$$\int \frac{(a+a\cot(c+dx))^3}{\sqrt{e\cot(c+dx)}}\,dx = \left[\frac{3\sqrt{2}a^3\sqrt{e}\log\left(-\frac{\sqrt{2}\sqrt{\frac{e\cos(2\,dx+2\,c)+e}{\sin(2\,dx+2\,c)}}(\cos(2\,dx+2\,c)-\sin(2\,dx+2\,c)-1)}{\sqrt{e}}+2\,\sin(2\,dx+2\,c)+1\right)\sin(2\,dx+2\,c)-2\,(c}{3\,de\sin(2\,dx+2\,c)}\right.$$
$$-\frac{2\left(3\sqrt{2}a^3e\sqrt{-\frac{1}{e}}\arctan\left(\frac{\sqrt{2}\sqrt{\frac{e\cos(2\,dx+2\,c)+e}{\sin(2\,dx+2\,c)}}\sqrt{-\frac{1}{e}}(\cos(2\,dx+2\,c)+\sin(2\,dx+2\,c)+1)}{2\,(\cos(2\,dx+2\,c)+1)}\right)\sin(2\,dx+2\,c)+(a^3\cos(2\,dx}{3\,de\sin(2\,dx+2\,c)}$$

```
[In] integrate((a+a*cot(d*x+c))^3/(e*cot(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] [1/3*(3*sqrt(2)*a^3*sqrt(e)*log(-sqrt(2)*sqrt((e*cos(2*d*x + 2*c) + e)/sin(
2*d*x + 2*c))*(cos(2*d*x + 2*c) - sin(2*d*x + 2*c) - 1)/sqrt(e) + 2*sin(2*d
*x + 2*c) + 1)*sin(2*d*x + 2*c) - 2*(a^3*cos(2*d*x + 2*c) + 9*a^3*sin(2*d*x
 + 2*c) + a^3)*sqrt((e*cos(2*d*x + 2*c) + e)/sin(2*d*x + 2*c)))/(d*e*sin(2*
d*x + 2*c)), -2/3*(3*sqrt(2)*a^3*e*sqrt(-1/e)*arctan(1/2*sqrt(2)*sqrt((e*co
s(2*d*x + 2*c) + e)/sin(2*d*x + 2*c))*sqrt(-1/e)*(cos(2*d*x + 2*c) + sin(2*
d*x + 2*c) + 1)/(cos(2*d*x + 2*c) + 1))*sin(2*d*x + 2*c) + (a^3*cos(2*d*x +
 2*c) + 9*a^3*sin(2*d*x + 2*c) + a^3)*sqrt((e*cos(2*d*x + 2*c) + e)/sin(2*d
*x + 2*c)))/(d*e*sin(2*d*x + 2*c))]
```

Sympy [F]

$$\int \frac{(a+a\cot(c+dx))^3}{\sqrt{e\cot(c+dx)}}\,dx = a^3\left(\int \frac{1}{\sqrt{e\cot(c+dx)}}\,dx+\int \frac{3\cot(c+dx)}{\sqrt{e\cot(c+dx)}}\,dx + \int \frac{3\cot^2(c+dx)}{\sqrt{e\cot(c+dx)}}\,dx+\int \frac{\cot^3(c+dx)}{\sqrt{e\cot(c+dx)}}\,dx\right)$$

```
[In] integrate((a+a*cot(d*x+c))**3/(e*cot(d*x+c))**(1/2),x)
```

```
[Out] a**3*(Integral(1/sqrt(e*cot(c + d*x)), x) + Integral(3*cot(c + d*x)/sqrt(e*
cot(c + d*x)), x) + Integral(3*cot(c + d*x)**2/sqrt(e*cot(c + d*x)), x) + I
ntegral(cot(c + d*x)**3/sqrt(e*cot(c + d*x)), x))
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{(a + a\cot(c + dx))^3}{\sqrt{e\cot(c + dx)}}\,dx = \text{Exception raised: ValueError}$$

```
[In] integrate((a+a*cot(d*x+c))^3/(e*cot(d*x+c))^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(e>0)', see `assume?` for more detai
ls)Is e
```

Giac [F]

$$\int \frac{(a + a\cot(c + dx))^3}{\sqrt{e\cot(c + dx)}}\,dx = \int \frac{(a\cot(dx + c) + a)^3}{\sqrt{e\cot(dx + c)}}\,dx$$

```
[In] integrate((a+a*cot(d*x+c))^3/(e*cot(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] integrate((a*cot(d*x + c) + a)^3/sqrt(e*cot(d*x + c)), x)
```

Mupad [B] (verification not implemented)

Time = 13.03 (sec) , antiderivative size = 100, normalized size of antiderivative = 0.85

$$\int \frac{(a + a\cot(c + dx))^3}{\sqrt{e\cot(c + dx)}}\,dx = \frac{2\sqrt{2}\,a^3\,\mathrm{atanh}\left(\frac{32\sqrt{2}\,a^6\,\sqrt{e}\,\sqrt{e\cot(c+d\,x)}}{32\,a^6\,e+32\,a^6\,e\cot(c+d\,x)}\right)}{d\,\sqrt{e}} - \frac{2\,a^3\,(e\cot(c+d\,x))^{3/2}}{3\,d\,e^2} - \frac{6\,a^3\,\sqrt{e\cot(c+d\,x)}}{d\,e}$$

```
[In] int((a + a*cot(c + d*x))^3/(e*cot(c + d*x))^(1/2),x)
```

```
[Out] (2*2^(1/2)*a^3*atanh((32*2^(1/2)*a^6*e^(1/2)*(e*cot(c + d*x))^(1/2))/(32*a^
6*e + 32*a^6*e*cot(c + d*x))))/(d*e^(1/2)) - (2*a^3*(e*cot(c + d*x))^(3/2))
/(3*d*e^2) - (6*a^3*(e*cot(c + d*x))^(1/2))/(d*e)
```

3.19 $\int \frac{(a+a\cot(c+dx))^3}{(e\cot(c+dx))^{3/2}}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 114

$$\int \frac{(a+a\cot(c+dx))^3}{(e\cot(c+dx))^{3/2}}\,dx = \frac{2\sqrt{2}a^3\arctan\left(\frac{\sqrt{e}-\sqrt{e}\cot(c+dx)}{\sqrt{2}\sqrt{e\cot(c+dx)}}\right)}{de^{3/2}} - \frac{4a^3\sqrt{e\cot(c+dx)}}{de^2} + \frac{2(a^3+a^3\cot(c+dx))}{de\sqrt{e\cot(c+dx)}}$$

```
[Out] 2*a^3*arctan(1/2*(e^(1/2)-cot(d*x+c)*e^(1/2))*2^(1/2)/(e*cot(d*x+c))^(1/2))
*2^(1/2)/d/e^(3/2)+2*(a^3+a^3*cot(d*x+c))/d/e/(e*cot(d*x+c))^(1/2)-4*a^3*(e
*cot(d*x+c))^(1/2)/d/e^2
```

Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 114, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.160, Rules used = {3646, 3711, 3613, 211}

$$\int \frac{(a+a\cot(c+dx))^3}{(e\cot(c+dx))^{3/2}}\,dx = \frac{2\sqrt{2}a^3\arctan\left(\frac{\sqrt{e}-\sqrt{e}\cot(c+dx)}{\sqrt{2}\sqrt{e\cot(c+dx)}}\right)}{de^{3/2}} - \frac{4a^3\sqrt{e\cot(c+dx)}}{de^2} + \frac{2(a^3\cot(c+dx)+a^3)}{de\sqrt{e\cot(c+dx)}}$$

```
[In] Int[(a + a*Cot[c + d*x])^3/(e*Cot[c + d*x])^(3/2),x]
```

```
[Out] (2*Sqrt[2]*a^3*ArcTan[(Sqrt[e] - Sqrt[e]*Cot[c + d*x])/(Sqrt[2]*Sqrt[e*Cot[
c + d*x]])])/(d*e^(3/2)) - (4*a^3*Sqrt[e*Cot[c + d*x]])/(d*e^2) + (2*(a^3 +
 a^3*Cot[c + d*x]))/(d*e*Sqrt[e*Cot[c + d*x]])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 3613

```
Int[((c_) + (d_.)*tan[(e_.) + (f_.)*(x_)])/Sqrt[(b_.)*tan[(e_.) + (f_.)*(x_
)]], x_Symbol] :> Dist[-2*(d^2/f), Subst[Int[1/(2*c*d + b*x^2), x], x, (c -
 d*Tan[e + f*x])/Sqrt[b*Tan[e + f*x]]], x] /; FreeQ[{b, c, d, e, f}, x] &&
EqQ[c^2 - d^2, 0]
```

Rule 3646

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*c - a*d)^2*(a + b*Tan[e + f*x])^(m
 - 2)*((c + d*Tan[e + f*x])^(n + 1)/(d*f*(n + 1)*(c^2 + d^2))), x] - Dist[1
/(d*(n + 1)*(c^2 + d^2)), Int[(a + b*Tan[e + f*x])^(m - 3)*(c + d*Tan[e + f
*x])^(n + 1)*Simp[a^2*d*(b*d*(m - 2) - a*c*(n + 1)) + b*(b*c - 2*a*d)*(b*c*
(m - 2) + a*d*(n + 1)) - d*(n + 1)*(3*a^2*b*c - b^3*c - a^3*d + 3*a*b^2*d)*
Tan[e + f*x] - b*(a*d*(2*b*c - a*d)*(m + n - 1) - b^2*(c^2*(m - 2) - d^2*(n
 + 1)))*Tan[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b
*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] && GtQ[m, 2] && LtQ[
n, -1] && IntegerQ[2*m]
```

Rule 3711

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*tan[(e_.)
+ (f_.)*(x_)] + (C_.)*tan[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[C*((a +
 b*Tan[e + f*x])^(m + 1)/(b*f*(m + 1))), x] + Int[(a + b*Tan[e + f*x])^m*Si
mp[A - C + B*Tan[e + f*x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] &&
NeQ[A*b^2 - a*b*B + a^2*C, 0] &&  !LeQ[m, -1]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{2(a^3+a^3\cot(c+dx))}{de\sqrt{e\cot(c+dx)}} - \frac{2\int \frac{-2a^3e^2-a^3e^2\cot(c+dx)-a^3e^2\cot^2(c+dx)}{\sqrt{e\cot(c+dx)}}\,dx}{e^3} \\
&= -\frac{4a^3\sqrt{e\cot(c+dx)}}{de^2} + \frac{2(a^3+a^3\cot(c+dx))}{de\sqrt{e\cot(c+dx)}} - \frac{2\int \frac{-a^3e^2-a^3e^2\cot(c+dx)}{\sqrt{e\cot(c+dx)}}\,dx}{e^3} \\
&= -\frac{4a^3\sqrt{e\cot(c+dx)}}{de^2} + \frac{2(a^3+a^3\cot(c+dx))}{de\sqrt{e\cot(c+dx)}} \\
&\quad + \frac{(4a^6e)\,\text{Subst}\left(\int \frac{1}{-2a^6e^4-ex^2}\,dx, x, \frac{-a^3e^2+a^3e^2\cot(c+dx)}{\sqrt{e\cot(c+dx)}}\right)}{d}
\end{aligned}$$

$$= \frac{2\sqrt{2}a^3 \arctan\left(\frac{\sqrt{e}-\sqrt{e}\cot(c+dx)}{\sqrt{2}\sqrt{e\cot(c+dx)}}\right)}{de^{3/2}} - \frac{4a^3\sqrt{e\cot(c+dx)}}{de^2} + \frac{2(a^3+a^3\cot(c+dx))}{de\sqrt{e\cot(c+dx)}}$$

Mathematica [B] (warning: unable to verify)

Leaf count is larger than twice the leaf count of optimal. 357 vs. 2(114) = 228.

Time = 3.12 (sec) , antiderivative size = 357, normalized size of antiderivative = 3.13

$$\int \frac{(a+a\cot(c+dx))^3}{(e\cot(c+dx))^{3/2}}\,dx = \frac{a^3(1+\cot(c+dx))^3\sin(c+dx)\left(-4\cos^2(c+dx)+4\arctan\left(\sqrt[4]{-\cot^2(c+dx}\right.\right.}{}$$

```
[In] Integrate[(a + a*Cot[c + d*x])^3/(e*Cot[c + d*x])^(3/2),x]
```

```
[Out] (a^3*(1 + Cot[c + d*x])^3*Sin[c + d*x]*(-4*Cos[c + d*x]^2 + 4*ArcTan[(-Cot[
c + d*x]^2)^(1/4)]*(-Cot[c + d*x])^(5/4)*Cot[c + d*x]^(1/4)*Sin[c + d*x]^2
+ 4*ArcTanh[(-Cot[c + d*x]^2)^(1/4)]*(-Cot[c + d*x])^(1/4)*Cot[c + d*x]^(5/
4)*Sin[c + d*x]^2 + 2*Sqrt[2]*ArcTan[1 - Sqrt[2]*Sqrt[Cot[c + d*x]]]*Cot[c
+ d*x]^(3/2)*Sin[c + d*x]^2 - 2*Sqrt[2]*ArcTan[1 + Sqrt[2]*Sqrt[Cot[c + d*x
]]]*Cot[c + d*x]^(3/2)*Sin[c + d*x]^2 + Sqrt[2]*Cot[c + d*x]^(3/2)*Log[1 -
Sqrt[2]*Sqrt[Cot[c + d*x]] + Cot[c + d*x]]*Sin[c + d*x]^2 - Sqrt[2]*Cot[c +
 d*x]^(3/2)*Log[1 + Sqrt[2]*Sqrt[Cot[c + d*x]] + Cot[c + d*x]]*Sin[c + d*x]
^2 + 2*Sin[2*(c + d*x)]))/(2*d*(e*Cot[c + d*x])^(3/2)*(Cos[c + d*x] + Sin[c
 + d*x])^3)
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 304 vs. 2(99) = 198.

Time = 0.04 (sec) , antiderivative size = 305, normalized size of antiderivative = 2.68

method	result
derivativedivides	$-\frac{2a^3\left(\sqrt{e\cot(dx+c)}+2e\left(\frac{(e^2)^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\frac{e\cot(dx+c)+(e^2)^{\frac{1}{4}}\sqrt{e\cot(dx+c)}\sqrt{2}+\sqrt{e^2}}{e\cot(dx+c)-(e^2)^{\frac{1}{4}}\sqrt{e\cot(dx+c)}\sqrt{2}+\sqrt{e^2}}\right)+2\arctan\left(\frac{\sqrt{2}\sqrt{e\cot(dx+c)}}{(e^2)^{\frac{1}{4}}}+1\right)-2\,a}{8e}\right.\right.}{}$
default	$-\frac{2a^3\left(\sqrt{e\cot(dx+c)}+2e\left(\frac{(e^2)^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\frac{e\cot(dx+c)+(e^2)^{\frac{1}{4}}\sqrt{e\cot(dx+c)}\sqrt{2}+\sqrt{e^2}}{e\cot(dx+c)-(e^2)^{\frac{1}{4}}\sqrt{e\cot(dx+c)}\sqrt{2}+\sqrt{e^2}}\right)+2\arctan\left(\frac{\sqrt{2}\sqrt{e\cot(dx+c)}}{(e^2)^{\frac{1}{4}}}+1\right)-2\,a}{8e}\right.\right.}{}$
parts	$-\frac{2a^3e\left(-\frac{\sqrt{2}\left(\ln\left(\frac{e\cot(dx+c)-(e^2)^{\frac{1}{4}}\sqrt{e\cot(dx+c)}\sqrt{2}+\sqrt{e^2}}{e\cot(dx+c)+(e^2)^{\frac{1}{4}}\sqrt{e\cot(dx+c)}\sqrt{2}+\sqrt{e^2}}\right)+2\arctan\left(\frac{\sqrt{2}\sqrt{e\cot(dx+c)}}{(e^2)^{\frac{1}{4}}}+1\right)-2\arctan\left(-\frac{\sqrt{2}\sqrt{e\cot(dx+c)}}{(e^2)^{\frac{1}{4}}}\right.\right.}{8e^2(e^2)^{\frac{1}{4}}}\right.}{d}$

```
[In] int((a+a*cot(d*x+c))^3/(e*cot(d*x+c))^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] -2/d*a^3/e^2*((e*cot(d*x+c))^(1/2)+2*e*(1/8/e*(e^2)^(1/4)*2^(1/2)*(ln((e*co
t(d*x+c)+(e^2)^(1/4)*(e*cot(d*x+c))^(1/2)*2^(1/2)+(e^2)^(1/2))/(e*cot(d*x+c
)-(e^2)^(1/4)*(e*cot(d*x+c))^(1/2)*2^(1/2)+(e^2)^(1/2)))+2*arctan(2^(1/2)/(
e^2)^(1/4)*(e*cot(d*x+c))^(1/2)+1)-2*arctan(-2^(1/2)/(e^2)^(1/4)*(e*cot(d*x
+c))^(1/2)+1))+1/8/(e^2)^(1/4)*2^(1/2)*(ln((e*cot(d*x+c)-(e^2)^(1/4)*(e*cot
(d*x+c))^(1/2)*2^(1/2)+(e^2)^(1/2))/(e*cot(d*x+c)+(e^2)^(1/4)*(e*cot(d*x+c)
)^(1/2)*2^(1/2)+(e^2)^(1/2)))+2*arctan(2^(1/2)/(e^2)^(1/4)*(e*cot(d*x+c))^(
1/2)+1)-2*arctan(-2^(1/2)/(e^2)^(1/4)*(e*cot(d*x+c))^(1/2)+1)))-e/(e*cot(d*
x+c))^(1/2))
```

Fricas [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 372, normalized size of antiderivative = 3.26

$$\int \frac{(a+a\cot(c+dx))^3}{(e\cot(c+dx))^{3/2}}\,dx = \left[\frac{\sqrt{2}(a^3e\cos(2\,dx+2\,c)+a^3e)\sqrt{-\frac{1}{e}}\log\left(-\sqrt{2}\sqrt{\frac{e\cos(2\,dx+2\,c)+e}{\sin(2\,dx+2\,c)}}\sqrt{-\frac{1}{e}}(\cos(2\,dx\right.}{}\right.$$

```
[In] integrate((a+a*cot(d*x+c))^3/(e*cot(d*x+c))^(3/2),x, algorithm="fricas")
```

```
[Out] [(sqrt(2)*(a^3*e*cos(2*d*x + 2*c) + a^3*e)*sqrt(-1/e)*log(-sqrt(2)*sqrt((e*
cos(2*d*x + 2*c) + e)/sin(2*d*x + 2*c))*sqrt(-1/e)*(cos(2*d*x + 2*c) + sin(
2*d*x + 2*c) - 1) - 2*sin(2*d*x + 2*c) + 1) - 2*(a^3*cos(2*d*x + 2*c) - a^3
*sin(2*d*x + 2*c) + a^3)*sqrt((e*cos(2*d*x + 2*c) + e)/sin(2*d*x + 2*c)))/(
d*e^2*cos(2*d*x + 2*c) + d*e^2), 2*(sqrt(2)*(a^3*e*cos(2*d*x + 2*c) + a^3*e
)*arctan(-1/2*sqrt(2)*sqrt((e*cos(2*d*x + 2*c) + e)/sin(2*d*x + 2*c))*(cos(
2*d*x + 2*c) - sin(2*d*x + 2*c) + 1)/(sqrt(e)*(cos(2*d*x + 2*c) + 1)))/sqrt
(e) - (a^3*cos(2*d*x + 2*c) - a^3*sin(2*d*x + 2*c) + a^3)*sqrt((e*cos(2*d*x
 + 2*c) + e)/sin(2*d*x + 2*c)))/(d*e^2*cos(2*d*x + 2*c) + d*e^2)]
```

Sympy [F]

$$\int \frac{(a+a\cot(c+dx))^3}{(e\cot(c+dx))^{3/2}}\,dx = a^3\left(\int \frac{1}{(e\cot(c+dx))^{\frac{3}{2}}}\,dx + \int \frac{3\cot(c+dx)}{(e\cot(c+dx))^{\frac{3}{2}}}\,dx + \int \frac{3\cot^2(c+dx)}{(e\cot(c+dx))^{\frac{3}{2}}}\,dx + \int \frac{\cot^3(c+dx)}{(e\cot(c+dx))^{\frac{3}{2}}}\,dx\right)$$

```
[In] integrate((a+a*cot(d*x+c))**3/(e*cot(d*x+c))**(3/2),x)
```

```
[Out] a**3*(Integral((e*cot(c + d*x))**(-3/2), x) + Integral(3*cot(c + d*x)/(e*co
t(c + d*x))**(3/2), x) + Integral(3*cot(c + d*x)**2/(e*cot(c + d*x))**(3/2)
, x) + Integral(cot(c + d*x)**3/(e*cot(c + d*x))**(3/2), x))
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{(a+a\cot(c+dx))^3}{(e\cot(c+dx))^{3/2}}\,dx = \text{Exception raised: ValueError}$$

```
[In] integrate((a+a*cot(d*x+c))^3/(e*cot(d*x+c))^(3/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(e>0)', see `assume?` for more detai
ls)Is e
```

Giac [F]

$$\int \frac{(a+a\cot(c+dx))^3}{(e\cot(c+dx))^{3/2}}\,dx = \int \frac{(a\cot\left(dx+c\right)+a)^3}{(e\cot\left(dx+c\right))^{\frac{3}{2}}}\,dx$$

```
[In] integrate((a+a*cot(d*x+c))^3/(e*cot(d*x+c))^(3/2),x, algorithm="giac")
```

```
[Out] integrate((a*cot(d*x + c) + a)^3/(e*cot(d*x + c))^(3/2), x)
```

Mupad [B] (verification not implemented)

Time = 12.45 (sec) , antiderivative size = 119, normalized size of antiderivative = 1.04

$$\int \frac{(a+a\cot(c+dx))^3}{(e\cot(c+dx))^{3/2}}\,dx = \frac{2\,a^3}{d\,e\,\sqrt{e\cot\left(c+d\,x\right)}} - \frac{2\,a^3\,\sqrt{e\cot\left(c+d\,x\right)}}{d\,e^2} - \frac{\sqrt{2}\,a^3\,\left(2\,\mathrm{atan}\left(\frac{\sqrt{2}\,\sqrt{e\cot(c+d\,x)}}{2\,\sqrt{e}}\right)+2\,\mathrm{atan}\left(\frac{\sqrt{2}\,\sqrt{e\cot(c+d\,x)}}{2\,\sqrt{e}}+\frac{\sqrt{2}\,(e\cot(c+d\,x))^{3/2}}{2\,e^{3/2}}\right)\right)}{d\,e^{3/2}}$$

```
[In] int((a + a*cot(c + d*x))^3/(e*cot(c + d*x))^(3/2),x)
```

```
[Out] (2*a^3)/(d*e*(e*cot(c + d*x))^(1/2)) - (2*a^3*(e*cot(c + d*x))^(1/2))/(d*e^
2) - (2^(1/2)*a^3*(2*atan((2^(1/2)*(e*cot(c + d*x))^(1/2))/(2*e^(1/2))) + 2
*atan((2^(1/2)*(e*cot(c + d*x))^(1/2))/(2*e^(1/2)) + (2^(1/2)*(e*cot(c + d*
x))^(3/2))/(2*e^(3/2)))))/(d*e^(3/2))
```

3.20 $\int \frac{(a+a\cot(c+dx))^3}{(e\cot(c+dx))^{5/2}}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 117

$$\int \frac{(a+a\cot(c+dx))^3}{(e\cot(c+dx))^{5/2}}\,dx = -\frac{2\sqrt{2}a^3\operatorname{arctanh}\left(\frac{\sqrt{e}+\sqrt{e}\cot(c+dx)}{\sqrt{2}\sqrt{e\cot(c+dx)}}\right)}{de^{5/2}} + \frac{16a^3}{3de^2\sqrt{e\cot(c+dx)}} + \frac{2(a^3+a^3\cot(c+dx))}{3de(e\cot(c+dx))^{3/2}}$$

```
[Out] 2/3*(a^3+a^3*cot(d*x+c))/d/e/(e*cot(d*x+c))^(3/2)-2*a^3*arctanh(1/2*(e^(1/2
)+cot(d*x+c)*e^(1/2))*2^(1/2)/(e*cot(d*x+c))^(1/2))*2^(1/2)/d/e^(5/2)+16/3*
a^3/d/e^2/(e*cot(d*x+c))^(1/2)
```

Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 117, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.160$, Rules used = {3646, 3709, 3613, 214}

$$\int \frac{(a+a\cot(c+dx))^3}{(e\cot(c+dx))^{5/2}}\,dx = -\frac{2\sqrt{2}a^3\operatorname{arctanh}\left(\frac{\sqrt{e}\cot(c+dx)+\sqrt{e}}{\sqrt{2}\sqrt{e\cot(c+dx)}}\right)}{de^{5/2}} + \frac{16a^3}{3de^2\sqrt{e\cot(c+dx)}} + \frac{2(a^3\cot(c+dx)+a^3)}{3de(e\cot(c+dx))^{3/2}}$$

```
[In] Int[(a + a*Cot[c + d*x])^3/(e*Cot[c + d*x])^(5/2),x]
```

```
[Out] (-2*Sqrt[2]*a^3*ArcTanh[(Sqrt[e] + Sqrt[e]*Cot[c + d*x])/(Sqrt[2]*Sqrt[e*Co
t[c + d*x]])])/(d*e^(5/2)) + (16*a^3)/(3*d*e^2*Sqrt[e*Cot[c + d*x]]) + (2*(
a^3 + a^3*Cot[c + d*x]))/(3*d*e*(e*Cot[c + d*x])^(3/2))
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 3613

```
Int[((c_) + (d_.)*tan[(e_.) + (f_.)*(x_)])/Sqrt[(b_.)*tan[(e_.) + (f_.)*(x_
)]], x_Symbol] :> Dist[-2*(d^2/f), Subst[Int[1/(2*c*d + b*x^2), x], x, (c -
 d*Tan[e + f*x])/Sqrt[b*Tan[e + f*x]]], x] /; FreeQ[{b, c, d, e, f}, x] &&
EqQ[c^2 - d^2, 0]
```

Rule 3646

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*c - a*d)^2*(a + b*Tan[e + f*x])^(m
 - 2)*((c + d*Tan[e + f*x])^(n + 1)/(d*f*(n + 1)*(c^2 + d^2))), x] - Dist[1
/(d*(n + 1)*(c^2 + d^2)), Int[(a + b*Tan[e + f*x])^(m - 3)*(c + d*Tan[e + f
*x])^(n + 1)*Simp[a^2*d*(b*d*(m - 2) - a*c*(n + 1)) + b*(b*c - 2*a*d)*(b*c*
(m - 2) + a*d*(n + 1)) - d*(n + 1)*(3*a^2*b*c - b^3*c - a^3*d + 3*a*b^2*d)*
Tan[e + f*x] - b*(a*d*(2*b*c - a*d)*(m + n - 1) - b^2*(c^2*(m - 2) - d^2*(n
 + 1)))*Tan[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b
*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] && GtQ[m, 2] && LtQ[
n, -1] && IntegerQ[2*m]
```

Rule 3709

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
 (f_.)*(x_)] + (C_.)*tan[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(A*b^2 -
 a*b*B + a^2*C)*((a + b*Tan[e + f*x])^(m + 1)/(b*f*(m + 1)*(a^2 + b^2))), x
] + Dist[1/(a^2 + b^2), Int[(a + b*Tan[e + f*x])^(m + 1)*Simp[b*B + a*(A -
C) - (A*b - a*B - b*C)*Tan[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B,
 C}, x] && NeQ[A*b^2 - a*b*B + a^2*C, 0] && LtQ[m, -1] && NeQ[a^2 + b^2, 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{2(a^3+a^3\cot(c+dx))}{3de(e\cot(c+dx))^{3/2}}-\frac{2\int\frac{-4a^3e^2-3a^3e^2\cot(c+dx)-a^3e^2\cot^2(c+dx)}{(e\cot(c+dx))^{3/2}}\,dx}{3e^3}\\
&= \frac{16a^3}{3de^2\sqrt{e\cot(c+dx)}}+\frac{2(a^3+a^3\cot(c+dx))}{3de(e\cot(c+dx))^{3/2}}-\frac{2\int\frac{-3a^3e^3+3a^3e^3\cot(c+dx)}{\sqrt{e\cot(c+dx)}}\,dx}{3e^5}\\
&= \frac{16a^3}{3de^2\sqrt{e\cot(c+dx)}}+\frac{2(a^3+a^3\cot(c+dx))}{3de(e\cot(c+dx))^{3/2}}\\
&\quad+\frac{(12a^6e)\,\text{Subst}\left(\int\frac{1}{18a^6e^6-ex^2}\,dx,x,\frac{-3a^3e^3-3a^3e^3\cot(c+dx)}{\sqrt{e\cot(c+dx)}}\right)}{d}
\end{aligned}$$

$$= -\frac{2\sqrt{2}a^3\text{arctanh}\left(\frac{\sqrt{e}+\sqrt{e}\cot(c+dx)}{\sqrt{2}\sqrt{e\cot(c+dx)}}\right)}{de^{5/2}} + \frac{16a^3}{3de^2\sqrt{e\cot(c+dx)}} + \frac{2(a^3+a^3\cot(c+dx))}{3de(e\cot(c+dx))^{3/2}}$$

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 599 vs. $2(117) = 234$.

Time = 6.21 (sec) , antiderivative size = 599, normalized size of antiderivative = 5.12

$$\int \frac{(a+a\cot(c+dx))^3}{(e\cot(c+dx))^{5/2}}\,dx = \frac{6\cos^2(c+dx)(a+a\cot(c+dx))^3\sin(c+dx)}{d(e\cot(c+dx))^{5/2}(\cos(c+dx)+\sin(c+dx))^3}$$
$$+ \frac{2\cos(c+dx)\left(1-\frac{3}{2}\arctan\left(\sqrt[4]{-\cot(c+dx)}\sqrt[4]{\cot(c+dx)}\right)(-\cot(c+dx))^{3/4}\cot^{\frac{3}{4}}(c+dx)-\frac{3}{2}\text{arctanh}\left(\sqrt[4]{\right.\right.}{3d(e\cot(c+dx))^{5/2}(\cos(c+ }$$
$$+ \frac{2\arctan\left(\sqrt[4]{-\cot(c+dx)}\sqrt[4]{\cot(c+dx)}\right)\sqrt[4]{-\cot(c+dx)}\cot^{\frac{9}{4}}(c+dx)(a+a\cot(c+dx))^3\sin^3(c+dx)}{d(e\cot(c+dx))^{5/2}(\cos(c+dx)+\sin(c+dx))^3}$$
$$- \frac{2\text{arctanh}\left(\sqrt[4]{-\cot(c+dx)}\sqrt[4]{\cot(c+dx)}\right)\sqrt[4]{-\cot(c+dx)}\cot^{\frac{9}{4}}(c+dx)(a+a\cot(c+dx))^3\sin^3(c+dx)}{d(e\cot(c+dx))^{5/2}(\cos(c+dx)+\sin(c+dx))^3}$$
$$+ \frac{3\cot^{\frac{5}{2}}(c+dx)(a+a\cot(c+dx))^3\left(2\sqrt{2}\arctan\left(1-\sqrt{2}\sqrt{\cot(c+dx)}\right)-2\sqrt{2}\arctan\left(1+\sqrt{2}\sqrt{\cot(c+d\right.\right.}{4d(e\cot(c+dx))^{5/2}(}$$

```
[In] Integrate[(a + a*Cot[c + d*x])^3/(e*Cot[c + d*x])^(5/2),x]
```

```
[Out] (6*Cos[c + d*x]^2*(a + a*Cot[c + d*x])^3*Sin[c + d*x])/(d*(e*Cot[c + d*x])^
(5/2)*(Cos[c + d*x] + Sin[c + d*x])^3) + (2*Cos[c + d*x]*(1 - (3*ArcTan[(-C
ot[c + d*x])^(1/4)*Cot[c + d*x]^(1/4)]*(-Cot[c + d*x])^(3/4)*Cot[c + d*x]^(
3/4))/2 - (3*ArcTanh[(-Cot[c + d*x])^(1/4)*Cot[c + d*x]^(1/4)]*(-Cot[c + d*
x])^(3/4)*Cot[c + d*x]^(3/4))/2)*(a + a*Cot[c + d*x])^3*Sin[c + d*x]^2)/(3*
d*(e*Cot[c + d*x])^(5/2)*(Cos[c + d*x] + Sin[c + d*x])^3) + (2*ArcTan[(-Cot
[c + d*x])^(1/4)*Cot[c + d*x]^(1/4)]*(-Cot[c + d*x])^(1/4)*Cot[c + d*x]^(9/
4)*(a + a*Cot[c + d*x])^3*Sin[c + d*x]^3)/(d*(e*Cot[c + d*x])^(5/2)*(Cos[c
+ d*x] + Sin[c + d*x])^3) - (2*ArcTanh[(-Cot[c + d*x])^(1/4)*Cot[c + d*x]^(
1/4)]*(-Cot[c + d*x])^(1/4)*Cot[c + d*x]^(9/4)*(a + a*Cot[c + d*x])^3*Sin[c
 + d*x]^3)/(d*(e*Cot[c + d*x])^(5/2)*(Cos[c + d*x] + Sin[c + d*x])^3) + (3*
Cot[c + d*x]^(5/2)*(a + a*Cot[c + d*x])^3*(2*Sqrt[2]*ArcTan[1 - Sqrt[2]*Sqr
t[Cot[c + d*x]]] - 2*Sqrt[2]*ArcTan[1 + Sqrt[2]*Sqrt[Cot[c + d*x]]] + Sqrt[
2]*Log[1 - Sqrt[2]*Sqrt[Cot[c + d*x]] + Cot[c + d*x]] - Sqrt[2]*Log[1 + Sqr
t[2]*Sqrt[Cot[c + d*x]] + Cot[c + d*x]])*Sin[c + d*x]^3)/(4*d*(e*Cot[c + d*
x])^(5/2)*(Cos[c + d*x] + Sin[c + d*x])^3)
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 302 vs. 2(98) = 196.

Time = 0.04 (sec) , antiderivative size = 303, normalized size of antiderivative = 2.59

method	result
derivativedivides	$-\frac{2a^3\left(\frac{(e^2)^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\frac{e\cot(dx+c)+(e^2)^{\frac{1}{4}}\sqrt{e\cot(dx+c)}\sqrt{2}+\sqrt{e^2}}{e\cot(dx+c)-(e^2)^{\frac{1}{4}}\sqrt{e\cot(dx+c)}\sqrt{2}+\sqrt{e^2}}\right)+2\arctan\left(\frac{\sqrt{2}\sqrt{e\cot(dx+c)}}{(e^2)^{\frac{1}{4}}}+1\right)-2\arctan\left(-\frac{\sqrt{2}\sqrt{e\cot(dx}{(e^2)^{\frac{1}{4}}}\right)\right)}{4e}\right)}{}$
default	$-\frac{2a^3\left(\frac{(e^2)^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\frac{e\cot(dx+c)+(e^2)^{\frac{1}{4}}\sqrt{e\cot(dx+c)}\sqrt{2}+\sqrt{e^2}}{e\cot(dx+c)-(e^2)^{\frac{1}{4}}\sqrt{e\cot(dx+c)}\sqrt{2}+\sqrt{e^2}}\right)+2\arctan\left(\frac{\sqrt{2}\sqrt{e\cot(dx+c)}}{(e^2)^{\frac{1}{4}}}+1\right)-2\arctan\left(-\frac{\sqrt{2}\sqrt{e\cot(dx}{(e^2)^{\frac{1}{4}}}\right)\right)}{4e}\right)}{}$
parts	$-\frac{2a^3e\left(-\frac{(e^2)^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\frac{e\cot(dx+c)+(e^2)^{\frac{1}{4}}\sqrt{e\cot(dx+c)}\sqrt{2}+\sqrt{e^2}}{e\cot(dx+c)-(e^2)^{\frac{1}{4}}\sqrt{e\cot(dx+c)}\sqrt{2}+\sqrt{e^2}}\right)+2\arctan\left(\frac{\sqrt{2}\sqrt{e\cot(dx+c)}}{(e^2)^{\frac{1}{4}}}+1\right)-2\arctan\left(-\frac{\sqrt{2}\sqrt{e\cot}{(e^2)}\right)\right)}{8e^4}\right)}{d}$

```
[In] int((a+a*cot(d*x+c))^3/(e*cot(d*x+c))^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] -2/d*a^3/e^2*(1/4/e*(e^2)^(1/4)*2^(1/2)*(ln((e*cot(d*x+c)+(e^2)^(1/4)*(e*co
t(d*x+c))^(1/2)*2^(1/2)+(e^2)^(1/2))/(e*cot(d*x+c)-(e^2)^(1/4)*(e*cot(d*x+c
))^(1/2)*2^(1/2)+(e^2)^(1/2)))+2*arctan(2^(1/2)/(e^2)^(1/4)*(e*cot(d*x+c))^
(1/2)+1)-2*arctan(-2^(1/2)/(e^2)^(1/4)*(e*cot(d*x+c))^(1/2)+1))-1/4/(e^2)^(
1/4)*2^(1/2)*(ln((e*cot(d*x+c)-(e^2)^(1/4)*(e*cot(d*x+c))^(1/2)*2^(1/2)+(e^
2)^(1/2))/(e*cot(d*x+c)+(e^2)^(1/4)*(e*cot(d*x+c))^(1/2)*2^(1/2)+(e^2)^(1/2
)))+2*arctan(2^(1/2)/(e^2)^(1/4)*(e*cot(d*x+c))^(1/2)+1)-2*arctan(-2^(1/2)/
(e^2)^(1/4)*(e*cot(d*x+c))^(1/2)+1))-1/3*e/(e*cot(d*x+c))^(3/2)-3/(e*cot(d*
x+c))^(1/2))
```

Fricas [A] (verification not implemented)

none

Time = 0.32 (sec) , antiderivative size = 378, normalized size of antiderivative = 3.23

$$\int \frac{(a+a\cot(c+dx))^3}{(e\cot(c+dx))^{5/2}}\,dx = \left[\frac{\frac{3\sqrt{2}\left(a^3e\cos(2\,dx+2\,c)+a^3e\right)\log\left(\frac{\sqrt{2}\sqrt{\frac{e\cos(2\,dx+2\,c)+e}{\sin(2\,dx+2\,c)}}(\cos(2\,dx+2\,c)-\sin(2\,dx+2\,c)-1)}{\sqrt{e}}+2\,\sin(2\,dx+2\,c)+1\right)}{\sqrt{e}}}{3\left(de^3\cos\left(2\,dx+\right.\right.}\right.$$

```
[In] integrate((a+a*cot(d*x+c))^3/(e*cot(d*x+c))^(5/2),x, algorithm="fricas")
```

```
[Out] [1/3*(3*sqrt(2)*(a^3*e*cos(2*d*x + 2*c) + a^3*e)*log(sqrt(2)*sqrt((e*cos(2*
d*x + 2*c) + e)/sin(2*d*x + 2*c))*(cos(2*d*x + 2*c) - sin(2*d*x + 2*c) - 1)
/sqrt(e) + 2*sin(2*d*x + 2*c) + 1)/sqrt(e) - 2*(a^3*cos(2*d*x + 2*c) - 9*a^
3*sin(2*d*x + 2*c) - a^3)*sqrt((e*cos(2*d*x + 2*c) + e)/sin(2*d*x + 2*c)))/
(d*e^3*cos(2*d*x + 2*c) + d*e^3), 2/3*(3*sqrt(2)*(a^3*e*cos(2*d*x + 2*c) +
a^3*e)*sqrt(-1/e)*arctan(1/2*sqrt(2)*sqrt((e*cos(2*d*x + 2*c) + e)/sin(2*d*
x + 2*c))*sqrt(-1/e)*(cos(2*d*x + 2*c) + sin(2*d*x + 2*c) + 1)/(cos(2*d*x +
 2*c) + 1)) - (a^3*cos(2*d*x + 2*c) - 9*a^3*sin(2*d*x + 2*c) - a^3)*sqrt((e
*cos(2*d*x + 2*c) + e)/sin(2*d*x + 2*c)))/(d*e^3*cos(2*d*x + 2*c) + d*e^3)]
```

Sympy [F]

$$\int \frac{(a+a\cot(c+dx))^3}{(e\cot(c+dx))^{5/2}}\,dx = a^3\left(\int \frac{1}{(e\cot(c+dx))^{\frac{5}{2}}}\,dx + \int \frac{3\cot(c+dx)}{(e\cot(c+dx))^{\frac{5}{2}}}\,dx + \int \frac{3\cot^2(c+dx)}{(e\cot(c+dx))^{\frac{5}{2}}}\,dx + \int \frac{\cot^3(c+dx)}{(e\cot(c+dx))^{\frac{5}{2}}}\,dx\right)$$

```
[In] integrate((a+a*cot(d*x+c))**3/(e*cot(d*x+c))**(5/2),x)
```

```
[Out] a**3*(Integral((e*cot(c + d*x))**(-5/2), x) + Integral(3*cot(c + d*x)/(e*co
t(c + d*x))**(5/2), x) + Integral(3*cot(c + d*x)**2/(e*cot(c + d*x))**(5/2)
, x) + Integral(cot(c + d*x)**3/(e*cot(c + d*x))**(5/2), x))
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{(a+a\cot(c+dx))^3}{(e\cot(c+dx))^{5/2}}\,dx = \text{Exception raised: ValueError}$$

```
[In] integrate((a+a*cot(d*x+c))^3/(e*cot(d*x+c))^(5/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(e>0)', see `assume?` for more detai
ls)Is e
```

Giac [F]

$$\int \frac{(a + a\cot(c + dx))^3}{(e\cot(c + dx))^{5/2}}\,dx = \int \frac{(a\cot(dx + c) + a)^3}{(e\cot(dx + c))^{\frac{5}{2}}}\,dx$$

```
[In] integrate((a+a*cot(d*x+c))^3/(e*cot(d*x+c))^(5/2),x, algorithm="giac")
[Out] integrate((a*cot(d*x + c) + a)^3/(e*cot(d*x + c))^(5/2), x)
```

Mupad [B] (verification not implemented)

Time = 12.31 (sec) , antiderivative size = 101, normalized size of antiderivative = 0.86

$$\int \frac{(a + a\cot(c + dx))^3}{(e\cot(c + dx))^{5/2}}\,dx = \frac{\frac{2a^3 e}{3} + 6a^3 e\cot(c + dx)}{d\,e^2\,(e\cot(c + dx))^{3/2}} - \frac{2\sqrt{2}\,a^3\,\mathrm{atanh}\left(\frac{32\sqrt{2}\,a^6\,d\,e^{5/2}\,\sqrt{e\cot(c+dx)}}{32\,a^6\,d\,e^3+32\,a^6\,d\,e^3\cot(c+dx)}\right)}{d\,e^{5/2}}$$

```
[In] int((a + a*cot(c + d*x))^3/(e*cot(c + d*x))^(5/2),x)
[Out] ((2*a^3*e)/3 + 6*a^3*e*cot(c + d*x))/(d*e^2*(e*cot(c + d*x))^(3/2)) - (2*2^
(1/2)*a^3*atanh((32*2^(1/2)*a^6*d*e^(5/2)*(e*cot(c + d*x))^(1/2))/(32*a^6*d
*e^3 + 32*a^6*d*e^3*cot(c + d*x))))/(d*e^(5/2))
```

3.21 $\int \frac{(a+a\cot(c+dx))^3}{(e\cot(c+dx))^{7/2}}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 141

$$\int \frac{(a+a\cot(c+dx))^3}{(e\cot(c+dx))^{7/2}}\,dx = -\frac{2\sqrt{2}a^3\arctan\left(\frac{\sqrt{e}-\sqrt{e}\cot(c+dx)}{\sqrt{2}\sqrt{e\cot(c+dx)}}\right)}{de^{7/2}} + \frac{8a^3}{5de^2(e\cot(c+dx))^{3/2}} + \frac{4a^3}{de^3\sqrt{e\cot(c+dx)}} + \frac{2(a^3+a^3\cot(c+dx))}{5de(e\cot(c+dx))^{5/2}}$$

```
[Out] 8/5*a^3/d/e^2/(e*cot(d*x+c))^(3/2)+2/5*(a^3+a^3*cot(d*x+c))/d/e/(e*cot(d*x+
c))^(5/2)-2*a^3*arctan(1/2*(e^(1/2)-cot(d*x+c)*e^(1/2))*2^(1/2)/(e*cot(d*x+
c))^(1/2))*2^(1/2)/d/e^(7/2)+4*a^3/d/e^3/(e*cot(d*x+c))^(1/2)
```

Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 141, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {3646, 3709, 3610, 3613, 211}

$$\int \frac{(a+a\cot(c+dx))^3}{(e\cot(c+dx))^{7/2}}\,dx = -\frac{2\sqrt{2}a^3\arctan\left(\frac{\sqrt{e}-\sqrt{e}\cot(c+dx)}{\sqrt{2}\sqrt{e\cot(c+dx)}}\right)}{de^{7/2}} + \frac{4a^3}{de^3\sqrt{e\cot(c+dx)}} + \frac{8a^3}{5de^2(e\cot(c+dx))^{3/2}} + \frac{2(a^3\cot(c+dx)+a^3)}{5de(e\cot(c+dx))^{5/2}}$$

```
[In] Int[(a + a*Cot[c + d*x])^3/(e*Cot[c + d*x])^(7/2),x]
```

```
[Out] (-2*Sqrt[2]*a^3*ArcTan[(Sqrt[e] - Sqrt[e]*Cot[c + d*x])/(Sqrt[2]*Sqrt[e*Cot
[c + d*x]])])/(d*e^(7/2)) + (8*a^3)/(5*d*e^2*(e*Cot[c + d*x])^(3/2)) + (4*a
```

```
^3)/(d*e^3*Sqrt[e*Cot[c + d*x]]) + (2*(a^3 + a^3*Cot[c + d*x]))/(5*d*e*(e*C
ot[c + d*x])^(5/2))
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 3610

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)]), x_Symbol] :> Simp[(b*c - a*d)*((a + b*Tan[e + f*x])^(m + 1)/
(f*(m + 1)*(a^2 + b^2))), x] + Dist[1/(a^2 + b^2), Int[(a + b*Tan[e + f*x])
^(m + 1)*Simp[a*c + b*d - (b*c - a*d)*Tan[e + f*x], x], x], x] /; FreeQ[{a,
 b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && LtQ[m, -1]
```

Rule 3613

```
Int[((c_) + (d_.)*tan[(e_.) + (f_.)*(x_)])/Sqrt[(b_.)*tan[(e_.) + (f_.)*(x_
)]], x_Symbol] :> Dist[-2*(d^2/f), Subst[Int[1/(2*c*d + b*x^2), x], x, (c -
 d*Tan[e + f*x])/Sqrt[b*Tan[e + f*x]]], x] /; FreeQ[{b, c, d, e, f}, x] &&
EqQ[c^2 - d^2, 0]
```

Rule 3646

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*c - a*d)^2*(a + b*Tan[e + f*x])^(m
 - 2)*((c + d*Tan[e + f*x])^(n + 1)/(d*f*(n + 1)*(c^2 + d^2))), x] - Dist[1
/(d*(n + 1)*(c^2 + d^2)), Int[(a + b*Tan[e + f*x])^(m - 3)*(c + d*Tan[e + f
*x])^(n + 1)*Simp[a^2*d*(b*d*(m - 2) - a*c*(n + 1)) + b*(b*c - 2*a*d)*(b*c*
(m - 2) + a*d*(n + 1)) - d*(n + 1)*(3*a^2*b*c - b^3*c - a^3*d + 3*a*b^2*d)*
Tan[e + f*x] - b*(a*d*(2*b*c - a*d)*(m + n - 1) - b^2*(c^2*(m - 2) - d^2*(n
 + 1)))*Tan[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b
*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] && GtQ[m, 2] && LtQ[
n, -1] && IntegerQ[2*m]
```

Rule 3709

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
 (f_.)*(x_)] + (C_.)*tan[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(A*b^2 -
 a*b*B + a^2*C)*((a + b*Tan[e + f*x])^(m + 1)/(b*f*(m + 1)*(a^2 + b^2))), x
] + Dist[1/(a^2 + b^2), Int[(a + b*Tan[e + f*x])^(m + 1)*Simp[b*B + a*(A -
C) - (A*b - a*B - b*C)*Tan[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B,
 C}, x] && NeQ[A*b^2 - a*b*B + a^2*C, 0] && LtQ[m, -1] && NeQ[a^2 + b^2, 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{2(a^3+a^3\cot(c+dx))}{5de(e\cot(c+dx))^{5/2}} - \frac{2\int \frac{-6a^3e^2-5a^3e^2\cot(c+dx)-a^3e^2\cot^2(c+dx)}{(e\cot(c+dx))^{5/2}}\,dx}{5e^3} \\
&= \frac{8a^3}{5de^2(e\cot(c+dx))^{3/2}} + \frac{2(a^3+a^3\cot(c+dx))}{5de(e\cot(c+dx))^{5/2}} - \frac{2\int \frac{-5a^3e^3+5a^3e^3\cot(c+dx)}{(e\cot(c+dx))^{3/2}}\,dx}{5e^5} \\
&= \frac{8a^3}{5de^2(e\cot(c+dx))^{3/2}} + \frac{4a^3}{de^3\sqrt{e\cot(c+dx)}} \\
&\quad + \frac{2(a^3+a^3\cot(c+dx))}{5de(e\cot(c+dx))^{5/2}} - \frac{2\int \frac{5a^3e^4+5a^3e^4\cot(c+dx)}{\sqrt{e\cot(c+dx)}}\,dx}{5e^7} \\
&= \frac{8a^3}{5de^2(e\cot(c+dx))^{3/2}} + \frac{4a^3}{de^3\sqrt{e\cot(c+dx)}} + \frac{2(a^3+a^3\cot(c+dx))}{5de(e\cot(c+dx))^{5/2}} \\
&\quad + \frac{(20a^6e)\,\text{Subst}\left(\int \frac{1}{-50a^6e^8-ex^2}\,dx, x, \frac{5a^3e^4-5a^3e^4\cot(c+dx)}{\sqrt{e\cot(c+dx)}}\right)}{d} \\
&= -\frac{2\sqrt{2}a^3\arctan\left(\frac{\sqrt{e}-\sqrt{e}\cot(c+dx)}{\sqrt{2}\sqrt{e\cot(c+dx)}}\right)}{de^{7/2}} + \frac{8a^3}{5de^2(e\cot(c+dx))^{3/2}} \\
&\quad + \frac{4a^3}{de^3\sqrt{e\cot(c+dx)}} + \frac{2(a^3+a^3\cot(c+dx))}{5de(e\cot(c+dx))^{5/2}}
\end{aligned}$$

Mathematica [B] (warning: unable to verify)

Leaf count is larger than twice the leaf count of optimal. 415 vs. $2(141) = 282$.

Time = 5.75 (sec) , antiderivative size = 415, normalized size of antiderivative = 2.94

$$\int \frac{(a+a\cot(c+dx))^3}{(e\cot(c+dx))^{7/2}}\,dx = \frac{a^3\Big(80\cos^3(c+dx)+40\cos^2(c+dx)\sin(c+dx)+8\cos(c+dx)\sin^2(c+dx)+}{}$$

```
[In] Integrate[(a + a*Cot[c + d*x])^3/(e*Cot[c + d*x])^(7/2),x]
```

```
[Out] (a^3*(80*Cos[c + d*x]^3 + 40*Cos[c + d*x]^2*Sin[c + d*x] + 8*Cos[c + d*x]*S
in[c + d*x]^2 + 10*Sqrt[2]*ArcTan[1 - Sqrt[2]*Sqrt[Cot[c + d*x]]]*Cot[c + d
*x]^(7/2)*Sin[c + d*x]^3 - 10*Sqrt[2]*ArcTan[1 + Sqrt[2]*Sqrt[Cot[c + d*x]]
]*Cot[c + d*x]^(7/2)*Sin[c + d*x]^3 - 20*ArcTanh[(-Cot[c + d*x]^2)^(1/4)]*(
2*(-Cot[c + d*x])^(1/4)*Cot[c + d*x]^(13/4) - 3*(-Cot[c + d*x]^2)^(7/4))*Si
n[c + d*x]^3 + 20*ArcTan[(-Cot[c + d*x]^2)^(1/4)]*(2*(-Cot[c + d*x])^(1/4)*
Cot[c + d*x]^(13/4) + 3*(-Cot[c + d*x]^2)^(7/4))*Sin[c + d*x]^3 + 5*Sqrt[2]
*Cot[c + d*x]^(7/2)*Log[1 - Sqrt[2]*Sqrt[Cot[c + d*x]] + Cot[c + d*x]]*Sin[
c + d*x]^3 - 5*Sqrt[2]*Cot[c + d*x]^(7/2)*Log[1 + Sqrt[2]*Sqrt[Cot[c + d*x]
] + Cot[c + d*x]]*Sin[c + d*x]^3)*(1 + Tan[c + d*x])^3)/(20*d*e^3*Sqrt[e*Co
t[c + d*x]]*(Cos[c + d*x] + Sin[c + d*x])^3)
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 322 vs. $2(120) = 240$.

Time = 0.05 (sec) , antiderivative size = 323, normalized size of antiderivative = 2.29

method	result
derivativedivides	$-\frac{2a^3\left(\frac{-\frac{(e^2)^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\frac{e\cot(dx+c)+(e^2)^{\frac{1}{4}}\sqrt{e\cot(dx+c)}\sqrt{2}+\sqrt{e^2}}{e\cot(dx+c)-(e^2)^{\frac{1}{4}}\sqrt{e\cot(dx+c)}\sqrt{2}+\sqrt{e^2}}\right)+2\arctan\left(\frac{\sqrt{2}\sqrt{e\cot(dx+c)}}{(e^2)^{\frac{1}{4}}}+1\right)-2\arctan\left(-\frac{\sqrt{2}\sqrt{e\cot(\cdots}}{(e^2)^{\frac{1}{4}}}\cdots\right.\right.}{4e}}{\cdots}\cdots\right.}{\cdots}$
default	$-\frac{2a^3\left(\frac{-\frac{(e^2)^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\frac{e\cot(dx+c)+(e^2)^{\frac{1}{4}}\sqrt{e\cot(dx+c)}\sqrt{2}+\sqrt{e^2}}{e\cot(dx+c)-(e^2)^{\frac{1}{4}}\sqrt{e\cot(dx+c)}\sqrt{2}+\sqrt{e^2}}\right)+2\arctan\left(\frac{\sqrt{2}\sqrt{e\cot(dx+c)}}{(e^2)^{\frac{1}{4}}}+1\right)-2\arctan\left(-\frac{\sqrt{2}\sqrt{e\cot(\cdots}}{(e^2)^{\frac{1}{4}}}\cdots\right.\right.}{4e}}{\cdots}\cdots\right.}{\cdots}$
parts	$-\frac{2a^3e\left(-\frac{1}{5e^2(e\cot(dx+c))^{\frac{5}{2}}}+\frac{1}{e^4\sqrt{e\cot(dx+c)}}+\frac{\sqrt{2}\left(\ln\left(\frac{e\cot(dx+c)-(e^2)^{\frac{1}{4}}\sqrt{e\cot(dx+c)}\sqrt{2}+\sqrt{e^2}}{e\cot(dx+c)+(e^2)^{\frac{1}{4}}\sqrt{e\cot(dx+c)}\sqrt{2}+\sqrt{e^2}}\right)+2\arctan\left(\frac{\sqrt{2}\sqrt{e\cot(\cdots}}{(e^2)^{\frac{1}{4}}}\cdots\right.\right.}{8e^4(e^2)^{\frac{1}{4}}}\cdots\right.}{d}$

[In] int((a+a*cot(d*x+c))^3/(e*cot(d*x+c))^(7/2),x,method=_RETURNVERBOSE)

[Out] -2/d*a^3/e^2*(1/e*(-1/4/e*(e^2)^(1/4)*2^(1/2)*(ln((e*cot(d*x+c)+(e^2)^(1/4)*(e*cot(d*x+c))^(1/2)*2^(1/2)+(e^2)^(1/2))/(e*cot(d*x+c)-(e^2)^(1/4)*(e*cot(d*x+c))^(1/2)*2^(1/2)+(e^2)^(1/2)))+2*arctan(2^(1/2)/(e^2)^(1/4)*(e*cot(d*x+c))^(1/2)+1)-2*arctan(-2^(1/2)/(e^2)^(1/4)*(e*cot(d*x+c))^(1/2)+1))-1/4/(e^2)^(1/4)*2^(1/2)*(ln((e*cot(d*x+c)-(e^2)^(1/4)*(e*cot(d*x+c))^(1/2)*2^(1/2)+(e^2)^(1/2))/(e*cot(d*x+c)+(e^2)^(1/4)*(e*cot(d*x+c))^(1/2)*2^(1/2)+(e^2)^(1/2)))+2*arctan(2^(1/2)/(e^2)^(1/4)*(e*cot(d*x+c))^(1/2)+1)-2*arctan(-2^(1/2)/(e^2)^(1/4)*(e*cot(d*x+c))^(1/2)+1)))-1/5*e/(e*cot(d*x+c))^(5/2)-1/(e*cot(d*x+c))^(3/2)-2/e/(e*cot(d*x+c))^(1/2))

Fricas [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 485, normalized size of antiderivative = 3.44

$$\int \frac{(a+a\cot(c+dx))^3}{(e\cot(c+dx))^{7/2}}\,dx = \left[\frac{5\sqrt{2}\left(a^3e\cos\left(2\,dx+2\,c\right)^2+2\,a^3e\cos\left(2\,dx+2\,c\right)+a^3e\right)\sqrt{-\frac{1}{e}}\log\left(\sqrt{2}\sqrt{\frac{e\cos(2\,}{\sin(2}}\right.}{}\right.$$

$$-\frac{2\left(\frac{5\sqrt{2}\left(a^3e\cos(2\,dx+2\,c)^2+2\,a^3e\cos(2\,dx+2\,c)+a^3e\right)\arctan\left(-\frac{\sqrt{2}\sqrt{\frac{e\cos(2\,dx+2\,c)+e}{\sin(2\,dx+2\,c)}}(\cos(2\,dx+2\,c)-\sin(2\,dx+2\,c)+1)}{2\sqrt{e}(\cos(2\,dx+2\,c)+1)}\right)}{\sqrt{e}}+\left(5\,a^3\cos\left(2\,dx+\right.\right.\right.}{5\left(de^4\cos\left(2\,dx+2\,c\right)^2+2\,de^4\cos\left(2\,dx+\right.\right.}$$

```
[In] integrate((a+a*cot(d*x+c))^3/(e*cot(d*x+c))^(7/2),x, algorithm="fricas")
```

```
[Out] [1/5*(5*sqrt(2)*(a^3*e*cos(2*d*x + 2*c)^2 + 2*a^3*e*cos(2*d*x + 2*c) + a^3*
e)*sqrt(-1/e)*log(sqrt(2)*sqrt((e*cos(2*d*x + 2*c) + e)/sin(2*d*x + 2*c))*s
qrt(-1/e)*(cos(2*d*x + 2*c) + sin(2*d*x + 2*c) - 1) - 2*sin(2*d*x + 2*c) +
1) - 2*(5*a^3*cos(2*d*x + 2*c)^2 - 5*a^3 - (9*a^3*cos(2*d*x + 2*c) + 11*a^3
)*sin(2*d*x + 2*c))*sqrt((e*cos(2*d*x + 2*c) + e)/sin(2*d*x + 2*c)))/(d*e^4
*cos(2*d*x + 2*c)^2 + 2*d*e^4*cos(2*d*x + 2*c) + d*e^4), -2/5*(5*sqrt(2)*(a
^3*e*cos(2*d*x + 2*c)^2 + 2*a^3*e*cos(2*d*x + 2*c) + a^3*e)*arctan(-1/2*sqr
t(2)*sqrt((e*cos(2*d*x + 2*c) + e)/sin(2*d*x + 2*c))*(cos(2*d*x + 2*c) - si
n(2*d*x + 2*c) + 1)/(sqrt(e)*(cos(2*d*x + 2*c) + 1)))/sqrt(e) + (5*a^3*cos(
2*d*x + 2*c)^2 - 5*a^3 - (9*a^3*cos(2*d*x + 2*c) + 11*a^3)*sin(2*d*x + 2*c)
)*sqrt((e*cos(2*d*x + 2*c) + e)/sin(2*d*x + 2*c)))/(d*e^4*cos(2*d*x + 2*c)^
2 + 2*d*e^4*cos(2*d*x + 2*c) + d*e^4)]
```

Sympy [F]

$$\int \frac{(a+a\cot(c+dx))^3}{(e\cot(c+dx))^{7/2}}\,dx = a^3\left(\int \frac{1}{(e\cot(c+dx))^{\frac{7}{2}}}\,dx + \int \frac{3\cot(c+dx)}{(e\cot(c+dx))^{\frac{7}{2}}}\,dx + \int \frac{3\cot^2(c+dx)}{(e\cot(c+dx))^{\frac{7}{2}}}\,dx + \int \frac{\cot^3(c+dx)}{(e\cot(c+dx))^{\frac{7}{2}}}\,dx\right)$$

```
[In] integrate((a+a*cot(d*x+c))**3/(e*cot(d*x+c))**(7/2),x)
```

```
[Out] a**3*(Integral((e*cot(c + d*x))**(-7/2), x) + Integral(3*cot(c + d*x)/(e*co
t(c + d*x))**(7/2), x) + Integral(3*cot(c + d*x)**2/(e*cot(c + d*x))**(7/2)
, x) + Integral(cot(c + d*x)**3/(e*cot(c + d*x))**(7/2), x))
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{(a+a\cot(c+dx))^3}{(e\cot(c+dx))^{7/2}}\,dx = \text{Exception raised: ValueError}$$

```
[In] integrate((a+a*cot(d*x+c))^3/(e*cot(d*x+c))^(7/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(e>0)', see `assume?` for more detai
ls)Is e
```

Giac [F(-1)]

Timed out.

$$\int \frac{(a+a\cot(c+dx))^3}{(e\cot(c+dx))^{7/2}}\,dx = \text{Timed out}$$

```
[In] integrate((a+a*cot(d*x+c))^3/(e*cot(d*x+c))^(7/2),x, algorithm="giac")
```

```
[Out] Timed out
```

Mupad [B] (verification not implemented)

Time = 13.54 (sec) , antiderivative size = 126, normalized size of antiderivative = 0.89

$$\int \frac{(a+a\cot(c+dx))^3}{(e\cot(c+dx))^{7/2}}\,dx = \frac{4\,e\,a^3\cot(c+d\,x)^2+2\,e\,a^3\cot(c+d\,x)+\frac{2\,e\,a^3}{5}}{d\,e^2\,(e\cot(c+d\,x))^{5/2}} + \frac{\sqrt{2}\,a^3\left(2\,\mathrm{atan}\left(\frac{\sqrt{2}\,\sqrt{e\cot(c+d\,x)}}{2\,\sqrt{e}}\right)+2\,\mathrm{atan}\left(\frac{\sqrt{2}\,\sqrt{e\cot(c+d\,x)}}{2\,\sqrt{e}}+\frac{\sqrt{2}\,(e\cot(c+d\,x))^{3/2}}{2\,e^{3/2}}\right)\right)}{d\,e^{7/2}}$$

```
[In] int((a + a*cot(c + d*x))^3/(e*cot(c + d*x))^(7/2),x)
```

```
[Out] ((2*a^3*e)/5 + 4*a^3*e*cot(c + d*x)^2 + 2*a^3*e*cot(c + d*x))/(d*e^2*(e*cot
(c + d*x))^(5/2)) + (2^(1/2)*a^3*(2*atan((2^(1/2)*(e*cot(c + d*x))^(1/2))/(
2*e^(1/2))) + 2*atan((2^(1/2)*(e*cot(c + d*x))^(1/2))/(2*e^(1/2)) + (2^(1/2
)*(e*cot(c + d*x))^(3/2))/(2*e^(3/2)))))/(d*e^(7/2))
```

3.22 $\int \frac{(a+a\cot(c+dx))^3}{(e\cot(c+dx))^{9/2}}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 165

$$\int \frac{(a+a\cot(c+dx))^3}{(e\cot(c+dx))^{9/2}}\,dx = \frac{2\sqrt{2}a^3\operatorname{arctanh}\left(\frac{\sqrt{e}+\sqrt{e}\cot(c+dx)}{\sqrt{2}\sqrt{e\cot(c+dx)}}\right)}{de^{9/2}} + \frac{32a^3}{35de^2(e\cot(c+dx))^{5/2}} + \frac{4a^3}{3de^3(e\cot(c+dx))^{3/2}} - \frac{4a^3}{de^4\sqrt{e\cot(c+dx)}} + \frac{2(a^3+a^3\cot(c+dx))}{7de(e\cot(c+dx))^{7/2}}$$

```
[Out] 32/35*a^3/d/e^2/(e*cot(d*x+c))^(5/2)+4/3*a^3/d/e^3/(e*cot(d*x+c))^(3/2)+2/7
*(a^3+a^3*cot(d*x+c))/d/e/(e*cot(d*x+c))^(7/2)+2*a^3*arctanh(1/2*(e^(1/2)+c
ot(d*x+c)*e^(1/2))*2^(1/2)/(e*cot(d*x+c))^(1/2))*2^(1/2)/d/e^(9/2)-4*a^3/d/
e^4/(e*cot(d*x+c))^(1/2)
```

Rubi [A] (verified)

Time = 0.34 (sec) , antiderivative size = 165, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {3646, 3709, 3610, 3613, 214}

$$\int \frac{(a+a\cot(c+dx))^3}{(e\cot(c+dx))^{9/2}}\,dx = \frac{2\sqrt{2}a^3\operatorname{arctanh}\left(\frac{\sqrt{e}\cot(c+dx)+\sqrt{e}}{\sqrt{2}\sqrt{e\cot(c+dx)}}\right)}{de^{9/2}} - \frac{4a^3}{de^4\sqrt{e\cot(c+dx)}} + \frac{4a^3}{3de^3(e\cot(c+dx))^{3/2}} + \frac{32a^3}{35de^2(e\cot(c+dx))^{5/2}} + \frac{2(a^3\cot(c+dx)+a^3)}{7de(e\cot(c+dx))^{7/2}}$$

```
[In] Int[(a + a*Cot[c + d*x])^3/(e*Cot[c + d*x])^(9/2),x]
```

```
[Out] (2*Sqrt[2]*a^3*ArcTanh[(Sqrt[e] + Sqrt[e]*Cot[c + d*x])/(Sqrt[2]*Sqrt[e*Cot
[c + d*x]])])/(d*e^(9/2)) + (32*a^3)/(35*d*e^2*(e*Cot[c + d*x])^(5/2)) + (4
```

```
*a^3)/(3*d*e^3*(e*Cot[c + d*x])^(3/2)) - (4*a^3)/(d*e^4*Sqrt[e*Cot[c + d*x]
]) + (2*(a^3 + a^3*Cot[c + d*x]))/(7*d*e*(e*Cot[c + d*x])^(7/2))
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 3610

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)]), x_Symbol] :> Simp[(b*c - a*d)*((a + b*Tan[e + f*x])^(m + 1)/
(f*(m + 1)*(a^2 + b^2))), x] + Dist[1/(a^2 + b^2), Int[(a + b*Tan[e + f*x])
^(m + 1)*Simp[a*c + b*d - (b*c - a*d)*Tan[e + f*x], x], x], x] /; FreeQ[{a,
 b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && LtQ[m, -1]
```

Rule 3613

```
Int[((c_) + (d_.)*tan[(e_.) + (f_.)*(x_)])/Sqrt[(b_.)*tan[(e_.) + (f_.)*(x_
)]], x_Symbol] :> Dist[-2*(d^2/f), Subst[Int[1/(2*c*d + b*x^2), x], x, (c -
 d*Tan[e + f*x])/Sqrt[b*Tan[e + f*x]]], x] /; FreeQ[{b, c, d, e, f}, x] &&
EqQ[c^2 - d^2, 0]
```

Rule 3646

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*c - a*d)^2*(a + b*Tan[e + f*x])^(m
 - 2)*((c + d*Tan[e + f*x])^(n + 1)/(d*f*(n + 1)*(c^2 + d^2))), x] - Dist[1
/(d*(n + 1)*(c^2 + d^2)), Int[(a + b*Tan[e + f*x])^(m - 3)*(c + d*Tan[e + f
*x])^(n + 1)*Simp[a^2*d*(b*d*(m - 2) - a*c*(n + 1)) + b*(b*c - 2*a*d)*(b*c*
(m - 2) + a*d*(n + 1)) - d*(n + 1)*(3*a^2*b*c - b^3*c - a^3*d + 3*a*b^2*d)*
Tan[e + f*x] - b*(a*d*(2*b*c - a*d)*(m + n - 1) - b^2*(c^2*(m - 2) - d^2*(n
 + 1)))*Tan[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b
*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] && GtQ[m, 2] && LtQ[
n, -1] && IntegerQ[2*m]
```

Rule 3709

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
 (f_.)*(x_)] + (C_.)*tan[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(A*b^2 -
 a*b*B + a^2*C)*((a + b*Tan[e + f*x])^(m + 1)/(b*f*(m + 1)*(a^2 + b^2))), x
] + Dist[1/(a^2 + b^2), Int[(a + b*Tan[e + f*x])^(m + 1)*Simp[b*B + a*(A -
C) - (A*b - a*B - b*C)*Tan[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B,
 C}, x] && NeQ[A*b^2 - a*b*B + a^2*C, 0] && LtQ[m, -1] && NeQ[a^2 + b^2, 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{2(a^3+a^3\cot(c+dx))}{7de(e\cot(c+dx))^{7/2}} - \frac{2\int \frac{-8a^3e^2-7a^3e^2\cot(c+dx)-a^3e^2\cot^2(c+dx)}{(e\cot(c+dx))^{7/2}}\,dx}{7e^3} \\
&= \frac{32a^3}{35de^2(e\cot(c+dx))^{5/2}} + \frac{2(a^3+a^3\cot(c+dx))}{7de(e\cot(c+dx))^{7/2}} - \frac{2\int \frac{-7a^3e^3+7a^3e^3\cot(c+dx)}{(e\cot(c+dx))^{5/2}}\,dx}{7e^5} \\
&= \frac{32a^3}{35de^2(e\cot(c+dx))^{5/2}} + \frac{4a^3}{3de^3(e\cot(c+dx))^{3/2}} \\
&\quad + \frac{2(a^3+a^3\cot(c+dx))}{7de(e\cot(c+dx))^{7/2}} - \frac{2\int \frac{7a^3e^4+7a^3e^4\cot(c+dx)}{(e\cot(c+dx))^{3/2}}\,dx}{7e^7} \\
&= \frac{32a^3}{35de^2(e\cot(c+dx))^{5/2}} + \frac{4a^3}{3de^3(e\cot(c+dx))^{3/2}} - \frac{4a^3}{de^4\sqrt{e\cot(c+dx)}} \\
&\quad + \frac{2(a^3+a^3\cot(c+dx))}{7de(e\cot(c+dx))^{7/2}} - \frac{2\int \frac{7a^3e^5-7a^3e^5\cot(c+dx)}{\sqrt{e\cot(c+dx)}}\,dx}{7e^9} \\
&= \frac{32a^3}{35de^2(e\cot(c+dx))^{5/2}} + \frac{4a^3}{3de^3(e\cot(c+dx))^{3/2}} - \frac{4a^3}{de^4\sqrt{e\cot(c+dx)}} \\
&\quad + \frac{2(a^3+a^3\cot(c+dx))}{7de(e\cot(c+dx))^{7/2}} + \frac{(28a^6e)\,\text{Subst}\left(\int \frac{1}{98a^6e^{10}-ex^2}\,dx, x, \frac{7a^3e^5+7a^3e^5\cot(c+dx)}{\sqrt{e\cot(c+dx)}}\right)}{d} \\
&= \frac{2\sqrt{2}a^3\text{arctanh}\left(\frac{\sqrt{e}+\sqrt{e}\cot(c+dx)}{\sqrt{2}\sqrt{e\cot(c+dx)}}\right)}{de^{9/2}} + \frac{32a^3}{35de^2(e\cot(c+dx))^{5/2}} \\
&\quad + \frac{4a^3}{3de^3(e\cot(c+dx))^{3/2}} - \frac{4a^3}{de^4\sqrt{e\cot(c+dx)}} + \frac{2(a^3+a^3\cot(c+dx))}{7de(e\cot(c+dx))^{7/2}}
\end{aligned}$$

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 625 vs. $2(165) = 330$.

Time = 6.31 (sec) , antiderivative size = 625, normalized size of antiderivative = 3.79

$$\int \frac{(a+a\cot(c+dx))^3}{(e\cot(c+dx))^{9/2}}\,dx = \frac{4\cos^3(c+dx)(a+a\cot(c+dx))^3}{3d(e\cot(c+dx))^{9/2}(\cos(c+dx)+\sin(c+dx))^3}$$
$$-\frac{4\cos^3(c+dx)\cot(c+dx)(a+a\cot(c+dx))^3}{d(e\cot(c+dx))^{9/2}(\cos(c+dx)+\sin(c+dx))^3}$$
$$+\frac{6\cos^2(c+dx)(a+a\cot(c+dx))^3\sin(c+dx)}{5d(e\cot(c+dx))^{9/2}(\cos(c+dx)+\sin(c+dx))^3}$$
$$+\frac{2\cos(c+dx)(a+a\cot(c+dx))^3\sin^2(c+dx)}{7d(e\cot(c+dx))^{9/2}(\cos(c+dx)+\sin(c+dx))^3}$$
$$-\frac{2\arctan\left(\sqrt[4]{-\cot(c+dx)}\sqrt[4]{\cot(c+dx)}\right)(-\cot(c+dx))^{3/4}\cot^{\frac{15}{4}}(c+dx)(a+a\cot(c+dx))^3\sin^3(c+dx)}{d(e\cot(c+dx))^{9/2}(\cos(c+dx)+\sin(c+dx))^3}$$
$$-\frac{2\text{arctanh}\left(\sqrt[4]{-\cot(c+dx)}\sqrt[4]{\cot(c+dx)}\right)(-\cot(c+dx))^{3/4}\cot^{\frac{15}{4}}(c+dx)(a+a\cot(c+dx))^3\sin^3(c+dx)}{d(e\cot(c+dx))^{9/2}(\cos(c+dx)+\sin(c+dx))^3}$$
$$-\frac{2\arctan\left(\sqrt[4]{-\cot(c+dx)}\sqrt[4]{\cot(c+dx)}\right)\sqrt[4]{-\cot(c+dx)}\cot^{\frac{17}{4}}(c+dx)(a+a\cot(c+dx))^3\sin^3(c+dx)}{d(e\cot(c+dx))^{9/2}(\cos(c+dx)+\sin(c+dx))^3}$$
$$+\frac{2\text{arctanh}\left(\sqrt[4]{-\cot(c+dx)}\sqrt[4]{\cot(c+dx)}\right)\sqrt[4]{-\cot(c+dx)}\cot^{\frac{17}{4}}(c+dx)(a+a\cot(c+dx))^3\sin^3(c+dx)}{d(e\cot(c+dx))^{9/2}(\cos(c+dx)+\sin(c+dx))^3}$$

```
[In] Integrate[(a + a*Cot[c + d*x])^3/(e*Cot[c + d*x])^(9/2),x]
```

```
[Out] (4*Cos[c + d*x]^3*(a + a*Cot[c + d*x])^3)/(3*d*(e*Cot[c + d*x])^(9/2)*(Cos[
c + d*x] + Sin[c + d*x])^3) - (4*Cos[c + d*x]^3*Cot[c + d*x]*(a + a*Cot[c +
 d*x])^3)/(d*(e*Cot[c + d*x])^(9/2)*(Cos[c + d*x] + Sin[c + d*x])^3) + (6*C
os[c + d*x]^2*(a + a*Cot[c + d*x])^3*Sin[c + d*x])/(5*d*(e*Cot[c + d*x])^(9
/2)*(Cos[c + d*x] + Sin[c + d*x])^3) + (2*Cos[c + d*x]*(a + a*Cot[c + d*x])
^3*Sin[c + d*x]^2)/(7*d*(e*Cot[c + d*x])^(9/2)*(Cos[c + d*x] + Sin[c + d*x]
)^3) - (2*ArcTan[(-Cot[c + d*x])^(1/4)*Cot[c + d*x]^(1/4)]*(-Cot[c + d*x])^
(3/4)*Cot[c + d*x]^(15/4)*(a + a*Cot[c + d*x])^3*Sin[c + d*x]^3)/(d*(e*Cot[
c + d*x])^(9/2)*(Cos[c + d*x] + Sin[c + d*x])^3) - (2*ArcTanh[(-Cot[c + d*x
])^(1/4)*Cot[c + d*x]^(1/4)]*(-Cot[c + d*x])^(3/4)*Cot[c + d*x]^(15/4)*(a +
 a*Cot[c + d*x])^3*Sin[c + d*x]^3)/(d*(e*Cot[c + d*x])^(9/2)*(Cos[c + d*x]
+ Sin[c + d*x])^3) - (2*ArcTan[(-Cot[c + d*x])^(1/4)*Cot[c + d*x]^(1/4)]*(-
Cot[c + d*x])^(1/4)*Cot[c + d*x]^(17/4)*(a + a*Cot[c + d*x])^3*Sin[c + d*x]
^3)/(d*(e*Cot[c + d*x])^(9/2)*(Cos[c + d*x] + Sin[c + d*x])^3) + (2*ArcTanh
[(-Cot[c + d*x])^(1/4)*Cot[c + d*x]^(1/4)]*(-Cot[c + d*x])^(1/4)*Cot[c + d*
x]^(17/4)*(a + a*Cot[c + d*x])^3*Sin[c + d*x]^3)/(d*(e*Cot[c + d*x])^(9/2)*
(Cos[c + d*x] + Sin[c + d*x])^3)
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 337 vs. $2(140) = 280$.

Time $= 0.09$ (sec) , antiderivative size $= 338$, normalized size of antiderivative $= 2.05$

method	result
derivativedivides	$-\frac{2a^3\Big(-\frac{(e^2)^{\frac{1}{4}}\sqrt{2}\big(\ln\big(\frac{e\cot(dx+c)+(e^2)^{\frac{1}{4}}\sqrt{e\cot(dx+c)}\,\sqrt{2}+\sqrt{e^2}}{e\cot(dx+c)-(e^2)^{\frac{1}{4}}\sqrt{e\cot(dx+c)}\,\sqrt{2}+\sqrt{e^2}}\big)+2\arctan\big(\frac{\sqrt{2}\sqrt{e\cot(dx+c)}}{(e^2)^{\frac{1}{4}}}+1\big)-2\arctan\big(-\frac{\sqrt{2}\sqrt{e\cot(\cdots}}{(e^2)^{\frac{1}{4}}}\cdots}{4e}\cdots}{\cdots}$
default	$-\frac{2a^3\Big(-\frac{(e^2)^{\frac{1}{4}}\sqrt{2}\big(\ln\big(\frac{e\cot(dx+c)+(e^2)^{\frac{1}{4}}\sqrt{e\cot(dx+c)}\,\sqrt{2}+\sqrt{e^2}}{e\cot(dx+c)-(e^2)^{\frac{1}{4}}\sqrt{e\cot(dx+c)}\,\sqrt{2}+\sqrt{e^2}}\big)+2\arctan\big(\frac{\sqrt{2}\sqrt{e\cot(dx+c)}}{(e^2)^{\frac{1}{4}}}+1\big)-2\arctan\big(-\frac{\sqrt{2}\sqrt{e\cot(\cdots}}{(e^2)^{\frac{1}{4}}}\cdots}{4e}\cdots}{\cdots}$
parts	$-\frac{2a^3e\Big(-\frac{1}{7e^2(e\cot(dx+c))^{\frac{7}{2}}}+\frac{1}{3e^4(e\cot(dx+c))^{\frac{3}{2}}}+\frac{(e^2)^{\frac{1}{4}}\sqrt{2}\big(\ln\big(\frac{e\cot(dx+c)+(e^2)^{\frac{1}{4}}\sqrt{e\cot(dx+c)}\,\sqrt{2}+\sqrt{e^2}}{e\cot(dx+c)-(e^2)^{\frac{1}{4}}\sqrt{e\cot(dx+c)}\,\sqrt{2}+\sqrt{e^2}}\big)+2\arctan\big(\frac{\sqrt{2}\cdots}{\cdots}}{8e^6}\cdots}{d}$

```
[In] int((a+a*cot(d*x+c))^3/(e*cot(d*x+c))^(9/2),x,method=_RETURNVERBOSE)
```

```
[Out] -2/d*a^3/e^2*(1/e^2*(-1/4/e*(e^2)^(1/4)*2^(1/2)*(ln((e*cot(d*x+c)+(e^2)^(1/
4)*(e*cot(d*x+c))^(1/2)*2^(1/2)+(e^2)^(1/2))/(e*cot(d*x+c)-(e^2)^(1/4)*(e*c
ot(d*x+c))^(1/2)*2^(1/2)+(e^2)^(1/2)))+2*arctan(2^(1/2)/(e^2)^(1/4)*(e*cot(
d*x+c))^(1/2)+1)-2*arctan(-2^(1/2)/(e^2)^(1/4)*(e*cot(d*x+c))^(1/2)+1))+1/4
/(e^2)^(1/4)*2^(1/2)*(ln((e*cot(d*x+c)-(e^2)^(1/4)*(e*cot(d*x+c))^(1/2)*2^(
1/2)+(e^2)^(1/2))/(e*cot(d*x+c)+(e^2)^(1/4)*(e*cot(d*x+c))^(1/2)*2^(1/2)+(e
^2)^(1/2)))+2*arctan(2^(1/2)/(e^2)^(1/4)*(e*cot(d*x+c))^(1/2)+1)-2*arctan(-
2^(1/2)/(e^2)^(1/4)*(e*cot(d*x+c))^(1/2)+1)))-1/7*e/(e*cot(d*x+c))^(7/2)-3/
5/(e*cot(d*x+c))^(5/2)+2/e^2/(e*cot(d*x+c))^(1/2)-2/3/e/(e*cot(d*x+c))^(3/2
))
```

Fricas [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 514, normalized size of antiderivative = 3.12

$$\int \frac{(a+a\cot(c+dx))^3}{(e\cot(c+dx))^{9/2}}\,dx = \left[\frac{\frac{105\sqrt{2}\left(a^3e\cos(2\,dx+2\,c)^2+2\,a^3e\cos(2\,dx+2\,c)+a^3e\right)\log\left(-\frac{\sqrt{2}\sqrt{\frac{e\cos(2\,dx+2\,c)+e}{\sin(2\,dx+2\,c)}}(\cos(2\,dx+2\,c)-\sin(2\,dx+2\,c}{\sqrt{e}}\right.}{\sqrt{e}}}{}\right.$$

$$-\frac{2\left(105\sqrt{2}\left(a^3e\cos(2\,dx+2\,c)^2+2\,a^3e\cos(2\,dx+2\,c)+a^3e\right)\sqrt{-\frac{1}{e}}\arctan\left(\frac{\sqrt{2}\sqrt{\frac{e\cos(2\,dx+2\,c)+e}{\sin(2\,dx+2\,c)}}\sqrt{-\frac{1}{e}}(\cos(2\,dx+2\,c}{2\,(\cos(2\,dx+2\,c)+1)}\right.\right.}{105\left(de^5\cos\right.}$$

```
[In] integrate((a+a*cot(d*x+c))^3/(e*cot(d*x+c))^(9/2),x, algorithm="fricas")
```

```
[Out] [1/105*(105*sqrt(2)*(a^3*e*cos(2*d*x + 2*c)^2 + 2*a^3*e*cos(2*d*x + 2*c) +
a^3*e)*log(-sqrt(2)*sqrt((e*cos(2*d*x + 2*c) + e)/sin(2*d*x + 2*c))*(cos(2*
d*x + 2*c) - sin(2*d*x + 2*c) - 1)/sqrt(e) + 2*sin(2*d*x + 2*c) + 1)/sqrt(e
) - 2*(55*a^3*cos(2*d*x + 2*c)^2 + 30*a^3*cos(2*d*x + 2*c) - 85*a^3 + 21*(1
3*a^3*cos(2*d*x + 2*c) + 7*a^3)*sin(2*d*x + 2*c))*sqrt((e*cos(2*d*x + 2*c)
+ e)/sin(2*d*x + 2*c)))/(d*e^5*cos(2*d*x + 2*c)^2 + 2*d*e^5*cos(2*d*x + 2*c
) + d*e^5), -2/105*(105*sqrt(2)*(a^3*e*cos(2*d*x + 2*c)^2 + 2*a^3*e*cos(2*d
*x + 2*c) + a^3*e)*sqrt(-1/e)*arctan(1/2*sqrt(2)*sqrt((e*cos(2*d*x + 2*c) +
 e)/sin(2*d*x + 2*c))*sqrt(-1/e)*(cos(2*d*x + 2*c) + sin(2*d*x + 2*c) + 1)/
(cos(2*d*x + 2*c) + 1)) + (55*a^3*cos(2*d*x + 2*c)^2 + 30*a^3*cos(2*d*x + 2
*c) - 85*a^3 + 21*(13*a^3*cos(2*d*x + 2*c) + 7*a^3)*sin(2*d*x + 2*c))*sqrt(
(e*cos(2*d*x + 2*c) + e)/sin(2*d*x + 2*c)))/(d*e^5*cos(2*d*x + 2*c)^2 + 2*d
*e^5*cos(2*d*x + 2*c) + d*e^5)]
```

Sympy [F]

$$\int \frac{(a+a\cot(c+dx))^3}{(e\cot(c+dx))^{9/2}}\,dx = a^3\left(\int \frac{1}{(e\cot(c+dx))^{\frac{9}{2}}}\,dx + \int \frac{3\cot(c+dx)}{(e\cot(c+dx))^{\frac{9}{2}}}\,dx + \int \frac{3\cot^2(c+dx)}{(e\cot(c+dx))^{\frac{9}{2}}}\,dx + \int \frac{\cot^3(c+dx)}{(e\cot(c+dx))^{\frac{9}{2}}}\,dx\right)$$

```
[In] integrate((a+a*cot(d*x+c))**3/(e*cot(d*x+c))**(9/2),x)
```

```
[Out] a**3*(Integral((e*cot(c + d*x))**(-9/2), x) + Integral(3*cot(c + d*x)/(e*co
t(c + d*x))**(9/2), x) + Integral(3*cot(c + d*x)**2/(e*cot(c + d*x))**(9/2)
, x) + Integral(cot(c + d*x)**3/(e*cot(c + d*x))**(9/2), x))
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{(a+a\cot(c+dx))^3}{(e\cot(c+dx))^{9/2}}\,dx = \text{Exception raised: ValueError}$$

```
[In] integrate((a+a*cot(d*x+c))^3/(e*cot(d*x+c))^(9/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(e>0)', see `assume?` for more detai
ls)Is e
```

Giac [F(-1)]

Timed out.

$$\int \frac{(a+a\cot(c+dx))^3}{(e\cot(c+dx))^{9/2}}\,dx = \text{Timed out}$$

```
[In] integrate((a+a*cot(d*x+c))^3/(e*cot(d*x+c))^(9/2),x, algorithm="giac")
```

```
[Out] Timed out
```

Mupad [B] (verification not implemented)

Time = 14.21 (sec) , antiderivative size = 129, normalized size of antiderivative = 0.78

$$\int \frac{(a+a\cot(c+dx))^3}{(e\cot(c+dx))^{9/2}}\,dx = \frac{-4\,e\,a^3\,\cot(c+d\,x)^3+\frac{4\,e\,a^3\,\cot(c+d\,x)^2}{3}+\frac{6\,e\,a^3\,\cot(c+d\,x)}{5}+\frac{2\,e\,a^3}{7}}{d\,e^2\,\left(e\cot\left(c+d\,x\right)\right)^{7/2}} + \frac{2\,\sqrt{2}\,a^3\,\mathrm{atanh}\left(\frac{32\,\sqrt{2}\,a^6\,d\,e^{9/2}\,\sqrt{e\cot(c+d\,x)}}{32\,a^6\,d\,e^5+32\,a^6\,d\,e^5\,\cot(c+d\,x)}\right)}{d\,e^{9/2}}$$

```
[In] int((a + a*cot(c + d*x))^3/(e*cot(c + d*x))^(9/2),x)
```

```
[Out] ((2*a^3*e)/7 + (4*a^3*e*cot(c + d*x)^2)/3 - 4*a^3*e*cot(c + d*x)^3 + (6*a^3
*e*cot(c + d*x))/5)/(d*e^2*(e*cot(c + d*x))^(7/2)) + (2*2^(1/2)*a^3*atanh((
32*2^(1/2)*a^6*d*e^(9/2)*(e*cot(c + d*x))^(1/2))/(32*a^6*d*e^5 + 32*a^6*d*e
^5*cot(c + d*x))))/(d*e^(9/2))
```

3.23 $\int \frac{(e\cot(c+dx))^{5/2}}{a+a\cot(c+dx)}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 111

$$\int \frac{(e\cot(c+dx))^{5/2}}{a+a\cot(c+dx)}\,dx = \frac{e^{5/2}\arctan\left(\frac{\sqrt{e\cot(c+dx)}}{\sqrt{e}}\right)}{ad} - \frac{e^{5/2}\arctan\left(\frac{\sqrt{e}-\sqrt{e}\cot(c+dx)}{\sqrt{2}\sqrt{e\cot(c+dx)}}\right)}{\sqrt{2}ad} - \frac{2e^2\sqrt{e\cot(c+dx)}}{ad}$$

```
[Out] e^(5/2)*arctan((e*cot(d*x+c))^(1/2)/e^(1/2))/a/d-1/2*e^(5/2)*arctan(1/2*(e^
(1/2)-cot(d*x+c)*e^(1/2))*2^(1/2)/(e*cot(d*x+c))^(1/2))/a/d*2^(1/2)-2*e^2*(
e*cot(d*x+c))^(1/2)/a/d
```

Rubi [A] (verified)

Time = 0.50 (sec) , antiderivative size = 111, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.240$, Rules used = {3647, 3734, 3613, 211, 3715, 65}

$$\int \frac{(e\cot(c+dx))^{5/2}}{a+a\cot(c+dx)}\,dx = \frac{e^{5/2}\arctan\left(\frac{\sqrt{e\cot(c+dx)}}{\sqrt{e}}\right)}{ad} - \frac{e^{5/2}\arctan\left(\frac{\sqrt{e}-\sqrt{e}\cot(c+dx)}{\sqrt{2}\sqrt{e\cot(c+dx)}}\right)}{\sqrt{2}ad} - \frac{2e^2\sqrt{e\cot(c+dx)}}{ad}$$

```
[In] Int[(e*Cot[c + d*x])^(5/2)/(a + a*Cot[c + d*x]),x]
```

```
[Out] (e^(5/2)*ArcTan[Sqrt[e*Cot[c + d*x]]/Sqrt[e]])/(a*d) - (e^(5/2)*ArcTan[(Sqr
t[e] - Sqrt[e]*Cot[c + d*x])/(Sqrt[2]*Sqrt[e*Cot[c + d*x]])])/(Sqrt[2]*a*d)
 - (2*e^2*Sqrt[e*Cot[c + d*x]])/(a*d)
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 3613

```
Int[((c_) + (d_.)*tan[(e_.) + (f_.)*(x_)])/Sqrt[(b_.)*tan[(e_.) + (f_.)*(x_
)]], x_Symbol] :> Dist[-2*(d^2/f), Subst[Int[1/(2*c*d + b*x^2), x], x, (c -
 d*Tan[e + f*x])/Sqrt[b*Tan[e + f*x]]], x] /; FreeQ[{b, c, d, e, f}, x] &&
EqQ[c^2 - d^2, 0]
```

Rule 3647

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)])^(n_), x_Symbol] :> Simp[b^2*(a + b*Tan[e + f*x])^(m - 2)*((c
+ d*Tan[e + f*x])^(n + 1)/(d*f*(m + n - 1))), x] + Dist[1/(d*(m + n - 1)),
Int[(a + b*Tan[e + f*x])^(m - 3)*(c + d*Tan[e + f*x])^n*Simp[a^3*d*(m + n -
 1) - b^2*(b*c*(m - 2) + a*d*(1 + n)) + b*d*(m + n - 1)*(3*a^2 - b^2)*Tan[e
 + f*x] - b^2*(b*c*(m - 2) - a*d*(3*m + 2*n - 4))*Tan[e + f*x]^2, x], x], x
] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2,
 0] && NeQ[c^2 + d^2, 0] && IntegerQ[2*m] && GtQ[m, 2] && (GeQ[n, -1] || In
tegerQ[m]) &&  !(IGtQ[n, 2] && ( !IntegerQ[m] || (EqQ[c, 0] && NeQ[a, 0])))
```

Rule 3715

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*tan[(e_.)
+ (f_.)*(x_)])^(n_.)*((A_) + (C_.)*tan[(e_.) + (f_.)*(x_)]^2), x_Symbol] :>
 Dist[A/f, Subst[Int[(a + b*x)^m*(c + d*x)^n, x], x, Tan[e + f*x]], x] /; F
reeQ[{a, b, c, d, e, f, A, C, m, n}, x] && EqQ[A, C]
```

Rule 3734

```
Int[(((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_)*((A_.) + (B_.)*tan[(e_.)
+ (f_.)*(x_)] + (C_.)*tan[(e_.) + (f_.)*(x_)]^2))/((a_.) + (b_.)*tan[(e_.)
+ (f_.)*(x_)]), x_Symbol] :> Dist[1/(a^2 + b^2), Int[(c + d*Tan[e + f*x])^n
*Simp[b*B + a*(A - C) + (a*B - b*(A - C))*Tan[e + f*x], x], x], x] + Dist[(
A*b^2 - a*b*B + a^2*C)/(a^2 + b^2), Int[(c + d*Tan[e + f*x])^n*((1 + Tan[e
```

```
+ f*x]^2)/(a + b*Tan[e + f*x])), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C
, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] &&
 !GtQ[n, 0] &&  !LeQ[n, -1]
```

Rubi steps

$$\text{integral} = -\frac{2e^2\sqrt{e\cot(c+dx)}}{ad} - \frac{2\int \frac{\frac{ae^3}{2}+\frac{1}{2}ae^3\cot(c+dx)+\frac{1}{2}ae^3\cot^2(c+dx)}{\sqrt{e\cot(c+dx)}(a+a\cot(c+dx))}\,dx}{a}$$

$$= -\frac{2e^2\sqrt{e\cot(c+dx)}}{ad} - \frac{\int \frac{\frac{a^2e^3}{2}+\frac{1}{2}a^2e^3\cot(c+dx)}{\sqrt{e\cot(c+dx)}}\,dx}{a^3} - \frac{1}{2}e^3\int \frac{1+\cot^2(c+dx)}{\sqrt{e\cot(c+dx)}(a+a\cot(c+dx))}\,dx$$

$$= -\frac{2e^2\sqrt{e\cot(c+dx)}}{ad} - \frac{e^3\text{Subst}\left(\int \frac{1}{\sqrt{-ex}(a-ax)}\,dx, x, -\cot(c+dx)\right)}{2d} + \frac{(ae^6)\,\text{Subst}\left(\int \frac{1}{-\frac{1}{2}a^4e^6-ex^2}\,dx, x, \frac{\frac{a^2e^3}{2}-\frac{1}{2}a^2e^3\cot(c+dx)}{\sqrt{e\cot(c+dx)}}\right)}{2d}$$

$$= -\frac{e^{5/2}\arctan\left(\frac{\sqrt{e}-\sqrt{e}\cot(c+dx)}{\sqrt{2}\sqrt{e\cot(c+dx)}}\right)}{\sqrt{2}ad} - \frac{2e^2\sqrt{e\cot(c+dx)}}{ad} + \frac{e^2\text{Subst}\left(\int \frac{1}{a+\frac{ax^2}{e}}\,dx, x, \sqrt{e\cot(c+dx)}\right)}{d}$$

$$= \frac{e^{5/2}\arctan\left(\frac{\sqrt{e\cot(c+dx)}}{\sqrt{e}}\right)}{ad} - \frac{e^{5/2}\arctan\left(\frac{\sqrt{e}-\sqrt{e}\cot(c+dx)}{\sqrt{2}\sqrt{e\cot(c+dx)}}\right)}{\sqrt{2}ad} - \frac{2e^2\sqrt{e\cot(c+dx)}}{ad}$$

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 296 vs. $2(111) = 222$.

Time = 0.92 (sec) , antiderivative size = 296, normalized size of antiderivative = 2.67

$$\int \frac{(e\cot(c+dx))^{5/2}}{a+a\cot(c+dx)}\,dx = \frac{e\left(8e^{3/2}\arctan\left(\frac{\sqrt{e\cot(c+dx)}}{\sqrt{e}}\right) - 4(-e^2)^{3/4}\arctan\left(\frac{\sqrt{e\cot(c+dx)}}{\sqrt[4]{-e^2}}\right) - 2\sqrt{2}e^{3/2}\arctan\left(\right.\right.}{}$$

```
[In] Integrate[(e*Cot[c + d*x])^(5/2)/(a + a*Cot[c + d*x]),x]
```

```
[Out] (e*(8*e^(3/2)*ArcTan[Sqrt[e*Cot[c + d*x]]/Sqrt[e]] - 4*(-e^2)^(3/4)*ArcTan[
Sqrt[e*Cot[c + d*x]]/(-e^2)^(1/4)] - 2*Sqrt[2]*e^(3/2)*ArcTan[1 - (Sqrt[2]*
Sqrt[e*Cot[c + d*x]])/Sqrt[e]] + 2*Sqrt[2]*e^(3/2)*ArcTan[1 + (Sqrt[2]*Sqrt
[e*Cot[c + d*x]])/Sqrt[e]] + 4*(-e^2)^(3/4)*ArcTanh[Sqrt[e*Cot[c + d*x]]/(-
e^2)^(1/4)] - 16*e*Sqrt[e*Cot[c + d*x]] - Sqrt[2]*e^(3/2)*Log[Sqrt[e] + Sqr
t[e]*Cot[c + d*x] - Sqrt[2]*Sqrt[e*Cot[c + d*x]]] + Sqrt[2]*e^(3/2)*Log[Sqr
t[e] + Sqrt[e]*Cot[c + d*x] + Sqrt[2]*Sqrt[e*Cot[c + d*x]]]))/(8*a*d)
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 311 vs. $2(92) = 184$.

Time = 0.10 (sec) , antiderivative size = 312, normalized size of antiderivative = 2.81

method	result
derivativedivides	$-\frac{2e^2\left(\sqrt{e\cot(dx+c)}-\frac{e\left(\frac{(e^2)^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\frac{e\cot(dx+c)+(e^2)^{\frac{1}{4}}\sqrt{e\cot(dx+c)}\sqrt{2}+\sqrt{e^2}}{e\cot(dx+c)-(e^2)^{\frac{1}{4}}\sqrt{e\cot(dx+c)}\sqrt{2}+\sqrt{e^2}}\right)+2\arctan\left(\frac{\sqrt{2}\sqrt{e\cot(dx+c)}}{(e^2)^{\frac{1}{4}}}+1\right)-2\operatorname{arc}\right.}{8e}}{}\right.}{}$
default	$-\frac{2e^2\left(\sqrt{e\cot(dx+c)}-\frac{e\left(\frac{(e^2)^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\frac{e\cot(dx+c)+(e^2)^{\frac{1}{4}}\sqrt{e\cot(dx+c)}\sqrt{2}+\sqrt{e^2}}{e\cot(dx+c)-(e^2)^{\frac{1}{4}}\sqrt{e\cot(dx+c)}\sqrt{2}+\sqrt{e^2}}\right)+2\arctan\left(\frac{\sqrt{2}\sqrt{e\cot(dx+c)}}{(e^2)^{\frac{1}{4}}}+1\right)-2\operatorname{arc}\right.}{8e}}{}\right.}{}$

```
[In] int((e*cot(d*x+c))^(5/2)/(a+a*cot(d*x+c)),x,method=_RETURNVERBOSE)

[Out] -2/d/a*e^2*((e*cot(d*x+c))^(1/2)-1/2*e*(1/8/e*(e^2)^(1/4)*2^(1/2)*(ln((e*co
t(d*x+c)+(e^2)^(1/4)*(e*cot(d*x+c))^(1/2)*2^(1/2)+(e^2)^(1/2))/(e*cot(d*x+c
)-(e^2)^(1/4)*(e*cot(d*x+c))^(1/2)*2^(1/2)+(e^2)^(1/2)))+2*arctan(2^(1/2)/(
e^2)^(1/4)*(e*cot(d*x+c))^(1/2)+1)-2*arctan(-2^(1/2)/(e^2)^(1/4)*(e*cot(d*x
+c))^(1/2)+1))+1/8/(e^2)^(1/4)*2^(1/2)*(ln((e*cot(d*x+c)-(e^2)^(1/4)*(e*cot
(d*x+c))^(1/2)*2^(1/2)+(e^2)^(1/2))/(e*cot(d*x+c)+(e^2)^(1/4)*(e*cot(d*x+c)
)^(1/2)*2^(1/2)+(e^2)^(1/2)))+2*arctan(2^(1/2)/(e^2)^(1/4)*(e*cot(d*x+c))^(
1/2)+1)-2*arctan(-2^(1/2)/(e^2)^(1/4)*(e*cot(d*x+c))^(1/2)+1)))-1/2*e^(1/2)
*arctan((e*cot(d*x+c))^(1/2)/e^(1/2)))
```

Fricas [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 400, normalized size of antiderivative = 3.60

$$\int \frac{(e\cot(c+dx))^{5/2}}{a+a\cot(c+dx)}\,dx = \left[\frac{\sqrt{2}\sqrt{-e}e^2\log\left(\left(\sqrt{2}\cos\left(2\,dx+2\,c\right)+\sqrt{2}\sin\left(2\,dx+2\,c\right)-\sqrt{2}\right)\sqrt{-e}\sqrt{\frac{e\cos(2\,dx+2}{\sin(2\,dx+2}}\right.}{}\right.$$

$$-\frac{\sqrt{2}e^{\frac{5}{2}}\arctan\left(-\frac{\left(\sqrt{2}\cos(2\,dx+2\,c)-\sqrt{2}\sin(2\,dx+2\,c)+\sqrt{2}\right)\sqrt{e}\sqrt{\frac{e\cos(2\,dx+2\,c)+e}{\sin(2\,dx+2\,c)}}}{2\,(e\cos(2\,dx+2\,c)+e)}\right)-2\,e^{\frac{5}{2}}\arctan\left(\frac{\sqrt{\frac{e\cos(2\,dx+2\,c)+e}{\sin(2\,dx+2\,c)}}}{\sqrt{e}}\right)+4\,e^2\sqrt{}}{2\,ad}$$

```
[In] integrate((e*cot(d*x+c))^(5/2)/(a+a*cot(d*x+c)),x, algorithm="fricas")
```

```
[Out] [1/4*(sqrt(2)*sqrt(-e)*e^2*log((sqrt(2)*cos(2*d*x + 2*c) + sqrt(2)*sin(2*d*
x + 2*c) - sqrt(2))*sqrt(-e)*sqrt((e*cos(2*d*x + 2*c) + e)/sin(2*d*x + 2*c)
) - 2*e*sin(2*d*x + 2*c) + e) + 2*sqrt(-e)*e^2*log((e*cos(2*d*x + 2*c) - e*
sin(2*d*x + 2*c) + 2*sqrt(-e)*sqrt((e*cos(2*d*x + 2*c) + e)/sin(2*d*x + 2*c
))*sin(2*d*x + 2*c) + e)/(cos(2*d*x + 2*c) + sin(2*d*x + 2*c) + 1)) - 8*e^2
*sqrt((e*cos(2*d*x + 2*c) + e)/sin(2*d*x + 2*c)))/(a*d), -1/2*(sqrt(2)*e^(5
/2)*arctan(-1/2*(sqrt(2)*cos(2*d*x + 2*c) - sqrt(2)*sin(2*d*x + 2*c) + sqrt
(2))*sqrt(e)*sqrt((e*cos(2*d*x + 2*c) + e)/sin(2*d*x + 2*c))/(e*cos(2*d*x +
 2*c) + e)) - 2*e^(5/2)*arctan(sqrt((e*cos(2*d*x + 2*c) + e)/sin(2*d*x + 2*
c))/sqrt(e)) + 4*e^2*sqrt((e*cos(2*d*x + 2*c) + e)/sin(2*d*x + 2*c)))/(a*d)
]
```

Sympy [F]

$$\int \frac{(e\cot(c+dx))^{5/2}}{a+a\cot(c+dx)}\,dx = \frac{\int \frac{(e\cot{(c+dx)})^{\frac{5}{2}}}{\cot{(c+dx)}+1}\,dx}{a}$$

```
[In] integrate((e*cot(d*x+c))**(5/2)/(a+a*cot(d*x+c)),x)
```

```
[Out] Integral((e*cot(c + d*x))**(5/2)/(cot(c + d*x) + 1), x)/a
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{(e\cot(c+dx))^{5/2}}{a+a\cot(c+dx)}\,dx = \text{Exception raised: ValueError}$$

[In] integrate((e*cot(d*x+c))^(5/2)/(a+a*cot(d*x+c)),x, algorithm="maxima")

[Out] Exception raised: ValueError >> Computation failed since Maxima requested a dditional constraints; using the 'assume' command before evaluation *may* h elp (example of legal syntax is 'assume(e>0)', see `assume?` for more detai ls)Is e

Giac [F]

$$\int \frac{(e\cot(c+dx))^{5/2}}{a+a\cot(c+dx)}\,dx = \int \frac{(e\cot(dx+c))^{\frac{5}{2}}}{a\cot(dx+c)+a}\,dx$$

[In] integrate((e*cot(d*x+c))^(5/2)/(a+a*cot(d*x+c)),x, algorithm="giac")

[Out] integrate((e*cot(d*x + c))^(5/2)/(a*cot(d*x + c) + a), x)

Mupad [B] (verification not implemented)

Time = 12.93 (sec) , antiderivative size = 123, normalized size of antiderivative = 1.11

$$\int \frac{(e\cot(c+dx))^{5/2}}{a+a\cot(c+dx)}\,dx = \frac{e^{5/2}\,\mathrm{atan}\left(\frac{\sqrt{e\cot(c+d\,x)}}{\sqrt{e}}\right)}{a\,d} - \frac{2\,e^2\,\sqrt{e\cot(c+d\,x)}}{a\,d} + \frac{\sqrt{2}\,e^{5/2}\left(2\,\mathrm{atan}\left(\frac{\sqrt{2}\,\sqrt{e\cot(c+d\,x)}}{2\,\sqrt{e}}\right)+2\,\mathrm{atan}\left(\frac{\sqrt{2}\,\sqrt{e\cot(c+d\,x)}}{2\,\sqrt{e}}+\frac{\sqrt{2}\,(e\cot(c+d\,x))^{3/2}}{2\,e^{3/2}}\right)\right)}{4\,a\,d}$$

[In] int((e*cot(c + d*x))^(5/2)/(a + a*cot(c + d*x)),x)

[Out] (e^(5/2)*atan((e*cot(c + d*x))^(1/2)/e^(1/2)))/(a*d) - (2*e^2*(e*cot(c + d* x))^(1/2))/(a*d) + (2^(1/2)*e^(5/2)*(2*atan((2^(1/2)*(e*cot(c + d*x))^(1/2))/(2*e^(1/2))) + 2*atan((2^(1/2)*(e*cot(c + d*x))^(1/2))/(2*e^(1/2)) + (2^(1/2)*(e*cot(c + d*x))^(3/2))/(2*e^(3/2)))))/(4*a*d)

3.24 $\int \frac{(e\cot(c+dx))^{3/2}}{a+a\cot(c+dx)}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 87

$$\int \frac{(e\cot(c+dx))^{3/2}}{a+a\cot(c+dx)}\,dx=-\frac{e^{3/2}\arctan\left(\frac{\sqrt{e\cot(c+dx)}}{\sqrt{e}}\right)}{ad}+\frac{e^{3/2}\text{arctanh}\left(\frac{\sqrt{e}+\sqrt{e}\cot(c+dx)}{\sqrt{2}\sqrt{e\cot(c+dx)}}\right)}{\sqrt{2}ad}$$

```
[Out] -e^(3/2)*arctan((e*cot(d*x+c))^(1/2)/e^(1/2))/a/d+1/2*e^(3/2)*arctanh(1/2*(
e^(1/2)+cot(d*x+c)*e^(1/2))*2^(1/2)/(e*cot(d*x+c))^(1/2))/a/d*2^(1/2)
```

Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 87, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.240, Rules used = {3654, 3613, 214, 3715, 65, 211}

$$\int \frac{(e\cot(c+dx))^{3/2}}{a+a\cot(c+dx)}\,dx=\frac{e^{3/2}\text{arctanh}\left(\frac{\sqrt{e}\cot(c+dx)+\sqrt{e}}{\sqrt{2}\sqrt{e\cot(c+dx)}}\right)}{\sqrt{2}ad}-\frac{e^{3/2}\arctan\left(\frac{\sqrt{e\cot(c+dx)}}{\sqrt{e}}\right)}{ad}$$

```
[In] Int[(e*Cot[c + d*x])^(3/2)/(a + a*Cot[c + d*x]),x]
```

```
[Out] -((e^(3/2)*ArcTan[Sqrt[e*Cot[c + d*x]]/Sqrt[e]])/(a*d)) + (e^(3/2)*ArcTanh[
(Sqrt[e] + Sqrt[e]*Cot[c + d*x])/(Sqrt[2]*Sqrt[e*Cot[c + d*x]])])/(Sqrt[2]*
a*d)
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
```

```
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 3613

```
Int[((c_) + (d_.)*tan[(e_.) + (f_.)*(x_)])/Sqrt[(b_.)*tan[(e_.) + (f_.)*(x_
)]], x_Symbol] :> Dist[-2*(d^2/f), Subst[Int[1/(2*c*d + b*x^2), x], x, (c -
 d*Tan[e + f*x])/Sqrt[b*Tan[e + f*x]]], x] /; FreeQ[{b, c, d, e, f}, x] &&
EqQ[c^2 - d^2, 0]
```

Rule 3654

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(3/2)/((c_.) + (d_.)*tan[(e_.)
+ (f_.)*(x_)]), x_Symbol] :> Dist[1/(c^2 + d^2), Int[Simp[a^2*c - b^2*c + 2
*a*b*d + (2*a*b*c - a^2*d + b^2*d)*Tan[e + f*x], x]/Sqrt[a + b*Tan[e + f*x]
], x], x] + Dist[(b*c - a*d)^2/(c^2 + d^2), Int[(1 + Tan[e + f*x]^2)/(Sqrt[
a + b*Tan[e + f*x]]*(c + d*Tan[e + f*x])), x], x] /; FreeQ[{a, b, c, d, e,
f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0]
```

Rule 3715

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*tan[(e_.)
+ (f_.)*(x_)])^(n_.)*((A_) + (C_.)*tan[(e_.) + (f_.)*(x_)]^2), x_Symbol] :>
 Dist[A/f, Subst[Int[(a + b*x)^m*(c + d*x)^n, x], x, Tan[e + f*x]], x] /; F
reeQ[{a, b, c, d, e, f, A, C, m, n}, x] && EqQ[A, C]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{\int \frac{-ae^2+ae^2\cot(c+dx)}{\sqrt{e\cot(c+dx)}}\,dx}{2a^2}+\frac{1}{2}e^2\int \frac{1+\cot^2(c+dx)}{\sqrt{e\cot(c+dx)}(a+a\cot(c+dx))}\,dx\\
&= \frac{e^2\text{Subst}\left(\int \frac{1}{\sqrt{-ex}(a-ax)}\,dx,x,-\cot(c+dx)\right)}{2d}-\frac{e^4\text{Subst}\left(\int \frac{1}{2a^2e^4-ex^2}\,dx,x,\frac{-ae^2-ae^2\cot(c+dx)}{\sqrt{e\cot(c+dx)}}\right)}{d}\\
&= \frac{e^{3/2}\text{arctanh}\left(\frac{\sqrt{e}+\sqrt{e}\cot(c+dx)}{\sqrt{2}\sqrt{e\cot(c+dx)}}\right)}{\sqrt{2}ad}-\frac{e\text{Subst}\left(\int \frac{1}{a+\frac{ax^2}{e}}\,dx,x,\sqrt{e\cot(c+dx)}\right)}{d}
\end{aligned}$$

$$= -\frac{e^{3/2}\arctan\left(\frac{\sqrt{e\cot(c+dx)}}{\sqrt{e}}\right)}{ad} + \frac{e^{3/2}\text{arctanh}\left(\frac{\sqrt{e}+\sqrt{e}\cot(c+dx)}{\sqrt{2}\sqrt{e\cot(c+dx)}}\right)}{\sqrt{2}ad}$$

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 280 vs. $2(87) = 174$.

Time = 0.55 (sec) , antiderivative size = 280, normalized size of antiderivative = 3.22

$$\int \frac{(e\cot(c+dx))^{3/2}}{a+a\cot(c+dx)}\,dx =$$

$$-\frac{8e^{3/2}\arctan\left(\frac{\sqrt{e\cot(c+dx)}}{\sqrt{e}}\right) - 4(-e^2)^{3/4}\arctan\left(\frac{\sqrt{e\cot(c+dx)}}{\sqrt[4]{-e^2}}\right) + 2\sqrt{2}e^{3/2}\arctan\left(1-\frac{\sqrt{2}\sqrt{e\cot(c+dx)}}{\sqrt{e}}\right) - 2\sqrt{2}e}{}$$

```
[In] Integrate[(e*Cot[c + d*x])^(3/2)/(a + a*Cot[c + d*x]),x]
```

```
[Out] -1/8*(8*e^(3/2)*ArcTan[Sqrt[e*Cot[c + d*x]]/Sqrt[e]] - 4*(-e^2)^(3/4)*ArcTa
n[Sqrt[e*Cot[c + d*x]]/(-e^2)^(1/4)] + 2*Sqrt[2]*e^(3/2)*ArcTan[1 - (Sqrt[2
]*Sqrt[e*Cot[c + d*x]])/Sqrt[e]] - 2*Sqrt[2]*e^(3/2)*ArcTan[1 + (Sqrt[2]*Sq
rt[e*Cot[c + d*x]])/Sqrt[e]] + 4*(-e^2)^(3/4)*ArcTanh[Sqrt[e*Cot[c + d*x]]/
(-e^2)^(1/4)] + Sqrt[2]*e^(3/2)*Log[Sqrt[e] + Sqrt[e]*Cot[c + d*x] - Sqrt[2
]*Sqrt[e*Cot[c + d*x]]] - Sqrt[2]*e^(3/2)*Log[Sqrt[e] + Sqrt[e]*Cot[c + d*x
] + Sqrt[2]*Sqrt[e*Cot[c + d*x]]])/(a*d)
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 297 vs. $2(71) = 142$.

Time = 0.05 (sec) , antiderivative size = 298, normalized size of antiderivative = 3.43

method	result
derivativedivides	$-\frac{2e^2\left(-\frac{(e^2)^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\frac{e\cot(dx+c)+(e^2)^{\frac{1}{4}}\sqrt{e\cot(dx+c)}\sqrt{2}+\sqrt{e^2}}{e\cot(dx+c)-(e^2)^{\frac{1}{4}}\sqrt{e\cot(dx+c)}\sqrt{2}+\sqrt{e^2}}\right)+2\arctan\left(\frac{\sqrt{2}\sqrt{e\cot(dx+c)}}{(e^2)^{\frac{1}{4}}}+1\right)-2\arctan\left(-\frac{\sqrt{2}\sqrt{e\cot(dx+}}{(e^2)^{\frac{1}{4}}}\right.\right.}{16e}\right.}{}$
default	$-\frac{2e^2\left(-\frac{(e^2)^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\frac{e\cot(dx+c)+(e^2)^{\frac{1}{4}}\sqrt{e\cot(dx+c)}\sqrt{2}+\sqrt{e^2}}{e\cot(dx+c)-(e^2)^{\frac{1}{4}}\sqrt{e\cot(dx+c)}\sqrt{2}+\sqrt{e^2}}\right)+2\arctan\left(\frac{\sqrt{2}\sqrt{e\cot(dx+c)}}{(e^2)^{\frac{1}{4}}}+1\right)-2\arctan\left(-\frac{\sqrt{2}\sqrt{e\cot(dx+}}{(e^2)^{\frac{1}{4}}}\right.\right.}{16e}\right.}{}$

```
[In] int((e*cot(d*x+c))^(3/2)/(a+a*cot(d*x+c)),x,method=_RETURNVERBOSE)
```

```
[Out] -2/d/a*e^2*(-1/16/e*(e^2)^(1/4)*2^(1/2)*(ln((e*cot(d*x+c)+(e^2)^(1/4)*(e*co
t(d*x+c))^(1/2)*2^(1/2)+(e^2)^(1/2))/(e*cot(d*x+c)-(e^2)^(1/4)*(e*cot(d*x+c
))^(1/2)*2^(1/2)+(e^2)^(1/2)))+2*arctan(2^(1/2)/(e^2)^(1/4)*(e*cot(d*x+c))^
(1/2)+1)-2*arctan(-2^(1/2)/(e^2)^(1/4)*(e*cot(d*x+c))^(1/2)+1))+1/16/(e^2)^
(1/4)*2^(1/2)*(ln((e*cot(d*x+c)-(e^2)^(1/4)*(e*cot(d*x+c))^(1/2)*2^(1/2)+(e
^2)^(1/2))/(e*cot(d*x+c)+(e^2)^(1/4)*(e*cot(d*x+c))^(1/2)*2^(1/2)+(e^2)^(1/
2)))+2*arctan(2^(1/2)/(e^2)^(1/4)*(e*cot(d*x+c))^(1/2)+1)-2*arctan(-2^(1/2)
/(e^2)^(1/4)*(e*cot(d*x+c))^(1/2)+1))+1/2/e^(1/2)*arctan((e*cot(d*x+c))^(1/
2)/e^(1/2)))
```

Fricas [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 333, normalized size of antiderivative = 3.83

$$\int \frac{(e\cot(c+dx))^{3/2}}{a+a\cot(c+dx)}\,dx = \left[-\frac{\sqrt{2}\sqrt{-e}e\arctan\left(\frac{\left(\sqrt{2}\cos(2\,dx+2\,c)+\sqrt{2}\sin(2\,dx+2\,c)+\sqrt{2}\right)\sqrt{-e}\sqrt{\frac{e\cos(2\,dx+2\,c)+e}{\sin(2\,dx+2\,c)}}}{2\,(e\cos(2\,dx+2\,c)+e)}\right)-\sqrt{-e}e\cdots}{2\,ad}\right.$$

```
[In] integrate((e*cot(d*x+c))^(3/2)/(a+a*cot(d*x+c)),x, algorithm="fricas")
```

```
[Out] [-1/2*(sqrt(2)*sqrt(-e)*e*arctan(1/2*(sqrt(2)*cos(2*d*x + 2*c) + sqrt(2)*si
n(2*d*x + 2*c) + sqrt(2))*sqrt(-e)*sqrt((e*cos(2*d*x + 2*c) + e)/sin(2*d*x
+ 2*c))/(e*cos(2*d*x + 2*c) + e)) - sqrt(-e)*e*log((e*cos(2*d*x + 2*c) - e*
sin(2*d*x + 2*c) - 2*sqrt(-e)*sqrt((e*cos(2*d*x + 2*c) + e)/sin(2*d*x + 2*c
))*sin(2*d*x + 2*c) + e)/(cos(2*d*x + 2*c) + sin(2*d*x + 2*c) + 1)))/(a*d),
 1/4*(sqrt(2)*e^(3/2)*log(-(sqrt(2)*cos(2*d*x + 2*c) - sqrt(2)*sin(2*d*x +
2*c) - sqrt(2))*sqrt(e)*sqrt((e*cos(2*d*x + 2*c) + e)/sin(2*d*x + 2*c)) + 2
*e*sin(2*d*x + 2*c) + e) - 4*e^(3/2)*arctan(sqrt((e*cos(2*d*x + 2*c) + e)/s
in(2*d*x + 2*c))/sqrt(e)))/(a*d)]
```

Sympy [F]

$$\int \frac{(e\cot(c+dx))^{3/2}}{a+a\cot(c+dx)}\,dx = \frac{\int \frac{(e\cot(c+dx))^{\frac{3}{2}}}{\cot(c+dx)+1}\,dx}{a}$$

```
[In] integrate((e*cot(d*x+c))**(3/2)/(a+a*cot(d*x+c)),x)
```

```
[Out] Integral((e*cot(c + d*x))**(3/2)/(cot(c + d*x) + 1), x)/a
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{(e\cot(c+dx))^{3/2}}{a+a\cot(c+dx)}\,dx = \text{Exception raised: ValueError}$$

```
[In] integrate((e*cot(d*x+c))^(3/2)/(a+a*cot(d*x+c)),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(e>0)', see `assume?` for more detai
ls)Is e
```

Giac [F]

$$\int \frac{(e\cot(c+dx))^{3/2}}{a+a\cot(c+dx)}\,dx = \int \frac{(e\cot(dx+c))^{\frac{3}{2}}}{a\cot(dx+c)+a}\,dx$$

```
[In] integrate((e*cot(d*x+c))^(3/2)/(a+a*cot(d*x+c)),x, algorithm="giac")
```

```
[Out] integrate((e*cot(d*x + c))^(3/2)/(a*cot(d*x + c) + a), x)
```

Mupad [B] (verification not implemented)

Time = 12.78 (sec) , antiderivative size = 79, normalized size of antiderivative = 0.91

$$\int \frac{(e\cot(c+dx))^{3/2}}{a+a\cot(c+dx)}\,dx = \frac{\sqrt{2}\,e^{3/2}\,\mathrm{atanh}\left(\frac{12\sqrt{2}\,e^{25/2}\,\sqrt{e\cot(c+d\,x)}}{12\,e^{13}\cot(c+d\,x)+12\,e^{13}}\right)}{2\,a\,d} - \frac{e^{3/2}\,\mathrm{atan}\left(\frac{\sqrt{e\cot(c+d\,x)}}{\sqrt{e}}\right)}{a\,d}$$

```
[In] int((e*cot(c + d*x))^(3/2)/(a + a*cot(c + d*x)),x)
```

```
[Out] (2^(1/2)*e^(3/2)*atanh((12*2^(1/2)*e^(25/2)*(e*cot(c + d*x))^(1/2))/(12*e^1
3*cot(c + d*x) + 12*e^13)))/(2*a*d) - (e^(3/2)*atan((e*cot(c + d*x))^(1/2)/
e^(1/2)))/(a*d)
```

3.25 $\int \frac{\sqrt{e\cot(c+dx)}}{a+a\cot(c+dx)}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 87

$$\int \frac{\sqrt{e\cot(c+dx)}}{a+a\cot(c+dx)}\,dx=\frac{\sqrt{e}\arctan\left(\frac{\sqrt{e\cot(c+dx)}}{\sqrt{e}}\right)}{ad}+\frac{\sqrt{e}\arctan\left(\frac{\sqrt{e}-\sqrt{e}\cot(c+dx)}{\sqrt{2}\sqrt{e\cot(c+dx)}}\right)}{\sqrt{2}ad}$$

```
[Out] arctan((e*cot(d*x+c))^(1/2)/e^(1/2))*e^(1/2)/a/d+1/2*arctan(1/2*(e^(1/2)-co
t(d*x+c)*e^(1/2))*2^(1/2)/(e*cot(d*x+c))^(1/2))*e^(1/2)/a/d*2^(1/2)
```

Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 87, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {3653, 3613, 211, 3715, 65}

$$\int \frac{\sqrt{e\cot(c+dx)}}{a+a\cot(c+dx)}\,dx=\frac{\sqrt{e}\arctan\left(\frac{\sqrt{e\cot(c+dx)}}{\sqrt{e}}\right)}{ad}+\frac{\sqrt{e}\arctan\left(\frac{\sqrt{e}-\sqrt{e}\cot(c+dx)}{\sqrt{2}\sqrt{e\cot(c+dx)}}\right)}{\sqrt{2}ad}$$

```
[In] Int[Sqrt[e*Cot[c + d*x]]/(a + a*Cot[c + d*x]),x]
```

```
[Out] (Sqrt[e]*ArcTan[Sqrt[e*Cot[c + d*x]]/Sqrt[e]])/(a*d) + (Sqrt[e]*ArcTan[(Sqr
t[e] - Sqrt[e]*Cot[c + d*x])/(Sqrt[2]*Sqrt[e*Cot[c + d*x]])])/(Sqrt[2]*a*d)
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
```

```
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 3613

```
Int[((c_) + (d_.)*tan[(e_.) + (f_.)*(x_)])/Sqrt[(b_.)*tan[(e_.) + (f_.)*(x_
)]], x_Symbol] :> Dist[-2*(d^2/f), Subst[Int[1/(2*c*d + b*x^2), x], x, (c -
 d*Tan[e + f*x])/Sqrt[b*Tan[e + f*x]]], x] /; FreeQ[{b, c, d, e, f}, x] &&
EqQ[c^2 - d^2, 0]
```

Rule 3653

```
Int[Sqrt[(a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)]]/((c_.) + (d_.)*tan[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Dist[1/(c^2 + d^2), Int[Simp[a*c + b*d + (b*c -
a*d)*Tan[e + f*x], x]/Sqrt[a + b*Tan[e + f*x]], x], x] - Dist[d*((b*c - a*d
)/(c^2 + d^2)), Int[(1 + Tan[e + f*x]^2)/(Sqrt[a + b*Tan[e + f*x]]*(c + d*T
an[e + f*x])), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && NeQ
[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0]
```

Rule 3715

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*tan[(e_.)
+ (f_.)*(x_)])^(n_.)*((A_) + (C_.)*tan[(e_.) + (f_.)*(x_)]^2), x_Symbol] :>
 Dist[A/f, Subst[Int[(a + b*x)^m*(c + d*x)^n, x], x, Tan[e + f*x]], x] /; F
reeQ[{a, b, c, d, e, f, A, C, m, n}, x] && EqQ[A, C]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{\int \frac{ae+ae\cot(c+dx)}{\sqrt{e\cot(c+dx)}}\,dx}{2a^2} - \frac{1}{2}e\int \frac{1+\cot^2(c+dx)}{\sqrt{e\cot(c+dx)}(a+a\cot(c+dx))}\,dx \\
&= -\frac{e\text{Subst}\left(\int \frac{1}{\sqrt{-ex}(a-ax)}\,dx, x, -\cot(c+dx)\right)}{2d} - \frac{e^2\text{Subst}\left(\int \frac{1}{-2a^2e^2-ex^2}\,dx, x, \frac{ae-ae\cot(c+dx)}{\sqrt{e\cot(c+dx)}}\right)}{d} \\
&= \frac{\sqrt{e}\arctan\left(\frac{\sqrt{e}-\sqrt{e}\cot(c+dx)}{\sqrt{2}\sqrt{e\cot(c+dx)}}\right)}{\sqrt{2}ad} + \frac{\text{Subst}\left(\int \frac{1}{a+\frac{ax^2}{e}}\,dx, x, \sqrt{e\cot(c+dx)}\right)}{d} \\
&= \frac{\sqrt{e}\arctan\left(\frac{\sqrt{e\cot(c+dx)}}{\sqrt{e}}\right)}{ad} + \frac{\sqrt{e}\arctan\left(\frac{\sqrt{e}-\sqrt{e}\cot(c+dx)}{\sqrt{2}\sqrt{e\cot(c+dx)}}\right)}{\sqrt{2}ad}
\end{aligned}
$$

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 283 vs. 2(87) = 174.

Time = 0.43 (sec) , antiderivative size = 283, normalized size of antiderivative = 3.25

$$\int \frac{\sqrt{e\cot(c+dx)}}{a+a\cot(c+dx)}\,dx$$
$$= \frac{8e^{3/2}\arctan\left(\frac{\sqrt{e\cot(c+dx)}}{\sqrt{e}}\right)+4(-e^2)^{3/4}\arctan\left(\frac{\sqrt{e\cot(c+dx)}}{\sqrt[4]{-e^2}}\right)+2\sqrt{2}e^{3/2}\arctan\left(1-\frac{\sqrt{2}\sqrt{e\cot(c+dx)}}{\sqrt{e}}\right)-2\sqrt{}}{}$$

```
[In] Integrate[Sqrt[e*Cot[c + d*x]]/(a + a*Cot[c + d*x]),x]
```

```
[Out] (8*e^(3/2)*ArcTan[Sqrt[e*Cot[c + d*x]]/Sqrt[e]] + 4*(-e^2)^(3/4)*ArcTan[Sqr
t[e*Cot[c + d*x]]/(-e^2)^(1/4)] + 2*Sqrt[2]*e^(3/2)*ArcTan[1 - (Sqrt[2]*Sqr
t[e*Cot[c + d*x]])/Sqrt[e]] - 2*Sqrt[2]*e^(3/2)*ArcTan[1 + (Sqrt[2]*Sqrt[e*
Cot[c + d*x]])/Sqrt[e]] - 4*(-e^2)^(3/4)*ArcTanh[Sqrt[e*Cot[c + d*x]]/(-e^2
)^(1/4)] + Sqrt[2]*e^(3/2)*Log[Sqrt[e] + Sqrt[e]*Cot[c + d*x] - Sqrt[2]*Sqr
t[e*Cot[c + d*x]]] - Sqrt[2]*e^(3/2)*Log[Sqrt[e] + Sqrt[e]*Cot[c + d*x] + S
qrt[2]*Sqrt[e*Cot[c + d*x]]])/(8*a*d*e)
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 303 vs. 2(71) = 142.

Time = 0.05 (sec) , antiderivative size = 304, normalized size of antiderivative = 3.49

method	result
derivativedivides	$-\frac{2e^2\left(\frac{(e^2)^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\frac{e\cot(dx+c)+(e^2)^{\frac{1}{4}}\sqrt{e\cot(dx+c)}\sqrt{2}+\sqrt{e^2}}{e\cot(dx+c)-(e^2)^{\frac{1}{4}}\sqrt{e\cot(dx+c)}\sqrt{2}+\sqrt{e^2}}\right)+2\arctan\left(\frac{\sqrt{2}\sqrt{e\cot(dx+c)}}{(e^2)^{\frac{1}{4}}}+1\right)-2\arctan\left(-\frac{\sqrt{2}\sqrt{e\cot(dx}}{(e^2)^{\frac{1}{4}}}\right.\right.}{8e}\right.}{}$
default	$-\frac{2e^2\left(\frac{(e^2)^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\frac{e\cot(dx+c)+(e^2)^{\frac{1}{4}}\sqrt{e\cot(dx+c)}\sqrt{2}+\sqrt{e^2}}{e\cot(dx+c)-(e^2)^{\frac{1}{4}}\sqrt{e\cot(dx+c)}\sqrt{2}+\sqrt{e^2}}\right)+2\arctan\left(\frac{\sqrt{2}\sqrt{e\cot(dx+c)}}{(e^2)^{\frac{1}{4}}}+1\right)-2\arctan\left(-\frac{\sqrt{2}\sqrt{e\cot(dx}}{(e^2)^{\frac{1}{4}}}\right.\right.}{8e}\right.}{}$

```
[In] int((e*cot(d*x+c))^(1/2)/(a+a*cot(d*x+c)),x,method=_RETURNVERBOSE)
```

```
[Out] -2/d/a*e^2*(1/2/e*(1/8/e*(e^2)^(1/4)*2^(1/2)*(ln((e*cot(d*x+c)+(e^2)^(1/4)*
(e*cot(d*x+c))^(1/2)*2^(1/2)+(e^2)^(1/2))/(e*cot(d*x+c)-(e^2)^(1/4)*(e*cot(
d*x+c))^(1/2)*2^(1/2)+(e^2)^(1/2)))+2*arctan(2^(1/2)/(e^2)^(1/4)*(e*cot(d*x
+c))^(1/2)+1)-2*arctan(-2^(1/2)/(e^2)^(1/4)*(e*cot(d*x+c))^(1/2)+1))+1/8/(e
^2)^(1/4)*2^(1/2)*(ln((e*cot(d*x+c)-(e^2)^(1/4)*(e*cot(d*x+c))^(1/2)*2^(1/2
)+(e^2)^(1/2))/(e*cot(d*x+c)+(e^2)^(1/4)*(e*cot(d*x+c))^(1/2)*2^(1/2)+(e^2)
^(1/2)))+2*arctan(2^(1/2)/(e^2)^(1/4)*(e*cot(d*x+c))^(1/2)+1)-2*arctan(-2^(
1/2)/(e^2)^(1/4)*(e*cot(d*x+c))^(1/2)+1)))-1/2/e^(3/2)*arctan((e*cot(d*x+c)
)^(1/2)/e^(1/2)))
```

Fricas [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 331, normalized size of antiderivative = 3.80

$$\int \frac{\sqrt{e\cot(c+dx)}}{a+a\cot(c+dx)}\,dx$$
$$= \left[\frac{\sqrt{2}\sqrt{-e}\log\left(-\left(\sqrt{2}\cos\left(2\,dx+2\,c\right)+\sqrt{2}\sin\left(2\,dx+2\,c\right)-\sqrt{2}\right)\sqrt{-e}\sqrt{\frac{e\cos(2\,dx+2\,c)+e}{\sin(2\,dx+2\,c)}}-2\,e\sin\left(2\,dx+2\,c\right)\right.}{4\,ad}\right.$$

```
[In] integrate((e*cot(d*x+c))^(1/2)/(a+a*cot(d*x+c)),x, algorithm="fricas")
```

```
[Out] [1/4*(sqrt(2)*sqrt(-e)*log(-(sqrt(2)*cos(2*d*x + 2*c) + sqrt(2)*sin(2*d*x +
 2*c) - sqrt(2))*sqrt(-e)*sqrt((e*cos(2*d*x + 2*c) + e)/sin(2*d*x + 2*c)) -
 2*e*sin(2*d*x + 2*c) + e) + 2*sqrt(-e)*log((e*cos(2*d*x + 2*c) - e*sin(2*d
*x + 2*c) + 2*sqrt(-e)*sqrt((e*cos(2*d*x + 2*c) + e)/sin(2*d*x + 2*c))*sin(
2*d*x + 2*c) + e)/(cos(2*d*x + 2*c) + sin(2*d*x + 2*c) + 1)))/(a*d), 1/2*(s
qrt(2)*sqrt(e)*arctan(-1/2*(sqrt(2)*cos(2*d*x + 2*c) - sqrt(2)*sin(2*d*x +
2*c) + sqrt(2))*sqrt(e)*sqrt((e*cos(2*d*x + 2*c) + e)/sin(2*d*x + 2*c))/(e*
cos(2*d*x + 2*c) + e)) + 2*sqrt(e)*arctan(sqrt((e*cos(2*d*x + 2*c) + e)/sin
(2*d*x + 2*c))/sqrt(e)))/(a*d)]
```

Sympy [F]

$$\int \frac{\sqrt{e\cot(c+dx)}}{a+a\cot(c+dx)}\,dx = \frac{\int \frac{\sqrt{e\cot{(c+dx)}}}{\cot{(c+dx)}+1}\,dx}{a}$$

```
[In] integrate((e*cot(d*x+c))**(1/2)/(a+a*cot(d*x+c)),x)
```

```
[Out] Integral(sqrt(e*cot(c + d*x))/(cot(c + d*x) + 1), x)/a
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{\sqrt{e\cot(c+dx)}}{a+a\cot(c+dx)}\,dx = \text{Exception raised: ValueError}$$

```
[In] integrate((e*cot(d*x+c))^(1/2)/(a+a*cot(d*x+c)),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(e>0)', see `assume?` for more detai
ls)Is e
```

Giac [F]

$$\int \frac{\sqrt{e\cot(c+dx)}}{a+a\cot(c+dx)}\,dx = \int \frac{\sqrt{e\cot(dx+c)}}{a\cot(dx+c)+a}\,dx$$

```
[In] integrate((e*cot(d*x+c))^(1/2)/(a+a*cot(d*x+c)),x, algorithm="giac")
```

```
[Out] integrate(sqrt(e*cot(d*x + c))/(a*cot(d*x + c) + a), x)
```

Mupad [B] (verification not implemented)

Time = 12.73 (sec) , antiderivative size = 102, normalized size of antiderivative = 1.17

$$\int \frac{\sqrt{e\cot(c+dx)}}{a+a\cot(c+dx)}\,dx = \frac{\sqrt{e}\,\mathrm{atan}\left(\frac{\sqrt{e\cot(c+dx)}}{\sqrt{e}}\right)}{a\,d} - \frac{\sqrt{2}\,\sqrt{e}\,\left(2\,\mathrm{atan}\left(\frac{\sqrt{2}\,\sqrt{e\cot(c+dx)}}{2\,\sqrt{e}}\right)+2\,\mathrm{atan}\left(\frac{\sqrt{2}\,\sqrt{e\cot(c+dx)}}{2\,\sqrt{e}}+\frac{\sqrt{2}\,{\left(e\cot(c+dx)\right)}^{3/2}}{2\,e^{3/2}}\right)\right)}{4\,a\,d}$$

```
[In] int((e*cot(c + d*x))^(1/2)/(a + a*cot(c + d*x)),x)
```

```
[Out] (e^(1/2)*atan((e*cot(c + d*x))^(1/2)/e^(1/2)))/(a*d) - (2^(1/2)*e^(1/2)*(2*
atan((2^(1/2)*(e*cot(c + d*x))^(1/2))/(2*e^(1/2))) + 2*atan((2^(1/2)*(e*cot
(c + d*x))^(1/2))/(2*e^(1/2)) + (2^(1/2)*(e*cot(c + d*x))^(3/2))/(2*e^(3/2)
))))/(4*a*d)
```

3.26 $\int \frac{1}{\sqrt{e\cot(c+dx)}(a+a\cot(c+dx))}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 83

$$\int \frac{1}{\sqrt{e\cot(c+dx)}(a+a\cot(c+dx))}\,dx=-\frac{\arctan\left(\frac{\sqrt{e\cot(c+dx)}}{\sqrt{e}}\right)}{ad\sqrt{e}}-\frac{\operatorname{arctanh}\left(\frac{\sqrt{e}(1+\cot(c+dx))}{\sqrt{2}\sqrt{e\cot(c+dx)}}\right)}{\sqrt{2}ad\sqrt{e}}$$

```
[Out] -arctan((e*cot(d*x+c))^(1/2)/e^(1/2))/a/d/e^(1/2)-1/2*arctanh(1/2*(1+cot(d*
x+c))*e^(1/2)*2^(1/2)/(e*cot(d*x+c))^(1/2))/a/d*2^(1/2)/e^(1/2)
```

Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 83, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.240, Rules used = {3655, 3613, 214, 3715, 65, 211}

$$\int \frac{1}{\sqrt{e\cot(c+dx)}(a+a\cot(c+dx))}\,dx=-\frac{\arctan\left(\frac{\sqrt{e\cot(c+dx)}}{\sqrt{e}}\right)}{ad\sqrt{e}}-\frac{\operatorname{arctanh}\left(\frac{\sqrt{e}(\cot(c+dx)+1)}{\sqrt{2}\sqrt{e\cot(c+dx)}}\right)}{\sqrt{2}ad\sqrt{e}}$$

```
[In] Int[1/(Sqrt[e*Cot[c + d*x]]*(a + a*Cot[c + d*x])),x]
```

```
[Out] -(ArcTan[Sqrt[e*Cot[c + d*x]]/Sqrt[e]]/(a*d*Sqrt[e])) - ArcTanh[(Sqrt[e]*(1
 + Cot[c + d*x]))/(Sqrt[2]*Sqrt[e*Cot[c + d*x]])]/(Sqrt[2]*a*d*Sqrt[e])
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
```

```
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 3613

```
Int[((c_) + (d_.)*tan[(e_.) + (f_.)*(x_)])/Sqrt[(b_.)*tan[(e_.) + (f_.)*(x_
)]], x_Symbol] :> Dist[-2*(d^2/f), Subst[Int[1/(2*c*d + b*x^2), x], x, (c -
 d*Tan[e + f*x])/Sqrt[b*Tan[e + f*x]]], x] /; FreeQ[{b, c, d, e, f}, x] &&
EqQ[c^2 - d^2, 0]
```

Rule 3655

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)/((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)]), x_Symbol] :> Dist[1/(c^2 + d^2), Int[(a + b*Tan[e + f*x])^m*
(c - d*Tan[e + f*x]), x], x] + Dist[d^2/(c^2 + d^2), Int[(a + b*Tan[e + f*x
])^m*((1 + Tan[e + f*x]^2)/(c + d*Tan[e + f*x])), x], x] /; FreeQ[{a, b, c,
 d, e, f, m}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2
, 0] &&  !IntegerQ[m]
```

Rule 3715

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*tan[(e_.)
+ (f_.)*(x_)])^(n_.)*((A_) + (C_.)*tan[(e_.) + (f_.)*(x_)]^2), x_Symbol] :>
 Dist[A/f, Subst[Int[(a + b*x)^m*(c + d*x)^n, x], x, Tan[e + f*x]], x] /; F
reeQ[{a, b, c, d, e, f, A, C, m, n}, x] && EqQ[A, C]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{1}{2}\int \frac{1+\cot^2(c+dx)}{\sqrt{e\cot(c+dx)}(a+a\cot(c+dx))}\,dx+\frac{\int \frac{a-a\cot(c+dx)}{\sqrt{e\cot(c+dx)}}\,dx}{2a^2}\\
&= \frac{\text{Subst}\left(\int \frac{1}{\sqrt{-ex}(a-ax)}\,dx,x,-\cot(c+dx)\right)}{2d}-\frac{\text{Subst}\left(\int \frac{1}{2a^2-ex^2}\,dx,x,\frac{a+a\cot(c+dx)}{\sqrt{e\cot(c+dx)}}\right)}{d}\\
&= -\frac{\operatorname{arctanh}\left(\frac{\sqrt{e}(1+\cot(c+dx))}{\sqrt{2}\sqrt{e\cot(c+dx)}}\right)}{\sqrt{2}ad\sqrt{e}}-\frac{\text{Subst}\left(\int \frac{1}{a+\frac{ax^2}{e}}\,dx,x,\sqrt{e\cot(c+dx)}\right)}{de}
\end{aligned}
$$

$$= -\frac{\arctan\left(\frac{\sqrt{e\cot(c+dx)}}{\sqrt{e}}\right)}{ad\sqrt{e}} - \frac{\operatorname{arctanh}\left(\frac{\sqrt{e}(1+\cot(c+dx))}{\sqrt{2}\sqrt{e\cot(c+dx)}}\right)}{\sqrt{2}ad\sqrt{e}}$$

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 283 vs. $2(83) = 166$.

Time = 0.52 (sec) , antiderivative size = 283, normalized size of antiderivative = 3.41

$$\int \frac{1}{\sqrt{e\cot(c+dx)}(a+a\cot(c+dx))}\,dx = -\frac{8e^{3/2}\arctan\left(\frac{\sqrt{e\cot(c+dx)}}{\sqrt{e}}\right) + 4(-e^2)^{3/4}\arctan\left(\frac{\sqrt{e\cot(c+dx)}}{\sqrt[4]{-e^2}}\right) - 2\sqrt{2}e^{3/2}\arctan\left(1 - \frac{\sqrt{2}\sqrt{e\cot(c+dx)}}{\sqrt{e}}\right) + 2\sqrt{}$$

```
[In] Integrate[1/(Sqrt[e*Cot[c + d*x]]*(a + a*Cot[c + d*x])),x]
```

```
[Out] -1/8*(8*e^(3/2)*ArcTan[Sqrt[e*Cot[c + d*x]]/Sqrt[e]] + 4*(-e^2)^(3/4)*ArcTa
n[Sqrt[e*Cot[c + d*x]]/(-e^2)^(1/4)] - 2*Sqrt[2]*e^(3/2)*ArcTan[1 - (Sqrt[2
]*Sqrt[e*Cot[c + d*x]])/Sqrt[e]] + 2*Sqrt[2]*e^(3/2)*ArcTan[1 + (Sqrt[2]*Sq
rt[e*Cot[c + d*x]])/Sqrt[e]] - 4*(-e^2)^(3/4)*ArcTanh[Sqrt[e*Cot[c + d*x]]/
(-e^2)^(1/4)] - Sqrt[2]*e^(3/2)*Log[Sqrt[e] + Sqrt[e]*Cot[c + d*x] - Sqrt[2
]*Sqrt[e*Cot[c + d*x]]] + Sqrt[2]*e^(3/2)*Log[Sqrt[e] + Sqrt[e]*Cot[c + d*x
] + Sqrt[2]*Sqrt[e*Cot[c + d*x]]])/(a*d*e^2)
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 303 vs. $2(68) = 136$.

Time = 0.04 (sec) , antiderivative size = 304, normalized size of antiderivative = 3.66

method	result
derivativedivides	$-\frac{2e^2\left(\frac{\frac{(e^2)^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\frac{e\cot(dx+c)+(e^2)^{\frac{1}{4}}\sqrt{e\cot(dx+c)}\sqrt{2}+\sqrt{e^2}}{e\cot(dx+c)-(e^2)^{\frac{1}{4}}\sqrt{e\cot(dx+c)}\sqrt{2}+\sqrt{e^2}}\right)+2\arctan\left(\frac{\sqrt{2}\sqrt{e\cot(dx+c)}}{(e^2)^{\frac{1}{4}}}+1\right)-2\arctan\left(-\frac{\sqrt{2}\sqrt{e\cot(dx}{(e^2)^{\frac{1}{4}}}\right.\right.}{8e}}{}\right.}{}$
default	$-\frac{2e^2\left(\frac{\frac{(e^2)^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\frac{e\cot(dx+c)+(e^2)^{\frac{1}{4}}\sqrt{e\cot(dx+c)}\sqrt{2}+\sqrt{e^2}}{e\cot(dx+c)-(e^2)^{\frac{1}{4}}\sqrt{e\cot(dx+c)}\sqrt{2}+\sqrt{e^2}}\right)+2\arctan\left(\frac{\sqrt{2}\sqrt{e\cot(dx+c)}}{(e^2)^{\frac{1}{4}}}+1\right)-2\arctan\left(-\frac{\sqrt{2}\sqrt{e\cot(dx}{(e^2)^{\frac{1}{4}}}\right.\right.}{8e}}{}\right.}{}$

```
[In] int(1/(e*cot(d*x+c))^(1/2)/(a+a*cot(d*x+c)),x,method=_RETURNVERBOSE)
```

```
[Out] -2/d/a*e^2*(1/2/e^2*(1/8/e*(e^2)^(1/4)*2^(1/2)*(ln((e*cot(d*x+c)+(e^2)^(1/4
)*(e*cot(d*x+c))^(1/2)*2^(1/2)+(e^2)^(1/2))/(e*cot(d*x+c)-(e^2)^(1/4)*(e*co
t(d*x+c))^(1/2)*2^(1/2)+(e^2)^(1/2)))+2*arctan(2^(1/2)/(e^2)^(1/4)*(e*cot(d
*x+c))^(1/2)+1)-2*arctan(-2^(1/2)/(e^2)^(1/4)*(e*cot(d*x+c))^(1/2)+1))-1/8/
(e^2)^(1/4)*2^(1/2)*(ln((e*cot(d*x+c)-(e^2)^(1/4)*(e*cot(d*x+c))^(1/2)*2^(1
/2)+(e^2)^(1/2))/(e*cot(d*x+c)+(e^2)^(1/4)*(e*cot(d*x+c))^(1/2)*2^(1/2)+(e^
2)^(1/2)))+2*arctan(2^(1/2)/(e^2)^(1/4)*(e*cot(d*x+c))^(1/2)+1)-2*arctan(-2
^(1/2)/(e^2)^(1/4)*(e*cot(d*x+c))^(1/2)+1)))+1/2/e^(5/2)*arctan((e*cot(d*x+
c))^(1/2)/e^(1/2)))
```

Fricas [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 321, normalized size of antiderivative = 3.87

$$\int \frac{1}{\sqrt{e\cot(c+dx)}(a+a\cot(c+dx))}\,dx = \left[\frac{\sqrt{2}\sqrt{-e}\arctan\left(\frac{\sqrt{2}\sqrt{-e}\sqrt{\frac{e\cos(2\,dx+2\,c)+e}{\sin(2\,dx+2\,c)}}(\cos(2\,dx+2\,c)+\sin(2\,dx+2\,c)+1)}{2\,(e\cos(2\,dx+2\,c)+e)}\right)-\sqrt{-e}\log\left(\frac{e\cos(2\,dx+2\,c)-e\sin(2\,dx+2\,c)+2\,\sqrt{}}{\cos(2\,dx+2\,c)+}\right.}{2\,ade}\right.$$

```
[In] integrate(1/(e*cot(d*x+c))^(1/2)/(a+a*cot(d*x+c)),x, algorithm="fricas")
```

```
[Out] [1/2*(sqrt(2)*sqrt(-e)*arctan(1/2*sqrt(2)*sqrt(-e)*sqrt((e*cos(2*d*x + 2*c)
 + e)/sin(2*d*x + 2*c))*(cos(2*d*x + 2*c) + sin(2*d*x + 2*c) + 1)/(e*cos(2*
```

```
d*x + 2*c) + e)) - sqrt(-e)*log((e*cos(2*d*x + 2*c) - e*sin(2*d*x + 2*c) +
2*sqrt(-e)*sqrt((e*cos(2*d*x + 2*c) + e)/sin(2*d*x + 2*c))*sin(2*d*x + 2*c)
 + e)/(cos(2*d*x + 2*c) + sin(2*d*x + 2*c) + 1)))/(a*d*e), 1/4*(sqrt(2)*sqr
t(e)*log(sqrt(2)*sqrt(e)*sqrt((e*cos(2*d*x + 2*c) + e)/sin(2*d*x + 2*c))*(c
os(2*d*x + 2*c) - sin(2*d*x + 2*c) - 1) + 2*e*sin(2*d*x + 2*c) + e) - 4*sqr
t(e)*arctan(sqrt((e*cos(2*d*x + 2*c) + e)/sin(2*d*x + 2*c))/sqrt(e)))/(a*d*
e)]
```

Sympy [F]

$$\int \frac{1}{\sqrt{e\cot(c+dx)}(a+a\cot(c+dx))}\,dx = \frac{\int \frac{1}{\sqrt{e\cot{(c+dx)}}\cot{(c+dx)}+\sqrt{e\cot{(c+dx)}}}\,dx}{a}$$

```
[In] integrate(1/(e*cot(d*x+c))**(1/2)/(a+a*cot(d*x+c)),x)
[Out] Integral(1/(sqrt(e*cot(c + d*x))*cot(c + d*x) + sqrt(e*cot(c + d*x))), x)/a
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{1}{\sqrt{e\cot(c+dx)}(a+a\cot(c+dx))}\,dx = \text{Exception raised: ValueError}$$

```
[In] integrate(1/(e*cot(d*x+c))^(1/2)/(a+a*cot(d*x+c)),x, algorithm="maxima")
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(e>0)', see `assume?` for more detai
ls)Is e
```

Giac [F]

$$\int \frac{1}{\sqrt{e\cot(c+dx)}(a+a\cot(c+dx))}\,dx = \int \frac{1}{(a\cot{(dx+c)}+a)\sqrt{e\cot{(dx+c)}}}\,dx$$

```
[In] integrate(1/(e*cot(d*x+c))^(1/2)/(a+a*cot(d*x+c)),x, algorithm="giac")
[Out] integrate(1/((a*cot(d*x + c) + a)*sqrt(e*cot(d*x + c))), x)
```

Mupad [B] (verification not implemented)

Time = 12.78 (sec) , antiderivative size = 79, normalized size of antiderivative = 0.95

$$\int \frac{1}{\sqrt{e\cot(c+dx)}(a+a\cot(c+dx))}\,dx = -\frac{\operatorname{atan}\left(\frac{\sqrt{e\cot(c+d\,x)}}{\sqrt{e}}\right)}{a\,d\,\sqrt{e}} - \frac{\sqrt{2}\operatorname{atanh}\left(\frac{12\,\sqrt{2}\,e^{9/2}\,\sqrt{e\cot(c+d\,x)}}{12\,e^5\cot(c+d\,x)+12\,e^5}\right)}{2\,a\,d\,\sqrt{e}}$$

```
[In] int(1/((e*cot(c + d*x))^(1/2)*(a + a*cot(c + d*x))),x)
```

```
[Out] - atan((e*cot(c + d*x))^(1/2)/e^(1/2))/(a*d*e^(1/2)) - (2^(1/2)*atanh((12*2
^(1/2)*e^(9/2)*(e*cot(c + d*x))^(1/2))/(12*e^5*cot(c + d*x) + 12*e^5)))/(2*
a*d*e^(1/2))
```

3.27 $\int \frac{1}{(e\cot(c+dx))^{3/2}(a+a\cot(c+dx))}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 111

$$\int \frac{1}{(e\cot(c+dx))^{3/2}(a+a\cot(c+dx))}\,dx=\frac{\arctan\left(\frac{\sqrt{e\cot(c+dx)}}{\sqrt{e}}\right)}{ade^{3/2}}-\frac{\arctan\left(\frac{\sqrt{e}-\sqrt{e}\cot(c+dx)}{\sqrt{2}\sqrt{e\cot(c+dx)}}\right)}{\sqrt{2}ade^{3/2}}+\frac{2}{ade\sqrt{e\cot(c+dx)}}$$

```
[Out] arctan((e*cot(d*x+c))^(1/2)/e^(1/2))/a/d/e^(3/2)-1/2*arctan(1/2*(e^(1/2)-co
t(d*x+c)*e^(1/2))*2^(1/2)/(e*cot(d*x+c))^(1/2))/a/d/e^(3/2)*2^(1/2)+2/a/d/e
/(e*cot(d*x+c))^(1/2)
```

Rubi [A] (verified)

Time = 0.51 (sec) , antiderivative size = 111, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.240, Rules used = {3650, 3734, 3613, 211, 3715, 65}

$$\int \frac{1}{(e\cot(c+dx))^{3/2}(a+a\cot(c+dx))}\,dx=\frac{\arctan\left(\frac{\sqrt{e\cot(c+dx)}}{\sqrt{e}}\right)}{ade^{3/2}}-\frac{\arctan\left(\frac{\sqrt{e}-\sqrt{e}\cot(c+dx)}{\sqrt{2}\sqrt{e\cot(c+dx)}}\right)}{\sqrt{2}ade^{3/2}}+\frac{2}{ade\sqrt{e\cot(c+dx)}}$$

```
[In] Int[1/((e*Cot[c + d*x])^(3/2)*(a + a*Cot[c + d*x])),x]
```

```
[Out] ArcTan[Sqrt[e*Cot[c + d*x]]/Sqrt[e]]/(a*d*e^(3/2)) - ArcTan[(Sqrt[e] - Sqrt
[e]*Cot[c + d*x])/(Sqrt[2]*Sqrt[e*Cot[c + d*x]])]/(Sqrt[2]*a*d*e^(3/2)) + 2
/(a*d*e*Sqrt[e*Cot[c + d*x]])
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 3613

```
Int[((c_) + (d_.)*tan[(e_.) + (f_.)*(x_)])/Sqrt[(b_.)*tan[(e_.) + (f_.)*(x_
)]], x_Symbol] :> Dist[-2*(d^2/f), Subst[Int[1/(2*c*d + b*x^2), x], x, (c -
 d*Tan[e + f*x])/Sqrt[b*Tan[e + f*x]]], x] /; FreeQ[{b, c, d, e, f}, x] &&
EqQ[c^2 - d^2, 0]
```

Rule 3650

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)])^(n_), x_Symbol] :> Simp[b^2*(a + b*Tan[e + f*x])^(m + 1)*((c
+ d*Tan[e + f*x])^(n + 1)/(f*(m + 1)*(a^2 + b^2)*(b*c - a*d))), x] + Dist[1
/((m + 1)*(a^2 + b^2)*(b*c - a*d)), Int[(a + b*Tan[e + f*x])^(m + 1)*(c + d
*Tan[e + f*x])^n*Simp[a*(b*c - a*d)*(m + 1) - b^2*d*(m + n + 2) - b*(b*c -
a*d)*(m + 1)*Tan[e + f*x] - b^2*d*(m + n + 2)*Tan[e + f*x]^2, x], x], x] /;
 FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0]
&& NeQ[c^2 + d^2, 0] && IntegerQ[2*m] && LtQ[m, -1] && (LtQ[n, 0] || Intege
rQ[m]) &&  !(ILtQ[n, -1] && ( !IntegerQ[m] || (EqQ[c, 0] && NeQ[a, 0])))
```

Rule 3715

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*tan[(e_.)
+ (f_.)*(x_)])^(n_.)*((A_) + (C_.)*tan[(e_.) + (f_.)*(x_)]^2), x_Symbol] :>
 Dist[A/f, Subst[Int[(a + b*x)^m*(c + d*x)^n, x], x, Tan[e + f*x]], x] /; F
reeQ[{a, b, c, d, e, f, A, C, m, n}, x] && EqQ[A, C]
```

Rule 3734

```
Int[(((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_)*((A_.) + (B_.)*tan[(e_.)
+ (f_.)*(x_)] + (C_.)*tan[(e_.) + (f_.)*(x_)]^2))/((a_.) + (b_.)*tan[(e_.)
+ (f_.)*(x_)]), x_Symbol] :> Dist[1/(a^2 + b^2), Int[(c + d*Tan[e + f*x])^n
*Simp[b*B + a*(A - C) + (a*B - b*(A - C))*Tan[e + f*x], x], x], x] + Dist[(
A*b^2 - a*b*B + a^2*C)/(a^2 + b^2), Int[(c + d*Tan[e + f*x])^n*((1 + Tan[e
```

```
+ f*x]^2)/(a + b*Tan[e + f*x])), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C
, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] &&
 !GtQ[n, 0] &&  !LeQ[n, -1]
```

Rubi steps

$$\text{integral} = \frac{2}{ade\sqrt{e\cot(c+dx)}} + \frac{2\int \frac{-\frac{ae^2}{2}-\frac{1}{2}ae^2\cot(c+dx)-\frac{1}{2}ae^2\cot^2(c+dx)}{\sqrt{e\cot(c+dx)}(a+a\cot(c+dx))}\,dx}{ae^3}$$

$$= \frac{2}{ade\sqrt{e\cot(c+dx)}} + \frac{\int \frac{-\frac{1}{2}a^2e^2-\frac{1}{2}a^2e^2\cot(c+dx)}{\sqrt{e\cot(c+dx)}}\,dx}{a^3e^3} - \frac{\int \frac{1+\cot^2(c+dx)}{\sqrt{e\cot(c+dx)}(a+a\cot(c+dx))}\,dx}{2e}$$

$$= \frac{2}{ade\sqrt{e\cot(c+dx)}} - \frac{\text{Subst}\left(\int \frac{1}{\sqrt{-ex}(a-ax)}\,dx, x, -\cot(c+dx)\right)}{2de} - \frac{(ae)\text{Subst}\left(\int \frac{1}{-\frac{1}{2}a^4e^4-ex^2}\,dx, x, \frac{-\frac{1}{2}a^2e^2+\frac{1}{2}a^2e^2\cot(c+dx)}{\sqrt{e\cot(c+dx)}}\right)}{2d}$$

$$= -\frac{\arctan\left(\frac{\sqrt{e}-\sqrt{e}\cot(c+dx)}{\sqrt{2}\sqrt{e\cot(c+dx)}}\right)}{\sqrt{2}ade^{3/2}} + \frac{2}{ade\sqrt{e\cot(c+dx)}} + \frac{\text{Subst}\left(\int \frac{1}{a+\frac{ax^2}{e}}\,dx, x, \sqrt{e\cot(c+dx)}\right)}{de^2}$$

$$= \frac{\arctan\left(\frac{\sqrt{e\cot(c+dx)}}{\sqrt{e}}\right)}{ade^{3/2}} - \frac{\arctan\left(\frac{\sqrt{e}-\sqrt{e}\cot(c+dx)}{\sqrt{2}\sqrt{e\cot(c+dx)}}\right)}{\sqrt{2}ade^{3/2}} + \frac{2}{ade\sqrt{e\cot(c+dx)}}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 0.82 (sec) , antiderivative size = 229, normalized size of antiderivative = 2.06

$$\int \frac{1}{(e\cot(c+dx))^{3/2}(a+a\cot(c+dx))}\,dx = \frac{8\sqrt{e}\,\text{Hypergeometric2F1}\left(-\frac{1}{2},1,\frac{1}{2},-\cot(c+dx)\right)+8\sqrt{e}\,\text{Hype}}{}$$

```
[In] Integrate[1/((e*Cot[c + d*x])^(3/2)*(a + a*Cot[c + d*x])),x]
```

```
[Out] (8*Sqrt[e]*Hypergeometric2F1[-1/2, 1, 1/2, -Cot[c + d*x]] + 8*Sqrt[e]*Hyper
geometric2F1[-1/4, 1, 3/4, -Cot[c + d*x]^2] + Sqrt[2]*Sqrt[e*Cot[c + d*x]]*
(-2*ArcTan[1 - (Sqrt[2]*Sqrt[e*Cot[c + d*x]])/Sqrt[e]] + 2*ArcTan[1 + (Sqrt
[2]*Sqrt[e*Cot[c + d*x]])/Sqrt[e]] - Log[Sqrt[e] + Sqrt[e]*Cot[c + d*x] - S
qrt[2]*Sqrt[e*Cot[c + d*x]]] + Log[Sqrt[e] + Sqrt[e]*Cot[c + d*x] + Sqrt[2]
*Sqrt[e*Cot[c + d*x]]]))/(8*a*d*e^(3/2)*Sqrt[e*Cot[c + d*x]])
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 318 vs. $2(92) = 184$.

Time = 0.04 (sec) , antiderivative size = 319, normalized size of antiderivative = 2.87

method	result
derivativedivides	$-\frac{2e^{2}\left(-\frac{\left(e^{2}\right)^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\frac{e\cot(dx+c)+\left(e^{2}\right)^{\frac{1}{4}}\sqrt{e\cot(dx+c)}\,\sqrt{2}+\sqrt{e^{2}}}{e\cot(dx+c)-\left(e^{2}\right)^{\frac{1}{4}}\sqrt{e\cot(dx+c)}\,\sqrt{2}+\sqrt{e^{2}}}\right)+2\arctan\left(\frac{\sqrt{2}\sqrt{e\cot(dx+c)}}{\left(e^{2}\right)^{\frac{1}{4}}}+1\right)-2\arctan\left(-\frac{\sqrt{2}\sqrt{e\cot(\cdots}}{\left(e^{2}\right)^{\frac{1}{4}}}\cdots\right.\right)}{8e}\cdots\right.}{\cdots}$
default	$-\frac{2e^{2}\left(-\frac{\left(e^{2}\right)^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\frac{e\cot(dx+c)+\left(e^{2}\right)^{\frac{1}{4}}\sqrt{e\cot(dx+c)}\,\sqrt{2}+\sqrt{e^{2}}}{e\cot(dx+c)-\left(e^{2}\right)^{\frac{1}{4}}\sqrt{e\cot(dx+c)}\,\sqrt{2}+\sqrt{e^{2}}}\right)+2\arctan\left(\frac{\sqrt{2}\sqrt{e\cot(dx+c)}}{\left(e^{2}\right)^{\frac{1}{4}}}+1\right)-2\arctan\left(-\frac{\sqrt{2}\sqrt{e\cot(\cdots}}{\left(e^{2}\right)^{\frac{1}{4}}}\cdots\right.\right)}{8e}\cdots\right.}{\cdots}$

```
[In] int(1/(e*cot(d*x+c))^(3/2)/(a+a*cot(d*x+c)),x,method=_RETURNVERBOSE)
```

```
[Out] -2/d/a*e^2*(1/2/e^3*(-1/8/e*(e^2)^(1/4)*2^(1/2)*(ln((e*cot(d*x+c)+(e^2)^(1/
4)*(e*cot(d*x+c))^(1/2)*2^(1/2)+(e^2)^(1/2))/(e*cot(d*x+c)-(e^2)^(1/4)*(e*c
ot(d*x+c))^(1/2)*2^(1/2)+(e^2)^(1/2)))+2*arctan(2^(1/2)/(e^2)^(1/4)*(e*cot(
d*x+c))^(1/2)+1)-2*arctan(-2^(1/2)/(e^2)^(1/4)*(e*cot(d*x+c))^(1/2)+1))-1/8
/(e^2)^(1/4)*2^(1/2)*(ln((e*cot(d*x+c)-(e^2)^(1/4)*(e*cot(d*x+c))^(1/2)*2^(
1/2)+(e^2)^(1/2))/(e*cot(d*x+c)+(e^2)^(1/4)*(e*cot(d*x+c))^(1/2)*2^(1/2)+(e
^2)^(1/2)))+2*arctan(2^(1/2)/(e^2)^(1/4)*(e*cot(d*x+c))^(1/2)+1)-2*arctan(-
2^(1/2)/(e^2)^(1/4)*(e*cot(d*x+c))^(1/2)+1)))-1/2/e^(7/2)*arctan((e*cot(d*x
+c))^(1/2)/e^(1/2))-1/e^3/(e*cot(d*x+c))^(1/2))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 204 vs. 2(93) = 186.

Time = 0.28 (sec) , antiderivative size = 472, normalized size of antiderivative = 4.25

$$\int \frac{1}{(e\cot(c+dx))^{3/2}(a+a\cot(c+dx))}\,dx = \left[-\frac{\sqrt{2}\sqrt{-e}(\cos(2\,dx+2\,c)+1)\log\left(-\sqrt{2}\sqrt{-e}\sqrt{\frac{e\cos(2\,dx+2\,c)+e}{\sin(2\,dx+2\,c)}}\right.}{}\right.$$

$$-\frac{\sqrt{2}\sqrt{e}(\cos(2\,dx+2\,c)+1)\arctan\left(-\frac{\sqrt{2}\sqrt{e}\sqrt{\frac{e\cos(2\,dx+2\,c)+e}{\sin(2\,dx+2\,c)}}(\cos(2\,dx+2\,c)-\sin(2\,dx+2\,c)+1)}{2\,(e\cos(2\,dx+2\,c)+e)}\right)-2\,\sqrt{e}(\cos(2\,dx+2\,c)}{2\,(ade^2\cos(2\,dx+2\,c)+ade^2)}$$

```
[In] integrate(1/(e*cot(d*x+c))^(3/2)/(a+a*cot(d*x+c)),x, algorithm="fricas")
```

```
[Out] [-1/4*(sqrt(2)*sqrt(-e)*(cos(2*d*x + 2*c) + 1)*log(-sqrt(2)*sqrt(-e)*sqrt((
e*cos(2*d*x + 2*c) + e)/sin(2*d*x + 2*c))*(cos(2*d*x + 2*c) + sin(2*d*x + 2
*c) - 1) - 2*e*sin(2*d*x + 2*c) + e) + 2*sqrt(-e)*(cos(2*d*x + 2*c) + 1)*lo
g((e*cos(2*d*x + 2*c) - e*sin(2*d*x + 2*c) - 2*sqrt(-e)*sqrt((e*cos(2*d*x +
 2*c) + e)/sin(2*d*x + 2*c))*sin(2*d*x + 2*c) + e)/(cos(2*d*x + 2*c) + sin(
2*d*x + 2*c) + 1)) - 8*sqrt((e*cos(2*d*x + 2*c) + e)/sin(2*d*x + 2*c))*sin(
2*d*x + 2*c))/(a*d*e^2*cos(2*d*x + 2*c) + a*d*e^2), -1/2*(sqrt(2)*sqrt(e)*(
cos(2*d*x + 2*c) + 1)*arctan(-1/2*sqrt(2)*sqrt(e)*sqrt((e*cos(2*d*x + 2*c)
+ e)/sin(2*d*x + 2*c))*(cos(2*d*x + 2*c) - sin(2*d*x + 2*c) + 1)/(e*cos(2*d
*x + 2*c) + e)) - 2*sqrt(e)*(cos(2*d*x + 2*c) + 1)*arctan(sqrt((e*cos(2*d*x
 + 2*c) + e)/sin(2*d*x + 2*c))/sqrt(e)) - 4*sqrt((e*cos(2*d*x + 2*c) + e)/s
in(2*d*x + 2*c))*sin(2*d*x + 2*c))/(a*d*e^2*cos(2*d*x + 2*c) + a*d*e^2)]
```

Sympy [F]

$$\int \frac{1}{(e\cot(c+dx))^{3/2}(a+a\cot(c+dx))}\,dx = \frac{\int \frac{1}{(e\cot(c+dx))^{\frac{3}{2}}\cot(c+dx)+(e\cot(c+dx))^{\frac{3}{2}}}\,dx}{a}$$

```
[In] integrate(1/(e*cot(d*x+c))**(3/2)/(a+a*cot(d*x+c)),x)
```

```
[Out] Integral(1/((e*cot(c + d*x))**(3/2)*cot(c + d*x) + (e*cot(c + d*x))**(3/2))
, x)/a
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{1}{(e\cot(c+dx))^{3/2}(a+a\cot(c+dx))}\,dx = \text{Exception raised: ValueError}$$

```
[In] integrate(1/(e*cot(d*x+c))^(3/2)/(a+a*cot(d*x+c)),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(e>0)', see `assume?` for more detai
ls)Is e
```

Giac [F]

$$\int \frac{1}{(e\cot(c+dx))^{3/2}(a+a\cot(c+dx))}\,dx = \int \frac{1}{(a\cot(dx+c)+a)\,(e\cot(dx+c))^{\frac{3}{2}}}\,dx$$

```
[In] integrate(1/(e*cot(d*x+c))^(3/2)/(a+a*cot(d*x+c)),x, algorithm="giac")
```

```
[Out] integrate(1/((a*cot(d*x + c) + a)*(e*cot(d*x + c))^(3/2)), x)
```

Mupad [B] (verification not implemented)

Time = 12.67 (sec) , antiderivative size = 123, normalized size of antiderivative = 1.11

$$\int \frac{1}{(e\cot(c+dx))^{3/2}(a+a\cot(c+dx))}\,dx = \frac{2}{a\,d\,e\,\sqrt{e\cot(c+d\,x)}} + \frac{\mathrm{atan}\left(\frac{\sqrt{e\cot(c+d\,x)}}{\sqrt{e}}\right)}{a\,d\,e^{3/2}} + \frac{\sqrt{2}\left(2\,\mathrm{atan}\left(\frac{\sqrt{2}\,\sqrt{e\cot(c+d\,x)}}{2\,\sqrt{e}}\right)+2\,\mathrm{atan}\left(\frac{\sqrt{2}\,\sqrt{e\cot(c+d\,x)}}{2\,\sqrt{e}}+\frac{\sqrt{2}\,(e\cot(c+d\,x))^{3/2}}{2\,e^{3/2}}\right)\right)}{4\,a\,d\,e^{3/2}}$$

```
[In] int(1/((e*cot(c + d*x))^(3/2)*(a + a*cot(c + d*x))),x)
```

```
[Out] 2/(a*d*e*(e*cot(c + d*x))^(1/2)) + atan((e*cot(c + d*x))^(1/2)/e^(1/2))/(a*
d*e^(3/2)) + (2^(1/2)*(2*atan((2^(1/2)*(e*cot(c + d*x))^(1/2))/(2*e^(1/2)))
 + 2*atan((2^(1/2)*(e*cot(c + d*x))^(1/2))/(2*e^(1/2)) + (2^(1/2)*(e*cot(c
+ d*x))^(3/2))/(2*e^(3/2)))))/(4*a*d*e^(3/2))
```

3.28 $\int \frac{1}{(e\cot(c+dx))^{5/2}(a+a\cot(c+dx))}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 135

$$\int \frac{1}{(e\cot(c+dx))^{5/2}(a+a\cot(c+dx))}\,dx = -\frac{\arctan\left(\frac{\sqrt{e\cot(c+dx)}}{\sqrt{e}}\right)}{ade^{5/2}} + \frac{\operatorname{arctanh}\left(\frac{\sqrt{e}+\sqrt{e}\cot(c+dx)}{\sqrt{2}\sqrt{e\cot(c+dx)}}\right)}{\sqrt{2}ade^{5/2}} + \frac{2}{3ade(e\cot(c+dx))^{3/2}} - \frac{2}{ade^2\sqrt{e\cot(c+dx)}}$$

```
[Out] -arctan((e*cot(d*x+c))^(1/2)/e^(1/2))/a/d/e^(5/2)+2/3/a/d/e/(e*cot(d*x+c))^
(3/2)+1/2*arctanh(1/2*(e^(1/2)+cot(d*x+c)*e^(1/2))*2^(1/2)/(e*cot(d*x+c))^(
1/2))/a/d/e^(5/2)*2^(1/2)-2/a/d/e^2/(e*cot(d*x+c))^(1/2)
```

Rubi [A] (verified)

Time = 0.65 (sec) , antiderivative size = 135, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.400, Rules used = {3650, 3730, 12, 16, 3654, 3613, 214, 3715, 65, 211}

$$\int \frac{1}{(e\cot(c+dx))^{5/2}(a+a\cot(c+dx))}\,dx = -\frac{\arctan\left(\frac{\sqrt{e\cot(c+dx)}}{\sqrt{e}}\right)}{ade^{5/2}} + \frac{\operatorname{arctanh}\left(\frac{\sqrt{e}\cot(c+dx)+\sqrt{e}}{\sqrt{2}\sqrt{e\cot(c+dx)}}\right)}{\sqrt{2}ade^{5/2}} - \frac{2}{ade^2\sqrt{e\cot(c+dx)}} + \frac{2}{3ade(e\cot(c+dx))^{3/2}}$$

```
[In] Int[1/((e*Cot[c + d*x])^(5/2)*(a + a*Cot[c + d*x])),x]
```

```
[Out] -(ArcTan[Sqrt[e*Cot[c + d*x]]/Sqrt[e]]/(a*d*e^(5/2))) + ArcTanh[(Sqrt[e] +
Sqrt[e]*Cot[c + d*x])/(Sqrt[2]*Sqrt[e*Cot[c + d*x]])]/(Sqrt[2]*a*d*e^(5/2))
 + 2/(3*a*d*e*(e*Cot[c + d*x])^(3/2)) - 2/(a*d*e^2*Sqrt[e*Cot[c + d*x]])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 16

```
Int[(u_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Dist[1/b^m, Int[u*(b*v)
^(m + n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 3613

```
Int[((c_) + (d_.)*tan[(e_.) + (f_.)*(x_)])/Sqrt[(b_.)*tan[(e_.) + (f_.)*(x_
)]], x_Symbol] :> Dist[-2*(d^2/f), Subst[Int[1/(2*c*d + b*x^2), x], x, (c -
 d*Tan[e + f*x])/Sqrt[b*Tan[e + f*x]]], x] /; FreeQ[{b, c, d, e, f}, x] &&
EqQ[c^2 - d^2, 0]
```

Rule 3650

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)])^(n_), x_Symbol] :> Simp[b^2*(a + b*Tan[e + f*x])^(m + 1)*((c
+ d*Tan[e + f*x])^(n + 1)/(f*(m + 1)*(a^2 + b^2)*(b*c - a*d))), x] + Dist[1
/((m + 1)*(a^2 + b^2)*(b*c - a*d)), Int[(a + b*Tan[e + f*x])^(m + 1)*(c + d
*Tan[e + f*x])^n*Simp[a*(b*c - a*d)*(m + 1) - b^2*d*(m + n + 2) - b*(b*c -
a*d)*(m + 1)*Tan[e + f*x] - b^2*d*(m + n + 2)*Tan[e + f*x]^2, x], x], x] /;
 FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0]
&& NeQ[c^2 + d^2, 0] && IntegerQ[2*m] && LtQ[m, -1] && (LtQ[n, 0] || Intege
rQ[m]) &&  !(ILtQ[n, -1] && ( !IntegerQ[m] || (EqQ[c, 0] && NeQ[a, 0])))
```

Rule 3654

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(3/2)/((c_.) + (d_.)*tan[(e_.)
+ (f_.)*(x_)]), x_Symbol] :> Dist[1/(c^2 + d^2), Int[Simp[a^2*c - b^2*c + 2
*a*b*d + (2*a*b*c - a^2*d + b^2*d)*Tan[e + f*x], x]/Sqrt[a + b*Tan[e + f*x]
], x], x] + Dist[(b*c - a*d)^2/(c^2 + d^2), Int[(1 + Tan[e + f*x]^2)/(Sqrt[
a + b*Tan[e + f*x]]*(c + d*Tan[e + f*x])), x], x] /; FreeQ[{a, b, c, d, e,
f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0]
```

Rule 3715

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*tan[(e_.)
+ (f_.)*(x_)])^(n_.)*((A_) + (C_.)*tan[(e_.) + (f_.)*(x_)]^2), x_Symbol] :>
 Dist[A/f, Subst[Int[(a + b*x)^m*(c + d*x)^n, x], x, Tan[e + f*x]], x] /; F
reeQ[{a, b, c, d, e, f, A, C, m, n}, x] && EqQ[A, C]
```

Rule 3730

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*tan[(e_.) + (f_.)*(x_)] + (C_.)*tan[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> Simp[(A*b^2 - a*(b*B - a*C))*(a + b*Tan[e +
 f*x])^(m + 1)*((c + d*Tan[e + f*x])^(n + 1)/(f*(m + 1)*(b*c - a*d)*(a^2 +
b^2))), x] + Dist[1/((m + 1)*(b*c - a*d)*(a^2 + b^2)), Int[(a + b*Tan[e + f
*x])^(m + 1)*(c + d*Tan[e + f*x])^n*Simp[A*(a*(b*c - a*d)*(m + 1) - b^2*d*(
m + n + 2)) + (b*B - a*C)*(b*c*(m + 1) + a*d*(n + 1)) - (m + 1)*(b*c - a*d)
*(A*b - a*B - b*C)*Tan[e + f*x] - d*(A*b^2 - a*(b*B - a*C))*(m + n + 2)*Tan
[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, n}, x] && NeQ[
b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] && LtQ[m, -1] &&  !
(ILtQ[n, -1] && ( !IntegerQ[m] || (EqQ[c, 0] && NeQ[a, 0])))
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{2}{3ade(e\cot(c+dx))^{3/2}} + \frac{2\int \frac{-\frac{3ae^2}{2}-\frac{3}{2}ae^2\cot(c+dx)-\frac{3}{2}ae^2\cot^2(c+dx)}{(e\cot(c+dx))^{3/2}(a+a\cot(c+dx))}\,dx}{3ae^3} \\
&= \frac{2}{3ade(e\cot(c+dx))^{3/2}} - \frac{2}{ade^2\sqrt{e\cot(c+dx)}} + \frac{4\int \frac{3a^2e^4\cot^2(c+dx)}{4\sqrt{e\cot(c+dx)}(a+a\cot(c+dx))}\,dx}{3a^2e^6} \\
&= \frac{2}{3ade(e\cot(c+dx))^{3/2}} - \frac{2}{ade^2\sqrt{e\cot(c+dx)}} + \frac{\int \frac{\cot^2(c+dx)}{\sqrt{e\cot(c+dx)}(a+a\cot(c+dx))}\,dx}{e^2} \\
&= \frac{2}{3ade(e\cot(c+dx))^{3/2}} - \frac{2}{ade^2\sqrt{e\cot(c+dx)}} + \frac{\int \frac{(e\cot(c+dx))^{3/2}}{a+a\cot(c+dx)}\,dx}{e^4}
\end{aligned}$$

$$= \frac{2}{3ade(e\cot(c+dx))^{3/2}} - \frac{2}{ade^2\sqrt{e\cot(c+dx)}} + \frac{\int \frac{-ae^2+ae^2\cot(c+dx)}{\sqrt{e\cot(c+dx)}}dx}{2a^2e^4} + \frac{\int \frac{1+\cot^2(c+dx)}{\sqrt{e\cot(c+dx)}(a+a\cot(c+dx))}dx}{2e^2}$$

$$= \frac{2}{3ade(e\cot(c+dx))^{3/2}} - \frac{2}{ade^2\sqrt{e\cot(c+dx)}} - \frac{\text{Subst}\left(\int \frac{1}{2a^2e^4-ex^2}dx, x, \frac{-ae^2-ae^2\cot(c+dx)}{\sqrt{e\cot(c+dx)}}\right)}{d} + \frac{\text{Subst}\left(\int \frac{1}{\sqrt{-ex}(a-ax)}dx, x, -\cot(c+dx)\right)}{2de^2}$$

$$= \frac{\text{arctanh}\left(\frac{\sqrt{e}+\sqrt{e}\cot(c+dx)}{\sqrt{2}\sqrt{e\cot(c+dx)}}\right)}{\sqrt{2}ade^{5/2}} + \frac{2}{3ade(e\cot(c+dx))^{3/2}} - \frac{2}{ade^2\sqrt{e\cot(c+dx)}} - \frac{\text{Subst}\left(\int \frac{1}{a+\frac{ax^2}{e}}dx, x, \sqrt{e\cot(c+dx)}\right)}{de^3}$$

$$= -\frac{\arctan\left(\frac{\sqrt{e\cot(c+dx)}}{\sqrt{e}}\right)}{ade^{5/2}} + \frac{\text{arctanh}\left(\frac{\sqrt{e}+\sqrt{e}\cot(c+dx)}{\sqrt{2}\sqrt{e\cot(c+dx)}}\right)}{\sqrt{2}ade^{5/2}} + \frac{2}{3ade(e\cot(c+dx))^{3/2}} - \frac{2}{ade^2\sqrt{e\cot(c+dx)}}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 0.62 (sec) , antiderivative size = 86, normalized size of antiderivative = 0.64

$$\int \frac{1}{(e\cot(c+dx))^{5/2}(a+a\cot(c+dx))}dx = \frac{\text{Hypergeometric2F1}\left(-\frac{3}{2},1,-\frac{1}{2},-\cot(c+dx)\right) + \text{Hypergeom}}{}$$

```
[In] Integrate[1/((e*Cot[c + d*x])^(5/2)*(a + a*Cot[c + d*x])),x]
[Out] (Hypergeometric2F1[-3/2, 1, -1/2, -Cot[c + d*x]] + Hypergeometric2F1[-3/4,
1, 1/4, -Cot[c + d*x]^2] - 3*Cot[c + d*x]*Hypergeometric2F1[-1/4, 1, 3/4, -
Cot[c + d*x]^2])/(3*a*d*e*(e*Cot[c + d*x])^(3/2))
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 332 vs. 2(113) = 226.

Time = 0.04 (sec) , antiderivative size = 333, normalized size of antiderivative = 2.47

method	result
derivativedivides	$-\frac{2e^2\left(-\frac{(e^2)^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\frac{e\cot(dx+c)+(e^2)^{\frac{1}{4}}\sqrt{e\cot(dx+c)}\sqrt{2}+\sqrt{e^2}}{e\cot(dx+c)-(e^2)^{\frac{1}{4}}\sqrt{e\cot(dx+c)}\sqrt{2}+\sqrt{e^2}}\right)+2\arctan\left(\frac{\sqrt{2}\sqrt{e\cot(dx+c)}}{(e^2)^{\frac{1}{4}}}+1\right)-2\arctan\left(-\frac{\sqrt{2}\sqrt{e\cot(dx+}}{(e^2)^{\frac{1}{4}}}\right.\right.}{8e}\right.}{}$
default	$-\frac{2e^2\left(-\frac{(e^2)^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\frac{e\cot(dx+c)+(e^2)^{\frac{1}{4}}\sqrt{e\cot(dx+c)}\sqrt{2}+\sqrt{e^2}}{e\cot(dx+c)-(e^2)^{\frac{1}{4}}\sqrt{e\cot(dx+c)}\sqrt{2}+\sqrt{e^2}}\right)+2\arctan\left(\frac{\sqrt{2}\sqrt{e\cot(dx+c)}}{(e^2)^{\frac{1}{4}}}+1\right)-2\arctan\left(-\frac{\sqrt{2}\sqrt{e\cot(dx+}}{(e^2)^{\frac{1}{4}}}\right.\right.}{8e}\right.}{}$

```
[In] int(1/(e*cot(d*x+c))^(5/2)/(a+a*cot(d*x+c)),x,method=_RETURNVERBOSE)
```

```
[Out] -2/d/a*e^2*(1/2/e^4*(-1/8/e*(e^2)^(1/4)*2^(1/2)*(ln((e*cot(d*x+c)+(e^2)^(1/
4)*(e*cot(d*x+c))^(1/2)*2^(1/2)+(e^2)^(1/2))/(e*cot(d*x+c)-(e^2)^(1/4)*(e*c
ot(d*x+c))^(1/2)*2^(1/2)+(e^2)^(1/2)))+2*arctan(2^(1/2)/(e^2)^(1/4)*(e*cot(
d*x+c))^(1/2)+1)-2*arctan(-2^(1/2)/(e^2)^(1/4)*(e*cot(d*x+c))^(1/2)+1))+1/8
/(e^2)^(1/4)*2^(1/2)*(ln((e*cot(d*x+c)-(e^2)^(1/4)*(e*cot(d*x+c))^(1/2)*2^(
1/2)+(e^2)^(1/2))/(e*cot(d*x+c)+(e^2)^(1/4)*(e*cot(d*x+c))^(1/2)*2^(1/2)+(e
^2)^(1/2)))+2*arctan(2^(1/2)/(e^2)^(1/4)*(e*cot(d*x+c))^(1/2)+1)-2*arctan(-
2^(1/2)/(e^2)^(1/4)*(e*cot(d*x+c))^(1/2)+1)))-1/3/e^3/(e*cot(d*x+c))^(3/2)+
1/e^4/(e*cot(d*x+c))^(1/2)+1/2/e^(9/2)*arctan((e*cot(d*x+c))^(1/2)/e^(1/2))
)
```

Fricas [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 500, normalized size of antiderivative = 3.70

$$\int \frac{1}{(e\cot(c+dx))^{5/2}(a+a\cot(c+dx))}\,dx = \left[-\frac{3\sqrt{2}\sqrt{-e}(\cos(2\,dx+2\,c)+1)\arctan\left(\frac{\sqrt{2}\sqrt{-e}\sqrt{\frac{e\cos(2\,dx+2\,c)+e}{\sin(2\,dx+2\,c)}}(}{2\,(e\cos(}\right.}{}\right.$$

[In] integrate(1/(e*cot(d*x+c))^(5/2)/(a+a*cot(d*x+c)),x, algorithm="fricas")

[Out] [-1/6*(3*sqrt(2)*sqrt(-e)*(cos(2*d*x + 2*c) + 1)*arctan(1/2*sqrt(2)*sqrt(-e)*sqrt((e*cos(2*d*x + 2*c) + e)/sin(2*d*x + 2*c))*(cos(2*d*x + 2*c) + sin(2*d*x + 2*c) + 1)/(e*cos(2*d*x + 2*c) + e)) + 3*sqrt(-e)*(cos(2*d*x + 2*c) + 1)*log((e*cos(2*d*x + 2*c) - e*sin(2*d*x + 2*c) + 2*sqrt(-e)*sqrt((e*cos(2*d*x + 2*c) + e)/sin(2*d*x + 2*c))*sin(2*d*x + 2*c) + e)/(cos(2*d*x + 2*c) + sin(2*d*x + 2*c) + 1)) + 4*sqrt((e*cos(2*d*x + 2*c) + e)/sin(2*d*x + 2*c))*(cos(2*d*x + 2*c) + 3*sin(2*d*x + 2*c) - 1))/(a*d*e^3*cos(2*d*x + 2*c) + a*d*e^3), 1/12*(3*sqrt(2)*sqrt(e)*(cos(2*d*x + 2*c) + 1)*log(-sqrt(2)*sqrt(e)*sqrt((e*cos(2*d*x + 2*c) + e)/sin(2*d*x + 2*c))*(cos(2*d*x + 2*c) - sin(2*d*x + 2*c) - 1) + 2*e*sin(2*d*x + 2*c) + e) - 12*sqrt(e)*(cos(2*d*x + 2*c) + 1)*arctan(sqrt((e*cos(2*d*x + 2*c) + e)/sin(2*d*x + 2*c))/sqrt(e)) - 8*sqrt((e*cos(2*d*x + 2*c) + e)/sin(2*d*x + 2*c))*(cos(2*d*x + 2*c) + 3*sin(2*d*x + 2*c) - 1))/(a*d*e^3*cos(2*d*x + 2*c) + a*d*e^3)]

Sympy [F]

$$\int \frac{1}{(e\cot(c+dx))^{5/2}(a+a\cot(c+dx))}\,dx = \frac{\int \frac{1}{(e\cot(c+dx))^{\frac{5}{2}}\cot(c+dx)+(e\cot(c+dx))^{\frac{5}{2}}}\,dx}{a}$$

[In] integrate(1/(e*cot(d*x+c))**(5/2)/(a+a*cot(d*x+c)),x)

[Out] Integral(1/((e*cot(c + d*x))**(5/2)*cot(c + d*x) + (e*cot(c + d*x))**(5/2)), x)/a

Maxima [F(-2)]

Exception generated.

$$\int \frac{1}{(e\cot(c+dx))^{5/2}(a+a\cot(c+dx))}\,dx = \text{Exception raised: ValueError}$$

[In] integrate(1/(e*cot(d*x+c))^(5/2)/(a+a*cot(d*x+c)),x, algorithm="maxima")

[Out] Exception raised: ValueError >> Computation failed since Maxima requested additional constraints; using the 'assume' command before evaluation *may* help (example of legal syntax is 'assume(e>0)', see `assume?` for more details)Is e

Giac [F]

$$\int \frac{1}{(e\cot(c+dx))^{5/2}(a+a\cot(c+dx))}\,dx=\int \frac{1}{(a\cot(dx+c)+a)\,(e\cot(dx+c))^{\frac{5}{2}}}\,dx$$

```
[In] integrate(1/(e*cot(d*x+c))^(5/2)/(a+a*cot(d*x+c)),x, algorithm="giac")
[Out] integrate(1/((a*cot(d*x + c) + a)*(e*cot(d*x + c))^(5/2)), x)
```

Mupad [B] (verification not implemented)

Time = 13.31 (sec) , antiderivative size = 132, normalized size of antiderivative = 0.98

$$\int \frac{1}{(e\cot(c+dx))^{5/2}(a+a\cot(c+dx))}\,dx=\frac{\sqrt{2}\,\mathrm{atanh}\left(\frac{12\sqrt{2}\,a^3\,d^3\,e^{21/2}\,\sqrt{e\cot(c+d\,x)}}{12\,a^3\,d^3\,e^{11}+12\,a^3\,d^3\,e^{11}\cot(c+d\,x)}\right)}{2\,a\,d\,e^{5/2}}-\frac{\mathrm{atan}\left(\frac{\sqrt{e\cot(c+d\,x)}}{\sqrt{e}}\right)}{a\,d\,e^{5/2}}-\frac{\frac{2\cot(c+d\,x)}{e}-\frac{2}{3\,e}}{a\,d\,(e\cot(c+d\,x))^{3/2}}$$

```
[In] int(1/((e*cot(c + d*x))^(5/2)*(a + a*cot(c + d*x))),x)
[Out] (2^(1/2)*atanh((12*2^(1/2)*a^3*d^3*e^(21/2)*(e*cot(c + d*x))^(1/2))/(12*a^3
*d^3*e^11 + 12*a^3*d^3*e^11*cot(c + d*x))))/(2*a*d*e^(5/2)) - atan((e*cot(c
 + d*x))^(1/2)/e^(1/2))/(a*d*e^(5/2)) - ((2*cot(c + d*x))/e - 2/(3*e))/(a*d
*(e*cot(c + d*x))^(3/2))
```

3.29 $\int \frac{(e\cot(c+dx))^{5/2}}{(a+a\cot(c+dx))^2}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 281

$$\int \frac{(e\cot(c+dx))^{5/2}}{(a+a\cot(c+dx))^2}\,dx = -\frac{3e^{5/2}\arctan\left(\frac{\sqrt{e\cot(c+dx)}}{\sqrt{e}}\right)}{2a^2d} - \frac{e^{5/2}\arctan\left(1-\frac{\sqrt{2}\sqrt{e\cot(c+dx)}}{\sqrt{e}}\right)}{2\sqrt{2}a^2d} + \frac{e^{5/2}\arctan\left(1+\frac{\sqrt{2}\sqrt{e\cot(c+dx)}}{\sqrt{e}}\right)}{2\sqrt{2}a^2d} + \frac{e^2\sqrt{e\cot(c+dx)}}{2d\,(a^2+a^2\cot(c+dx))} - \frac{e^{5/2}\log\left(\sqrt{e}+\sqrt{e}\cot(c+dx)-\sqrt{2}\sqrt{e\cot(c+dx)}\right)}{4\sqrt{2}a^2d} + \frac{e^{5/2}\log\left(\sqrt{e}+\sqrt{e}\cot(c+dx)+\sqrt{2}\sqrt{e\cot(c+dx)}\right)}{4\sqrt{2}a^2d}$$

```
[Out] -3/2*e^(5/2)*arctan((e*cot(d*x+c))^(1/2)/e^(1/2))/a^2/d-1/4*e^(5/2)*arctan(
1-2^(1/2)*(e*cot(d*x+c))^(1/2)/e^(1/2))/a^2/d*2^(1/2)+1/4*e^(5/2)*arctan(1+
2^(1/2)*(e*cot(d*x+c))^(1/2)/e^(1/2))/a^2/d*2^(1/2)-1/8*e^(5/2)*ln(e^(1/2)+
cot(d*x+c)*e^(1/2)-2^(1/2)*(e*cot(d*x+c))^(1/2))/a^2/d*2^(1/2)+1/8*e^(5/2)*
ln(e^(1/2)+cot(d*x+c)*e^(1/2)+2^(1/2)*(e*cot(d*x+c))^(1/2))/a^2/d*2^(1/2)+1
/2*e^2*(e*cot(d*x+c))^(1/2)/d/(a^2+a^2*cot(d*x+c))
```

Rubi [A] (verified)

Time = 0.66 (sec) , antiderivative size = 281, normalized size of antiderivative = 1.00, number of steps used = 17, number of rules used = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.560, Rules used = {3646, 3734, 12, 3557, 335, 217, 1179, 642, 1176, 631, 210, 3715, 65, 211}

$$\int \frac{(e\cot(c+dx))^{5/2}}{(a+a\cot(c+dx))^2}\,dx = -\frac{3e^{5/2}\arctan\left(\frac{\sqrt{e\cot(c+dx)}}{\sqrt{e}}\right)}{2a^2d} - \frac{e^{5/2}\arctan\left(1-\frac{\sqrt{2}\sqrt{e\cot(c+dx)}}{\sqrt{e}}\right)}{2\sqrt{2}a^2d} + \frac{e^{5/2}\arctan\left(\frac{\sqrt{2}\sqrt{e\cot(c+dx)}}{\sqrt{e}}+1\right)}{2\sqrt{2}a^2d} - \frac{e^{5/2}\log\left(\sqrt{e}\cot(c+dx)-\sqrt{2}\sqrt{e\cot(c+dx)}+\sqrt{e}\right)}{4\sqrt{2}a^2d} + \frac{e^{5/2}\log\left(\sqrt{e}\cot(c+dx)+\sqrt{2}\sqrt{e\cot(c+dx)}+\sqrt{e}\right)}{4\sqrt{2}a^2d} + \frac{e^2\sqrt{e\cot(c+dx)}}{2d\left(a^2\cot(c+dx)+a^2\right)}$$

```
[In] Int[(e*Cot[c + d*x])^(5/2)/(a + a*Cot[c + d*x])^2,x]
```

```
[Out] (-3*e^(5/2)*ArcTan[Sqrt[e*Cot[c + d*x]]/Sqrt[e]])/(2*a^2*d) - (e^(5/2)*ArcT
an[1 - (Sqrt[2]*Sqrt[e*Cot[c + d*x]])/Sqrt[e]])/(2*Sqrt[2]*a^2*d) + (e^(5/2
)*ArcTan[1 + (Sqrt[2]*Sqrt[e*Cot[c + d*x]])/Sqrt[e]])/(2*Sqrt[2]*a^2*d) + (
e^2*Sqrt[e*Cot[c + d*x]])/(2*d*(a^2 + a^2*Cot[c + d*x])) - (e^(5/2)*Log[Sqr
t[e] + Sqrt[e]*Cot[c + d*x] - Sqrt[2]*Sqrt[e*Cot[c + d*x]]])/(4*Sqrt[2]*a^2
*d) + (e^(5/2)*Log[Sqrt[e] + Sqrt[e]*Cot[c + d*x] + Sqrt[2]*Sqrt[e*Cot[c +
d*x]]])/(4*Sqrt[2]*a^2*d)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 217

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4),
 x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b
}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &&
AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1179

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 3557

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[b/d, Subst[Int[
x^n/(b^2 + x^2), x], x, b*Tan[c + d*x]], x] /; FreeQ[{b, c, d, n}, x] &&  !
IntegerQ[n]
```

Rule 3646

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*c - a*d)^2*(a + b*Tan[e + f*x])^(m
 - 2)*((c + d*Tan[e + f*x])^(n + 1)/(d*f*(n + 1)*(c^2 + d^2))), x] - Dist[1
/(d*(n + 1)*(c^2 + d^2)), Int[(a + b*Tan[e + f*x])^(m - 3)*(c + d*Tan[e + f
*x])^(n + 1)*Simp[a^2*d*(b*d*(m - 2) - a*c*(n + 1)) + b*(b*c - 2*a*d)*(b*c*
(m - 2) + a*d*(n + 1)) - d*(n + 1)*(3*a^2*b*c - b^3*c - a^3*d + 3*a*b^2*d)*
Tan[e + f*x] - b*(a*d*(2*b*c - a*d)*(m + n - 1) - b^2*(c^2*(m - 2) - d^2*(n
 + 1)))*Tan[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b
*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] && GtQ[m, 2] && LtQ[
n, -1] && IntegerQ[2*m]
```

Rule 3715

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*tan[(e_.)
+ (f_.)*(x_)])^(n_.)*((A_) + (C_.)*tan[(e_.) + (f_.)*(x_)]^2), x_Symbol] :>
 Dist[A/f, Subst[Int[(a + b*x)^m*(c + d*x)^n, x], x, Tan[e + f*x]], x] /; F
reeQ[{a, b, c, d, e, f, A, C, m, n}, x] && EqQ[A, C]
```

Rule 3734

```
Int[(((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_)*((A_.) + (B_.)*tan[(e_.)
+ (f_.)*(x_)] + (C_.)*tan[(e_.) + (f_.)*(x_)]^2))/((a_.) + (b_.)*tan[(e_.)
+ (f_.)*(x_)]), x_Symbol] :> Dist[1/(a^2 + b^2), Int[(c + d*Tan[e + f*x])^n
*Simp[b*B + a*(A - C) + (a*B - b*(A - C))*Tan[e + f*x], x], x], x] + Dist[(
A*b^2 - a*b*B + a^2*C)/(a^2 + b^2), Int[(c + d*Tan[e + f*x])^n*((1 + Tan[e
+ f*x]^2)/(a + b*Tan[e + f*x])), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C
, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] &&
 !GtQ[n, 0] &&  !LeQ[n, -1]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \frac{e^2\sqrt{e\cot(c+dx)}}{2d\left(a^2+a^2\cot(c+dx)\right)}-\frac{\int \frac{-\frac{1}{2}a^2e^3+a^2e^3\cot(c+dx)-\frac{3}{2}a^2e^3\cot^2(c+dx)}{\sqrt{e\cot(c+dx)}(a+a\cot(c+dx))}\,dx}{2a^3}\\ &= \frac{e^2\sqrt{e\cot(c+dx)}}{2d\left(a^2+a^2\cot(c+dx)\right)}-\frac{\int \frac{2a^3e^3}{\sqrt{e\cot(c+dx)}}\,dx}{4a^5}+\frac{(3e^3)\int \frac{1+\cot^2(c+dx)}{\sqrt{e\cot(c+dx)}(a+a\cot(c+dx))}\,dx}{4a}\end{aligned}$$

$$= \frac{e^2\sqrt{e\cot(c+dx)}}{2d\left(a^2+a^2\cot(c+dx)\right)} - \frac{e^3\int\frac{1}{\sqrt{e\cot(c+dx)}}\,dx}{2a^2} + \frac{(3e^3)\,\text{Subst}\left(\int\frac{1}{\sqrt{-ex}(a-ax)}\,dx, x, -\cot(c+dx)\right)}{4ad}$$

$$= \frac{e^2\sqrt{e\cot(c+dx)}}{2d\left(a^2+a^2\cot(c+dx)\right)} - \frac{(3e^2)\,\text{Subst}\left(\int\frac{1}{a+\frac{ax^2}{e}}\,dx, x, \sqrt{e\cot(c+dx)}\right)}{2ad} + \frac{e^4\text{Subst}\left(\int\frac{1}{\sqrt{x}(e^2+x^2)}\,dx, x, e\cot(c+dx)\right)}{2a^2d}$$

$$= -\frac{3e^{5/2}\arctan\left(\frac{\sqrt{e\cot(c+dx)}}{\sqrt{e}}\right)}{2a^2d} + \frac{e^2\sqrt{e\cot(c+dx)}}{2d\left(a^2+a^2\cot(c+dx)\right)} + \frac{e^4\text{Subst}\left(\int\frac{1}{e^2+x^4}\,dx, x, \sqrt{e\cot(c+dx)}\right)}{a^2d}$$

$$= -\frac{3e^{5/2}\arctan\left(\frac{\sqrt{e\cot(c+dx)}}{\sqrt{e}}\right)}{2a^2d} + \frac{e^2\sqrt{e\cot(c+dx)}}{2d\left(a^2+a^2\cot(c+dx)\right)} + \frac{e^3\text{Subst}\left(\int\frac{e-x^2}{e^2+x^4}\,dx, x, \sqrt{e\cot(c+dx)}\right)}{2a^2d} + \frac{e^3\text{Subst}\left(\int\frac{e+x^2}{e^2+x^4}\,dx, x, \sqrt{e\cot(c+dx)}\right)}{2a^2d}$$

$$= -\frac{3e^{5/2}\arctan\left(\frac{\sqrt{e\cot(c+dx)}}{\sqrt{e}}\right)}{2a^2d} + \frac{e^2\sqrt{e\cot(c+dx)}}{2d\left(a^2+a^2\cot(c+dx)\right)} - \frac{e^{5/2}\text{Subst}\left(\int\frac{\sqrt{2}\sqrt{e}+2x}{-e-\sqrt{2}\sqrt{e}x-x^2}\,dx, x, \sqrt{e\cot(c+dx)}\right)}{4\sqrt{2}a^2d} - \frac{e^{5/2}\text{Subst}\left(\int\frac{\sqrt{2}\sqrt{e}-2x}{-e+\sqrt{2}\sqrt{e}x-x^2}\,dx, x, \sqrt{e\cot(c+dx)}\right)}{4\sqrt{2}a^2d} + \frac{e^3\text{Subst}\left(\int\frac{1}{e-\sqrt{2}\sqrt{e}x+x^2}\,dx, x, \sqrt{e\cot(c+dx)}\right)}{4a^2d} + \frac{e^3\text{Subst}\left(\int\frac{1}{e+\sqrt{2}\sqrt{e}x+x^2}\,dx, x, \sqrt{e\cot(c+dx)}\right)}{4a^2d}$$

$$
\begin{aligned}
&= -\frac{3e^{5/2}\arctan\left(\frac{\sqrt{e\cot(c+dx)}}{\sqrt{e}}\right)}{2a^2d} + \frac{e^2\sqrt{e\cot(c+dx)}}{2d\,(a^2+a^2\cot(c+dx))} \\
&\quad - \frac{e^{5/2}\log\left(\sqrt{e}+\sqrt{e}\cot(c+dx)-\sqrt{2}\sqrt{e\cot(c+dx)}\right)}{4\sqrt{2}a^2d} \\
&\quad + \frac{e^{5/2}\log\left(\sqrt{e}+\sqrt{e}\cot(c+dx)+\sqrt{2}\sqrt{e\cot(c+dx)}\right)}{4\sqrt{2}a^2d} \\
&\quad + \frac{e^{5/2}\text{Subst}\left(\int\frac{1}{-1-x^2}\,dx, x, 1-\frac{\sqrt{2}\sqrt{e\cot(c+dx)}}{\sqrt{e}}\right)}{2\sqrt{2}a^2d} \\
&\quad - \frac{e^{5/2}\text{Subst}\left(\int\frac{1}{-1-x^2}\,dx, x, 1+\frac{\sqrt{2}\sqrt{e\cot(c+dx)}}{\sqrt{e}}\right)}{2\sqrt{2}a^2d} \\
&= -\frac{3e^{5/2}\arctan\left(\frac{\sqrt{e\cot(c+dx)}}{\sqrt{e}}\right)}{2a^2d} - \frac{e^{5/2}\arctan\left(1-\frac{\sqrt{2}\sqrt{e\cot(c+dx)}}{\sqrt{e}}\right)}{2\sqrt{2}a^2d} \\
&\quad + \frac{e^{5/2}\arctan\left(1+\frac{\sqrt{2}\sqrt{e\cot(c+dx)}}{\sqrt{e}}\right)}{2\sqrt{2}a^2d} + \frac{e^2\sqrt{e\cot(c+dx)}}{2d\,(a^2+a^2\cot(c+dx))} \\
&\quad - \frac{e^{5/2}\log\left(\sqrt{e}+\sqrt{e}\cot(c+dx)-\sqrt{2}\sqrt{e\cot(c+dx)}\right)}{4\sqrt{2}a^2d} \\
&\quad + \frac{e^{5/2}\log\left(\sqrt{e}+\sqrt{e}\cot(c+dx)+\sqrt{2}\sqrt{e\cot(c+dx)}\right)}{4\sqrt{2}a^2d}
\end{aligned}
$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 2.38 (sec) , antiderivative size = 288, normalized size of antiderivative = 1.02

$$
\int \frac{(e\cot(c+dx))^{5/2}}{(a+a\cot(c+dx))^2}\,dx = \frac{e^2\left(-24\cot^3(c+dx)\sqrt{e\cot(c+dx)}\,\text{Hypergeometric2F1}\left(2,\frac{7}{2},\frac{9}{2},-\cot(c+dx)\right)\right.}{}
$$

```
[In] Integrate[(e*Cot[c + d*x])^(5/2)/(a + a*Cot[c + d*x])^2,x]
```

```
[Out] (e^2*(-24*Cot[c + d*x]^3*Sqrt[e*Cot[c + d*x]]*Hypergeometric2F1[2, 7/2, 9/2
, -Cot[c + d*x]] + 7*(24*Sqrt[e]*ArcTan[Sqrt[e*Cot[c + d*x]]/Sqrt[e]] - 6*S
qrt[2]*Sqrt[e]*ArcTan[1 - (Sqrt[2]*Sqrt[e*Cot[c + d*x]])/Sqrt[e]] + 6*Sqrt[
2]*Sqrt[e]*ArcTan[1 + (Sqrt[2]*Sqrt[e*Cot[c + d*x]])/Sqrt[e]] - 48*Sqrt[e*C
ot[c + d*x]] + (8*(e*Cot[c + d*x])^(3/2))/e - 3*Sqrt[2]*Sqrt[e]*Log[Sqrt[e]
 + Sqrt[e]*Cot[c + d*x] - Sqrt[2]*Sqrt[e*Cot[c + d*x]]] + 3*Sqrt[2]*Sqrt[e]
*Log[Sqrt[e] + Sqrt[e]*Cot[c + d*x] + Sqrt[2]*Sqrt[e*Cot[c + d*x]]])))/(168
*a^2*d)
```

Maple [A] (verified)

Time = 0.32 (sec) , antiderivative size = 191, normalized size of antiderivative = 0.68

method	result
derivativedivides	$-\frac{2e^3\left(-\frac{(e^2)^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\frac{e\cot(dx+c)+(e^2)^{\frac{1}{4}}\sqrt{e\cot(dx+c)}\sqrt{2}+\sqrt{e^2}}{e\cot(dx+c)-(e^2)^{\frac{1}{4}}\sqrt{e\cot(dx+c)}\sqrt{2}+\sqrt{e^2}}\right)+2\arctan\left(\frac{\sqrt{2}\sqrt{e\cot(dx+c)}}{(e^2)^{\frac{1}{4}}}+1\right)-2\arctan\left(-\frac{\sqrt{2}\sqrt{e\cot(c}}{(e^2)^{\frac{1}{4}}}\right.\right.}{16e}\right.}{d\,a^2}$
default	$-\frac{2e^3\left(-\frac{(e^2)^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\frac{e\cot(dx+c)+(e^2)^{\frac{1}{4}}\sqrt{e\cot(dx+c)}\sqrt{2}+\sqrt{e^2}}{e\cot(dx+c)-(e^2)^{\frac{1}{4}}\sqrt{e\cot(dx+c)}\sqrt{2}+\sqrt{e^2}}\right)+2\arctan\left(\frac{\sqrt{2}\sqrt{e\cot(dx+c)}}{(e^2)^{\frac{1}{4}}}+1\right)-2\arctan\left(-\frac{\sqrt{2}\sqrt{e\cot(c}}{(e^2)^{\frac{1}{4}}}\right.\right.}{16e}\right.}{d\,a^2}$

```
[In] int((e*cot(d*x+c))^(5/2)/(a+a*cot(d*x+c))^2,x,method=_RETURNVERBOSE)
```

```
[Out] -2/d/a^2*e^3*(-1/16/e*(e^2)^(1/4)*2^(1/2)*(ln((e*cot(d*x+c)+(e^2)^(1/4)*(e*
cot(d*x+c))^(1/2)*2^(1/2)+(e^2)^(1/2))/(e*cot(d*x+c)-(e^2)^(1/4)*(e*cot(d*x
+c))^(1/2)*2^(1/2)+(e^2)^(1/2)))+2*arctan(2^(1/2)/(e^2)^(1/4)*(e*cot(d*x+c)
)^(1/2)+1)-2*arctan(-2^(1/2)/(e^2)^(1/4)*(e*cot(d*x+c))^(1/2)+1))-1/4*(e*co
t(d*x+c))^(1/2)/(e*cot(d*x+c)+e)+3/4/e^(1/2)*arctan((e*cot(d*x+c))^(1/2)/e^
(1/2)))
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.30 (sec) , antiderivative size = 1173, normalized size of antiderivative = 4.17

$$\int \frac{(e\cot(c+dx))^{5/2}}{(a+a\cot(c+dx))^2}\,dx = \text{Too large to display}$$

```
[In] integrate((e*cot(d*x+c))^(5/2)/(a+a*cot(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] [1/4*(2*e^2*sqrt((e*cos(2*d*x + 2*c) + e)/sin(2*d*x + 2*c))*sin(2*d*x + 2*c
) + 3*(e^2*cos(2*d*x + 2*c) + e^2*sin(2*d*x + 2*c) + e^2)*sqrt(-e)*log((e*c
os(2*d*x + 2*c) - e*sin(2*d*x + 2*c) - 2*sqrt(-e)*sqrt((e*cos(2*d*x + 2*c)
+ e)/sin(2*d*x + 2*c))*sin(2*d*x + 2*c) + e)/(cos(2*d*x + 2*c) + sin(2*d*x
+ 2*c) + 1)) + (a^2*d*cos(2*d*x + 2*c) + a^2*d*sin(2*d*x + 2*c) + a^2*d)*(-
e^10/(a^8*d^4))^(1/4)*log(a^2*d*(-e^10/(a^8*d^4))^(1/4) + e^2*sqrt((e*cos(2
*d*x + 2*c) + e)/sin(2*d*x + 2*c))) - (-I*a^2*d*cos(2*d*x + 2*c) - I*a^2*d*
sin(2*d*x + 2*c) - I*a^2*d)*(-e^10/(a^8*d^4))^(1/4)*log(I*a^2*d*(-e^10/(a^8
*d^4))^(1/4) + e^2*sqrt((e*cos(2*d*x + 2*c) + e)/sin(2*d*x + 2*c))) - (I*a^
2*d*cos(2*d*x + 2*c) + I*a^2*d*sin(2*d*x + 2*c) + I*a^2*d)*(-e^10/(a^8*d^4)
```

```
)^(1/4)*log(-I*a^2*d*(-e^10/(a^8*d^4))^(1/4) + e^2*sqrt((e*cos(2*d*x + 2*c)
 + e)/sin(2*d*x + 2*c))) - (a^2*d*cos(2*d*x + 2*c) + a^2*d*sin(2*d*x + 2*c)
 + a^2*d)*(-e^10/(a^8*d^4))^(1/4)*log(-a^2*d*(-e^10/(a^8*d^4))^(1/4) + e^2*
sqrt((e*cos(2*d*x + 2*c) + e)/sin(2*d*x + 2*c))))/(a^2*d*cos(2*d*x + 2*c) +
 a^2*d*sin(2*d*x + 2*c) + a^2*d), 1/4*(2*e^2*sqrt((e*cos(2*d*x + 2*c) + e)/
sin(2*d*x + 2*c))*sin(2*d*x + 2*c) - 6*(e^2*cos(2*d*x + 2*c) + e^2*sin(2*d*
x + 2*c) + e^2)*sqrt(e)*arctan(sqrt((e*cos(2*d*x + 2*c) + e)/sin(2*d*x + 2*
c))/sqrt(e)) + (a^2*d*cos(2*d*x + 2*c) + a^2*d*sin(2*d*x + 2*c) + a^2*d)*(-
e^10/(a^8*d^4))^(1/4)*log(a^2*d*(-e^10/(a^8*d^4))^(1/4) + e^2*sqrt((e*cos(2
*d*x + 2*c) + e)/sin(2*d*x + 2*c))) - (-I*a^2*d*cos(2*d*x + 2*c) - I*a^2*d*
sin(2*d*x + 2*c) - I*a^2*d)*(-e^10/(a^8*d^4))^(1/4)*log(I*a^2*d*(-e^10/(a^8
*d^4))^(1/4) + e^2*sqrt((e*cos(2*d*x + 2*c) + e)/sin(2*d*x + 2*c))) - (I*a^
2*d*cos(2*d*x + 2*c) + I*a^2*d*sin(2*d*x + 2*c) + I*a^2*d)*(-e^10/(a^8*d^4)
)^(1/4)*log(-I*a^2*d*(-e^10/(a^8*d^4))^(1/4) + e^2*sqrt((e*cos(2*d*x + 2*c)
 + e)/sin(2*d*x + 2*c))) - (a^2*d*cos(2*d*x + 2*c) + a^2*d*sin(2*d*x + 2*c)
 + a^2*d)*(-e^10/(a^8*d^4))^(1/4)*log(-a^2*d*(-e^10/(a^8*d^4))^(1/4) + e^2*
sqrt((e*cos(2*d*x + 2*c) + e)/sin(2*d*x + 2*c))))/(a^2*d*cos(2*d*x + 2*c) +
 a^2*d*sin(2*d*x + 2*c) + a^2*d)]
```

Sympy [F]

$$\int \frac{(e\cot(c+dx))^{5/2}}{(a+a\cot(c+dx))^2}\,dx = \frac{\int \frac{(e\cot(c+dx))^{\frac{5}{2}}}{\cot^2(c+dx)+2\cot(c+dx)+1}\,dx}{a^2}$$

[In] integrate((e*cot(d*x+c))**(5/2)/(a+a*cot(d*x+c))**2,x)

[Out] Integral((e*cot(c + d*x))**(5/2)/(cot(c + d*x)**2 + 2*cot(c + d*x) + 1), x)
/a**2

Maxima [F(-2)]

Exception generated.

$$\int \frac{(e\cot(c+dx))^{5/2}}{(a+a\cot(c+dx))^2}\,dx = \text{Exception raised: ValueError}$$

[In] integrate((e*cot(d*x+c))^(5/2)/(a+a*cot(d*x+c))^2,x, algorithm="maxima")

[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(e>0)', see `assume?` for more detai
ls)Is e

Giac [F]

$$\int \frac{(e\cot(c+dx))^{5/2}}{(a+a\cot(c+dx))^2}\,dx = \int \frac{(e\cot(dx+c))^{\frac{5}{2}}}{(a\cot(dx+c)+a)^2}\,dx$$

[In] integrate((e*cot(d*x+c))^(5/2)/(a+a*cot(d*x+c))^2,x, algorithm="giac")

[Out] integrate((e*cot(d*x + c))^(5/2)/(a*cot(d*x + c) + a)^2, x)

Mupad [B] (verification not implemented)

Time = 13.30 (sec) , antiderivative size = 375, normalized size of antiderivative = 1.33

$$\int \frac{(e\cot(c+dx))^{5/2}}{(a+a\cot(c+dx))^2}\,dx = \frac{e^3\sqrt{e\cot(c+dx)}}{2\left(a^2de+a^2de\cot(c+dx)\right)} - \operatorname{atan}\left(\frac{e^{20}\sqrt{e\cot(c+dx)}\left(-\frac{e^{10}}{256a^8d^4}\right)^{1/4}16\mathrm{i}}{\frac{36e^{23}}{a^2d}+64a^2de^{18}\sqrt{-\frac{e^{10}}{256a^8d^4}}} - \frac{e^{15}\sqrt{e\cot(c+dx)}\left(-\frac{e^{10}}{256a^8d^4}\right)^{3/4}2304\mathrm{i}}{\frac{36e^{23}}{a^6d^3}+\frac{64e^{18}\sqrt{-\frac{e^{10}}{256a^8d^4}}}{a^2d}}\right)\left(-\frac{e^{10}}{256a^8d^4}\right)^{1/4}2\mathrm{i} - \frac{\operatorname{atan}\left(\frac{4e^{20}\sqrt{e\cot(c+dx)}\left(-\frac{e^{10}}{a^8d^4}\right)^{1/4}}{\frac{36e^{23}}{a^2d}-4a^2de^{18}\sqrt{-\frac{e^{10}}{a^8d^4}}} + \frac{36e^{15}\sqrt{\ldots}}{\ldots}\right)\ldots}{2}\ldots$$

[In] int((e*cot(c + d*x))^(5/2)/(a + a*cot(c + d*x))^2,x)

[Out] (e^3*(e*cot(c + d*x))^(1/2))/(2*(a^2*d*e + a^2*d*e*cot(c + d*x))) - atan((e^20*(e*cot(c + d*x))^(1/2)*(-e^10/(256*a^8*d^4))^(1/4)*16i)/((36*e^23)/(a^2*d) + 64*a^2*d*e^18*(-e^10/(256*a^8*d^4))^(1/2)) - (e^15*(e*cot(c + d*x))^(1/2)*(-e^10/(256*a^8*d^4))^(3/4)*2304i)/((36*e^23)/(a^6*d^3) + (64*e^18*(-e^10/(256*a^8*d^4))^(1/2))/(a^2*d)))*(-e^10/(256*a^8*d^4))^(1/4)*2i - (atan((4*e^20*(e*cot(c + d*x))^(1/2)*(-e^10/(a^8*d^4))^(1/4))/((36*e^23)/(a^2*d) - 4*a^2*d*e^18*(-e^10/(a^8*d^4))^(1/2)) + (36*e^15*(e*cot(c + d*x))^(1/2)*(-e^10/(a^8*d^4))^(3/4))/((36*e^23)/(a^6*d^3) - (4*e^18*(-e^10/(a^8*d^4))^(1/2))/(a^2*d)))*(-e^10/(a^8*d^4))^(1/4))/2 - (atan(((e*cot(c + d*x))^(1/2)*(-e^5)^(1/2)*1i)/e^3)*(-e^5)^(1/2)*3i)/(2*a^2*d)

3.30 $\int \frac{(e\cot(c+dx))^{3/2}}{(a+a\cot(c+dx))^2}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 279

$$\int \frac{(e\cot(c+dx))^{3/2}}{(a+a\cot(c+dx))^2}\,dx = \frac{e^{3/2}\arctan\left(\frac{\sqrt{e\cot(c+dx)}}{\sqrt{e}}\right)}{2a^2d} + \frac{e^{3/2}\arctan\left(1-\frac{\sqrt{2}\sqrt{e\cot(c+dx)}}{\sqrt{e}}\right)}{2\sqrt{2}a^2d} - \frac{e^{3/2}\arctan\left(1+\frac{\sqrt{2}\sqrt{e\cot(c+dx)}}{\sqrt{e}}\right)}{2\sqrt{2}a^2d} - \frac{e\sqrt{e\cot(c+dx)}}{2d\,(a^2+a^2\cot(c+dx))} - \frac{e^{3/2}\log\left(\sqrt{e}+\sqrt{e}\cot(c+dx)-\sqrt{2}\sqrt{e\cot(c+dx)}\right)}{4\sqrt{2}a^2d} + \frac{e^{3/2}\log\left(\sqrt{e}+\sqrt{e}\cot(c+dx)+\sqrt{2}\sqrt{e\cot(c+dx)}\right)}{4\sqrt{2}a^2d}$$

```
[Out] 1/2*e^(3/2)*arctan((e*cot(d*x+c))^(1/2)/e^(1/2))/a^2/d+1/4*e^(3/2)*arctan(1
-2^(1/2)*(e*cot(d*x+c))^(1/2)/e^(1/2))/a^2/d*2^(1/2)-1/4*e^(3/2)*arctan(1+2
^(1/2)*(e*cot(d*x+c))^(1/2)/e^(1/2))/a^2/d*2^(1/2)-1/8*e^(3/2)*ln(e^(1/2)+c
ot(d*x+c)*e^(1/2)-2^(1/2)*(e*cot(d*x+c))^(1/2))/a^2/d*2^(1/2)+1/8*e^(3/2)*l
n(e^(1/2)+cot(d*x+c)*e^(1/2)+2^(1/2)*(e*cot(d*x+c))^(1/2))/a^2/d*2^(1/2)-1/
2*e*(e*cot(d*x+c))^(1/2)/d/(a^2+a^2*cot(d*x+c))
```

Rubi [A] (verified)

Time = 0.64 (sec) , antiderivative size = 279, normalized size of antiderivative = 1.00, number of steps used = 18, number of rules used = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.600, Rules used = {3648, 3734, 12, 16, 3557, 335, 303, 1176, 631, 210, 1179, 642, 3715, 65, 211}

$$\int \frac{(e\cot(c+dx))^{3/2}}{(a+a\cot(c+dx))^2}dx = \frac{e^{3/2}\arctan\left(\frac{\sqrt{e\cot(c+dx)}}{\sqrt{e}}\right)}{2a^2d} + \frac{e^{3/2}\arctan\left(1-\frac{\sqrt{2}\sqrt{e\cot(c+dx)}}{\sqrt{e}}\right)}{2\sqrt{2}a^2d} - \frac{e^{3/2}\arctan\left(\frac{\sqrt{2}\sqrt{e\cot(c+dx)}}{\sqrt{e}}+1\right)}{2\sqrt{2}a^2d} - \frac{e^{3/2}\log\left(\sqrt{e}\cot(c+dx)-\sqrt{2}\sqrt{e\cot(c+dx)}+\sqrt{e}\right)}{4\sqrt{2}a^2d} + \frac{e^{3/2}\log\left(\sqrt{e}\cot(c+dx)+\sqrt{2}\sqrt{e\cot(c+dx)}+\sqrt{e}\right)}{4\sqrt{2}a^2d} - \frac{e\sqrt{e\cot(c+dx)}}{2d\left(a^2\cot(c+dx)+a^2\right)}$$

```
[In] Int[(e*Cot[c + d*x])^(3/2)/(a + a*Cot[c + d*x])^2,x]
```

```
[Out] (e^(3/2)*ArcTan[Sqrt[e*Cot[c + d*x]]/Sqrt[e]])/(2*a^2*d) + (e^(3/2)*ArcTan[
1 - (Sqrt[2]*Sqrt[e*Cot[c + d*x]])/Sqrt[e]])/(2*Sqrt[2]*a^2*d) - (e^(3/2)*A
rcTan[1 + (Sqrt[2]*Sqrt[e*Cot[c + d*x]])/Sqrt[e]])/(2*Sqrt[2]*a^2*d) - (e*S
qrt[e*Cot[c + d*x]])/(2*d*(a^2 + a^2*Cot[c + d*x])) - (e^(3/2)*Log[Sqrt[e]
+ Sqrt[e]*Cot[c + d*x] - Sqrt[2]*Sqrt[e*Cot[c + d*x]]])/(4*Sqrt[2]*a^2*d) +
 (e^(3/2)*Log[Sqrt[e] + Sqrt[e]*Cot[c + d*x] + Sqrt[2]*Sqrt[e*Cot[c + d*x]]
])/(4*Sqrt[2]*a^2*d)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 16

```
Int[(u_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Dist[1/b^m, Int[u*(b*v)
^(m + n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 303

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*s), Int[(r + s*x^2)/(a + b*x^4
), x], x] - Dist[1/(2*s), Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1179

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 3557

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[b/d, Subst[Int[
x^n/(b^2 + x^2), x], x, b*Tan[c + d*x]], x] /; FreeQ[{b, c, d, n}, x] &&  !
IntegerQ[n]
```

Rule 3648

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*c - a*d)*(a + b*Tan[e + f*x])^(m +
 1)*((c + d*Tan[e + f*x])^(n - 1)/(f*(m + 1)*(a^2 + b^2))), x] + Dist[1/((m
 + 1)*(a^2 + b^2)), Int[(a + b*Tan[e + f*x])^(m + 1)*(c + d*Tan[e + f*x])^(
n - 2)*Simp[a*c^2*(m + 1) + a*d^2*(n - 1) + b*c*d*(m - n + 2) - (b*c^2 - 2*
a*c*d - b*d^2)*(m + 1)*Tan[e + f*x] - d*(b*c - a*d)*(m + n)*Tan[e + f*x]^2,
 x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2
 + b^2, 0] && NeQ[c^2 + d^2, 0] && LtQ[m, -1] && LtQ[1, n, 2] && IntegerQ[2
*m]
```

Rule 3715

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*tan[(e_.)
+ (f_.)*(x_)])^(n_.)*((A_) + (C_.)*tan[(e_.) + (f_.)*(x_)]^2), x_Symbol] :>
 Dist[A/f, Subst[Int[(a + b*x)^m*(c + d*x)^n, x], x, Tan[e + f*x]], x] /; F
reeQ[{a, b, c, d, e, f, A, C, m, n}, x] && EqQ[A, C]
```

Rule 3734

```
Int[(((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_)*((A_.) + (B_.)*tan[(e_.)
+ (f_.)*(x_)] + (C_.)*tan[(e_.) + (f_.)*(x_)]^2))/((a_.) + (b_.)*tan[(e_.)
+ (f_.)*(x_)]), x_Symbol] :> Dist[1/(a^2 + b^2), Int[(c + d*Tan[e + f*x])^n
*Simp[b*B + a*(A - C) + (a*B - b*(A - C))*Tan[e + f*x], x], x], x] + Dist[(
A*b^2 - a*b*B + a^2*C)/(a^2 + b^2), Int[(c + d*Tan[e + f*x])^n*((1 + Tan[e
+ f*x]^2)/(a + b*Tan[e + f*x])), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C
, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] &&
 !GtQ[n, 0] &&  !LeQ[n, -1]
```

Rubi steps

$$\text{integral} = -\frac{e\sqrt{e\cot(c+dx)}}{2d\left(a^2+a^2\cot(c+dx)\right)} - \frac{\int \frac{\frac{ae^2}{2}-ae^2\cot(c+dx)-\frac{1}{2}ae^2\cot^2(c+dx)}{\sqrt{e\cot(c+dx)}(a+a\cot(c+dx))}\,dx}{2a^2}$$

$$= -\frac{e\sqrt{e\cot(c+dx)}}{2d\left(a^2+a^2\cot(c+dx)\right)} - \frac{\int -\frac{2a^2e^2\cot(c+dx)}{\sqrt{e\cot(c+dx)}}\,dx}{4a^4} - \frac{e^2\int \frac{1+\cot^2(c+dx)}{\sqrt{e\cot(c+dx)}(a+a\cot(c+dx))}\,dx}{4a}$$

$$= -\frac{e\sqrt{e\cot(c+dx)}}{2d\left(a^2+a^2\cot(c+dx)\right)} + \frac{e^2\int \frac{\cot(c+dx)}{\sqrt{e\cot(c+dx)}}\,dx}{2a^2} - \frac{e^2\text{Subst}\left(\int \frac{1}{\sqrt{-ex}(a-ax)}\,dx, x, -\cot(c+dx)\right)}{4ad}$$

$$= -\frac{e\sqrt{e\cot(c+dx)}}{2d\left(a^2+a^2\cot(c+dx)\right)} + \frac{e\int \sqrt{e\cot(c+dx)}\,dx}{2a^2} + \frac{e\text{Subst}\left(\int \frac{1}{a+\frac{ax^2}{e}}\,dx, x, \sqrt{e\cot(c+dx)}\right)}{2ad}$$

$$= \frac{e^{3/2}\arctan\left(\frac{\sqrt{e\cot(c+dx)}}{\sqrt{e}}\right)}{2a^2d} - \frac{e\sqrt{e\cot(c+dx)}}{2d\left(a^2+a^2\cot(c+dx)\right)} - \frac{e^2\text{Subst}\left(\int \frac{\sqrt{x}}{e^2+x^2}\,dx, x, e\cot(c+dx)\right)}{2a^2d}$$

$$= \frac{e^{3/2}\arctan\left(\frac{\sqrt{e\cot(c+dx)}}{\sqrt{e}}\right)}{2a^2d} - \frac{e\sqrt{e\cot(c+dx)}}{2d\left(a^2+a^2\cot(c+dx)\right)} - \frac{e^2\text{Subst}\left(\int \frac{x^2}{e^2+x^4}\,dx, x, \sqrt{e\cot(c+dx)}\right)}{a^2d}$$

$$= \frac{e^{3/2}\arctan\left(\frac{\sqrt{e\cot(c+dx)}}{\sqrt{e}}\right)}{2a^2d} - \frac{e\sqrt{e\cot(c+dx)}}{2d\left(a^2+a^2\cot(c+dx)\right)} + \frac{e^2\text{Subst}\left(\int \frac{e-x^2}{e^2+x^4}\,dx, x, \sqrt{e\cot(c+dx)}\right)}{2a^2d} - \frac{e^2\text{Subst}\left(\int \frac{e+x^2}{e^2+x^4}\,dx, x, \sqrt{e\cot(c+dx)}\right)}{2a^2d}$$

$$= \frac{e^{3/2}\arctan\left(\frac{\sqrt{e\cot(c+dx)}}{\sqrt{e}}\right)}{2a^2d} - \frac{e\sqrt{e\cot(c+dx)}}{2d\left(a^2+a^2\cot(c+dx)\right)} - \frac{e^{3/2}\text{Subst}\left(\int \frac{\sqrt{2}\sqrt{e}+2x}{-e-\sqrt{2}\sqrt{e}x-x^2}\,dx, x, \sqrt{e\cot(c+dx)}\right)}{4\sqrt{2}a^2d} - \frac{e^{3/2}\text{Subst}\left(\int \frac{\sqrt{2}\sqrt{e}-2x}{-e+\sqrt{2}\sqrt{e}x-x^2}\,dx, x, \sqrt{e\cot(c+dx)}\right)}{4\sqrt{2}a^2d} - \frac{e^2\text{Subst}\left(\int \frac{1}{e-\sqrt{2}\sqrt{e}x+x^2}\,dx, x, \sqrt{e\cot(c+dx)}\right)}{4a^2d} - \frac{e^2\text{Subst}\left(\int \frac{1}{e+\sqrt{2}\sqrt{e}x+x^2}\,dx, x, \sqrt{e\cot(c+dx)}\right)}{4a^2d}$$

$$
\begin{aligned}
&= \frac{e^{3/2}\arctan\left(\frac{\sqrt{e\cot(c+dx)}}{\sqrt{e}}\right)}{2a^2d} - \frac{e\sqrt{e\cot(c+dx)}}{2d\left(a^2+a^2\cot(c+dx)\right)} \\
&\quad - \frac{e^{3/2}\log\left(\sqrt{e}+\sqrt{e}\cot(c+dx)-\sqrt{2}\sqrt{e\cot(c+dx)}\right)}{4\sqrt{2}a^2d} \\
&\quad + \frac{e^{3/2}\log\left(\sqrt{e}+\sqrt{e}\cot(c+dx)+\sqrt{2}\sqrt{e\cot(c+dx)}\right)}{4\sqrt{2}a^2d} \\
&\quad - \frac{e^{3/2}\text{Subst}\left(\int\frac{1}{-1-x^2}\,dx,x,1-\frac{\sqrt{2}\sqrt{e\cot(c+dx)}}{\sqrt{e}}\right)}{2\sqrt{2}a^2d} \\
&\quad + \frac{e^{3/2}\text{Subst}\left(\int\frac{1}{-1-x^2}\,dx,x,1+\frac{\sqrt{2}\sqrt{e\cot(c+dx)}}{\sqrt{e}}\right)}{2\sqrt{2}a^2d} \\
&= \frac{e^{3/2}\arctan\left(\frac{\sqrt{e\cot(c+dx)}}{\sqrt{e}}\right)}{2a^2d} + \frac{e^{3/2}\arctan\left(1-\frac{\sqrt{2}\sqrt{e\cot(c+dx)}}{\sqrt{e}}\right)}{2\sqrt{2}a^2d} \\
&\quad - \frac{e^{3/2}\arctan\left(1+\frac{\sqrt{2}\sqrt{e\cot(c+dx)}}{\sqrt{e}}\right)}{2\sqrt{2}a^2d} - \frac{e\sqrt{e\cot(c+dx)}}{2d\left(a^2+a^2\cot(c+dx)\right)} \\
&\quad - \frac{e^{3/2}\log\left(\sqrt{e}+\sqrt{e}\cot(c+dx)-\sqrt{2}\sqrt{e\cot(c+dx)}\right)}{4\sqrt{2}a^2d} \\
&\quad + \frac{e^{3/2}\log\left(\sqrt{e}+\sqrt{e}\cot(c+dx)+\sqrt{2}\sqrt{e\cot(c+dx)}\right)}{4\sqrt{2}a^2d}
\end{aligned}
$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 1.22 (sec) , antiderivative size = 155, normalized size of antiderivative = 0.56

$$
\int \frac{(e\cot(c+dx))^{3/2}}{(a+a\cot(c+dx))^2}\,dx = \frac{5\left(-2e^{3/2}\arctan\left(\frac{\sqrt{e\cot(c+dx)}}{\sqrt{e}}\right)+(-e^2)^{3/4}\arctan\left(\frac{\sqrt{e\cot(c+dx)}}{\sqrt[4]{-e^2}}\right)-(-e^2)^{3/4}\text{arc}
$$

```
[In] Integrate[(e*Cot[c + d*x])^(3/2)/(a + a*Cot[c + d*x])^2,x]
```

```
[Out] (5*(-2*e^(3/2)*ArcTan[Sqrt[e*Cot[c + d*x]]/Sqrt[e]] + (-e^2)^(3/4)*ArcTan[S
qrt[e*Cot[c + d*x]]/(-e^2)^(1/4)] - (-e^2)^(3/4)*ArcTanh[Sqrt[e*Cot[c + d*x
]]/(-e^2)^(1/4)] + 2*e*Sqrt[e*Cot[c + d*x]]) - (2*(e*Cot[c + d*x])^(5/2)*Hy
pergeometric2F1[2, 5/2, 7/2, -Cot[c + d*x]])/e)/(10*a^2*d)
```

Maple [A] (verified)

Time = 0.04 (sec) , antiderivative size = 197, normalized size of antiderivative = 0.71

method	result
derivativedivides	$-\frac{2e^3\left(\frac{\sqrt{2}\left(\ln\left(\frac{e\cot(dx+c)-(e^2)^{\frac{1}{4}}\sqrt{e\cot(dx+c)}\,\sqrt{2}+\sqrt{e^2}}{e\cot(dx+c)+(e^2)^{\frac{1}{4}}\sqrt{e\cot(dx+c)}\,\sqrt{2}+\sqrt{e^2}}\right)+2\arctan\left(\frac{\sqrt{2}\,\sqrt{e\cot(dx+c)}}{(e^2)^{\frac{1}{4}}}+1\right)-2\arctan\left(-\frac{\sqrt{2}\,\sqrt{e\cot(dx+c)}}{(e^2)^{\frac{1}{4}}}+1\right)\right)}{16e(e^2)^{\frac{1}{4}}}$
default	$-\frac{2e^3\left(\frac{\sqrt{2}\left(\ln\left(\frac{e\cot(dx+c)-(e^2)^{\frac{1}{4}}\sqrt{e\cot(dx+c)}\,\sqrt{2}+\sqrt{e^2}}{e\cot(dx+c)+(e^2)^{\frac{1}{4}}\sqrt{e\cot(dx+c)}\,\sqrt{2}+\sqrt{e^2}}\right)+2\arctan\left(\frac{\sqrt{2}\,\sqrt{e\cot(dx+c)}}{(e^2)^{\frac{1}{4}}}+1\right)-2\arctan\left(-\frac{\sqrt{2}\,\sqrt{e\cot(dx+c)}}{(e^2)^{\frac{1}{4}}}+1\right)\right)}{16e(e^2)^{\frac{1}{4}}}$

```
[In] int((e*cot(d*x+c))^(3/2)/(a+a*cot(d*x+c))^2,x,method=_RETURNVERBOSE)
```

```
[Out] -2/d/a^2*e^3*(1/16/e/(e^2)^(1/4)*2^(1/2)*(ln((e*cot(d*x+c)-(e^2)^(1/4)*(e*c
ot(d*x+c))^(1/2)*2^(1/2)+(e^2)^(1/2))/(e*cot(d*x+c)+(e^2)^(1/4)*(e*cot(d*x+
c))^(1/2)*2^(1/2)+(e^2)^(1/2)))+2*arctan(2^(1/2)/(e^2)^(1/4)*(e*cot(d*x+c))
^(1/2)+1)-2*arctan(-2^(1/2)/(e^2)^(1/4)*(e*cot(d*x+c))^(1/2)+1))-1/2/e*(-1/
2*(e*cot(d*x+c))^(1/2)/(e*cot(d*x+c)+e)+1/2/e^(1/2)*arctan((e*cot(d*x+c))^(
1/2)/e^(1/2))))
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.30 (sec) , antiderivative size = 1172, normalized size of antiderivative = 4.20

$$\int \frac{(e\cot(c+dx))^{3/2}}{(a+a\cot(c+dx))^2}\,dx = \text{Too large to display}$$

```
[In] integrate((e*cot(d*x+c))^(3/2)/(a+a*cot(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] [1/4*((e*cos(2*d*x + 2*c) + e*sin(2*d*x + 2*c) + e)*sqrt(-e)*log((e*cos(2*d
*x + 2*c) - e*sin(2*d*x + 2*c) + 2*sqrt(-e)*sqrt((e*cos(2*d*x + 2*c) + e)/s
in(2*d*x + 2*c))*sin(2*d*x + 2*c) + e)/(cos(2*d*x + 2*c) + sin(2*d*x + 2*c)
 + 1)) - 2*e*sqrt((e*cos(2*d*x + 2*c) + e)/sin(2*d*x + 2*c))*sin(2*d*x + 2*
c) - (a^2*d*cos(2*d*x + 2*c) + a^2*d*sin(2*d*x + 2*c) + a^2*d)*(-e^6/(a^8*d
^4))^(1/4)*log(a^6*d^3*(-e^6/(a^8*d^4))^(3/4) + e^4*sqrt((e*cos(2*d*x + 2*c
) + e)/sin(2*d*x + 2*c))) - (-I*a^2*d*cos(2*d*x + 2*c) - I*a^2*d*sin(2*d*x
+ 2*c) - I*a^2*d)*(-e^6/(a^8*d^4))^(1/4)*log(I*a^6*d^3*(-e^6/(a^8*d^4))^(3/
4) + e^4*sqrt((e*cos(2*d*x + 2*c) + e)/sin(2*d*x + 2*c))) - (I*a^2*d*cos(2*
d*x + 2*c) + I*a^2*d*sin(2*d*x + 2*c) + I*a^2*d)*(-e^6/(a^8*d^4))^(1/4)*log
```

```
(-I*a^6*d^3*(-e^6/(a^8*d^4))^(3/4) + e^4*sqrt((e*cos(2*d*x + 2*c) + e)/sin(
2*d*x + 2*c))) + (a^2*d*cos(2*d*x + 2*c) + a^2*d*sin(2*d*x + 2*c) + a^2*d)*
(-e^6/(a^8*d^4))^(1/4)*log(-a^6*d^3*(-e^6/(a^8*d^4))^(3/4) + e^4*sqrt((e*co
s(2*d*x + 2*c) + e)/sin(2*d*x + 2*c))))/(a^2*d*cos(2*d*x + 2*c) + a^2*d*sin
(2*d*x + 2*c) + a^2*d), 1/4*(2*(e*cos(2*d*x + 2*c) + e*sin(2*d*x + 2*c) + e
)*sqrt(e)*arctan(sqrt((e*cos(2*d*x + 2*c) + e)/sin(2*d*x + 2*c))/sqrt(e)) -
 2*e*sqrt((e*cos(2*d*x + 2*c) + e)/sin(2*d*x + 2*c))*sin(2*d*x + 2*c) - (a^
2*d*cos(2*d*x + 2*c) + a^2*d*sin(2*d*x + 2*c) + a^2*d)*(-e^6/(a^8*d^4))^(1/
4)*log(a^6*d^3*(-e^6/(a^8*d^4))^(3/4) + e^4*sqrt((e*cos(2*d*x + 2*c) + e)/s
in(2*d*x + 2*c))) - (-I*a^2*d*cos(2*d*x + 2*c) - I*a^2*d*sin(2*d*x + 2*c) -
 I*a^2*d)*(-e^6/(a^8*d^4))^(1/4)*log(I*a^6*d^3*(-e^6/(a^8*d^4))^(3/4) + e^4
*sqrt((e*cos(2*d*x + 2*c) + e)/sin(2*d*x + 2*c))) - (I*a^2*d*cos(2*d*x + 2*
c) + I*a^2*d*sin(2*d*x + 2*c) + I*a^2*d)*(-e^6/(a^8*d^4))^(1/4)*log(-I*a^6*
d^3*(-e^6/(a^8*d^4))^(3/4) + e^4*sqrt((e*cos(2*d*x + 2*c) + e)/sin(2*d*x +
2*c))) + (a^2*d*cos(2*d*x + 2*c) + a^2*d*sin(2*d*x + 2*c) + a^2*d)*(-e^6/(a
^8*d^4))^(1/4)*log(-a^6*d^3*(-e^6/(a^8*d^4))^(3/4) + e^4*sqrt((e*cos(2*d*x
+ 2*c) + e)/sin(2*d*x + 2*c))))/(a^2*d*cos(2*d*x + 2*c) + a^2*d*sin(2*d*x +
 2*c) + a^2*d)]
```

Sympy [F]

$$\int \frac{(e\cot(c+dx))^{3/2}}{(a+a\cot(c+dx))^2}\,dx = \frac{\int \frac{(e\cot{(c+dx)})^{\frac{3}{2}}}{\cot^2{(c+dx)}+2\cot{(c+dx)}+1}\,dx}{a^2}$$

```
[In] integrate((e*cot(d*x+c))**(3/2)/(a+a*cot(d*x+c))**2,x)
```

```
[Out] Integral((e*cot(c + d*x))**(3/2)/(cot(c + d*x)**2 + 2*cot(c + d*x) + 1), x)
/a**2
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{(e\cot(c+dx))^{3/2}}{(a+a\cot(c+dx))^2}\,dx = \text{Exception raised: ValueError}$$

```
[In] integrate((e*cot(d*x+c))^(3/2)/(a+a*cot(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(e>0)', see `assume?` for more detai
ls)Is e
```

Giac [F]

$$\int \frac{(e\cot(c+dx))^{3/2}}{(a+a\cot(c+dx))^2}\,dx = \int \frac{(e\cot(dx+c))^{\frac{3}{2}}}{(a\cot(dx+c)+a)^2}\,dx$$

```
[In] integrate((e*cot(d*x+c))^(3/2)/(a+a*cot(d*x+c))^2,x, algorithm="giac")
[Out] integrate((e*cot(d*x + c))^(3/2)/(a*cot(d*x + c) + a)^2, x)
```

Mupad [B] (verification not implemented)

Time = 13.44 (sec) , antiderivative size = 376, normalized size of antiderivative = 1.35

$$\int \frac{(e\cot(c+dx))^{3/2}}{(a+a\cot(c+dx))^2}\,dx = -\frac{\operatorname{atan}\left(\frac{4e^{16}\sqrt{e\cot(c+dx)}\left(-\frac{e^6}{a^8d^4}\right)^{1/4}}{\frac{4e^{18}}{a^2d}+4a^2de^{15}\sqrt{-\frac{e^6}{a^8d^4}}}+\frac{4e^{13}\sqrt{e\cot(c+dx)}\left(-\frac{e^6}{a^8d^4}\right)^{3/4}}{\frac{4e^{18}}{a^6d^3}+\frac{4e^{15}\sqrt{-\frac{e^6}{a^8d^4}}}{a^2d}}\right)\left(-\frac{e^6}{a^8d^4}\right)^{1/4}}{2} - \operatorname{atan}\left(\frac{e^{16}\sqrt{e\cot(c+dx)}\left(-\frac{e^6}{256a^8d^4}\right)^{1/4}16\mathrm{i}}{\frac{4e^{18}}{a^2d}-64a^2de^{15}\sqrt{-\frac{e^6}{256a^8d^4}}}-\frac{e^{13}\sqrt{e\cot(c+dx)}\left(-\frac{e^6}{256a^8d^4}\right)^{3/4}256\mathrm{i}}{\frac{4e^{18}}{a^6d^3}-\frac{64e^{15}\sqrt{-\frac{e^6}{256a^8d^4}}}{a^2d}}\right)\left(-\frac{e^6}{256a^8d^4}\right)^{1/4}$$

```
[In] int((e*cot(c + d*x))^(3/2)/(a + a*cot(c + d*x))^2,x)
[Out] (atan(((e*cot(c + d*x))^(1/2)*(-e^3)^(1/2)*1i)/e^2)*(-e^3)^(1/2)*1i)/(2*a^2
*d) - atan((e^16*(e*cot(c + d*x))^(1/2)*(-e^6/(256*a^8*d^4))^(1/4)*16i)/((4
*e^18)/(a^2*d) - 64*a^2*d*e^15*(-e^6/(256*a^8*d^4))^(1/2)) - (e^13*(e*cot(c
 + d*x))^(1/2)*(-e^6/(256*a^8*d^4))^(3/4)*256i)/((4*e^18)/(a^6*d^3) - (64*e
^15*(-e^6/(256*a^8*d^4))^(1/2))/(a^2*d)))*(-e^6/(256*a^8*d^4))^(1/4)*2i - (
e^2*(e*cot(c + d*x))^(1/2))/(2*(a^2*d*e + a^2*d*e*cot(c + d*x))) - (atan((4
*e^16*(e*cot(c + d*x))^(1/2)*(-e^6/(a^8*d^4))^(1/4))/((4*e^18)/(a^2*d) + 4*
a^2*d*e^15*(-e^6/(a^8*d^4))^(1/2)) + (4*e^13*(e*cot(c + d*x))^(1/2)*(-e^6/(
a^8*d^4))^(3/4))/((4*e^18)/(a^6*d^3) + (4*e^15*(-e^6/(a^8*d^4))^(1/2))/(a^2
*d)))*(-e^6/(a^8*d^4))^(1/4))/2
```

3.31 $\int \frac{\sqrt{e\cot(c+dx)}}{(a+a\cot(c+dx))^2}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 278

$$\begin{aligned}\int \frac{\sqrt{e\cot(c+dx)}}{(a+a\cot(c+dx))^2}\,dx &= \frac{\sqrt{e}\arctan\left(\frac{\sqrt{e\cot(c+dx)}}{\sqrt{e}}\right)}{2a^2d} + \frac{\sqrt{e}\arctan\left(1-\frac{\sqrt{2}\sqrt{e\cot(c+dx)}}{\sqrt{e}}\right)}{2\sqrt{2}a^2d} \\ &\quad - \frac{\sqrt{e}\arctan\left(1+\frac{\sqrt{2}\sqrt{e\cot(c+dx)}}{\sqrt{e}}\right)}{2\sqrt{2}a^2d} + \frac{\sqrt{e\cot(c+dx)}}{2d\,(a^2+a^2\cot(c+dx))} \\ &\quad + \frac{\sqrt{e}\log\left(\sqrt{e}+\sqrt{e}\cot(c+dx)-\sqrt{2}\sqrt{e\cot(c+dx)}\right)}{4\sqrt{2}a^2d} \\ &\quad - \frac{\sqrt{e}\log\left(\sqrt{e}+\sqrt{e}\cot(c+dx)+\sqrt{2}\sqrt{e\cot(c+dx)}\right)}{4\sqrt{2}a^2d}\end{aligned}$$

```
[Out] 1/2*arctan((e*cot(d*x+c))^(1/2)/e^(1/2))*e^(1/2)/a^2/d+1/4*arctan(1-2^(1/2)
*(e*cot(d*x+c))^(1/2)/e^(1/2))*e^(1/2)/a^2/d*2^(1/2)-1/4*arctan(1+2^(1/2)*(
e*cot(d*x+c))^(1/2)/e^(1/2))*e^(1/2)/a^2/d*2^(1/2)+1/8*ln(e^(1/2)+cot(d*x+c
)*e^(1/2)-2^(1/2)*(e*cot(d*x+c))^(1/2))*e^(1/2)/a^2/d*2^(1/2)-1/8*ln(e^(1/2
)+cot(d*x+c)*e^(1/2)+2^(1/2)*(e*cot(d*x+c))^(1/2))*e^(1/2)/a^2/d*2^(1/2)+1/
2*(e*cot(d*x+c))^(1/2)/d/(a^2+a^2*cot(d*x+c))
```

Rubi [A] (verified)

Time = 0.61 (sec) , antiderivative size = 278, normalized size of antiderivative = 1.00, number of steps used = 17, number of rules used = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.560, Rules used = {3649, 3734, 12, 3557, 335, 217, 1179, 642, 1176, 631, 210, 3715, 65, 211}

$$\int \frac{\sqrt{e\cot(c+dx)}}{(a+a\cot(c+dx))^2}\,dx = \frac{\sqrt{e}\arctan\left(\frac{\sqrt{e\cot(c+dx)}}{\sqrt{e}}\right)}{2a^2d}+\frac{\sqrt{e}\arctan\left(1-\frac{\sqrt{2}\sqrt{e\cot(c+dx)}}{\sqrt{e}}\right)}{2\sqrt{2}a^2d}-\frac{\sqrt{e}\arctan\left(\frac{\sqrt{2}\sqrt{e\cot(c+dx)}}{\sqrt{e}}+1\right)}{2\sqrt{2}a^2d}+\frac{\sqrt{e\cot(c+dx)}}{2d\left(a^2\cot(c+dx)+a^2\right)}+\frac{\sqrt{e}\log\left(\sqrt{e}\cot(c+dx)-\sqrt{2}\sqrt{e\cot(c+dx)}+\sqrt{e}\right)}{4\sqrt{2}a^2d}-\frac{\sqrt{e}\log\left(\sqrt{e}\cot(c+dx)+\sqrt{2}\sqrt{e\cot(c+dx)}+\sqrt{e}\right)}{4\sqrt{2}a^2d}$$

```
[In] Int[Sqrt[e*Cot[c + d*x]]/(a + a*Cot[c + d*x])^2,x]
```

```
[Out] (Sqrt[e]*ArcTan[Sqrt[e*Cot[c + d*x]]/Sqrt[e]])/(2*a^2*d) + (Sqrt[e]*ArcTan[
1 - (Sqrt[2]*Sqrt[e*Cot[c + d*x]])/Sqrt[e]])/(2*Sqrt[2]*a^2*d) - (Sqrt[e]*A
rcTan[1 + (Sqrt[2]*Sqrt[e*Cot[c + d*x]])/Sqrt[e]])/(2*Sqrt[2]*a^2*d) + Sqrt
[e*Cot[c + d*x]]/(2*d*(a^2 + a^2*Cot[c + d*x])) + (Sqrt[e]*Log[Sqrt[e] + Sq
rt[e]*Cot[c + d*x] - Sqrt[2]*Sqrt[e*Cot[c + d*x]]])/(4*Sqrt[2]*a^2*d) - (Sq
rt[e]*Log[Sqrt[e] + Sqrt[e]*Cot[c + d*x] + Sqrt[2]*Sqrt[e*Cot[c + d*x]]])/(
4*Sqrt[2]*a^2*d)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 217

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4),
 x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b
}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &&
AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1179

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 3557

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[b/d, Subst[Int[
x^n/(b^2 + x^2), x], x, b*Tan[c + d*x]], x] /; FreeQ[{b, c, d, n}, x] &&  !
IntegerQ[n]
```

Rule 3649

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)])^(n_), x_Symbol] :> Simp[b*(a + b*Tan[e + f*x])^(m + 1)*((c +
d*Tan[e + f*x])^n/(f*(m + 1)*(a^2 + b^2))), x] + Dist[1/((m + 1)*(a^2 + b^2
)), Int[(a + b*Tan[e + f*x])^(m + 1)*(c + d*Tan[e + f*x])^(n - 1)*Simp[a*c*
(m + 1) - b*d*n - (b*c - a*d)*(m + 1)*Tan[e + f*x] - b*d*(m + n + 1)*Tan[e
+ f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] &
& NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] && LtQ[m, -1] && GtQ[n, 0] && Inte
gerQ[2*m]
```

Rule 3715

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*tan[(e_.)
+ (f_.)*(x_)])^(n_.)*((A_) + (C_.)*tan[(e_.) + (f_.)*(x_)]^2), x_Symbol] :>
 Dist[A/f, Subst[Int[(a + b*x)^m*(c + d*x)^n, x], x, Tan[e + f*x]], x] /; F
reeQ[{a, b, c, d, e, f, A, C, m, n}, x] && EqQ[A, C]
```

Rule 3734

```
Int[(((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_)*((A_.) + (B_.)*tan[(e_.)
+ (f_.)*(x_)] + (C_.)*tan[(e_.) + (f_.)*(x_)]^2))/((a_.) + (b_.)*tan[(e_.)
+ (f_.)*(x_)]), x_Symbol] :> Dist[1/(a^2 + b^2), Int[(c + d*Tan[e + f*x])^n
*Simp[b*B + a*(A - C) + (a*B - b*(A - C))*Tan[e + f*x], x], x], x] + Dist[(
A*b^2 - a*b*B + a^2*C)/(a^2 + b^2), Int[(c + d*Tan[e + f*x])^n*((1 + Tan[e
+ f*x]^2)/(a + b*Tan[e + f*x])), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C
, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] &&
 !GtQ[n, 0] &&  !LeQ[n, -1]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{\sqrt{e\cot(c+dx)}}{2d\left(a^2+a^2\cot(c+dx)\right)}-\frac{\int \frac{-\frac{ae}{2}-ae\cot(c+dx)+\frac{1}{2}ae\cot^2(c+dx)}{\sqrt{e\cot(c+dx)}(a+a\cot(c+dx))}\,dx}{2a^2}\\
&= \frac{\sqrt{e\cot(c+dx)}}{2d\left(a^2+a^2\cot(c+dx)\right)}-\frac{\int -\frac{2a^2e}{\sqrt{e\cot(c+dx)}}\,dx}{4a^4}-\frac{e\int \frac{1+\cot^2(c+dx)}{\sqrt{e\cot(c+dx)}(a+a\cot(c+dx))}\,dx}{4a}\\
&= \frac{\sqrt{e\cot(c+dx)}}{2d\left(a^2+a^2\cot(c+dx)\right)}+\frac{e\int \frac{1}{\sqrt{e\cot(c+dx)}}\,dx}{2a^2}-\frac{e\text{Subst}\left(\int \frac{1}{\sqrt{-ex}(a-ax)}\,dx,x,-\cot(c+dx)\right)}{4ad}
\end{aligned}$$

$$= \frac{\sqrt{e\cot(c+dx)}}{2d\,(a^2+a^2\cot(c+dx))} + \frac{\text{Subst}\left(\int \frac{1}{a+\frac{ax^2}{e}}\,dx, x, \sqrt{e\cot(c+dx)}\right)}{2ad} - \frac{e^2\text{Subst}\left(\int \frac{1}{\sqrt{x}(e^2+x^2)}\,dx, x, e\cot(c+dx)\right)}{2a^2d}$$

$$= \frac{\sqrt{e}\arctan\left(\frac{\sqrt{e\cot(c+dx)}}{\sqrt{e}}\right)}{2a^2d} + \frac{\sqrt{e\cot(c+dx)}}{2d\,(a^2+a^2\cot(c+dx))} - \frac{e^2\text{Subst}\left(\int \frac{1}{e^2+x^4}\,dx, x, \sqrt{e\cot(c+dx)}\right)}{a^2d}$$

$$= \frac{\sqrt{e}\arctan\left(\frac{\sqrt{e\cot(c+dx)}}{\sqrt{e}}\right)}{2a^2d} + \frac{\sqrt{e\cot(c+dx)}}{2d\,(a^2+a^2\cot(c+dx))} - \frac{e\text{Subst}\left(\int \frac{e-x^2}{e^2+x^4}\,dx, x, \sqrt{e\cot(c+dx)}\right)}{2a^2d} - \frac{e\text{Subst}\left(\int \frac{e+x^2}{e^2+x^4}\,dx, x, \sqrt{e\cot(c+dx)}\right)}{2a^2d}$$

$$= \frac{\sqrt{e}\arctan\left(\frac{\sqrt{e\cot(c+dx)}}{\sqrt{e}}\right)}{2a^2d} + \frac{\sqrt{e\cot(c+dx)}}{2d\,(a^2+a^2\cot(c+dx))} + \frac{\sqrt{e}\text{Subst}\left(\int \frac{\sqrt{2}\sqrt{e}+2x}{-e-\sqrt{2}\sqrt{e}x-x^2}\,dx, x, \sqrt{e\cot(c+dx)}\right)}{4\sqrt{2}a^2d} + \frac{\sqrt{e}\text{Subst}\left(\int \frac{\sqrt{2}\sqrt{e}-2x}{-e+\sqrt{2}\sqrt{e}x-x^2}\,dx, x, \sqrt{e\cot(c+dx)}\right)}{4\sqrt{2}a^2d} - \frac{e\text{Subst}\left(\int \frac{1}{e-\sqrt{2}\sqrt{e}x+x^2}\,dx, x, \sqrt{e\cot(c+dx)}\right)}{4a^2d} - \frac{e\text{Subst}\left(\int \frac{1}{e+\sqrt{2}\sqrt{e}x+x^2}\,dx, x, \sqrt{e\cot(c+dx)}\right)}{4a^2d}$$

$$= \frac{\sqrt{e}\arctan\left(\frac{\sqrt{e\cot(c+dx)}}{\sqrt{e}}\right)}{2a^2d} + \frac{\sqrt{e\cot(c+dx)}}{2d\,(a^2+a^2\cot(c+dx))} + \frac{\sqrt{e}\log\left(\sqrt{e}+\sqrt{e}\cot(c+dx)-\sqrt{2}\sqrt{e\cot(c+dx)}\right)}{4\sqrt{2}a^2d} - \frac{\sqrt{e}\log\left(\sqrt{e}+\sqrt{e}\cot(c+dx)+\sqrt{2}\sqrt{e\cot(c+dx)}\right)}{4\sqrt{2}a^2d} - \frac{\sqrt{e}\text{Subst}\left(\int \frac{1}{-1-x^2}\,dx, x, 1-\frac{\sqrt{2}\sqrt{e\cot(c+dx)}}{\sqrt{e}}\right)}{2\sqrt{2}a^2d} + \frac{\sqrt{e}\text{Subst}\left(\int \frac{1}{-1-x^2}\,dx, x, 1+\frac{\sqrt{2}\sqrt{e\cot(c+dx)}}{\sqrt{e}}\right)}{2\sqrt{2}a^2d}$$

$$= \frac{\sqrt{e}\arctan\left(\frac{\sqrt{e\cot(c+dx)}}{\sqrt{e}}\right)}{2a^2d} + \frac{\sqrt{e}\arctan\left(1-\frac{\sqrt{2}\sqrt{e\cot(c+dx)}}{\sqrt{e}}\right)}{2\sqrt{2}a^2d} - \frac{\sqrt{e}\arctan\left(1+\frac{\sqrt{2}\sqrt{e\cot(c+dx)}}{\sqrt{e}}\right)}{2\sqrt{2}a^2d} + \frac{\sqrt{e\cot(c+dx)}}{2d\left(a^2+a^2\cot(c+dx)\right)} + \frac{\sqrt{e}\log\left(\sqrt{e}+\sqrt{e}\cot(c+dx)-\sqrt{2}\sqrt{e\cot(c+dx)}\right)}{4\sqrt{2}a^2d} - \frac{\sqrt{e}\log\left(\sqrt{e}+\sqrt{e}\cot(c+dx)+\sqrt{2}\sqrt{e\cot(c+dx)}\right)}{4\sqrt{2}a^2d}$$

Mathematica [A] (verified)

Time = 2.20 (sec) , antiderivative size = 221, normalized size of antiderivative = 0.79

$$\int \frac{\sqrt{e\cot(c+dx)}}{(a+a\cot(c+dx))^2}\,dx = -\frac{\sqrt{e}\left(-4\arctan\left(\frac{\sqrt{e\cot(c+dx)}}{\sqrt{e}}\right) - 2\sqrt{2}\arctan\left(1-\frac{\sqrt{2}\sqrt{e\cot(c+dx)}}{\sqrt{e}}\right) + 2\sqrt{2}\arctan\left(1+\frac{\sqrt{2}\sqrt{e\cot(c+dx)}}{\sqrt{e}}\right) - \frac{4\sqrt{\ }}{\sqrt{e}(}\right.}{}$$

```
[In] Integrate[Sqrt[e*Cot[c + d*x]]/(a + a*Cot[c + d*x])^2,x]
```

```
[Out] -1/8*(Sqrt[e]*(-4*ArcTan[Sqrt[e*Cot[c + d*x]]/Sqrt[e]] - 2*Sqrt[2]*ArcTan[1
 - (Sqrt[2]*Sqrt[e*Cot[c + d*x]])/Sqrt[e]] + 2*Sqrt[2]*ArcTan[1 + (Sqrt[2]*
Sqrt[e*Cot[c + d*x]])/Sqrt[e]] - (4*Sqrt[e*Cot[c + d*x]])/(Sqrt[e]*(1 + Cot
[c + d*x])) - Sqrt[2]*Log[Sqrt[e] + Sqrt[e]*Cot[c + d*x] - Sqrt[2]*Sqrt[e*C
ot[c + d*x]]] + Sqrt[2]*Log[Sqrt[e] + Sqrt[e]*Cot[c + d*x] + Sqrt[2]*Sqrt[e
*Cot[c + d*x]]]))/(a^2*d)
```

Maple [A] (verified)

Time = 0.04 (sec) , antiderivative size = 200, normalized size of antiderivative = 0.72

method	result
derivativedivides	$-\frac{2e^3\left(\frac{\left(e^2\right)^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\frac{e\cot(dx+c)+\left(e^2\right)^{\frac{1}{4}}\sqrt{e\cot(dx+c)}\,\sqrt{2}+\sqrt{e^2}}{e\cot(dx+c)-\left(e^2\right)^{\frac{1}{4}}\sqrt{e\cot(dx+c)}\,\sqrt{2}+\sqrt{e^2}}\right)+2\arctan\left(\frac{\sqrt{2}\,\sqrt{e\cot(dx+c)}}{\left(e^2\right)^{\frac{1}{4}}}+1\right)-2\arctan\left(-\frac{\sqrt{2}\,\sqrt{e\cot(dx}{\left(e^2\right)^{\frac{1}{4}}}\right.\right.}{16e^3}\right.}{d\,a^2}$
default	$-\frac{2e^3\left(\frac{\left(e^2\right)^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\frac{e\cot(dx+c)+\left(e^2\right)^{\frac{1}{4}}\sqrt{e\cot(dx+c)}\,\sqrt{2}+\sqrt{e^2}}{e\cot(dx+c)-\left(e^2\right)^{\frac{1}{4}}\sqrt{e\cot(dx+c)}\,\sqrt{2}+\sqrt{e^2}}\right)+2\arctan\left(\frac{\sqrt{2}\,\sqrt{e\cot(dx+c)}}{\left(e^2\right)^{\frac{1}{4}}}+1\right)-2\arctan\left(-\frac{\sqrt{2}\,\sqrt{e\cot(dx}{\left(e^2\right)^{\frac{1}{4}}}\right.\right.}{16e^3}\right.}{d\,a^2}$

```
[In] int((e*cot(d*x+c))^(1/2)/(a+a*cot(d*x+c))^2,x,method=_RETURNVERBOSE)
```

```
[Out] -2/d/a^2*e^3*(1/16/e^3*(e^2)^(1/4)*2^(1/2)*(ln((e*cot(d*x+c)+(e^2)^(1/4)*(e
*cot(d*x+c))^(1/2)*2^(1/2)+(e^2)^(1/2))/(e*cot(d*x+c)-(e^2)^(1/4)*(e*cot(d*
x+c))^(1/2)*2^(1/2)+(e^2)^(1/2)))+2*arctan(2^(1/2)/(e^2)^(1/4)*(e*cot(d*x+c
))^(1/2)+1)-2*arctan(-2^(1/2)/(e^2)^(1/4)*(e*cot(d*x+c))^(1/2)+1))-1/2/e*(1
/2*(e*cot(d*x+c))^(1/2)/e/(e*cot(d*x+c)+e)+1/2/e^(3/2)*arctan((e*cot(d*x+c)
)^(1/2)/e^(1/2))))
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.33 (sec) , antiderivative size = 1114, normalized size of antiderivative = 4.01

$$\int \frac{\sqrt{e\cot(c+dx)}}{(a+a\cot(c+dx))^2}\,dx = \text{Too large to display}$$

```
[In] integrate((e*cot(d*x+c))^(1/2)/(a+a*cot(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] [1/4*(sqrt(-e)*(cos(2*d*x + 2*c) + sin(2*d*x + 2*c) + 1)*log((e*cos(2*d*x +
 2*c) - e*sin(2*d*x + 2*c) + 2*sqrt(-e)*sqrt((e*cos(2*d*x + 2*c) + e)/sin(2
*d*x + 2*c))*sin(2*d*x + 2*c) + e)/(cos(2*d*x + 2*c) + sin(2*d*x + 2*c) + 1
)) - (a^2*d*cos(2*d*x + 2*c) + a^2*d*sin(2*d*x + 2*c) + a^2*d)*(-e^2/(a^8*d
^4))^(1/4)*log(a^2*d*(-e^2/(a^8*d^4))^(1/4) + sqrt((e*cos(2*d*x + 2*c) + e)
/sin(2*d*x + 2*c))) - (I*a^2*d*cos(2*d*x + 2*c) + I*a^2*d*sin(2*d*x + 2*c)
+ I*a^2*d)*(-e^2/(a^8*d^4))^(1/4)*log(I*a^2*d*(-e^2/(a^8*d^4))^(1/4) + sqrt
((e*cos(2*d*x + 2*c) + e)/sin(2*d*x + 2*c))) - (-I*a^2*d*cos(2*d*x + 2*c) -
 I*a^2*d*sin(2*d*x + 2*c) - I*a^2*d)*(-e^2/(a^8*d^4))^(1/4)*log(-I*a^2*d*(-
e^2/(a^8*d^4))^(1/4) + sqrt((e*cos(2*d*x + 2*c) + e)/sin(2*d*x + 2*c))) + (
```

```
a^2*d*cos(2*d*x + 2*c) + a^2*d*sin(2*d*x + 2*c) + a^2*d)*(-e^2/(a^8*d^4))^(
1/4)*log(-a^2*d*(-e^2/(a^8*d^4))^(1/4) + sqrt((e*cos(2*d*x + 2*c) + e)/sin(
2*d*x + 2*c))) + 2*sqrt((e*cos(2*d*x + 2*c) + e)/sin(2*d*x + 2*c))*sin(2*d*
x + 2*c))/(a^2*d*cos(2*d*x + 2*c) + a^2*d*sin(2*d*x + 2*c) + a^2*d), 1/4*(2
*sqrt(e)*(cos(2*d*x + 2*c) + sin(2*d*x + 2*c) + 1)*arctan(sqrt((e*cos(2*d*x
 + 2*c) + e)/sin(2*d*x + 2*c))/sqrt(e)) - (a^2*d*cos(2*d*x + 2*c) + a^2*d*s
in(2*d*x + 2*c) + a^2*d)*(-e^2/(a^8*d^4))^(1/4)*log(a^2*d*(-e^2/(a^8*d^4))^
(1/4) + sqrt((e*cos(2*d*x + 2*c) + e)/sin(2*d*x + 2*c))) - (I*a^2*d*cos(2*d
*x + 2*c) + I*a^2*d*sin(2*d*x + 2*c) + I*a^2*d)*(-e^2/(a^8*d^4))^(1/4)*log(
I*a^2*d*(-e^2/(a^8*d^4))^(1/4) + sqrt((e*cos(2*d*x + 2*c) + e)/sin(2*d*x +
2*c))) - (-I*a^2*d*cos(2*d*x + 2*c) - I*a^2*d*sin(2*d*x + 2*c) - I*a^2*d)*(
-e^2/(a^8*d^4))^(1/4)*log(-I*a^2*d*(-e^2/(a^8*d^4))^(1/4) + sqrt((e*cos(2*d
*x + 2*c) + e)/sin(2*d*x + 2*c))) + (a^2*d*cos(2*d*x + 2*c) + a^2*d*sin(2*d
*x + 2*c) + a^2*d)*(-e^2/(a^8*d^4))^(1/4)*log(-a^2*d*(-e^2/(a^8*d^4))^(1/4)
 + sqrt((e*cos(2*d*x + 2*c) + e)/sin(2*d*x + 2*c))) + 2*sqrt((e*cos(2*d*x +
 2*c) + e)/sin(2*d*x + 2*c))*sin(2*d*x + 2*c))/(a^2*d*cos(2*d*x + 2*c) + a^
2*d*sin(2*d*x + 2*c) + a^2*d)]
```

Sympy [F]

$$\int \frac{\sqrt{e\cot(c+dx)}}{(a+a\cot(c+dx))^2}\,dx = \frac{\int \frac{\sqrt{e\cot(c+dx)}}{\cot^2(c+dx)+2\cot(c+dx)+1}\,dx}{a^2}$$

```
[In] integrate((e*cot(d*x+c))**(1/2)/(a+a*cot(d*x+c))**2,x)
```

```
[Out] Integral(sqrt(e*cot(c + d*x))/(cot(c + d*x)**2 + 2*cot(c + d*x) + 1), x)/a*
*2
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{\sqrt{e\cot(c+dx)}}{(a+a\cot(c+dx))^2}\,dx = \text{Exception raised: ValueError}$$

```
[In] integrate((e*cot(d*x+c))^(1/2)/(a+a*cot(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(e>0)', see `assume?` for more detai
ls)Is e
```

Giac [F]

$$\int \frac{\sqrt{e\cot(c+dx)}}{(a+a\cot(c+dx))^2}\,dx = \int \frac{\sqrt{e\cot(dx+c)}}{(a\cot(dx+c)+a)^2}\,dx$$

[In] integrate((e*cot(d*x+c))^(1/2)/(a+a*cot(d*x+c))^2,x, algorithm="giac")

[Out] integrate(sqrt(e*cot(d*x + c))/(a*cot(d*x + c) + a)^2, x)

Mupad [B] (verification not implemented)

Time = 13.13 (sec) , antiderivative size = 366, normalized size of antiderivative = 1.32

$$\int \frac{\sqrt{e\cot(c+dx)}}{(a+a\cot(c+dx))^2}\,dx = \frac{\mathrm{atan}\left(\frac{4e^{12}\sqrt{e\cot(c+dx)}\left(-\frac{e^2}{a^8d^4}\right)^{1/4}}{\frac{4e^{13}}{a^2d}-4a^2de^{12}\sqrt{-\frac{e^2}{a^8d^4}}}+\frac{4e^{11}\sqrt{e\cot(c+dx)}\left(-\frac{e^2}{a^8d^4}\right)^{3/4}}{\frac{4e^{13}}{a^6d^3}-\frac{4e^{12}\sqrt{-\frac{e^2}{a^8d^4}}}{a^2d}}\right)\left(-\frac{e^2}{a^8d^4}\right)^{1/4}}{2} + \mathrm{atan}\left(\frac{e^{12}\sqrt{e\cot(c+dx)}\left(-\frac{e^2}{256a^8d^4}\right)^{1/4}16\mathrm{i}}{\frac{4e^{13}}{a^2d}+64a^2de^{12}\sqrt{-\frac{e^2}{256a^8d^4}}}-\frac{e^{11}\sqrt{e\cot(c+dx)}\left(-\frac{e^2}{256a^8d^4}\right)^{3/4}256\mathrm{i}}{\frac{4e^{13}}{a^6d^3}+\frac{64e^{12}\sqrt{-\frac{e^2}{256a^8d^4}}}{a^2d}}\right)\left(-\frac{e^2}{256a^8d^4}\right.$$

[In] int((e*cot(c + d*x))^(1/2)/(a + a*cot(c + d*x))^2,x)

[Out] (atan((4*e^12*(e*cot(c + d*x))^(1/2)*(-e^2/(a^8*d^4))^(1/4))/((4*e^13)/(a^2*d) - 4*a^2*d*e^12*(-e^2/(a^8*d^4))^(1/2)) + (4*e^11*(e*cot(c + d*x))^(1/2)*(-e^2/(a^8*d^4))^(3/4))/((4*e^13)/(a^6*d^3) - (4*e^12*(-e^2/(a^8*d^4))^(1/2))/(a^2*d)))*(-e^2/(a^8*d^4))^(1/4))/2 + atan((e^12*(e*cot(c + d*x))^(1/2)*(-e^2/(256*a^8*d^4))^(1/4)*16i)/((4*e^13)/(a^2*d) + 64*a^2*d*e^12*(-e^2/(256*a^8*d^4))^(1/2)) - (e^11*(e*cot(c + d*x))^(1/2)*(-e^2/(256*a^8*d^4))^(3/4)*256i)/((4*e^13)/(a^6*d^3) + (64*e^12*(-e^2/(256*a^8*d^4))^(1/2))/(a^2*d)))*(-e^2/(256*a^8*d^4))^(1/4)*2i + (e*(e*cot(c + d*x))^(1/2))/(2*(a^2*d*e + a^2*d*e*cot(c + d*x))) - ((-e)^(1/2)*atan(((e*cot(c + d*x))^(1/2)*1i)/(-e)^(1/2))*1i)/(2*a^2*d)

3.32 $\int \frac{1}{\sqrt{e\cot(c+dx)}(a+a\cot(c+dx))^2}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 281

$$\begin{aligned}\int \frac{1}{\sqrt{e\cot(c+dx)}(a+a\cot(c+dx))^2}\,dx &= -\frac{3\arctan\left(\frac{\sqrt{e\cot(c+dx)}}{\sqrt{e}}\right)}{2a^2d\sqrt{e}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt{e\cot(c+dx)}}{\sqrt{e}}\right)}{2\sqrt{2}a^2d\sqrt{e}}+\frac{\arctan\left(1+\frac{\sqrt{2}\sqrt{e\cot(c+dx)}}{\sqrt{e}}\right)}{2\sqrt{2}a^2d\sqrt{e}}\\ &\quad-\frac{\sqrt{e\cot(c+dx)}}{2de\left(a^2+a^2\cot(c+dx)\right)}+\frac{\log\left(\sqrt{e}+\sqrt{e}\cot(c+dx)-\sqrt{2}\sqrt{e\cot(c+dx)}\right)}{4\sqrt{2}a^2d\sqrt{e}}\\ &\quad-\frac{\log\left(\sqrt{e}+\sqrt{e}\cot(c+dx)+\sqrt{2}\sqrt{e\cot(c+dx)}\right)}{4\sqrt{2}a^2d\sqrt{e}}\end{aligned}$$

```
[Out] -3/2*arctan((e*cot(d*x+c))^(1/2)/e^(1/2))/a^2/d/e^(1/2)-1/4*arctan(1-2^(1/2
)*(e*cot(d*x+c))^(1/2)/e^(1/2))/a^2/d*2^(1/2)/e^(1/2)+1/4*arctan(1+2^(1/2)*
(e*cot(d*x+c))^(1/2)/e^(1/2))/a^2/d*2^(1/2)/e^(1/2)+1/8*ln(e^(1/2)+cot(d*x+
c)*e^(1/2)-2^(1/2)*(e*cot(d*x+c))^(1/2))/a^2/d*2^(1/2)/e^(1/2)-1/8*ln(e^(1/
2)+cot(d*x+c)*e^(1/2)+2^(1/2)*(e*cot(d*x+c))^(1/2))/a^2/d*2^(1/2)/e^(1/2)-1
/2*(e*cot(d*x+c))^(1/2)/d/e/(a^2+a^2*cot(d*x+c))
```

Rubi [A] (verified)

Time = 0.67 (sec) , antiderivative size = 281, normalized size of antiderivative = 1.00, number of steps used = 18, number of rules used = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.600, Rules used = {3650, 3734, 12, 16, 3557, 335, 303, 1176, 631, 210, 1179, 642, 3715, 65, 211}

$$\int \frac{1}{\sqrt{e\cot(c+dx)}(a+a\cot(c+dx))^2}\,dx$$
$$= -\frac{3\arctan\left(\frac{\sqrt{e\cot(c+dx)}}{\sqrt{e}}\right)}{2a^2d\sqrt{e}} - \frac{\arctan\left(1-\frac{\sqrt{2}\sqrt{e\cot(c+dx)}}{\sqrt{e}}\right)}{2\sqrt{2}a^2d\sqrt{e}} + \frac{\arctan\left(\frac{\sqrt{2}\sqrt{e\cot(c+dx)}}{\sqrt{e}}+1\right)}{2\sqrt{2}a^2d\sqrt{e}}$$
$$- \frac{\sqrt{e\cot(c+dx)}}{2de\left(a^2\cot(c+dx)+a^2\right)} + \frac{\log\left(\sqrt{e}\cot(c+dx)-\sqrt{2}\sqrt{e\cot(c+dx)}+\sqrt{e}\right)}{4\sqrt{2}a^2d\sqrt{e}}$$
$$- \frac{\log\left(\sqrt{e}\cot(c+dx)+\sqrt{2}\sqrt{e\cot(c+dx)}+\sqrt{e}\right)}{4\sqrt{2}a^2d\sqrt{e}}$$

```
[In] Int[1/(Sqrt[e*Cot[c + d*x]]*(a + a*Cot[c + d*x])^2),x]
```

```
[Out] (-3*ArcTan[Sqrt[e*Cot[c + d*x]]/Sqrt[e]])/(2*a^2*d*Sqrt[e]) - ArcTan[1 - (S
qrt[2]*Sqrt[e*Cot[c + d*x]])/Sqrt[e]]/(2*Sqrt[2]*a^2*d*Sqrt[e]) + ArcTan[1
+ (Sqrt[2]*Sqrt[e*Cot[c + d*x]])/Sqrt[e]]/(2*Sqrt[2]*a^2*d*Sqrt[e]) - Sqrt[
e*Cot[c + d*x]]/(2*d*e*(a^2 + a^2*Cot[c + d*x])) + Log[Sqrt[e] + Sqrt[e]*Co
t[c + d*x] - Sqrt[2]*Sqrt[e*Cot[c + d*x]]]/(4*Sqrt[2]*a^2*d*Sqrt[e]) - Log[
Sqrt[e] + Sqrt[e]*Cot[c + d*x] + Sqrt[2]*Sqrt[e*Cot[c + d*x]]]/(4*Sqrt[2]*a
^2*d*Sqrt[e])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 16

```
Int[(u_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Dist[1/b^m, Int[u*(b*v)
^(m + n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 303

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*s), Int[(r + s*x^2)/(a + b*x^4
), x], x] - Dist[1/(2*s), Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1179

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 3557

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[b/d, Subst[Int[
x^n/(b^2 + x^2), x], x, b*Tan[c + d*x]], x] /; FreeQ[{b, c, d, n}, x] &&  !
IntegerQ[n]
```

Rule 3650

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)])^(n_), x_Symbol] :> Simp[b^2*(a + b*Tan[e + f*x])^(m + 1)*((c
+ d*Tan[e + f*x])^(n + 1)/(f*(m + 1)*(a^2 + b^2)*(b*c - a*d))), x] + Dist[1
/((m + 1)*(a^2 + b^2)*(b*c - a*d)), Int[(a + b*Tan[e + f*x])^(m + 1)*(c + d
*Tan[e + f*x])^n*Simp[a*(b*c - a*d)*(m + 1) - b^2*d*(m + n + 2) - b*(b*c -
a*d)*(m + 1)*Tan[e + f*x] - b^2*d*(m + n + 2)*Tan[e + f*x]^2, x], x], x] /;
 FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0]
&& NeQ[c^2 + d^2, 0] && IntegerQ[2*m] && LtQ[m, -1] && (LtQ[n, 0] || Intege
rQ[m]) &&  !(ILtQ[n, -1] && ( !IntegerQ[m] || (EqQ[c, 0] && NeQ[a, 0])))
```

Rule 3715

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*tan[(e_.)
+ (f_.)*(x_)])^(n_.)*((A_) + (C_.)*tan[(e_.) + (f_.)*(x_)]^2), x_Symbol] :>
 Dist[A/f, Subst[Int[(a + b*x)^m*(c + d*x)^n, x], x, Tan[e + f*x]], x] /; F
reeQ[{a, b, c, d, e, f, A, C, m, n}, x] && EqQ[A, C]
```

Rule 3734

```
Int[(((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_)*((A_.) + (B_.)*tan[(e_.)
+ (f_.)*(x_)] + (C_.)*tan[(e_.) + (f_.)*(x_)]^2))/((a_.) + (b_.)*tan[(e_.)
+ (f_.)*(x_)]), x_Symbol] :> Dist[1/(a^2 + b^2), Int[(c + d*Tan[e + f*x])^n
*Simp[b*B + a*(A - C) + (a*B - b*(A - C))*Tan[e + f*x], x], x], x] + Dist[(
A*b^2 - a*b*B + a^2*C)/(a^2 + b^2), Int[(c + d*Tan[e + f*x])^n*((1 + Tan[e
+ f*x]^2)/(a + b*Tan[e + f*x])), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C
, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] &&
 !GtQ[n, 0] &&  !LeQ[n, -1]
```

Rubi steps

$$\text{integral} = -\frac{\sqrt{e\cot(c+dx)}}{2de\,(a^2+a^2\cot(c+dx))} - \frac{\int \frac{-\frac{3a^2e}{2}+a^2e\cot(c+dx)-\frac{1}{2}a^2e\cot^2(c+dx)}{\sqrt{e\cot(c+dx)}(a+a\cot(c+dx))}\,dx}{2a^3e}$$

$$= -\frac{\sqrt{e\cot(c+dx)}}{2de\left(a^2+a^2\cot(c+dx)\right)} + \frac{3\int \frac{1+\cot^2(c+dx)}{\sqrt{e\cot(c+dx)}(a+a\cot(c+dx))}\,dx}{4a} - \frac{\int \frac{2a^3e\cot(c+dx)}{\sqrt{e\cot(c+dx)}}\,dx}{4a^5e}$$

$$= -\frac{\sqrt{e\cot(c+dx)}}{2de\left(a^2+a^2\cot(c+dx)\right)} - \frac{\int \frac{\cot(c+dx)}{\sqrt{e\cot(c+dx)}}\,dx}{2a^2} + \frac{3\text{Subst}\left(\int \frac{1}{\sqrt{-ex}(a-ax)}\,dx, x, -\cot(c+dx)\right)}{4ad}$$

$$= -\frac{\sqrt{e\cot(c+dx)}}{2de\left(a^2+a^2\cot(c+dx)\right)} - \frac{\int \sqrt{e\cot(c+dx)}\,dx}{2a^2e} - \frac{3\text{Subst}\left(\int \frac{1}{a+\frac{ax^2}{e}}\,dx, x, \sqrt{e\cot(c+dx)}\right)}{2ade}$$

$$= -\frac{3\arctan\left(\frac{\sqrt{e\cot(c+dx)}}{\sqrt{e}}\right)}{2a^2d\sqrt{e}} - \frac{\sqrt{e\cot(c+dx)}}{2de\left(a^2+a^2\cot(c+dx)\right)} + \frac{\text{Subst}\left(\int \frac{\sqrt{x}}{e^2+x^2}\,dx, x, e\cot(c+dx)\right)}{2a^2d}$$

$$= -\frac{3\arctan\left(\frac{\sqrt{e\cot(c+dx)}}{\sqrt{e}}\right)}{2a^2d\sqrt{e}} - \frac{\sqrt{e\cot(c+dx)}}{2de\left(a^2+a^2\cot(c+dx)\right)} + \frac{\text{Subst}\left(\int \frac{x^2}{e^2+x^4}\,dx, x, \sqrt{e\cot(c+dx)}\right)}{a^2d}$$

$$= -\frac{3\arctan\left(\frac{\sqrt{e\cot(c+dx)}}{\sqrt{e}}\right)}{2a^2d\sqrt{e}} - \frac{\sqrt{e\cot(c+dx)}}{2de\left(a^2+a^2\cot(c+dx)\right)} - \frac{\text{Subst}\left(\int \frac{e-x^2}{e^2+x^4}\,dx, x, \sqrt{e\cot(c+dx)}\right)}{2a^2d} + \frac{\text{Subst}\left(\int \frac{e+x^2}{e^2+x^4}\,dx, x, \sqrt{e\cot(c+dx)}\right)}{2a^2d}$$

$$= -\frac{3\arctan\left(\frac{\sqrt{e\cot(c+dx)}}{\sqrt{e}}\right)}{2a^2d\sqrt{e}} - \frac{\sqrt{e\cot(c+dx)}}{2de\left(a^2+a^2\cot(c+dx)\right)} + \frac{\text{Subst}\left(\int \frac{1}{e-\sqrt{2}\sqrt{e}x+x^2}\,dx, x, \sqrt{e\cot(c+dx)}\right)}{4a^2d} + \frac{\text{Subst}\left(\int \frac{1}{e+\sqrt{2}\sqrt{e}x+x^2}\,dx, x, \sqrt{e\cot(c+dx)}\right)}{4a^2d} + \frac{\text{Subst}\left(\int \frac{\sqrt{2}\sqrt{e}+2x}{-e-\sqrt{2}\sqrt{e}x-x^2}\,dx, x, \sqrt{e\cot(c+dx)}\right)}{4\sqrt{2}a^2d\sqrt{e}} + \frac{\text{Subst}\left(\int \frac{\sqrt{2}\sqrt{e}-2x}{-e+\sqrt{2}\sqrt{e}x-x^2}\,dx, x, \sqrt{e\cot(c+dx)}\right)}{4\sqrt{2}a^2d\sqrt{e}}$$

$$= -\frac{3\arctan\left(\frac{\sqrt{e\cot(c+dx)}}{\sqrt{e}}\right)}{2a^2d\sqrt{e}} - \frac{\sqrt{e\cot(c+dx)}}{2de\left(a^2+a^2\cot(c+dx)\right)} + \frac{\log\left(\sqrt{e}+\sqrt{e}\cot(c+dx)-\sqrt{2}\sqrt{e\cot(c+dx)}\right)}{4\sqrt{2}a^2d\sqrt{e}} - \frac{\log\left(\sqrt{e}+\sqrt{e}\cot(c+dx)+\sqrt{2}\sqrt{e\cot(c+dx)}\right)}{4\sqrt{2}a^2d\sqrt{e}} + \frac{\text{Subst}\left(\int\frac{1}{-1-x^2}\,dx,x,1-\frac{\sqrt{2}\sqrt{e\cot(c+dx)}}{\sqrt{e}}\right)}{2\sqrt{2}a^2d\sqrt{e}} - \frac{\text{Subst}\left(\int\frac{1}{-1-x^2}\,dx,x,1+\frac{\sqrt{2}\sqrt{e\cot(c+dx)}}{\sqrt{e}}\right)}{2\sqrt{2}a^2d\sqrt{e}}$$

$$= -\frac{3\arctan\left(\frac{\sqrt{e\cot(c+dx)}}{\sqrt{e}}\right)}{2a^2d\sqrt{e}} - \frac{\arctan\left(1-\frac{\sqrt{2}\sqrt{e\cot(c+dx)}}{\sqrt{e}}\right)}{2\sqrt{2}a^2d\sqrt{e}} + \frac{\arctan\left(1+\frac{\sqrt{2}\sqrt{e\cot(c+dx)}}{\sqrt{e}}\right)}{2\sqrt{2}a^2d\sqrt{e}} - \frac{\sqrt{e\cot(c+dx)}}{2de\left(a^2+a^2\cot(c+dx)\right)} + \frac{\log\left(\sqrt{e}+\sqrt{e}\cot(c+dx)-\sqrt{2}\sqrt{e\cot(c+dx)}\right)}{4\sqrt{2}a^2d\sqrt{e}} - \frac{\log\left(\sqrt{e}+\sqrt{e}\cot(c+dx)+\sqrt{2}\sqrt{e\cot(c+dx)}\right)}{4\sqrt{2}a^2d\sqrt{e}}$$

Mathematica [A] (verified)

Time = 1.60 (sec) , antiderivative size = 131, normalized size of antiderivative = 0.47

$$\int\frac{1}{\sqrt{e\cot(c+dx)}(a+a\cot(c+dx))^2}\,dx = -\frac{3e^{3/2}\arctan\left(\frac{\sqrt{e\cot(c+dx)}}{\sqrt{e}}\right)+\left(-e^2\right)^{3/4}\arctan\left(\frac{\sqrt{e\cot(c+dx)}}{\sqrt[4]{-e^2}}\right)-\left(-e^2\right)^{3/4}\text{arctanh}\left(\frac{\sqrt{e\cot(c+dx)}}{\sqrt[4]{-e^2}}\right)+\frac{e\sqrt{e\cot(c+dx)}}{1+\cot(c+dx)}}{2a^2de^2}$$

```
[In] Integrate[1/(Sqrt[e*Cot[c + d*x]]*(a + a*Cot[c + d*x])^2),x]

[Out] -1/2*(3*e^(3/2)*ArcTan[Sqrt[e*Cot[c + d*x]]/Sqrt[e]] + (-e^2)^(3/4)*ArcTan[
Sqrt[e*Cot[c + d*x]]/(-e^2)^(1/4)] - (-e^2)^(3/4)*ArcTanh[Sqrt[e*Cot[c + d*
x]]/(-e^2)^(1/4)] + (e*Sqrt[e*Cot[c + d*x]])/(1 + Cot[c + d*x]))/(a^2*d*e^2
)
```

Maple [A] (verified)

Time = 0.04 (sec) , antiderivative size = 197, normalized size of antiderivative = 0.70

method	result
derivativedivides	$-\frac{2e^3\left(-\frac{\sqrt{2}\left(\ln\left(\frac{e\cot(dx+c)-(e^2)^{\frac{1}{4}}\sqrt{e\cot(dx+c)}\,\sqrt{2}+\sqrt{e^2}}{e\cot(dx+c)+(e^2)^{\frac{1}{4}}\sqrt{e\cot(dx+c)}\,\sqrt{2}+\sqrt{e^2}}\right)+2\arctan\left(\frac{\sqrt{2}\,\sqrt{e\cot(dx+c)}}{(e^2)^{\frac{1}{4}}}+1\right)-2\arctan\left(-\frac{\sqrt{2}\,\sqrt{e\cot(dx+c)}}{(e^2)^{\frac{1}{4}}}+1\right.}{16e^3(e^2)^{\frac{1}{4}}}\right.}{d\,a^2}$
default	$-\frac{2e^3\left(-\frac{\sqrt{2}\left(\ln\left(\frac{e\cot(dx+c)-(e^2)^{\frac{1}{4}}\sqrt{e\cot(dx+c)}\,\sqrt{2}+\sqrt{e^2}}{e\cot(dx+c)+(e^2)^{\frac{1}{4}}\sqrt{e\cot(dx+c)}\,\sqrt{2}+\sqrt{e^2}}\right)+2\arctan\left(\frac{\sqrt{2}\,\sqrt{e\cot(dx+c)}}{(e^2)^{\frac{1}{4}}}+1\right)-2\arctan\left(-\frac{\sqrt{2}\,\sqrt{e\cot(dx+c)}}{(e^2)^{\frac{1}{4}}}+1\right.}{16e^3(e^2)^{\frac{1}{4}}}\right.}{d\,a^2}$

```
[In] int(1/(e*cot(d*x+c))^(1/2)/(a+a*cot(d*x+c))^2,x,method=_RETURNVERBOSE)
```

```
[Out] -2/d/a^2*e^3*(-1/16/e^3/(e^2)^(1/4)*2^(1/2)*(ln((e*cot(d*x+c)-(e^2)^(1/4)*(
e*cot(d*x+c))^(1/2)*2^(1/2)+(e^2)^(1/2))/(e*cot(d*x+c)+(e^2)^(1/4)*(e*cot(d
*x+c))^(1/2)*2^(1/2)+(e^2)^(1/2)))+2*arctan(2^(1/2)/(e^2)^(1/4)*(e*cot(d*x+
c))^(1/2)+1)-2*arctan(-2^(1/2)/(e^2)^(1/4)*(e*cot(d*x+c))^(1/2)+1))+1/2/e^3
*(1/2*(e*cot(d*x+c))^(1/2)/(e*cot(d*x+c)+e)+3/2/e^(1/2)*arctan((e*cot(d*x+c
))^(1/2)/e^(1/2))))
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.30 (sec) , antiderivative size = 1181, normalized size of antiderivative = 4.20

$$\int \frac{1}{\sqrt{e\cot(c+dx)}(a+a\cot(c+dx))^2}\,dx = \text{Too large to display}$$

```
[In] integrate(1/(e*cot(d*x+c))^(1/2)/(a+a*cot(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] [-1/4*(3*sqrt(-e)*(cos(2*d*x + 2*c) + sin(2*d*x + 2*c) + 1)*log((e*cos(2*d*
x + 2*c) - e*sin(2*d*x + 2*c) + 2*sqrt(-e)*sqrt((e*cos(2*d*x + 2*c) + e)/si
n(2*d*x + 2*c))*sin(2*d*x + 2*c) + e)/(cos(2*d*x + 2*c) + sin(2*d*x + 2*c)
+ 1)) - (a^2*d*e*cos(2*d*x + 2*c) + a^2*d*e*sin(2*d*x + 2*c) + a^2*d*e)*(-1
/(a^8*d^4*e^2))^(1/4)*log(a^6*d^3*e^2*(-1/(a^8*d^4*e^2))^(3/4) + sqrt((e*co
s(2*d*x + 2*c) + e)/sin(2*d*x + 2*c))) + (I*a^2*d*e*cos(2*d*x + 2*c) + I*a^
2*d*e*sin(2*d*x + 2*c) + I*a^2*d*e)*(-1/(a^8*d^4*e^2))^(1/4)*log(I*a^6*d^3*
e^2*(-1/(a^8*d^4*e^2))^(3/4) + sqrt((e*cos(2*d*x + 2*c) + e)/sin(2*d*x + 2*
c))) + (-I*a^2*d*e*cos(2*d*x + 2*c) - I*a^2*d*e*sin(2*d*x + 2*c) - I*a^2*d*
e)*(-1/(a^8*d^4*e^2))^(1/4)*log(-I*a^6*d^3*e^2*(-1/(a^8*d^4*e^2))^(3/4) + s
```

```
qrt((e*cos(2*d*x + 2*c) + e)/sin(2*d*x + 2*c))) + (a^2*d*e*cos(2*d*x + 2*c)
 + a^2*d*e*sin(2*d*x + 2*c) + a^2*d*e)*(-1/(a^8*d^4*e^2))^(1/4)*log(-a^6*d^
3*e^2*(-1/(a^8*d^4*e^2))^(3/4) + sqrt((e*cos(2*d*x + 2*c) + e)/sin(2*d*x +
2*c))) + 2*sqrt((e*cos(2*d*x + 2*c) + e)/sin(2*d*x + 2*c))*sin(2*d*x + 2*c)
)/(a^2*d*e*cos(2*d*x + 2*c) + a^2*d*e*sin(2*d*x + 2*c) + a^2*d*e), -1/4*(6*
sqrt(e)*(cos(2*d*x + 2*c) + sin(2*d*x + 2*c) + 1)*arctan(sqrt((e*cos(2*d*x
+ 2*c) + e)/sin(2*d*x + 2*c))/sqrt(e)) - (a^2*d*e*cos(2*d*x + 2*c) + a^2*d*
e*sin(2*d*x + 2*c) + a^2*d*e)*(-1/(a^8*d^4*e^2))^(1/4)*log(a^6*d^3*e^2*(-1/
(a^8*d^4*e^2))^(3/4) + sqrt((e*cos(2*d*x + 2*c) + e)/sin(2*d*x + 2*c))) + (
I*a^2*d*e*cos(2*d*x + 2*c) + I*a^2*d*e*sin(2*d*x + 2*c) + I*a^2*d*e)*(-1/(a
^8*d^4*e^2))^(1/4)*log(I*a^6*d^3*e^2*(-1/(a^8*d^4*e^2))^(3/4) + sqrt((e*cos
(2*d*x + 2*c) + e)/sin(2*d*x + 2*c))) + (-I*a^2*d*e*cos(2*d*x + 2*c) - I*a^
2*d*e*sin(2*d*x + 2*c) - I*a^2*d*e)*(-1/(a^8*d^4*e^2))^(1/4)*log(-I*a^6*d^3
*e^2*(-1/(a^8*d^4*e^2))^(3/4) + sqrt((e*cos(2*d*x + 2*c) + e)/sin(2*d*x + 2
*c))) + (a^2*d*e*cos(2*d*x + 2*c) + a^2*d*e*sin(2*d*x + 2*c) + a^2*d*e)*(-1
/(a^8*d^4*e^2))^(1/4)*log(-a^6*d^3*e^2*(-1/(a^8*d^4*e^2))^(3/4) + sqrt((e*c
os(2*d*x + 2*c) + e)/sin(2*d*x + 2*c))) + 2*sqrt((e*cos(2*d*x + 2*c) + e)/s
in(2*d*x + 2*c))*sin(2*d*x + 2*c))/(a^2*d*e*cos(2*d*x + 2*c) + a^2*d*e*sin(
2*d*x + 2*c) + a^2*d*e)]
```

Sympy [F]

$$\int \frac{1}{\sqrt{e\cot(c+dx)}(a+a\cot(c+dx))^2}\,dx = \frac{\int \frac{1}{\sqrt{e\cot(c+dx)}\cot^2(c+dx)+2\sqrt{e\cot(c+dx)}\cot(c+dx)+\sqrt{e\cot(c+dx)}}\,dx}{a^2}$$

```
[In] integrate(1/(e*cot(d*x+c))**(1/2)/(a+a*cot(d*x+c))**2,x)
[Out] Integral(1/(sqrt(e*cot(c + d*x))*cot(c + d*x)**2 + 2*sqrt(e*cot(c + d*x))*c
ot(c + d*x) + sqrt(e*cot(c + d*x))), x)/a**2
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{1}{\sqrt{e\cot(c+dx)}(a+a\cot(c+dx))^2}\,dx = \text{Exception raised: ValueError}$$

```
[In] integrate(1/(e*cot(d*x+c))^(1/2)/(a+a*cot(d*x+c))^2,x, algorithm="maxima")
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(e>0)', see `assume?` for more detai
ls)Is e
```

Giac [F]

$$\int \frac{1}{\sqrt{e\cot(c+dx)}(a+a\cot(c+dx))^2}\,dx = \int \frac{1}{(a\cot(dx+c)+a)^2\sqrt{e\cot(dx+c)}}\,dx$$

[In] integrate(1/(e*cot(d*x+c))^(1/2)/(a+a*cot(d*x+c))^2,x, algorithm="giac")

[Out] integrate(1/((a*cot(d*x + c) + a)^2*sqrt(e*cot(d*x + c))), x)

Mupad [B] (verification not implemented)

Time = 13.14 (sec) , antiderivative size = 366, normalized size of antiderivative = 1.30

$$\int \frac{1}{\sqrt{e\cot(c+dx)}(a+a\cot(c+dx))^2}\,dx = \frac{\operatorname{atan}\left(\frac{4e^8\sqrt{e\cot(c+dx)}\left(-\frac{1}{a^8d^4e^2}\right)^{1/4}}{\frac{4e^8}{a^2d}+36a^2de^9\sqrt{-\frac{1}{a^8d^4e^2}}}+\frac{36e^9\sqrt{e\cot(c+dx)}\left(-\frac{1}{a^8d^4e^2}\right)^{3/4}}{\frac{4e^8}{a^6d^3}+\frac{36e^9\sqrt{-\frac{1}{a^8d^4e^2}}}{a^2d}}\right)\left(-\frac{1}{a^8d^4e^2}\right)^{1/4}}{2} + \operatorname{atan}\left(\frac{e^8\sqrt{e\cot(c+dx)}\left(-\frac{1}{256a^8d^4e^2}\right)^{1/4}16i}{\frac{4e^8}{a^2d}-576a^2de^9\sqrt{-\frac{1}{256a^8d^4e^2}}}-\frac{e^9\sqrt{e\cot(c+dx)}\left(-\frac{1}{256a^8d^4e^2}\right)^{3/4}2304i}{\frac{4e^8}{a^6d^3}-\frac{576e^9\sqrt{-\frac{1}{256a^8d^4e^2}}}{a^2d}}\right)\left(-\frac{1}{256a^8d^4}\right.$$

[In] int(1/((e*cot(c + d*x))^(1/2)*(a + a*cot(c + d*x))^2),x)

[Out] (atan((4*e^8*(e*cot(c + d*x))^(1/2)*(-1/(a^8*d^4*e^2))^(1/4))/((4*e^8)/(a^2*d) + 36*a^2*d*e^9*(-1/(a^8*d^4*e^2))^(1/2)) + (36*e^9*(e*cot(c + d*x))^(1/2)*(-1/(a^8*d^4*e^2))^(3/4))/((4*e^8)/(a^6*d^3) + (36*e^9*(-1/(a^8*d^4*e^2))^(1/2))/(a^2*d)))*(-1/(a^8*d^4*e^2))^(1/4))/2 + atan((e^8*(e*cot(c + d*x))^(1/2)*(-1/(256*a^8*d^4*e^2))^(1/4)*16i)/((4*e^8)/(a^2*d) - 576*a^2*d*e^9*(-1/(256*a^8*d^4*e^2))^(1/2)) - (e^9*(e*cot(c + d*x))^(1/2)*(-1/(256*a^8*d^4*e^2))^(3/4)*2304i)/((4*e^8)/(a^6*d^3) - (576*e^9*(-1/(256*a^8*d^4*e^2))^(1/2))/(a^2*d)))*(-1/(256*a^8*d^4*e^2))^(1/4)*2i - (e*cot(c + d*x))^(1/2)/(2*(a^2*d*e + a^2*d*e*cot(c + d*x))) - (atan(((e*cot(c + d*x))^(1/2)*1i)/(-e)^(1/2))*3i)/(2*a^2*d*(-e)^(1/2))

3.33 $\int \frac{1}{(e\cot(c+dx))^{3/2}(a+a\cot(c+dx))^2}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 306

$$\begin{aligned}\int \frac{1}{(e\cot(c+dx))^{3/2}(a+a\cot(c+dx))^2}\,dx &= \frac{5\arctan\left(\frac{\sqrt{e\cot(c+dx)}}{\sqrt{e}}\right)}{2a^2de^{3/2}} \\ &- \frac{\arctan\left(1-\frac{\sqrt{2}\sqrt{e\cot(c+dx)}}{\sqrt{e}}\right)}{2\sqrt{2}a^2de^{3/2}} + \frac{\arctan\left(1+\frac{\sqrt{2}\sqrt{e\cot(c+dx)}}{\sqrt{e}}\right)}{2\sqrt{2}a^2de^{3/2}} \\ &+ \frac{5}{2a^2de\sqrt{e\cot(c+dx)}} - \frac{1}{2de\sqrt{e\cot(c+dx)}\,(a^2+a^2\cot(c+dx))} \\ &- \frac{\log\left(\sqrt{e}+\sqrt{e}\cot(c+dx)-\sqrt{2}\sqrt{e\cot(c+dx)}\right)}{4\sqrt{2}a^2de^{3/2}} \\ &+ \frac{\log\left(\sqrt{e}+\sqrt{e}\cot(c+dx)+\sqrt{2}\sqrt{e\cot(c+dx)}\right)}{4\sqrt{2}a^2de^{3/2}}\end{aligned}$$

```
[Out] 5/2*arctan((e*cot(d*x+c))^(1/2)/e^(1/2))/a^2/d/e^(3/2)-1/4*arctan(1-2^(1/2)
*(e*cot(d*x+c))^(1/2)/e^(1/2))/a^2/d/e^(3/2)*2^(1/2)+1/4*arctan(1+2^(1/2)*(
e*cot(d*x+c))^(1/2)/e^(1/2))/a^2/d/e^(3/2)*2^(1/2)-1/8*ln(e^(1/2)+cot(d*x+c
)*e^(1/2)-2^(1/2)*(e*cot(d*x+c))^(1/2))/a^2/d/e^(3/2)*2^(1/2)+1/8*ln(e^(1/2
)+cot(d*x+c)*e^(1/2)+2^(1/2)*(e*cot(d*x+c))^(1/2))/a^2/d/e^(3/2)*2^(1/2)+5/
2/a^2/d/e/(e*cot(d*x+c))^(1/2)-1/2/d/e/(a^2+a^2*cot(d*x+c))/(e*cot(d*x+c))^
(1/2)
```

Rubi [A] (verified)

Time = 0.90 (sec) , antiderivative size = 306, normalized size of antiderivative = 1.00, number of steps used = 18, number of rules used = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.600, Rules used = {3650, 3730, 3734, 12, 3557, 335, 217, 1179, 642, 1176, 631, 210, 3715, 65, 211}

$$\int \frac{1}{(e\cot(c+dx))^{3/2}(a+a\cot(c+dx))^2}\,dx=\frac{5\arctan\left(\frac{\sqrt{e\cot(c+dx)}}{\sqrt{e}}\right)}{2a^2de^{3/2}} - \frac{\arctan\left(1-\frac{\sqrt{2}\sqrt{e\cot(c+dx)}}{\sqrt{e}}\right)}{2\sqrt{2}a^2de^{3/2}} + \frac{\arctan\left(\frac{\sqrt{2}\sqrt{e\cot(c+dx)}}{\sqrt{e}}+1\right)}{2\sqrt{2}a^2de^{3/2}} - \frac{\log\left(\sqrt{e}\cot(c+dx)-\sqrt{2}\sqrt{e\cot(c+dx)}+\sqrt{e}\right)}{4\sqrt{2}a^2de^{3/2}} + \frac{\log\left(\sqrt{e}\cot(c+dx)+\sqrt{2}\sqrt{e\cot(c+dx)}+\sqrt{e}\right)}{4\sqrt{2}a^2de^{3/2}} + \frac{5}{2a^2de\sqrt{e\cot(c+dx)}} - \frac{1}{2de\left(a^2\cot(c+dx)+a^2\right)\sqrt{e\cot(c+dx)}}$$

```
[In] Int[1/((e*Cot[c + d*x])^(3/2)*(a + a*Cot[c + d*x])^2),x]
```

```
[Out] (5*ArcTan[Sqrt[e*Cot[c + d*x]]/Sqrt[e]])/(2*a^2*d*e^(3/2)) - ArcTan[1 - (Sq
rt[2]*Sqrt[e*Cot[c + d*x]])/Sqrt[e]]/(2*Sqrt[2]*a^2*d*e^(3/2)) + ArcTan[1 +
 (Sqrt[2]*Sqrt[e*Cot[c + d*x]])/Sqrt[e]]/(2*Sqrt[2]*a^2*d*e^(3/2)) + 5/(2*a
^2*d*e*Sqrt[e*Cot[c + d*x]]) - 1/(2*d*e*Sqrt[e*Cot[c + d*x]]*(a^2 + a^2*Cot
[c + d*x])) - Log[Sqrt[e] + Sqrt[e]*Cot[c + d*x] - Sqrt[2]*Sqrt[e*Cot[c + d
*x]]]/(4*Sqrt[2]*a^2*d*e^(3/2)) + Log[Sqrt[e] + Sqrt[e]*Cot[c + d*x] + Sqrt
[2]*Sqrt[e*Cot[c + d*x]]]/(4*Sqrt[2]*a^2*d*e^(3/2))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
```

```
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 217

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4),
 x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b
}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &&
AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1179

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
```

```
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 3557

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[b/d, Subst[Int[
x^n/(b^2 + x^2), x], x, b*Tan[c + d*x]], x] /; FreeQ[{b, c, d, n}, x] &&  !
IntegerQ[n]
```

Rule 3650

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)])^(n_), x_Symbol] :> Simp[b^2*(a + b*Tan[e + f*x])^(m + 1)*((c
+ d*Tan[e + f*x])^(n + 1)/(f*(m + 1)*(a^2 + b^2)*(b*c - a*d))), x] + Dist[1
/((m + 1)*(a^2 + b^2)*(b*c - a*d)), Int[(a + b*Tan[e + f*x])^(m + 1)*(c + d
*Tan[e + f*x])^n*Simp[a*(b*c - a*d)*(m + 1) - b^2*d*(m + n + 2) - b*(b*c -
a*d)*(m + 1)*Tan[e + f*x] - b^2*d*(m + n + 2)*Tan[e + f*x]^2, x], x], x] /;
 FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0]
&& NeQ[c^2 + d^2, 0] && IntegerQ[2*m] && LtQ[m, -1] && (LtQ[n, 0] || Intege
rQ[m]) &&  !(ILtQ[n, -1] && ( !IntegerQ[m] || (EqQ[c, 0] && NeQ[a, 0])))
```

Rule 3715

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*tan[(e_.)
+ (f_.)*(x_)])^(n_.)*((A_) + (C_.)*tan[(e_.) + (f_.)*(x_)]^2), x_Symbol] :>
 Dist[A/f, Subst[Int[(a + b*x)^m*(c + d*x)^n, x], x, Tan[e + f*x]], x] /; F
reeQ[{a, b, c, d, e, f, A, C, m, n}, x] && EqQ[A, C]
```

Rule 3730

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*tan[(e_.) + (f_.)*(x_)] + (C_.)*tan[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> Simp[(A*b^2 - a*(b*B - a*C))*(a + b*Tan[e +
 f*x])^(m + 1)*((c + d*Tan[e + f*x])^(n + 1)/(f*(m + 1)*(b*c - a*d)*(a^2 +
b^2))), x] + Dist[1/((m + 1)*(b*c - a*d)*(a^2 + b^2)), Int[(a + b*Tan[e + f
*x])^(m + 1)*(c + d*Tan[e + f*x])^n*Simp[A*(a*(b*c - a*d)*(m + 1) - b^2*d*(
m + n + 2)) + (b*B - a*C)*(b*c*(m + 1) + a*d*(n + 1)) - (m + 1)*(b*c - a*d)
*(A*b - a*B - b*C)*Tan[e + f*x] - d*(A*b^2 - a*(b*B - a*C))*(m + n + 2)*Tan
[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, n}, x] && NeQ[
b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] && LtQ[m, -1] &&  !
(ILtQ[n, -1] && ( !IntegerQ[m] || (EqQ[c, 0] && NeQ[a, 0])))
```

Rule 3734

```
Int[(((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_)*((A_.) + (B_.)*tan[(e_.)
+ (f_.)*(x_)] + (C_.)*tan[(e_.) + (f_.)*(x_)]^2))/((a_.) + (b_.)*tan[(e_.)
```

```
+ (f_.)*(x_)]), x_Symbol] :> Dist[1/(a^2 + b^2), Int[(c + d*Tan[e + f*x])^n
*Simp[b*B + a*(A - C) + (a*B - b*(A - C))*Tan[e + f*x], x], x], x] + Dist[(
A*b^2 - a*b*B + a^2*C)/(a^2 + b^2), Int[(c + d*Tan[e + f*x])^n*((1 + Tan[e
+ f*x]^2)/(a + b*Tan[e + f*x])), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C
, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] &&
 !GtQ[n, 0] &&  !LeQ[n, -1]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= -\frac{1}{2de\sqrt{e\cot(c+dx)}\,(a^2+a^2\cot(c+dx))}-\frac{\int\frac{-\frac{5a^2e}{2}+a^2e\cot(c+dx)-\frac{3}{2}a^2e\cot^2(c+dx)}{(e\cot(c+dx))^{3/2}(a+a\cot(c+dx))}\,dx}{2a^3e}\\
&= \frac{5}{2a^2de\sqrt{e\cot(c+dx)}}-\frac{1}{2de\sqrt{e\cot(c+dx)}\,(a^2+a^2\cot(c+dx))}\\
&\quad-\frac{\int\frac{\frac{7a^3e^3}{4}+\frac{1}{2}a^3e^3\cot(c+dx)+\frac{5}{4}a^3e^3\cot^2(c+dx)}{\sqrt{e\cot(c+dx)}(a+a\cot(c+dx))}\,dx}{a^4e^4}\\
&= \frac{5}{2a^2de\sqrt{e\cot(c+dx)}}-\frac{1}{2de\sqrt{e\cot(c+dx)}\,(a^2+a^2\cot(c+dx))}\\
&\quad-\frac{\int\frac{a^4e^3}{\sqrt{e\cot(c+dx)}}\,dx}{2a^6e^4}-\frac{5\int\frac{1+\cot^2(c+dx)}{\sqrt{e\cot(c+dx)}(a+a\cot(c+dx))}\,dx}{4ae}\\
&= \frac{5}{2a^2de\sqrt{e\cot(c+dx)}}-\frac{1}{2de\sqrt{e\cot(c+dx)}\,(a^2+a^2\cot(c+dx))}\\
&\quad-\frac{\int\frac{1}{\sqrt{e\cot(c+dx)}}\,dx}{2a^2e}-\frac{5\text{Subst}\left(\int\frac{1}{\sqrt{-ex}(a-ax)}\,dx,x,-\cot(c+dx)\right)}{4ade}\\
&= \frac{5}{2a^2de\sqrt{e\cot(c+dx)}}-\frac{1}{2de\sqrt{e\cot(c+dx)}\,(a^2+a^2\cot(c+dx))}\\
&\quad+\frac{\text{Subst}\left(\int\frac{1}{\sqrt{x}(e^2+x^2)}\,dx,x,e\cot(c+dx)\right)}{2a^2d}\\
&\quad+\frac{5\text{Subst}\left(\int\frac{1}{a+\frac{ax^2}{e}}\,dx,x,\sqrt{e\cot(c+dx)}\right)}{2ade^2}\\
&= \frac{5\arctan\left(\frac{\sqrt{e\cot(c+dx)}}{\sqrt{e}}\right)}{2a^2de^{3/2}}+\frac{5}{2a^2de\sqrt{e\cot(c+dx)}}\\
&\quad-\frac{1}{2de\sqrt{e\cot(c+dx)}\,(a^2+a^2\cot(c+dx))}\\
&\quad+\frac{\text{Subst}\left(\int\frac{1}{e^2+x^4}\,dx,x,\sqrt{e\cot(c+dx)}\right)}{a^2d}
\end{aligned}
$$

$$= \frac{5\arctan\left(\frac{\sqrt{e\cot(c+dx)}}{\sqrt{e}}\right)}{2a^2de^{3/2}} + \frac{5}{2a^2de\sqrt{e\cot(c+dx)}} - \frac{1}{2de\sqrt{e\cot(c+dx)}\,(a^2+a^2\cot(c+dx))} + \frac{\text{Subst}\left(\int \frac{e-x^2}{e^2+x^4}\,dx, x, \sqrt{e\cot(c+dx)}\right)}{2a^2de} + \frac{\text{Subst}\left(\int \frac{e+x^2}{e^2+x^4}\,dx, x, \sqrt{e\cot(c+dx)}\right)}{2a^2de}$$

$$= \frac{5\arctan\left(\frac{\sqrt{e\cot(c+dx)}}{\sqrt{e}}\right)}{2a^2de^{3/2}} + \frac{5}{2a^2de\sqrt{e\cot(c+dx)}} - \frac{1}{2de\sqrt{e\cot(c+dx)}\,(a^2+a^2\cot(c+dx))} - \frac{\text{Subst}\left(\int \frac{\sqrt{2}\sqrt{e}+2x}{-e-\sqrt{2}\sqrt{e}x-x^2}\,dx, x, \sqrt{e\cot(c+dx)}\right)}{4\sqrt{2}a^2de^{3/2}} - \frac{\text{Subst}\left(\int \frac{\sqrt{2}\sqrt{e}-2x}{-e+\sqrt{2}\sqrt{e}x-x^2}\,dx, x, \sqrt{e\cot(c+dx)}\right)}{4\sqrt{2}a^2de^{3/2}} + \frac{\text{Subst}\left(\int \frac{1}{e-\sqrt{2}\sqrt{e}x+x^2}\,dx, x, \sqrt{e\cot(c+dx)}\right)}{4a^2de} + \frac{\text{Subst}\left(\int \frac{1}{e+\sqrt{2}\sqrt{e}x+x^2}\,dx, x, \sqrt{e\cot(c+dx)}\right)}{4a^2de}$$

$$= \frac{5\arctan\left(\frac{\sqrt{e\cot(c+dx)}}{\sqrt{e}}\right)}{2a^2de^{3/2}} + \frac{5}{2a^2de\sqrt{e\cot(c+dx)}} - \frac{1}{2de\sqrt{e\cot(c+dx)}\,(a^2+a^2\cot(c+dx))} - \frac{\log\left(\sqrt{e}+\sqrt{e}\cot(c+dx)-\sqrt{2}\sqrt{e\cot(c+dx)}\right)}{4\sqrt{2}a^2de^{3/2}} + \frac{\log\left(\sqrt{e}+\sqrt{e}\cot(c+dx)+\sqrt{2}\sqrt{e\cot(c+dx)}\right)}{4\sqrt{2}a^2de^{3/2}} + \frac{\text{Subst}\left(\int \frac{1}{-1-x^2}\,dx, x, 1-\frac{\sqrt{2}\sqrt{e\cot(c+dx)}}{\sqrt{e}}\right)}{2\sqrt{2}a^2de^{3/2}} - \frac{\text{Subst}\left(\int \frac{1}{-1-x^2}\,dx, x, 1+\frac{\sqrt{2}\sqrt{e\cot(c+dx)}}{\sqrt{e}}\right)}{2\sqrt{2}a^2de^{3/2}}$$

$$= \frac{5\arctan\left(\frac{\sqrt{e\cot(c+dx)}}{\sqrt{e}}\right)}{2a^2de^{3/2}} - \frac{\arctan\left(1-\frac{\sqrt{2}\sqrt{e\cot(c+dx)}}{\sqrt{e}}\right)}{2\sqrt{2}a^2de^{3/2}} + \frac{\arctan\left(1+\frac{\sqrt{2}\sqrt{e\cot(c+dx)}}{\sqrt{e}}\right)}{2\sqrt{2}a^2de^{3/2}} + \frac{5}{2a^2de\sqrt{e\cot(c+dx)}} - \frac{1}{2de\sqrt{e\cot(c+dx)}\,(a^2+a^2\cot(c+dx))} - \frac{\log\left(\sqrt{e}+\sqrt{e}\cot(c+dx)-\sqrt{2}\sqrt{e\cot(c+dx)}\right)}{4\sqrt{2}a^2de^{3/2}} + \frac{\log\left(\sqrt{e}+\sqrt{e}\cot(c+dx)+\sqrt{2}\sqrt{e\cot(c+dx)}\right)}{4\sqrt{2}a^2de^{3/2}}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 1.17 (sec) , antiderivative size = 227, normalized size of antiderivative = 0.74

$$\int \frac{1}{(e\cot(c+dx))^{3/2}(a+a\cot(c+dx))^2}\,dx = \frac{8\sqrt{e}\,\text{Hypergeometric2F1}\left(-\frac{1}{2},1,\frac{1}{2},-\cot(c+dx)\right)+8\sqrt{e}\,\text{Hy}}{}$$

```
[In] Integrate[1/((e*Cot[c + d*x])^(3/2)*(a + a*Cot[c + d*x])^2),x]
```

```
[Out] (8*Sqrt[e]*Hypergeometric2F1[-1/2, 1, 1/2, -Cot[c + d*x]] + 8*Sqrt[e]*Hyper
geometric2F1[-1/2, 2, 1/2, -Cot[c + d*x]] + Sqrt[2]*Sqrt[e*Cot[c + d*x]]*(-
2*ArcTan[1 - (Sqrt[2]*Sqrt[e*Cot[c + d*x]])/Sqrt[e]] + 2*ArcTan[1 + (Sqrt[2
]*Sqrt[e*Cot[c + d*x]])/Sqrt[e]] - Log[Sqrt[e] + Sqrt[e]*Cot[c + d*x] - Sqr
t[2]*Sqrt[e*Cot[c + d*x]]] + Log[Sqrt[e] + Sqrt[e]*Cot[c + d*x] + Sqrt[2]*S
qrt[e*Cot[c + d*x]]]))/(8*a^2*d*e^(3/2)*Sqrt[e*Cot[c + d*x]])
```

Maple [A] (verified)

Time = 0.04 (sec) , antiderivative size = 212, normalized size of antiderivative = 0.69

method	result
derivativedivides	$-\frac{2e^3\left(-\frac{(e^2)^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\frac{e\cot(dx+c)+(e^2)^{\frac{1}{4}}\sqrt{e\cot(dx+c)}\sqrt{2}+\sqrt{e^2}}{e\cot(dx+c)-(e^2)^{\frac{1}{4}}\sqrt{e\cot(dx+c)}\sqrt{2}+\sqrt{e^2}}\right)+2\arctan\left(\frac{\sqrt{2}\sqrt{e\cot(dx+c)}}{(e^2)^{\frac{1}{4}}}+1\right)-2\arctan\left(-\frac{\sqrt{2}\sqrt{e\cot(c}}{(e^2)^{\frac{1}{4}}}\right.\right.}{16e^5}\right.}{d\,a^2}$
default	$-\frac{2e^3\left(-\frac{(e^2)^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\frac{e\cot(dx+c)+(e^2)^{\frac{1}{4}}\sqrt{e\cot(dx+c)}\sqrt{2}+\sqrt{e^2}}{e\cot(dx+c)-(e^2)^{\frac{1}{4}}\sqrt{e\cot(dx+c)}\sqrt{2}+\sqrt{e^2}}\right)+2\arctan\left(\frac{\sqrt{2}\sqrt{e\cot(dx+c)}}{(e^2)^{\frac{1}{4}}}+1\right)-2\arctan\left(-\frac{\sqrt{2}\sqrt{e\cot(c}}{(e^2)^{\frac{1}{4}}}\right.\right.}{16e^5}\right.}{d\,a^2}$

```
[In] int(1/(e*cot(d*x+c))^(3/2)/(a+a*cot(d*x+c))^2,x,method=_RETURNVERBOSE)
```

```
[Out] -2/d/a^2*e^3*(-1/16/e^5*(e^2)^(1/4)*2^(1/2)*(ln((e*cot(d*x+c)+(e^2)^(1/4)*(
e*cot(d*x+c))^(1/2)*2^(1/2)+(e^2)^(1/2))/(e*cot(d*x+c)-(e^2)^(1/4)*(e*cot(d
*x+c))^(1/2)*2^(1/2)+(e^2)^(1/2)))+2*arctan(2^(1/2)/(e^2)^(1/4)*(e*cot(d*x+
c))^(1/2)+1)-2*arctan(-2^(1/2)/(e^2)^(1/4)*(e*cot(d*x+c))^(1/2)+1))-1/e^4/(
e*cot(d*x+c))^(1/2)-1/2/e^4*(1/2*(e*cot(d*x+c))^(1/2)/(e*cot(d*x+c)+e)+5/2/
e^(1/2)*arctan((e*cot(d*x+c))^(1/2)/e^(1/2))))
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.32 (sec) , antiderivative size = 1257, normalized size of antiderivative = 4.11

$$\int \frac{1}{(e\cot(c+dx))^{3/2}(a+a\cot(c+dx))^2}\,dx = \text{Too large to display}$$

```
[In] integrate(1/(e*cot(d*x+c))^(3/2)/(a+a*cot(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] [-1/4*(5*sqrt(-e)*(cos(2*d*x + 2*c) + sin(2*d*x + 2*c) + 1)*log((e*cos(2*d*
x + 2*c) - e*sin(2*d*x + 2*c) - 2*sqrt(-e)*sqrt((e*cos(2*d*x + 2*c) + e)/si
n(2*d*x + 2*c))*sin(2*d*x + 2*c) + e)/(cos(2*d*x + 2*c) + sin(2*d*x + 2*c)
+ 1)) - (a^2*d*e^2*cos(2*d*x + 2*c) + a^2*d*e^2*sin(2*d*x + 2*c) + a^2*d*e^
2)*(-1/(a^8*d^4*e^6))^(1/4)*log(a^2*d*e^2*(-1/(a^8*d^4*e^6))^(1/4) + sqrt((
e*cos(2*d*x + 2*c) + e)/sin(2*d*x + 2*c))) + (-I*a^2*d*e^2*cos(2*d*x + 2*c)
 - I*a^2*d*e^2*sin(2*d*x + 2*c) - I*a^2*d*e^2)*(-1/(a^8*d^4*e^6))^(1/4)*log
(I*a^2*d*e^2*(-1/(a^8*d^4*e^6))^(1/4) + sqrt((e*cos(2*d*x + 2*c) + e)/sin(2
*d*x + 2*c))) + (I*a^2*d*e^2*cos(2*d*x + 2*c) + I*a^2*d*e^2*sin(2*d*x + 2*c
) + I*a^2*d*e^2)*(-1/(a^8*d^4*e^6))^(1/4)*log(-I*a^2*d*e^2*(-1/(a^8*d^4*e^6
))^(1/4) + sqrt((e*cos(2*d*x + 2*c) + e)/sin(2*d*x + 2*c))) + (a^2*d*e^2*co
s(2*d*x + 2*c) + a^2*d*e^2*sin(2*d*x + 2*c) + a^2*d*e^2)*(-1/(a^8*d^4*e^6))
^(1/4)*log(-a^2*d*e^2*(-1/(a^8*d^4*e^6))^(1/4) + sqrt((e*cos(2*d*x + 2*c) +
 e)/sin(2*d*x + 2*c))) + 2*sqrt((e*cos(2*d*x + 2*c) + e)/sin(2*d*x + 2*c))*
(4*cos(2*d*x + 2*c) - 5*sin(2*d*x + 2*c) - 4))/(a^2*d*e^2*cos(2*d*x + 2*c)
+ a^2*d*e^2*sin(2*d*x + 2*c) + a^2*d*e^2), 1/4*(10*sqrt(e)*(cos(2*d*x + 2*c
) + sin(2*d*x + 2*c) + 1)*arctan(sqrt((e*cos(2*d*x + 2*c) + e)/sin(2*d*x +
2*c))/sqrt(e)) + (a^2*d*e^2*cos(2*d*x + 2*c) + a^2*d*e^2*sin(2*d*x + 2*c) +
 a^2*d*e^2)*(-1/(a^8*d^4*e^6))^(1/4)*log(a^2*d*e^2*(-1/(a^8*d^4*e^6))^(1/4)
 + sqrt((e*cos(2*d*x + 2*c) + e)/sin(2*d*x + 2*c))) - (-I*a^2*d*e^2*cos(2*d
*x + 2*c) - I*a^2*d*e^2*sin(2*d*x + 2*c) - I*a^2*d*e^2)*(-1/(a^8*d^4*e^6))^
(1/4)*log(I*a^2*d*e^2*(-1/(a^8*d^4*e^6))^(1/4) + sqrt((e*cos(2*d*x + 2*c) +
 e)/sin(2*d*x + 2*c))) - (I*a^2*d*e^2*cos(2*d*x + 2*c) + I*a^2*d*e^2*sin(2*
d*x + 2*c) + I*a^2*d*e^2)*(-1/(a^8*d^4*e^6))^(1/4)*log(-I*a^2*d*e^2*(-1/(a^
8*d^4*e^6))^(1/4) + sqrt((e*cos(2*d*x + 2*c) + e)/sin(2*d*x + 2*c))) - (a^2
*d*e^2*cos(2*d*x + 2*c) + a^2*d*e^2*sin(2*d*x + 2*c) + a^2*d*e^2)*(-1/(a^8*
```

```
d^4*e^6))^(1/4)*log(-a^2*d*e^2*(-1/(a^8*d^4*e^6))^(1/4) + sqrt((e*cos(2*d*x
 + 2*c) + e)/sin(2*d*x + 2*c))) - 2*sqrt((e*cos(2*d*x + 2*c) + e)/sin(2*d*x
 + 2*c))*(4*cos(2*d*x + 2*c) - 5*sin(2*d*x + 2*c) - 4))/(a^2*d*e^2*cos(2*d*
x + 2*c) + a^2*d*e^2*sin(2*d*x + 2*c) + a^2*d*e^2)]
```

Sympy [F]

$$\int \frac{1}{(e\cot(c+dx))^{3/2}(a+a\cot(c+dx))^2}\,dx = \frac{\int \frac{1}{(e\cot(c+dx))^{\frac{3}{2}}\cot^2(c+dx)+2(e\cot(c+dx))^{\frac{3}{2}}\cot(c+dx)+(e\cot(c+dx))^{\frac{3}{2}}}\,dx}{a^2}$$

```
[In] integrate(1/(e*cot(d*x+c))**(3/2)/(a+a*cot(d*x+c))**2,x)
```

```
[Out] Integral(1/((e*cot(c + d*x))**(3/2)*cot(c + d*x)**2 + 2*(e*cot(c + d*x))**(
3/2)*cot(c + d*x) + (e*cot(c + d*x))**(3/2)), x)/a**2
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{1}{(e\cot(c+dx))^{3/2}(a+a\cot(c+dx))^2}\,dx = \text{Exception raised: ValueError}$$

```
[In] integrate(1/(e*cot(d*x+c))^(3/2)/(a+a*cot(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(e>0)', see `assume?` for more detai
ls)Is e
```

Giac [F]

$$\int \frac{1}{(e\cot(c+dx))^{3/2}(a+a\cot(c+dx))^2}\,dx = \int \frac{1}{(a\cot(dx+c)+a)^2(e\cot(dx+c))^{\frac{3}{2}}}\,dx$$

```
[In] integrate(1/(e*cot(d*x+c))^(3/2)/(a+a*cot(d*x+c))^2,x, algorithm="giac")
```

```
[Out] integrate(1/((a*cot(d*x + c) + a)^2*(e*cot(d*x + c))^(3/2)), x)
```

Mupad [B] (verification not implemented)

Time = 13.24 (sec) , antiderivative size = 414, normalized size of antiderivative = 1.35

$$\int \frac{1}{(e\cot(c+dx))^{3/2}(a+a\cot(c+dx))^2}\,dx = \frac{\frac{5\cot(c+d\,x)}{2}+2}{a^2\,d\,(e\cot(c+d\,x))^{3/2}+a^2\,d\,e\,\sqrt{e\cot(c+d\,x)}} - \frac{\operatorname{atan}\left(\frac{2048\,a^{10}\,d^5\,e^{13}\,\sqrt{e\cot(c+d\,x)}\,\left(-\frac{1}{a^8\,d^4\,e^6}\right)^{1/4}}{51200\,a^8\,d^4\,e^{12}-2048\,a^{12}\,d^6\,e^{15}\,\sqrt{-\frac{1}{a^8\,d^4\,e^6}}}+\frac{51200\,a^{14}\,d^7\,e^{16}\,\sqrt{e\cot(c+d\,x)}\,\left(-\frac{1}{a^8\,d^4\,e^6}\right)^{3/4}}{51200\,a^8\,d^4\,e^{12}-2048\,a^{12}\,d^6\,e^{15}\,\sqrt{-\frac{1}{a^8\,d^4\,e^6}}}\right)\,\left(-\frac{1}{a^8\,d^4\,e^6}\right)^{1/4}}{2}$$

$$-\operatorname{atan}\left(\frac{a^{10}\,d^5\,e^{13}\,\sqrt{e\cot(c+d\,x)}\,\left(-\frac{1}{256\,a^8\,d^4\,e^6}\right)^{1/4}\,8192\mathrm{i}}{51200\,a^8\,d^4\,e^{12}+32768\,a^{12}\,d^6\,e^{15}\,\sqrt{-\frac{1}{256\,a^8\,d^4\,e^6}}}-\frac{a^{14}\,d^7\,e^{16}\,\sqrt{e\cot(c+d\,x)}\,\left(-\frac{1}{256\,a^8\,d^4\,e^6}\right)^{3/4}\,3276800}{51200\,a^8\,d^4\,e^{12}+32768\,a^{12}\,d^6\,e^{15}\,\sqrt{-\frac{1}{256\,a^8\,d^4\,e^6}}}\right.$$

```
[In] int(1/((e*cot(c + d*x))^(3/2)*(a + a*cot(c + d*x))^2),x)
```

```
[Out] ((5*cot(c + d*x))/2 + 2)/(a^2*d*(e*cot(c + d*x))^(3/2) + a^2*d*e*(e*cot(c +
 d*x))^(1/2)) - (atan((2048*a^10*d^5*e^13*(e*cot(c + d*x))^(1/2)*(-1/(a^8*d
^4*e^6))^(1/4))/(51200*a^8*d^4*e^12 - 2048*a^12*d^6*e^15*(-1/(a^8*d^4*e^6))
^(1/2)) + (51200*a^14*d^7*e^16*(e*cot(c + d*x))^(1/2)*(-1/(a^8*d^4*e^6))^(3
/4))/(51200*a^8*d^4*e^12 - 2048*a^12*d^6*e^15*(-1/(a^8*d^4*e^6))^(1/2)))*(-
1/(a^8*d^4*e^6))^(1/4))/2 - atan((a^10*d^5*e^13*(e*cot(c + d*x))^(1/2)*(-1/
(256*a^8*d^4*e^6))^(1/4)*8192i)/(51200*a^8*d^4*e^12 + 32768*a^12*d^6*e^15*(
-1/(256*a^8*d^4*e^6))^(1/2)) - (a^14*d^7*e^16*(e*cot(c + d*x))^(1/2)*(-1/(2
56*a^8*d^4*e^6))^(3/4)*3276800i)/(51200*a^8*d^4*e^12 + 32768*a^12*d^6*e^15*
(-1/(256*a^8*d^4*e^6))^(1/2)))*(-1/(256*a^8*d^4*e^6))^(1/4)*2i + (atan(((e*
cot(c + d*x))^(1/2)*(-e^3)^(1/2)*1i)/e^2)*(-e^3)^(1/2)*5i)/(2*a^2*d*e^3)
```

3.34 $\int \frac{1}{(e\cot(c+dx))^{5/2}(a+a\cot(c+dx))^2}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 331

$$\begin{aligned}\int \frac{1}{(e\cot(c+dx))^{5/2}(a+a\cot(c+dx))^2}\,dx &= -\frac{7\arctan\left(\frac{\sqrt{e\cot(c+dx)}}{\sqrt{e}}\right)}{2a^2de^{5/2}}\\
&+\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt{e\cot(c+dx)}}{\sqrt{e}}\right)}{2\sqrt{2}a^2de^{5/2}}-\frac{\arctan\left(1+\frac{\sqrt{2}\sqrt{e\cot(c+dx)}}{\sqrt{e}}\right)}{2\sqrt{2}a^2de^{5/2}}+\frac{7}{6a^2de(e\cot(c+dx))^{3/2}}\\
&-\frac{9}{2a^2de^2\sqrt{e\cot(c+dx)}}-\frac{1}{2de(e\cot(c+dx))^{3/2}\,(a^2+a^2\cot(c+dx))}\\
&-\frac{\log\left(\sqrt{e}+\sqrt{e}\cot(c+dx)-\sqrt{2}\sqrt{e\cot(c+dx)}\right)}{4\sqrt{2}a^2de^{5/2}}\\
&+\frac{\log\left(\sqrt{e}+\sqrt{e}\cot(c+dx)+\sqrt{2}\sqrt{e\cot(c+dx)}\right)}{4\sqrt{2}a^2de^{5/2}}\end{aligned}$$

```
[Out] -7/2*arctan((e*cot(d*x+c))^(1/2)/e^(1/2))/a^2/d/e^(5/2)+7/6/a^2/d/e/(e*cot(
d*x+c))^(3/2)-1/2/d/e/(e*cot(d*x+c))^(3/2)/(a^2+a^2*cot(d*x+c))+1/4*arctan(
1-2^(1/2)*(e*cot(d*x+c))^(1/2)/e^(1/2))/a^2/d/e^(5/2)*2^(1/2)-1/4*arctan(1+
2^(1/2)*(e*cot(d*x+c))^(1/2)/e^(1/2))/a^2/d/e^(5/2)*2^(1/2)-1/8*ln(e^(1/2)+
cot(d*x+c)*e^(1/2)-2^(1/2)*(e*cot(d*x+c))^(1/2))/a^2/d/e^(5/2)*2^(1/2)+1/8*
ln(e^(1/2)+cot(d*x+c)*e^(1/2)+2^(1/2)*(e*cot(d*x+c))^(1/2))/a^2/d/e^(5/2)*2
^(1/2)-9/2/a^2/d/e^2/(e*cot(d*x+c))^(1/2)
```

Rubi [A] (verified)

Time = 1.18 (sec) , antiderivative size = 331, normalized size of antiderivative = 1.00, number of steps used = 20, number of rules used = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.640, Rules used = {3650, 3730, 3734, 12, 16, 3557, 335, 303, 1176, 631, 210, 1179, 642, 3715, 65, 211}

$$\int \frac{1}{(e\cot(c+dx))^{5/2}(a+a\cot(c+dx))^2}\,dx = -\frac{7\arctan\left(\frac{\sqrt{e\cot(c+dx)}}{\sqrt{e}}\right)}{2a^2de^{5/2}} + \frac{\arctan\left(1-\frac{\sqrt{2}\sqrt{e\cot(c+dx)}}{\sqrt{e}}\right)}{2\sqrt{2}a^2de^{5/2}} - \frac{\arctan\left(\frac{\sqrt{2}\sqrt{e\cot(c+dx)}}{\sqrt{e}}+1\right)}{2\sqrt{2}a^2de^{5/2}} - \frac{\log\left(\sqrt{e}\cot(c+dx)-\sqrt{2}\sqrt{e\cot(c+dx)}+\sqrt{e}\right)}{4\sqrt{2}a^2de^{5/2}} + \frac{\log\left(\sqrt{e}\cot(c+dx)+\sqrt{2}\sqrt{e\cot(c+dx)}+\sqrt{e}\right)}{4\sqrt{2}a^2de^{5/2}} - \frac{9}{2a^2de^2\sqrt{e\cot(c+dx)}} - \frac{1}{2de\left(a^2\cot(c+dx)+a^2\right)(e\cot(c+dx))^{3/2}} + \frac{7}{6a^2de(e\cot(c+dx))^{3/2}}$$

```
[In] Int[1/((e*Cot[c + d*x])^(5/2)*(a + a*Cot[c + d*x])^2),x]
```

```
[Out] (-7*ArcTan[Sqrt[e*Cot[c + d*x]]/Sqrt[e]])/(2*a^2*d*e^(5/2)) + ArcTan[1 - (S
qrt[2]*Sqrt[e*Cot[c + d*x]])/Sqrt[e]]/(2*Sqrt[2]*a^2*d*e^(5/2)) - ArcTan[1
+ (Sqrt[2]*Sqrt[e*Cot[c + d*x]])/Sqrt[e]]/(2*Sqrt[2]*a^2*d*e^(5/2)) + 7/(6*
a^2*d*e*(e*Cot[c + d*x])^(3/2)) - 9/(2*a^2*d*e^2*Sqrt[e*Cot[c + d*x]]) - 1/
(2*d*e*(e*Cot[c + d*x])^(3/2)*(a^2 + a^2*Cot[c + d*x])) - Log[Sqrt[e] + Sqr
t[e]*Cot[c + d*x] - Sqrt[2]*Sqrt[e*Cot[c + d*x]]]/(4*Sqrt[2]*a^2*d*e^(5/2))
 + Log[Sqrt[e] + Sqrt[e]*Cot[c + d*x] + Sqrt[2]*Sqrt[e*Cot[c + d*x]]]/(4*Sq
rt[2]*a^2*d*e^(5/2))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 16

```
Int[(u_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Dist[1/b^m, Int[u*(b*v)
^(m + n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
```

```
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 303

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*s), Int[(r + s*x^2)/(a + b*x^4
), x], x] - Dist[1/(2*s), Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
```

```
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1179

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 3557

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[b/d, Subst[Int[
x^n/(b^2 + x^2), x], x, b*Tan[c + d*x]], x] /; FreeQ[{b, c, d, n}, x] &&  !
IntegerQ[n]
```

Rule 3650

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)])^(n_), x_Symbol] :> Simp[b^2*(a + b*Tan[e + f*x])^(m + 1)*((c
+ d*Tan[e + f*x])^(n + 1)/(f*(m + 1)*(a^2 + b^2)*(b*c - a*d))), x] + Dist[1
/((m + 1)*(a^2 + b^2)*(b*c - a*d)), Int[(a + b*Tan[e + f*x])^(m + 1)*(c + d
*Tan[e + f*x])^n*Simp[a*(b*c - a*d)*(m + 1) - b^2*d*(m + n + 2) - b*(b*c -
a*d)*(m + 1)*Tan[e + f*x] - b^2*d*(m + n + 2)*Tan[e + f*x]^2, x], x], x] /;
 FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0]
&& NeQ[c^2 + d^2, 0] && IntegerQ[2*m] && LtQ[m, -1] && (LtQ[n, 0] || Intege
rQ[m]) &&  !(ILtQ[n, -1] && ( !IntegerQ[m] || (EqQ[c, 0] && NeQ[a, 0])))
```

Rule 3715

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*tan[(e_.)
+ (f_.)*(x_)])^(n_.)*((A_) + (C_.)*tan[(e_.) + (f_.)*(x_)]^2), x_Symbol] :>
 Dist[A/f, Subst[Int[(a + b*x)^m*(c + d*x)^n, x], x, Tan[e + f*x]], x] /; F
reeQ[{a, b, c, d, e, f, A, C, m, n}, x] && EqQ[A, C]
```

Rule 3730

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*tan[(e_.) + (f_.)*(x_)] + (C_.)*tan[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> Simp[(A*b^2 - a*(b*B - a*C))*(a + b*Tan[e +
 f*x])^(m + 1)*((c + d*Tan[e + f*x])^(n + 1)/(f*(m + 1)*(b*c - a*d)*(a^2 +
b^2))), x] + Dist[1/((m + 1)*(b*c - a*d)*(a^2 + b^2)), Int[(a + b*Tan[e + f
*x])^(m + 1)*(c + d*Tan[e + f*x])^n*Simp[A*(a*(b*c - a*d)*(m + 1) - b^2*d*(
m + n + 2)) + (b*B - a*C)*(b*c*(m + 1) + a*d*(n + 1)) - (m + 1)*(b*c - a*d)
*(A*b - a*B - b*C)*Tan[e + f*x] - d*(A*b^2 - a*(b*B - a*C))*(m + n + 2)*Tan
[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, n}, x] && NeQ[
b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] && LtQ[m, -1] &&  !
```

```
(ILtQ[n, -1] && ( !IntegerQ[m] || (EqQ[c, 0] && NeQ[a, 0])))
```

Rule 3734

```
Int[(((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_)*((A_.) + (B_.)*tan[(e_.)
+ (f_.)*(x_)] + (C_.)*tan[(e_.) + (f_.)*(x_)]^2))/((a_.) + (b_.)*tan[(e_.)
+ (f_.)*(x_)]), x_Symbol] :> Dist[1/(a^2 + b^2), Int[(c + d*Tan[e + f*x])^n
*Simp[b*B + a*(A - C) + (a*B - b*(A - C))*Tan[e + f*x], x], x], x] + Dist[(
A*b^2 - a*b*B + a^2*C)/(a^2 + b^2), Int[(c + d*Tan[e + f*x])^n*((1 + Tan[e
+ f*x]^2)/(a + b*Tan[e + f*x])), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C
, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] &&
 !GtQ[n, 0] &&  !LeQ[n, -1]
```

Rubi steps

$$\text{integral} = -\frac{1}{2de(e\cot(c+dx))^{3/2}\left(a^2+a^2\cot(c+dx)\right)} - \frac{\int \frac{-\frac{7a^2e}{2}+a^2e\cot(c+dx)-\frac{5}{2}a^2e\cot^2(c+dx)}{(e\cot(c+dx))^{5/2}(a+a\cot(c+dx))}\,dx}{2a^3e}$$

$$= \frac{7}{6a^2de(e\cot(c+dx))^{3/2}} - \frac{1}{2de(e\cot(c+dx))^{3/2}\left(a^2+a^2\cot(c+dx)\right)} - \frac{\int \frac{\frac{27a^3e^3}{4}+\frac{3}{2}a^3e^3\cot(c+dx)+\frac{21}{4}a^3e^3\cot^2(c+dx)}{(e\cot(c+dx))^{3/2}(a+a\cot(c+dx))}\,dx}{3a^4e^4}$$

$$= \frac{7}{6a^2de(e\cot(c+dx))^{3/2}} - \frac{9}{2a^2de^2\sqrt{e\cot(c+dx)}} - \frac{1}{2de(e\cot(c+dx))^{3/2}\left(a^2+a^2\cot(c+dx)\right)} - \frac{2\int \frac{-\frac{21}{8}a^4e^5-\frac{3}{4}a^4e^5\cot(c+dx)-\frac{27}{8}a^4e^5\cot^2(c+dx)}{\sqrt{e\cot(c+dx)}(a+a\cot(c+dx))}\,dx}{3a^5e^7}$$

$$= \frac{7}{6a^2de(e\cot(c+dx))^{3/2}} - \frac{9}{2a^2de^2\sqrt{e\cot(c+dx)}} - \frac{1}{2de(e\cot(c+dx))^{3/2}\left(a^2+a^2\cot(c+dx)\right)} - \frac{\int -\frac{3a^5e^5\cot(c+dx)}{2\sqrt{e\cot(c+dx)}}\,dx}{3a^7e^7} + \frac{7\int \frac{1+\cot^2(c+dx)}{\sqrt{e\cot(c+dx)}(a+a\cot(c+dx))}\,dx}{4ae^2}$$

$$= \frac{7}{6a^2de(e\cot(c+dx))^{3/2}} - \frac{9}{2a^2de^2\sqrt{e\cot(c+dx)}} - \frac{1}{2de(e\cot(c+dx))^{3/2}\left(a^2+a^2\cot(c+dx)\right)} + \frac{\int \frac{\cot(c+dx)}{\sqrt{e\cot(c+dx)}}\,dx}{2a^2e^2} + \frac{7\text{Subst}\left(\int \frac{1}{\sqrt{-ex}(a-ax)}\,dx, x, -\cot(c+dx)\right)}{4ade^2}$$

$$= \frac{7}{6a^2de(e\cot(c+dx))^{3/2}} - \frac{9}{2a^2de^2\sqrt{e\cot(c+dx)}} - \frac{1}{2de(e\cot(c+dx))^{3/2}\left(a^2+a^2\cot(c+dx)\right)} + \frac{\int\sqrt{e\cot(c+dx)}\,dx}{2a^2e^3} - \frac{7\text{Subst}\left(\int\frac{1}{a+\frac{ax^2}{e}}\,dx, x, \sqrt{e\cot(c+dx)}\right)}{2ade^3}$$

$$= -\frac{7\arctan\left(\frac{\sqrt{e\cot(c+dx)}}{\sqrt{e}}\right)}{2a^2de^{5/2}} + \frac{7}{6a^2de(e\cot(c+dx))^{3/2}} - \frac{9}{2a^2de^2\sqrt{e\cot(c+dx)}} - \frac{1}{2de(e\cot(c+dx))^{3/2}\left(a^2+a^2\cot(c+dx)\right)} - \frac{\text{Subst}\left(\int\frac{\sqrt{x}}{e^2+x^2}\,dx, x, e\cot(c+dx)\right)}{2a^2de^2}$$

$$= -\frac{7\arctan\left(\frac{\sqrt{e\cot(c+dx)}}{\sqrt{e}}\right)}{2a^2de^{5/2}} + \frac{7}{6a^2de(e\cot(c+dx))^{3/2}} - \frac{9}{2a^2de^2\sqrt{e\cot(c+dx)}} - \frac{1}{2de(e\cot(c+dx))^{3/2}\left(a^2+a^2\cot(c+dx)\right)} - \frac{\text{Subst}\left(\int\frac{x^2}{e^2+x^4}\,dx, x, \sqrt{e\cot(c+dx)}\right)}{a^2de^2}$$

$$= -\frac{7\arctan\left(\frac{\sqrt{e\cot(c+dx)}}{\sqrt{e}}\right)}{2a^2de^{5/2}} + \frac{7}{6a^2de(e\cot(c+dx))^{3/2}} - \frac{9}{2a^2de^2\sqrt{e\cot(c+dx)}} - \frac{1}{2de(e\cot(c+dx))^{3/2}\left(a^2+a^2\cot(c+dx)\right)} + \frac{\text{Subst}\left(\int\frac{e-x^2}{e^2+x^4}\,dx, x, \sqrt{e\cot(c+dx)}\right)}{2a^2de^2} - \frac{\text{Subst}\left(\int\frac{e+x^2}{e^2+x^4}\,dx, x, \sqrt{e\cot(c+dx)}\right)}{2a^2de^2}$$

$$= -\frac{7\arctan\left(\frac{\sqrt{e\cot(c+dx)}}{\sqrt{e}}\right)}{2a^2de^{5/2}} + \frac{7}{6a^2de(e\cot(c+dx))^{3/2}} - \frac{9}{2a^2de^2\sqrt{e\cot(c+dx)}} - \frac{1}{2de(e\cot(c+dx))^{3/2}\left(a^2+a^2\cot(c+dx)\right)} - \frac{\text{Subst}\left(\int\frac{\sqrt{2}\sqrt{e}+2x}{-e-\sqrt{2}\sqrt{e}x-x^2}\,dx, x, \sqrt{e\cot(c+dx)}\right)}{4\sqrt{2}a^2de^{5/2}} - \frac{\text{Subst}\left(\int\frac{\sqrt{2}\sqrt{e}-2x}{-e+\sqrt{2}\sqrt{e}x-x^2}\,dx, x, \sqrt{e\cot(c+dx)}\right)}{4\sqrt{2}a^2de^{5/2}} - \frac{\text{Subst}\left(\int\frac{1}{e-\sqrt{2}\sqrt{e}x+x^2}\,dx, x, \sqrt{e\cot(c+dx)}\right)}{4a^2de^2} - \frac{\text{Subst}\left(\int\frac{1}{e+\sqrt{2}\sqrt{e}x+x^2}\,dx, x, \sqrt{e\cot(c+dx)}\right)}{4a^2de^2}$$

$$
\begin{aligned}
&=-\frac{7\arctan\left(\frac{\sqrt{e\cot(c+dx)}}{\sqrt{e}}\right)}{2a^2de^{5/2}}+\frac{7}{6a^2de(e\cot(c+dx))^{3/2}}\\
&-\frac{9}{2a^2de^2\sqrt{e\cot(c+dx)}}-\frac{1}{2de(e\cot(c+dx))^{3/2}\left(a^2+a^2\cot(c+dx)\right)}\\
&-\frac{\log\left(\sqrt{e}+\sqrt{e}\cot(c+dx)-\sqrt{2}\sqrt{e\cot(c+dx)}\right)}{4\sqrt{2}a^2de^{5/2}}\\
&+\frac{\log\left(\sqrt{e}+\sqrt{e}\cot(c+dx)+\sqrt{2}\sqrt{e\cot(c+dx)}\right)}{4\sqrt{2}a^2de^{5/2}}\\
&-\frac{\text{Subst}\left(\int\frac{1}{-1-x^2}\,dx,x,1-\frac{\sqrt{2}\sqrt{e\cot(c+dx)}}{\sqrt{e}}\right)}{2\sqrt{2}a^2de^{5/2}}\\
&+\frac{\text{Subst}\left(\int\frac{1}{-1-x^2}\,dx,x,1+\frac{\sqrt{2}\sqrt{e\cot(c+dx)}}{\sqrt{e}}\right)}{2\sqrt{2}a^2de^{5/2}}\\
&=-\frac{7\arctan\left(\frac{\sqrt{e\cot(c+dx)}}{\sqrt{e}}\right)}{2a^2de^{5/2}}+\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt{e\cot(c+dx)}}{\sqrt{e}}\right)}{2\sqrt{2}a^2de^{5/2}}\\
&-\frac{\arctan\left(1+\frac{\sqrt{2}\sqrt{e\cot(c+dx)}}{\sqrt{e}}\right)}{2\sqrt{2}a^2de^{5/2}}+\frac{7}{6a^2de(e\cot(c+dx))^{3/2}}\\
&-\frac{9}{2a^2de^2\sqrt{e\cot(c+dx)}}-\frac{1}{2de(e\cot(c+dx))^{3/2}\left(a^2+a^2\cot(c+dx)\right)}\\
&-\frac{\log\left(\sqrt{e}+\sqrt{e}\cot(c+dx)-\sqrt{2}\sqrt{e\cot(c+dx)}\right)}{4\sqrt{2}a^2de^{5/2}}\\
&+\frac{\log\left(\sqrt{e}+\sqrt{e}\cot(c+dx)+\sqrt{2}\sqrt{e\cot(c+dx)}\right)}{4\sqrt{2}a^2de^{5/2}}
\end{aligned}
$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 1.03 (sec) , antiderivative size = 84, normalized size of antiderivative = 0.25

$$
\int\frac{1}{(e\cot(c+dx))^{5/2}(a+a\cot(c+dx))^2}\,dx=\frac{\text{Hypergeometric2F1}\left(-\frac{3}{2},1,-\frac{1}{2},-\cot(c+dx)\right)+\text{Hypergeor}}{}
$$

```
[In] Integrate[1/((e*Cot[c + d*x])^(5/2)*(a + a*Cot[c + d*x])^2),x]
```

```
[Out] (Hypergeometric2F1[-3/2, 1, -1/2, -Cot[c + d*x]] + Hypergeometric2F1[-3/2,
2, -1/2, -Cot[c + d*x]] - 3*Cot[c + d*x]*Hypergeometric2F1[-1/4, 1, 3/4, -C
ot[c + d*x]^2])/(3*a^2*d*e*(e*Cot[c + d*x])^(3/2))
```

Maple [A] (verified)

Time = 0.04 (sec) , antiderivative size = 227, normalized size of antiderivative = 0.69

method	result
derivativedivides	$-\frac{2e^3\left(\frac{\sqrt{2}\left(\ln\left(\frac{e\cot(dx+c)-(e^2)^{\frac{1}{4}}\sqrt{e\cot(dx+c)}\sqrt{2}+\sqrt{e^2}}{e\cot(dx+c)+(e^2)^{\frac{1}{4}}\sqrt{e\cot(dx+c)}\sqrt{2}+\sqrt{e^2}}\right)+2\arctan\left(\frac{\sqrt{2}\sqrt{e\cot(dx+c)}}{(e^2)^{\frac{1}{4}}}+1\right)-2\arctan\left(-\frac{\sqrt{2}\sqrt{e\cot(dx+c)}}{(e^2)^{\frac{1}{4}}}+1\right)\right)}{16e^5(e^2)^{\frac{1}{4}}}$
default	$-\frac{2e^3\left(\frac{\sqrt{2}\left(\ln\left(\frac{e\cot(dx+c)-(e^2)^{\frac{1}{4}}\sqrt{e\cot(dx+c)}\sqrt{2}+\sqrt{e^2}}{e\cot(dx+c)+(e^2)^{\frac{1}{4}}\sqrt{e\cot(dx+c)}\sqrt{2}+\sqrt{e^2}}\right)+2\arctan\left(\frac{\sqrt{2}\sqrt{e\cot(dx+c)}}{(e^2)^{\frac{1}{4}}}+1\right)-2\arctan\left(-\frac{\sqrt{2}\sqrt{e\cot(dx+c)}}{(e^2)^{\frac{1}{4}}}+1\right)\right)}{16e^5(e^2)^{\frac{1}{4}}}$

```
[In] int(1/(e*cot(d*x+c))^(5/2)/(a+a*cot(d*x+c))^2,x,method=_RETURNVERBOSE)
```

```
[Out] -2/d/a^2*e^3*(1/16/e^5/(e^2)^(1/4)*2^(1/2)*(ln((e*cot(d*x+c)-(e^2)^(1/4)*(e
*cot(d*x+c))^(1/2)*2^(1/2)+(e^2)^(1/2))/(e*cot(d*x+c)+(e^2)^(1/4)*(e*cot(d*
x+c))^(1/2)*2^(1/2)+(e^2)^(1/2)))+2*arctan(2^(1/2)/(e^2)^(1/4)*(e*cot(d*x+c
))^(1/2)+1)-2*arctan(-2^(1/2)/(e^2)^(1/4)*(e*cot(d*x+c))^(1/2)+1))-1/3/e^4/
(e*cot(d*x+c))^(3/2)+2/e^5/(e*cot(d*x+c))^(1/2)+1/2/e^5*(1/2*(e*cot(d*x+c))
^(1/2)/(e*cot(d*x+c)+e)+7/2/e^(1/2)*arctan((e*cot(d*x+c))^(1/2)/e^(1/2))))
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.33 (sec) , antiderivative size = 1751, normalized size of antiderivative = 5.29

$$\int \frac{1}{(e\cot(c+dx))^{5/2}(a+a\cot(c+dx))^2}\,dx = \text{Too large to display}$$

```
[In] integrate(1/(e*cot(d*x+c))^(5/2)/(a+a*cot(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] [-1/12*(21*(cos(2*d*x + 2*c)^2 + (cos(2*d*x + 2*c) + 1)*sin(2*d*x + 2*c) +
2*cos(2*d*x + 2*c) + 1)*sqrt(-e)*log((e*cos(2*d*x + 2*c) - e*sin(2*d*x + 2*
c) + 2*sqrt(-e)*sqrt((e*cos(2*d*x + 2*c) + e)/sin(2*d*x + 2*c))*sin(2*d*x +
 2*c) + e)/(cos(2*d*x + 2*c) + sin(2*d*x + 2*c) + 1)) + 3*(a^2*d*e^3*cos(2*
d*x + 2*c)^2 + 2*a^2*d*e^3*cos(2*d*x + 2*c) + a^2*d*e^3 + (a^2*d*e^3*cos(2*
d*x + 2*c) + a^2*d*e^3)*sin(2*d*x + 2*c))*(-1/(a^8*d^4*e^10))^(1/4)*log(a^6
*d^3*e^8*(-1/(a^8*d^4*e^10))^(3/4) + sqrt((e*cos(2*d*x + 2*c) + e)/sin(2*d*
x + 2*c))) + 3*(-I*a^2*d*e^3*cos(2*d*x + 2*c)^2 - 2*I*a^2*d*e^3*cos(2*d*x +
 2*c) - I*a^2*d*e^3 + (-I*a^2*d*e^3*cos(2*d*x + 2*c) - I*a^2*d*e^3)*sin(2*d
*x + 2*c))*(-1/(a^8*d^4*e^10))^(1/4)*log(I*a^6*d^3*e^8*(-1/(a^8*d^4*e^10))^
```

```
(3/4) + sqrt((e*cos(2*d*x + 2*c) + e)/sin(2*d*x + 2*c))) + 3*(I*a^2*d*e^3*c
os(2*d*x + 2*c)^2 + 2*I*a^2*d*e^3*cos(2*d*x + 2*c) + I*a^2*d*e^3 + (I*a^2*d
*e^3*cos(2*d*x + 2*c) + I*a^2*d*e^3)*sin(2*d*x + 2*c))*(-1/(a^8*d^4*e^10))^
(1/4)*log(-I*a^6*d^3*e^8*(-1/(a^8*d^4*e^10))^(3/4) + sqrt((e*cos(2*d*x + 2*
c) + e)/sin(2*d*x + 2*c))) - 3*(a^2*d*e^3*cos(2*d*x + 2*c)^2 + 2*a^2*d*e^3*
cos(2*d*x + 2*c) + a^2*d*e^3 + (a^2*d*e^3*cos(2*d*x + 2*c) + a^2*d*e^3)*sin
(2*d*x + 2*c))*(-1/(a^8*d^4*e^10))^(1/4)*log(-a^6*d^3*e^8*(-1/(a^8*d^4*e^10
))^(3/4) + sqrt((e*cos(2*d*x + 2*c) + e)/sin(2*d*x + 2*c))) - 2*(20*cos(2*d
*x + 2*c)^2 - (31*cos(2*d*x + 2*c) + 23)*sin(2*d*x + 2*c) - 20)*sqrt((e*cos
(2*d*x + 2*c) + e)/sin(2*d*x + 2*c)))/(a^2*d*e^3*cos(2*d*x + 2*c)^2 + 2*a^2
*d*e^3*cos(2*d*x + 2*c) + a^2*d*e^3 + (a^2*d*e^3*cos(2*d*x + 2*c) + a^2*d*e
^3)*sin(2*d*x + 2*c)), -1/12*(42*(cos(2*d*x + 2*c)^2 + (cos(2*d*x + 2*c) +
1)*sin(2*d*x + 2*c) + 2*cos(2*d*x + 2*c) + 1)*sqrt(e)*arctan(sqrt((e*cos(2*
d*x + 2*c) + e)/sin(2*d*x + 2*c))/sqrt(e)) + 3*(a^2*d*e^3*cos(2*d*x + 2*c)^
2 + 2*a^2*d*e^3*cos(2*d*x + 2*c) + a^2*d*e^3 + (a^2*d*e^3*cos(2*d*x + 2*c)
+ a^2*d*e^3)*sin(2*d*x + 2*c))*(-1/(a^8*d^4*e^10))^(1/4)*log(a^6*d^3*e^8*(-
1/(a^8*d^4*e^10))^(3/4) + sqrt((e*cos(2*d*x + 2*c) + e)/sin(2*d*x + 2*c)))
+ 3*(-I*a^2*d*e^3*cos(2*d*x + 2*c)^2 - 2*I*a^2*d*e^3*cos(2*d*x + 2*c) - I*a
^2*d*e^3 + (-I*a^2*d*e^3*cos(2*d*x + 2*c) - I*a^2*d*e^3)*sin(2*d*x + 2*c))*
(-1/(a^8*d^4*e^10))^(1/4)*log(I*a^6*d^3*e^8*(-1/(a^8*d^4*e^10))^(3/4) + sqr
t((e*cos(2*d*x + 2*c) + e)/sin(2*d*x + 2*c))) + 3*(I*a^2*d*e^3*cos(2*d*x +
2*c)^2 + 2*I*a^2*d*e^3*cos(2*d*x + 2*c) + I*a^2*d*e^3 + (I*a^2*d*e^3*cos(2*
d*x + 2*c) + I*a^2*d*e^3)*sin(2*d*x + 2*c))*(-1/(a^8*d^4*e^10))^(1/4)*log(-
I*a^6*d^3*e^8*(-1/(a^8*d^4*e^10))^(3/4) + sqrt((e*cos(2*d*x + 2*c) + e)/sin
(2*d*x + 2*c))) - 3*(a^2*d*e^3*cos(2*d*x + 2*c)^2 + 2*a^2*d*e^3*cos(2*d*x +
 2*c) + a^2*d*e^3 + (a^2*d*e^3*cos(2*d*x + 2*c) + a^2*d*e^3)*sin(2*d*x + 2*
c))*(-1/(a^8*d^4*e^10))^(1/4)*log(-a^6*d^3*e^8*(-1/(a^8*d^4*e^10))^(3/4) +
sqrt((e*cos(2*d*x + 2*c) + e)/sin(2*d*x + 2*c))) - 2*(20*cos(2*d*x + 2*c)^2
 - (31*cos(2*d*x + 2*c) + 23)*sin(2*d*x + 2*c) - 20)*sqrt((e*cos(2*d*x + 2*
c) + e)/sin(2*d*x + 2*c)))/(a^2*d*e^3*cos(2*d*x + 2*c)^2 + 2*a^2*d*e^3*cos(
2*d*x + 2*c) + a^2*d*e^3 + (a^2*d*e^3*cos(2*d*x + 2*c) + a^2*d*e^3)*sin(2*d
*x + 2*c))]
```

Sympy [F]

$$\int \frac{1}{(e\cot(c+dx))^{5/2}(a+a\cot(c+dx))^2}\,dx = \frac{\int \frac{1}{(e\cot(c+dx))^{\frac{5}{2}}\cot^2(c+dx)+2(e\cot(c+dx))^{\frac{5}{2}}\cot(c+dx)+(e\cot(c+dx))^{\frac{5}{2}}}\,dx}{a^2}$$

```
[In] integrate(1/(e*cot(d*x+c))**(5/2)/(a+a*cot(d*x+c))**2,x)
```

```
[Out] Integral(1/((e*cot(c + d*x))**(5/2)*cot(c + d*x)**2 + 2*(e*cot(c + d*x))**(
5/2)*cot(c + d*x) + (e*cot(c + d*x))**(5/2)), x)/a**2
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{1}{(e\cot(c+dx))^{5/2}(a+a\cot(c+dx))^2}\,dx = \text{Exception raised: ValueError}$$

```
[In] integrate(1/(e*cot(d*x+c))^(5/2)/(a+a*cot(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(e>0)', see `assume?` for more detai
ls)Is e
```

Giac [F]

$$\int \frac{1}{(e\cot(c+dx))^{5/2}(a+a\cot(c+dx))^2}\,dx = \int \frac{1}{(a\cot(dx+c)+a)^2(e\cot(dx+c))^{\frac{5}{2}}}\,dx$$

```
[In] integrate(1/(e*cot(d*x+c))^(5/2)/(a+a*cot(d*x+c))^2,x, algorithm="giac")
```

```
[Out] integrate(1/((a*cot(d*x + c) + a)^2*(e*cot(d*x + c))^(5/2)), x)
```

Mupad [B] (verification not implemented)

Time = 13.57 (sec) , antiderivative size = 425, normalized size of antiderivative = 1.28

$$\int \frac{1}{(e\cot(c+dx))^{5/2}(a+a\cot(c+dx))^2}\,dx =$$
$$-\frac{\operatorname{atan}\left(\frac{2048\,a^{10}\,d^5\,e^{18}\,\sqrt{e\cot(c+d\,x)}\,\left(-\frac{1}{a^8\,d^4\,e^{10}}\right)^{1/4}}{2048\,a^8\,d^4\,e^{16}+100352\,a^{12}\,d^6\,e^{21}\,\sqrt{-\frac{1}{a^8\,d^4\,e^{10}}}}+\frac{100352\,a^{14}\,d^7\,e^{23}\,\sqrt{e\cot(c+d\,x)}\,\left(-\frac{1}{a^8\,d^4\,e^{10}}\right)^{3/4}}{2048\,a^8\,d^4\,e^{16}+100352\,a^{12}\,d^6\,e^{21}\,\sqrt{-\frac{1}{a^8\,d^4\,e^{10}}}}\right)\,\left(-\frac{1}{a^8\,d^4\,e^{10}}\right)^{1/4}}{2}$$
$$-\operatorname{atan}\left(\frac{a^{10}\,d^5\,e^{18}\,\sqrt{e\cot\left(c+d\,x\right)}\,\left(-\frac{1}{256\,a^8\,d^4\,e^{10}}\right)^{1/4}\,8192\mathrm{i}}{2048\,a^8\,d^4\,e^{16}-1605632\,a^{12}\,d^6\,e^{21}\,\sqrt{-\frac{1}{256\,a^8\,d^4\,e^{10}}}}-\frac{a^{14}\,d^7\,e^{23}\,\sqrt{e\cot\left(c+d\,x\right)}\,\left(-\frac{1}{256\,a^8\,d^4\,e^{10}}\right)^{3/4}\,642252}{2048\,a^8\,d^4\,e^{16}-1605632\,a^{12}\,d^6\,e^{21}\,\sqrt{-\frac{1}{256\,a^8\,d^4\,e^{10}}}}\right.$$

```
[In] int(1/((e*cot(c + d*x))^(5/2)*(a + a*cot(c + d*x))^2),x)
```

```
[Out] - (atan((2048*a^10*d^5*e^18*(e*cot(c + d*x))^(1/2)*(-1/(a^8*d^4*e^10))^(1/4
))/(2048*a^8*d^4*e^16 + 100352*a^12*d^6*e^21*(-1/(a^8*d^4*e^10))^(1/2)) + (
100352*a^14*d^7*e^23*(e*cot(c + d*x))^(1/2)*(-1/(a^8*d^4*e^10))^(3/4))/(204
8*a^8*d^4*e^16 + 100352*a^12*d^6*e^21*(-1/(a^8*d^4*e^10))^(1/2)))*(-1/(a^8*
d^4*e^10))^(1/4))/2 - atan((a^10*d^5*e^18*(e*cot(c + d*x))^(1/2)*(-1/(256*a
```

```
^8*d^4*e^10))^(1/4)*8192i)/(2048*a^8*d^4*e^16 - 1605632*a^12*d^6*e^21*(-1/(
256*a^8*d^4*e^10))^(1/2)) - (a^14*d^7*e^23*(e*cot(c + d*x))^(1/2)*(-1/(256*
a^8*d^4*e^10))^(3/4)*6422528i)/(2048*a^8*d^4*e^16 - 1605632*a^12*d^6*e^21*(
-1/(256*a^8*d^4*e^10))^(1/2)))*(-1/(256*a^8*d^4*e^10))^(1/4)*2i - ((10*cot(
c + d*x))/3 + (9*cot(c + d*x)^2)/2 - 2/3)/(a^2*d*(e*cot(c + d*x))^(5/2) + a
^2*d*e*(e*cot(c + d*x))^(3/2)) - (atan(((e*cot(c + d*x))^(1/2)*(-e^5)^(1/2)
*1i)/e^3)*(-e^5)^(1/2)*7i)/(2*a^2*d*e^5)
```

3.35 $\int \frac{(e\cot(c+dx))^{5/2}}{(a+a\cot(c+dx))^3}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 164

$$\int \frac{(e\cot(c+dx))^{5/2}}{(a+a\cot(c+dx))^3}\,dx = -\frac{e^{5/2}\arctan\left(\frac{\sqrt{e\cot(c+dx)}}{\sqrt{e}}\right)}{8a^3d} + \frac{e^{5/2}\text{arctanh}\left(\frac{\sqrt{e}+\sqrt{e}\cot(c+dx)}{\sqrt{2}\sqrt{e\cot(c+dx)}}\right)}{2\sqrt{2}a^3d} - \frac{5e^2\sqrt{e\cot(c+dx)}}{8a^3d(1+\cot(c+dx))} + \frac{e^2\sqrt{e\cot(c+dx)}}{4ad(a+a\cot(c+dx))^2}$$

```
[Out] -1/8*e^(5/2)*arctan((e*cot(d*x+c))^(1/2)/e^(1/2))/a^3/d+1/4*e^(5/2)*arctanh
(1/2*(e^(1/2)+cot(d*x+c)*e^(1/2))*2^(1/2)/(e*cot(d*x+c))^(1/2))/a^3/d*2^(1/
2)-5/8*e^2*(e*cot(d*x+c))^(1/2)/a^3/d/(1+cot(d*x+c))+1/4*e^2*(e*cot(d*x+c))
^(1/2)/a/d/(a+a*cot(d*x+c))^2
```

Rubi [A] (verified)

Time = 0.68 (sec) , antiderivative size = 164, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.320, Rules used = {3646, 3730, 3735, 3613, 214, 3715, 65, 211}

$$\int \frac{(e\cot(c+dx))^{5/2}}{(a+a\cot(c+dx))^3}\,dx = -\frac{e^{5/2}\arctan\left(\frac{\sqrt{e\cot(c+dx)}}{\sqrt{e}}\right)}{8a^3d} + \frac{e^{5/2}\text{arctanh}\left(\frac{\sqrt{e}\cot(c+dx)+\sqrt{e}}{\sqrt{2}\sqrt{e\cot(c+dx)}}\right)}{2\sqrt{2}a^3d} - \frac{5e^2\sqrt{e\cot(c+dx)}}{8a^3d(\cot(c+dx)+1)} + \frac{e^2\sqrt{e\cot(c+dx)}}{4ad(a\cot(c+dx)+a)^2}$$

```
[In] Int[(e*Cot[c + d*x])^(5/2)/(a + a*Cot[c + d*x])^3,x]
```

```
[Out] -1/8*(e^(5/2)*ArcTan[Sqrt[e*Cot[c + d*x]]/Sqrt[e]])/(a^3*d) + (e^(5/2)*ArcT
anh[(Sqrt[e] + Sqrt[e]*Cot[c + d*x])/(Sqrt[2]*Sqrt[e*Cot[c + d*x]])])/(2*Sq
```

```
rt[2]*a^3*d) - (5*e^2*Sqrt[e*Cot[c + d*x]])/(8*a^3*d*(1 + Cot[c + d*x])) +
(e^2*Sqrt[e*Cot[c + d*x]])/(4*a*d*(a + a*Cot[c + d*x])^2)
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 3613

```
Int[((c_) + (d_.)*tan[(e_.) + (f_.)*(x_)])/Sqrt[(b_.)*tan[(e_.) + (f_.)*(x_
)]], x_Symbol] :> Dist[-2*(d^2/f), Subst[Int[1/(2*c*d + b*x^2), x], x, (c -
 d*Tan[e + f*x])/Sqrt[b*Tan[e + f*x]]], x] /; FreeQ[{b, c, d, e, f}, x] &&
EqQ[c^2 - d^2, 0]
```

Rule 3646

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*c - a*d)^2*(a + b*Tan[e + f*x])^(m
 - 2)*((c + d*Tan[e + f*x])^(n + 1)/(d*f*(n + 1)*(c^2 + d^2))), x] - Dist[1
/(d*(n + 1)*(c^2 + d^2)), Int[(a + b*Tan[e + f*x])^(m - 3)*(c + d*Tan[e + f
*x])^(n + 1)*Simp[a^2*d*(b*d*(m - 2) - a*c*(n + 1)) + b*(b*c - 2*a*d)*(b*c*
(m - 2) + a*d*(n + 1)) - d*(n + 1)*(3*a^2*b*c - b^3*c - a^3*d + 3*a*b^2*d)*
Tan[e + f*x] - b*(a*d*(2*b*c - a*d)*(m + n - 1) - b^2*(c^2*(m - 2) - d^2*(n
 + 1)))*Tan[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b
*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] && GtQ[m, 2] && LtQ[
n, -1] && IntegerQ[2*m]
```

Rule 3715

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*tan[(e_.)
+ (f_.)*(x_)])^(n_.)*((A_) + (C_.)*tan[(e_.) + (f_.)*(x_)]^2), x_Symbol] :>
 Dist[A/f, Subst[Int[(a + b*x)^m*(c + d*x)^n, x], x, Tan[e + f*x]], x] /; F
reeQ[{a, b, c, d, e, f, A, C, m, n}, x] && EqQ[A, C]
```

Rule 3730

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*tan[(e_.) + (f_.)*(x_)] + (C_.)*tan[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> Simp[(A*b^2 - a*(b*B - a*C))*(a + b*Tan[e +
 f*x])^(m + 1)*((c + d*Tan[e + f*x])^(n + 1)/(f*(m + 1)*(b*c - a*d)*(a^2 +
b^2))), x] + Dist[1/((m + 1)*(b*c - a*d)*(a^2 + b^2)), Int[(a + b*Tan[e + f
*x])^(m + 1)*(c + d*Tan[e + f*x])^n*Simp[A*(a*(b*c - a*d)*(m + 1) - b^2*d*(
m + n + 2)) + (b*B - a*C)*(b*c*(m + 1) + a*d*(n + 1)) - (m + 1)*(b*c - a*d)
*(A*b - a*B - b*C)*Tan[e + f*x] - d*(A*b^2 - a*(b*B - a*C))*(m + n + 2)*Tan
[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, n}, x] && NeQ[
b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] && LtQ[m, -1] &&  !
(ILtQ[n, -1] && ( !IntegerQ[m] || (EqQ[c, 0] && NeQ[a, 0])))
```

Rule 3735

```
Int[(((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_)*((A_.) + (C_.)*tan[(e_.)
+ (f_.)*(x_)]^2))/((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dis
t[1/(a^2 + b^2), Int[(c + d*Tan[e + f*x])^n*Simp[a*(A - C) - (A*b - b*C)*Ta
n[e + f*x], x], x], x] + Dist[(A*b^2 + a^2*C)/(a^2 + b^2), Int[(c + d*Tan[e
 + f*x])^n*((1 + Tan[e + f*x]^2)/(a + b*Tan[e + f*x])), x], x] /; FreeQ[{a,
 b, c, d, e, f, A, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && N
eQ[c^2 + d^2, 0] &&  !GtQ[n, 0] &&  !LeQ[n, -1]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{e^2\sqrt{e\cot(c+dx)}}{4ad(a+a\cot(c+dx))^2} - \frac{\int \frac{-\frac{1}{2}a^2e^3+2a^2e^3\cot(c+dx)-\frac{5}{2}a^2e^3\cot^2(c+dx)}{\sqrt{e\cot(c+dx)}(a+a\cot(c+dx))^2}\,dx}{4a^3} \\
&= -\frac{5e^2\sqrt{e\cot(c+dx)}}{8a^3d(1+\cot(c+dx))} + \frac{e^2\sqrt{e\cot(c+dx)}}{4ad(a+a\cot(c+dx))^2} + \frac{\int \frac{-\frac{3}{2}a^4e^4+\frac{5}{2}a^4e^4\cot^2(c+dx)}{\sqrt{e\cot(c+dx)}(a+a\cot(c+dx))}\,dx}{8a^6e} \\
&= -\frac{5e^2\sqrt{e\cot(c+dx)}}{8a^3d(1+\cot(c+dx))} + \frac{e^2\sqrt{e\cot(c+dx)}}{4ad(a+a\cot(c+dx))^2} \\
&\quad + \frac{\int \frac{-4a^5e^4+4a^5e^4\cot(c+dx)}{\sqrt{e\cot(c+dx)}}\,dx}{16a^8e} + \frac{e^3\int \frac{1+\cot^2(c+dx)}{\sqrt{e\cot(c+dx)}(a+a\cot(c+dx))}\,dx}{16a^2} \\
&= -\frac{5e^2\sqrt{e\cot(c+dx)}}{8a^3d(1+\cot(c+dx))} + \frac{e^2\sqrt{e\cot(c+dx)}}{4ad(a+a\cot(c+dx))^2} \\
&\quad + \frac{e^3\text{Subst}\left(\int \frac{1}{\sqrt{-ex}(a-ax)}\,dx, x, -\cot(c+dx)\right)}{16a^2d} \\
&\quad - \frac{(2a^2e^7)\,\text{Subst}\left(\int \frac{1}{32a^{10}e^8-ex^2}\,dx, x, \frac{-4a^5e^4-4a^5e^4\cot(c+dx)}{\sqrt{e\cot(c+dx)}}\right)}{d}
\end{aligned}
$$

$$
\begin{aligned}
&= \frac{e^{5/2}\operatorname{arctanh}\left(\frac{\sqrt{e}+\sqrt{e}\cot(c+dx)}{\sqrt{2}\sqrt{e\cot(c+dx)}}\right)}{2\sqrt{2}a^3d} - \frac{5e^2\sqrt{e\cot(c+dx)}}{8a^3d(1+\cot(c+dx))} \\
&\quad + \frac{e^2\sqrt{e\cot(c+dx)}}{4ad(a+a\cot(c+dx))^2} - \frac{e^2\text{Subst}\left(\int \frac{1}{a+\frac{ax^2}{e}}\,dx, x, \sqrt{e\cot(c+dx)}\right)}{8a^2d} \\
&= -\frac{e^{5/2}\arctan\left(\frac{\sqrt{e\cot(c+dx)}}{\sqrt{e}}\right)}{8a^3d} + \frac{e^{5/2}\operatorname{arctanh}\left(\frac{\sqrt{e}+\sqrt{e}\cot(c+dx)}{\sqrt{2}\sqrt{e\cot(c+dx)}}\right)}{2\sqrt{2}a^3d} \\
&\quad - \frac{5e^2\sqrt{e\cot(c+dx)}}{8a^3d(1+\cot(c+dx))} + \frac{e^2\sqrt{e\cot(c+dx)}}{4ad(a+a\cot(c+dx))^2}
\end{aligned}
$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 3.68 (sec) , antiderivative size = 390, normalized size of antiderivative = 2.38

$$
\int \frac{(e\cot(c+dx))^{5/2}}{(a+a\cot(c+dx))^3}\,dx = \frac{e\left(-48\cot^2(c+dx)(e\cot(c+dx))^{3/2}\,\text{Hypergeometric2F1}\left(2,\frac{7}{2},\frac{9}{2},-\cot(c+d\right.\right.}{}
$$

```
[In] Integrate[(e*Cot[c + d*x])^(5/2)/(a + a*Cot[c + d*x])^3,x]
```

```
[Out] (e*(-48*Cot[c + d*x]^2*(e*Cot[c + d*x])^(3/2)*Hypergeometric2F1[2, 7/2, 9/2
, -Cot[c + d*x]] - 48*Cot[c + d*x]^2*(e*Cot[c + d*x])^(3/2)*Hypergeometric2
F1[3, 7/2, 9/2, -Cot[c + d*x]] + 7*(24*e^(3/2)*ArcTan[Sqrt[e*Cot[c + d*x]]/
Sqrt[e]] + 12*(-e^2)^(3/4)*ArcTan[Sqrt[e*Cot[c + d*x]]/(-e^2)^(1/4)] - 6*Sq
rt[2]*e^(3/2)*ArcTan[1 - (Sqrt[2]*Sqrt[e*Cot[c + d*x]])/Sqrt[e]] + 6*Sqrt[2
]*e^(3/2)*ArcTan[1 + (Sqrt[2]*Sqrt[e*Cot[c + d*x]])/Sqrt[e]] - 12*(-e^2)^(3
/4)*ArcTanh[Sqrt[e*Cot[c + d*x]]/(-e^2)^(1/4)] - 48*e*Sqrt[e*Cot[c + d*x]]
+ 16*(e*Cot[c + d*x])^(3/2) - 3*Sqrt[2]*e^(3/2)*Log[Sqrt[e] + Sqrt[e]*Cot[c
 + d*x] - Sqrt[2]*Sqrt[e*Cot[c + d*x]]] + 3*Sqrt[2]*e^(3/2)*Log[Sqrt[e] + S
qrt[e]*Cot[c + d*x] + Sqrt[2]*Sqrt[e*Cot[c + d*x]]])))/(336*a^3*d)
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 348 vs. 2(135) = 270.

Time = 1.23 (sec) , antiderivative size = 349, normalized size of antiderivative = 2.13

method	result
derivativedivides	$-\frac{2e^4\left(-\frac{(e^2)^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\frac{e\cot(dx+c)+(e^2)^{\frac{1}{4}}\sqrt{e\cot(dx+c)}\sqrt{2}+\sqrt{e^2}}{e\cot(dx+c)-(e^2)^{\frac{1}{4}}\sqrt{e\cot(dx+c)}\sqrt{2}+\sqrt{e^2}}\right)+2\arctan\left(\frac{\sqrt{2}\sqrt{e\cot(dx+c)}}{(e^2)^{\frac{1}{4}}}+1\right)-2\arctan\left(-\frac{\sqrt{2}\sqrt{e\cot(dx+c)}}{(e^2)^{\frac{1}{4}}}\right.\right.}{8e}\right.}{}$
default	$-\frac{2e^4\left(-\frac{(e^2)^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\frac{e\cot(dx+c)+(e^2)^{\frac{1}{4}}\sqrt{e\cot(dx+c)}\sqrt{2}+\sqrt{e^2}}{e\cot(dx+c)-(e^2)^{\frac{1}{4}}\sqrt{e\cot(dx+c)}\sqrt{2}+\sqrt{e^2}}\right)+2\arctan\left(\frac{\sqrt{2}\sqrt{e\cot(dx+c)}}{(e^2)^{\frac{1}{4}}}+1\right)-2\arctan\left(-\frac{\sqrt{2}\sqrt{e\cot(dx+c)}}{(e^2)^{\frac{1}{4}}}\right.\right.}{8e}\right.}{}$

```
[In] int((e*cot(d*x+c))^(5/2)/(a+a*cot(d*x+c))^3,x,method=_RETURNVERBOSE)
```

```
[Out] -2/d/a^3*e^4*(1/4/e*(-1/8/e*(e^2)^(1/4)*2^(1/2)*(ln((e*cot(d*x+c)+(e^2)^(1/
4)*(e*cot(d*x+c))^(1/2)*2^(1/2)+(e^2)^(1/2))/(e*cot(d*x+c)-(e^2)^(1/4)*(e*c
ot(d*x+c))^(1/2)*2^(1/2)+(e^2)^(1/2)))+2*arctan(2^(1/2)/(e^2)^(1/4)*(e*cot(
d*x+c))^(1/2)+1)-2*arctan(-2^(1/2)/(e^2)^(1/4)*(e*cot(d*x+c))^(1/2)+1))+1/8
/(e^2)^(1/4)*2^(1/2)*(ln((e*cot(d*x+c)-(e^2)^(1/4)*(e*cot(d*x+c))^(1/2)*2^(
1/2)+(e^2)^(1/2))/(e*cot(d*x+c)+(e^2)^(1/4)*(e*cot(d*x+c))^(1/2)*2^(1/2)+(e
^2)^(1/2)))+2*arctan(2^(1/2)/(e^2)^(1/4)*(e*cot(d*x+c))^(1/2)+1)-2*arctan(-
2^(1/2)/(e^2)^(1/4)*(e*cot(d*x+c))^(1/2)+1)))+1/4/e*((5/4*(e*cot(d*x+c))^(3
/2)+3/4*e*(e*cot(d*x+c))^(1/2))/(e*cot(d*x+c)+e)^2+1/4/e^(1/2)*arctan((e*co
t(d*x+c))^(1/2)/e^(1/2))))
```

Fricas [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 567, normalized size of antiderivative = 3.46

$$\int \frac{(e\cot(c+dx))^{5/2}}{(a+a\cot(c+dx))^3}\,dx = \left[-\frac{4\left(\sqrt{2}e^2\sin(2\,dx+2\,c)+\sqrt{2}e^2\right)\sqrt{-e}\arctan\left(\frac{\left(\sqrt{2}\cos(2\,dx+2\,c)+\sqrt{2}\sin(2\,dx+2\,c)+\sqrt{}\right.}{2\,(e\cos(2\,dx+2\,c)}\right.}{}\right.$$

$$-\frac{2\left(e^2\sin(2\,dx+2\,c)+e^2\right)\sqrt{e}\arctan\left(\frac{\sqrt{\frac{e\cos(2\,dx+2\,c)+e}{\sin(2\,dx+2\,c)}}}{\sqrt{e}}\right)-2\left(\sqrt{2}e^2\sin(2\,dx+2\,c)+\sqrt{2}e^2\right)\sqrt{e}\log\left(-\left(\sqrt{2}\cos\right.\right.}{}$$

```
[In] integrate((e*cot(d*x+c))^(5/2)/(a+a*cot(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] [-1/16*(4*(sqrt(2)*e^2*sin(2*d*x + 2*c) + sqrt(2)*e^2)*sqrt(-e)*arctan(1/2*
(sqrt(2)*cos(2*d*x + 2*c) + sqrt(2)*sin(2*d*x + 2*c) + sqrt(2))*sqrt(-e)*sq
rt((e*cos(2*d*x + 2*c) + e)/sin(2*d*x + 2*c))/(e*cos(2*d*x + 2*c) + e)) - (
e^2*sin(2*d*x + 2*c) + e^2)*sqrt(-e)*log((e*cos(2*d*x + 2*c) - e*sin(2*d*x
+ 2*c) - 2*sqrt(-e)*sqrt((e*cos(2*d*x + 2*c) + e)/sin(2*d*x + 2*c))*sin(2*d
*x + 2*c) + e)/(cos(2*d*x + 2*c) + sin(2*d*x + 2*c) + 1)) - (3*e^2*cos(2*d*
x + 2*c) - 5*e^2*sin(2*d*x + 2*c) - 3*e^2)*sqrt((e*cos(2*d*x + 2*c) + e)/si
n(2*d*x + 2*c)))/(a^3*d*sin(2*d*x + 2*c) + a^3*d), -1/16*(2*(e^2*sin(2*d*x
+ 2*c) + e^2)*sqrt(e)*arctan(sqrt((e*cos(2*d*x + 2*c) + e)/sin(2*d*x + 2*c)
)/sqrt(e)) - 2*(sqrt(2)*e^2*sin(2*d*x + 2*c) + sqrt(2)*e^2)*sqrt(e)*log(-(s
qrt(2)*cos(2*d*x + 2*c) - sqrt(2)*sin(2*d*x + 2*c) - sqrt(2))*sqrt(e)*sqrt(
(e*cos(2*d*x + 2*c) + e)/sin(2*d*x + 2*c)) + 2*e*sin(2*d*x + 2*c) + e) - (3
*e^2*cos(2*d*x + 2*c) - 5*e^2*sin(2*d*x + 2*c) - 3*e^2)*sqrt((e*cos(2*d*x +
 2*c) + e)/sin(2*d*x + 2*c)))/(a^3*d*sin(2*d*x + 2*c) + a^3*d)]
```

Sympy [F]

$$\int \frac{(e\cot(c+dx))^{5/2}}{(a+a\cot(c+dx))^3}\,dx = \frac{\int \frac{(e\cot(c+dx))^{\frac{5}{2}}}{\cot^3(c+dx)+3\cot^2(c+dx)+3\cot(c+dx)+1}\,dx}{a^3}$$

```
[In] integrate((e*cot(d*x+c))**(5/2)/(a+a*cot(d*x+c))**3,x)
```

```
[Out] Integral((e*cot(c + d*x))**(5/2)/(cot(c + d*x)**3 + 3*cot(c + d*x)**2 + 3*c
ot(c + d*x) + 1), x)/a**3
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{(e\cot(c+dx))^{5/2}}{(a+a\cot(c+dx))^3}\,dx = \text{Exception raised: ValueError}$$

```
[In] integrate((e*cot(d*x+c))^(5/2)/(a+a*cot(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(e>0)', see `assume?` for more detai
ls)Is e
```

Giac [F]

$$\int \frac{(e\cot(c+dx))^{5/2}}{(a+a\cot(c+dx))^3}\,dx = \int \frac{(e\cot(dx+c))^{\frac{5}{2}}}{(a\cot(dx+c)+a)^3}\,dx$$

[In] integrate((e*cot(d*x+c))^(5/2)/(a+a*cot(d*x+c))^3,x, algorithm="giac")

[Out] integrate((e*cot(d*x + c))^(5/2)/(a*cot(d*x + c) + a)^3, x)

Mupad [B] (verification not implemented)

Time = 13.53 (sec) , antiderivative size = 154, normalized size of antiderivative = 0.94

$$\int \frac{(e\cot(c+dx))^{5/2}}{(a+a\cot(c+dx))^3}\,dx = \frac{\sqrt{2}\,e^{5/2}\,\mathrm{atanh}\left(\frac{9\sqrt{2}\,e^{33/2}\,\sqrt{e\cot(c+d\,x)}}{32\left(\frac{9\,e^{17}\cot(c+d\,x)}{32}+\frac{9\,e^{17}}{32}\right)}\right)}{4\,a^3\,d} - \frac{e^{5/2}\,\mathrm{atan}\left(\frac{\sqrt{e\cot(c+d\,x)}}{\sqrt{e}}\right)}{8\,a^3\,d} - \frac{\frac{3\,e^4\,\sqrt{e\cot(c+d\,x)}}{8}+\frac{5\,e^3\,(e\cot(c+d\,x))^{3/2}}{8}}{d\,a^3\,e^2\cot(c+d\,x)^2+2\,d\,a^3\,e^2\cot(c+d\,x)+d\,a^3\,e^2}$$

[In] int((e*cot(c + d*x))^(5/2)/(a + a*cot(c + d*x))^3,x)

[Out] (2^(1/2)*e^(5/2)*atanh((9*2^(1/2)*e^(33/2)*(e*cot(c + d*x))^(1/2))/(32*((9*e^17*cot(c + d*x))/32 + (9*e^17)/32))))/(4*a^3*d) - (e^(5/2)*atan((e*cot(c + d*x))^(1/2)/e^(1/2)))/(8*a^3*d) - ((3*e^4*(e*cot(c + d*x))^(1/2))/8 + (5*e^3*(e*cot(c + d*x))^(3/2))/8)/(a^3*d*e^2 + a^3*d*e^2*cot(c + d*x)^2 + 2*a^3*d*e^2*cot(c + d*x))

3.36 $\int \frac{(e\cot(c+dx))^{3/2}}{(a+a\cot(c+dx))^3}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 164

$$\int \frac{(e\cot(c+dx))^{3/2}}{(a+a\cot(c+dx))^3}\,dx = \frac{5e^{3/2}\arctan\left(\frac{\sqrt{e\cot(c+dx)}}{\sqrt{e}}\right)}{8a^3d} + \frac{e^{3/2}\arctan\left(\frac{\sqrt{e}-\sqrt{e}\cot(c+dx)}{\sqrt{2}\sqrt{e\cot(c+dx)}}\right)}{2\sqrt{2}a^3d} - \frac{e\sqrt{e\cot(c+dx)}}{4ad(a+a\cot(c+dx))^2} + \frac{e\sqrt{e\cot(c+dx)}}{8d\left(a^3+a^3\cot(c+dx)\right)}$$

```
[Out] 5/8*e^(3/2)*arctan((e*cot(d*x+c))^(1/2)/e^(1/2))/a^3/d+1/4*e^(3/2)*arctan(1
/2*(e^(1/2)-cot(d*x+c)*e^(1/2))*2^(1/2)/(e*cot(d*x+c))^(1/2))/a^3/d*2^(1/2)
-1/4*e*(e*cot(d*x+c))^(1/2)/a/d/(a+a*cot(d*x+c))^2+1/8*e*(e*cot(d*x+c))^(1/
2)/d/(a^3+a^3*cot(d*x+c))
```

Rubi [A] (verified)

Time = 0.74 (sec) , antiderivative size = 164, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.280, Rules used = {3648, 3730, 3734, 3613, 211, 3715, 65}

$$\int \frac{(e\cot(c+dx))^{3/2}}{(a+a\cot(c+dx))^3}\,dx = \frac{5e^{3/2}\arctan\left(\frac{\sqrt{e\cot(c+dx)}}{\sqrt{e}}\right)}{8a^3d} + \frac{e^{3/2}\arctan\left(\frac{\sqrt{e}-\sqrt{e}\cot(c+dx)}{\sqrt{2}\sqrt{e\cot(c+dx)}}\right)}{2\sqrt{2}a^3d} + \frac{e\sqrt{e\cot(c+dx)}}{8d\left(a^3\cot(c+dx)+a^3\right)} - \frac{e\sqrt{e\cot(c+dx)}}{4ad(a\cot(c+dx)+a)^2}$$

```
[In] Int[(e*Cot[c + d*x])^(3/2)/(a + a*Cot[c + d*x])^3,x]
```

```
[Out] (5*e^(3/2)*ArcTan[Sqrt[e*Cot[c + d*x]]/Sqrt[e]])/(8*a^3*d) + (e^(3/2)*ArcTa
n[(Sqrt[e] - Sqrt[e]*Cot[c + d*x])/(Sqrt[2]*Sqrt[e*Cot[c + d*x]])])/(2*Sqrt
```

```
[2]*a^3*d) - (e*Sqrt[e*Cot[c + d*x]])/(4*a*d*(a + a*Cot[c + d*x])^2) + (e*S
qrt[e*Cot[c + d*x]])/(8*d*(a^3 + a^3*Cot[c + d*x]))
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 3613

```
Int[((c_) + (d_.)*tan[(e_.) + (f_.)*(x_)])/Sqrt[(b_.)*tan[(e_.) + (f_.)*(x_
)]], x_Symbol] :> Dist[-2*(d^2/f), Subst[Int[1/(2*c*d + b*x^2), x], x, (c -
 d*Tan[e + f*x])/Sqrt[b*Tan[e + f*x]]], x] /; FreeQ[{b, c, d, e, f}, x] &&
EqQ[c^2 - d^2, 0]
```

Rule 3648

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*c - a*d)*(a + b*Tan[e + f*x])^(m +
 1)*((c + d*Tan[e + f*x])^(n - 1)/(f*(m + 1)*(a^2 + b^2))), x] + Dist[1/((m
 + 1)*(a^2 + b^2)), Int[(a + b*Tan[e + f*x])^(m + 1)*(c + d*Tan[e + f*x])^(
n - 2)*Simp[a*c^2*(m + 1) + a*d^2*(n - 1) + b*c*d*(m - n + 2) - (b*c^2 - 2*
a*c*d - b*d^2)*(m + 1)*Tan[e + f*x] - d*(b*c - a*d)*(m + n)*Tan[e + f*x]^2,
 x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2
 + b^2, 0] && NeQ[c^2 + d^2, 0] && LtQ[m, -1] && LtQ[1, n, 2] && IntegerQ[2
*m]
```

Rule 3715

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*tan[(e_.)
+ (f_.)*(x_)])^(n_.)*((A_) + (C_.)*tan[(e_.) + (f_.)*(x_)]^2), x_Symbol] :>
 Dist[A/f, Subst[Int[(a + b*x)^m*(c + d*x)^n, x], x, Tan[e + f*x]], x] /; F
reeQ[{a, b, c, d, e, f, A, C, m, n}, x] && EqQ[A, C]
```

Rule 3730

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*tan[(e_.) + (f_.)*(x_)] + (C_.)*tan[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> Simp[(A*b^2 - a*(b*B - a*C))*(a + b*Tan[e +
```

```
 f*x])^(m + 1)*((c + d*Tan[e + f*x])^(n + 1)/(f*(m + 1)*(b*c - a*d)*(a^2 +
b^2))), x] + Dist[1/((m + 1)*(b*c - a*d)*(a^2 + b^2)), Int[(a + b*Tan[e + f
*x])^(m + 1)*(c + d*Tan[e + f*x])^n*Simp[A*(a*(b*c - a*d)*(m + 1) - b^2*d*(
m + n + 2)) + (b*B - a*C)*(b*c*(m + 1) + a*d*(n + 1)) - (m + 1)*(b*c - a*d)
*(A*b - a*B - b*C)*Tan[e + f*x] - d*(A*b^2 - a*(b*B - a*C))*(m + n + 2)*Tan
[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, n}, x] && NeQ[
b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] && LtQ[m, -1] &&  !
(ILtQ[n, -1] && ( !IntegerQ[m] || (EqQ[c, 0] && NeQ[a, 0])))
```

Rule 3734

```
Int[(((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_)*((A_.) + (B_.)*tan[(e_.)
+ (f_.)*(x_)] + (C_.)*tan[(e_.) + (f_.)*(x_)]^2))/((a_.) + (b_.)*tan[(e_.)
+ (f_.)*(x_)]), x_Symbol] :> Dist[1/(a^2 + b^2), Int[(c + d*Tan[e + f*x])^n
*Simp[b*B + a*(A - C) + (a*B - b*(A - C))*Tan[e + f*x], x], x], x] + Dist[(
A*b^2 - a*b*B + a^2*C)/(a^2 + b^2), Int[(c + d*Tan[e + f*x])^n*((1 + Tan[e
+ f*x]^2)/(a + b*Tan[e + f*x])), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C
, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] &&
 !GtQ[n, 0] &&  !LeQ[n, -1]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= -\frac{e\sqrt{e\cot(c+dx)}}{4ad(a+a\cot(c+dx))^2} - \frac{\int \frac{\frac{ae^2}{2}-2ae^2\cot(c+dx)-\frac{3}{2}ae^2\cot^2(c+dx)}{\sqrt{e\cot(c+dx)}(a+a\cot(c+dx))^2}\,dx}{4a^2} \\
&= -\frac{e\sqrt{e\cot(c+dx)}}{4ad(a+a\cot(c+dx))^2} + \frac{e\sqrt{e\cot(c+dx)}}{8d\left(a^3+a^3\cot(c+dx)\right)} \\
&\quad + \frac{\int \frac{-\frac{1}{2}a^3e^3+4a^3e^3\cot(c+dx)-\frac{1}{2}a^3e^3\cot^2(c+dx)}{\sqrt{e\cot(c+dx)}(a+a\cot(c+dx))}\,dx}{8a^5e} \\
&= -\frac{e\sqrt{e\cot(c+dx)}}{4ad(a+a\cot(c+dx))^2} + \frac{e\sqrt{e\cot(c+dx)}}{8d\left(a^3+a^3\cot(c+dx)\right)} \\
&\quad + \frac{\int \frac{4a^4e^3+4a^4e^3\cot(c+dx)}{\sqrt{e\cot(c+dx)}}\,dx}{16a^7e} - \frac{(5e^2)\int \frac{1+\cot^2(c+dx)}{\sqrt{e\cot(c+dx)}(a+a\cot(c+dx))}\,dx}{16a^2} \\
&= -\frac{e\sqrt{e\cot(c+dx)}}{4ad(a+a\cot(c+dx))^2} + \frac{e\sqrt{e\cot(c+dx)}}{8d\left(a^3+a^3\cot(c+dx)\right)} \\
&\quad - \frac{(5e^2)\,\text{Subst}\left(\int \frac{1}{\sqrt{-ex}(a-ax)}\,dx, x, -\cot(c+dx)\right)}{16a^2d} \\
&\quad - \frac{(2ae^5)\,\text{Subst}\left(\int \frac{1}{-32a^8e^6-ex^2}\,dx, x, \frac{4a^4e^3-4a^4e^3\cot(c+dx)}{\sqrt{e\cot(c+dx)}}\right)}{d}
\end{aligned}
$$

$$= \frac{e^{3/2}\arctan\left(\frac{\sqrt{e}-\sqrt{e}\cot(c+dx)}{\sqrt{2}\sqrt{e\cot(c+dx)}}\right)}{2\sqrt{2}a^3d} - \frac{e\sqrt{e\cot(c+dx)}}{4ad(a+a\cot(c+dx))^2} + \frac{e\sqrt{e\cot(c+dx)}}{8d\left(a^3+a^3\cot(c+dx)\right)} + \frac{(5e)\text{Subst}\left(\int\frac{1}{a+\frac{ax^2}{e}}\,dx, x, \sqrt{e\cot(c+dx)}\right)}{8a^2d}$$

$$= \frac{5e^{3/2}\arctan\left(\frac{\sqrt{e\cot(c+dx)}}{\sqrt{e}}\right)}{8a^3d} + \frac{e^{3/2}\arctan\left(\frac{\sqrt{e}-\sqrt{e}\cot(c+dx)}{\sqrt{2}\sqrt{e\cot(c+dx)}}\right)}{2\sqrt{2}a^3d} - \frac{e\sqrt{e\cot(c+dx)}}{4ad(a+a\cot(c+dx))^2} + \frac{e\sqrt{e\cot(c+dx)}}{8d\left(a^3+a^3\cot(c+dx)\right)}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 4.13 (sec) , antiderivative size = 365, normalized size of antiderivative = 2.23

$$\int\frac{(e\cot(c+dx))^{3/2}}{(a+a\cot(c+dx))^3}\,dx = -\frac{e\left(70\sqrt{e}\arctan\left(\frac{\sqrt{e\cot(c+dx)}}{\sqrt{e}}\right) + \frac{20e\arctan\left(\frac{\sqrt{e\cot(c+dx)}}{\sqrt[4]{-e^2}}\right)}{\sqrt[4]{-e^2}} - 10\sqrt{2}\sqrt{e}\arctan\left(1-\frac{\sqrt{2}\sqrt{e\cot(c+dx)}}{\sqrt{e}}\right) + 10\sqrt{2}\sqrt{e}\,\text{arc}\right.}{}$$

```
[In] Integrate[(e*Cot[c + d*x])^(3/2)/(a + a*Cot[c + d*x])^3,x]
```

```
[Out] -1/80*(e*(70*Sqrt[e]*ArcTan[Sqrt[e*Cot[c + d*x]]/Sqrt[e]] + (20*e*ArcTan[Sq
rt[e*Cot[c + d*x]]/(-e^2)^(1/4)])/(-e^2)^(1/4) - 10*Sqrt[2]*Sqrt[e]*ArcTan[
1 - (Sqrt[2]*Sqrt[e*Cot[c + d*x]])/Sqrt[e]] + 10*Sqrt[2]*Sqrt[e]*ArcTan[1 +
 (Sqrt[2]*Sqrt[e*Cot[c + d*x]])/Sqrt[e]] - (20*e*ArcTanh[Sqrt[e*Cot[c + d*x
]]/(-e^2)^(1/4)])/(-e^2)^(1/4) - 80*Sqrt[e*Cot[c + d*x]] - (10*Sqrt[e*Cot[c
 + d*x]]*(3 + 5*Cot[c + d*x]))/(1 + Cot[c + d*x])^2 + (16*(e*Cot[c + d*x])^
(5/2)*Hypergeometric2F1[2, 5/2, 7/2, -Cot[c + d*x]])/e^2 - 5*Sqrt[2]*Sqrt[e
]*Log[Sqrt[e] + Sqrt[e]*Cot[c + d*x] - Sqrt[2]*Sqrt[e*Cot[c + d*x]]] + 5*Sq
rt[2]*Sqrt[e]*Log[Sqrt[e] + Sqrt[e]*Cot[c + d*x] + Sqrt[2]*Sqrt[e*Cot[c + d
*x]]]))/(a^3*d)
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 348 vs. $2(135) = 270$.

Time = 0.04 (sec) , antiderivative size = 349, normalized size of antiderivative = 2.13

method	result
derivativedivides	$-\frac{2e^4\left(\frac{(e^2)^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\frac{e\cot(dx+c)+(e^2)^{\frac{1}{4}}\sqrt{e\cot(dx+c)}\sqrt{2}+\sqrt{e^2}}{e\cot(dx+c)-(e^2)^{\frac{1}{4}}\sqrt{e\cot(dx+c)}\sqrt{2}+\sqrt{e^2}}\right)+2\arctan\left(\frac{\sqrt{2}\sqrt{e\cot(dx+c)}}{(e^2)^{\frac{1}{4}}}+1\right)-2\arctan\left(-\frac{\sqrt{2}\sqrt{e\cot(dx}}{(e^2)^{\frac{1}{4}}}\right.\right.}{8e}\right.}{}$
default	$-\frac{2e^4\left(\frac{(e^2)^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\frac{e\cot(dx+c)+(e^2)^{\frac{1}{4}}\sqrt{e\cot(dx+c)}\sqrt{2}+\sqrt{e^2}}{e\cot(dx+c)-(e^2)^{\frac{1}{4}}\sqrt{e\cot(dx+c)}\sqrt{2}+\sqrt{e^2}}\right)+2\arctan\left(\frac{\sqrt{2}\sqrt{e\cot(dx+c)}}{(e^2)^{\frac{1}{4}}}+1\right)-2\arctan\left(-\frac{\sqrt{2}\sqrt{e\cot(dx}}{(e^2)^{\frac{1}{4}}}\right.\right.}{8e}\right.}{}$

```
[In] int((e*cot(d*x+c))^(3/2)/(a+a*cot(d*x+c))^3,x,method=_RETURNVERBOSE)
```

```
[Out] -2/d/a^3*e^4*(1/4/e^2*(1/8/e*(e^2)^(1/4)*2^(1/2)*(ln((e*cot(d*x+c)+(e^2)^(1
/4)*(e*cot(d*x+c))^(1/2)*2^(1/2)+(e^2)^(1/2))/(e*cot(d*x+c)-(e^2)^(1/4)*(e*
cot(d*x+c))^(1/2)*2^(1/2)+(e^2)^(1/2)))+2*arctan(2^(1/2)/(e^2)^(1/4)*(e*cot
(d*x+c))^(1/2)+1)-2*arctan(-2^(1/2)/(e^2)^(1/4)*(e*cot(d*x+c))^(1/2)+1))+1/
8/(e^2)^(1/4)*2^(1/2)*(ln((e*cot(d*x+c)-(e^2)^(1/4)*(e*cot(d*x+c))^(1/2)*2^
(1/2)+(e^2)^(1/2))/(e*cot(d*x+c)+(e^2)^(1/4)*(e*cot(d*x+c))^(1/2)*2^(1/2)+(
e^2)^(1/2)))+2*arctan(2^(1/2)/(e^2)^(1/4)*(e*cot(d*x+c))^(1/2)+1)-2*arctan(
-2^(1/2)/(e^2)^(1/4)*(e*cot(d*x+c))^(1/2)+1)))-1/4/e^2*((1/4*(e*cot(d*x+c))
^(3/2)-1/4*e*(e*cot(d*x+c))^(1/2))/(e*cot(d*x+c)+e)^2+5/4/e^(1/2)*arctan((e
*cot(d*x+c))^(1/2)/e^(1/2))))
```

Fricas [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 533, normalized size of antiderivative = 3.25

$$\int \frac{(e\cot(c+dx))^{3/2}}{(a+a\cot(c+dx))^3}\,dx = \left[\frac{2\left(\sqrt{2}e\sin(2\,dx+2\,c)+\sqrt{2}e\right)\sqrt{-e}\log\left(-(\sqrt{2}\cos(2\,dx+2\,c)+\sqrt{2}\sin(2\,dx\right.}{}\right.$$

```
[In] integrate((e*cot(d*x+c))^(3/2)/(a+a*cot(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] [1/16*(2*(sqrt(2)*e*sin(2*d*x + 2*c) + sqrt(2)*e)*sqrt(-e)*log(-(sqrt(2)*co
s(2*d*x + 2*c) + sqrt(2)*sin(2*d*x + 2*c) - sqrt(2))*sqrt(-e)*sqrt((e*cos(2
*d*x + 2*c) + e)/sin(2*d*x + 2*c)) - 2*e*sin(2*d*x + 2*c) + e) + 5*(e*sin(2
*d*x + 2*c) + e)*sqrt(-e)*log((e*cos(2*d*x + 2*c) - e*sin(2*d*x + 2*c) + 2*
sqrt(-e)*sqrt((e*cos(2*d*x + 2*c) + e)/sin(2*d*x + 2*c))*sin(2*d*x + 2*c) +
 e)/(cos(2*d*x + 2*c) + sin(2*d*x + 2*c) + 1)) + (e*cos(2*d*x + 2*c) + e*si
n(2*d*x + 2*c) - e)*sqrt((e*cos(2*d*x + 2*c) + e)/sin(2*d*x + 2*c)))/(a^3*d
*sin(2*d*x + 2*c) + a^3*d), 1/16*(4*(sqrt(2)*e*sin(2*d*x + 2*c) + sqrt(2)*e
)*sqrt(e)*arctan(-1/2*(sqrt(2)*cos(2*d*x + 2*c) - sqrt(2)*sin(2*d*x + 2*c)
+ sqrt(2))*sqrt(e)*sqrt((e*cos(2*d*x + 2*c) + e)/sin(2*d*x + 2*c))/(e*cos(2
*d*x + 2*c) + e)) + 10*(e*sin(2*d*x + 2*c) + e)*sqrt(e)*arctan(sqrt((e*cos(
2*d*x + 2*c) + e)/sin(2*d*x + 2*c))/sqrt(e)) + (e*cos(2*d*x + 2*c) + e*sin(
2*d*x + 2*c) - e)*sqrt((e*cos(2*d*x + 2*c) + e)/sin(2*d*x + 2*c)))/(a^3*d*s
in(2*d*x + 2*c) + a^3*d)]
```

Sympy [F]

$$\int \frac{(e\cot(c+dx))^{3/2}}{(a+a\cot(c+dx))^3}\,dx = \frac{\int \frac{(e\cot(c+dx))^{\frac{3}{2}}}{\cot^3(c+dx)+3\cot^2(c+dx)+3\cot(c+dx)+1}\,dx}{a^3}$$

```
[In] integrate((e*cot(d*x+c))**(3/2)/(a+a*cot(d*x+c))**3,x)
```

```
[Out] Integral((e*cot(c + d*x))**(3/2)/(cot(c + d*x)**3 + 3*cot(c + d*x)**2 + 3*c
ot(c + d*x) + 1), x)/a**3
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{(e\cot(c+dx))^{3/2}}{(a+a\cot(c+dx))^3}\,dx = \text{Exception raised: ValueError}$$

```
[In] integrate((e*cot(d*x+c))^(3/2)/(a+a*cot(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(e>0)', see `assume?` for more detai
ls)Is e
```

Giac [F]

$$\int \frac{(e\cot(c+dx))^{3/2}}{(a+a\cot(c+dx))^3}\,dx = \int \frac{(e\cot(dx+c))^{\frac{3}{2}}}{(a\cot(dx+c)+a)^3}\,dx$$

```
[In] integrate((e*cot(d*x+c))^(3/2)/(a+a*cot(d*x+c))^3,x, algorithm="giac")
[Out] integrate((e*cot(d*x + c))^(3/2)/(a*cot(d*x + c) + a)^3, x)
```

Mupad [B] (verification not implemented)

Time = 13.27 (sec) , antiderivative size = 178, normalized size of antiderivative = 1.09

$$\int \frac{(e\cot(c+dx))^{3/2}}{(a+a\cot(c+dx))^3}\,dx = \frac{5\,e^{3/2}\,\mathrm{atan}\left(\frac{\sqrt{e\cot(c+d\,x)}}{\sqrt{e}}\right)}{8\,a^3\,d} - \frac{\frac{e^3\,\sqrt{e\cot(c+d\,x)}}{8} - \frac{e^2\,(e\cot(c+d\,x))^{3/2}}{8}}{d\,a^3\,e^2\cot\left(c+d\,x\right)^2 + 2\,d\,a^3\,e^2\cot\left(c+d\,x\right) + d\,a^3\,e^2} - \frac{\sqrt{2}\,e^{3/2}\left(2\,\mathrm{atan}\left(\frac{\sqrt{2}\,\sqrt{e\cot(c+d\,x)}}{2\,\sqrt{e}}\right) + 2\,\mathrm{atan}\left(\frac{\sqrt{2}\,\sqrt{e\cot(c+d\,x)}}{2\,\sqrt{e}} + \frac{\sqrt{2}\,(e\cot(c+d\,x))^{3/2}}{2\,e^{3/2}}\right)\right)}{8\,a^3\,d}$$

```
[In] int((e*cot(c + d*x))^(3/2)/(a + a*cot(c + d*x))^3,x)
[Out] (5*e^(3/2)*atan((e*cot(c + d*x))^(1/2)/e^(1/2)))/(8*a^3*d) - ((e^3*(e*cot(c
 + d*x))^(1/2))/8 - (e^2*(e*cot(c + d*x))^(3/2))/8)/(a^3*d*e^2 + a^3*d*e^2*
cot(c + d*x)^2 + 2*a^3*d*e^2*cot(c + d*x)) - (2^(1/2)*e^(3/2)*(2*atan((2^(1
/2)*(e*cot(c + d*x))^(1/2))/(2*e^(1/2))) + 2*atan((2^(1/2)*(e*cot(c + d*x))
^(1/2))/(2*e^(1/2)) + (2^(1/2)*(e*cot(c + d*x))^(3/2))/(2*e^(3/2)))))/(8*a^
3*d)
```

3.37 $\int \frac{\sqrt{e \cot(c+dx)}}{(a+a \cot(c+dx))^3} \, dx$

Optimal result

Integrand size = 25, antiderivative size = 161

$$\int \frac{\sqrt{e \cot(c+dx)}}{(a+a \cot(c+dx))^3} \, dx = -\frac{\sqrt{e} \arctan\left(\frac{\sqrt{e \cot(c+dx)}}{\sqrt{e}}\right)}{8a^3 d} - \frac{\sqrt{e} \operatorname{arctanh}\left(\frac{\sqrt{e}+\sqrt{e}\cot(c+dx)}{\sqrt{2}\sqrt{e \cot(c+dx)}}\right)}{2\sqrt{2}a^3 d} + \frac{\sqrt{e \cot(c+dx)}}{4ad(a+a\cot(c+dx))^2} + \frac{3\sqrt{e \cot(c+dx)}}{8d\left(a^3+a^3\cot(c+dx)\right)}$$

```
[Out] -1/8*arctan((e*cot(d*x+c))^(1/2)/e^(1/2))*e^(1/2)/a^3/d-1/4*arctanh(1/2*(e^
(1/2)+cot(d*x+c)*e^(1/2))*2^(1/2)/(e*cot(d*x+c))^(1/2))*e^(1/2)/a^3/d*2^(1/
2)+1/4*(e*cot(d*x+c))^(1/2)/a/d/(a+a*cot(d*x+c))^2+3/8*(e*cot(d*x+c))^(1/2)
/d/(a^3+a^3*cot(d*x+c))
```

Rubi [A] (verified)

Time = 0.66 (sec) , antiderivative size = 161, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.320$, Rules used = {3649, 3730, 3735, 3613, 214, 3715, 65, 211}

$$\int \frac{\sqrt{e \cot(c+dx)}}{(a+a \cot(c+dx))^3} \, dx = -\frac{\sqrt{e} \arctan\left(\frac{\sqrt{e \cot(c+dx)}}{\sqrt{e}}\right)}{8a^3 d} - \frac{\sqrt{e} \operatorname{arctanh}\left(\frac{\sqrt{e}\cot(c+dx)+\sqrt{e}}{\sqrt{2}\sqrt{e \cot(c+dx)}}\right)}{2\sqrt{2}a^3 d} + \frac{3\sqrt{e \cot(c+dx)}}{8d\left(a^3\cot(c+dx)+a^3\right)} + \frac{\sqrt{e \cot(c+dx)}}{4ad(a\cot(c+dx)+a)^2}$$

```
[In] Int[Sqrt[e*Cot[c + d*x]]/(a + a*Cot[c + d*x])^3,x]
```

```
[Out] -1/8*(Sqrt[e]*ArcTan[Sqrt[e*Cot[c + d*x]]/Sqrt[e]])/(a^3*d) - (Sqrt[e]*ArcT
anh[(Sqrt[e] + Sqrt[e]*Cot[c + d*x])/(Sqrt[2]*Sqrt[e*Cot[c + d*x]])])/(2*Sq
```

```
rt[2]*a^3*d) + Sqrt[e*Cot[c + d*x]]/(4*a*d*(a + a*Cot[c + d*x])^2) + (3*Sqr
t[e*Cot[c + d*x]])/(8*d*(a^3 + a^3*Cot[c + d*x]))
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 3613

```
Int[((c_) + (d_.)*tan[(e_.) + (f_.)*(x_)])/Sqrt[(b_.)*tan[(e_.) + (f_.)*(x_
)]], x_Symbol] :> Dist[-2*(d^2/f), Subst[Int[1/(2*c*d + b*x^2), x], x, (c -
 d*Tan[e + f*x])/Sqrt[b*Tan[e + f*x]]], x] /; FreeQ[{b, c, d, e, f}, x] &&
EqQ[c^2 - d^2, 0]
```

Rule 3649

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)])^(n_), x_Symbol] :> Simp[b*(a + b*Tan[e + f*x])^(m + 1)*((c +
d*Tan[e + f*x])^n/(f*(m + 1)*(a^2 + b^2))), x] + Dist[1/((m + 1)*(a^2 + b^2
)), Int[(a + b*Tan[e + f*x])^(m + 1)*(c + d*Tan[e + f*x])^(n - 1)*Simp[a*c*
(m + 1) - b*d*n - (b*c - a*d)*(m + 1)*Tan[e + f*x] - b*d*(m + n + 1)*Tan[e
+ f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] &
& NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] && LtQ[m, -1] && GtQ[n, 0] && Inte
gerQ[2*m]
```

Rule 3715

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*tan[(e_.)
+ (f_.)*(x_)])^(n_.)*((A_) + (C_.)*tan[(e_.) + (f_.)*(x_)]^2), x_Symbol] :>
 Dist[A/f, Subst[Int[(a + b*x)^m*(c + d*x)^n, x], x, Tan[e + f*x]], x] /; F
reeQ[{a, b, c, d, e, f, A, C, m, n}, x] && EqQ[A, C]
```

Rule 3730

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*tan[(e_.) + (f_.)*(x_)] + (C_.)*tan[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> Simp[(A*b^2 - a*(b*B - a*C))*(a + b*Tan[e +
 f*x])^(m + 1)*((c + d*Tan[e + f*x])^(n + 1)/(f*(m + 1)*(b*c - a*d)*(a^2 +
b^2))), x] + Dist[1/((m + 1)*(b*c - a*d)*(a^2 + b^2)), Int[(a + b*Tan[e + f
*x])^(m + 1)*(c + d*Tan[e + f*x])^n*Simp[A*(a*(b*c - a*d)*(m + 1) - b^2*d*(
m + n + 2)) + (b*B - a*C)*(b*c*(m + 1) + a*d*(n + 1)) - (m + 1)*(b*c - a*d)
*(A*b - a*B - b*C)*Tan[e + f*x] - d*(A*b^2 - a*(b*B - a*C))*(m + n + 2)*Tan
[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, n}, x] && NeQ[
b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] && LtQ[m, -1] &&  !
(ILtQ[n, -1] && ( !IntegerQ[m] || (EqQ[c, 0] && NeQ[a, 0])))
```

Rule 3735

```
Int[(((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_)*((A_.) + (C_.)*tan[(e_.)
+ (f_.)*(x_)]^2))/((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dis
t[1/(a^2 + b^2), Int[(c + d*Tan[e + f*x])^n*Simp[a*(A - C) - (A*b - b*C)*Ta
n[e + f*x], x], x], x] + Dist[(A*b^2 + a^2*C)/(a^2 + b^2), Int[(c + d*Tan[e
 + f*x])^n*((1 + Tan[e + f*x]^2)/(a + b*Tan[e + f*x])), x], x] /; FreeQ[{a,
 b, c, d, e, f, A, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && N
eQ[c^2 + d^2, 0] &&  !GtQ[n, 0] &&  !LeQ[n, -1]
```

Rubi steps

$$\text{integral} = \frac{\sqrt{e\cot(c+dx)}}{4ad(a+a\cot(c+dx))^2} - \frac{\int \frac{-\frac{ae}{2}-2ae\cot(c+dx)+\frac{3}{2}ae\cot^2(c+dx)}{\sqrt{e\cot(c+dx)}(a+a\cot(c+dx))^2}\,dx}{4a^2}$$

$$= \frac{\sqrt{e\cot(c+dx)}}{4ad(a+a\cot(c+dx))^2} + \frac{3\sqrt{e\cot(c+dx)}}{8d\left(a^3+a^3\cot(c+dx)\right)} + \frac{\int \frac{\frac{5a^3e^2}{2}-\frac{3}{2}a^3e^2\cot^2(c+dx)}{\sqrt{e\cot(c+dx)}(a+a\cot(c+dx))}\,dx}{8a^5e}$$

$$= \frac{\sqrt{e\cot(c+dx)}}{4ad(a+a\cot(c+dx))^2} + \frac{3\sqrt{e\cot(c+dx)}}{8d\left(a^3+a^3\cot(c+dx)\right)} + \frac{\int \frac{4a^4e^2-4a^4e^2\cot(c+dx)}{\sqrt{e\cot(c+dx)}}\,dx}{16a^7e} + \frac{e\int \frac{1+\cot^2(c+dx)}{\sqrt{e\cot(c+dx)}(a+a\cot(c+dx))}\,dx}{16a^2}$$

$$= \frac{\sqrt{e\cot(c+dx)}}{4ad(a+a\cot(c+dx))^2} + \frac{3\sqrt{e\cot(c+dx)}}{8d\left(a^3+a^3\cot(c+dx)\right)} + \frac{e\,\text{Subst}\left(\int \frac{1}{\sqrt{-ex}(a-ax)}\,dx, x, -\cot(c+dx)\right)}{16a^2d} - \frac{(2ae^3)\,\text{Subst}\left(\int \frac{1}{32a^8e^4-ex^2}\,dx, x, \frac{4a^4e^2+4a^4e^2\cot(c+dx)}{\sqrt{e\cot(c+dx)}}\right)}{d}$$

$$= -\frac{\sqrt{e}\,\text{arctanh}\left(\frac{\sqrt{e}+\sqrt{e}\cot(c+dx)}{\sqrt{2}\sqrt{e\cot(c+dx)}}\right)}{2\sqrt{2}a^3d} + \frac{\sqrt{e\cot(c+dx)}}{4ad(a+a\cot(c+dx))^2} + \frac{3\sqrt{e\cot(c+dx)}}{8d\left(a^3+a^3\cot(c+dx)\right)} - \frac{\text{Subst}\left(\int \frac{1}{a+\frac{ax^2}{e}}\,dx, x, \sqrt{e\cot(c+dx)}\right)}{8a^2d}$$

$$= -\frac{\sqrt{e}\arctan\left(\frac{\sqrt{e\cot(c+dx)}}{\sqrt{e}}\right)}{8a^3d} - \frac{\sqrt{e}\,\text{arctanh}\left(\frac{\sqrt{e}+\sqrt{e}\cot(c+dx)}{\sqrt{2}\sqrt{e\cot(c+dx)}}\right)}{2\sqrt{2}a^3d} + \frac{\sqrt{e\cot(c+dx)}}{4ad(a+a\cot(c+dx))^2} + \frac{3\sqrt{e\cot(c+dx)}}{8d\left(a^3+a^3\cot(c+dx)\right)}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 6.92 (sec) , antiderivative size = 443, normalized size of antiderivative = 2.75

$$\int \frac{\sqrt{e\cot(c+dx)}}{(a+a\cot(c+dx))^3}\,dx =$$

$$-\frac{e\left(-\frac{\arctan\left(\frac{\sqrt{e\cot(c+dx)}}{\sqrt{e}}\right)}{2a^3\sqrt{e}} - \frac{\arctan\left(\frac{\sqrt{e\cot(c+dx)}}{\sqrt[4]{-e^2}}\right)}{4a^3\sqrt[4]{-e^2}} - \frac{\arctan\left(1-\frac{\sqrt{2}\sqrt{e\cot(c+dx)}}{\sqrt{e}}\right)}{4\sqrt{2}a^3\sqrt{e}} + \frac{\arctan\left(1+\frac{\sqrt{2}\sqrt{e\cot(c+dx)}}{\sqrt{e}}\right)}{4\sqrt{2}a^3\sqrt{e}} + \frac{\text{arctanh}\left(\right.}{4a^3}\right.}{}$$

```
[In] Integrate[Sqrt[e*Cot[c + d*x]]/(a + a*Cot[c + d*x])^3,x]
```

```
[Out] -((e*(-1/2*ArcTan[Sqrt[e*Cot[c + d*x]]/Sqrt[e]]/(a^3*Sqrt[e]) - ArcTan[Sqrt
[e*Cot[c + d*x]]/(-e^2)^(1/4)]/(4*a^3*(-e^2)^(1/4)) - ArcTan[1 - (Sqrt[2]*S
qrt[e*Cot[c + d*x]])/Sqrt[e]]/(4*Sqrt[2]*a^3*Sqrt[e]) + ArcTan[1 + (Sqrt[2]
*Sqrt[e*Cot[c + d*x]])/Sqrt[e]]/(4*Sqrt[2]*a^3*Sqrt[e]) + ArcTanh[Sqrt[e*Co
t[c + d*x]]/(-e^2)^(1/4)]/(4*a^3*(-e^2)^(1/4)) - (-(e*ArcTan[Sqrt[e*Cot[c +
 d*x]]/Sqrt[e]]) - e*ArcTan[Sqrt[e*Cot[c + d*x]]/Sqrt[e]]*Cot[c + d*x] + Sq
rt[e]*Sqrt[e*Cot[c + d*x]])/(2*a^3*Sqrt[e]*(e + e*Cot[c + d*x])) + ((e*Cot[
c + d*x])^(3/2)*Hypergeometric2F1[3/2, 3, 5/2, -Cot[c + d*x]])/(3*a^3*e^2)
- Log[Sqrt[e] + Sqrt[e]*Cot[c + d*x] - Sqrt[2]*Sqrt[e*Cot[c + d*x]]]/(8*Sqr
t[2]*a^3*Sqrt[e]) + Log[Sqrt[e] + Sqrt[e]*Cot[c + d*x] + Sqrt[2]*Sqrt[e*Cot
[c + d*x]]]/(8*Sqrt[2]*a^3*Sqrt[e])))/d)
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 348 vs. $2(132) = 264$.

Time = 0.05 (sec) , antiderivative size = 349, normalized size of antiderivative = 2.17

method	result
derivativedivides	$-\frac{2e^4\left(\frac{(e^2)^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\frac{e\cot(dx+c)+(e^2)^{\frac{1}{4}}\sqrt{e\cot(dx+c)}\sqrt{2}+\sqrt{e^2}}{e\cot(dx+c)-(e^2)^{\frac{1}{4}}\sqrt{e\cot(dx+c)}\sqrt{2}+\sqrt{e^2}}\right)+2\arctan\left(\frac{\sqrt{2}\sqrt{e\cot(dx+c)}}{(e^2)^{\frac{1}{4}}}+1\right)-2\arctan\left(-\frac{\sqrt{2}\sqrt{e\cot(dx+c}}{(e^2)^{\frac{1}{4}}}\cdots\right)\right)}{8e}\cdots\right)}{\cdots}$
default	$-\frac{2e^4\left(\frac{(e^2)^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\frac{e\cot(dx+c)+(e^2)^{\frac{1}{4}}\sqrt{e\cot(dx+c)}\sqrt{2}+\sqrt{e^2}}{e\cot(dx+c)-(e^2)^{\frac{1}{4}}\sqrt{e\cot(dx+c)}\sqrt{2}+\sqrt{e^2}}\right)+2\arctan\left(\frac{\sqrt{2}\sqrt{e\cot(dx+c)}}{(e^2)^{\frac{1}{4}}}+1\right)-2\arctan\left(-\frac{\sqrt{2}\sqrt{e\cot(dx+c}}{(e^2)^{\frac{1}{4}}}\cdots\right)\right)}{8e}\cdots\right)}{\cdots}$

```
[In] int((e*cot(d*x+c))^(1/2)/(a+a*cot(d*x+c))^3,x,method=_RETURNVERBOSE)
```

```
[Out] -2/d/a^3*e^4*(1/4/e^3*(1/8/e*(e^2)^(1/4)*2^(1/2)*(ln((e*cot(d*x+c)+(e^2)^(1
/4)*(e*cot(d*x+c))^(1/2)*2^(1/2)+(e^2)^(1/2))/(e*cot(d*x+c)-(e^2)^(1/4)*(e*
cot(d*x+c))^(1/2)*2^(1/2)+(e^2)^(1/2)))+2*arctan(2^(1/2)/(e^2)^(1/4)*(e*cot
(d*x+c))^(1/2)+1)-2*arctan(-2^(1/2)/(e^2)^(1/4)*(e*cot(d*x+c))^(1/2)+1))-1/
8/(e^2)^(1/4)*2^(1/2)*(ln((e*cot(d*x+c)-(e^2)^(1/4)*(e*cot(d*x+c))^(1/2)*2^
(1/2)+(e^2)^(1/2))/(e*cot(d*x+c)+(e^2)^(1/4)*(e*cot(d*x+c))^(1/2)*2^(1/2)+(
e^2)^(1/2)))+2*arctan(2^(1/2)/(e^2)^(1/4)*(e*cot(d*x+c))^(1/2)+1)-2*arctan(
-2^(1/2)/(e^2)^(1/4)*(e*cot(d*x+c))^(1/2)+1)))-1/4/e^3*((3/4*(e*cot(d*x+c))
^(3/2)+5/4*e*(e*cot(d*x+c))^(1/2))/(e*cot(d*x+c)+e)^2-1/4/e^(1/2)*arctan((e
*cot(d*x+c))^(1/2)/e^(1/2))))
```

Fricas [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 518, normalized size of antiderivative = 3.22

$$\int \frac{\sqrt{e\cot(c+dx)}}{(a+a\cot(c+dx))^3}\,dx$$

$$= \left[\frac{4\left(\sqrt{2}\sin\left(2\,dx+2\,c\right)+\sqrt{2}\right)\sqrt{-e}\arctan\left(\frac{\left(\sqrt{2}\cos(2\,dx+2\,c)+\sqrt{2}\sin(2\,dx+2\,c)+\sqrt{2}\right)\sqrt{-e}\sqrt{\frac{e\cos(2\,dx+2\,c)+e}{\sin(2\,dx+2\,c)}}}{2\,(e\cos(2\,dx+2\,c)+e)}\right)+\sqrt{-e}(\mathrm{si}}{} \right.$$

$$-\frac{2\sqrt{e}(\sin\left(2\,dx+2\,c\right)+1)\arctan\left(\frac{\sqrt{\frac{e\cos(2\,dx+2\,c)+e}{\sin(2\,dx+2\,c)}}}{\sqrt{e}}\right)-2\left(\sqrt{2}\sin\left(2\,dx+2\,c\right)+\sqrt{2}\right)\sqrt{e}\log\left(\left(\sqrt{2}\cos\left(2\,d\right.\right.\right.}{}$$

```
[In] integrate((e*cot(d*x+c))^(1/2)/(a+a*cot(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] [1/16*(4*(sqrt(2)*sin(2*d*x + 2*c) + sqrt(2))*sqrt(-e)*arctan(1/2*(sqrt(2)*
cos(2*d*x + 2*c) + sqrt(2)*sin(2*d*x + 2*c) + sqrt(2))*sqrt(-e)*sqrt((e*cos
(2*d*x + 2*c) + e)/sin(2*d*x + 2*c))/(e*cos(2*d*x + 2*c) + e)) + sqrt(-e)*(
sin(2*d*x + 2*c) + 1)*log((e*cos(2*d*x + 2*c) - e*sin(2*d*x + 2*c) - 2*sqrt
(-e)*sqrt((e*cos(2*d*x + 2*c) + e)/sin(2*d*x + 2*c))*sin(2*d*x + 2*c) + e)/
(cos(2*d*x + 2*c) + sin(2*d*x + 2*c) + 1)) - sqrt((e*cos(2*d*x + 2*c) + e)/
sin(2*d*x + 2*c))*(5*cos(2*d*x + 2*c) - 3*sin(2*d*x + 2*c) - 5))/(a^3*d*sin
(2*d*x + 2*c) + a^3*d), -1/16*(2*sqrt(e)*(sin(2*d*x + 2*c) + 1)*arctan(sqrt
((e*cos(2*d*x + 2*c) + e)/sin(2*d*x + 2*c))/sqrt(e)) - 2*(sqrt(2)*sin(2*d*x
 + 2*c) + sqrt(2))*sqrt(e)*log((sqrt(2)*cos(2*d*x + 2*c) - sqrt(2)*sin(2*d*
x + 2*c) - sqrt(2))*sqrt(e)*sqrt((e*cos(2*d*x + 2*c) + e)/sin(2*d*x + 2*c))
 + 2*e*sin(2*d*x + 2*c) + e) + sqrt((e*cos(2*d*x + 2*c) + e)/sin(2*d*x + 2*
c))*(5*cos(2*d*x + 2*c) - 3*sin(2*d*x + 2*c) - 5))/(a^3*d*sin(2*d*x + 2*c)
+ a^3*d)]
```

Sympy [F]

$$\int \frac{\sqrt{e\cot(c+dx)}}{(a+a\cot(c+dx))^3}\,dx = \frac{\int \frac{\sqrt{e\cot{(c+dx)}}}{\cot^3{(c+dx)}+3\cot^2{(c+dx)}+3\cot{(c+dx)}+1}\,dx}{a^3}$$

```
[In] integrate((e*cot(d*x+c))**(1/2)/(a+a*cot(d*x+c))**3,x)
```

```
[Out] Integral(sqrt(e*cot(c + d*x))/(cot(c + d*x)**3 + 3*cot(c + d*x)**2 + 3*cot(
c + d*x) + 1), x)/a**3
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{\sqrt{e\cot(c+dx)}}{(a+a\cot(c+dx))^3}\,dx = \text{Exception raised: ValueError}$$

```
[In] integrate((e*cot(d*x+c))^(1/2)/(a+a*cot(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(e>0)', see `assume?` for more detai
ls)Is e
```

Giac [F]

$$\int \frac{\sqrt{e\cot(c+dx)}}{(a+a\cot(c+dx))^3}\,dx = \int \frac{\sqrt{e\cot(dx+c)}}{(a\cot(dx+c)+a)^3}\,dx$$

```
[In] integrate((e*cot(d*x+c))^(1/2)/(a+a*cot(d*x+c))^3,x, algorithm="giac")
```

```
[Out] integrate(sqrt(e*cot(d*x + c))/(a*cot(d*x + c) + a)^3, x)
```

Mupad [B] (verification not implemented)

Time = 13.34 (sec) , antiderivative size = 151, normalized size of antiderivative = 0.94

$$\int \frac{\sqrt{e\cot(c+dx)}}{(a+a\cot(c+dx))^3}\,dx = \frac{\frac{3\,e\,(e\cot(c+d\,x))^{3/2}}{8}+\frac{5\,e^2\,\sqrt{e\cot(c+d\,x)}}{8}}{d\,a^3\,e^2\cot\left(c+d\,x\right)^2+2\,d\,a^3\,e^2\cot\left(c+d\,x\right)+d\,a^3\,e^2} - \frac{\sqrt{e}\,\mathrm{atan}\left(\frac{\sqrt{e\cot(c+d\,x)}}{\sqrt{e}}\right)}{8\,a^3\,d} - \frac{\sqrt{2}\,\sqrt{e}\,\mathrm{atanh}\left(\frac{9\,\sqrt{2}\,e^{17/2}\,\sqrt{e\cot(c+d\,x)}}{32\,\left(\frac{9\,e^9\cot(c+d\,x)}{32}+\frac{9\,e^9}{32}\right)}\right)}{4\,a^3\,d}$$

```
[In] int((e*cot(c + d*x))^(1/2)/(a + a*cot(c + d*x))^3,x)
```

```
[Out] ((3*e*(e*cot(c + d*x))^(3/2))/8 + (5*e^2*(e*cot(c + d*x))^(1/2))/8)/(a^3*d*
e^2 + a^3*d*e^2*cot(c + d*x)^2 + 2*a^3*d*e^2*cot(c + d*x)) - (e^(1/2)*atan(
(e*cot(c + d*x))^(1/2)/e^(1/2)))/(8*a^3*d) - (2^(1/2)*e^(1/2)*atanh((9*2^(1
/2)*e^(17/2)*(e*cot(c + d*x))^(1/2))/(32*((9*e^9*cot(c + d*x))/32 + (9*e^9)
/32))))/(4*a^3*d)
```

3.38 $\int \frac{1}{\sqrt{e\cot(c+dx)}(a+a\cot(c+dx))^3}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 165

$$\int \frac{1}{\sqrt{e\cot(c+dx)}(a+a\cot(c+dx))^3}\,dx = -\frac{11\arctan\left(\frac{\sqrt{e\cot(c+dx)}}{\sqrt{e}}\right)}{8a^3d\sqrt{e}} - \frac{\arctan\left(\frac{\sqrt{e}-\sqrt{e}\cot(c+dx)}{\sqrt{2}\sqrt{e\cot(c+dx)}}\right)}{2\sqrt{2}a^3d\sqrt{e}} - \frac{7\sqrt{e\cot(c+dx)}}{8a^3de(1+\cot(c+dx))} - \frac{\sqrt{e\cot(c+dx)}}{4ade(a+a\cot(c+dx))^2}$$

```
[Out] -11/8*arctan((e*cot(d*x+c))^(1/2)/e^(1/2))/a^3/d/e^(1/2)-1/4*arctan(1/2*(e^
(1/2)-cot(d*x+c)*e^(1/2))*2^(1/2)/(e*cot(d*x+c))^(1/2))/a^3/d*2^(1/2)/e^(1/
2)-7/8*(e*cot(d*x+c))^(1/2)/a^3/d/e/(1+cot(d*x+c))-1/4*(e*cot(d*x+c))^(1/2)
/a/d/e/(a+a*cot(d*x+c))^2
```

Rubi [A] (verified)

Time = 0.73 (sec) , antiderivative size = 165, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.280, Rules used

$= \{3650, 3730, 3734, 3613, 211, 3715, 65\}$

$$\int \frac{1}{\sqrt{e\cot(c+dx)}(a+a\cot(c+dx))^3}\,dx = -\frac{11\arctan\left(\frac{\sqrt{e\cot(c+dx)}}{\sqrt{e}}\right)}{8a^3d\sqrt{e}} - \frac{\arctan\left(\frac{\sqrt{e}-\sqrt{e}\cot(c+dx)}{\sqrt{2}\sqrt{e\cot(c+dx)}}\right)}{2\sqrt{2}a^3d\sqrt{e}} - \frac{7\sqrt{e\cot(c+dx)}}{8a^3de(\cot(c+dx)+1)} - \frac{\sqrt{e\cot(c+dx)}}{4ade(a\cot(c+dx)+a)^2}$$

```
[In] Int[1/(Sqrt[e*Cot[c + d*x]]*(a + a*Cot[c + d*x])^3),x]
```

```
[Out] (-11*ArcTan[Sqrt[e*Cot[c + d*x]]/Sqrt[e]])/(8*a^3*d*Sqrt[e]) - ArcTan[(Sqrt
[e] - Sqrt[e]*Cot[c + d*x])/(Sqrt[2]*Sqrt[e*Cot[c + d*x]])]/(2*Sqrt[2]*a^3*
d*Sqrt[e]) - (7*Sqrt[e*Cot[c + d*x]])/(8*a^3*d*e*(1 + Cot[c + d*x])) - Sqrt
[e*Cot[c + d*x]]/(4*a*d*e*(a + a*Cot[c + d*x])^2)
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 3613

```
Int[((c_) + (d_.)*tan[(e_.) + (f_.)*(x_)])/Sqrt[(b_.)*tan[(e_.) + (f_.)*(x_
)]], x_Symbol] :> Dist[-2*(d^2/f), Subst[Int[1/(2*c*d + b*x^2), x], x, (c -
 d*Tan[e + f*x])/Sqrt[b*Tan[e + f*x]]], x] /; FreeQ[{b, c, d, e, f}, x] &&
EqQ[c^2 - d^2, 0]
```

Rule 3650

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)])^(n_), x_Symbol] :> Simp[b^2*(a + b*Tan[e + f*x])^(m + 1)*((c
+ d*Tan[e + f*x])^(n + 1)/(f*(m + 1)*(a^2 + b^2)*(b*c - a*d))), x] + Dist[1
/((m + 1)*(a^2 + b^2)*(b*c - a*d)), Int[(a + b*Tan[e + f*x])^(m + 1)*(c + d
*Tan[e + f*x])^n*Simp[a*(b*c - a*d)*(m + 1) - b^2*d*(m + n + 2) - b*(b*c -
```

```
a*d)*(m + 1)*Tan[e + f*x] - b^2*d*(m + n + 2)*Tan[e + f*x]^2, x], x], x] /;
 FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0]
&& NeQ[c^2 + d^2, 0] && IntegerQ[2*m] && LtQ[m, -1] && (LtQ[n, 0] || Intege
rQ[m]) &&  !(ILtQ[n, -1] && ( !IntegerQ[m] || (EqQ[c, 0] && NeQ[a, 0])))
```

Rule 3715

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*tan[(e_.)
+ (f_.)*(x_)])^(n_.)*((A_) + (C_.)*tan[(e_.) + (f_.)*(x_)]^2), x_Symbol] :>
 Dist[A/f, Subst[Int[(a + b*x)^m*(c + d*x)^n, x], x, Tan[e + f*x]], x] /; F
reeQ[{a, b, c, d, e, f, A, C, m, n}, x] && EqQ[A, C]
```

Rule 3730

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*tan[(e_.) + (f_.)*(x_)] + (C_.)*tan[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> Simp[(A*b^2 - a*(b*B - a*C))*(a + b*Tan[e +
 f*x])^(m + 1)*((c + d*Tan[e + f*x])^(n + 1)/(f*(m + 1)*(b*c - a*d)*(a^2 +
b^2))), x] + Dist[1/((m + 1)*(b*c - a*d)*(a^2 + b^2)), Int[(a + b*Tan[e + f
*x])^(m + 1)*(c + d*Tan[e + f*x])^n*Simp[A*(a*(b*c - a*d)*(m + 1) - b^2*d*(
m + n + 2)) + (b*B - a*C)*(b*c*(m + 1) + a*d*(n + 1)) - (m + 1)*(b*c - a*d)
*(A*b - a*B - b*C)*Tan[e + f*x] - d*(A*b^2 - a*(b*B - a*C))*(m + n + 2)*Tan
[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, n}, x] && NeQ[
b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] && LtQ[m, -1] &&  !
(ILtQ[n, -1] && ( !IntegerQ[m] || (EqQ[c, 0] && NeQ[a, 0])))
```

Rule 3734

```
Int[(((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_)*((A_.) + (B_.)*tan[(e_.)
+ (f_.)*(x_)] + (C_.)*tan[(e_.) + (f_.)*(x_)]^2))/((a_.) + (b_.)*tan[(e_.)
+ (f_.)*(x_)]), x_Symbol] :> Dist[1/(a^2 + b^2), Int[(c + d*Tan[e + f*x])^n
*Simp[b*B + a*(A - C) + (a*B - b*(A - C))*Tan[e + f*x], x], x], x] + Dist[(
A*b^2 - a*b*B + a^2*C)/(a^2 + b^2), Int[(c + d*Tan[e + f*x])^n*((1 + Tan[e
+ f*x]^2)/(a + b*Tan[e + f*x])), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C
, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] &&
 !GtQ[n, 0] &&  !LeQ[n, -1]
```

Rubi steps

$$\text{integral} = -\frac{\sqrt{e\cot(c+dx)}}{4ade(a+a\cot(c+dx))^2} - \frac{\int \frac{-\frac{7a^2e}{2}+2a^2e\cot(c+dx)-\frac{3}{2}a^2e\cot^2(c+dx)}{\sqrt{e\cot(c+dx)}(a+a\cot(c+dx))^2}\,dx}{4a^3e}$$

$$= -\frac{7\sqrt{e\cot(c+dx)}}{8a^3de(1+\cot(c+dx))} - \frac{\sqrt{e\cot(c+dx)}}{4ade(a+a\cot(c+dx))^2} + \frac{\int \frac{\frac{7a^4e^2}{2}-4a^4e^2\cot(c+dx)+\frac{7}{2}a^4e^2\cot^2(c+dx)}{\sqrt{e\cot(c+dx)}(a+a\cot(c+dx))}\,dx}{8a^6e^2}$$

$$= -\frac{7\sqrt{e\cot(c+dx)}}{8a^3de(1+\cot(c+dx))} - \frac{\sqrt{e\cot(c+dx)}}{4ade(a+a\cot(c+dx))^2} + \frac{11\int \frac{1+\cot^2(c+dx)}{\sqrt{e\cot(c+dx)}(a+a\cot(c+dx))}\,dx}{16a^2} + \frac{\int \frac{-4a^5e^2-4a^5e^2\cot(c+dx)}{\sqrt{e\cot(c+dx)}}\,dx}{16a^8e^2}$$

$$= -\frac{7\sqrt{e\cot(c+dx)}}{8a^3de(1+\cot(c+dx))} - \frac{\sqrt{e\cot(c+dx)}}{4ade(a+a\cot(c+dx))^2} + \frac{11\text{Subst}\left(\int \frac{1}{\sqrt{-ex}(a-ax)}\,dx, x, -\cot(c+dx)\right)}{16a^2d} - \frac{(2a^2e^2)\,\text{Subst}\left(\int \frac{1}{-32a^{10}e^4-ex^2}\,dx, x, \frac{-4a^5e^2+4a^5e^2\cot(c+dx)}{\sqrt{e\cot(c+dx)}}\right)}{d}$$

$$= -\frac{\arctan\left(\frac{\sqrt{e}-\sqrt{e}\cot(c+dx)}{\sqrt{2}\sqrt{e\cot(c+dx)}}\right)}{2\sqrt{2}a^3d\sqrt{e}} - \frac{7\sqrt{e\cot(c+dx)}}{8a^3de(1+\cot(c+dx))} - \frac{\sqrt{e\cot(c+dx)}}{4ade(a+a\cot(c+dx))^2} - \frac{11\text{Subst}\left(\int \frac{1}{a+\frac{ax^2}{e}}\,dx, x, \sqrt{e\cot(c+dx)}\right)}{8a^2de}$$

$$= -\frac{11\arctan\left(\frac{\sqrt{e\cot(c+dx)}}{\sqrt{e}}\right)}{8a^3d\sqrt{e}} - \frac{\arctan\left(\frac{\sqrt{e}-\sqrt{e}\cot(c+dx)}{\sqrt{2}\sqrt{e\cot(c+dx)}}\right)}{2\sqrt{2}a^3d\sqrt{e}} - \frac{7\sqrt{e\cot(c+dx)}}{8a^3de(1+\cot(c+dx))} - \frac{\sqrt{e\cot(c+dx)}}{4ade(a+a\cot(c+dx))^2}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 3.39 (sec) , antiderivative size = 339, normalized size of antiderivative = 2.05

$$\int \frac{1}{\sqrt{e\cot(c+dx)}(a+a\cot(c+dx))^3}\,dx =$$

$$-\frac{16e^{3/2}\arctan\left(\frac{\sqrt{e\cot(c+dx)}}{\sqrt{e}}\right) + 4(-e^2)^{3/4}\arctan\left(\frac{\sqrt{e\cot(c+dx)}}{\sqrt[4]{-e^2}}\right) + 2\sqrt{2}e^{3/2}\arctan\left(1-\frac{\sqrt{2}\sqrt{e\cot(c+dx)}}{\sqrt{e}}\right) - 2}{}$$

```
[In] Integrate[1/(Sqrt[e*Cot[c + d*x]]*(a + a*Cot[c + d*x])^3),x]
```

```
[Out] -1/16*(16*e^(3/2)*ArcTan[Sqrt[e*Cot[c + d*x]]/Sqrt[e]] + 4*(-e^2)^(3/4)*Arc
Tan[Sqrt[e*Cot[c + d*x]]/(-e^2)^(1/4)] + 2*Sqrt[2]*e^(3/2)*ArcTan[1 - (Sqrt
[2]*Sqrt[e*Cot[c + d*x]])/Sqrt[e]] - 2*Sqrt[2]*e^(3/2)*ArcTan[1 + (Sqrt[2]*
Sqrt[e*Cot[c + d*x]])/Sqrt[e]] - 4*(-e^2)^(3/4)*ArcTanh[Sqrt[e*Cot[c + d*x]
]/(-e^2)^(1/4)] + (8*e*Sqrt[e*Cot[c + d*x]])/(1 + Cot[c + d*x]) + 16*e*Sqrt
[e*Cot[c + d*x]]*Hypergeometric2F1[1/2, 3, 3/2, -Cot[c + d*x]] + Sqrt[2]*e^
```

```
(3/2)*Log[Sqrt[e] + Sqrt[e]*Cot[c + d*x] - Sqrt[2]*Sqrt[e*Cot[c + d*x]]] -
Sqrt[2]*e^(3/2)*Log[Sqrt[e] + Sqrt[e]*Cot[c + d*x] + Sqrt[2]*Sqrt[e*Cot[c +
 d*x]]])/(a^3*d*e^2)
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 348 vs. $2(136) = 272$.

Time = 0.06 (sec) , antiderivative size = 349, normalized size of antiderivative = 2.12

method	result
derivativedivides	$-\frac{2e^4\left(-\frac{(e^2)^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\frac{e\cot(dx+c)+(e^2)^{\frac{1}{4}}\sqrt{e\cot(dx+c)}\sqrt{2}+\sqrt{e^2}}{e\cot(dx+c)-(e^2)^{\frac{1}{4}}\sqrt{e\cot(dx+c)}\sqrt{2}+\sqrt{e^2}}\right)+2\arctan\left(\frac{\sqrt{2}\sqrt{e\cot(dx+c)}}{(e^2)^{\frac{1}{4}}}+1\right)-2\arctan\left(-\frac{\sqrt{2}\sqrt{e\cot(\cdots}{(e^2)^{\frac{1}{4}}}\cdots\right.\right.}{8e}\cdots\right.}{\cdots}\cdots$
default	$-\frac{2e^4\left(-\frac{(e^2)^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\frac{e\cot(dx+c)+(e^2)^{\frac{1}{4}}\sqrt{e\cot(dx+c)}\sqrt{2}+\sqrt{e^2}}{e\cot(dx+c)-(e^2)^{\frac{1}{4}}\sqrt{e\cot(dx+c)}\sqrt{2}+\sqrt{e^2}}\right)+2\arctan\left(\frac{\sqrt{2}\sqrt{e\cot(dx+c)}}{(e^2)^{\frac{1}{4}}}+1\right)-2\arctan\left(-\frac{\sqrt{2}\sqrt{e\cot(\cdots}{(e^2)^{\frac{1}{4}}}\cdots\right.\right.}{8e}\cdots\right.}{\cdots}\cdots$

```
[In] int(1/(e*cot(d*x+c))^(1/2)/(a+a*cot(d*x+c))^3,x,method=_RETURNVERBOSE)
```

```
[Out] -2/d/a^3*e^4*(1/4/e^4*(-1/8/e*(e^2)^(1/4)*2^(1/2)*(ln((e*cot(d*x+c)+(e^2)^(
1/4)*(e*cot(d*x+c))^(1/2)*2^(1/2)+(e^2)^(1/2))/(e*cot(d*x+c)-(e^2)^(1/4)*(e
*cot(d*x+c))^(1/2)*2^(1/2)+(e^2)^(1/2)))+2*arctan(2^(1/2)/(e^2)^(1/4)*(e*co
t(d*x+c))^(1/2)+1)-2*arctan(-2^(1/2)/(e^2)^(1/4)*(e*cot(d*x+c))^(1/2)+1))-1
/8/(e^2)^(1/4)*2^(1/2)*(ln((e*cot(d*x+c)-(e^2)^(1/4)*(e*cot(d*x+c))^(1/2)*2
^(1/2)+(e^2)^(1/2))/(e*cot(d*x+c)+(e^2)^(1/4)*(e*cot(d*x+c))^(1/2)*2^(1/2)+
(e^2)^(1/2)))+2*arctan(2^(1/2)/(e^2)^(1/4)*(e*cot(d*x+c))^(1/2)+1)-2*arctan
(-2^(1/2)/(e^2)^(1/4)*(e*cot(d*x+c))^(1/2)+1)))+1/4/e^4*((7/4*(e*cot(d*x+c)
)^(3/2)+9/4*e*(e*cot(d*x+c))^(1/2))/(e*cot(d*x+c)+e)^2+11/4/e^(1/2)*arctan(
(e*cot(d*x+c))^(1/2)/e^(1/2))))
```

Fricas [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 504, normalized size of antiderivative = 3.05

$$\int \frac{1}{\sqrt{e\cot(c+dx)}(a+a\cot(c+dx))^3}\,dx$$

$$= \left[-\frac{2\sqrt{2}\sqrt{-e}(\sin(2\,dx+2\,c)+1)\log\left(-\sqrt{2}\sqrt{-e}\sqrt{\frac{e\cos(2\,dx+2\,c)+e}{\sin(2\,dx+2\,c)}}(\cos(2\,dx+2\,c)+\sin(2\,dx+2\,c)-1)-\right.}{}\right.$$

$$-\frac{4\sqrt{2}\sqrt{e}(\sin(2\,dx+2\,c)+1)\arctan\left(-\frac{\sqrt{2}\sqrt{e}\sqrt{\frac{e\cos(2\,dx+2\,c)+e}{\sin(2\,dx+2\,c)}}(\cos(2\,dx+2\,c)-\sin(2\,dx+2\,c)+1)}{2\,(e\cos(2\,dx+2\,c)+e)}\right)+22\sqrt{e}(\sin(2\,dx+}{16\,(a^3de\sin(2\,dx+}$$

```
[In] integrate(1/(e*cot(d*x+c))^(1/2)/(a+a*cot(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] [-1/16*(2*sqrt(2)*sqrt(-e)*(sin(2*d*x + 2*c) + 1)*log(-sqrt(2)*sqrt(-e)*sqr
t((e*cos(2*d*x + 2*c) + e)/sin(2*d*x + 2*c))*(cos(2*d*x + 2*c) + sin(2*d*x
+ 2*c) - 1) - 2*e*sin(2*d*x + 2*c) + e) + 11*sqrt(-e)*(sin(2*d*x + 2*c) + 1
)*log((e*cos(2*d*x + 2*c) - e*sin(2*d*x + 2*c) + 2*sqrt(-e)*sqrt((e*cos(2*d
*x + 2*c) + e)/sin(2*d*x + 2*c))*sin(2*d*x + 2*c) + e)/(cos(2*d*x + 2*c) +
sin(2*d*x + 2*c) + 1)) - sqrt((e*cos(2*d*x + 2*c) + e)/sin(2*d*x + 2*c))*(9
*cos(2*d*x + 2*c) - 7*sin(2*d*x + 2*c) - 9))/(a^3*d*e*sin(2*d*x + 2*c) + a^
3*d*e), -1/16*(4*sqrt(2)*sqrt(e)*(sin(2*d*x + 2*c) + 1)*arctan(-1/2*sqrt(2)
*sqrt(e)*sqrt((e*cos(2*d*x + 2*c) + e)/sin(2*d*x + 2*c))*(cos(2*d*x + 2*c)
- sin(2*d*x + 2*c) + 1)/(e*cos(2*d*x + 2*c) + e)) + 22*sqrt(e)*(sin(2*d*x +
 2*c) + 1)*arctan(sqrt((e*cos(2*d*x + 2*c) + e)/sin(2*d*x + 2*c))/sqrt(e))
- sqrt((e*cos(2*d*x + 2*c) + e)/sin(2*d*x + 2*c))*(9*cos(2*d*x + 2*c) - 7*s
in(2*d*x + 2*c) - 9))/(a^3*d*e*sin(2*d*x + 2*c) + a^3*d*e)]
```

Sympy [F]

$$\int \frac{1}{\sqrt{e\cot(c+dx)}(a+a\cot(c+dx))^3}\,dx$$

$$= \frac{\int \frac{1}{\sqrt{e\cot(c+dx)}\cot^3(c+dx)+3\sqrt{e\cot(c+dx)}\cot^2(c+dx)+3\sqrt{e\cot(c+dx)}\cot(c+dx)+\sqrt{e\cot(c+dx)}}\,dx}{a^3}$$

```
[In] integrate(1/(e*cot(d*x+c))**(1/2)/(a+a*cot(d*x+c))**3,x)
```

```
[Out] Integral(1/(sqrt(e*cot(c + d*x))*cot(c + d*x)**3 + 3*sqrt(e*cot(c + d*x))*c
ot(c + d*x)**2 + 3*sqrt(e*cot(c + d*x))*cot(c + d*x) + sqrt(e*cot(c + d*x))
), x)/a**3
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{1}{\sqrt{e\cot(c+dx)}(a+a\cot(c+dx))^3}\,dx = \text{Exception raised: ValueError}$$

```
[In] integrate(1/(e*cot(d*x+c))^(1/2)/(a+a*cot(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(e>0)', see `assume?` for more detai
ls)Is e
```

Giac [F]

$$\int \frac{1}{\sqrt{e\cot(c+dx)}(a+a\cot(c+dx))^3}\,dx = \int \frac{1}{(a\cot(dx+c)+a)^3\sqrt{e\cot(dx+c)}}\,dx$$

```
[In] integrate(1/(e*cot(d*x+c))^(1/2)/(a+a*cot(d*x+c))^3,x, algorithm="giac")
```

```
[Out] integrate(1/((a*cot(d*x + c) + a)^3*sqrt(e*cot(d*x + c))), x)
```

Mupad [B] (verification not implemented)

Time = 13.41 (sec) , antiderivative size = 173, normalized size of antiderivative = 1.05

$$\begin{aligned}\int \frac{1}{\sqrt{e\cot(c+dx)}(a+a\cot(c+dx))^3}\,dx &= \frac{\sqrt{2}\left(2\,\mathrm{atan}\left(\frac{\sqrt{2}\sqrt{e\cot(c+d\,x)}}{2\sqrt{e}}\right)+2\,\mathrm{atan}\left(\frac{\sqrt{2}\sqrt{e\cot(c+d\,x)}}{2\sqrt{e}}+\frac{\sqrt{2}\,(e\cot(c+d\,x))^{3/2}}{2\,e^{3/2}}\right)\right)}{8\,a^3\,d\sqrt{e}} \\ &\quad - \frac{11\,\mathrm{atan}\left(\frac{\sqrt{e\cot(c+d\,x)}}{\sqrt{e}}\right)}{8\,a^3\,d\sqrt{e}} - \frac{\frac{9\,e\sqrt{e\cot(c+d\,x)}}{8}+\frac{7\,(e\cot(c+d\,x))^{3/2}}{8}}{d\,a^3\,e^2\cot(c+d\,x)^2+2\,d\,a^3\,e^2\cot(c+d\,x)+d\,a^3\,e^2}\end{aligned}$$

```
[In] int(1/((e*cot(c + d*x))^(1/2)*(a + a*cot(c + d*x))^3),x)
```

```
[Out] (2^(1/2)*(2*atan((2^(1/2)*(e*cot(c + d*x))^(1/2))/(2*e^(1/2))) + 2*atan((2^
(1/2)*(e*cot(c + d*x))^(1/2))/(2*e^(1/2)) + (2^(1/2)*(e*cot(c + d*x))^(3/2)
)/(2*e^(3/2)))))/(8*a^3*d*e^(1/2)) - (11*atan((e*cot(c + d*x))^(1/2)/e^(1/2
)))/(8*a^3*d*e^(1/2)) - ((9*e*(e*cot(c + d*x))^(1/2))/8 + (7*(e*cot(c + d*x
))^(3/2))/8)/(a^3*d*e^2 + a^3*d*e^2*cot(c + d*x)^2 + 2*a^3*d*e^2*cot(c + d*
x))
```

3.39 $\int \frac{1}{(e\cot(c+dx))^{3/2}(a+a\cot(c+dx))^3}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 189

$$\int \frac{1}{(e\cot(c+dx))^{3/2}(a+a\cot(c+dx))^3}\,dx = \frac{31\arctan\left(\frac{\sqrt{e\cot(c+dx)}}{\sqrt{e}}\right)}{8a^3de^{3/2}} + \frac{\operatorname{arctanh}\left(\frac{\sqrt{e}+\sqrt{e}\cot(c+dx)}{\sqrt{2}\sqrt{e\cot(c+dx)}}\right)}{2\sqrt{2}a^3de^{3/2}} + \frac{27}{8a^3de\sqrt{e\cot(c+dx)}} - \frac{9}{8a^3de\sqrt{e\cot(c+dx)}(1+\cot(c+dx))} - \frac{1}{4ade\sqrt{e\cot(c+dx)}(a+a\cot(c+dx))^2}$$

```
[Out] 31/8*arctan((e*cot(d*x+c))^(1/2)/e^(1/2))/a^3/d/e^(3/2)+1/4*arctanh(1/2*(e^
(1/2)+cot(d*x+c)*e^(1/2))*2^(1/2)/(e*cot(d*x+c))^(1/2))/a^3/d/e^(3/2)*2^(1/
2)+27/8/a^3/d/e/(e*cot(d*x+c))^(1/2)-9/8/a^3/d/e/(1+cot(d*x+c))/(e*cot(d*x+
c))^(1/2)-1/4/a/d/e/(a+a*cot(d*x+c))^2/(e*cot(d*x+c))^(1/2)
```

Rubi [A] (verified)

Time = 1.13 (sec) , antiderivative size = 189, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.320, Rules used = {3650, 3730, 3735, 3613, 214, 3715, 65, 211}

$$\int \frac{1}{(e\cot(c+dx))^{3/2}(a+a\cot(c+dx))^3}\,dx = \frac{31\arctan\left(\frac{\sqrt{e\cot(c+dx)}}{\sqrt{e}}\right)}{8a^3de^{3/2}} + \frac{\operatorname{arctanh}\left(\frac{\sqrt{e}\cot(c+dx)+\sqrt{e}}{\sqrt{2}\sqrt{e\cot(c+dx)}}\right)}{2\sqrt{2}a^3de^{3/2}} + \frac{27}{8a^3de\sqrt{e\cot(c+dx)}} - \frac{9}{8a^3de(\cot(c+dx)+1)\sqrt{e\cot(c+dx)}} - \frac{1}{4ade(a\cot(c+dx)+a)^2\sqrt{e\cot(c+dx)}}$$

```
[In] Int[1/((e*Cot[c + d*x])^(3/2)*(a + a*Cot[c + d*x])^3),x]
```

```
[Out] (31*ArcTan[Sqrt[e*Cot[c + d*x]]/Sqrt[e]])/(8*a^3*d*e^(3/2)) + ArcTanh[(Sqrt
[e] + Sqrt[e]*Cot[c + d*x])/(Sqrt[2]*Sqrt[e*Cot[c + d*x]])]/(2*Sqrt[2]*a^3*
d*e^(3/2)) + 27/(8*a^3*d*e*Sqrt[e*Cot[c + d*x]]) - 9/(8*a^3*d*e*Sqrt[e*Cot[
c + d*x]]*(1 + Cot[c + d*x])) - 1/(4*a*d*e*Sqrt[e*Cot[c + d*x]]*(a + a*Cot[
c + d*x])^2)
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 3613

```
Int[((c_) + (d_.)*tan[(e_.) + (f_.)*(x_)])/Sqrt[(b_.)*tan[(e_.) + (f_.)*(x_
)]], x_Symbol] :> Dist[-2*(d^2/f), Subst[Int[1/(2*c*d + b*x^2), x], x, (c -
 d*Tan[e + f*x])/Sqrt[b*Tan[e + f*x]]], x] /; FreeQ[{b, c, d, e, f}, x] &&
EqQ[c^2 - d^2, 0]
```

Rule 3650

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)])^(n_), x_Symbol] :> Simp[b^2*(a + b*Tan[e + f*x])^(m + 1)*((c
+ d*Tan[e + f*x])^(n + 1)/(f*(m + 1)*(a^2 + b^2)*(b*c - a*d))), x] + Dist[1
/((m + 1)*(a^2 + b^2)*(b*c - a*d)), Int[(a + b*Tan[e + f*x])^(m + 1)*(c + d
*Tan[e + f*x])^n*Simp[a*(b*c - a*d)*(m + 1) - b^2*d*(m + n + 2) - b*(b*c -
a*d)*(m + 1)*Tan[e + f*x] - b^2*d*(m + n + 2)*Tan[e + f*x]^2, x], x], x] /;
 FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0]
&& NeQ[c^2 + d^2, 0] && IntegerQ[2*m] && LtQ[m, -1] && (LtQ[n, 0] || Intege
rQ[m]) &&  !(ILtQ[n, -1] && ( !IntegerQ[m] || (EqQ[c, 0] && NeQ[a, 0])))
```

Rule 3715

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*tan[(e_.)
+ (f_.)*(x_)])^(n_.)*((A_) + (C_.)*tan[(e_.) + (f_.)*(x_)]^2), x_Symbol] :>
 Dist[A/f, Subst[Int[(a + b*x)^m*(c + d*x)^n, x], x, Tan[e + f*x]], x] /; F
reeQ[{a, b, c, d, e, f, A, C, m, n}, x] && EqQ[A, C]
```

Rule 3730

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*tan[(e_.) + (f_.)*(x_)] + (C_.)*tan[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> Simp[(A*b^2 - a*(b*B - a*C))*(a + b*Tan[e +
 f*x])^(m + 1)*((c + d*Tan[e + f*x])^(n + 1)/(f*(m + 1)*(b*c - a*d)*(a^2 +
b^2))), x] + Dist[1/((m + 1)*(b*c - a*d)*(a^2 + b^2)), Int[(a + b*Tan[e + f
*x])^(m + 1)*(c + d*Tan[e + f*x])^n*Simp[A*(a*(b*c - a*d)*(m + 1) - b^2*d*(
m + n + 2)) + (b*B - a*C)*(b*c*(m + 1) + a*d*(n + 1)) - (m + 1)*(b*c - a*d)
*(A*b - a*B - b*C)*Tan[e + f*x] - d*(A*b^2 - a*(b*B - a*C))*(m + n + 2)*Tan
[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, n}, x] && NeQ[
b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] && LtQ[m, -1] &&  !
(ILtQ[n, -1] && ( !IntegerQ[m] || (EqQ[c, 0] && NeQ[a, 0])))
```

Rule 3735

```
Int[(((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_)*((A_.) + (C_.)*tan[(e_.)
+ (f_.)*(x_)]^2))/((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dis
t[1/(a^2 + b^2), Int[(c + d*Tan[e + f*x])^n*Simp[a*(A - C) - (A*b - b*C)*Ta
n[e + f*x], x], x], x] + Dist[(A*b^2 + a^2*C)/(a^2 + b^2), Int[(c + d*Tan[e
 + f*x])^n*((1 + Tan[e + f*x]^2)/(a + b*Tan[e + f*x])), x], x] /; FreeQ[{a,
 b, c, d, e, f, A, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && N
eQ[c^2 + d^2, 0] &&  !GtQ[n, 0] &&  !LeQ[n, -1]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\frac{1}{4ade\sqrt{e\cot(c+dx)}(a+a\cot(c+dx))^2} - \frac{\int \frac{-\frac{9a^2e}{2}+2a^2e\cot(c+dx)-\frac{5}{2}a^2e\cot^2(c+dx)}{(e\cot(c+dx))^{3/2}(a+a\cot(c+dx))^2}\,dx}{4a^3e} \\
&= -\frac{9}{8a^3de\sqrt{e\cot(c+dx)}(1+\cot(c+dx))} - \frac{1}{4ade\sqrt{e\cot(c+dx)}(a+a\cot(c+dx))^2} \\
&\quad + \frac{\int \frac{\frac{27a^4e^2}{2}-4a^4e^2\cot(c+dx)+\frac{27}{2}a^4e^2\cot^2(c+dx)}{(e\cot(c+dx))^{3/2}(a+a\cot(c+dx))}\,dx}{8a^6e^2} \\
&= \frac{27}{8a^3de\sqrt{e\cot(c+dx)}} - \frac{9}{8a^3de\sqrt{e\cot(c+dx)}(1+\cot(c+dx))} \\
&\quad - \frac{1}{4ade\sqrt{e\cot(c+dx)}(a+a\cot(c+dx))^2} + \frac{\int \frac{-\frac{35}{4}a^5e^4-\frac{27}{4}a^5e^4\cot^2(c+dx)}{\sqrt{e\cot(c+dx)}(a+a\cot(c+dx))}\,dx}{4a^7e^5}
\end{aligned}$$

$$= \frac{27}{8a^3de\sqrt{e\cot(c+dx)}} - \frac{9}{8a^3de\sqrt{e\cot(c+dx)}(1+\cot(c+dx))} - \frac{1}{4ade\sqrt{e\cot(c+dx)}(a+a\cot(c+dx))^2} + \frac{\int \frac{-2a^6e^4+2a^6e^4\cot(c+dx)}{\sqrt{e\cot(c+dx)}}dx}{8a^9e^5} - \frac{31\int \frac{1+\cot^2(c+dx)}{\sqrt{e\cot(c+dx)}(a+a\cot(c+dx))}dx}{16a^2e}$$

$$= \frac{27}{8a^3de\sqrt{e\cot(c+dx)}} - \frac{9}{8a^3de\sqrt{e\cot(c+dx)}(1+\cot(c+dx))} - \frac{1}{4ade\sqrt{e\cot(c+dx)}(a+a\cot(c+dx))^2} - \frac{31\text{Subst}\left(\int \frac{1}{\sqrt{-ex}(a-ax)}dx, x, -\cot(c+dx)\right)}{16a^2de} - \frac{(a^3e^3)\,\text{Subst}\left(\int \frac{1}{8a^{12}e^8-ex^2}dx, x, \frac{-2a^6e^4-2a^6e^4\cot(c+dx)}{\sqrt{e\cot(c+dx)}}\right)}{d}$$

$$= \frac{\operatorname{arctanh}\left(\frac{\sqrt{e}+\sqrt{e}\cot(c+dx)}{\sqrt{2}\sqrt{e\cot(c+dx)}}\right)}{2\sqrt{2}a^3de^{3/2}} + \frac{27}{8a^3de\sqrt{e\cot(c+dx)}} - \frac{9}{8a^3de\sqrt{e\cot(c+dx)}(1+\cot(c+dx))} - \frac{1}{4ade\sqrt{e\cot(c+dx)}(a+a\cot(c+dx))^2} + \frac{31\text{Subst}\left(\int \frac{1}{a+\frac{ax^2}{e}}dx, x, \sqrt{e\cot(c+dx)}\right)}{8a^2de^2}$$

$$= \frac{31\arctan\left(\frac{\sqrt{e\cot(c+dx)}}{\sqrt{e}}\right)}{8a^3de^{3/2}} + \frac{\operatorname{arctanh}\left(\frac{\sqrt{e}+\sqrt{e}\cot(c+dx)}{\sqrt{2}\sqrt{e\cot(c+dx)}}\right)}{2\sqrt{2}a^3de^{3/2}} + \frac{27}{8a^3de\sqrt{e\cot(c+dx)}} - \frac{9}{8a^3de\sqrt{e\cot(c+dx)}(1+\cot(c+dx))} - \frac{1}{4ade\sqrt{e\cot(c+dx)}(a+a\cot(c+dx))^2}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 1.28 (sec) , antiderivative size = 325, normalized size of antiderivative = 1.72

$$\int \frac{1}{(e\cot(c+dx))^{3/2}(a+a\cot(c+dx))^3}dx = \frac{-2\sqrt{2}\arctan\left(1-\frac{\sqrt{2}\sqrt{e\cot(c+dx)}}{\sqrt{e}}\right)\sqrt{e\cot(c+dx)}+2\sqrt{2}\,\text{arct}}{}$$

```
[In] Integrate[1/((e*Cot[c + d*x])^(3/2)*(a + a*Cot[c + d*x])^3),x]
[Out] (-2*Sqrt[2]*ArcTan[1 - (Sqrt[2]*Sqrt[e*Cot[c + d*x]])/Sqrt[e]]*Sqrt[e*Cot[c
 + d*x]] + 2*Sqrt[2]*ArcTan[1 + (Sqrt[2]*Sqrt[e*Cot[c + d*x]])/Sqrt[e]]*Sqr
t[e*Cot[c + d*x]] + 8*Sqrt[e]*Hypergeometric2F1[-1/2, 1, 1/2, -Cot[c + d*x]
] + 16*Sqrt[e]*Hypergeometric2F1[-1/2, 2, 1/2, -Cot[c + d*x]] + 16*Sqrt[e]*
```

```
Hypergeometric2F1[-1/2, 3, 1/2, -Cot[c + d*x]] - 8*Sqrt[e]*Hypergeometric2F
1[-1/4, 1, 3/4, -Cot[c + d*x]^2] - Sqrt[2]*Sqrt[e*Cot[c + d*x]]*Log[Sqrt[e]
 + Sqrt[e]*Cot[c + d*x] - Sqrt[2]*Sqrt[e*Cot[c + d*x]]] + Sqrt[2]*Sqrt[e*Co
t[c + d*x]]*Log[Sqrt[e] + Sqrt[e]*Cot[c + d*x] + Sqrt[2]*Sqrt[e*Cot[c + d*x
]]])/(16*a^3*d*e^(3/2)*Sqrt[e*Cot[c + d*x]])
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 363 vs. $2(156) = 312$.

Time = 0.05 (sec) , antiderivative size = 364, normalized size of antiderivative = 1.93

method	result
derivativedivides	$-\frac{2e^4\left(-\frac{\left(e^2\right)^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\frac{e\cot(dx+c)+\left(e^2\right)^{\frac{1}{4}}\sqrt{e\cot(dx+c)}\sqrt{2}+\sqrt{e^2}}{e\cot(dx+c)-\left(e^2\right)^{\frac{1}{4}}\sqrt{e\cot(dx+c)}\sqrt{2}+\sqrt{e^2}}\right)+2\arctan\left(\frac{\sqrt{2}\sqrt{e\cot(dx+c)}}{\left(e^2\right)^{\frac{1}{4}}}+1\right)-2\arctan\left(-\frac{\sqrt{2}\sqrt{e\cot(dx\cdots}}{\left(e^2\right)^{\frac{1}{4}}}\cdots\right.\right.}{8e}\cdots\right.}{\cdots}$
default	$-\frac{2e^4\left(-\frac{\left(e^2\right)^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\frac{e\cot(dx+c)+\left(e^2\right)^{\frac{1}{4}}\sqrt{e\cot(dx+c)}\sqrt{2}+\sqrt{e^2}}{e\cot(dx+c)-\left(e^2\right)^{\frac{1}{4}}\sqrt{e\cot(dx+c)}\sqrt{2}+\sqrt{e^2}}\right)+2\arctan\left(\frac{\sqrt{2}\sqrt{e\cot(dx+c)}}{\left(e^2\right)^{\frac{1}{4}}}+1\right)-2\arctan\left(-\frac{\sqrt{2}\sqrt{e\cot(dx\cdots}}{\left(e^2\right)^{\frac{1}{4}}}\cdots\right.\right.}{8e}\cdots\right.}{\cdots}$

```
[In] int(1/(e*cot(d*x+c))^(3/2)/(a+a*cot(d*x+c))^3,x,method=_RETURNVERBOSE)
```

```
[Out] -2/d/a^3*e^4*(1/4/e^5*(-1/8/e*(e^2)^(1/4)*2^(1/2)*(ln((e*cot(d*x+c)+(e^2)^(
1/4)*(e*cot(d*x+c))^(1/2)*2^(1/2)+(e^2)^(1/2))/(e*cot(d*x+c)-(e^2)^(1/4)*(e
*cot(d*x+c))^(1/2)*2^(1/2)+(e^2)^(1/2)))+2*arctan(2^(1/2)/(e^2)^(1/4)*(e*co
t(d*x+c))^(1/2)+1)-2*arctan(-2^(1/2)/(e^2)^(1/4)*(e*cot(d*x+c))^(1/2)+1))+1
/8/(e^2)^(1/4)*2^(1/2)*(ln((e*cot(d*x+c)-(e^2)^(1/4)*(e*cot(d*x+c))^(1/2)*2
^(1/2)+(e^2)^(1/2))/(e*cot(d*x+c)+(e^2)^(1/4)*(e*cot(d*x+c))^(1/2)*2^(1/2)+
(e^2)^(1/2)))+2*arctan(2^(1/2)/(e^2)^(1/4)*(e*cot(d*x+c))^(1/2)+1)-2*arctan
(-2^(1/2)/(e^2)^(1/4)*(e*cot(d*x+c))^(1/2)+1)))-1/e^5/(e*cot(d*x+c))^(1/2)-
1/4/e^5*((11/4*(e*cot(d*x+c))^(3/2)+13/4*e*(e*cot(d*x+c))^(1/2))/(e*cot(d*x
+c)+e)^2+31/4/e^(1/2)*arctan((e*cot(d*x+c))^(1/2)/e^(1/2))))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 316 vs. $2(156) = 312$.

Time = 0.29 (sec) , antiderivative size = 697, normalized size of antiderivative = 3.69

$$\int \frac{1}{(e\cot(c+dx))^{3/2}(a+a\cot(c+dx))^3}\,dx = \left[-\frac{4\sqrt{2}((\cos(2\,dx+2\,c)+1)\sin(2\,dx+2\,c)+\cos(2\,dx+2\right.$$

```
[In] integrate(1/(e*cot(d*x+c))^(3/2)/(a+a*cot(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] [-1/16*(4*sqrt(2)*((cos(2*d*x + 2*c) + 1)*sin(2*d*x + 2*c) + cos(2*d*x + 2*
c) + 1)*sqrt(-e)*arctan(1/2*sqrt(2)*sqrt(-e)*sqrt((e*cos(2*d*x + 2*c) + e)/
sin(2*d*x + 2*c))*(cos(2*d*x + 2*c) + sin(2*d*x + 2*c) + 1)/(e*cos(2*d*x +
2*c) + e)) + 31*((cos(2*d*x + 2*c) + 1)*sin(2*d*x + 2*c) + cos(2*d*x + 2*c)
 + 1)*sqrt(-e)*log((e*cos(2*d*x + 2*c) - e*sin(2*d*x + 2*c) - 2*sqrt(-e)*sq
rt((e*cos(2*d*x + 2*c) + e)/sin(2*d*x + 2*c))*sin(2*d*x + 2*c) + e)/(cos(2*
d*x + 2*c) + sin(2*d*x + 2*c) + 1)) + (45*cos(2*d*x + 2*c)^2 - (11*cos(2*d*
x + 2*c) + 43)*sin(2*d*x + 2*c) - 45)*sqrt((e*cos(2*d*x + 2*c) + e)/sin(2*d
*x + 2*c)))/(a^3*d*e^2*cos(2*d*x + 2*c) + a^3*d*e^2 + (a^3*d*e^2*cos(2*d*x
+ 2*c) + a^3*d*e^2)*sin(2*d*x + 2*c)), 1/16*(2*sqrt(2)*((cos(2*d*x + 2*c) +
 1)*sin(2*d*x + 2*c) + cos(2*d*x + 2*c) + 1)*sqrt(e)*log(-sqrt(2)*sqrt(e)*s
qrt((e*cos(2*d*x + 2*c) + e)/sin(2*d*x + 2*c))*(cos(2*d*x + 2*c) - sin(2*d*
x + 2*c) - 1) + 2*e*sin(2*d*x + 2*c) + e) + 62*((cos(2*d*x + 2*c) + 1)*sin(
2*d*x + 2*c) + cos(2*d*x + 2*c) + 1)*sqrt(e)*arctan(sqrt((e*cos(2*d*x + 2*c
) + e)/sin(2*d*x + 2*c))/sqrt(e)) - (45*cos(2*d*x + 2*c)^2 - (11*cos(2*d*x
+ 2*c) + 43)*sin(2*d*x + 2*c) - 45)*sqrt((e*cos(2*d*x + 2*c) + e)/sin(2*d*x
 + 2*c)))/(a^3*d*e^2*cos(2*d*x + 2*c) + a^3*d*e^2 + (a^3*d*e^2*cos(2*d*x +
2*c) + a^3*d*e^2)*sin(2*d*x + 2*c))]
```

Sympy [F]

$$\int \frac{1}{(e\cot(c+dx))^{3/2}(a+a\cot(c+dx))^3}\,dx = \frac{\int \frac{1}{(e\cot(c+dx))^{\frac{3}{2}}\cot^3(c+dx)+3(e\cot(c+dx))^{\frac{3}{2}}\cot^2(c+dx)+3(e\cot(c+dx))^{\frac{3}{2}}\text{co}}}{a^3}$$

```
[In] integrate(1/(e*cot(d*x+c))**(3/2)/(a+a*cot(d*x+c))**3,x)
```

```
[Out] Integral(1/((e*cot(c + d*x))**(3/2)*cot(c + d*x)**3 + 3*(e*cot(c + d*x))**(
3/2)*cot(c + d*x)**2 + 3*(e*cot(c + d*x))**(3/2)*cot(c + d*x) + (e*cot(c +
d*x))**(3/2)), x)/a**3
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{1}{(e\cot(c+dx))^{3/2}(a+a\cot(c+dx))^3}\,dx = \text{Exception raised: ValueError}$$

[In] integrate(1/(e*cot(d*x+c))^(3/2)/(a+a*cot(d*x+c))^3,x, algorithm="maxima")

[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(e>0)', see `assume?` for more detai
ls)Is e

Giac [F]

$$\int \frac{1}{(e\cot(c+dx))^{3/2}(a+a\cot(c+dx))^3}\,dx = \int \frac{1}{(a\cot(dx+c)+a)^3\,(e\cot(dx+c))^{\frac{3}{2}}}\,dx$$

[In] integrate(1/(e*cot(d*x+c))^(3/2)/(a+a*cot(d*x+c))^3,x, algorithm="giac")

[Out] integrate(1/((a*cot(d*x + c) + a)^3*(e*cot(d*x + c))^(3/2)), x)

Mupad [B] (verification not implemented)

Time = 13.34 (sec) , antiderivative size = 175, normalized size of antiderivative = 0.93

$$\int \frac{1}{(e\cot(c+dx))^{3/2}(a+a\cot(c+dx))^3}\,dx = \frac{\frac{27\,e\cot(c+d\,x)^2}{8}+\frac{45\,e\cot(c+d\,x)}{8}+2\,e}{a^3\,d\,(e\cot(c+d\,x))^{5/2}+2\,a^3\,d\,e\,(e\cot(c+d\,x))^{3/2}+a^3\,d\,e^2\,\sqrt{e\cot(c+d\,x)}} + \frac{31\,\mathrm{atan}\left(\frac{\sqrt{e\cot(c+d\,x)}}{\sqrt{e}}\right)}{8\,a^3\,d\,e^{3/2}} + \frac{\sqrt{2}\,\mathrm{atanh}\left(\frac{63504384\,\sqrt{2}\,a^9\,d^3\,e^{15/2}\,\sqrt{e\cot(c+d\,x)}}{63504384\,a^9\,d^3\,e^8+63504384\,a^9\,d^3\,e^8\cot(c+d\,x)}\right)}{4\,a^3\,d\,e^{3/2}}$$

[In] int(1/((e*cot(c + d*x))^(3/2)*(a + a*cot(c + d*x))^3),x)

[Out] (2*e + (45*e*cot(c + d*x))/8 + (27*e*cot(c + d*x)^2)/8)/(a^3*d*(e*cot(c + d
*x))^(5/2) + 2*a^3*d*e*(e*cot(c + d*x))^(3/2) + a^3*d*e^2*(e*cot(c + d*x))^
(1/2)) + (31*atan((e*cot(c + d*x))^(1/2)/e^(1/2)))/(8*a^3*d*e^(3/2)) + (2^(
1/2)*atanh((63504384*2^(1/2)*a^9*d^3*e^(15/2)*(e*cot(c + d*x))^(1/2))/(6350
4384*a^9*d^3*e^8 + 63504384*a^9*d^3*e^8*cot(c + d*x))))/(4*a^3*d*e^(3/2))

3.40 $\int \frac{1}{(e\cot(c+dx))^{5/2}(a+a\cot(c+dx))^3}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 215

$$\int \frac{1}{(e\cot(c+dx))^{5/2}(a+a\cot(c+dx))^3}\,dx = -\frac{59\arctan\left(\frac{\sqrt{e\cot(c+dx)}}{\sqrt{e}}\right)}{8a^3de^{5/2}} + \frac{\arctan\left(\frac{\sqrt{e}-\sqrt{e}\cot(c+dx)}{\sqrt{2}\sqrt{e\cot(c+dx)}}\right)}{2\sqrt{2}a^3de^{5/2}} + \frac{55}{24a^3de(e\cot(c+dx))^{3/2}} - \frac{63}{8a^3de^2\sqrt{e\cot(c+dx)}} - \frac{11}{8a^3de(e\cot(c+dx))^{3/2}(1+\cot(c+dx))} - \frac{1}{4ade(e\cot(c+dx))^{3/2}(a+a\cot(c+dx))^2}$$

```
[Out] -59/8*arctan((e*cot(d*x+c))^(1/2)/e^(1/2))/a^3/d/e^(5/2)+55/24/a^3/d/e/(e*c
ot(d*x+c))^(3/2)-11/8/a^3/d/e/(e*cot(d*x+c))^(3/2)/(1+cot(d*x+c))-1/4/a/d/e
/(e*cot(d*x+c))^(3/2)/(a+a*cot(d*x+c))^2+1/4*arctan(1/2*(e^(1/2)-cot(d*x+c)
*e^(1/2))*2^(1/2)/(e*cot(d*x+c))^(1/2))/a^3/d/e^(5/2)*2^(1/2)-63/8/a^3/d/e^
2/(e*cot(d*x+c))^(1/2)
```

Rubi [A] (verified)

Time = 1.81 (sec) , antiderivative size = 215, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.320, Rules used

$= \{3650, 3730, 3731, 3734, 3613, 211, 3715, 65\}$

$$\int \frac{1}{(e\cot(c+dx))^{5/2}(a+a\cot(c+dx))^3}\,dx = -\frac{59\arctan\left(\frac{\sqrt{e\cot(c+dx)}}{\sqrt{e}}\right)}{8a^3de^{5/2}} + \frac{\arctan\left(\frac{\sqrt{e}-\sqrt{e}\cot(c+dx)}{\sqrt{2}\sqrt{e\cot(c+dx)}}\right)}{2\sqrt{2}a^3de^{5/2}} - \frac{63}{8a^3de^2\sqrt{e\cot(c+dx)}} - \frac{11}{8a^3de(\cot(c+dx)+1)(e\cot(c+dx))^{3/2}} + \frac{55}{24a^3de(e\cot(c+dx))^{3/2}} - \frac{1}{4ade(a\cot(c+dx)+a)^2(e\cot(c+dx))^{3/2}}$$

```
[In] Int[1/((e*Cot[c + d*x])^(5/2)*(a + a*Cot[c + d*x])^3),x]
```

```
[Out] (-59*ArcTan[Sqrt[e*Cot[c + d*x]]/Sqrt[e]])/(8*a^3*d*e^(5/2)) + ArcTan[(Sqrt
[e] - Sqrt[e]*Cot[c + d*x])/(Sqrt[2]*Sqrt[e*Cot[c + d*x]])]/(2*Sqrt[2]*a^3*
d*e^(5/2)) + 55/(24*a^3*d*e*(e*Cot[c + d*x])^(3/2)) - 63/(8*a^3*d*e^2*Sqrt[
e*Cot[c + d*x]]) - 11/(8*a^3*d*e*(e*Cot[c + d*x])^(3/2)*(1 + Cot[c + d*x]))
 - 1/(4*a*d*e*(e*Cot[c + d*x])^(3/2)*(a + a*Cot[c + d*x])^2)
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 3613

```
Int[((c_) + (d_.)*tan[(e_.) + (f_.)*(x_)])/Sqrt[(b_.)*tan[(e_.) + (f_.)*(x_
)]], x_Symbol] :> Dist[-2*(d^2/f), Subst[Int[1/(2*c*d + b*x^2), x], x, (c -
 d*Tan[e + f*x])/Sqrt[b*Tan[e + f*x]]], x] /; FreeQ[{b, c, d, e, f}, x] &&
EqQ[c^2 - d^2, 0]
```

Rule 3650

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)])^(n_), x_Symbol] :> Simp[b^2*(a + b*Tan[e + f*x])^(m + 1)*((c
+ d*Tan[e + f*x])^(n + 1)/(f*(m + 1)*(a^2 + b^2)*(b*c - a*d))), x] + Dist[1
/((m + 1)*(a^2 + b^2)*(b*c - a*d)), Int[(a + b*Tan[e + f*x])^(m + 1)*(c + d
*Tan[e + f*x])^n*Simp[a*(b*c - a*d)*(m + 1) - b^2*d*(m + n + 2) - b*(b*c -
```

```
a*d)*(m + 1)*Tan[e + f*x] - b^2*d*(m + n + 2)*Tan[e + f*x]^2, x], x], x] /;
 FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0]
&& NeQ[c^2 + d^2, 0] && IntegerQ[2*m] && LtQ[m, -1] && (LtQ[n, 0] || Intege
rQ[m]) &&  !(ILtQ[n, -1] && ( !IntegerQ[m] || (EqQ[c, 0] && NeQ[a, 0])))
```

Rule 3715

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*tan[(e_.)
+ (f_.)*(x_)])^(n_.)*((A_) + (C_.)*tan[(e_.) + (f_.)*(x_)]^2), x_Symbol] :>
 Dist[A/f, Subst[Int[(a + b*x)^m*(c + d*x)^n, x], x, Tan[e + f*x]], x] /; F
reeQ[{a, b, c, d, e, f, A, C, m, n}, x] && EqQ[A, C]
```

Rule 3730

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*tan[(e_.) + (f_.)*(x_)] + (C_.)*tan[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> Simp[(A*b^2 - a*(b*B - a*C))*(a + b*Tan[e +
 f*x])^(m + 1)*((c + d*Tan[e + f*x])^(n + 1)/(f*(m + 1)*(b*c - a*d)*(a^2 +
b^2))), x] + Dist[1/((m + 1)*(b*c - a*d)*(a^2 + b^2)), Int[(a + b*Tan[e + f
*x])^(m + 1)*(c + d*Tan[e + f*x])^n*Simp[A*(a*(b*c - a*d)*(m + 1) - b^2*d*(
m + n + 2)) + (b*B - a*C)*(b*c*(m + 1) + a*d*(n + 1)) - (m + 1)*(b*c - a*d)
*(A*b - a*B - b*C)*Tan[e + f*x] - d*(A*b^2 - a*(b*B - a*C))*(m + n + 2)*Tan
[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, n}, x] && NeQ[
b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] && LtQ[m, -1] &&  !
(ILtQ[n, -1] && ( !IntegerQ[m] || (EqQ[c, 0] && NeQ[a, 0])))
```

Rule 3731

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (C_.)*tan[(e_.) + (f_.)*(x_)]^2), x_Symbol] :>
Simp[(A*b^2 + a^2*C)*(a + b*Tan[e + f*x])^(m + 1)*((c + d*Tan[e + f*x])^(n
+ 1)/(f*(m + 1)*(b*c - a*d)*(a^2 + b^2))), x] + Dist[1/((m + 1)*(b*c - a*d)
*(a^2 + b^2)), Int[(a + b*Tan[e + f*x])^(m + 1)*(c + d*Tan[e + f*x])^n*Simp
[A*(a*(b*c - a*d)*(m + 1) - b^2*d*(m + n + 2)) - a*C*(b*c*(m + 1) + a*d*(n
+ 1)) - (m + 1)*(b*c - a*d)*(A*b - b*C)*Tan[e + f*x] - d*(A*b^2 + a^2*C)*(m
 + n + 2)*Tan[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, C, n},
x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] && LtQ[m,
 -1] &&  !(ILtQ[n, -1] && ( !IntegerQ[m] || (EqQ[c, 0] && NeQ[a, 0])))
```

Rule 3734

```
Int[(((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_)*((A_.) + (B_.)*tan[(e_.)
+ (f_.)*(x_)] + (C_.)*tan[(e_.) + (f_.)*(x_)]^2))/((a_.) + (b_.)*tan[(e_.)
+ (f_.)*(x_)]), x_Symbol] :> Dist[1/(a^2 + b^2), Int[(c + d*Tan[e + f*x])^n
*Simp[b*B + a*(A - C) + (a*B - b*(A - C))*Tan[e + f*x], x], x], x] + Dist[(
A*b^2 - a*b*B + a^2*C)/(a^2 + b^2), Int[(c + d*Tan[e + f*x])^n*((1 + Tan[e
```

```
+ f*x]^2)/(a + b*Tan[e + f*x])), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C
, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] &&
 !GtQ[n, 0] &&  !LeQ[n, -1]
```

Rubi steps

integral

$$= -\frac{1}{4ade(e\cot(c+dx))^{3/2}(a+a\cot(c+dx))^2} - \frac{\int \frac{-\frac{11a^2e}{2}+2a^2e\cot(c+dx)-\frac{7}{2}a^2e\cot^2(c+dx)}{(e\cot(c+dx))^{5/2}(a+a\cot(c+dx))^2}\,dx}{4a^3e}$$

$$= -\frac{11}{8a^3de(e\cot(c+dx))^{3/2}(1+\cot(c+dx))} - \frac{1}{4ade(e\cot(c+dx))^{3/2}(a+a\cot(c+dx))^2} + \frac{\int \frac{\frac{55a^4e^2}{2}-4a^4e^2\cot(c+dx)+\frac{55}{2}a^4e^2\cot^2(c+dx)}{(e\cot(c+dx))^{5/2}(a+a\cot(c+dx))}\,dx}{8a^6e^2}$$

$$= \frac{55}{24a^3de(e\cot(c+dx))^{3/2}} - \frac{11}{8a^3de(e\cot(c+dx))^{3/2}(1+\cot(c+dx))} - \frac{1}{4ade(e\cot(c+dx))^{3/2}(a+a\cot(c+dx))^2} + \frac{\int \frac{-\frac{189}{4}a^5e^4-\frac{165}{4}a^5e^4\cot^2(c+dx)}{(e\cot(c+dx))^{3/2}(a+a\cot(c+dx))}\,dx}{12a^7e^5}$$

$$= \frac{55}{24a^3de(e\cot(c+dx))^{3/2}} - \frac{63}{8a^3de^2\sqrt{e\cot(c+dx)}} - \frac{11}{8a^3de(e\cot(c+dx))^{3/2}(1+\cot(c+dx))} - \frac{1}{4ade(e\cot(c+dx))^{3/2}(a+a\cot(c+dx))^2} + \frac{\int \frac{\frac{189a^6e^6}{8}+3a^6e^6\cot(c+dx)+\frac{189}{8}a^6e^6\cot^2(c+dx)}{\sqrt{e\cot(c+dx)}(a+a\cot(c+dx))}\,dx}{6a^8e^8}$$

$$= \frac{55}{24a^3de(e\cot(c+dx))^{3/2}} - \frac{63}{8a^3de^2\sqrt{e\cot(c+dx)}} - \frac{11}{8a^3de(e\cot(c+dx))^{3/2}(1+\cot(c+dx))} - \frac{1}{4ade(e\cot(c+dx))^{3/2}(a+a\cot(c+dx))^2} + \frac{\int \frac{3a^7e^6+3a^7e^6\cot(c+dx)}{\sqrt{e\cot(c+dx)}}\,dx}{12a^{10}e^8} + \frac{59\int \frac{1+\cot^2(c+dx)}{\sqrt{e\cot(c+dx)}(a+a\cot(c+dx))}\,dx}{16a^2e^2}$$

$$= \frac{55}{24a^3de(e\cot(c+dx))^{3/2}} - \frac{63}{8a^3de^2\sqrt{e\cot(c+dx)}} - \frac{11}{8a^3de(e\cot(c+dx))^{3/2}(1+\cot(c+dx))} - \frac{1}{4ade(e\cot(c+dx))^{3/2}(a+a\cot(c+dx))^2} + \frac{59\text{Subst}\left(\int \frac{1}{\sqrt{-ex}(a-ax)}\,dx, x, -\cot(c+dx)\right)}{16a^2de^2} - \frac{(3a^4e^4)\,\text{Subst}\left(\int \frac{1}{-18a^{14}e^{12}-ex^2}\,dx, x, \frac{3a^7e^6-3a^7e^6\cot(c+dx)}{\sqrt{e\cot(c+dx)}}\right)}{2d}$$

$$= \frac{\arctan\left(\frac{\sqrt{e}-\sqrt{e}\cot(c+dx)}{\sqrt{2}\sqrt{e\cot(c+dx)}}\right)}{2\sqrt{2}a^3de^{5/2}} + \frac{55}{24a^3de(e\cot(c+dx))^{3/2}} - \frac{63}{8a^3de^2\sqrt{e\cot(c+dx)}} - \frac{11}{8a^3de(e\cot(c+dx))^{3/2}(1+\cot(c+dx))} - \frac{1}{4ade(e\cot(c+dx))^{3/2}(a+a\cot(c+dx))^2} - \frac{59\text{Subst}\left(\int \frac{1}{a+\frac{ax^2}{e}}\,dx, x, \sqrt{e\cot(c+dx)}\right)}{8a^2de^3}$$

$$= -\frac{59\arctan\left(\frac{\sqrt{e\cot(c+dx)}}{\sqrt{e}}\right)}{8a^3de^{5/2}} + \frac{\arctan\left(\frac{\sqrt{e}-\sqrt{e}\cot(c+dx)}{\sqrt{2}\sqrt{e\cot(c+dx)}}\right)}{2\sqrt{2}a^3de^{5/2}} + \frac{55}{24a^3de(e\cot(c+dx))^{3/2}} - \frac{63}{8a^3de^2\sqrt{e\cot(c+dx)}} - \frac{11}{8a^3de(e\cot(c+dx))^{3/2}(1+\cot(c+dx))} - \frac{1}{4ade(e\cot(c+dx))^{3/2}(a+a\cot(c+dx))^2}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 1.11 (sec) , antiderivative size = 124, normalized size of antiderivative = 0.58

$$\int \frac{1}{(e\cot(c+dx))^{5/2}(a+a\cot(c+dx))^3}\,dx = \frac{\text{Hypergeometric2F1}\left(-\frac{3}{2},1,-\frac{1}{2},-\cot(c+dx)\right)+2\,\text{Hyperge}}{}$$

```
[In] Integrate[1/((e*Cot[c + d*x])^(5/2)*(a + a*Cot[c + d*x])^3),x]
```

```
[Out] (Hypergeometric2F1[-3/2, 1, -1/2, -Cot[c + d*x]] + 2*Hypergeometric2F1[-3/2
, 2, -1/2, -Cot[c + d*x]] + 2*Hypergeometric2F1[-3/2, 3, -1/2, -Cot[c + d*x
]] - Hypergeometric2F1[-3/4, 1, 1/4, -Cot[c + d*x]^2] - 3*Cot[c + d*x]*Hype
rgeometric2F1[-1/4, 1, 3/4, -Cot[c + d*x]^2])/(6*a^3*d*e*(e*Cot[c + d*x])^(
3/2))
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 378 vs. 2(178) = 356.

Time = 0.05 (sec) , antiderivative size = 379, normalized size of antiderivative = 1.76

method	result
derivativedivides	$-\frac{2e^4\left(\frac{\left(e^2\right)^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\frac{e\cot(dx+c)+\left(e^2\right)^{\frac{1}{4}}\sqrt{e\cot(dx+c)}\,\sqrt{2}+\sqrt{e^2}}{e\cot(dx+c)-\left(e^2\right)^{\frac{1}{4}}\sqrt{e\cot(dx+c)}\,\sqrt{2}+\sqrt{e^2}}\right)+2\arctan\left(\frac{\sqrt{2}\,\sqrt{e\cot(dx+c)}}{\left(e^2\right)^{\frac{1}{4}}}+1\right)-2\arctan\left(-\frac{\sqrt{2}\,\sqrt{e\cot(dx+c)}}{\left(e^2\right)^{\frac{1}{4}}}\cdots\right.\right.}{8e}\cdots\right.}{\cdots}$
default	$-\frac{2e^4\left(\frac{\left(e^2\right)^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\frac{e\cot(dx+c)+\left(e^2\right)^{\frac{1}{4}}\sqrt{e\cot(dx+c)}\,\sqrt{2}+\sqrt{e^2}}{e\cot(dx+c)-\left(e^2\right)^{\frac{1}{4}}\sqrt{e\cot(dx+c)}\,\sqrt{2}+\sqrt{e^2}}\right)+2\arctan\left(\frac{\sqrt{2}\,\sqrt{e\cot(dx+c)}}{\left(e^2\right)^{\frac{1}{4}}}+1\right)-2\arctan\left(-\frac{\sqrt{2}\,\sqrt{e\cot(dx+c)}}{\left(e^2\right)^{\frac{1}{4}}}\cdots\right.\right.}{8e}\cdots\right.}{\cdots}$

```
[In] int(1/(e*cot(d*x+c))^(5/2)/(a+a*cot(d*x+c))^3,x,method=_RETURNVERBOSE)
```

```
[Out] -2/d/a^3*e^4*(1/4/e^6*(1/8/e*(e^2)^(1/4)*2^(1/2)*(ln((e*cot(d*x+c)+(e^2)^(1
/4)*(e*cot(d*x+c))^(1/2)*2^(1/2)+(e^2)^(1/2))/(e*cot(d*x+c)-(e^2)^(1/4)*(e*
cot(d*x+c))^(1/2)*2^(1/2)+(e^2)^(1/2)))+2*arctan(2^(1/2)/(e^2)^(1/4)*(e*cot
(d*x+c))^(1/2)+1)-2*arctan(-2^(1/2)/(e^2)^(1/4)*(e*cot(d*x+c))^(1/2)+1))+1/
8/(e^2)^(1/4)*2^(1/2)*(ln((e*cot(d*x+c)-(e^2)^(1/4)*(e*cot(d*x+c))^(1/2)*2^
(1/2)+(e^2)^(1/2))/(e*cot(d*x+c)+(e^2)^(1/4)*(e*cot(d*x+c))^(1/2)*2^(1/2)+(
e^2)^(1/2)))+2*arctan(2^(1/2)/(e^2)^(1/4)*(e*cot(d*x+c))^(1/2)+1)-2*arctan(
-2^(1/2)/(e^2)^(1/4)*(e*cot(d*x+c))^(1/2)+1)))-1/3/e^5/(e*cot(d*x+c))^(3/2)
+3/e^6/(e*cot(d*x+c))^(1/2)+1/4/e^6*((15/4*(e*cot(d*x+c))^(3/2)+17/4*e*(e*c
ot(d*x+c))^(1/2))/(e*cot(d*x+c)+e)^2+59/4/e^(1/2)*arctan((e*cot(d*x+c))^(1/
2)/e^(1/2))))
```

Fricas [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 718, normalized size of antiderivative = 3.34

$$\int \frac{1}{(e\cot(c+dx))^{5/2}(a+a\cot(c+dx))^3}\,dx = \left[-\frac{6\sqrt{2}((\cos(2\,dx+2\,c)+1)\sin(2\,dx+2\,c)+\cos(2\,dx+2\,c\cdots}{\cdots}\right.$$

```
[In] integrate(1/(e*cot(d*x+c))^(5/2)/(a+a*cot(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] [-1/48*(6*sqrt(2)*((cos(2*d*x + 2*c) + 1)*sin(2*d*x + 2*c) + cos(2*d*x + 2*
c) + 1)*sqrt(-e)*log(sqrt(2)*sqrt(-e)*sqrt((e*cos(2*d*x + 2*c) + e)/sin(2*d
```

```
*x + 2*c))*(cos(2*d*x + 2*c) + sin(2*d*x + 2*c) - 1) - 2*e*sin(2*d*x + 2*c)
 + e) + 177*((cos(2*d*x + 2*c) + 1)*sin(2*d*x + 2*c) + cos(2*d*x + 2*c) + 1
)*sqrt(-e)*log((e*cos(2*d*x + 2*c) - e*sin(2*d*x + 2*c) + 2*sqrt(-e)*sqrt((
e*cos(2*d*x + 2*c) + e)/sin(2*d*x + 2*c))*sin(2*d*x + 2*c) + e)/(cos(2*d*x
+ 2*c) + sin(2*d*x + 2*c) + 1)) - (339*cos(2*d*x + 2*c)^2 - 7*(11*cos(2*d*x
 + 2*c) + 43)*sin(2*d*x + 2*c) - 32*cos(2*d*x + 2*c) - 307)*sqrt((e*cos(2*d
*x + 2*c) + e)/sin(2*d*x + 2*c)))/(a^3*d*e^3*cos(2*d*x + 2*c) + a^3*d*e^3 +
 (a^3*d*e^3*cos(2*d*x + 2*c) + a^3*d*e^3)*sin(2*d*x + 2*c)), 1/48*(12*sqrt(
2)*((cos(2*d*x + 2*c) + 1)*sin(2*d*x + 2*c) + cos(2*d*x + 2*c) + 1)*sqrt(e)
*arctan(-1/2*sqrt(2)*sqrt(e)*sqrt((e*cos(2*d*x + 2*c) + e)/sin(2*d*x + 2*c)
)*(cos(2*d*x + 2*c) - sin(2*d*x + 2*c) + 1)/(e*cos(2*d*x + 2*c) + e)) - 354
*((cos(2*d*x + 2*c) + 1)*sin(2*d*x + 2*c) + cos(2*d*x + 2*c) + 1)*sqrt(e)*a
rctan(sqrt((e*cos(2*d*x + 2*c) + e)/sin(2*d*x + 2*c))/sqrt(e)) + (339*cos(2
*d*x + 2*c)^2 - 7*(11*cos(2*d*x + 2*c) + 43)*sin(2*d*x + 2*c) - 32*cos(2*d*
x + 2*c) - 307)*sqrt((e*cos(2*d*x + 2*c) + e)/sin(2*d*x + 2*c)))/(a^3*d*e^3
*cos(2*d*x + 2*c) + a^3*d*e^3 + (a^3*d*e^3*cos(2*d*x + 2*c) + a^3*d*e^3)*si
n(2*d*x + 2*c))]
```

Sympy [F]

$$\int \frac{1}{(e\cot(c+dx))^{5/2}(a+a\cot(c+dx))^3}\,dx = \frac{\int \frac{1}{(e\cot(c+dx))^{\frac{5}{2}}\cot^3(c+dx)+3(e\cot(c+dx))^{\frac{5}{2}}\cot^2(c+dx)+3(e\cot(c+dx))^{\frac{5}{2}}\,\mathrm{co}}}{a^3}$$

```
[In] integrate(1/(e*cot(d*x+c))**(5/2)/(a+a*cot(d*x+c))**3,x)
```

```
[Out] Integral(1/((e*cot(c + d*x))**(5/2)*cot(c + d*x)**3 + 3*(e*cot(c + d*x))**(
5/2)*cot(c + d*x)**2 + 3*(e*cot(c + d*x))**(5/2)*cot(c + d*x) + (e*cot(c +
d*x))**(5/2)), x)/a**3
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{1}{(e\cot(c+dx))^{5/2}(a+a\cot(c+dx))^3}\,dx = \text{Exception raised: ValueError}$$

```
[In] integrate(1/(e*cot(d*x+c))^(5/2)/(a+a*cot(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(e>0)', see `assume?` for more detai
ls)Is e
```

Giac [F]

$$\int \frac{1}{(e\cot(c+dx))^{5/2}(a+a\cot(c+dx))^3}\,dx = \int \frac{1}{\left(a\cot\left(dx+c\right)+a\right)^3\left(e\cot\left(dx+c\right)\right)^{\frac{5}{2}}}\,dx$$

```
[In] integrate(1/(e*cot(d*x+c))^(5/2)/(a+a*cot(d*x+c))^3,x, algorithm="giac")
[Out] integrate(1/((a*cot(d*x + c) + a)^3*(e*cot(d*x + c))^(5/2)), x)
```

Mupad [B] (verification not implemented)

Time = 13.53 (sec) , antiderivative size = 193, normalized size of antiderivative = 0.90

$$\int \frac{1}{(e\cot(c+dx))^{5/2}(a+a\cot(c+dx))^3}\,dx = -\frac{\frac{63\,e\cot(c+d\,x)^3}{8}+\frac{323\,e\cot(c+d\,x)^2}{24}+\frac{14\,e\cot(c+d\,x)}{3}-\frac{2\,e}{3}}{a^3\,d\left(e\cot\left(c+d\,x\right)\right)^{7/2}+2\,a^3\,d\,e\left(e\cot\left(c+d\,x\right)\right)^{5/2}+a^3\,d\,e^2\left(e\cot\left(c+d\,x\right)\right)^{3/2}} - \frac{59\,\mathrm{atan}\left(\frac{\sqrt{e\cot(c+d\,x)}}{\sqrt{e}}\right)}{8\,a^3\,d\,e^{5/2}} - \frac{\sqrt{2}\left(2\,\mathrm{atan}\left(\frac{\sqrt{2}\,\sqrt{e\cot(c+d\,x)}}{2\,\sqrt{e}}\right)+2\,\mathrm{atan}\left(\frac{\sqrt{2}\,\sqrt{e\cot(c+d\,x)}}{2\,\sqrt{e}}+\frac{\sqrt{2}\,(e\cot(c+d\,x))^{3/2}}{2\,e^{3/2}}\right)\right)}{8\,a^3\,d\,e^{5/2}}$$

```
[In] int(1/((e*cot(c + d*x))^(5/2)*(a + a*cot(c + d*x))^3),x)
[Out] - ((14*e*cot(c + d*x))/3 - (2*e)/3 + (323*e*cot(c + d*x)^2)/24 + (63*e*cot(
c + d*x)^3)/8)/(a^3*d*(e*cot(c + d*x))^(7/2) + 2*a^3*d*e*(e*cot(c + d*x))^(
5/2) + a^3*d*e^2*(e*cot(c + d*x))^(3/2)) - (59*atan((e*cot(c + d*x))^(1/2)/
e^(1/2)))/(8*a^3*d*e^(5/2)) - (2^(1/2)*(2*atan((2^(1/2)*(e*cot(c + d*x))^(1
/2))/(2*e^(1/2))) + 2*atan((2^(1/2)*(e*cot(c + d*x))^(1/2))/(2*e^(1/2)) + (
2^(1/2)*(e*cot(c + d*x))^(3/2))/(2*e^(3/2)))))/(8*a^3*d*e^(5/2))
```

3.41 $\int \cot^2(x)\sqrt{1+\cot(x)}\,dx$

Optimal result

Integrand size = 13, antiderivative size = 223

$$\begin{aligned}\int \cot^2(x)\sqrt{1+\cot(x)}\,dx = &-\sqrt{\frac{1}{2}\left(1+\sqrt{2}\right)}\arctan\left(\frac{\sqrt{2\left(1+\sqrt{2}\right)}-2\sqrt{1+\cot(x)}}{\sqrt{2\left(-1+\sqrt{2}\right)}}\right)\\ &+\sqrt{\frac{1}{2}\left(1+\sqrt{2}\right)}\arctan\left(\frac{\sqrt{2\left(1+\sqrt{2}\right)}+2\sqrt{1+\cot(x)}}{\sqrt{2\left(-1+\sqrt{2}\right)}}\right)\\ &-\frac{2}{3}(1+\cot(x))^{3/2}\\ &+\frac{\log\left(1+\sqrt{2}+\cot(x)-\sqrt{2\left(1+\sqrt{2}\right)}\sqrt{1+\cot(x)}\right)}{2\sqrt{2\left(1+\sqrt{2}\right)}}\\ &-\frac{\log\left(1+\sqrt{2}+\cot(x)+\sqrt{2\left(1+\sqrt{2}\right)}\sqrt{1+\cot(x)}\right)}{2\sqrt{2\left(1+\sqrt{2}\right)}}\end{aligned}$$

```
[Out] -2/3*(1+cot(x))^(3/2)-1/2*arctan((-2*(1+cot(x))^(1/2)+(2+2*2^(1/2))^(1/2))/
(-2+2*2^(1/2))^(1/2))*(2+2*2^(1/2))^(1/2)+1/2*arctan((2*(1+cot(x))^(1/2)+(2
+2*2^(1/2))^(1/2))/(-2+2*2^(1/2))^(1/2))*(2+2*2^(1/2))^(1/2)+1/2*ln(1+cot(x
)+2^(1/2)-(1+cot(x))^(1/2)*(2+2*2^(1/2))^(1/2))/(2+2*2^(1/2))^(1/2)-1/2*ln(
1+cot(x)+2^(1/2)+(1+cot(x))^(1/2)*(2+2*2^(1/2))^(1/2))/(2+2*2^(1/2))^(1/2)
```

Rubi [A] (verified)

Time = 0.34 (sec) , antiderivative size = 223, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.692, Rules used = {3624, 3566, 714, 1141, 1175, 632, 210, 1178, 642}

$$\int \cot^2(x)\sqrt{1+\cot(x)}\,dx = -\sqrt{\frac{1}{2}\left(1+\sqrt{2}\right)}\arctan\left(\frac{\sqrt{2\left(1+\sqrt{2}\right)}-2\sqrt{\cot(x)+1}}{\sqrt{2\left(\sqrt{2}-1\right)}}\right) + \sqrt{\frac{1}{2}\left(1+\sqrt{2}\right)}\arctan\left(\frac{2\sqrt{\cot(x)+1}+\sqrt{2\left(1+\sqrt{2}\right)}}{\sqrt{2\left(\sqrt{2}-1\right)}}\right) - \frac{2}{3}(\cot(x)+1)^{3/2} + \frac{\log\left(\cot(x)-\sqrt{2\left(1+\sqrt{2}\right)}\sqrt{\cot(x)+1}+\sqrt{2}+1\right)}{2\sqrt{2\left(1+\sqrt{2}\right)}} - \frac{\log\left(\cot(x)+\sqrt{2\left(1+\sqrt{2}\right)}\sqrt{\cot(x)+1}+\sqrt{2}+1\right)}{2\sqrt{2\left(1+\sqrt{2}\right)}}$$

```
[In] Int[Cot[x]^2*Sqrt[1 + Cot[x]],x]
```

```
[Out] -(Sqrt[(1 + Sqrt[2])/2]*ArcTan[(Sqrt[2*(1 + Sqrt[2])] - 2*Sqrt[1 + Cot[x]])
/Sqrt[2*(-1 + Sqrt[2])]]) + Sqrt[(1 + Sqrt[2])/2]*ArcTan[(Sqrt[2*(1 + Sqrt[
2])] + 2*Sqrt[1 + Cot[x]])/Sqrt[2*(-1 + Sqrt[2])]] - (2*(1 + Cot[x])^(3/2))
/3 + Log[1 + Sqrt[2] + Cot[x] - Sqrt[2*(1 + Sqrt[2])]*Sqrt[1 + Cot[x]]]/(2*
Sqrt[2*(1 + Sqrt[2])]) - Log[1 + Sqrt[2] + Cot[x] + Sqrt[2*(1 + Sqrt[2])]*S
qrt[1 + Cot[x]]]/(2*Sqrt[2*(1 + Sqrt[2])])
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 714

```
Int[Sqrt[(d_) + (e_.)*(x_)]/((a_) + (c_.)*(x_)^2), x_Symbol] :> Dist[2*e, S
ubst[Int[x^2/(c*d^2 + a*e^2 - 2*c*d*x^2 + c*x^4), x], x, Sqrt[d + e*x]], x]
 /; FreeQ[{a, c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0]
```

Rule 1141

```
Int[(x_)^2/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
a/c, 2]}, Dist[1/2, Int[(q + x^2)/(a + b*x^2 + c*x^4), x], x] - Dist[1/2, I
nt[(q - x^2)/(a + b*x^2 + c*x^4), x], x]] /; FreeQ[{a, b, c}, x] && LtQ[b^2
 - 4*a*c, 0] && PosQ[a*c]
```

Rule 1175

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[2*(d/e) - b/c, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2
, x], x], x] + Dist[e/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; Fre
eQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - a*e^2, 0] && (
GtQ[2*(d/e) - b/c, 0] || ( !LtQ[2*(d/e) - b/c, 0] && EqQ[d - e*Rt[a/c, 2],
0]))
```

Rule 1178

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[-2*(d/e) - b/c, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e
+ q*x - x^2, x], x], x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^
2, x], x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c
*d^2 - a*e^2, 0] &&  !GtQ[b^2 - 4*a*c, 0]
```

Rule 3566

```
Int[((a_) + (b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[b/d, Sub
st[Int[(a + x)^n/(b^2 + x^2), x], x, b*Tan[c + d*x]], x] /; FreeQ[{a, b, c,
 d, n}, x] && NeQ[a^2 + b^2, 0]
```

Rule 3624

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)])^2, x_Symbol] :> Simp[d^2*((a + b*Tan[e + f*x])^(m + 1)/(b*f*(
m + 1))), x] + Int[(a + b*Tan[e + f*x])^m*Simp[c^2 - d^2 + 2*c*d*Tan[e + f*
x], x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b*c - a*d, 0] &&  !LeQ
```

```
[m, -1] &&  !(EqQ[m, 2] && EqQ[a, 0])
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= -\frac{2}{3}(1+\cot(x))^{3/2}-\int\sqrt{1+\cot(x)}\,dx\\
&= -\frac{2}{3}(1+\cot(x))^{3/2}+\text{Subst}\left(\int\frac{\sqrt{1+x}}{1+x^2}\,dx,x,\cot(x)\right)\\
&= -\frac{2}{3}(1+\cot(x))^{3/2}+2\text{Subst}\left(\int\frac{x^2}{2-2x^2+x^4}\,dx,x,\sqrt{1+\cot(x)}\right)\\
&= -\frac{2}{3}(1+\cot(x))^{3/2}-\text{Subst}\left(\int\frac{\sqrt{2}-x^2}{2-2x^2+x^4}\,dx,x,\sqrt{1+\cot(x)}\right)\\
&\quad+\text{Subst}\left(\int\frac{\sqrt{2}+x^2}{2-2x^2+x^4}\,dx,x,\sqrt{1+\cot(x)}\right)\\
&= -\frac{2}{3}(1+\cot(x))^{3/2}+\frac{1}{2}\text{Subst}\left(\int\frac{1}{\sqrt{2}-\sqrt{2\left(1+\sqrt{2}\right)}x+x^2}\,dx,x,\sqrt{1+\cot(x)}\right)\\
&\quad+\frac{1}{2}\text{Subst}\left(\int\frac{1}{\sqrt{2}+\sqrt{2\left(1+\sqrt{2}\right)}x+x^2}\,dx,x,\sqrt{1+\cot(x)}\right)\\
&\quad+\frac{\text{Subst}\left(\int\frac{\sqrt{2\left(1+\sqrt{2}\right)}+2x}{-\sqrt{2}-\sqrt{2\left(1+\sqrt{2}\right)}x-x^2}\,dx,x,\sqrt{1+\cot(x)}\right)}{2\sqrt{2\left(1+\sqrt{2}\right)}}\\
&\quad+\frac{\text{Subst}\left(\int\frac{\sqrt{2\left(1+\sqrt{2}\right)}-2x}{-\sqrt{2}+\sqrt{2\left(1+\sqrt{2}\right)}x-x^2}\,dx,x,\sqrt{1+\cot(x)}\right)}{2\sqrt{2\left(1+\sqrt{2}\right)}}
\end{aligned}
$$

$$= -\frac{2}{3}(1+\cot(x))^{3/2} + \frac{\log\left(1+\sqrt{2}+\cot(x)-\sqrt{2\left(1+\sqrt{2}\right)}\sqrt{1+\cot(x)}\right)}{2\sqrt{2\left(1+\sqrt{2}\right)}}$$
$$- \frac{\log\left(1+\sqrt{2}+\cot(x)+\sqrt{2\left(1+\sqrt{2}\right)}\sqrt{1+\cot(x)}\right)}{2\sqrt{2\left(1+\sqrt{2}\right)}}$$
$$- \text{Subst}\left(\int \frac{1}{2\left(1-\sqrt{2}\right)-x^2}\,dx, x, -\sqrt{2\left(1+\sqrt{2}\right)}+2\sqrt{1+\cot(x)}\right)$$
$$- \text{Subst}\left(\int \frac{1}{2\left(1-\sqrt{2}\right)-x^2}\,dx, x, \sqrt{2\left(1+\sqrt{2}\right)}+2\sqrt{1+\cot(x)}\right)$$

$$= \frac{\arctan\left(\frac{-\sqrt{2\left(1+\sqrt{2}\right)}+2\sqrt{1+\cot(x)}}{\sqrt{2\left(-1+\sqrt{2}\right)}}\right)}{\sqrt{2\left(-1+\sqrt{2}\right)}} + \frac{\arctan\left(\frac{\sqrt{2\left(1+\sqrt{2}\right)}+2\sqrt{1+\cot(x)}}{\sqrt{2\left(-1+\sqrt{2}\right)}}\right)}{\sqrt{2\left(-1+\sqrt{2}\right)}}$$
$$- \frac{2}{3}(1+\cot(x))^{3/2} + \frac{\log\left(1+\sqrt{2}+\cot(x)-\sqrt{2\left(1+\sqrt{2}\right)}\sqrt{1+\cot(x)}\right)}{2\sqrt{2\left(1+\sqrt{2}\right)}}$$
$$- \frac{\log\left(1+\sqrt{2}+\cot(x)+\sqrt{2\left(1+\sqrt{2}\right)}\sqrt{1+\cot(x)}\right)}{2\sqrt{2\left(1+\sqrt{2}\right)}}$$

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.52 (sec) , antiderivative size = 69, normalized size of antiderivative = 0.31

$$\int \cot^2(x)\sqrt{1+\cot(x)}\,dx = -i\sqrt{1-i}\,\text{arctanh}\left(\frac{\sqrt{1+\cot(x)}}{\sqrt{1-i}}\right) + i\sqrt{1+i}\,\text{arctanh}\left(\frac{\sqrt{1+\cot(x)}}{\sqrt{1+i}}\right) - \frac{2}{3}(1+\cot(x))^{3/2}$$

```
[In] Integrate[Cot[x]^2*Sqrt[1 + Cot[x]],x]
```

```
[Out] (-I)*Sqrt[1 - I]*ArcTanh[Sqrt[1 + Cot[x]]/Sqrt[1 - I]] + I*Sqrt[1 + I]*ArcT
anh[Sqrt[1 + Cot[x]]/Sqrt[1 + I]] - (2*(1 + Cot[x])^(3/2))/3
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 355 vs. 2(160) = 320.

Time = 0.15 (sec) , antiderivative size = 356, normalized size of antiderivative = 1.60

method	result
derivativedivides	$-\frac{2(1+\cot(x))^{\frac{3}{2}}}{3}+\frac{\sqrt{2+2\sqrt{2}}\,\sqrt{2}\,\ln\left(1+\cot(x)+\sqrt{2}-\sqrt{1+\cot(x)}\,\sqrt{2+2\sqrt{2}}\right)}{4}+\frac{\sqrt{2}\left(2+2\sqrt{2}\right)\arctan\left(\frac{2\sqrt{1+\cot(x)}-\sqrt{2}}{\sqrt{-2+2\sqrt{2}}}\right)}{2\sqrt{-2+2\sqrt{2}}}$
default	$-\frac{2(1+\cot(x))^{\frac{3}{2}}}{3}+\frac{\sqrt{2+2\sqrt{2}}\,\sqrt{2}\,\ln\left(1+\cot(x)+\sqrt{2}-\sqrt{1+\cot(x)}\,\sqrt{2+2\sqrt{2}}\right)}{4}+\frac{\sqrt{2}\left(2+2\sqrt{2}\right)\arctan\left(\frac{2\sqrt{1+\cot(x)}-\sqrt{2}}{\sqrt{-2+2\sqrt{2}}}\right)}{2\sqrt{-2+2\sqrt{2}}}$

```
[In] int(cot(x)^2*(1+cot(x))^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -2/3*(1+cot(x))^(3/2)+1/4*(2+2*2^(1/2))^(1/2)*2^(1/2)*ln(1+cot(x)+2^(1/2)-(
1+cot(x))^(1/2)*(2+2*2^(1/2))^(1/2))+1/2*2^(1/2)*(2+2*2^(1/2))/(-2+2*2^(1/2
))^(1/2)*arctan((2*(1+cot(x))^(1/2)-(2+2*2^(1/2))^(1/2))/(-2+2*2^(1/2))^(1/
2))-1/4*(2+2*2^(1/2))^(1/2)*ln(1+cot(x)+2^(1/2)-(1+cot(x))^(1/2)*(2+2*2^(1/
2))^(1/2))-1/2*(2+2*2^(1/2))/(-2+2*2^(1/2))^(1/2)*arctan((2*(1+cot(x))^(1/2
)-(2+2*2^(1/2))^(1/2))/(-2+2*2^(1/2))^(1/2))-1/4*(2+2*2^(1/2))^(1/2)*2^(1/2
)*ln(1+cot(x)+2^(1/2)+(1+cot(x))^(1/2)*(2+2*2^(1/2))^(1/2))+1/2*2^(1/2)*(2+
2*2^(1/2))/(-2+2*2^(1/2))^(1/2)*arctan((2*(1+cot(x))^(1/2)+(2+2*2^(1/2))^(1
/2))/(-2+2*2^(1/2))^(1/2))+1/4*(2+2*2^(1/2))^(1/2)*ln(1+cot(x)+2^(1/2)+(1+c
ot(x))^(1/2)*(2+2*2^(1/2))^(1/2))-1/2*(2+2*2^(1/2))/(-2+2*2^(1/2))^(1/2)*ar
ctan((2*(1+cot(x))^(1/2)+(2+2*2^(1/2))^(1/2))/(-2+2*2^(1/2))^(1/2))
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.27 (sec) , antiderivative size = 180, normalized size of antiderivative = 0.81

$$\int \cot^2(x)\sqrt{1+\cot(x)}\,dx$$
$$=\frac{3\sqrt{i-1}\log\left(i\sqrt{i-1}+\sqrt{\frac{\cos(2x)+\sin(2x)+1}{\sin(2x)}}\right)\sin(2x)-3\sqrt{i-1}\log\left(-i\sqrt{i-1}+\sqrt{\frac{\cos(2x)+\sin(2x)+1}{\sin(2x)}}\right)\sin(2}{}$$

```
[In] integrate(cot(x)^2*(1+cot(x))^(1/2),x, algorithm="fricas")
```

```
[Out] 1/6*(3*sqrt(I - 1)*log(I*sqrt(I - 1) + sqrt((cos(2*x) + sin(2*x) + 1)/sin(2
*x)))*sin(2*x) - 3*sqrt(I - 1)*log(-I*sqrt(I - 1) + sqrt((cos(2*x) + sin(2*
x) + 1)/sin(2*x)))*sin(2*x) - 3*sqrt(-I - 1)*log(I*sqrt(-I - 1) + sqrt((cos
(2*x) + sin(2*x) + 1)/sin(2*x)))*sin(2*x) + 3*sqrt(-I - 1)*log(-I*sqrt(-I -
 1) + sqrt((cos(2*x) + sin(2*x) + 1)/sin(2*x)))*sin(2*x) - 4*sqrt((cos(2*x)
 + sin(2*x) + 1)/sin(2*x))*(cos(2*x) + sin(2*x) + 1))/sin(2*x)
```

Sympy [F]

$$\int \cot^2(x)\sqrt{1+\cot(x)}\,dx = \int \sqrt{\cot(x)+1}\cot^2(x)\,dx$$

```
[In] integrate(cot(x)**2*(1+cot(x))**(1/2),x)
[Out] Integral(sqrt(cot(x) + 1)*cot(x)**2, x)
```

Maxima [F]

$$\int \cot^2(x)\sqrt{1+\cot(x)}\,dx = \int \sqrt{\cot(x)+1}\cot(x)^2\,dx$$

```
[In] integrate(cot(x)^2*(1+cot(x))^(1/2),x, algorithm="maxima")
[Out] integrate(sqrt(cot(x) + 1)*cot(x)^2, x)
```

Giac [F]

$$\int \cot^2(x)\sqrt{1+\cot(x)}\,dx = \int \sqrt{\cot(x)+1}\cot(x)^2\,dx$$

```
[In] integrate(cot(x)^2*(1+cot(x))^(1/2),x, algorithm="giac")
[Out] integrate(sqrt(cot(x) + 1)*cot(x)^2, x)
```

Mupad [B] (verification not implemented)

Time = 12.98 (sec) , antiderivative size = 119, normalized size of antiderivative = 0.53

$$\begin{aligned}
&\int \cot^2(x)\sqrt{1+\cot(x)}\,dx \\
&= \operatorname{atanh}\left(4\sqrt{\cot(x)+1}\left(\sqrt{-\frac{\sqrt{2}}{8}-\frac{1}{8}}+\sqrt{\frac{\sqrt{2}}{8}-\frac{1}{8}}\right)^3\right)\left(2\sqrt{-\frac{\sqrt{2}}{8}-\frac{1}{8}}\right. \\
&\quad \left.+2\sqrt{\frac{\sqrt{2}}{8}-\frac{1}{8}}\right)-\frac{2(\cot(x)+1)^{3/2}}{3} \\
&\quad +\operatorname{atanh}\left(4\sqrt{\cot(x)+1}\left(\sqrt{-\frac{\sqrt{2}}{8}-\frac{1}{8}}-\sqrt{\frac{\sqrt{2}}{8}-\frac{1}{8}}\right)^3\right)\left(2\sqrt{-\frac{\sqrt{2}}{8}-\frac{1}{8}}\right. \\
&\quad \left.-2\sqrt{\frac{\sqrt{2}}{8}-\frac{1}{8}}\right)
\end{aligned}$$

```
[In] int(cot(x)^2*(cot(x) + 1)^(1/2),x)
```

```
[Out] atanh(4*(cot(x) + 1)^(1/2)*((- 2^(1/2)/8 - 1/8)^(1/2) + (2^(1/2)/8 - 1/8)^(
1/2))^3)*(2*(- 2^(1/2)/8 - 1/8)^(1/2) + 2*(2^(1/2)/8 - 1/8)^(1/2)) - (2*(co
t(x) + 1)^(3/2))/3 + atanh(4*(cot(x) + 1)^(1/2)*((- 2^(1/2)/8 - 1/8)^(1/2)
- (2^(1/2)/8 - 1/8)^(1/2))^3)*(2*(- 2^(1/2)/8 - 1/8)^(1/2) - 2*(2^(1/2)/8 -
 1/8)^(1/2))
```

3.42 $\int \cot(x)\sqrt{1+\cot(x)}\,dx$

Optimal result

Integrand size = 11, antiderivative size = 135

$$\int \cot(x)\sqrt{1+\cot(x)}\,dx = \sqrt{\frac{1}{2}\left(-1+\sqrt{2}\right)}\arctan\left(\frac{4-3\sqrt{2}+\left(2-\sqrt{2}\right)\cot(x)}{2\sqrt{-7+5\sqrt{2}}\sqrt{1+\cot(x)}}\right)+\sqrt{\frac{1}{2}\left(1+\sqrt{2}\right)}\operatorname{arctanh}\left(\frac{4+3\sqrt{2}+\left(2+\sqrt{2}\right)\cot(x)}{2\sqrt{7+5\sqrt{2}}\sqrt{1+\cot(x)}}\right)-2\sqrt{1+\cot(x)}$$

```
[Out] -2*(1+cot(x))^(1/2)+1/2*arctan(1/2*(4+cot(x)*(2-2^(1/2))-3*2^(1/2))/(1+cot(
x))^(1/2)/(-7+5*2^(1/2))^(1/2))*(-2+2*2^(1/2))^(1/2)+1/2*arctanh(1/2*(4+3*2
^(1/2)+cot(x)*(2+2^(1/2)))/(1+cot(x))^(1/2)/(7+5*2^(1/2))^(1/2))*(2+2*2^(1/
2))^(1/2)
```

Rubi [A] (verified)

Time = 0.31 (sec) , antiderivative size = 135, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.455, Rules used = {3609, 3617, 3616, 209, 213}

$$\int \cot(x)\sqrt{1+\cot(x)}\,dx = \sqrt{\frac{1}{2}\left(\sqrt{2}-1\right)}\arctan\left(\frac{\left(2-\sqrt{2}\right)\cot(x)-3\sqrt{2}+4}{2\sqrt{5\sqrt{2}-7}\sqrt{\cot(x)+1}}\right)+\sqrt{\frac{1}{2}\left(1+\sqrt{2}\right)}\operatorname{arctanh}\left(\frac{\left(2+\sqrt{2}\right)\cot(x)+3\sqrt{2}+4}{2\sqrt{7+5\sqrt{2}}\sqrt{\cot(x)+1}}\right)-2\sqrt{\cot(x)+1}$$

```
[In] Int[Cot[x]*Sqrt[1 + Cot[x]],x]
```

```
[Out] Sqrt[(-1 + Sqrt[2])/2]*ArcTan[(4 - 3*Sqrt[2] + (2 - Sqrt[2])*Cot[x])/(2*Sqr
t[-7 + 5*Sqrt[2]]*Sqrt[1 + Cot[x]])] + Sqrt[(1 + Sqrt[2])/2]*ArcTanh[(4 + 3
*Sqrt[2] + (2 + Sqrt[2])*Cot[x])/(2*Sqrt[7 + 5*Sqrt[2]]*Sqrt[1 + Cot[x]])]
- 2*Sqrt[1 + Cot[x]]
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 213

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[b, 2])^(-
1))*ArcTanh[Rt[b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] &&
 (LtQ[a, 0] || GtQ[b, 0])
```

Rule 3609

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)]), x_Symbol] :> Simp[d*((a + b*Tan[e + f*x])^m/(f*m)), x] + Int
[(a + b*Tan[e + f*x])^(m - 1)*Simp[a*c - b*d + (b*c + a*d)*Tan[e + f*x], x]
, x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2,
 0] && GtQ[m, 0]
```

Rule 3616

```
Int[((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])/Sqrt[(a_) + (b_.)*tan[(e_.) + (
f_.)*(x_)]], x_Symbol] :> Dist[-2*(d^2/f), Subst[Int[1/(2*b*c*d - 4*a*d^2 +
 x^2), x], x, (b*c - 2*a*d - b*d*Tan[e + f*x])/Sqrt[a + b*Tan[e + f*x]]], x
] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0]
 && NeQ[c^2 + d^2, 0] && EqQ[2*a*c*d - b*(c^2 - d^2), 0]
```

Rule 3617

```
Int[((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])/Sqrt[(a_) + (b_.)*tan[(e_.) + (
f_.)*(x_)]], x_Symbol] :> With[{q = Rt[a^2 + b^2, 2]}, Dist[1/(2*q), Int[(a
*c + b*d + c*q + (b*c - a*d + d*q)*Tan[e + f*x])/Sqrt[a + b*Tan[e + f*x]],
x], x] - Dist[1/(2*q), Int[(a*c + b*d - c*q + (b*c - a*d - d*q)*Tan[e + f*x
])/Sqrt[a + b*Tan[e + f*x]], x], x]] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ
[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] && NeQ[2*a*c*d - b
*(c^2 - d^2), 0] && (PerfectSquareQ[a^2 + b^2] || RationalQ[a, b, c, d])
```

Rubi steps

$$\text{integral} = -2\sqrt{1+\cot(x)} - \int \frac{1-\cot(x)}{\sqrt{1+\cot(x)}}\,dx$$

$$= -2\sqrt{1+\cot(x)} + \frac{\int \frac{-\sqrt{2}-\left(-2-\sqrt{2}\right)\cot(x)}{\sqrt{1+\cot(x)}}dx}{2\sqrt{2}} - \frac{\int \frac{\sqrt{2}-\left(-2+\sqrt{2}\right)\cot(x)}{\sqrt{1+\cot(x)}}dx}{2\sqrt{2}}$$

$$= -2\sqrt{1+\cot(x)} + \left(-4 +3\sqrt{2}\right)\text{Subst}\left(\int \frac{1}{-2\sqrt{2}\left(-2+\sqrt{2}\right)-4\left(-2+\sqrt{2}\right)^2+x^2}dx, x, \frac{-\sqrt{2}-2(-2+\sqrt{2})-(-2+}{\sqrt{1+\cot(x)}}\right.$$
$$- \left(4 +3\sqrt{2}\right)\text{Subst}\left(\int \frac{1}{2\sqrt{2}\left(-2-\sqrt{2}\right)-4\left(-2-\sqrt{2}\right)^2+x^2}dx, x, \frac{\sqrt{2}-2(-2-\sqrt{2})-(-2-\sqrt{2})}{\sqrt{1+\cot(x)}}\right.$$

$$= \sqrt{\frac{1}{2}\left(-1+\sqrt{2}\right)}\arctan\left(\frac{4-3\sqrt{2}+\left(2-\sqrt{2}\right)\cot(x)}{2\sqrt{-7+5\sqrt{2}}\sqrt{1+\cot(x)}}\right) + \sqrt{\frac{1}{2}\left(1+\sqrt{2}\right)}\text{arctanh}\left(\frac{4+3\sqrt{2}+\left(2+\sqrt{2}\right)\cot(x)}{2\sqrt{7+5\sqrt{2}}\sqrt{1+\cot(x)}}\right) - 2\sqrt{1+\cot(x)}$$

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.32 (sec) , antiderivative size = 61, normalized size of antiderivative = 0.45

$$\int \cot(x)\sqrt{1+\cot(x)}\,dx = \sqrt{1-i}\,\text{arctanh}\left(\frac{\sqrt{1+\cot(x)}}{\sqrt{1-i}}\right) + \sqrt{1+i}\,\text{arctanh}\left(\frac{\sqrt{1+\cot(x)}}{\sqrt{1+i}}\right) - 2\sqrt{1+\cot(x)}$$

```
[In] Integrate[Cot[x]*Sqrt[1 + Cot[x]],x]
```

```
[Out] Sqrt[1 - I]*ArcTanh[Sqrt[1 + Cot[x]]/Sqrt[1 - I]] + Sqrt[1 + I]*ArcTanh[Sqr
t[1 + Cot[x]]/Sqrt[1 + I]] - 2*Sqrt[1 + Cot[x]]
```

Maple [A] (verified)

Time = 0.04 (sec) , antiderivative size = 174, normalized size of antiderivative = 1.29

method	result
derivativedivides	$-2\sqrt{1+\cot(x)}-\frac{\sqrt{2+2\sqrt{2}}\,\ln\left(1+\cot(x)+\sqrt{2}-\sqrt{1+\cot(x)}\,\sqrt{2+2\sqrt{2}}\right)}{4}+\frac{\left(\sqrt{2}-1\right)\arctan\left(\frac{2\sqrt{1+\cot(x)}-\sqrt{2+2\sqrt{2}}}{\sqrt{-2+2\sqrt{2}}}\right)}{\sqrt{-2+2\sqrt{2}}}$
default	$-2\sqrt{1+\cot(x)}-\frac{\sqrt{2+2\sqrt{2}}\,\ln\left(1+\cot(x)+\sqrt{2}-\sqrt{1+\cot(x)}\,\sqrt{2+2\sqrt{2}}\right)}{4}+\frac{\left(\sqrt{2}-1\right)\arctan\left(\frac{2\sqrt{1+\cot(x)}-\sqrt{2+2\sqrt{2}}}{\sqrt{-2+2\sqrt{2}}}\right)}{\sqrt{-2+2\sqrt{2}}}$

```
[In] int(cot(x)*(1+cot(x))^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -2*(1+cot(x))^(1/2)-1/4*(2+2*2^(1/2))^(1/2)*ln(1+cot(x)+2^(1/2)-(1+cot(x))^
(1/2)*(2+2*2^(1/2))^(1/2))+(2^(1/2)-1)/(-2+2*2^(1/2))^(1/2)*arctan((2*(1+co
t(x))^(1/2)-(2+2*2^(1/2))^(1/2))/(-2+2*2^(1/2))^(1/2))+1/4*(2+2*2^(1/2))^(1
/2)*ln(1+cot(x)+2^(1/2)+(1+cot(x))^(1/2)*(2+2*2^(1/2))^(1/2))-(1-2^(1/2))/(
-2+2*2^(1/2))^(1/2)*arctan((2*(1+cot(x))^(1/2)+(2+2*2^(1/2))^(1/2))/(-2+2*2
^(1/2))^(1/2))
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.27 (sec) , antiderivative size = 142, normalized size of antiderivative = 1.05

$$\begin{aligned}\int \cot(x)\sqrt{1+\cot(x)}\,dx &= \frac{1}{2}\sqrt{i+1}\log\left(\sqrt{i+1}+\sqrt{\frac{\cos(2x)+\sin(2x)+1}{\sin(2x)}}\right) \\ &\quad -\frac{1}{2}\sqrt{i+1}\log\left(-\sqrt{i+1}+\sqrt{\frac{\cos(2x)+\sin(2x)+1}{\sin(2x)}}\right) \\ &\quad +\frac{1}{2}\sqrt{-i+1}\log\left(\sqrt{-i+1}+\sqrt{\frac{\cos(2x)+\sin(2x)+1}{\sin(2x)}}\right) \\ &\quad -\frac{1}{2}\sqrt{-i+1}\log\left(-\sqrt{-i+1}+\sqrt{\frac{\cos(2x)+\sin(2x)+1}{\sin(2x)}}\right) \\ &\quad -2\sqrt{\frac{\cos(2x)+\sin(2x)+1}{\sin(2x)}}\end{aligned}$$

```
[In] integrate(cot(x)*(1+cot(x))^(1/2),x, algorithm="fricas")
```

```
[Out] 1/2*sqrt(I + 1)*log(sqrt(I + 1) + sqrt((cos(2*x) + sin(2*x) + 1)/sin(2*x)))
 - 1/2*sqrt(I + 1)*log(-sqrt(I + 1) + sqrt((cos(2*x) + sin(2*x) + 1)/sin(2*
x))) + 1/2*sqrt(-I + 1)*log(sqrt(-I + 1) + sqrt((cos(2*x) + sin(2*x) + 1)/s
in(2*x))) - 1/2*sqrt(-I + 1)*log(-sqrt(-I + 1) + sqrt((cos(2*x) + sin(2*x)
+ 1)/sin(2*x))) - 2*sqrt((cos(2*x) + sin(2*x) + 1)/sin(2*x))
```

Sympy [F]

$$\int \cot(x)\sqrt{1+\cot(x)}\,dx = \int \sqrt{\cot{(x)}+1}\cot{(x)}\,dx$$

```
[In] integrate(cot(x)*(1+cot(x))**(1/2),x)
[Out] Integral(sqrt(cot(x) + 1)*cot(x), x)
```

Maxima [F]

$$\int \cot(x)\sqrt{1+\cot(x)}\,dx = \int \sqrt{\cot{(x)}+1}\cot{(x)}\;dx$$

```
[In] integrate(cot(x)*(1+cot(x))^(1/2),x, algorithm="maxima")
[Out] integrate(sqrt(cot(x) + 1)*cot(x), x)
```

Giac [F]

$$\int \cot(x)\sqrt{1+\cot(x)}\,dx = \int \sqrt{\cot{(x)}+1}\cot{(x)}\;dx$$

```
[In] integrate(cot(x)*(1+cot(x))^(1/2),x, algorithm="giac")
[Out] integrate(sqrt(cot(x) + 1)*cot(x), x)
```

Mupad [B] (verification not implemented)

Time = 12.26 (sec) , antiderivative size = 210, normalized size of antiderivative = 1.56

$$\begin{aligned}\int \cot(x)\sqrt{1+\cot(x)}\,dx = {} & \operatorname{atanh}\left(\frac{\sqrt{\cot{(x)}+1}}{4\sqrt{\frac{1}{8}-\frac{\sqrt{2}}{8}}}+\frac{\sqrt{\cot{(x)}+1}}{4\sqrt{\frac{\sqrt{2}}{8}+\frac{1}{8}}}-\frac{\sqrt{2}\sqrt{\cot{(x)}+1}}{8\sqrt{\frac{1}{8}-\frac{\sqrt{2}}{8}}}\right.\\ & \left.+\frac{\sqrt{2}\sqrt{\cot{(x)}+1}}{8\sqrt{\frac{\sqrt{2}}{8}+\frac{1}{8}}}\right)\left(2\sqrt{\frac{1}{8}-\frac{\sqrt{2}}{8}}+2\sqrt{\frac{\sqrt{2}}{8}+\frac{1}{8}}\right)\\ & -\operatorname{atanh}\left(\frac{\sqrt{\cot{(x)}+1}}{4\sqrt{\frac{\sqrt{2}}{8}+\frac{1}{8}}}-\frac{\sqrt{\cot{(x)}+1}}{4\sqrt{\frac{1}{8}-\frac{\sqrt{2}}{8}}}+\frac{\sqrt{2}\sqrt{\cot{(x)}+1}}{8\sqrt{\frac{1}{8}-\frac{\sqrt{2}}{8}}}\right.\\ & \left.+\frac{\sqrt{2}\sqrt{\cot{(x)}+1}}{8\sqrt{\frac{\sqrt{2}}{8}+\frac{1}{8}}}\right)\left(2\sqrt{\frac{1}{8}-\frac{\sqrt{2}}{8}}-2\sqrt{\frac{\sqrt{2}}{8}+\frac{1}{8}}\right)\\ & -2\sqrt{\cot{(x)}+1}\end{aligned}$$

```
[In] int(cot(x)*(cot(x) + 1)^(1/2),x)
```

```
[Out] atanh((cot(x) + 1)^(1/2)/(4*(1/8 - 2^(1/2)/8)^(1/2)) + (cot(x) + 1)^(1/2)/(
4*(2^(1/2)/8 + 1/8)^(1/2)) - (2^(1/2)*(cot(x) + 1)^(1/2))/(8*(1/8 - 2^(1/2)
/8)^(1/2)) + (2^(1/2)*(cot(x) + 1)^(1/2))/(8*(2^(1/2)/8 + 1/8)^(1/2)))*(2*(
1/8 - 2^(1/2)/8)^(1/2) + 2*(2^(1/2)/8 + 1/8)^(1/2)) - atanh((cot(x) + 1)^(1
/2)/(4*(2^(1/2)/8 + 1/8)^(1/2)) - (cot(x) + 1)^(1/2)/(4*(1/8 - 2^(1/2)/8)^(
1/2)) + (2^(1/2)*(cot(x) + 1)^(1/2))/(8*(1/8 - 2^(1/2)/8)^(1/2)) + (2^(1/2)
*(cot(x) + 1)^(1/2))/(8*(2^(1/2)/8 + 1/8)^(1/2)))*(2*(1/8 - 2^(1/2)/8)^(1/2
) - 2*(2^(1/2)/8 + 1/8)^(1/2)) - 2*(cot(x) + 1)^(1/2)
```

3.43 $\int \cot^2(x)(1+\cot(x))^{3/2}\,dx$

Optimal result

Integrand size = 13, antiderivative size = 139

$$\int \cot^2(x)(1+\cot(x))^{3/2}\,dx = -\sqrt{-1+\sqrt{2}}\arctan\left(\frac{3-2\sqrt{2}+\left(1-\sqrt{2}\right)\cot(x)}{\sqrt{2\left(-7+5\sqrt{2}\right)}\sqrt{1+\cot(x)}}\right) - \sqrt{1+\sqrt{2}}\operatorname{arctanh}\left(\frac{3+2\sqrt{2}+\left(1+\sqrt{2}\right)\cot(x)}{\sqrt{2\left(7+5\sqrt{2}\right)}\sqrt{1+\cot(x)}}\right) + 2\sqrt{1+\cot(x)} - \frac{2}{5}(1+\cot(x))^{5/2}$$

```
[Out] -2/5*(1+cot(x))^(5/2)+2*(1+cot(x))^(1/2)-arctan((3+cot(x)*(1-2^(1/2))-2*2^(
1/2))/(1+cot(x))^(1/2)/(-14+10*2^(1/2))^(1/2))*(2^(1/2)-1)^(1/2)-arctanh((3
+2*2^(1/2)+cot(x)*(1+2^(1/2)))/(1+cot(x))^(1/2)/(14+10*2^(1/2))^(1/2))*(1+2
^(1/2))^(1/2)
```

Rubi [A] (verified)

Time = 0.31 (sec) , antiderivative size = 139, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.538$, Rules used = {3624, 3563, 12, 3617, 3616, 209, 213}

$$\int \cot^2(x)(1+\cot(x))^{3/2}\,dx = -\sqrt{\sqrt{2}-1}\arctan\left(\frac{\left(1-\sqrt{2}\right)\cot(x)-2\sqrt{2}+3}{\sqrt{2\left(5\sqrt{2}-7\right)}\sqrt{\cot(x)+1}}\right) - \sqrt{1+\sqrt{2}}\operatorname{arctanh}\left(\frac{\left(1+\sqrt{2}\right)\cot(x)+2\sqrt{2}+3}{\sqrt{2\left(7+5\sqrt{2}\right)}\sqrt{\cot(x)+1}}\right) - \frac{2}{5}(\cot(x)+1)^{5/2} + 2\sqrt{\cot(x)+1}$$

```
[In] Int[Cot[x]^2*(1 + Cot[x])^(3/2),x]
```

```
[Out] -(Sqrt[-1 + Sqrt[2]]*ArcTan[(3 - 2*Sqrt[2] + (1 - Sqrt[2])*Cot[x])/(Sqrt[2*
(-7 + 5*Sqrt[2])]*Sqrt[1 + Cot[x]])]) - Sqrt[1 + Sqrt[2]]*ArcTanh[(3 + 2*Sq
rt[2] + (1 + Sqrt[2])*Cot[x])/(Sqrt[2*(7 + 5*Sqrt[2])]*Sqrt[1 + Cot[x]])] +
 2*Sqrt[1 + Cot[x]] - (2*(1 + Cot[x])^(5/2))/5
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 213

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[b, 2])^(-
1))*ArcTanh[Rt[b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] &&
 (LtQ[a, 0] || GtQ[b, 0])
```

Rule 3563

```
Int[((a_) + (b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[b*((a +
b*Tan[c + d*x])^(n - 1)/(d*(n - 1))), x] + Int[(a^2 - b^2 + 2*a*b*Tan[c + d
*x])*(a + b*Tan[c + d*x])^(n - 2), x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2
+ b^2, 0] && GtQ[n, 1]
```

Rule 3616

```
Int[((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])/Sqrt[(a_) + (b_.)*tan[(e_.) + (
f_.)*(x_)]], x_Symbol] :> Dist[-2*(d^2/f), Subst[Int[1/(2*b*c*d - 4*a*d^2 +
 x^2), x], x, (b*c - 2*a*d - b*d*Tan[e + f*x])/Sqrt[a + b*Tan[e + f*x]]], x
] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0]
 && NeQ[c^2 + d^2, 0] && EqQ[2*a*c*d - b*(c^2 - d^2), 0]
```

Rule 3617

```
Int[((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])/Sqrt[(a_) + (b_.)*tan[(e_.) + (
f_.)*(x_)]], x_Symbol] :> With[{q = Rt[a^2 + b^2, 2]}, Dist[1/(2*q), Int[(a
*c + b*d + c*q + (b*c - a*d + d*q)*Tan[e + f*x])/Sqrt[a + b*Tan[e + f*x]],
x], x] - Dist[1/(2*q), Int[(a*c + b*d - c*q + (b*c - a*d - d*q)*Tan[e + f*x
])/Sqrt[a + b*Tan[e + f*x]], x], x]] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ
[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] && NeQ[2*a*c*d - b
*(c^2 - d^2), 0] && (PerfectSquareQ[a^2 + b^2] || RationalQ[a, b, c, d])
```

Rule 3624

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)])^2, x_Symbol] :> Simp[d^2*((a + b*Tan[e + f*x])^(m + 1)/(b*f*(
m + 1))), x] + Int[(a + b*Tan[e + f*x])^m*Simp[c^2 - d^2 + 2*c*d*Tan[e + f*
x], x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b*c - a*d, 0] &&  !LeQ
[m, -1] &&  !(EqQ[m, 2] && EqQ[a, 0])
```

Rubi steps

$$\text{integral} = -\frac{2}{5}(1+\cot(x))^{5/2} - \int (1+\cot(x))^{3/2}\,dx$$

$$= 2\sqrt{1+\cot(x)} - \frac{2}{5}(1+\cot(x))^{5/2} - \int \frac{2\cot(x)}{\sqrt{1+\cot(x)}}\,dx$$

$$= 2\sqrt{1+\cot(x)} - \frac{2}{5}(1+\cot(x))^{5/2} - 2\int \frac{\cot(x)}{\sqrt{1+\cot(x)}}\,dx$$

$$= 2\sqrt{1+\cot(x)} - \frac{2}{5}(1+\cot(x))^{5/2} - \frac{\int \frac{-1-\left(-1-\sqrt{2}\right)\cot(x)}{\sqrt{1+\cot(x)}}\,dx}{\sqrt{2}} + \frac{\int \frac{-1-\left(-1+\sqrt{2}\right)\cot(x)}{\sqrt{1+\cot(x)}}\,dx}{\sqrt{2}}$$

$$= 2\sqrt{1+\cot(x)} - \frac{2}{5}(1+\cot(x))^{5/2} - \Big(-4$$

$$+3\sqrt{2}\Big)\,\text{Subst}\left(\int \frac{1}{2\left(-1+\sqrt{2}\right)-4\left(-1+\sqrt{2}\right)^2+x^2}\,dx, x, \frac{1-2(-1+\sqrt{2})-(-1+\sqrt{2})\cot(}{\sqrt{1+\cot(x)}}\right.$$

$$+\Big(4$$

$$+3\sqrt{2}\Big)\,\text{Subst}\left(\int \frac{1}{2\left(-1-\sqrt{2}\right)-4\left(-1-\sqrt{2}\right)^2+x^2}\,dx, x, \frac{1-2(-1-\sqrt{2})-(-1-\sqrt{2})\cot(}{\sqrt{1+\cot(x)}}\right.$$

$$= -\sqrt{-1+\sqrt{2}}\arctan\left(\frac{3-2\sqrt{2}+\left(1-\sqrt{2}\right)\cot(x)}{\sqrt{2\left(-7+5\sqrt{2}\right)}\sqrt{1+\cot(x)}}\right)$$

$$-\sqrt{1+\sqrt{2}}\,\text{arctanh}\left(\frac{3+2\sqrt{2}+\left(1+\sqrt{2}\right)\cot(x)}{\sqrt{2\left(7+5\sqrt{2}\right)}\sqrt{1+\cot(x)}}\right)$$

$$+2\sqrt{1+\cot(x)} - \frac{2}{5}(1+\cot(x))^{5/2}$$

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.57 (sec) , antiderivative size = 75, normalized size of antiderivative = 0.54

$$\int \cot^2(x)(1+\cot(x))^{3/2}\,dx = -\frac{2\operatorname{arctanh}\left(\frac{\sqrt{1+\cot(x)}}{\sqrt{1-i}}\right)}{\sqrt{1-i}} - \frac{2\operatorname{arctanh}\left(\frac{\sqrt{1+\cot(x)}}{\sqrt{1+i}}\right)}{\sqrt{1+i}} + 2\sqrt{1+\cot(x)} - \frac{2}{5}(1+\cot(x))^{5/2}$$

```
[In] Integrate[Cot[x]^2*(1 + Cot[x])^(3/2),x]
```

```
[Out] (-2*ArcTanh[Sqrt[1 + Cot[x]]/Sqrt[1 - I]])/Sqrt[1 - I] - (2*ArcTanh[Sqrt[1
+ Cot[x]]/Sqrt[1 + I]])/Sqrt[1 + I] + 2*Sqrt[1 + Cot[x]] - (2*(1 + Cot[x])^
(5/2))/5
```

Maple [A] (verified)

Time = 0.04 (sec) , antiderivative size = 197, normalized size of antiderivative = 1.42

method	result
derivativedivides	$-\frac{2(1+\cot(x))^{\frac{5}{2}}}{5} + 2\sqrt{1+\cot(x)} - \frac{\sqrt{2}\left(-\frac{\sqrt{2+2\sqrt{2}}\,\ln\left(1+\cot(x)+\sqrt{2}-\sqrt{1+\cot(x)}\,\sqrt{2+2\sqrt{2}}\right)}{2} + \frac{2\left(1-\sqrt{2}\right)\arctan\left(\frac{2\sqrt{1}}{}\right.}{\sqrt{-2+}}\right.}{2}$
default	$-\frac{2(1+\cot(x))^{\frac{5}{2}}}{5} + 2\sqrt{1+\cot(x)} - \frac{\sqrt{2}\left(-\frac{\sqrt{2+2\sqrt{2}}\,\ln\left(1+\cot(x)+\sqrt{2}-\sqrt{1+\cot(x)}\,\sqrt{2+2\sqrt{2}}\right)}{2} + \frac{2\left(1-\sqrt{2}\right)\arctan\left(\frac{2\sqrt{1}}{}\right.}{\sqrt{-2+}}\right.}{2}$

```
[In] int(cot(x)^2*(1+cot(x))^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] -2/5*(1+cot(x))^(5/2)+2*(1+cot(x))^(1/2)-1/2*2^(1/2)*(-1/2*(2+2*2^(1/2))^(1
/2)*ln(1+cot(x)+2^(1/2)-(1+cot(x))^(1/2)*(2+2*2^(1/2))^(1/2))+2*(1-2^(1/2))
/(-2+2*2^(1/2))^(1/2)*arctan((2*(1+cot(x))^(1/2)-(2+2*2^(1/2))^(1/2))/(-2+2
*2^(1/2))^(1/2)))-1/2*2^(1/2)*(1/2*(2+2*2^(1/2))^(1/2)*ln(1+cot(x)+2^(1/2)+
(1+cot(x))^(1/2)*(2+2*2^(1/2))^(1/2))+2*(1-2^(1/2))/(-2+2*2^(1/2))^(1/2)*ar
ctan((2*(1+cot(x))^(1/2)+(2+2*2^(1/2))^(1/2))/(-2+2*2^(1/2))^(1/2)))
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.27 (sec) , antiderivative size = 202, normalized size of antiderivative = 1.45

$$\int \cot^2(x)(1+\cot(x))^{3/2}\,dx =$$

$$-\frac{5\sqrt{2i+2}(\cos(2x)-1)\log\left(-(i-1)\sqrt{2i+2}+2\sqrt{\frac{\cos(2x)+\sin(2x)+1}{\sin(2x)}}\right)-5\sqrt{2i+2}(\cos(2x)-1)\log\left((i-\right.}{}$$

```
[In] integrate(cot(x)^2*(1+cot(x))^(3/2),x, algorithm="fricas")
```

```
[Out] -1/10*(5*sqrt(2*I + 2)*(cos(2*x) - 1)*log(-(I - 1)*sqrt(2*I + 2) + 2*sqrt((
cos(2*x) + sin(2*x) + 1)/sin(2*x))) - 5*sqrt(2*I + 2)*(cos(2*x) - 1)*log((I
 - 1)*sqrt(2*I + 2) + 2*sqrt((cos(2*x) + sin(2*x) + 1)/sin(2*x))) + 5*sqrt(
-2*I + 2)*(cos(2*x) - 1)*log((I + 1)*sqrt(-2*I + 2) + 2*sqrt((cos(2*x) + si
n(2*x) + 1)/sin(2*x))) - 5*sqrt(-2*I + 2)*(cos(2*x) - 1)*log(-(I + 1)*sqrt(
-2*I + 2) + 2*sqrt((cos(2*x) + sin(2*x) + 1)/sin(2*x))) - 4*sqrt((cos(2*x)
+ sin(2*x) + 1)/sin(2*x))*(5*cos(2*x) + 2*sin(2*x) - 3))/(cos(2*x) - 1)
```

Sympy [F]

$$\int \cot^2(x)(1+\cot(x))^{3/2}\,dx = \int (\cot(x)+1)^{\frac{3}{2}}\cot^2(x)\,dx$$

```
[In] integrate(cot(x)**2*(1+cot(x))**(3/2),x)
```

```
[Out] Integral((cot(x) + 1)**(3/2)*cot(x)**2, x)
```

Maxima [F(-1)]

Timed out.

$$\int \cot^2(x)(1+\cot(x))^{3/2}\,dx = \text{Timed out}$$

```
[In] integrate(cot(x)^2*(1+cot(x))^(3/2),x, algorithm="maxima")
```

```
[Out] Timed out
```

Giac [F]

$$\int \cot^2(x)(1+\cot(x))^{3/2}\,dx = \int (\cot(x)+1)^{\frac{3}{2}}\cot(x)^2\,dx$$

```
[In] integrate(cot(x)^2*(1+cot(x))^(3/2),x, algorithm="giac")
[Out] integrate((cot(x) + 1)^(3/2)*cot(x)^2, x)
```

Mupad [B] (verification not implemented)

Time = 12.86 (sec) , antiderivative size = 254, normalized size of antiderivative = 1.83

$$\int \cot^2(x)(1+\cot(x))^{3/2}\,dx = \operatorname{atan}\left(\frac{\sqrt{2}\sqrt{\frac{1}{4}-\frac{\sqrt{2}}{4}}\sqrt{\cot(x)+1}\,64\mathrm{i}}{256\sqrt{\frac{1}{4}-\frac{\sqrt{2}}{4}}\sqrt{\frac{\sqrt{2}}{4}+\frac{1}{4}}-64} - \frac{\sqrt{2}\sqrt{\frac{\sqrt{2}}{4}+\frac{1}{4}}\sqrt{\cot(x)+1}\,64\mathrm{i}}{256\sqrt{\frac{1}{4}-\frac{\sqrt{2}}{4}}\sqrt{\frac{\sqrt{2}}{4}+\frac{1}{4}}-64}\right)\left(\sqrt{\frac{1}{4}-\frac{\sqrt{2}}{4}}\,2\mathrm{i}+\sqrt{\frac{\sqrt{2}}{4}+\frac{1}{4}}\,2\mathrm{i}\right) - \operatorname{atan}\left(\frac{\sqrt{2}\sqrt{\frac{1}{4}-\frac{\sqrt{2}}{4}}\sqrt{\cot(x)+1}\,64\mathrm{i}}{256\sqrt{\frac{1}{4}-\frac{\sqrt{2}}{4}}\sqrt{\frac{\sqrt{2}}{4}+\frac{1}{4}}+64} + \frac{\sqrt{2}\sqrt{\frac{\sqrt{2}}{4}+\frac{1}{4}}\sqrt{\cot(x)+1}\,64\mathrm{i}}{256\sqrt{\frac{1}{4}-\frac{\sqrt{2}}{4}}\sqrt{\frac{\sqrt{2}}{4}+\frac{1}{4}}+64}\right)\left(\sqrt{\frac{1}{4}-\frac{\sqrt{2}}{4}}\,2\mathrm{i}-\sqrt{\frac{\sqrt{2}}{4}+\frac{1}{4}}\,2\mathrm{i}\right) + 2\sqrt{\cot(x)+1} - \frac{2(\cot(x)+1)^{5/2}}{5}$$

```
[In] int(cot(x)^2*(cot(x) + 1)^(3/2),x)
[Out] atan((2^(1/2)*(1/4 - 2^(1/2)/4)^(1/2)*(cot(x) + 1)^(1/2)*64i)/(256*(1/4 - 2
^(1/2)/4)^(1/2)*(2^(1/2)/4 + 1/4)^(1/2) - 64) - (2^(1/2)*(2^(1/2)/4 + 1/4)^
(1/2)*(cot(x) + 1)^(1/2)*64i)/(256*(1/4 - 2^(1/2)/4)^(1/2)*(2^(1/2)/4 + 1/4
)^(1/2) - 64))*((1/4 - 2^(1/2)/4)^(1/2)*2i + (2^(1/2)/4 + 1/4)^(1/2)*2i) -
atan((2^(1/2)*(1/4 - 2^(1/2)/4)^(1/2)*(cot(x) + 1)^(1/2)*64i)/(256*(1/4 - 2
^(1/2)/4)^(1/2)*(2^(1/2)/4 + 1/4)^(1/2) + 64) + (2^(1/2)*(2^(1/2)/4 + 1/4)^
(1/2)*(cot(x) + 1)^(1/2)*64i)/(256*(1/4 - 2^(1/2)/4)^(1/2)*(2^(1/2)/4 + 1/4
)^(1/2) + 64))*((1/4 - 2^(1/2)/4)^(1/2)*2i - (2^(1/2)/4 + 1/4)^(1/2)*2i) +
2*(cot(x) + 1)^(1/2) - (2*(cot(x) + 1)^(5/2))/5
```

3.44 $\int \cot(x)(1+\cot(x))^{3/2}\,dx$

Optimal result

Integrand size = 11, antiderivative size = 221

$$\begin{aligned}\int \cot(x)(1+\cot(x))^{3/2}\,dx &= -\sqrt{1+\sqrt{2}}\arctan\left(\frac{\sqrt{2\left(1+\sqrt{2}\right)}-2\sqrt{1+\cot(x)}}{\sqrt{2\left(-1+\sqrt{2}\right)}}\right)\\ &+\sqrt{1+\sqrt{2}}\arctan\left(\frac{\sqrt{2\left(1+\sqrt{2}\right)}+2\sqrt{1+\cot(x)}}{\sqrt{2\left(-1+\sqrt{2}\right)}}\right)-2\sqrt{1+\cot(x)}\\ &-\frac{2}{3}(1+\cot(x))^{3/2}-\frac{\log\left(1+\sqrt{2}+\cot(x)-\sqrt{2\left(1+\sqrt{2}\right)}\sqrt{1+\cot(x)}\right)}{2\sqrt{1+\sqrt{2}}}\\ &+\frac{\log\left(1+\sqrt{2}+\cot(x)+\sqrt{2\left(1+\sqrt{2}\right)}\sqrt{1+\cot(x)}\right)}{2\sqrt{1+\sqrt{2}}}\end{aligned}$$

```
[Out] -2/3*(1+cot(x))^(3/2)-2*(1+cot(x))^(1/2)-1/2*ln(1+cot(x)+2^(1/2)-(1+cot(x))
^(1/2)*(2+2*2^(1/2))^(1/2))/(1+2^(1/2))^(1/2)+1/2*ln(1+cot(x)+2^(1/2)+(1+co
t(x))^(1/2)*(2+2*2^(1/2))^(1/2))/(1+2^(1/2))^(1/2)-arctan((-2*(1+cot(x))^(1
/2)+(2+2*2^(1/2))^(1/2))/(-2+2*2^(1/2))^(1/2))*(1+2^(1/2))^(1/2)+arctan((2*
(1+cot(x))^(1/2)+(2+2*2^(1/2))^(1/2))/(-2+2*2^(1/2))^(1/2))*(1+2^(1/2))^(1/
2)
```

Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 221, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.818, Rules used = {3609, 12, 3566, 722, 1108, 648, 632, 210, 642}

$$\int \cot(x)(1+\cot(x))^{3/2}\,dx = -\sqrt{1+\sqrt{2}}\arctan\left(\frac{\sqrt{2\left(1+\sqrt{2}\right)}-2\sqrt{\cot(x)+1}}{\sqrt{2\left(\sqrt{2}-1\right)}}\right) + \sqrt{1+\sqrt{2}}\arctan\left(\frac{2\sqrt{\cot(x)+1}+\sqrt{2\left(1+\sqrt{2}\right)}}{\sqrt{2\left(\sqrt{2}-1\right)}}\right) - \frac{2}{3}(\cot(x)+1)^{3/2} - 2\sqrt{\cot(x)+1} - \frac{\log\left(\cot(x)-\sqrt{2\left(1+\sqrt{2}\right)}\sqrt{\cot(x)+1}+\sqrt{2}+1\right)}{2\sqrt{1+\sqrt{2}}} + \frac{\log\left(\cot(x)+\sqrt{2\left(1+\sqrt{2}\right)}\sqrt{\cot(x)+1}+\sqrt{2}+1\right)}{2\sqrt{1+\sqrt{2}}}$$

```
[In] Int[Cot[x]*(1 + Cot[x])^(3/2),x]
```

```
[Out] -(Sqrt[1 + Sqrt[2]]*ArcTan[(Sqrt[2*(1 + Sqrt[2])] - 2*Sqrt[1 + Cot[x]])/Sqr
t[2*(-1 + Sqrt[2])]]) + Sqrt[1 + Sqrt[2]]*ArcTan[(Sqrt[2*(1 + Sqrt[2])] + 2
*Sqrt[1 + Cot[x]])/Sqrt[2*(-1 + Sqrt[2])]] - 2*Sqrt[1 + Cot[x]] - (2*(1 + C
ot[x])^(3/2))/3 - Log[1 + Sqrt[2] + Cot[x] - Sqrt[2*(1 + Sqrt[2])]*Sqrt[1 +
 Cot[x]]]/(2*Sqrt[1 + Sqrt[2]]) + Log[1 + Sqrt[2] + Cot[x] + Sqrt[2*(1 + Sq
rt[2])]*Sqrt[1 + Cot[x]]]/(2*Sqrt[1 + Sqrt[2]])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 722

```
Int[1/(Sqrt[(d_) + (e_.)*(x_)]*((a_) + (c_.)*(x_)^2)), x_Symbol] :> Dist[2*
e, Subst[Int[1/(c*d^2 + a*e^2 - 2*c*d*x^2 + c*x^4), x], x, Sqrt[d + e*x]],
x] /; FreeQ[{a, c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0]
```

Rule 1108

```
Int[((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(-1), x_Symbol] :> With[{q = Rt[a/
c, 2]}, With[{r = Rt[2*q - b/c, 2]}, Dist[1/(2*c*q*r), Int[(r - x)/(q - r*x
 + x^2), x], x] + Dist[1/(2*c*q*r), Int[(r + x)/(q + r*x + x^2), x], x]] /
; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && NegQ[b^2 - 4*a*c]
```

Rule 3566

```
Int[((a_) + (b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[b/d, Sub
st[Int[(a + x)^n/(b^2 + x^2), x], x, b*Tan[c + d*x]], x] /; FreeQ[{a, b, c,
 d, n}, x] && NeQ[a^2 + b^2, 0]
```

Rule 3609

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)]), x_Symbol] :> Simp[d*((a + b*Tan[e + f*x])^m/(f*m)), x] + Int
[(a + b*Tan[e + f*x])^(m - 1)*Simp[a*c - b*d + (b*c + a*d)*Tan[e + f*x], x]
, x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2,
 0] && GtQ[m, 0]
```

Rubi steps

$$\begin{aligned}\text{integral} &= -\frac{2}{3}(1+\cot(x))^{3/2} - \int (1-\cot(x))\sqrt{1+\cot(x)}\,dx\\ &= -2\sqrt{1+\cot(x)} - \frac{2}{3}(1+\cot(x))^{3/2} - \int \frac{2}{\sqrt{1+\cot(x)}}\,dx\end{aligned}$$

$$= -2\sqrt{1+\cot(x)} - \frac{2}{3}(1+\cot(x))^{3/2} - 2\int \frac{1}{\sqrt{1+\cot(x)}}\,dx$$

$$= -2\sqrt{1+\cot(x)} - \frac{2}{3}(1+\cot(x))^{3/2} + 2\text{Subst}\left(\int \frac{1}{\sqrt{1+x}\,(1+x^2)}\,dx, x, \cot(x)\right)$$

$$= -2\sqrt{1+\cot(x)} - \frac{2}{3}(1+\cot(x))^{3/2} + 4\text{Subst}\left(\int \frac{1}{2-2x^2+x^4}\,dx, x, \sqrt{1+\cot(x)}\right)$$

$$= -2\sqrt{1+\cot(x)} - \frac{2}{3}(1+\cot(x))^{3/2} + \frac{\text{Subst}\left(\int \frac{\sqrt{2\left(1+\sqrt{2}\right)}-x}{\sqrt{2}-\sqrt{2\left(1+\sqrt{2}\right)}x+x^2}\,dx, x, \sqrt{1+\cot(x)}\right)}{\sqrt{1+\sqrt{2}}} + \frac{\text{Subst}\left(\int \frac{\sqrt{2\left(1+\sqrt{2}\right)}+x}{\sqrt{2}+\sqrt{2\left(1+\sqrt{2}\right)}x+x^2}\,dx, x, \sqrt{1+\cot(x)}\right)}{\sqrt{1+\sqrt{2}}}$$

$$= -2\sqrt{1+\cot(x)} - \frac{2}{3}(1+\cot(x))^{3/2} + \frac{\text{Subst}\left(\int \frac{1}{\sqrt{2}-\sqrt{2\left(1+\sqrt{2}\right)}x+x^2}\,dx, x, \sqrt{1+\cot(x)}\right)}{\sqrt{2}} + \frac{\text{Subst}\left(\int \frac{1}{\sqrt{2}+\sqrt{2\left(1+\sqrt{2}\right)}x+x^2}\,dx, x, \sqrt{1+\cot(x)}\right)}{\sqrt{2}} - \frac{\text{Subst}\left(\int \frac{-\sqrt{2\left(1+\sqrt{2}\right)}+2x}{\sqrt{2}-\sqrt{2\left(1+\sqrt{2}\right)}x+x^2}\,dx, x, \sqrt{1+\cot(x)}\right)}{2\sqrt{1+\sqrt{2}}} + \frac{\text{Subst}\left(\int \frac{\sqrt{2\left(1+\sqrt{2}\right)}+2x}{\sqrt{2}+\sqrt{2\left(1+\sqrt{2}\right)}x+x^2}\,dx, x, \sqrt{1+\cot(x)}\right)}{2\sqrt{1+\sqrt{2}}}$$

$$= -2\sqrt{1+\cot(x)} - \frac{2}{3}(1+\cot(x))^{3/2} - \frac{\log\left(1+\sqrt{2}+\cot(x)-\sqrt{2\left(1+\sqrt{2}\right)}\sqrt{1+\cot(x)}\right)}{2\sqrt{1+\sqrt{2}}} + \frac{\log\left(1+\sqrt{2}+\cot(x)+\sqrt{2\left(1+\sqrt{2}\right)}\sqrt{1+\cot(x)}\right)}{2\sqrt{1+\sqrt{2}}} - \sqrt{2}\text{Subst}\left(\int \frac{1}{2\left(1-\sqrt{2}\right)-x^2}\,dx, x, -\sqrt{2\left(1+\sqrt{2}\right)}+2\sqrt{1+\cot(x)}\right) - \sqrt{2}\text{Subst}\left(\int \frac{1}{2\left(1-\sqrt{2}\right)-x^2}\,dx, x, \sqrt{2\left(1+\sqrt{2}\right)}+2\sqrt{1+\cot(x)}\right)$$

$$= -\frac{\arctan\left(\frac{\sqrt{2\left(1+\sqrt{2}\right)}-2\sqrt{1+\cot(x)}}{\sqrt{2\left(-1+\sqrt{2}\right)}}\right)}{\sqrt{-1+\sqrt{2}}} + \frac{\arctan\left(\frac{\sqrt{2\left(1+\sqrt{2}\right)}+2\sqrt{1+\cot(x)}}{\sqrt{2\left(-1+\sqrt{2}\right)}}\right)}{\sqrt{-1+\sqrt{2}}} - 2\sqrt{1+\cot(x)} - \frac{2}{3}(1+\cot(x))^{3/2} - \frac{\log\left(1+\sqrt{2}+\cot(x)-\sqrt{2\left(1+\sqrt{2}\right)}\sqrt{1+\cot(x)}\right)}{2\sqrt{1+\sqrt{2}}} + \frac{\log\left(1+\sqrt{2}+\cot(x)+\sqrt{2\left(1+\sqrt{2}\right)}\sqrt{1+\cot(x)}\right)}{2\sqrt{1+\sqrt{2}}}$$

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.44 (sec) , antiderivative size = 67, normalized size of antiderivative = 0.30

$$\int \cot(x)(1+\cot(x))^{3/2}\,dx = (1-i)^{3/2}\text{arctanh}\left(\frac{\sqrt{1+\cot(x)}}{\sqrt{1-i}}\right) + (1+i)^{3/2}\text{arctanh}\left(\frac{\sqrt{1+\cot(x)}}{\sqrt{1+i}}\right) - \frac{2}{3}\sqrt{1+\cot(x)}(4+\cot(x))$$

```
[In] Integrate[Cot[x]*(1 + Cot[x])^(3/2),x]
```

```
[Out] (1 - I)^(3/2)*ArcTanh[Sqrt[1 + Cot[x]]/Sqrt[1 - I]] + (1 + I)^(3/2)*ArcTanh
[Sqrt[1 + Cot[x]]/Sqrt[1 + I]] - (2*Sqrt[1 + Cot[x]]*(4 + Cot[x]))/3
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 451 vs. $2(159) = 318$.

Time = 0.04 (sec) , antiderivative size = 452, normalized size of antiderivative = 2.05

method	result
derivativedivides	$-\frac{2(1+\cot(x))^{\frac{3}{2}}}{3}-2\sqrt{1+\cot(x)}+\frac{\sqrt{2+2\sqrt{2}}\,\sqrt{2}\,\ln\left(1+\cot(x)+\sqrt{2}-\sqrt{1+\cot(x)}\,\sqrt{2+2\sqrt{2}}\right)}{4}-\frac{\sqrt{2+2\sqrt{2}}\,\ln\left(\right.}{}$
default	$-\frac{2(1+\cot(x))^{\frac{3}{2}}}{3}-2\sqrt{1+\cot(x)}+\frac{\sqrt{2+2\sqrt{2}}\,\sqrt{2}\,\ln\left(1+\cot(x)+\sqrt{2}-\sqrt{1+\cot(x)}\,\sqrt{2+2\sqrt{2}}\right)}{4}-\frac{\sqrt{2+2\sqrt{2}}\,\ln\left(\right.}{}$

```
[In] int(cot(x)*(1+cot(x))^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] -2/3*(1+cot(x))^(3/2)-2*(1+cot(x))^(1/2)+1/4*(2+2*2^(1/2))^(1/2)*2^(1/2)*ln
(1+cot(x)+2^(1/2)-(1+cot(x))^(1/2)*(2+2*2^(1/2))^(1/2))-1/2*(2+2*2^(1/2))^(
1/2)*ln(1+cot(x)+2^(1/2)-(1+cot(x))^(1/2)*(2+2*2^(1/2))^(1/2))+1/2*2^(1/2)*
(2+2*2^(1/2))/(-2+2*2^(1/2))^(1/2)*arctan((2*(1+cot(x))^(1/2)-(2+2*2^(1/2))
^(1/2))/(-2+2*2^(1/2))^(1/2))-(2+2*2^(1/2))/(-2+2*2^(1/2))^(1/2)*arctan((2*
(1+cot(x))^(1/2)-(2+2*2^(1/2))^(1/2))/(-2+2*2^(1/2))^(1/2))+2/(-2+2*2^(1/2)
)^(1/2)*arctan((2*(1+cot(x))^(1/2)-(2+2*2^(1/2))^(1/2))/(-2+2*2^(1/2))^(1/2
))*2^(1/2)-1/4*(2+2*2^(1/2))^(1/2)*2^(1/2)*ln(1+cot(x)+2^(1/2)+(1+cot(x))^(
1/2)*(2+2*2^(1/2))^(1/2))+1/2*(2+2*2^(1/2))^(1/2)*ln(1+cot(x)+2^(1/2)+(1+co
t(x))^(1/2)*(2+2*2^(1/2))^(1/2))+1/2*2^(1/2)*(2+2*2^(1/2))/(-2+2*2^(1/2))^(
1/2)*arctan((2*(1+cot(x))^(1/2)+(2+2*2^(1/2))^(1/2))/(-2+2*2^(1/2))^(1/2))-
(2+2*2^(1/2))/(-2+2*2^(1/2))^(1/2)*arctan((2*(1+cot(x))^(1/2)+(2+2*2^(1/2))
^(1/2))/(-2+2*2^(1/2))^(1/2))+2/(-2+2*2^(1/2))^(1/2)*arctan((2*(1+cot(x))^(
1/2)+(2+2*2^(1/2))^(1/2))/(-2+2*2^(1/2))^(1/2))*2^(1/2)
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.26 (sec) , antiderivative size = 190, normalized size of antiderivative = 0.86

$$\int \cot(x)(1+\cot(x))^{3/2}\,dx = \frac{3\sqrt{2i-2}\log\left(-(i-1)\,\sqrt{2i-2}+2\sqrt{\frac{\cos(2x)+\sin(2x)+1}{\sin(2x)}}\right)\sin(2x)-3\sqrt{2i-2}\log\left((i-1)\,\sqrt{\right.}}{}$$

```
[In] integrate(cot(x)*(1+cot(x))^(3/2),x, algorithm="fricas")
```

```
[Out] 1/6*(3*sqrt(2*I - 2)*log(-(I - 1)*sqrt(2*I - 2) + 2*sqrt((cos(2*x) + sin(2*
x) + 1)/sin(2*x)))*sin(2*x) - 3*sqrt(2*I - 2)*log((I - 1)*sqrt(2*I - 2) + 2
*sqrt((cos(2*x) + sin(2*x) + 1)/sin(2*x)))*sin(2*x) + 3*sqrt(-2*I - 2)*log(
```

```
(I + 1)*sqrt(-2*I - 2) + 2*sqrt((cos(2*x) + sin(2*x) + 1)/sin(2*x)))*sin(2*
x) - 3*sqrt(-2*I - 2)*log(-(I + 1)*sqrt(-2*I - 2) + 2*sqrt((cos(2*x) + sin(
2*x) + 1)/sin(2*x)))*sin(2*x) - 4*sqrt((cos(2*x) + sin(2*x) + 1)/sin(2*x))*
(cos(2*x) + 4*sin(2*x) + 1))/sin(2*x)
```

Sympy [F]

$$\int \cot(x)(1+\cot(x))^{3/2}\,dx = \int (\cot(x)+1)^{\frac{3}{2}}\cot(x)\,dx$$

```
[In] integrate(cot(x)*(1+cot(x))**(3/2),x)
[Out] Integral((cot(x) + 1)**(3/2)*cot(x), x)
```

Maxima [F]

$$\int \cot(x)(1+\cot(x))^{3/2}\,dx = \int (\cot(x)+1)^{\frac{3}{2}}\cot(x)\,dx$$

```
[In] integrate(cot(x)*(1+cot(x))^(3/2),x, algorithm="maxima")
[Out] integrate((cot(x) + 1)^(3/2)*cot(x), x)
```

Giac [F]

$$\int \cot(x)(1+\cot(x))^{3/2}\,dx = \int (\cot(x)+1)^{\frac{3}{2}}\cot(x)\,dx$$

```
[In] integrate(cot(x)*(1+cot(x))^(3/2),x, algorithm="giac")
[Out] integrate((cot(x) + 1)^(3/2)*cot(x), x)
```

Mupad [B] (verification not implemented)

Time = 12.59 (sec) , antiderivative size = 254, normalized size of antiderivative = 1.15

$$\int \cot(x)(1+\cot(x))^{3/2}\,dx = -\mathrm{atan}\left(\frac{\sqrt{2}\,\sqrt{-\frac{\sqrt{2}}{4}-\frac{1}{4}}\,\sqrt{\cot\left(x\right)+1}\,64\mathrm{i}}{256\,\sqrt{\frac{\sqrt{2}}{4}-\frac{1}{4}}\,\sqrt{-\frac{\sqrt{2}}{4}-\frac{1}{4}}-64} - \frac{\sqrt{2}\,\sqrt{\frac{\sqrt{2}}{4}-\frac{1}{4}}\,\sqrt{\cot\left(x\right)+1}\,64\mathrm{i}}{256\,\sqrt{\frac{\sqrt{2}}{4}-\frac{1}{4}}\,\sqrt{-\frac{\sqrt{2}}{4}-\frac{1}{4}}-64}\right)\left(\sqrt{-\frac{\sqrt{2}}{4}-\frac{1}{4}}\,2\mathrm{i}+\sqrt{\frac{\sqrt{2}}{4}-\frac{1}{4}}\,2\mathrm{i}\right) + \mathrm{atan}\left(\frac{\sqrt{2}\,\sqrt{-\frac{\sqrt{2}}{4}-\frac{1}{4}}\,\sqrt{\cot\left(x\right)+1}\,64\mathrm{i}}{256\,\sqrt{\frac{\sqrt{2}}{4}-\frac{1}{4}}\,\sqrt{-\frac{\sqrt{2}}{4}-\frac{1}{4}}+64} + \frac{\sqrt{2}\,\sqrt{\frac{\sqrt{2}}{4}-\frac{1}{4}}\,\sqrt{\cot\left(x\right)+1}\,64\mathrm{i}}{256\,\sqrt{\frac{\sqrt{2}}{4}-\frac{1}{4}}\,\sqrt{-\frac{\sqrt{2}}{4}-\frac{1}{4}}+64}\right)\left(\sqrt{-\frac{\sqrt{2}}{4}-\frac{1}{4}}\,2\mathrm{i}-\sqrt{\frac{\sqrt{2}}{4}-\frac{1}{4}}\,2\mathrm{i}\right) - 2\,\sqrt{\cot\left(x\right)+1} - \frac{2\,{\left(\cot(x)+1\right)}^{3/2}}{3}$$

```
[In] int(cot(x)*(cot(x) + 1)^(3/2),x)
```

```
[Out] atan((2^(1/2)*(- 2^(1/2)/4 - 1/4)^(1/2)*(cot(x) + 1)^(1/2)*64i)/(256*(2^(1/
2)/4 - 1/4)^(1/2)*(- 2^(1/2)/4 - 1/4)^(1/2) + 64) + (2^(1/2)*(2^(1/2)/4 - 1
/4)^(1/2)*(cot(x) + 1)^(1/2)*64i)/(256*(2^(1/2)/4 - 1/4)^(1/2)*(- 2^(1/2)/4
 - 1/4)^(1/2) + 64))*((- 2^(1/2)/4 - 1/4)^(1/2)*2i - (2^(1/2)/4 - 1/4)^(1/2
)*2i) - atan((2^(1/2)*(- 2^(1/2)/4 - 1/4)^(1/2)*(cot(x) + 1)^(1/2)*64i)/(25
6*(2^(1/2)/4 - 1/4)^(1/2)*(- 2^(1/2)/4 - 1/4)^(1/2) - 64) - (2^(1/2)*(2^(1/
2)/4 - 1/4)^(1/2)*(cot(x) + 1)^(1/2)*64i)/(256*(2^(1/2)/4 - 1/4)^(1/2)*(- 2
^(1/2)/4 - 1/4)^(1/2) - 64))*((- 2^(1/2)/4 - 1/4)^(1/2)*2i + (2^(1/2)/4 - 1
/4)^(1/2)*2i) - 2*(cot(x) + 1)^(1/2) - (2*(cot(x) + 1)^(3/2))/3
```

3.45 $\int \frac{\cot^2(x)}{\sqrt{1+\cot(x)}} dx$

Optimal result

Integrand size = 13, antiderivative size = 214

$$\begin{aligned}\int \frac{\cot^2(x)}{\sqrt{1+\cot(x)}}\,dx = & -\frac{1}{2}\sqrt{1+\sqrt{2}}\arctan\left(\frac{\sqrt{2\left(1+\sqrt{2}\right)}-2\sqrt{1+\cot(x)}}{\sqrt{2\left(-1+\sqrt{2}\right)}}\right)\\ & +\frac{1}{2}\sqrt{1+\sqrt{2}}\arctan\left(\frac{\sqrt{2\left(1+\sqrt{2}\right)}+2\sqrt{1+\cot(x)}}{\sqrt{2\left(-1+\sqrt{2}\right)}}\right)\\ & -2\sqrt{1+\cot(x)}\\ & -\frac{\log\left(1+\sqrt{2}+\cot(x)-\sqrt{2\left(1+\sqrt{2}\right)}\sqrt{1+\cot(x)}\right)}{4\sqrt{1+\sqrt{2}}}\\ & +\frac{\log\left(1+\sqrt{2}+\cot(x)+\sqrt{2\left(1+\sqrt{2}\right)}\sqrt{1+\cot(x)}\right)}{4\sqrt{1+\sqrt{2}}}\end{aligned}$$

```
[Out] -2*(1+cot(x))^(1/2)-1/4*ln(1+cot(x)+2^(1/2)-(1+cot(x))^(1/2)*(2+2*2^(1/2))^
(1/2))/(1+2^(1/2))^(1/2)+1/4*ln(1+cot(x)+2^(1/2)+(1+cot(x))^(1/2)*(2+2*2^(1
/2))^(1/2))/(1+2^(1/2))^(1/2)-1/2*arctan((-2*(1+cot(x))^(1/2)+(2+2*2^(1/2))
^(1/2))/(-2+2*2^(1/2))^(1/2))*(1+2^(1/2))^(1/2)+1/2*arctan((2*(1+cot(x))^(1
/2)+(2+2*2^(1/2))^(1/2))/(-2+2*2^(1/2))^(1/2))*(1+2^(1/2))^(1/2)
```

Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 214, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.615$, Rules used = {3624, 3566, 722, 1108, 648, 632, 210, 642}

$$\int \frac{\cot^2(x)}{\sqrt{1+\cot(x)}}\,dx = -\frac{1}{2}\sqrt{1+\sqrt{2}}\arctan\left(\frac{\sqrt{2\left(1+\sqrt{2}\right)}-2\sqrt{\cot(x)+1}}{\sqrt{2\left(\sqrt{2}-1\right)}}\right) + \frac{1}{2}\sqrt{1+\sqrt{2}}\arctan\left(\frac{2\sqrt{\cot(x)+1}+\sqrt{2\left(1+\sqrt{2}\right)}}{\sqrt{2\left(\sqrt{2}-1\right)}}\right) - 2\sqrt{\cot(x)+1} - \frac{\log\left(\cot(x)-\sqrt{2\left(1+\sqrt{2}\right)}\sqrt{\cot(x)+1}+\sqrt{2}+1\right)}{4\sqrt{1+\sqrt{2}}} + \frac{\log\left(\cot(x)+\sqrt{2\left(1+\sqrt{2}\right)}\sqrt{\cot(x)+1}+\sqrt{2}+1\right)}{4\sqrt{1+\sqrt{2}}}$$

```
[In] Int[Cot[x]^2/Sqrt[1 + Cot[x]],x]
```

```
[Out] -1/2*(Sqrt[1 + Sqrt[2]]*ArcTan[(Sqrt[2*(1 + Sqrt[2])] - 2*Sqrt[1 + Cot[x]])
/Sqrt[2*(-1 + Sqrt[2])]]) + (Sqrt[1 + Sqrt[2]]*ArcTan[(Sqrt[2*(1 + Sqrt[2])
] + 2*Sqrt[1 + Cot[x]])/Sqrt[2*(-1 + Sqrt[2])]])/2 - 2*Sqrt[1 + Cot[x]] - L
og[1 + Sqrt[2] + Cot[x] - Sqrt[2*(1 + Sqrt[2])]*Sqrt[1 + Cot[x]]]/(4*Sqrt[1
 + Sqrt[2]]) + Log[1 + Sqrt[2] + Cot[x] + Sqrt[2*(1 + Sqrt[2])]*Sqrt[1 + Co
t[x]]]/(4*Sqrt[1 + Sqrt[2]])
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
```

```
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 722

```
Int[1/(Sqrt[(d_) + (e_.)*(x_)]*((a_) + (c_.)*(x_)^2)), x_Symbol] :> Dist[2*
e, Subst[Int[1/(c*d^2 + a*e^2 - 2*c*d*x^2 + c*x^4), x], x, Sqrt[d + e*x]],
x] /; FreeQ[{a, c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0]
```

Rule 1108

```
Int[((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(-1), x_Symbol] :> With[{q = Rt[a/
c, 2]}, With[{r = Rt[2*q - b/c, 2]}, Dist[1/(2*c*q*r), Int[(r - x)/(q - r*x
 + x^2), x], x] + Dist[1/(2*c*q*r), Int[(r + x)/(q + r*x + x^2), x], x]]] /
; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && NegQ[b^2 - 4*a*c]
```

Rule 3566

```
Int[((a_) + (b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[b/d, Sub
st[Int[(a + x)^n/(b^2 + x^2), x], x, b*Tan[c + d*x]], x] /; FreeQ[{a, b, c,
 d, n}, x] && NeQ[a^2 + b^2, 0]
```

Rule 3624

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)])^2, x_Symbol] :> Simp[d^2*((a + b*Tan[e + f*x])^(m + 1)/(b*f*(
m + 1))), x] + Int[(a + b*Tan[e + f*x])^m*Simp[c^2 - d^2 + 2*c*d*Tan[e + f*
x], x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b*c - a*d, 0] &&  !LeQ
[m, -1] &&  !(EqQ[m, 2] && EqQ[a, 0])
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -2\sqrt{1+\cot(x)} - \int \frac{1}{\sqrt{1+\cot(x)}}\,dx \\
&= -2\sqrt{1+\cot(x)} + \text{Subst}\left(\int \frac{1}{\sqrt{1+x}\,(1+x^2)}\,dx, x, \cot(x)\right) \\
&= -2\sqrt{1+\cot(x)} + 2\text{Subst}\left(\int \frac{1}{2-2x^2+x^4}\,dx, x, \sqrt{1+\cot(x)}\right)
\end{aligned}$$

$$= -2\sqrt{1+\cot(x)} + \frac{\text{Subst}\left(\int \frac{\sqrt{2\left(1+\sqrt{2}\right)}-x}{\sqrt{2}-\sqrt{2\left(1+\sqrt{2}\right)}x+x^2}\,dx, x, \sqrt{1+\cot(x)}\right)}{2\sqrt{1+\sqrt{2}}} + \frac{\text{Subst}\left(\int \frac{\sqrt{2\left(1+\sqrt{2}\right)}+x}{\sqrt{2}+\sqrt{2\left(1+\sqrt{2}\right)}x+x^2}\,dx, x, \sqrt{1+\cot(x)}\right)}{2\sqrt{1+\sqrt{2}}}$$

$$= -2\sqrt{1+\cot(x)} + \frac{\text{Subst}\left(\int \frac{1}{\sqrt{2}-\sqrt{2\left(1+\sqrt{2}\right)}x+x^2}\,dx, x, \sqrt{1+\cot(x)}\right)}{2\sqrt{2}} + \frac{\text{Subst}\left(\int \frac{1}{\sqrt{2}+\sqrt{2\left(1+\sqrt{2}\right)}x+x^2}\,dx, x, \sqrt{1+\cot(x)}\right)}{2\sqrt{2}} - \frac{\text{Subst}\left(\int \frac{-\sqrt{2\left(1+\sqrt{2}\right)}+2x}{\sqrt{2}-\sqrt{2\left(1+\sqrt{2}\right)}x+x^2}\,dx, x, \sqrt{1+\cot(x)}\right)}{4\sqrt{1+\sqrt{2}}} + \frac{\text{Subst}\left(\int \frac{\sqrt{2\left(1+\sqrt{2}\right)}+2x}{\sqrt{2}+\sqrt{2\left(1+\sqrt{2}\right)}x+x^2}\,dx, x, \sqrt{1+\cot(x)}\right)}{4\sqrt{1+\sqrt{2}}}$$

$$= -2\sqrt{1+\cot(x)} - \frac{\log\left(1+\sqrt{2}+\cot(x)-\sqrt{2\left(1+\sqrt{2}\right)}\sqrt{1+\cot(x)}\right)}{4\sqrt{1+\sqrt{2}}} + \frac{\log\left(1+\sqrt{2}+\cot(x)+\sqrt{2\left(1+\sqrt{2}\right)}\sqrt{1+\cot(x)}\right)}{4\sqrt{1+\sqrt{2}}} - \frac{\text{Subst}\left(\int \frac{1}{2\left(1-\sqrt{2}\right)-x^2}\,dx, x, -\sqrt{2\left(1+\sqrt{2}\right)}+2\sqrt{1+\cot(x)}\right)}{\sqrt{2}} - \frac{\text{Subst}\left(\int \frac{1}{2\left(1-\sqrt{2}\right)-x^2}\,dx, x, \sqrt{2\left(1+\sqrt{2}\right)}+2\sqrt{1+\cot(x)}\right)}{\sqrt{2}}$$

$$= -\frac{\arctan\left(\frac{\sqrt{2\left(1+\sqrt{2}\right)}-2\sqrt{1+\cot(x)}}{\sqrt{2\left(-1+\sqrt{2}\right)}}\right)}{2\sqrt{-1+\sqrt{2}}} + \frac{\arctan\left(\frac{\sqrt{2\left(1+\sqrt{2}\right)}+2\sqrt{1+\cot(x)}}{\sqrt{2\left(-1+\sqrt{2}\right)}}\right)}{2\sqrt{-1+\sqrt{2}}} - 2\sqrt{1+\cot(x)} - \frac{\log\left(1+\sqrt{2}+\cot(x)-\sqrt{2\left(1+\sqrt{2}\right)}\sqrt{1+\cot(x)}\right)}{4\sqrt{1+\sqrt{2}}} + \frac{\log\left(1+\sqrt{2}+\cot(x)+\sqrt{2\left(1+\sqrt{2}\right)}\sqrt{1+\cot(x)}\right)}{4\sqrt{1+\sqrt{2}}}$$

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.47 (sec) , antiderivative size = 67, normalized size of antiderivative = 0.31

$$\int \frac{\cot^2(x)}{\sqrt{1+\cot(x)}}\,dx = \frac{1}{2}(1-i)^{3/2}\text{arctanh}\left(\frac{\sqrt{1+\cot(x)}}{\sqrt{1-i}}\right) + \frac{1}{2}(1+i)^{3/2}\text{arctanh}\left(\frac{\sqrt{1+\cot(x)}}{\sqrt{1+i}}\right) - 2\sqrt{1+\cot(x)}$$

```
[In] Integrate[Cot[x]^2/Sqrt[1 + Cot[x]],x]
[Out] ((1 - I)^(3/2)*ArcTanh[Sqrt[1 + Cot[x]]/Sqrt[1 - I]])/2 + ((1 + I)^(3/2)*Ar
cTanh[Sqrt[1 + Cot[x]]/Sqrt[1 + I]])/2 - 2*Sqrt[1 + Cot[x]]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 441 vs. 2(152) = 304.

Time = 0.04 (sec) , antiderivative size = 442, normalized size of antiderivative = 2.07

method	result
derivativedivides	$-2\sqrt{1+\cot(x)} - \frac{\sqrt{2+2\sqrt{2}}\,\ln\left(1+\cot(x)+\sqrt{2}-\sqrt{1+\cot(x)}\,\sqrt{2+2\sqrt{2}}\right)}{4} + \frac{\sqrt{2+2\sqrt{2}}\,\sqrt{2}\,\ln\left(1+\cot(x)+\sqrt{2}-\right.}{8}$
default	$-2\sqrt{1+\cot(x)} - \frac{\sqrt{2+2\sqrt{2}}\,\ln\left(1+\cot(x)+\sqrt{2}-\sqrt{1+\cot(x)}\,\sqrt{2+2\sqrt{2}}\right)}{4} + \frac{\sqrt{2+2\sqrt{2}}\,\sqrt{2}\,\ln\left(1+\cot(x)+\sqrt{2}-\right.}{8}$

```
[In] int(cot(x)^2/(1+cot(x))^(1/2),x,method=_RETURNVERBOSE)
[Out] -2*(1+cot(x))^(1/2)-1/4*(2+2*2^(1/2))^(1/2)*ln(1+cot(x)+2^(1/2)-(1+cot(x))^
(1/2)*(2+2*2^(1/2))^(1/2))+1/8*(2+2*2^(1/2))^(1/2)*2^(1/2)*ln(1+cot(x)+2^(1
```

```
/2)-(1+cot(x))^(1/2)*(2+2*2^(1/2))^(1/2))+1/4*2^(1/2)*(2+2*2^(1/2))/(-2+2*2
^(1/2))^(1/2)*arctan((2*(1+cot(x))^(1/2)-(2+2*2^(1/2))^(1/2))/(-2+2*2^(1/2)
)^(1/2))-1/2*(2+2*2^(1/2))/(-2+2*2^(1/2))^(1/2)*arctan((2*(1+cot(x))^(1/2)-
(2+2*2^(1/2))^(1/2))/(-2+2*2^(1/2))^(1/2))+1/(-2+2*2^(1/2))^(1/2)*arctan((2
*(1+cot(x))^(1/2)-(2+2*2^(1/2))^(1/2))/(-2+2*2^(1/2))^(1/2))*2^(1/2)+1/4*(2
+2*2^(1/2))^(1/2)*ln(1+cot(x)+2^(1/2)+(1+cot(x))^(1/2)*(2+2*2^(1/2))^(1/2))
-1/8*(2+2*2^(1/2))^(1/2)*2^(1/2)*ln(1+cot(x)+2^(1/2)+(1+cot(x))^(1/2)*(2+2*
2^(1/2))^(1/2))+1/4*2^(1/2)*(2+2*2^(1/2))/(-2+2*2^(1/2))^(1/2)*arctan((2*(1
+cot(x))^(1/2)+(2+2*2^(1/2))^(1/2))/(-2+2*2^(1/2))^(1/2))-1/2*(2+2*2^(1/2))
/(-2+2*2^(1/2))^(1/2)*arctan((2*(1+cot(x))^(1/2)+(2+2*2^(1/2))^(1/2))/(-2+2
*2^(1/2))^(1/2))+1/(-2+2*2^(1/2))^(1/2)*arctan((2*(1+cot(x))^(1/2)+(2+2*2^(
1/2))^(1/2))/(-2+2*2^(1/2))^(1/2))*2^(1/2)
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.29 (sec) , antiderivative size = 178, normalized size of antiderivative = 0.83

$$\int \frac{\cot^2(x)}{\sqrt{1+\cot(x)}}\,dx = \frac{1}{4}\sqrt{2}\sqrt{i-1}\log\left(-(i-1)\sqrt{2}\sqrt{i-1}+2\sqrt{\frac{\cos(2x)+\sin(2x)+1}{\sin(2x)}}\right) - \frac{1}{4}\sqrt{2}\sqrt{i-1}\log\left((i-1)\sqrt{2}\sqrt{i-1}+2\sqrt{\frac{\cos(2x)+\sin(2x)+1}{\sin(2x)}}\right) + \frac{1}{4}\sqrt{2}\sqrt{-i-1}\log\left((i+1)\sqrt{2}\sqrt{-i-1}+2\sqrt{\frac{\cos(2x)+\sin(2x)+1}{\sin(2x)}}\right) - \frac{1}{4}\sqrt{2}\sqrt{-i-1}\log\left(-(i+1)\sqrt{2}\sqrt{-i-1}+2\sqrt{\frac{\cos(2x)+\sin(2x)+1}{\sin(2x)}}\right) - 2\sqrt{\frac{\cos(2x)+\sin(2x)+1}{\sin(2x)}}$$

```
[In] integrate(cot(x)^2/(1+cot(x))^(1/2),x, algorithm="fricas")
```

```
[Out] 1/4*sqrt(2)*sqrt(I - 1)*log(-(I - 1)*sqrt(2)*sqrt(I - 1) + 2*sqrt((cos(2*x)
 + sin(2*x) + 1)/sin(2*x))) - 1/4*sqrt(2)*sqrt(I - 1)*log((I - 1)*sqrt(2)*s
qrt(I - 1) + 2*sqrt((cos(2*x) + sin(2*x) + 1)/sin(2*x))) + 1/4*sqrt(2)*sqrt
(-I - 1)*log((I + 1)*sqrt(2)*sqrt(-I - 1) + 2*sqrt((cos(2*x) + sin(2*x) + 1
)/sin(2*x))) - 1/4*sqrt(2)*sqrt(-I - 1)*log(-(I + 1)*sqrt(2)*sqrt(-I - 1) +
 2*sqrt((cos(2*x) + sin(2*x) + 1)/sin(2*x))) - 2*sqrt((cos(2*x) + sin(2*x)
+ 1)/sin(2*x))
```

Sympy [F]

$$\int \frac{\cot^2(x)}{\sqrt{1+\cot(x)}}\,dx = \int \frac{\cot^2(x)}{\sqrt{\cot(x)+1}}\,dx$$

```
[In] integrate(cot(x)**2/(1+cot(x))**(1/2),x)
[Out] Integral(cot(x)**2/sqrt(cot(x) + 1), x)
```

Maxima [F]

$$\int \frac{\cot^2(x)}{\sqrt{1+\cot(x)}}\,dx = \int \frac{\cot(x)^2}{\sqrt{\cot(x)+1}}\,dx$$

```
[In] integrate(cot(x)^2/(1+cot(x))^(1/2),x, algorithm="maxima")
[Out] integrate(cot(x)^2/sqrt(cot(x) + 1), x)
```

Giac [F]

$$\int \frac{\cot^2(x)}{\sqrt{1+\cot(x)}}\,dx = \int \frac{\cot(x)^2}{\sqrt{\cot(x)+1}}\,dx$$

```
[In] integrate(cot(x)^2/(1+cot(x))^(1/2),x, algorithm="giac")
[Out] integrate(cot(x)^2/sqrt(cot(x) + 1), x)
```

Mupad [B] (verification not implemented)

Time = 12.46 (sec) , antiderivative size = 238, normalized size of antiderivative = 1.11

$$\int \frac{\cot^2(x)}{\sqrt{1+\cot(x)}}\,dx = \operatorname{atanh}\left(\frac{16\sqrt{2}\sqrt{-\frac{\sqrt{2}}{16}-\frac{1}{16}}\sqrt{\cot(x)+1}}{128\sqrt{\frac{\sqrt{2}}{16}-\frac{1}{16}}\sqrt{-\frac{\sqrt{2}}{16}-\frac{1}{16}}-8} - \frac{16\sqrt{2}\sqrt{\frac{\sqrt{2}}{16}-\frac{1}{16}}\sqrt{\cot(x)+1}}{128\sqrt{\frac{\sqrt{2}}{16}-\frac{1}{16}}\sqrt{-\frac{\sqrt{2}}{16}-\frac{1}{16}}-8}\right)\left(2\sqrt{-\frac{\sqrt{2}}{16}-\frac{1}{16}} + 2\sqrt{\frac{\sqrt{2}}{16}-\frac{1}{16}}\right) - \operatorname{atanh}\left(\frac{16\sqrt{2}\sqrt{-\frac{\sqrt{2}}{16}-\frac{1}{16}}\sqrt{\cot(x)+1}}{128\sqrt{\frac{\sqrt{2}}{16}-\frac{1}{16}}\sqrt{-\frac{\sqrt{2}}{16}-\frac{1}{16}}+8} + \frac{16\sqrt{2}\sqrt{\frac{\sqrt{2}}{16}-\frac{1}{16}}\sqrt{\cot(x)+1}}{128\sqrt{\frac{\sqrt{2}}{16}-\frac{1}{16}}\sqrt{-\frac{\sqrt{2}}{16}-\frac{1}{16}}+8}\right)\left(2\sqrt{-\frac{\sqrt{2}}{16}-\frac{1}{16}} - 2\sqrt{\frac{\sqrt{2}}{16}-\frac{1}{16}}\right) - 2\sqrt{\cot(x)+1}$$

```
[In] int(cot(x)^2/(cot(x) + 1)^(1/2),x)
```

```
[Out] atanh((16*2^(1/2)*(- 2^(1/2)/16 - 1/16)^(1/2)*(cot(x) + 1)^(1/2))/(128*(2^(
1/2)/16 - 1/16)^(1/2)*(- 2^(1/2)/16 - 1/16)^(1/2) - 8) - (16*2^(1/2)*(2^(1/
2)/16 - 1/16)^(1/2)*(cot(x) + 1)^(1/2))/(128*(2^(1/2)/16 - 1/16)^(1/2)*(- 2
^(1/2)/16 - 1/16)^(1/2) - 8))*(2*(- 2^(1/2)/16 - 1/16)^(1/2) + 2*(2^(1/2)/1
6 - 1/16)^(1/2)) - atanh((16*2^(1/2)*(- 2^(1/2)/16 - 1/16)^(1/2)*(cot(x) +
1)^(1/2))/(128*(2^(1/2)/16 - 1/16)^(1/2)*(- 2^(1/2)/16 - 1/16)^(1/2) + 8) +
 (16*2^(1/2)*(2^(1/2)/16 - 1/16)^(1/2)*(cot(x) + 1)^(1/2))/(128*(2^(1/2)/16
 - 1/16)^(1/2)*(- 2^(1/2)/16 - 1/16)^(1/2) + 8))*(2*(- 2^(1/2)/16 - 1/16)^(
1/2) - 2*(2^(1/2)/16 - 1/16)^(1/2)) - 2*(cot(x) + 1)^(1/2)
```

3.46 $\int \frac{\cot(x)}{\sqrt{1+\cot(x)}}\,dx$

Optimal result

Integrand size = 11, antiderivative size = 121

$$\int \frac{\cot(x)}{\sqrt{1+\cot(x)}}\,dx = \frac{1}{2}\sqrt{-1+\sqrt{2}}\arctan\left(\frac{3-2\sqrt{2}+\left(1-\sqrt{2}\right)\cot(x)}{\sqrt{2\left(-7+5\sqrt{2}\right)}\sqrt{1+\cot(x)}}\right) + \frac{1}{2}\sqrt{1+\sqrt{2}}\text{arctanh}\left(\frac{3+2\sqrt{2}+\left(1+\sqrt{2}\right)\cot(x)}{\sqrt{2\left(7+5\sqrt{2}\right)}\sqrt{1+\cot(x)}}\right)$$

```
[Out] 1/2*arctan((3+cot(x)*(1-2^(1/2))-2*2^(1/2))/(1+cot(x))^(1/2)/(-14+10*2^(1/2
))^(1/2))*(2^(1/2)-1)^(1/2)+1/2*arctanh((3+2*2^(1/2)+cot(x)*(1+2^(1/2)))/(1
+cot(x))^(1/2)/(14+10*2^(1/2))^(1/2))*(1+2^(1/2))^(1/2)
```

Rubi [A] (verified)

Time = 0.15 (sec) , antiderivative size = 121, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.364, Rules used = {3617, 3616, 209, 213}

$$\int \frac{\cot(x)}{\sqrt{1+\cot(x)}}\,dx = \frac{1}{2}\sqrt{\sqrt{2}-1}\arctan\left(\frac{\left(1-\sqrt{2}\right)\cot(x)-2\sqrt{2}+3}{\sqrt{2\left(5\sqrt{2}-7\right)}\sqrt{\cot(x)+1}}\right) + \frac{1}{2}\sqrt{1+\sqrt{2}}\text{arctanh}\left(\frac{\left(1+\sqrt{2}\right)\cot(x)+2\sqrt{2}+3}{\sqrt{2\left(7+5\sqrt{2}\right)}\sqrt{\cot(x)+1}}\right)$$

```
[In] Int[Cot[x]/Sqrt[1 + Cot[x]],x]
```

```
[Out] (Sqrt[-1 + Sqrt[2]]*ArcTan[(3 - 2*Sqrt[2] + (1 - Sqrt[2])*Cot[x])/(Sqrt[2*(
-7 + 5*Sqrt[2])]*Sqrt[1 + Cot[x]])])/2 + (Sqrt[1 + Sqrt[2]]*ArcTanh[(3 + 2*
Sqrt[2] + (1 + Sqrt[2])*Cot[x])/(Sqrt[2*(7 + 5*Sqrt[2])]*Sqrt[1 + Cot[x]])]
)/2
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 213

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[b, 2])^(-
1))*ArcTanh[Rt[b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] &&
 (LtQ[a, 0] || GtQ[b, 0])
```

Rule 3616

```
Int[((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])/Sqrt[(a_) + (b_.)*tan[(e_.) + (
f_.)*(x_)]], x_Symbol] :> Dist[-2*(d^2/f), Subst[Int[1/(2*b*c*d - 4*a*d^2 +
 x^2), x], x, (b*c - 2*a*d - b*d*Tan[e + f*x])/Sqrt[a + b*Tan[e + f*x]]], x
] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0]
 && NeQ[c^2 + d^2, 0] && EqQ[2*a*c*d - b*(c^2 - d^2), 0]
```

Rule 3617

```
Int[((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])/Sqrt[(a_) + (b_.)*tan[(e_.) + (
f_.)*(x_)]], x_Symbol] :> With[{q = Rt[a^2 + b^2, 2]}, Dist[1/(2*q), Int[(a
*c + b*d + c*q + (b*c - a*d + d*q)*Tan[e + f*x])/Sqrt[a + b*Tan[e + f*x]],
x], x] - Dist[1/(2*q), Int[(a*c + b*d - c*q + (b*c - a*d - d*q)*Tan[e + f*x
])/Sqrt[a + b*Tan[e + f*x]], x], x]] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ
[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] && NeQ[2*a*c*d - b
*(c^2 - d^2), 0] && (PerfectSquareQ[a^2 + b^2] || RationalQ[a, b, c, d])
```

Rubi steps

$$\text{integral} = \frac{\int \frac{-1-\left(-1-\sqrt{2}\right)\cot(x)}{\sqrt{1+\cot(x)}}\,dx}{2\sqrt{2}} - \frac{\int \frac{-1-\left(-1+\sqrt{2}\right)\cot(x)}{\sqrt{1+\cot(x)}}\,dx}{2\sqrt{2}}$$

$$= \frac{1}{2}\Big(-4 +3\sqrt{2}\Big)\,\mathrm{Subst}\left(\int \frac{1}{2\left(-1+\sqrt{2}\right)-4\left(-1+\sqrt{2}\right)^2+x^2}\,dx, x, \frac{1-2\big(-1+\sqrt{2}\big)-\big(-1+\sqrt{2}\big)\cot(x}{\sqrt{1+\cot(x)}}\right.$$

$$-\frac{1}{2}\Big(4 +3\sqrt{2}\Big)\,\mathrm{Subst}\left(\int \frac{1}{2\left(-1-\sqrt{2}\right)-4\left(-1-\sqrt{2}\right)^2+x^2}\,dx, x, \frac{1-2\big(-1-\sqrt{2}\big)-\big(-1-\sqrt{2}\big)\cot(x}{\sqrt{1+\cot(x)}}\right.$$

$$= \frac{1}{2}\sqrt{-1+\sqrt{2}}\arctan\left(\frac{3-2\sqrt{2}+\left(1-\sqrt{2}\right)\cot(x)}{\sqrt{2\left(-7+5\sqrt{2}\right)}\sqrt{1+\cot(x)}}\right) + \frac{1}{2}\sqrt{1+\sqrt{2}}\,\mathrm{arctanh}\left(\frac{3+2\sqrt{2}+\left(1+\sqrt{2}\right)\cot(x)}{\sqrt{2\left(7+5\sqrt{2}\right)}\sqrt{1+\cot(x)}}\right)$$

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.26 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.42

$$\int \frac{\cot(x)}{\sqrt{1+\cot(x)}}\,dx = \frac{\mathrm{arctanh}\left(\frac{\sqrt{1+\cot(x)}}{\sqrt{1-i}}\right)}{\sqrt{1-i}} + \frac{\mathrm{arctanh}\left(\frac{\sqrt{1+\cot(x)}}{\sqrt{1+i}}\right)}{\sqrt{1+i}}$$

```
[In] Integrate[Cot[x]/Sqrt[1 + Cot[x]],x]
```

```
[Out] ArcTanh[Sqrt[1 + Cot[x]]/Sqrt[1 - I]]/Sqrt[1 - I] + ArcTanh[Sqrt[1 + Cot[x]
]/Sqrt[1 + I]]/Sqrt[1 + I]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 180 vs. 2(85) = 170.

Time = 0.09 (sec) , antiderivative size = 181, normalized size of antiderivative = 1.50

method	result
derivativedivides	$\frac{\sqrt{2}\left(-\frac{\sqrt{2+2\sqrt{2}}\,\ln\left(1+\cot(x)+\sqrt{2}-\sqrt{1+\cot(x)}\,\sqrt{2+2\sqrt{2}}\right)}{2}+\frac{2\left(1-\sqrt{2}\right)\arctan\left(\frac{2\sqrt{1+\cot(x)}-\sqrt{2+2\sqrt{2}}}{\sqrt{-2+2\sqrt{2}}}\right)}{\sqrt{-2+2\sqrt{2}}}\right)}{4}+\frac{\sqrt{2}\left(\frac{\sqrt{2+2\sqrt{2}}\,\ln\left(\right.\right.}{}\right.}{}$
default	$\frac{\sqrt{2}\left(-\frac{\sqrt{2+2\sqrt{2}}\,\ln\left(1+\cot(x)+\sqrt{2}-\sqrt{1+\cot(x)}\,\sqrt{2+2\sqrt{2}}\right)}{2}+\frac{2\left(1-\sqrt{2}\right)\arctan\left(\frac{2\sqrt{1+\cot(x)}-\sqrt{2+2\sqrt{2}}}{\sqrt{-2+2\sqrt{2}}}\right)}{\sqrt{-2+2\sqrt{2}}}\right)}{4}+\frac{\sqrt{2}\left(\frac{\sqrt{2+2\sqrt{2}}\,\ln\left(\right.\right.}{}\right.}{}$

```
[In] int(cot(x)/(1+cot(x))^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/4*2^(1/2)*(-1/2*(2+2*2^(1/2))^(1/2)*ln(1+cot(x)+2^(1/2)-(1+cot(x))^(1/2)*
(2+2*2^(1/2))^(1/2))+2*(1-2^(1/2))/(-2+2*2^(1/2))^(1/2)*arctan((2*(1+cot(x)
)^(1/2)-(2+2*2^(1/2))^(1/2))/(-2+2*2^(1/2))^(1/2)))+1/4*2^(1/2)*(1/2*(2+2*2
^(1/2))^(1/2)*ln(1+cot(x)+2^(1/2)+(1+cot(x))^(1/2)*(2+2*2^(1/2))^(1/2))+2*(
1-2^(1/2))/(-2+2*2^(1/2))^(1/2)*arctan((2*(1+cot(x))^(1/2)+(2+2*2^(1/2))^(1
/2))/(-2+2*2^(1/2))^(1/2)))
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.25 (sec) , antiderivative size = 157, normalized size of antiderivative = 1.30

$$\int \frac{\cot(x)}{\sqrt{1+\cot(x)}}\,dx = \frac{1}{4}\sqrt{2}\sqrt{i+1}\log\left(-(i-1)\sqrt{2}\sqrt{i+1}+2\sqrt{\frac{\cos(2x)+\sin(2x)+1}{\sin(2x)}}\right) - \frac{1}{4}\sqrt{2}\sqrt{i+1}\log\left((i-1)\sqrt{2}\sqrt{i+1}+2\sqrt{\frac{\cos(2x)+\sin(2x)+1}{\sin(2x)}}\right) + \frac{1}{4}\sqrt{2}\sqrt{-i+1}\log\left((i+1)\sqrt{2}\sqrt{-i+1}+2\sqrt{\frac{\cos(2x)+\sin(2x)+1}{\sin(2x)}}\right) - \frac{1}{4}\sqrt{2}\sqrt{-i+1}\log\left(-(i+1)\sqrt{2}\sqrt{-i+1}+2\sqrt{\frac{\cos(2x)+\sin(2x)+1}{\sin(2x)}}\right)$$

```
[In] integrate(cot(x)/(1+cot(x))^(1/2),x, algorithm="fricas")
```

```
[Out] 1/4*sqrt(2)*sqrt(I + 1)*log(-(I - 1)*sqrt(2)*sqrt(I + 1) + 2*sqrt((cos(2*x)
 + sin(2*x) + 1)/sin(2*x))) - 1/4*sqrt(2)*sqrt(I + 1)*log((I - 1)*sqrt(2)*s
qrt(I + 1) + 2*sqrt((cos(2*x) + sin(2*x) + 1)/sin(2*x))) + 1/4*sqrt(2)*sqrt
(-I + 1)*log((I + 1)*sqrt(2)*sqrt(-I + 1) + 2*sqrt((cos(2*x) + sin(2*x) + 1
)/sin(2*x))) - 1/4*sqrt(2)*sqrt(-I + 1)*log(-(I + 1)*sqrt(2)*sqrt(-I + 1) +
 2*sqrt((cos(2*x) + sin(2*x) + 1)/sin(2*x)))
```

Sympy [F]

$$\int \frac{\cot(x)}{\sqrt{1+\cot(x)}}\,dx = \int \frac{\cot(x)}{\sqrt{\cot(x)+1}}\,dx$$

```
[In] integrate(cot(x)/(1+cot(x))**(1/2),x)
[Out] Integral(cot(x)/sqrt(cot(x) + 1), x)
```

Maxima [F]

$$\int \frac{\cot(x)}{\sqrt{1+\cot(x)}}\,dx = \int \frac{\cot(x)}{\sqrt{\cot(x)+1}}\,dx$$

```
[In] integrate(cot(x)/(1+cot(x))^(1/2),x, algorithm="maxima")
[Out] integrate(cot(x)/sqrt(cot(x) + 1), x)
```

Giac [F]

$$\int \frac{\cot(x)}{\sqrt{1+\cot(x)}}\,dx = \int \frac{\cot(x)}{\sqrt{\cot(x)+1}}\,dx$$

```
[In] integrate(cot(x)/(1+cot(x))^(1/2),x, algorithm="giac")
[Out] integrate(cot(x)/sqrt(cot(x) + 1), x)
```

Mupad [B] (verification not implemented)

Time = 12.87 (sec) , antiderivative size = 230, normalized size of antiderivative = 1.90

$$\int \frac{\cot(x)}{\sqrt{1+\cot(x)}}\,dx = \operatorname{atanh}\left(\frac{16\sqrt{2}\sqrt{\frac{1}{16}-\frac{\sqrt{2}}{16}}\sqrt{\cot(x)+1}}{128\sqrt{\frac{1}{16}-\frac{\sqrt{2}}{16}}\sqrt{\frac{\sqrt{2}}{16}+\frac{1}{16}}-8} - \frac{16\sqrt{2}\sqrt{\frac{\sqrt{2}}{16}+\frac{1}{16}}\sqrt{\cot(x)+1}}{128\sqrt{\frac{1}{16}-\frac{\sqrt{2}}{16}}\sqrt{\frac{\sqrt{2}}{16}+\frac{1}{16}}-8}\right)\left(2\sqrt{\frac{1}{16}-\frac{\sqrt{2}}{16}}+2\sqrt{\frac{\sqrt{2}}{16}+\frac{1}{16}}\right) - \operatorname{atanh}\left(\frac{16\sqrt{2}\sqrt{\frac{1}{16}-\frac{\sqrt{2}}{16}}\sqrt{\cot(x)+1}}{128\sqrt{\frac{1}{16}-\frac{\sqrt{2}}{16}}\sqrt{\frac{\sqrt{2}}{16}+\frac{1}{16}}+8} + \frac{16\sqrt{2}\sqrt{\frac{\sqrt{2}}{16}+\frac{1}{16}}\sqrt{\cot(x)+1}}{128\sqrt{\frac{1}{16}-\frac{\sqrt{2}}{16}}\sqrt{\frac{\sqrt{2}}{16}+\frac{1}{16}}+8}\right)\left(2\sqrt{\frac{1}{16}-\frac{\sqrt{2}}{16}}-2\sqrt{\frac{\sqrt{2}}{16}+\frac{1}{16}}\right)$$

```
[In] int(cot(x)/(cot(x) + 1)^(1/2),x)
```

```
[Out] atanh((16*2^(1/2)*(1/16 - 2^(1/2)/16)^(1/2)*(cot(x) + 1)^(1/2))/(128*(1/16
- 2^(1/2)/16)^(1/2)*(2^(1/2)/16 + 1/16)^(1/2) - 8) - (16*2^(1/2)*(2^(1/2)/1
6 + 1/16)^(1/2)*(cot(x) + 1)^(1/2))/(128*(1/16 - 2^(1/2)/16)^(1/2)*(2^(1/2)
/16 + 1/16)^(1/2) - 8))*(2*(1/16 - 2^(1/2)/16)^(1/2) + 2*(2^(1/2)/16 + 1/16
)^(1/2)) - atanh((16*2^(1/2)*(1/16 - 2^(1/2)/16)^(1/2)*(cot(x) + 1)^(1/2))/
(128*(1/16 - 2^(1/2)/16)^(1/2)*(2^(1/2)/16 + 1/16)^(1/2) + 8) + (16*2^(1/2)
*(2^(1/2)/16 + 1/16)^(1/2)*(cot(x) + 1)^(1/2))/(128*(1/16 - 2^(1/2)/16)^(1/
2)*(2^(1/2)/16 + 1/16)^(1/2) + 8))*(2*(1/16 - 2^(1/2)/16)^(1/2) - 2*(2^(1/2
)/16 + 1/16)^(1/2))
```

3.47 $\int \frac{\cot^2(x)}{(1+\cot(x))^{3/2}}\,dx$

Optimal result

Integrand size = 13, antiderivative size = 139

$$\int \frac{\cot^2(x)}{(1+\cot(x))^{3/2}}\,dx = \frac{1}{2}\sqrt{\frac{1}{2}\left(-1+\sqrt{2}\right)}\arctan\left(\frac{4-3\sqrt{2}+\left(2-\sqrt{2}\right)\cot(x)}{2\sqrt{-7+5\sqrt{2}}\sqrt{1+\cot(x)}}\right) + \frac{1}{2}\sqrt{\frac{1}{2}\left(1+\sqrt{2}\right)}\operatorname{arctanh}\left(\frac{4+3\sqrt{2}+\left(2+\sqrt{2}\right)\cot(x)}{2\sqrt{7+5\sqrt{2}}\sqrt{1+\cot(x)}}\right) + \frac{1}{\sqrt{1+\cot(x)}}$$

```
[Out] 1/(1+cot(x))^(1/2)+1/4*arctan(1/2*(4+cot(x)*(2-2^(1/2))-3*2^(1/2))/(1+cot(x
))^(1/2)/(-7+5*2^(1/2))^(1/2))*(-2+2*2^(1/2))^(1/2)+1/4*arctanh(1/2*(4+3*2^
(1/2)+cot(x)*(2+2^(1/2)))/(1+cot(x))^(1/2)/(7+5*2^(1/2))^(1/2))*(2+2*2^(1/2
))^(1/2)
```

Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 139, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.385, Rules used = {3623, 3617, 3616, 209, 213}

$$\int \frac{\cot^2(x)}{(1+\cot(x))^{3/2}}\,dx = \frac{1}{2}\sqrt{\frac{1}{2}\left(\sqrt{2}-1\right)}\arctan\left(\frac{\left(2-\sqrt{2}\right)\cot(x)-3\sqrt{2}+4}{2\sqrt{5\sqrt{2}-7}\sqrt{\cot(x)+1}}\right) + \frac{1}{2}\sqrt{\frac{1}{2}\left(1+\sqrt{2}\right)}\operatorname{arctanh}\left(\frac{\left(2+\sqrt{2}\right)\cot(x)+3\sqrt{2}+4}{2\sqrt{7+5\sqrt{2}}\sqrt{\cot(x)+1}}\right) + \frac{1}{\sqrt{\cot(x)+1}}$$

```
[In] Int[Cot[x]^2/(1 + Cot[x])^(3/2),x]
```

```
[Out] (Sqrt[(-1 + Sqrt[2])/2]*ArcTan[(4 - 3*Sqrt[2] + (2 - Sqrt[2])*Cot[x])/(2*Sq
rt[-7 + 5*Sqrt[2]]*Sqrt[1 + Cot[x]])])/2 + (Sqrt[(1 + Sqrt[2])/2]*ArcTanh[(
```

```
4 + 3*Sqrt[2] + (2 + Sqrt[2])*Cot[x])/(2*Sqrt[7 + 5*Sqrt[2]]*Sqrt[1 + Cot[x
]])])/2 + 1/Sqrt[1 + Cot[x]]
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 213

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[b, 2])^(-
1))*ArcTanh[Rt[b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] &&
 (LtQ[a, 0] || GtQ[b, 0])
```

Rule 3616

```
Int[((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])/Sqrt[(a_) + (b_.)*tan[(e_.) + (
f_.)*(x_)]], x_Symbol] :> Dist[-2*(d^2/f), Subst[Int[1/(2*b*c*d - 4*a*d^2 +
 x^2), x], x, (b*c - 2*a*d - b*d*Tan[e + f*x])/Sqrt[a + b*Tan[e + f*x]]], x
] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0]
 && NeQ[c^2 + d^2, 0] && EqQ[2*a*c*d - b*(c^2 - d^2), 0]
```

Rule 3617

```
Int[((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])/Sqrt[(a_) + (b_.)*tan[(e_.) + (
f_.)*(x_)]], x_Symbol] :> With[{q = Rt[a^2 + b^2, 2]}, Dist[1/(2*q), Int[(a
*c + b*d + c*q + (b*c - a*d + d*q)*Tan[e + f*x])/Sqrt[a + b*Tan[e + f*x]],
x], x] - Dist[1/(2*q), Int[(a*c + b*d - c*q + (b*c - a*d - d*q)*Tan[e + f*x
])/Sqrt[a + b*Tan[e + f*x]], x], x]] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ
[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] && NeQ[2*a*c*d - b
*(c^2 - d^2), 0] && (PerfectSquareQ[a^2 + b^2] || RationalQ[a, b, c, d])
```

Rule 3623

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)])^2, x_Symbol] :> Simp[(b*c - a*d)^2*((a + b*Tan[e + f*x])^(m +
 1)/(b*f*(m + 1)*(a^2 + b^2))), x] + Dist[1/(a^2 + b^2), Int[(a + b*Tan[e +
 f*x])^(m + 1)*Simp[a*c^2 + 2*b*c*d - a*d^2 - (b*c^2 - 2*a*c*d - b*d^2)*Tan
[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
&& LtQ[m, -1] && NeQ[a^2 + b^2, 0]
```

Rubi steps

$$\text{integral} = \frac{1}{\sqrt{1+\cot(x)}} + \frac{1}{2}\int \frac{-1+\cot(x)}{\sqrt{1+\cot(x)}}\,dx$$

$$= \frac{1}{\sqrt{1+\cot(x)}} + \frac{\int \frac{-\sqrt{2}-\left(2-\sqrt{2}\right)\cot(x)}{\sqrt{1+\cot(x)}}\,dx}{4\sqrt{2}} - \frac{\int \frac{\sqrt{2}-\left(2+\sqrt{2}\right)\cot(x)}{\sqrt{1+\cot(x)}}\,dx}{4\sqrt{2}}$$

$$= \frac{1}{\sqrt{1+\cot(x)}} - \frac{1}{2}\Bigg(-4 +3\sqrt{2}\Bigg)\text{Subst}\left(\int \frac{1}{2\sqrt{2}\left(2-\sqrt{2}\right)-4\left(2-\sqrt{2}\right)^2+x^2}\,dx, x, \frac{\sqrt{2}-2(2-\sqrt{2})-(2-\sqrt{2})\cot(x)}{\sqrt{1+\cot(x)}}\right.$$

$$+\frac{1}{2}\Bigg(4 +3\sqrt{2}\Bigg)\text{Subst}\left(\int \frac{1}{-2\sqrt{2}\left(2+\sqrt{2}\right)-4\left(2+\sqrt{2}\right)^2+x^2}\,dx, x, \frac{-\sqrt{2}-2(2+\sqrt{2})-(2+\sqrt{2})\cot}{\sqrt{1+\cot(x)}}\right.$$

$$= \frac{1}{2}\sqrt{\frac{1}{2}\left(-1+\sqrt{2}\right)}\arctan\left(\frac{4-3\sqrt{2}+\left(2-\sqrt{2}\right)\cot(x)}{2\sqrt{-7+5\sqrt{2}}\sqrt{1+\cot(x)}}\right) + \frac{1}{2}\sqrt{\frac{1}{2}\left(1+\sqrt{2}\right)}\text{arctanh}\left(\frac{4+3\sqrt{2}+\left(2+\sqrt{2}\right)\cot(x)}{2\sqrt{7+5\sqrt{2}}\sqrt{1+\cot(x)}}\right) + \frac{1}{\sqrt{1+\cot(x)}}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 0.45 (sec) , antiderivative size = 62, normalized size of antiderivative = 0.45

$$\int \frac{\cot^2(x)}{(1+\cot(x))^{3/2}}\,dx = \frac{4-(1+i)\,\text{Hypergeometric2F1}\left(-\frac{1}{2},1,\frac{1}{2},\left(\frac{1}{2}-\frac{i}{2}\right)(1+\cot(x))\right)-(1-i)\,\text{Hypergeo}}{2\sqrt{1+\cot(x)}}$$

```
[In] Integrate[Cot[x]^2/(1 + Cot[x])^(3/2),x]
[Out] (4 - (1 + I)*Hypergeometric2F1[-1/2, 1, 1/2, (1/2 - I/2)*(1 + Cot[x])] - (1
 - I)*Hypergeometric2F1[-1/2, 1, 1/2, (1/2 + I/2)*(1 + Cot[x])])/(2*Sqrt[1
+ Cot[x]])
```

Maple [A] (verified)

Time = 0.04 (sec) , antiderivative size = 173, normalized size of antiderivative = 1.24

method	result
derivativedivides	$\frac{1}{\sqrt{1+\cot(x)}} - \frac{\sqrt{2+2\sqrt{2}}\,\ln\left(1+\cot(x)+\sqrt{2}-\sqrt{1+\cot(x)}\,\sqrt{2+2\sqrt{2}}\right)}{8} + \frac{\left(\sqrt{2}-1\right)\arctan\left(\frac{2\sqrt{1+\cot(x)}-\sqrt{2+2\sqrt{2}}}{\sqrt{-2+2\sqrt{2}}}\right)}{2\sqrt{-2+2\sqrt{2}}} +$
default	$\frac{1}{\sqrt{1+\cot(x)}} - \frac{\sqrt{2+2\sqrt{2}}\,\ln\left(1+\cot(x)+\sqrt{2}-\sqrt{1+\cot(x)}\,\sqrt{2+2\sqrt{2}}\right)}{8} + \frac{\left(\sqrt{2}-1\right)\arctan\left(\frac{2\sqrt{1+\cot(x)}-\sqrt{2+2\sqrt{2}}}{\sqrt{-2+2\sqrt{2}}}\right)}{2\sqrt{-2+2\sqrt{2}}} +$

```
[In] int(cot(x)^2/(1+cot(x))^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/(1+cot(x))^(1/2)-1/8*(2+2*2^(1/2))^(1/2)*ln(1+cot(x)+2^(1/2)-(1+cot(x))^(
1/2)*(2+2*2^(1/2))^(1/2))+1/2*(2^(1/2)-1)/(-2+2*2^(1/2))^(1/2)*arctan((2*(1
+cot(x))^(1/2)-(2+2*2^(1/2))^(1/2))/(-2+2*2^(1/2))^(1/2))+1/8*(2+2*2^(1/2))
^(1/2)*ln(1+cot(x)+2^(1/2)+(1+cot(x))^(1/2)*(2+2*2^(1/2))^(1/2))-1/2*(1-2^(
1/2))/(-2+2*2^(1/2))^(1/2)*arctan((2*(1+cot(x))^(1/2)+(2+2*2^(1/2))^(1/2))/
(-2+2*2^(1/2))^(1/2))
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.26 (sec) , antiderivative size = 198, normalized size of antiderivative = 1.42

$$\int \frac{\cot^2(x)}{(1+\cot(x))^{3/2}}\,dx = \frac{\sqrt{i+1}(\cos(2x)+\sin(2x)+1)\log\left(\sqrt{i+1}+\sqrt{\frac{\cos(2x)+\sin(2x)+1}{\sin(2x)}}\right)-\sqrt{i+1}(\cos(2x)}{}$$

```
[In] integrate(cot(x)^2/(1+cot(x))^(3/2),x, algorithm="fricas")
```

```
[Out] 1/4*(sqrt(I + 1)*(cos(2*x) + sin(2*x) + 1)*log(sqrt(I + 1) + sqrt((cos(2*x)
 + sin(2*x) + 1)/sin(2*x))) - sqrt(I + 1)*(cos(2*x) + sin(2*x) + 1)*log(-sq
rt(I + 1) + sqrt((cos(2*x) + sin(2*x) + 1)/sin(2*x))) + sqrt(-I + 1)*(cos(2
*x) + sin(2*x) + 1)*log(sqrt(-I + 1) + sqrt((cos(2*x) + sin(2*x) + 1)/sin(2
*x))) - sqrt(-I + 1)*(cos(2*x) + sin(2*x) + 1)*log(-sqrt(-I + 1) + sqrt((co
s(2*x) + sin(2*x) + 1)/sin(2*x))) + 4*sqrt((cos(2*x) + sin(2*x) + 1)/sin(2*
x))*sin(2*x))/(cos(2*x) + sin(2*x) + 1)
```

Sympy [F]

$$\int \frac{\cot^2(x)}{(1+\cot(x))^{3/2}}\,dx = \int \frac{\cot^2(x)}{(\cot(x)+1)^{\frac{3}{2}}}\,dx$$

```
[In] integrate(cot(x)**2/(1+cot(x))**(3/2),x)
```

```
[Out] Integral(cot(x)**2/(cot(x) + 1)**(3/2), x)
```

Maxima [F]

$$\int \frac{\cot^2(x)}{(1+\cot(x))^{3/2}}\,dx = \int \frac{\cot(x)^2}{(\cot(x)+1)^{\frac{3}{2}}}\,dx$$

```
[In] integrate(cot(x)^2/(1+cot(x))^(3/2),x, algorithm="maxima")
[Out] integrate(cot(x)^2/(cot(x) + 1)^(3/2), x)
```

Giac [F]

$$\int \frac{\cot^2(x)}{(1+\cot(x))^{3/2}}\,dx = \int \frac{\cot(x)^2}{(\cot(x)+1)^{\frac{3}{2}}}\,dx$$

```
[In] integrate(cot(x)^2/(1+cot(x))^(3/2),x, algorithm="giac")
[Out] integrate(cot(x)^2/(cot(x) + 1)^(3/2), x)
```

Mupad [B] (verification not implemented)

Time = 13.08 (sec) , antiderivative size = 208, normalized size of antiderivative = 1.50

$$\begin{aligned}\int \frac{\cot^2(x)}{(1+\cot(x))^{3/2}}\,dx &= \frac{1}{\sqrt{\cot(x)+1}} - \operatorname{atanh}\left(\frac{\sqrt{\cot(x)+1}}{8\sqrt{\frac{\sqrt{2}}{32}+\frac{1}{32}}} - \frac{\sqrt{\cot(x)+1}}{8\sqrt{\frac{1}{32}-\frac{\sqrt{2}}{32}}}\right.\\&\left.+\frac{\sqrt{2}\sqrt{\cot(x)+1}}{16\sqrt{\frac{1}{32}-\frac{\sqrt{2}}{32}}}+\frac{\sqrt{2}\sqrt{\cot(x)+1}}{16\sqrt{\frac{\sqrt{2}}{32}+\frac{1}{32}}}\right)\left(2\sqrt{\frac{1}{32}-\frac{\sqrt{2}}{32}}-2\sqrt{\frac{\sqrt{2}}{32}+\frac{1}{32}}\right)\\&+\operatorname{atanh}\left(\frac{\sqrt{\cot(x)+1}}{8\sqrt{\frac{1}{32}-\frac{\sqrt{2}}{32}}}+\frac{\sqrt{\cot(x)+1}}{8\sqrt{\frac{\sqrt{2}}{32}+\frac{1}{32}}}-\frac{\sqrt{2}\sqrt{\cot(x)+1}}{16\sqrt{\frac{1}{32}-\frac{\sqrt{2}}{32}}}\right.\\&\left.+\frac{\sqrt{2}\sqrt{\cot(x)+1}}{16\sqrt{\frac{\sqrt{2}}{32}+\frac{1}{32}}}\right)\left(2\sqrt{\frac{1}{32}-\frac{\sqrt{2}}{32}}+2\sqrt{\frac{\sqrt{2}}{32}+\frac{1}{32}}\right)\end{aligned}$$

```
[In] int(cot(x)^2/(cot(x) + 1)^(3/2),x)
[Out] 1/(cot(x) + 1)^(1/2) - atanh((cot(x) + 1)^(1/2)/(8*(2^(1/2)/32 + 1/32)^(1/2
)) - (cot(x) + 1)^(1/2)/(8*(1/32 - 2^(1/2)/32)^(1/2)) + (2^(1/2)*(cot(x) +
1)^(1/2))/(16*(1/32 - 2^(1/2)/32)^(1/2)) + (2^(1/2)*(cot(x) + 1)^(1/2))/(16
*(2^(1/2)/32 + 1/32)^(1/2)))*(2*(1/32 - 2^(1/2)/32)^(1/2) - 2*(2^(1/2)/32 +
```

```
 1/32)^(1/2)) + atanh((cot(x) + 1)^(1/2)/(8*(1/32 - 2^(1/2)/32)^(1/2)) + (c
ot(x) + 1)^(1/2)/(8*(2^(1/2)/32 + 1/32)^(1/2)) - (2^(1/2)*(cot(x) + 1)^(1/2
))/(16*(1/32 - 2^(1/2)/32)^(1/2)) + (2^(1/2)*(cot(x) + 1)^(1/2))/(16*(2^(1/
2)/32 + 1/32)^(1/2)))*(2*(1/32 - 2^(1/2)/32)^(1/2) + 2*(2^(1/2)/32 + 1/32)^
(1/2))
```

3.48 $\int \frac{\cot(x)}{(1+\cot(x))^{3/2}}\,dx$

Optimal result

Integrand size = 11, antiderivative size = 226

$$\begin{aligned}\int \frac{\cot(x)}{(1+\cot(x))^{3/2}}\,dx &= \frac{1}{2}\sqrt{\frac{1}{2}\left(1+\sqrt{2}\right)}\arctan\left(\frac{\sqrt{2\left(1+\sqrt{2}\right)}-2\sqrt{1+\cot(x)}}{\sqrt{2\left(-1+\sqrt{2}\right)}}\right)\\
&-\frac{1}{2}\sqrt{\frac{1}{2}\left(1+\sqrt{2}\right)}\arctan\left(\frac{\sqrt{2\left(1+\sqrt{2}\right)}+2\sqrt{1+\cot(x)}}{\sqrt{2\left(-1+\sqrt{2}\right)}}\right)\\
&-\frac{1}{\sqrt{1+\cot(x)}}-\frac{\log\left(1+\sqrt{2}+\cot(x)-\sqrt{2\left(1+\sqrt{2}\right)}\sqrt{1+\cot(x)}\right)}{4\sqrt{2\left(1+\sqrt{2}\right)}}\\
&+\frac{\log\left(1+\sqrt{2}+\cot(x)+\sqrt{2\left(1+\sqrt{2}\right)}\sqrt{1+\cot(x)}\right)}{4\sqrt{2\left(1+\sqrt{2}\right)}}\end{aligned}$$

```
[Out] -1/(1+cot(x))^(1/2)+1/4*arctan((-2*(1+cot(x))^(1/2)+(2+2*2^(1/2))^(1/2))/(-
2+2*2^(1/2))^(1/2))*(2+2*2^(1/2))^(1/2)-1/4*arctan((2*(1+cot(x))^(1/2)+(2+2
*2^(1/2))^(1/2))/(-2+2*2^(1/2))^(1/2))*(2+2*2^(1/2))^(1/2)-1/4*ln(1+cot(x)+
2^(1/2)-(1+cot(x))^(1/2)*(2+2*2^(1/2))^(1/2))/(2+2*2^(1/2))^(1/2)+1/4*ln(1+
cot(x)+2^(1/2)+(1+cot(x))^(1/2)*(2+2*2^(1/2))^(1/2))/(2+2*2^(1/2))^(1/2)
```

Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 226, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.909, Rules used = {3610, 21, 3566, 714, 1141, 1175, 632, 210, 1178, 642}

$$\int \frac{\cot(x)}{(1+\cot(x))^{3/2}}\,dx = \frac{1}{2}\sqrt{\frac{1}{2}\left(1+\sqrt{2}\right)}\arctan\left(\frac{\sqrt{2\left(1+\sqrt{2}\right)}-2\sqrt{\cot(x)+1}}{\sqrt{2\left(\sqrt{2}-1\right)}}\right) - \frac{1}{2}\sqrt{\frac{1}{2}\left(1+\sqrt{2}\right)}\arctan\left(\frac{2\sqrt{\cot(x)+1}+\sqrt{2\left(1+\sqrt{2}\right)}}{\sqrt{2\left(\sqrt{2}-1\right)}}\right) - \frac{1}{\sqrt{\cot(x)+1}} - \frac{\log\left(\cot(x)-\sqrt{2\left(1+\sqrt{2}\right)}\sqrt{\cot(x)+1}+\sqrt{2}+1\right)}{4\sqrt{2\left(1+\sqrt{2}\right)}} + \frac{\log\left(\cot(x)+\sqrt{2\left(1+\sqrt{2}\right)}\sqrt{\cot(x)+1}+\sqrt{2}+1\right)}{4\sqrt{2\left(1+\sqrt{2}\right)}}$$

```
[In] Int[Cot[x]/(1 + Cot[x])^(3/2),x]
```

```
[Out] (Sqrt[(1 + Sqrt[2])/2]*ArcTan[(Sqrt[2*(1 + Sqrt[2])] - 2*Sqrt[1 + Cot[x]])/
Sqrt[2*(-1 + Sqrt[2])]])/2 - (Sqrt[(1 + Sqrt[2])/2]*ArcTan[(Sqrt[2*(1 + Sqr
t[2])] + 2*Sqrt[1 + Cot[x]])/Sqrt[2*(-1 + Sqrt[2])]])/2 - 1/Sqrt[1 + Cot[x]
] - Log[1 + Sqrt[2] + Cot[x] - Sqrt[2*(1 + Sqrt[2])]*Sqrt[1 + Cot[x]]]/(4*S
qrt[2*(1 + Sqrt[2])]) + Log[1 + Sqrt[2] + Cot[x] + Sqrt[2*(1 + Sqrt[2])]*Sq
rt[1 + Cot[x]]]/(4*Sqrt[2*(1 + Sqrt[2])])
```

Rule 21

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_.)*((c_) + (d_.)*(v_))^(n_.), x_Symbol] :>
 Dist[(b/d)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && EqQ[b*c - a*d, 0] && IntegerQ[m] && ( !IntegerQ[n] || SimplerQ[c + d*x,
 a + b*x])
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
```

```
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 714

```
Int[Sqrt[(d_) + (e_.)*(x_)]/((a_) + (c_.)*(x_)^2), x_Symbol] :> Dist[2*e, S
ubst[Int[x^2/(c*d^2 + a*e^2 - 2*c*d*x^2 + c*x^4), x], x, Sqrt[d + e*x]], x]
 /; FreeQ[{a, c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0]
```

Rule 1141

```
Int[(x_)^2/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
a/c, 2]}, Dist[1/2, Int[(q + x^2)/(a + b*x^2 + c*x^4), x], x] - Dist[1/2, I
nt[(q - x^2)/(a + b*x^2 + c*x^4), x], x]] /; FreeQ[{a, b, c}, x] && LtQ[b^2
 - 4*a*c, 0] && PosQ[a*c]
```

Rule 1175

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[2*(d/e) - b/c, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2
, x], x], x] + Dist[e/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; Fre
eQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - a*e^2, 0] && (
GtQ[2*(d/e) - b/c, 0] || ( !LtQ[2*(d/e) - b/c, 0] && EqQ[d - e*Rt[a/c, 2],
0]))
```

Rule 1178

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[-2*(d/e) - b/c, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e
+ q*x - x^2, x], x], x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^
2, x], x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c
*d^2 - a*e^2, 0] &&  !GtQ[b^2 - 4*a*c, 0]
```

Rule 3566

```
Int[((a_) + (b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[b/d, Sub
st[Int[(a + x)^n/(b^2 + x^2), x], x, b*Tan[c + d*x]], x] /; FreeQ[{a, b, c,
 d, n}, x] && NeQ[a^2 + b^2, 0]
```

Rule 3610

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)]), x_Symbol] :> Simp[(b*c - a*d)*((a + b*Tan[e + f*x])^(m + 1)/
(f*(m + 1)*(a^2 + b^2))), x] + Dist[1/(a^2 + b^2), Int[(a + b*Tan[e + f*x])
^(m + 1)*Simp[a*c + b*d - (b*c - a*d)*Tan[e + f*x], x], x], x] /; FreeQ[{a,
 b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && LtQ[m, -1]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\frac{1}{\sqrt{1+\cot(x)}} - \frac{1}{2}\int \frac{-1-\cot(x)}{\sqrt{1+\cot(x)}}\,dx \\
&= -\frac{1}{\sqrt{1+\cot(x)}} + \frac{1}{2}\int \sqrt{1+\cot(x)}\,dx \\
&= -\frac{1}{\sqrt{1+\cot(x)}} - \frac{1}{2}\text{Subst}\left(\int \frac{\sqrt{1+x}}{1+x^2}\,dx, x, \cot(x)\right) \\
&= -\frac{1}{\sqrt{1+\cot(x)}} - \text{Subst}\left(\int \frac{x^2}{2-2x^2+x^4}\,dx, x, \sqrt{1+\cot(x)}\right) \\
&= -\frac{1}{\sqrt{1+\cot(x)}} + \frac{1}{2}\text{Subst}\left(\int \frac{\sqrt{2}-x^2}{2-2x^2+x^4}\,dx, x, \sqrt{1+\cot(x)}\right) \\
&\quad - \frac{1}{2}\text{Subst}\left(\int \frac{\sqrt{2}+x^2}{2-2x^2+x^4}\,dx, x, \sqrt{1+\cot(x)}\right) \\
&= -\frac{1}{\sqrt{1+\cot(x)}} - \frac{1}{4}\text{Subst}\left(\int \frac{1}{\sqrt{2}-\sqrt{2\left(1+\sqrt{2}\right)}x+x^2}\,dx, x, \sqrt{1+\cot(x)}\right) \\
&\quad - \frac{1}{4}\text{Subst}\left(\int \frac{1}{\sqrt{2}+\sqrt{2\left(1+\sqrt{2}\right)}x+x^2}\,dx, x, \sqrt{1+\cot(x)}\right) \\
&\quad - \frac{\text{Subst}\left(\int \frac{\sqrt{2\left(1+\sqrt{2}\right)}+2x}{-\sqrt{2}-\sqrt{2\left(1+\sqrt{2}\right)}x-x^2}\,dx, x, \sqrt{1+\cot(x)}\right)}{4\sqrt{2\left(1+\sqrt{2}\right)}} \\
&\quad - \frac{\text{Subst}\left(\int \frac{\sqrt{2\left(1+\sqrt{2}\right)}-2x}{-\sqrt{2}+\sqrt{2\left(1+\sqrt{2}\right)}x-x^2}\,dx, x, \sqrt{1+\cot(x)}\right)}{4\sqrt{2\left(1+\sqrt{2}\right)}}
\end{aligned}$$

$$= -\frac{1}{\sqrt{1+\cot(x)}} - \frac{\log\left(1+\sqrt{2}+\cot(x)-\sqrt{2\left(1+\sqrt{2}\right)}\sqrt{1+\cot(x)}\right)}{4\sqrt{2\left(1+\sqrt{2}\right)}} + \frac{\log\left(1+\sqrt{2}+\cot(x)+\sqrt{2\left(1+\sqrt{2}\right)}\sqrt{1+\cot(x)}\right)}{4\sqrt{2\left(1+\sqrt{2}\right)}} + \frac{1}{2}\text{Subst}\left(\int \frac{1}{2\left(1-\sqrt{2}\right)-x^2}\,dx, x, -\sqrt{2\left(1+\sqrt{2}\right)}+2\sqrt{1+\cot(x)}\right) + \frac{1}{2}\text{Subst}\left(\int \frac{1}{2\left(1-\sqrt{2}\right)-x^2}\,dx, x, \sqrt{2\left(1+\sqrt{2}\right)}+2\sqrt{1+\cot(x)}\right)$$

$$= \frac{\arctan\left(\frac{\sqrt{2\left(1+\sqrt{2}\right)}-2\sqrt{1+\cot(x)}}{\sqrt{2\left(-1+\sqrt{2}\right)}}\right)}{2\sqrt{2\left(-1+\sqrt{2}\right)}} - \frac{\arctan\left(\frac{\sqrt{2\left(1+\sqrt{2}\right)}+2\sqrt{1+\cot(x)}}{\sqrt{2\left(-1+\sqrt{2}\right)}}\right)}{2\sqrt{2\left(-1+\sqrt{2}\right)}} - \frac{1}{\sqrt{1+\cot(x)}} - \frac{\log\left(1+\sqrt{2}+\cot(x)-\sqrt{2\left(1+\sqrt{2}\right)}\sqrt{1+\cot(x)}\right)}{4\sqrt{2\left(1+\sqrt{2}\right)}} + \frac{\log\left(1+\sqrt{2}+\cot(x)+\sqrt{2\left(1+\sqrt{2}\right)}\sqrt{1+\cot(x)}\right)}{4\sqrt{2\left(1+\sqrt{2}\right)}}$$

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.58 (sec) , antiderivative size = 71, normalized size of antiderivative = 0.31

$$\int \frac{\cot(x)}{(1+\cot(x))^{3/2}}\,dx = \frac{1}{2}i\sqrt{1-i}\,\text{arctanh}\left(\frac{\sqrt{1+\cot(x)}}{\sqrt{1-i}}\right) - \frac{1}{2}i\sqrt{1+i}\,\text{arctanh}\left(\frac{\sqrt{1+\cot(x)}}{\sqrt{1+i}}\right) - \frac{1}{\sqrt{1+\cot(x)}}$$

```
[In] Integrate[Cot[x]/(1 + Cot[x])^(3/2),x]
```

```
[Out] (I/2)*Sqrt[1 - I]*ArcTanh[Sqrt[1 + Cot[x]]/Sqrt[1 - I]] - (I/2)*Sqrt[1 + I]
*ArcTanh[Sqrt[1 + Cot[x]]/Sqrt[1 + I]] - 1/Sqrt[1 + Cot[x]]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 355 vs. 2(160) = 320.

Time = 0.04 (sec) , antiderivative size = 356, normalized size of antiderivative = 1.58

method	result
derivativedivides	$-\frac{1}{\sqrt{1+\cot(x)}}-\frac{\sqrt{2+2\sqrt{2}}\,\sqrt{2}\,\ln\left(1+\cot(x)+\sqrt{2}-\sqrt{1+\cot(x)}\,\sqrt{2+2\sqrt{2}}\right)}{8}-\frac{\sqrt{2}\left(2+2\sqrt{2}\right)\arctan\left(\frac{2\sqrt{1+\cot(x)}-\sqrt{2+2}}{\sqrt{-2+2\sqrt{2}}}\right.}{4\sqrt{-2+2\sqrt{2}}}$
default	$-\frac{1}{\sqrt{1+\cot(x)}}-\frac{\sqrt{2+2\sqrt{2}}\,\sqrt{2}\,\ln\left(1+\cot(x)+\sqrt{2}-\sqrt{1+\cot(x)}\,\sqrt{2+2\sqrt{2}}\right)}{8}-\frac{\sqrt{2}\left(2+2\sqrt{2}\right)\arctan\left(\frac{2\sqrt{1+\cot(x)}-\sqrt{2+2}}{\sqrt{-2+2\sqrt{2}}}\right.}{4\sqrt{-2+2\sqrt{2}}}$

```
[In] int(cot(x)/(1+cot(x))^(3/2),x,method=_RETURNVERBOSE)
[Out] -1/(1+cot(x))^(1/2)-1/8*(2+2*2^(1/2))^(1/2)*2^(1/2)*ln(1+cot(x)+2^(1/2)-(1+
cot(x))^(1/2)*(2+2*2^(1/2))^(1/2))-1/4*2^(1/2)*(2+2*2^(1/2))/(-2+2*2^(1/2))
^(1/2)*arctan((2*(1+cot(x))^(1/2)-(2+2*2^(1/2))^(1/2))/(-2+2*2^(1/2))^(1/2)
)+1/8*(2+2*2^(1/2))^(1/2)*ln(1+cot(x)+2^(1/2)-(1+cot(x))^(1/2)*(2+2*2^(1/2)
)^(1/2))+1/4*(2+2*2^(1/2))/(-2+2*2^(1/2))^(1/2)*arctan((2*(1+cot(x))^(1/2)-
(2+2*2^(1/2))^(1/2))/(-2+2*2^(1/2))^(1/2))+1/8*(2+2*2^(1/2))^(1/2)*2^(1/2)*
ln(1+cot(x)+2^(1/2)+(1+cot(x))^(1/2)*(2+2*2^(1/2))^(1/2))-1/4*2^(1/2)*(2+2*
2^(1/2))/(-2+2*2^(1/2))^(1/2)*arctan((2*(1+cot(x))^(1/2)+(2+2*2^(1/2))^(1/2
))/(-2+2*2^(1/2))^(1/2))-1/8*(2+2*2^(1/2))^(1/2)*ln(1+cot(x)+2^(1/2)+(1+cot
(x))^(1/2)*(2+2*2^(1/2))^(1/2))+1/4*(2+2*2^(1/2))/(-2+2*2^(1/2))^(1/2)*arct
an((2*(1+cot(x))^(1/2)+(2+2*2^(1/2))^(1/2))/(-2+2*2^(1/2))^(1/2))
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.26 (sec) , antiderivative size = 202, normalized size of antiderivative = 0.89

$$\int \frac{\cot(x)}{(1+\cot(x))^{3/2}}\,dx =$$

$$-\frac{\sqrt{i-1}(\cos(2x)+\sin(2x)+1)\log\left(i\sqrt{i-1}+\sqrt{\frac{\cos(2x)+\sin(2x)+1}{\sin(2x)}}\right)-\sqrt{i-1}(\cos(2x)+\sin(2x)+1)\log}{}$$

```
[In] integrate(cot(x)/(1+cot(x))^(3/2),x, algorithm="fricas")
[Out] -1/4*(sqrt(I - 1)*(cos(2*x) + sin(2*x) + 1)*log(I*sqrt(I - 1) + sqrt((cos(2
*x) + sin(2*x) + 1)/sin(2*x))) - sqrt(I - 1)*(cos(2*x) + sin(2*x) + 1)*log(
-I*sqrt(I - 1) + sqrt((cos(2*x) + sin(2*x) + 1)/sin(2*x))) - sqrt(-I - 1)*(
cos(2*x) + sin(2*x) + 1)*log(I*sqrt(-I - 1) + sqrt((cos(2*x) + sin(2*x) + 1
)/sin(2*x))) + sqrt(-I - 1)*(cos(2*x) + sin(2*x) + 1)*log(-I*sqrt(-I - 1) +
 sqrt((cos(2*x) + sin(2*x) + 1)/sin(2*x))) + 4*sqrt((cos(2*x) + sin(2*x) +
1)/sin(2*x))*sin(2*x))/(cos(2*x) + sin(2*x) + 1)
```

Sympy [F]

$$\int \frac{\cot(x)}{(1+\cot(x))^{3/2}}\,dx = \int \frac{\cot(x)}{(\cot(x)+1)^{\frac{3}{2}}}\,dx$$

```
[In] integrate(cot(x)/(1+cot(x))**(3/2),x)
[Out] Integral(cot(x)/(cot(x) + 1)**(3/2), x)
```

Maxima [F]

$$\int \frac{\cot(x)}{(1+\cot(x))^{3/2}}\,dx = \int \frac{\cot(x)}{(\cot(x)+1)^{\frac{3}{2}}}\,dx$$

```
[In] integrate(cot(x)/(1+cot(x))^(3/2),x, algorithm="maxima")
[Out] integrate(cot(x)/(cot(x) + 1)^(3/2), x)
```

Giac [F]

$$\int \frac{\cot(x)}{(1+\cot(x))^{3/2}}\,dx = \int \frac{\cot(x)}{(\cot(x)+1)^{\frac{3}{2}}}\,dx$$

```
[In] integrate(cot(x)/(1+cot(x))^(3/2),x, algorithm="giac")
[Out] integrate(cot(x)/(cot(x) + 1)^(3/2), x)
```

Mupad [B] (verification not implemented)

Time = 12.84 (sec) , antiderivative size = 121, normalized size of antiderivative = 0.54

$$\int \frac{\cot(x)}{(1+\cot(x))^{3/2}}\,dx = -\mathrm{atanh}\left(32\sqrt{\cot(x)+1}\left(\sqrt{-\frac{\sqrt{2}}{32}-\frac{1}{32}}+\sqrt{\frac{\sqrt{2}}{32}-\frac{1}{32}}\right)^3\right)\left(2\sqrt{-\frac{\sqrt{2}}{32}-\frac{1}{32}}+2\sqrt{\frac{\sqrt{2}}{32}-\frac{1}{32}}\right)-\frac{1}{\sqrt{\cot(x)+1}}-\mathrm{atanh}\left(32\sqrt{\cot(x)+1}\left(\sqrt{-\frac{\sqrt{2}}{32}-\frac{1}{32}}-\sqrt{\frac{\sqrt{2}}{32}-\frac{1}{32}}\right)^3\right)\left(2\sqrt{-\frac{\sqrt{2}}{32}-\frac{1}{32}}-2\sqrt{\frac{\sqrt{2}}{32}-\frac{1}{32}}\right)$$

```
[In] int(cot(x)/(cot(x) + 1)^(3/2),x)
```

```
[Out] - atanh(32*(cot(x) + 1)^(1/2)*((- 2^(1/2)/32 - 1/32)^(1/2) + (2^(1/2)/32 -
1/32)^(1/2))^3)*(2*(- 2^(1/2)/32 - 1/32)^(1/2) + 2*(2^(1/2)/32 - 1/32)^(1/2
)) - 1/(cot(x) + 1)^(1/2) - atanh(32*(cot(x) + 1)^(1/2)*((- 2^(1/2)/32 - 1/
32)^(1/2) - (2^(1/2)/32 - 1/32)^(1/2))^3)*(2*(- 2^(1/2)/32 - 1/32)^(1/2) -
2*(2^(1/2)/32 - 1/32)^(1/2))
```

3.49 $\int \frac{\cot^2(x)}{(1+\cot(x))^{5/2}}\,dx$

Optimal result

Integrand size = 13, antiderivative size = 143

$$\int \frac{\cot^2(x)}{(1+\cot(x))^{5/2}}\,dx = \frac{1}{4}\sqrt{-1+\sqrt{2}}\arctan\left(\frac{3-2\sqrt{2}+\left(1-\sqrt{2}\right)\cot(x)}{\sqrt{2\left(-7+5\sqrt{2}\right)}\sqrt{1+\cot(x)}}\right) + \frac{1}{4}\sqrt{1+\sqrt{2}}\operatorname{arctanh}\left(\frac{3+2\sqrt{2}+\left(1+\sqrt{2}\right)\cot(x)}{\sqrt{2\left(7+5\sqrt{2}\right)}\sqrt{1+\cot(x)}}\right) + \frac{1}{3(1+\cot(x))^{3/2}} - \frac{1}{\sqrt{1+\cot(x)}}$$

```
[Out] 1/3/(1+cot(x))^(3/2)-1/(1+cot(x))^(1/2)+1/4*arctan((3+cot(x)*(1-2^(1/2))-2*
2^(1/2))/(1+cot(x))^(1/2)/(-14+10*2^(1/2))^(1/2))*(2^(1/2)-1)^(1/2)+1/4*arc
tanh((3+2*2^(1/2)+cot(x)*(1+2^(1/2)))/(1+cot(x))^(1/2)/(14+10*2^(1/2))^(1/2
))*(1+2^(1/2))^(1/2)
```

Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 143, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.538, Rules used = {3623, 3610, 12, 3617, 3616, 209, 213}

$$\int \frac{\cot^2(x)}{(1+\cot(x))^{5/2}}\,dx = \frac{1}{4}\sqrt{\sqrt{2}-1}\arctan\left(\frac{\left(1-\sqrt{2}\right)\cot(x)-2\sqrt{2}+3}{\sqrt{2\left(5\sqrt{2}-7\right)}\sqrt{\cot(x)+1}}\right) + \frac{1}{4}\sqrt{1+\sqrt{2}}\operatorname{arctanh}\left(\frac{\left(1+\sqrt{2}\right)\cot(x)+2\sqrt{2}+3}{\sqrt{2\left(7+5\sqrt{2}\right)}\sqrt{\cot(x)+1}}\right) - \frac{1}{\sqrt{\cot(x)+1}} + \frac{1}{3(\cot(x)+1)^{3/2}}$$

```
[In] Int[Cot[x]^2/(1 + Cot[x])^(5/2),x]
```

```
[Out] (Sqrt[-1 + Sqrt[2]]*ArcTan[(3 - 2*Sqrt[2] + (1 - Sqrt[2])*Cot[x])/(Sqrt[2*(
-7 + 5*Sqrt[2])]*Sqrt[1 + Cot[x]])])/4 + (Sqrt[1 + Sqrt[2]]*ArcTanh[(3 + 2*
Sqrt[2] + (1 + Sqrt[2])*Cot[x])/(Sqrt[2*(7 + 5*Sqrt[2])]*Sqrt[1 + Cot[x]])]
)/4 + 1/(3*(1 + Cot[x])^(3/2)) - 1/Sqrt[1 + Cot[x]]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 213

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[b, 2])^(-
1))*ArcTanh[Rt[b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] &&
 (LtQ[a, 0] || GtQ[b, 0])
```

Rule 3610

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)]), x_Symbol] :> Simp[(b*c - a*d)*((a + b*Tan[e + f*x])^(m + 1)/
(f*(m + 1)*(a^2 + b^2))), x] + Dist[1/(a^2 + b^2), Int[(a + b*Tan[e + f*x])
^(m + 1)*Simp[a*c + b*d - (b*c - a*d)*Tan[e + f*x], x], x], x] /; FreeQ[{a,
 b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && LtQ[m, -1]
```

Rule 3616

```
Int[((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])/Sqrt[(a_) + (b_.)*tan[(e_.) + (
f_.)*(x_)]], x_Symbol] :> Dist[-2*(d^2/f), Subst[Int[1/(2*b*c*d - 4*a*d^2 +
 x^2), x], x, (b*c - 2*a*d - b*d*Tan[e + f*x])/Sqrt[a + b*Tan[e + f*x]]], x
] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0]
 && NeQ[c^2 + d^2, 0] && EqQ[2*a*c*d - b*(c^2 - d^2), 0]
```

Rule 3617

```
Int[((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])/Sqrt[(a_) + (b_.)*tan[(e_.) + (
f_.)*(x_)]], x_Symbol] :> With[{q = Rt[a^2 + b^2, 2]}, Dist[1/(2*q), Int[(a
*c + b*d + c*q + (b*c - a*d + d*q)*Tan[e + f*x])/Sqrt[a + b*Tan[e + f*x]],
x], x] - Dist[1/(2*q), Int[(a*c + b*d - c*q + (b*c - a*d - d*q)*Tan[e + f*x
])/Sqrt[a + b*Tan[e + f*x]], x], x]] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ
[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] && NeQ[2*a*c*d - b
*(c^2 - d^2), 0] && (PerfectSquareQ[a^2 + b^2] || RationalQ[a, b, c, d])
```

Rule 3623

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)])^2, x_Symbol] :> Simp[(b*c - a*d)^2*((a + b*Tan[e + f*x])^(m +
 1)/(b*f*(m + 1)*(a^2 + b^2))), x] + Dist[1/(a^2 + b^2), Int[(a + b*Tan[e +
 f*x])^(m + 1)*Simp[a*c^2 + 2*b*c*d - a*d^2 - (b*c^2 - 2*a*c*d - b*d^2)*Tan
[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
&& LtQ[m, -1] && NeQ[a^2 + b^2, 0]
```

Rubi steps

$$\text{integral} = \frac{1}{3(1+\cot(x))^{3/2}} + \frac{1}{2}\int \frac{-1+\cot(x)}{(1+\cot(x))^{3/2}}\,dx$$

$$= \frac{1}{3(1+\cot(x))^{3/2}} - \frac{1}{\sqrt{1+\cot(x)}} + \frac{1}{4}\int \frac{2\cot(x)}{\sqrt{1+\cot(x)}}\,dx$$

$$= \frac{1}{3(1+\cot(x))^{3/2}} - \frac{1}{\sqrt{1+\cot(x)}} + \frac{1}{2}\int \frac{\cot(x)}{\sqrt{1+\cot(x)}}\,dx$$

$$= \frac{1}{3(1+\cot(x))^{3/2}} - \frac{1}{\sqrt{1+\cot(x)}} + \frac{\int \frac{-1-\left(-1-\sqrt{2}\right)\cot(x)}{\sqrt{1+\cot(x)}}\,dx}{4\sqrt{2}} - \frac{\int \frac{-1-\left(-1+\sqrt{2}\right)\cot(x)}{\sqrt{1+\cot(x)}}\,dx}{4\sqrt{2}}$$

$$= \frac{1}{3(1+\cot(x))^{3/2}} - \frac{1}{\sqrt{1+\cot(x)}} + \frac{1}{4}\Big(-4$$
$$+3\sqrt{2}\Big)\,\text{Subst}\left(\int \frac{1}{2\left(-1+\sqrt{2}\right)-4\left(-1+\sqrt{2}\right)^2+x^2}\,dx, x, \frac{1-2(-1+\sqrt{2})-(-1+\sqrt{2})\cot(}{\sqrt{1+\cot(x)}}\right.$$
$$-\frac{1}{4}\Big(4$$
$$+3\sqrt{2}\Big)\,\text{Subst}\left(\int \frac{1}{2\left(-1-\sqrt{2}\right)-4\left(-1-\sqrt{2}\right)^2+x^2}\,dx, x, \frac{1-2(-1-\sqrt{2})-(-1-\sqrt{2})\cot(}{\sqrt{1+\cot(x)}}\right.$$

$$= \frac{1}{4}\sqrt{-1+\sqrt{2}}\arctan\left(\frac{3-2\sqrt{2}+\left(1-\sqrt{2}\right)\cot(x)}{\sqrt{2\left(-7+5\sqrt{2}\right)}\sqrt{1+\cot(x)}}\right)$$
$$+\frac{1}{4}\sqrt{1+\sqrt{2}}\,\text{arctanh}\left(\frac{3+2\sqrt{2}+\left(1+\sqrt{2}\right)\cot(x)}{\sqrt{2\left(7+5\sqrt{2}\right)}\sqrt{1+\cot(x)}}\right)$$
$$+\frac{1}{3(1+\cot(x))^{3/2}} - \frac{1}{\sqrt{1+\cot(x)}}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 0.54 (sec) , antiderivative size = 62, normalized size of antiderivative = 0.43

$$\int \frac{\cot^2(x)}{(1+\cot(x))^{5/2}}\,dx = \frac{4-(1+i)\,\text{Hypergeometric2F1}\left(-\frac{3}{2},1,-\frac{1}{2},\left(\frac{1}{2}-\frac{i}{2}\right)(1+\cot(x))\right)-(1-i)\,\text{Hyperge}}{6(1+\cot(x))^{3/2}}$$

```
[In] Integrate[Cot[x]^2/(1 + Cot[x])^(5/2),x]
```

```
[Out] (4 - (1 + I)*Hypergeometric2F1[-3/2, 1, -1/2, (1/2 - I/2)*(1 + Cot[x])] - (
1 - I)*Hypergeometric2F1[-3/2, 1, -1/2, (1/2 + I/2)*(1 + Cot[x])])/(6*(1 +
Cot[x])^(3/2))
```

Maple [A] (verified)

Time = 0.04 (sec) , antiderivative size = 195, normalized size of antiderivative = 1.36

method	result
derivativedivides	$\frac{1}{3(1+\cot(x))^{\frac{3}{2}}}-\frac{1}{\sqrt{1+\cot(x)}}-\frac{\sqrt{2}\left(\frac{\sqrt{2+2\sqrt{2}}\,\ln\left(1+\cot(x)+\sqrt{2}-\sqrt{1+\cot(x)}\,\sqrt{2+2\sqrt{2}}\right)}{2}+\frac{2\left(\sqrt{2}-1\right)\arctan\left(\frac{2\sqrt{1+\cot(x)}-\sqrt{2}}{\sqrt{-2+2\sqrt{2}}}\right.}{\sqrt{-2+2\sqrt{2}}}\right.}{8}$
default	$\frac{1}{3(1+\cot(x))^{\frac{3}{2}}}-\frac{1}{\sqrt{1+\cot(x)}}-\frac{\sqrt{2}\left(\frac{\sqrt{2+2\sqrt{2}}\,\ln\left(1+\cot(x)+\sqrt{2}-\sqrt{1+\cot(x)}\,\sqrt{2+2\sqrt{2}}\right)}{2}+\frac{2\left(\sqrt{2}-1\right)\arctan\left(\frac{2\sqrt{1+\cot(x)}-\sqrt{2}}{\sqrt{-2+2\sqrt{2}}}\right.}{\sqrt{-2+2\sqrt{2}}}\right.}{8}$

```
[In] int(cot(x)^2/(1+cot(x))^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/3/(1+cot(x))^(3/2)-1/(1+cot(x))^(1/2)-1/8*2^(1/2)*(1/2*(2+2*2^(1/2))^(1/2
)*ln(1+cot(x)+2^(1/2)-(1+cot(x))^(1/2)*(2+2*2^(1/2))^(1/2))+2*(2^(1/2)-1)/(
-2+2*2^(1/2))^(1/2)*arctan((2*(1+cot(x))^(1/2)-(2+2*2^(1/2))^(1/2))/(-2+2*2
^(1/2))^(1/2)))+1/8*2^(1/2)*(1/2*(2+2*2^(1/2))^(1/2)*ln(1+cot(x)+2^(1/2)+(1
+cot(x))^(1/2)*(2+2*2^(1/2))^(1/2))+2*(1-2^(1/2))/(-2+2*2^(1/2))^(1/2)*arct
an((2*(1+cot(x))^(1/2)+(2+2*2^(1/2))^(1/2))/(-2+2*2^(1/2))^(1/2)))
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.26 (sec) , antiderivative size = 238, normalized size of antiderivative = 1.66

$$\int \frac{\cot^2(x)}{(1+\cot(x))^{5/2}}\,dx = \frac{3\sqrt{i+1}\left(\sqrt{2}\sin(2x)+\sqrt{2}\right)\log\left(-(i-1)\sqrt{2}\sqrt{i+1}+2\sqrt{\frac{\cos(2x)+\sin(2x)+1}{\sin(2x)}}\right)-3\sqrt{}$$

```
[In] integrate(cot(x)^2/(1+cot(x))^(5/2),x, algorithm="fricas")
```

```
[Out] 1/24*(3*sqrt(I + 1)*(sqrt(2)*sin(2*x) + sqrt(2))*log(-(I - 1)*sqrt(2)*sqrt(
I + 1) + 2*sqrt((cos(2*x) + sin(2*x) + 1)/sin(2*x))) - 3*sqrt(I + 1)*(sqrt(
2)*sin(2*x) + sqrt(2))*log((I - 1)*sqrt(2)*sqrt(I + 1) + 2*sqrt((cos(2*x) +
 sin(2*x) + 1)/sin(2*x))) + 3*sqrt(-I + 1)*(sqrt(2)*sin(2*x) + sqrt(2))*log
((I + 1)*sqrt(2)*sqrt(-I + 1) + 2*sqrt((cos(2*x) + sin(2*x) + 1)/sin(2*x)))
 - 3*sqrt(-I + 1)*(sqrt(2)*sin(2*x) + sqrt(2))*log(-(I + 1)*sqrt(2)*sqrt(-I
 + 1) + 2*sqrt((cos(2*x) + sin(2*x) + 1)/sin(2*x))) + 4*sqrt((cos(2*x) + si
n(2*x) + 1)/sin(2*x))*(2*cos(2*x) - 3*sin(2*x) - 2))/(sin(2*x) + 1)
```

Sympy [F]

$$\int \frac{\cot^2(x)}{(1+\cot(x))^{5/2}}\,dx = \int \frac{\cot^2(x)}{(\cot(x)+1)^{\frac{5}{2}}}\,dx$$

```
[In] integrate(cot(x)**2/(1+cot(x))**(5/2),x)
```

```
[Out] Integral(cot(x)**2/(cot(x) + 1)**(5/2), x)
```

Maxima [F]

$$\int \frac{\cot^2(x)}{(1+\cot(x))^{5/2}}\,dx = \int \frac{\cot(x)^2}{(\cot(x)+1)^{\frac{5}{2}}}\,dx$$

```
[In] integrate(cot(x)^2/(1+cot(x))^(5/2),x, algorithm="maxima")
```

```
[Out] integrate(cot(x)^2/(cot(x) + 1)^(5/2), x)
```

Giac [F]

$$\int \frac{\cot^2(x)}{(1+\cot(x))^{5/2}}\,dx = \int \frac{\cot(x)^2}{(\cot(x)+1)^{\frac{5}{2}}}\,dx$$

```
[In] integrate(cot(x)^2/(1+cot(x))^(5/2),x, algorithm="giac")
[Out] integrate(cot(x)^2/(cot(x) + 1)^(5/2), x)
```

Mupad [B] (verification not implemented)

Time = 13.18 (sec) , antiderivative size = 242, normalized size of antiderivative = 1.69

$$\int \frac{\cot^2(x)}{(1+\cot(x))^{5/2}}\,dx = \operatorname{atanh}\left(\frac{4\sqrt{2}\sqrt{\frac{1}{64}-\frac{\sqrt{2}}{64}}\sqrt{\cot(x)+1}}{64\sqrt{\frac{1}{64}-\frac{\sqrt{2}}{64}}\sqrt{\frac{\sqrt{2}}{64}+\frac{1}{64}}-1} - \frac{4\sqrt{2}\sqrt{\frac{\sqrt{2}}{64}+\frac{1}{64}}\sqrt{\cot(x)+1}}{64\sqrt{\frac{1}{64}-\frac{\sqrt{2}}{64}}\sqrt{\frac{\sqrt{2}}{64}+\frac{1}{64}}-1}\right)\left(2\sqrt{\frac{1}{64}-\frac{\sqrt{2}}{64}}+2\sqrt{\frac{\sqrt{2}}{64}+\frac{1}{64}}\right) - \operatorname{atanh}\left(\frac{4\sqrt{2}\sqrt{\frac{1}{64}-\frac{\sqrt{2}}{64}}\sqrt{\cot(x)+1}}{64\sqrt{\frac{1}{64}-\frac{\sqrt{2}}{64}}\sqrt{\frac{\sqrt{2}}{64}+\frac{1}{64}}+1} + \frac{4\sqrt{2}\sqrt{\frac{\sqrt{2}}{64}+\frac{1}{64}}\sqrt{\cot(x)+1}}{64\sqrt{\frac{1}{64}-\frac{\sqrt{2}}{64}}\sqrt{\frac{\sqrt{2}}{64}+\frac{1}{64}}+1}\right)\left(2\sqrt{\frac{1}{64}-\frac{\sqrt{2}}{64}}-2\sqrt{\frac{\sqrt{2}}{64}+\frac{1}{64}}\right) - \frac{\cot(x)+\frac{2}{3}}{(\cot(x)+1)^{3/2}}$$

```
[In] int(cot(x)^2/(cot(x) + 1)^(5/2),x)
[Out] atanh((4*2^(1/2)*(1/64 - 2^(1/2)/64)^(1/2)*(cot(x) + 1)^(1/2))/(64*(1/64 -
2^(1/2)/64)^(1/2)*(2^(1/2)/64 + 1/64)^(1/2) - 1) - (4*2^(1/2)*(2^(1/2)/64 +
 1/64)^(1/2)*(cot(x) + 1)^(1/2))/(64*(1/64 - 2^(1/2)/64)^(1/2)*(2^(1/2)/64
+ 1/64)^(1/2) - 1))*(2*(1/64 - 2^(1/2)/64)^(1/2) + 2*(2^(1/2)/64 + 1/64)^(1
/2)) - atanh((4*2^(1/2)*(1/64 - 2^(1/2)/64)^(1/2)*(cot(x) + 1)^(1/2))/(64*(
1/64 - 2^(1/2)/64)^(1/2)*(2^(1/2)/64 + 1/64)^(1/2) + 1) + (4*2^(1/2)*(2^(1/
2)/64 + 1/64)^(1/2)*(cot(x) + 1)^(1/2))/(64*(1/64 - 2^(1/2)/64)^(1/2)*(2^(1
/2)/64 + 1/64)^(1/2) + 1))*(2*(1/64 - 2^(1/2)/64)^(1/2) - 2*(2^(1/2)/64 + 1
/64)^(1/2)) - (cot(x) + 2/3)/(cot(x) + 1)^(3/2)
```

3.50 $\int \frac{\cot(x)}{(1+\cot(x))^{5/2}}\,dx$

Optimal result

Integrand size = 11, antiderivative size = 216

$$\begin{aligned}\int \frac{\cot(x)}{(1+\cot(x))^{5/2}}\,dx &= \frac{1}{4}\sqrt{1+\sqrt{2}}\arctan\left(\frac{\sqrt{2\left(1+\sqrt{2}\right)}-2\sqrt{1+\cot(x)}}{\sqrt{2\left(-1+\sqrt{2}\right)}}\right)\\
&-\frac{1}{4}\sqrt{1+\sqrt{2}}\arctan\left(\frac{\sqrt{2\left(1+\sqrt{2}\right)}+2\sqrt{1+\cot(x)}}{\sqrt{2\left(-1+\sqrt{2}\right)}}\right)\\
&-\frac{1}{3(1+\cot(x))^{3/2}}+\frac{\log\left(1+\sqrt{2}+\cot(x)-\sqrt{2\left(1+\sqrt{2}\right)}\sqrt{1+\cot(x)}\right)}{8\sqrt{1+\sqrt{2}}}\\
&-\frac{\log\left(1+\sqrt{2}+\cot(x)+\sqrt{2\left(1+\sqrt{2}\right)}\sqrt{1+\cot(x)}\right)}{8\sqrt{1+\sqrt{2}}}\end{aligned}$$

```
[Out] -1/3/(1+cot(x))^(3/2)+1/8*ln(1+cot(x)+2^(1/2)-(1+cot(x))^(1/2)*(2+2*2^(1/2)
)^(1/2))/(1+2^(1/2))^(1/2)-1/8*ln(1+cot(x)+2^(1/2)+(1+cot(x))^(1/2)*(2+2*2^
(1/2))^(1/2))/(1+2^(1/2))^(1/2)+1/4*arctan((-2*(1+cot(x))^(1/2)+(2+2*2^(1/2
))^(1/2))/(-2+2*2^(1/2))^(1/2))*(1+2^(1/2))^(1/2)-1/4*arctan((2*(1+cot(x))^
(1/2)+(2+2*2^(1/2))^(1/2))/(-2+2*2^(1/2))^(1/2))*(1+2^(1/2))^(1/2)
```

Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 216, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.818$, Rules used = {3610, 21, 3566, 722, 1108, 648, 632, 210, 642}

$$\int \frac{\cot(x)}{(1+\cot(x))^{5/2}}\,dx = \frac{1}{4}\sqrt{1+\sqrt{2}}\arctan\left(\frac{\sqrt{2\left(1+\sqrt{2}\right)}-2\sqrt{\cot(x)+1}}{\sqrt{2\left(\sqrt{2}-1\right)}}\right) - \frac{1}{4}\sqrt{1+\sqrt{2}}\arctan\left(\frac{2\sqrt{\cot(x)+1}+\sqrt{2\left(1+\sqrt{2}\right)}}{\sqrt{2\left(\sqrt{2}-1\right)}}\right) - \frac{1}{3(\cot(x)+1)^{3/2}} + \frac{\log\left(\cot(x)-\sqrt{2\left(1+\sqrt{2}\right)}\sqrt{\cot(x)+1}+\sqrt{2}+1\right)}{8\sqrt{1+\sqrt{2}}} - \frac{\log\left(\cot(x)+\sqrt{2\left(1+\sqrt{2}\right)}\sqrt{\cot(x)+1}+\sqrt{2}+1\right)}{8\sqrt{1+\sqrt{2}}}$$

```
[In] Int[Cot[x]/(1 + Cot[x])^(5/2),x]
```

```
[Out] (Sqrt[1 + Sqrt[2]]*ArcTan[(Sqrt[2*(1 + Sqrt[2])] - 2*Sqrt[1 + Cot[x]])/Sqrt
[2*(-1 + Sqrt[2])]])/4 - (Sqrt[1 + Sqrt[2]]*ArcTan[(Sqrt[2*(1 + Sqrt[2])] +
 2*Sqrt[1 + Cot[x]])/Sqrt[2*(-1 + Sqrt[2])]])/4 - 1/(3*(1 + Cot[x])^(3/2))
+ Log[1 + Sqrt[2] + Cot[x] - Sqrt[2*(1 + Sqrt[2])]*Sqrt[1 + Cot[x]]]/(8*Sqr
t[1 + Sqrt[2]]) - Log[1 + Sqrt[2] + Cot[x] + Sqrt[2*(1 + Sqrt[2])]*Sqrt[1 +
 Cot[x]]]/(8*Sqrt[1 + Sqrt[2]])
```

Rule 21

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_.)*((c_) + (d_.)*(v_))^(n_.), x_Symbol] :>
 Dist[(b/d)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && EqQ[b*c - a*d, 0] && IntegerQ[m] && ( !IntegerQ[n] || SimplerQ[c + d*x,
 a + b*x])
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
```

```
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 722

```
Int[1/(Sqrt[(d_) + (e_.)*(x_)]*((a_) + (c_.)*(x_)^2)), x_Symbol] :> Dist[2*
e, Subst[Int[1/(c*d^2 + a*e^2 - 2*c*d*x^2 + c*x^4), x], x, Sqrt[d + e*x]],
x] /; FreeQ[{a, c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0]
```

Rule 1108

```
Int[((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(-1), x_Symbol] :> With[{q = Rt[a/
c, 2]}, With[{r = Rt[2*q - b/c, 2]}, Dist[1/(2*c*q*r), Int[(r - x)/(q - r*x
 + x^2), x], x] + Dist[1/(2*c*q*r), Int[(r + x)/(q + r*x + x^2), x], x]]] /
; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && NegQ[b^2 - 4*a*c]
```

Rule 3566

```
Int[((a_) + (b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[b/d, Sub
st[Int[(a + x)^n/(b^2 + x^2), x], x, b*Tan[c + d*x]], x] /; FreeQ[{a, b, c,
 d, n}, x] && NeQ[a^2 + b^2, 0]
```

Rule 3610

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)]), x_Symbol] :> Simp[(b*c - a*d)*((a + b*Tan[e + f*x])^(m + 1)/
(f*(m + 1)*(a^2 + b^2))), x] + Dist[1/(a^2 + b^2), Int[(a + b*Tan[e + f*x])
^(m + 1)*Simp[a*c + b*d - (b*c - a*d)*Tan[e + f*x], x], x], x] /; FreeQ[{a,
 b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && LtQ[m, -1]
```

Rubi steps

$$\text{integral} = -\frac{1}{3(1+\cot(x))^{3/2}} - \frac{1}{2}\int \frac{-1-\cot(x)}{(1+\cot(x))^{3/2}}\,dx$$

$$= -\frac{1}{3(1+\cot(x))^{3/2}} + \frac{1}{2}\int \frac{1}{\sqrt{1+\cot(x)}}\,dx$$

$$= -\frac{1}{3(1+\cot(x))^{3/2}} - \frac{1}{2}\text{Subst}\left(\int \frac{1}{\sqrt{1+x}\,(1+x^2)}\,dx, x, \cot(x)\right)$$

$$= -\frac{1}{3(1+\cot(x))^{3/2}} - \text{Subst}\left(\int \frac{1}{2-2x^2+x^4}\,dx, x, \sqrt{1+\cot(x)}\right)$$

$$= -\frac{1}{3(1+\cot(x))^{3/2}} - \frac{\text{Subst}\left(\int \frac{\sqrt{2\left(1+\sqrt{2}\right)}-x}{\sqrt{2}-\sqrt{2\left(1+\sqrt{2}\right)}x+x^2}\,dx, x, \sqrt{1+\cot(x)}\right)}{4\sqrt{1+\sqrt{2}}}$$

$$- \frac{\text{Subst}\left(\int \frac{\sqrt{2\left(1+\sqrt{2}\right)}+x}{\sqrt{2}+\sqrt{2\left(1+\sqrt{2}\right)}x+x^2}\,dx, x, \sqrt{1+\cot(x)}\right)}{4\sqrt{1+\sqrt{2}}}$$

$$= -\frac{1}{3(1+\cot(x))^{3/2}} - \frac{\text{Subst}\left(\int \frac{1}{\sqrt{2}-\sqrt{2\left(1+\sqrt{2}\right)}x+x^2}\,dx, x, \sqrt{1+\cot(x)}\right)}{4\sqrt{2}}$$

$$- \frac{\text{Subst}\left(\int \frac{1}{\sqrt{2}+\sqrt{2\left(1+\sqrt{2}\right)}x+x^2}\,dx, x, \sqrt{1+\cot(x)}\right)}{4\sqrt{2}}$$

$$+ \frac{\text{Subst}\left(\int \frac{-\sqrt{2\left(1+\sqrt{2}\right)}+2x}{\sqrt{2}-\sqrt{2\left(1+\sqrt{2}\right)}x+x^2}\,dx, x, \sqrt{1+\cot(x)}\right)}{8\sqrt{1+\sqrt{2}}}$$

$$- \frac{\text{Subst}\left(\int \frac{\sqrt{2\left(1+\sqrt{2}\right)}+2x}{\sqrt{2}+\sqrt{2\left(1+\sqrt{2}\right)}x+x^2}\,dx, x, \sqrt{1+\cot(x)}\right)}{8\sqrt{1+\sqrt{2}}}$$

$$= -\frac{1}{3(1+\cot(x))^{3/2}} + \frac{\log\left(1+\sqrt{2}+\cot(x)-\sqrt{2\left(1+\sqrt{2}\right)}\sqrt{1+\cot(x)}\right)}{8\sqrt{1+\sqrt{2}}} - \frac{\log\left(1+\sqrt{2}+\cot(x)+\sqrt{2\left(1+\sqrt{2}\right)}\sqrt{1+\cot(x)}\right)}{8\sqrt{1+\sqrt{2}}} + \frac{\text{Subst}\left(\int \frac{1}{2\left(1-\sqrt{2}\right)-x^2}\,dx, x, -\sqrt{2\left(1+\sqrt{2}\right)}+2\sqrt{1+\cot(x)}\right)}{2\sqrt{2}} + \frac{\text{Subst}\left(\int \frac{1}{2\left(1-\sqrt{2}\right)-x^2}\,dx, x, \sqrt{2\left(1+\sqrt{2}\right)}+2\sqrt{1+\cot(x)}\right)}{2\sqrt{2}}$$

$$= \frac{\arctan\left(\frac{\sqrt{2\left(1+\sqrt{2}\right)}-2\sqrt{1+\cot(x)}}{\sqrt{2\left(-1+\sqrt{2}\right)}}\right)}{4\sqrt{-1+\sqrt{2}}} - \frac{\arctan\left(\frac{\sqrt{2\left(1+\sqrt{2}\right)}+2\sqrt{1+\cot(x)}}{\sqrt{2\left(-1+\sqrt{2}\right)}}\right)}{4\sqrt{-1+\sqrt{2}}} - \frac{1}{3(1+\cot(x))^{3/2}} + \frac{\log\left(1+\sqrt{2}+\cot(x)-\sqrt{2\left(1+\sqrt{2}\right)}\sqrt{1+\cot(x)}\right)}{8\sqrt{1+\sqrt{2}}} - \frac{\log\left(1+\sqrt{2}+\cot(x)+\sqrt{2\left(1+\sqrt{2}\right)}\sqrt{1+\cot(x)}\right)}{8\sqrt{1+\sqrt{2}}}$$

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.45 (sec) , antiderivative size = 69, normalized size of antiderivative = 0.32

$$\int \frac{\cot(x)}{(1+\cot(x))^{5/2}}\,dx = -\frac{1}{4}(1-i)^{3/2}\text{arctanh}\left(\frac{\sqrt{1+\cot(x)}}{\sqrt{1-i}}\right) - \frac{1}{4}(1+i)^{3/2}\text{arctanh}\left(\frac{\sqrt{1+\cot(x)}}{\sqrt{1+i}}\right) - \frac{1}{3(1+\cot(x))^{3/2}}$$

```
[In] Integrate[Cot[x]/(1 + Cot[x])^(5/2),x]
```

```
[Out] -1/4*((1 - I)^(3/2)*ArcTanh[Sqrt[1 + Cot[x]]/Sqrt[1 - I]]) - ((1 + I)^(3/2)
*ArcTanh[Sqrt[1 + Cot[x]]/Sqrt[1 + I]])/4 - 1/(3*(1 + Cot[x])^(3/2))
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 443 vs. $2(152) = 304$.

Time = 0.03 (sec) , antiderivative size = 444, normalized size of antiderivative = 2.06

method	result
derivativedivides	$-\frac{1}{3(1+\cot(x))^{\frac{3}{2}}}-\frac{\sqrt{2+2\sqrt{2}}\,\sqrt{2}\,\ln\left(1+\cot(x)+\sqrt{2}-\sqrt{1+\cot(x)}\,\sqrt{2+2\sqrt{2}}\right)}{16}+\frac{\sqrt{2+2\sqrt{2}}\,\ln\left(1+\cot(x)+\sqrt{2}-\sqrt{1+\cot}\right.}{8}$
default	$-\frac{1}{3(1+\cot(x))^{\frac{3}{2}}}-\frac{\sqrt{2+2\sqrt{2}}\,\sqrt{2}\,\ln\left(1+\cot(x)+\sqrt{2}-\sqrt{1+\cot(x)}\,\sqrt{2+2\sqrt{2}}\right)}{16}+\frac{\sqrt{2+2\sqrt{2}}\,\ln\left(1+\cot(x)+\sqrt{2}-\sqrt{1+\cot}\right.}{8}$

```
[In] int(cot(x)/(1+cot(x))^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/3/(1+cot(x))^(3/2)-1/16*(2+2*2^(1/2))^(1/2)*2^(1/2)*ln(1+cot(x)+2^(1/2)-
(1+cot(x))^(1/2)*(2+2*2^(1/2))^(1/2))+1/8*(2+2*2^(1/2))^(1/2)*ln(1+cot(x)+2
^(1/2)-(1+cot(x))^(1/2)*(2+2*2^(1/2))^(1/2))-1/8*2^(1/2)*(2+2*2^(1/2))/(-2+
2*2^(1/2))^(1/2)*arctan((2*(1+cot(x))^(1/2)-(2+2*2^(1/2))^(1/2))/(-2+2*2^(1
/2))^(1/2))+1/4*(2+2*2^(1/2))/(-2+2*2^(1/2))^(1/2)*arctan((2*(1+cot(x))^(1/
2)-(2+2*2^(1/2))^(1/2))/(-2+2*2^(1/2))^(1/2))-1/2/(-2+2*2^(1/2))^(1/2)*arct
an((2*(1+cot(x))^(1/2)-(2+2*2^(1/2))^(1/2))/(-2+2*2^(1/2))^(1/2))*2^(1/2)+1
/16*(2+2*2^(1/2))^(1/2)*2^(1/2)*ln(1+cot(x)+2^(1/2)+(1+cot(x))^(1/2)*(2+2*2
^(1/2))^(1/2))-1/8*(2+2*2^(1/2))^(1/2)*ln(1+cot(x)+2^(1/2)+(1+cot(x))^(1/2)
*(2+2*2^(1/2))^(1/2))-1/8*2^(1/2)*(2+2*2^(1/2))/(-2+2*2^(1/2))^(1/2)*arctan
((2*(1+cot(x))^(1/2)+(2+2*2^(1/2))^(1/2))/(-2+2*2^(1/2))^(1/2))+1/4*(2+2*2^
(1/2))/(-2+2*2^(1/2))^(1/2)*arctan((2*(1+cot(x))^(1/2)+(2+2*2^(1/2))^(1/2))
/(-2+2*2^(1/2))^(1/2))-1/2/(-2+2*2^(1/2))^(1/2)*arctan((2*(1+cot(x))^(1/2)+
(2+2*2^(1/2))^(1/2))/(-2+2*2^(1/2))^(1/2))*2^(1/2)
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.27 (sec) , antiderivative size = 230, normalized size of antiderivative = 1.06

$$\int \frac{\cot(x)}{(1+\cot(x))^{5/2}}\,dx =$$
$$-\frac{3\sqrt{i-1}\left(\sqrt{2}\sin(2x)+\sqrt{2}\right)\log\left(-(i-1)\,\sqrt{2}\sqrt{i-1}+2\sqrt{\frac{\cos(2x)+\sin(2x)+1}{\sin(2x)}}\right)-3\sqrt{i-1}\left(\sqrt{2}\sin(2x)+\sqrt{2}\right.}{}$$

```
[In] integrate(cot(x)/(1+cot(x))^(5/2),x, algorithm="fricas")
```

```
[Out] -1/24*(3*sqrt(I - 1)*(sqrt(2)*sin(2*x) + sqrt(2))*log(-(I - 1)*sqrt(2)*sqrt
(I - 1) + 2*sqrt((cos(2*x) + sin(2*x) + 1)/sin(2*x))) - 3*sqrt(I - 1)*(sqrt
(2)*sin(2*x) + sqrt(2))*log((I - 1)*sqrt(2)*sqrt(I - 1) + 2*sqrt((cos(2*x)
```

```
+ sin(2*x) + 1)/sin(2*x))) + 3*sqrt(-I - 1)*(sqrt(2)*sin(2*x) + sqrt(2))*lo
g((I + 1)*sqrt(2)*sqrt(-I - 1) + 2*sqrt((cos(2*x) + sin(2*x) + 1)/sin(2*x))
) - 3*sqrt(-I - 1)*(sqrt(2)*sin(2*x) + sqrt(2))*log(-(I + 1)*sqrt(2)*sqrt(-
I - 1) + 2*sqrt((cos(2*x) + sin(2*x) + 1)/sin(2*x))) - 4*sqrt((cos(2*x) + s
in(2*x) + 1)/sin(2*x))*(cos(2*x) - 1))/(sin(2*x) + 1)
```

Sympy [F]

$$\int \frac{\cot(x)}{(1+\cot(x))^{5/2}}\,dx = \int \frac{\cot(x)}{(\cot(x)+1)^{\frac{5}{2}}}\,dx$$

```
[In] integrate(cot(x)/(1+cot(x))**(5/2),x)
[Out] Integral(cot(x)/(cot(x) + 1)**(5/2), x)
```

Maxima [F]

$$\int \frac{\cot(x)}{(1+\cot(x))^{5/2}}\,dx = \int \frac{\cot(x)}{(\cot(x)+1)^{\frac{5}{2}}}\,dx$$

```
[In] integrate(cot(x)/(1+cot(x))^(5/2),x, algorithm="maxima")
[Out] integrate(cot(x)/(cot(x) + 1)^(5/2), x)
```

Giac [F]

$$\int \frac{\cot(x)}{(1+\cot(x))^{5/2}}\,dx = \int \frac{\cot(x)}{(\cot(x)+1)^{\frac{5}{2}}}\,dx$$

```
[In] integrate(cot(x)/(1+cot(x))^(5/2),x, algorithm="giac")
[Out] integrate(cot(x)/(cot(x) + 1)^(5/2), x)
```

Mupad [B] (verification not implemented)

Time = 13.10 (sec) , antiderivative size = 238, normalized size of antiderivative = 1.10

$$\int \frac{\cot(x)}{(1+\cot(x))^{5/2}}\,dx = \operatorname{atanh}\left(\frac{4\sqrt{2}\sqrt{-\frac{\sqrt{2}}{64}-\frac{1}{64}}\sqrt{\cot(x)+1}}{64\sqrt{\frac{\sqrt{2}}{64}-\frac{1}{64}}\sqrt{-\frac{\sqrt{2}}{64}-\frac{1}{64}}+1} + \frac{4\sqrt{2}\sqrt{\frac{\sqrt{2}}{64}-\frac{1}{64}}\sqrt{\cot(x)+1}}{64\sqrt{\frac{\sqrt{2}}{64}-\frac{1}{64}}\sqrt{-\frac{\sqrt{2}}{64}-\frac{1}{64}}+1}\right)\left(2\sqrt{-\frac{\sqrt{2}}{64}-\frac{1}{64}}-2\sqrt{\frac{\sqrt{2}}{64}-\frac{1}{64}}\right) - \operatorname{atanh}\left(\frac{4\sqrt{2}\sqrt{-\frac{\sqrt{2}}{64}-\frac{1}{64}}\sqrt{\cot(x)+1}}{64\sqrt{\frac{\sqrt{2}}{64}-\frac{1}{64}}\sqrt{-\frac{\sqrt{2}}{64}-\frac{1}{64}}-1} - \frac{4\sqrt{2}\sqrt{\frac{\sqrt{2}}{64}-\frac{1}{64}}\sqrt{\cot(x)+1}}{64\sqrt{\frac{\sqrt{2}}{64}-\frac{1}{64}}\sqrt{-\frac{\sqrt{2}}{64}-\frac{1}{64}}-1}\right)\left(2\sqrt{-\frac{\sqrt{2}}{64}-\frac{1}{64}}+2\sqrt{\frac{\sqrt{2}}{64}-\frac{1}{64}}\right) - \frac{1}{3\left(\cot(x)+1\right)^{3/2}}$$

```
[In] int(cot(x)/(cot(x) + 1)^(5/2),x)
```

```
[Out] atanh((4*2^(1/2)*(- 2^(1/2)/64 - 1/64)^(1/2)*(cot(x) + 1)^(1/2))/(64*(2^(1/
2)/64 - 1/64)^(1/2)*(- 2^(1/2)/64 - 1/64)^(1/2) + 1) + (4*2^(1/2)*(2^(1/2)/
64 - 1/64)^(1/2)*(cot(x) + 1)^(1/2))/(64*(2^(1/2)/64 - 1/64)^(1/2)*(- 2^(1/
2)/64 - 1/64)^(1/2) + 1))*(2*(- 2^(1/2)/64 - 1/64)^(1/2) - 2*(2^(1/2)/64 -
1/64)^(1/2)) - atanh((4*2^(1/2)*(- 2^(1/2)/64 - 1/64)^(1/2)*(cot(x) + 1)^(1
/2))/(64*(2^(1/2)/64 - 1/64)^(1/2)*(- 2^(1/2)/64 - 1/64)^(1/2) - 1) - (4*2^
(1/2)*(2^(1/2)/64 - 1/64)^(1/2)*(cot(x) + 1)^(1/2))/(64*(2^(1/2)/64 - 1/64)
^(1/2)*(- 2^(1/2)/64 - 1/64)^(1/2) - 1))*(2*(- 2^(1/2)/64 - 1/64)^(1/2) + 2
*(2^(1/2)/64 - 1/64)^(1/2)) - 1/(3*(cot(x) + 1)^(3/2))
```

3.51 $\int (e\cot(c+dx))^{3/2}(a+b\cot(c+dx))\,dx$

Optimal result

Integrand size = 23, antiderivative size = 247

$$\int (e\cot(c+dx))^{3/2}(a+b\cot(c+dx))\,dx = -\frac{(a+b)e^{3/2}\arctan\left(1-\frac{\sqrt{2}\sqrt{e\cot(c+dx)}}{\sqrt{e}}\right)}{\sqrt{2}d} + \frac{(a+b)e^{3/2}\arctan\left(1+\frac{\sqrt{2}\sqrt{e\cot(c+dx)}}{\sqrt{e}}\right)}{\sqrt{2}d} - \frac{2ae\sqrt{e\cot(c+dx)}}{d} - \frac{2b(e\cot(c+dx))^{3/2}}{3d} - \frac{(a-b)e^{3/2}\log\left(\sqrt{e}+\sqrt{e}\cot(c+dx)-\sqrt{2}\sqrt{e\cot(c+dx)}\right)}{2\sqrt{2}d} + \frac{(a-b)e^{3/2}\log\left(\sqrt{e}+\sqrt{e}\cot(c+dx)+\sqrt{2}\sqrt{e\cot(c+dx)}\right)}{2\sqrt{2}d}$$

```
[Out] -2/3*b*(e*cot(d*x+c))^(3/2)/d-1/2*(a+b)*e^(3/2)*arctan(1-2^(1/2)*(e*cot(d*x
+c))^(1/2)/e^(1/2))/d*2^(1/2)+1/2*(a+b)*e^(3/2)*arctan(1+2^(1/2)*(e*cot(d*x
+c))^(1/2)/e^(1/2))/d*2^(1/2)-1/4*(a-b)*e^(3/2)*ln(e^(1/2)+cot(d*x+c)*e^(1/
2)-2^(1/2)*(e*cot(d*x+c))^(1/2))/d*2^(1/2)+1/4*(a-b)*e^(3/2)*ln(e^(1/2)+cot
(d*x+c)*e^(1/2)+2^(1/2)*(e*cot(d*x+c))^(1/2))/d*2^(1/2)-2*a*e*(e*cot(d*x+c)
)^(1/2)/d
```

Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 247, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.348, Rules used = {3609, 3615, 1182, 1176, 631, 210, 1179, 642}

$$\int (e\cot(c+dx))^{3/2}(a+b\cot(c+dx))\,dx = -\frac{e^{3/2}(a+b)\arctan\left(1-\frac{\sqrt{2}\sqrt{e\cot(c+dx)}}{\sqrt{e}}\right)}{\sqrt{2}d}+\frac{e^{3/2}(a+b)\arctan\left(\frac{\sqrt{2}\sqrt{e\cot(c+dx)}}{\sqrt{e}}+1\right)}{\sqrt{2}d} - \frac{e^{3/2}(a-b)\log\left(\sqrt{e}\cot(c+dx)-\sqrt{2}\sqrt{e\cot(c+dx)}+\sqrt{e}\right)}{2\sqrt{2}d} + \frac{e^{3/2}(a-b)\log\left(\sqrt{e}\cot(c+dx)+\sqrt{2}\sqrt{e\cot(c+dx)}+\sqrt{e}\right)}{2\sqrt{2}d} - \frac{2ae\sqrt{e\cot(c+dx)}}{d}-\frac{2b(e\cot(c+dx))^{3/2}}{3d}$$

```
[In] Int[(e*Cot[c + d*x])^(3/2)*(a + b*Cot[c + d*x]),x]
```

```
[Out] -(((a + b)*e^(3/2)*ArcTan[1 - (Sqrt[2]*Sqrt[e*Cot[c + d*x]])/Sqrt[e]])/(Sqr
t[2]*d)) + ((a + b)*e^(3/2)*ArcTan[1 + (Sqrt[2]*Sqrt[e*Cot[c + d*x]])/Sqrt[
e]])/(Sqrt[2]*d) - (2*a*e*Sqrt[e*Cot[c + d*x]])/d - (2*b*(e*Cot[c + d*x])^(
3/2))/(3*d) - ((a - b)*e^(3/2)*Log[Sqrt[e] + Sqrt[e]*Cot[c + d*x] - Sqrt[2]
*Sqrt[e*Cot[c + d*x]]])/(2*Sqrt[2]*d) + ((a - b)*e^(3/2)*Log[Sqrt[e] + Sqrt
[e]*Cot[c + d*x] + Sqrt[2]*Sqrt[e*Cot[c + d*x]]])/(2*Sqrt[2]*d)
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1179

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 1182

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
a*c, 2]}, Dist[(d*q + a*e)/(2*a*c), Int[(q + c*x^2)/(a + c*x^4), x], x] + D
ist[(d*q - a*e)/(2*a*c), Int[(q - c*x^2)/(a + c*x^4), x], x]] /; FreeQ[{a,
c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0] && NeQ[c*d^2 - a*e^2, 0] && NegQ[(-a)
*c]
```

Rule 3609

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)]), x_Symbol] :> Simp[d*((a + b*Tan[e + f*x])^m/(f*m)), x] + Int
[(a + b*Tan[e + f*x])^(m - 1)*Simp[a*c - b*d + (b*c + a*d)*Tan[e + f*x], x]
, x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2,
 0] && GtQ[m, 0]
```

Rule 3615

```
Int[((c_) + (d_.)*tan[(e_.) + (f_.)*(x_)])/Sqrt[(b_.)*tan[(e_.) + (f_.)*(x_
)]], x_Symbol] :> Dist[2/f, Subst[Int[(b*c + d*x^2)/(b^2 + x^4), x], x, Sqr
t[b*Tan[e + f*x]]], x] /; FreeQ[{b, c, d, e, f}, x] && NeQ[c^2 - d^2, 0] &&
 NeQ[c^2 + d^2, 0]
```

Rubi steps

$$\begin{aligned}\text{integral} &= -\frac{2b(e\cot(c+dx))^{3/2}}{3d} + \int \sqrt{e\cot(c+dx)}(-be+ae\cot(c+dx))\,dx \\ &= -\frac{2ae\sqrt{e\cot(c+dx)}}{d} - \frac{2b(e\cot(c+dx))^{3/2}}{3d} + \int \frac{-ae^2-be^2\cot(c+dx)}{\sqrt{e\cot(c+dx)}}\,dx \\ &= -\frac{2ae\sqrt{e\cot(c+dx)}}{d} - \frac{2b(e\cot(c+dx))^{3/2}}{3d} + \frac{2\text{Subst}\left(\int \frac{ae^3+be^2x^2}{e^2+x^4}\,dx, x, \sqrt{e\cot(c+dx)}\right)}{d}\end{aligned}$$

$$= -\frac{2ae\sqrt{e\cot(c+dx)}}{d} - \frac{2b(e\cot(c+dx))^{3/2}}{3d} + \frac{((a-b)e^2)\,\text{Subst}\left(\int \frac{e-x^2}{e^2+x^4}\,dx, x, \sqrt{e\cot(c+dx)}\right)}{d} + \frac{((a+b)e^2)\,\text{Subst}\left(\int \frac{e+x^2}{e^2+x^4}\,dx, x, \sqrt{e\cot(c+dx)}\right)}{d}$$

$$= -\frac{2ae\sqrt{e\cot(c+dx)}}{d} - \frac{2b(e\cot(c+dx))^{3/2}}{3d} - \frac{((a-b)e^{3/2})\,\text{Subst}\left(\int \frac{\sqrt{2}\sqrt{e}+2x}{-e-\sqrt{2}\sqrt{e}x-x^2}\,dx, x, \sqrt{e\cot(c+dx)}\right)}{2\sqrt{2}d} - \frac{((a-b)e^{3/2})\,\text{Subst}\left(\int \frac{\sqrt{2}\sqrt{e}-2x}{-e+\sqrt{2}\sqrt{e}x-x^2}\,dx, x, \sqrt{e\cot(c+dx)}\right)}{2\sqrt{2}d} + \frac{((a+b)e^2)\,\text{Subst}\left(\int \frac{1}{e-\sqrt{2}\sqrt{e}x+x^2}\,dx, x, \sqrt{e\cot(c+dx)}\right)}{2d} + \frac{((a+b)e^2)\,\text{Subst}\left(\int \frac{1}{e+\sqrt{2}\sqrt{e}x+x^2}\,dx, x, \sqrt{e\cot(c+dx)}\right)}{2d}$$

$$= -\frac{2ae\sqrt{e\cot(c+dx)}}{d} - \frac{2b(e\cot(c+dx))^{3/2}}{3d} - \frac{(a-b)e^{3/2}\log\left(\sqrt{e}+\sqrt{e}\cot(c+dx)-\sqrt{2}\sqrt{e\cot(c+dx)}\right)}{2\sqrt{2}d} + \frac{(a-b)e^{3/2}\log\left(\sqrt{e}+\sqrt{e}\cot(c+dx)+\sqrt{2}\sqrt{e\cot(c+dx)}\right)}{2\sqrt{2}d} + \frac{((a+b)e^{3/2})\,\text{Subst}\left(\int \frac{1}{-1-x^2}\,dx, x, 1-\frac{\sqrt{2}\sqrt{e\cot(c+dx)}}{\sqrt{e}}\right)}{\sqrt{2}d} - \frac{((a+b)e^{3/2})\,\text{Subst}\left(\int \frac{1}{-1-x^2}\,dx, x, 1+\frac{\sqrt{2}\sqrt{e\cot(c+dx)}}{\sqrt{e}}\right)}{\sqrt{2}d}$$

$$= -\frac{(a+b)e^{3/2}\arctan\left(1-\frac{\sqrt{2}\sqrt{e\cot(c+dx)}}{\sqrt{e}}\right)}{\sqrt{2}d} + \frac{(a+b)e^{3/2}\arctan\left(1+\frac{\sqrt{2}\sqrt{e\cot(c+dx)}}{\sqrt{e}}\right)}{\sqrt{2}d} - \frac{2ae\sqrt{e\cot(c+dx)}}{d} - \frac{2b(e\cot(c+dx))^{3/2}}{3d} - \frac{(a-b)e^{3/2}\log\left(\sqrt{e}+\sqrt{e}\cot(c+dx)-\sqrt{2}\sqrt{e\cot(c+dx)}\right)}{2\sqrt{2}d} + \frac{(a-b)e^{3/2}\log\left(\sqrt{e}+\sqrt{e}\cot(c+dx)+\sqrt{2}\sqrt{e\cot(c+dx)}\right)}{2\sqrt{2}d}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 0.18 (sec) , antiderivative size = 68, normalized size of antiderivative = 0.28

$$\int (e\cot(c+dx))^{3/2}(a+b\cot(c+dx))\,dx =$$
$$-\frac{2e\sqrt{e\cot(c+dx)}\left(b\cot(c+dx)\,\text{Hypergeometric2F1}\left(-\frac{3}{4},1,\frac{1}{4},-\tan^2(c+dx)\right)+3a\,\text{Hypergeometric2F1}\right.}{3d}$$

```
[In] Integrate[(e*Cot[c + d*x])^(3/2)*(a + b*Cot[c + d*x]),x]
```

```
[Out] (-2*e*Sqrt[e*Cot[c + d*x]]*(b*Cot[c + d*x]*Hypergeometric2F1[-3/4, 1, 1/4,
-Tan[c + d*x]^2] + 3*a*Hypergeometric2F1[-1/4, 1, 3/4, -Tan[c + d*x]^2]))/(
3*d)
```

Maple [A] (verified)

Time = 0.04 (sec) , antiderivative size = 303, normalized size of antiderivative = 1.23

method	result
parts	$-\frac{2ae\left(\sqrt{e\cot(dx+c)}-\frac{(e^2)^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\frac{e\cot(dx+c)+(e^2)^{\frac{1}{4}}\sqrt{e\cot(dx+c)}\sqrt{2}+\sqrt{e^2}}{e\cot(dx+c)-(e^2)^{\frac{1}{4}}\sqrt{e\cot(dx+c)}\sqrt{2}+\sqrt{e^2}}\right)+2\arctan\left(\frac{\sqrt{2}\sqrt{e\cot(dx+c)}}{(e^2)^{\frac{1}{4}}}+1\right)-2\arctan\right.}{8}\right.}{d}$
derivativedivides	$\frac{-\frac{2b(e\cot(dx+c))^{\frac{3}{2}}}{3}-2ae\sqrt{e\cot(dx+c)}+2e^2\left(\frac{a(e^2)^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\frac{e\cot(dx+c)+(e^2)^{\frac{1}{4}}\sqrt{e\cot(dx+c)}\sqrt{2}+\sqrt{e^2}}{e\cot(dx+c)-(e^2)^{\frac{1}{4}}\sqrt{e\cot(dx+c)}\sqrt{2}+\sqrt{e^2}}\right)+2\arctan\left(\frac{\sqrt{2}\sqrt{e}}{(e}\right.\right.}{8e}\right.}{}$
default	$\frac{-\frac{2b(e\cot(dx+c))^{\frac{3}{2}}}{3}-2ae\sqrt{e\cot(dx+c)}+2e^2\left(\frac{a(e^2)^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\frac{e\cot(dx+c)+(e^2)^{\frac{1}{4}}\sqrt{e\cot(dx+c)}\sqrt{2}+\sqrt{e^2}}{e\cot(dx+c)-(e^2)^{\frac{1}{4}}\sqrt{e\cot(dx+c)}\sqrt{2}+\sqrt{e^2}}\right)+2\arctan\left(\frac{\sqrt{2}\sqrt{e}}{(e}\right.\right.}{8e}\right.}{}$

```
[In] int((e*cot(d*x+c))^(3/2)*(a+b*cot(d*x+c)),x,method=_RETURNVERBOSE)
```

```
[Out] -2*a/d*e*((e*cot(d*x+c))^(1/2)-1/8*(e^2)^(1/4)*2^(1/2)*(ln((e*cot(d*x+c)+(e
^2)^(1/4)*(e*cot(d*x+c))^(1/2)*2^(1/2)+(e^2)^(1/2))/(e*cot(d*x+c)-(e^2)^(1/
4)*(e*cot(d*x+c))^(1/2)*2^(1/2)+(e^2)^(1/2)))+2*arctan(2^(1/2)/(e^2)^(1/4)*
(e*cot(d*x+c))^(1/2)+1)-2*arctan(-2^(1/2)/(e^2)^(1/4)*(e*cot(d*x+c))^(1/2)+
1)))+b/d*(-2/3*(e*cot(d*x+c))^(3/2)+1/4*e^2/(e^2)^(1/4)*2^(1/2)*(ln((e*cot(
d*x+c)-(e^2)^(1/4)*(e*cot(d*x+c))^(1/2)*2^(1/2)+(e^2)^(1/2))/(e*cot(d*x+c)+
```

```
(e^2)^(1/4)*(e*cot(d*x+c))^(1/2)*2^(1/2)+(e^2)^(1/2)))+2*arctan(2^(1/2)/(e^
2)^(1/4)*(e*cot(d*x+c))^(1/2)+1)-2*arctan(-2^(1/2)/(e^2)^(1/4)*(e*cot(d*x+c
))^(1/2)+1)))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 843 vs. $2(194) = 388$.

Time = 0.28 (sec) , antiderivative size = 843, normalized size of antiderivative = 3.41

$$\int (e\cot(c+dx))^{3/2}(a+b\cot(c+dx))\,dx =$$

$$-\frac{3\,d\sqrt{-\frac{2\,abe^3+\sqrt{-\frac{\left(a^4-2\,a^2b^2+b^4\right)e^6}{d^4}}d^2}{d^2}}\log\left(-(a^4-b^4)e^4\sqrt{\frac{e\cos(2\,dx+2\,c)+e}{\sin(2\,dx+2\,c)}}+\left((a^3-ab^2)de^3+\sqrt{-\frac{(a^4-2\,a^2b^2+b^4)e^6}{d^4}}bd\right.\right.}{}$$

```
[In] integrate((e*cot(d*x+c))^(3/2)*(a+b*cot(d*x+c)),x, algorithm="fricas")
```

```
[Out] -1/6*(3*d*sqrt(-(2*a*b*e^3 + sqrt(-(a^4 - 2*a^2*b^2 + b^4)*e^6/d^4)*d^2)/d^
2)*log(-(a^4 - b^4)*e^4*sqrt((e*cos(2*d*x + 2*c) + e)/sin(2*d*x + 2*c)) + (
(a^3 - a*b^2)*d*e^3 + sqrt(-(a^4 - 2*a^2*b^2 + b^4)*e^6/d^4)*b*d^3)*sqrt(-(
2*a*b*e^3 + sqrt(-(a^4 - 2*a^2*b^2 + b^4)*e^6/d^4)*d^2)/d^2))*sin(2*d*x + 2
*c) - 3*d*sqrt(-(2*a*b*e^3 + sqrt(-(a^4 - 2*a^2*b^2 + b^4)*e^6/d^4)*d^2)/d^
2)*log(-(a^4 - b^4)*e^4*sqrt((e*cos(2*d*x + 2*c) + e)/sin(2*d*x + 2*c)) - (
(a^3 - a*b^2)*d*e^3 + sqrt(-(a^4 - 2*a^2*b^2 + b^4)*e^6/d^4)*b*d^3)*sqrt(-(
2*a*b*e^3 + sqrt(-(a^4 - 2*a^2*b^2 + b^4)*e^6/d^4)*d^2)/d^2))*sin(2*d*x + 2
*c) + 3*d*sqrt(-(2*a*b*e^3 - sqrt(-(a^4 - 2*a^2*b^2 + b^4)*e^6/d^4)*d^2)/d^
2)*log(-(a^4 - b^4)*e^4*sqrt((e*cos(2*d*x + 2*c) + e)/sin(2*d*x + 2*c)) + (
(a^3 - a*b^2)*d*e^3 - sqrt(-(a^4 - 2*a^2*b^2 + b^4)*e^6/d^4)*b*d^3)*sqrt(-(
2*a*b*e^3 - sqrt(-(a^4 - 2*a^2*b^2 + b^4)*e^6/d^4)*d^2)/d^2))*sin(2*d*x + 2
*c) - 3*d*sqrt(-(2*a*b*e^3 - sqrt(-(a^4 - 2*a^2*b^2 + b^4)*e^6/d^4)*d^2)/d^
2)*log(-(a^4 - b^4)*e^4*sqrt((e*cos(2*d*x + 2*c) + e)/sin(2*d*x + 2*c)) - (
(a^3 - a*b^2)*d*e^3 - sqrt(-(a^4 - 2*a^2*b^2 + b^4)*e^6/d^4)*b*d^3)*sqrt(-(
2*a*b*e^3 - sqrt(-(a^4 - 2*a^2*b^2 + b^4)*e^6/d^4)*d^2)/d^2))*sin(2*d*x + 2
*c) + 4*(b*e*cos(2*d*x + 2*c) + 3*a*e*sin(2*d*x + 2*c) + b*e)*sqrt((e*cos(2
*d*x + 2*c) + e)/sin(2*d*x + 2*c)))/(d*sin(2*d*x + 2*c))
```

Sympy [F]

$$\int (e\cot(c+dx))^{3/2}(a+b\cot(c+dx))\,dx = \int (e\cot(c+dx))^{\frac{3}{2}}\,(a+b\cot(c+dx))\,\,dx$$

```
[In] integrate((e*cot(d*x+c))**(3/2)*(a+b*cot(d*x+c)),x)
[Out] Integral((e*cot(c + d*x))**(3/2)*(a + b*cot(c + d*x)), x)
```

Maxima [F(-2)]

Exception generated.

$$\int (e\cot(c+dx))^{3/2}(a+b\cot(c+dx))\,dx = \text{Exception raised: ValueError}$$

```
[In] integrate((e*cot(d*x+c))^(3/2)*(a+b*cot(d*x+c)),x, algorithm="maxima")
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(e>0)', see `assume?` for more detai
ls)Is e
```

Giac [F]

$$\int (e\cot(c+dx))^{3/2}(a+b\cot(c+dx))\,dx = \int (b\cot(dx+c)+a)(e\cot(dx+c))^{\frac{3}{2}}\,\,dx$$

```
[In] integrate((e*cot(d*x+c))^(3/2)*(a+b*cot(d*x+c)),x, algorithm="giac")
[Out] integrate((b*cot(d*x + c) + a)*(e*cot(d*x + c))^(3/2), x)
```

Mupad [B] (verification not implemented)

Time = 13.86 (sec) , antiderivative size = 153, normalized size of antiderivative = 0.62

$$\int (e\cot(c+dx))^{3/2}(a+b\cot(c+dx))\,dx = \frac{(-1)^{1/4}\,b\,e^{3/2}\,\mathrm{atan}\left(\frac{(-1)^{1/4}\,\sqrt{e\cot(c+d\,x)}}{\sqrt{e}}\right)}{d} - \frac{2\,a\,e\,\sqrt{e\cot(c+d\,x)}}{d} - \frac{2\,b\,(e\cot(c+d\,x))^{3/2}}{3\,d} - \frac{(-1)^{1/4}\,b\,e^{3/2}\,\mathrm{atanh}\left(\frac{(-1)^{1/4}\,\sqrt{e\cot(c+d\,x)}}{\sqrt{e}}\right)}{d} - \frac{(-1)^{1/4}\,a\,e^{3/2}\,\mathrm{atan}\left(\frac{(-1)^{1/4}\,\sqrt{e\cot(c+d\,x)}}{\sqrt{e}}\right)\,1{}\mathrm{i}}{d} - \frac{(-1)^{1/4}\,a\,e^{3/2}\,\mathrm{atanh}\left(\frac{(-1)^{1/4}\,\sqrt{e\cot(c+d\,x)}}{\sqrt{e}}\right)\,1{}\mathrm{i}}{d}$$

```
[In] int((e*cot(c + d*x))^(3/2)*(a + b*cot(c + d*x)),x)
```

```
[Out] ((-1)^(1/4)*b*e^(3/2)*atan(((-1)^(1/4)*(e*cot(c + d*x))^(1/2))/e^(1/2)))/d
- (2*a*e*(e*cot(c + d*x))^(1/2))/d - ((-1)^(1/4)*a*e^(3/2)*atan(((-1)^(1/4)
*(e*cot(c + d*x))^(1/2))/e^(1/2))*1i)/d - ((-1)^(1/4)*a*e^(3/2)*atanh(((-1)
^(1/4)*(e*cot(c + d*x))^(1/2))/e^(1/2))*1i)/d - (2*b*(e*cot(c + d*x))^(3/2)
)/(3*d) - ((-1)^(1/4)*b*e^(3/2)*atanh(((-1)^(1/4)*(e*cot(c + d*x))^(1/2))/e
^(1/2)))/d
```

3.52 $\int \sqrt{e\cot(c+dx)}(a+b\cot(c+dx))\,dx$

Optimal result

Integrand size = 23, antiderivative size = 226

$$\int \sqrt{e\cot(c+dx)}(a+b\cot(c+dx))\,dx = \frac{(a-b)\sqrt{e}\arctan\left(1-\frac{\sqrt{2}\sqrt{e\cot(c+dx)}}{\sqrt{e}}\right)}{\sqrt{2}d} - \frac{(a-b)\sqrt{e}\arctan\left(1+\frac{\sqrt{2}\sqrt{e\cot(c+dx)}}{\sqrt{e}}\right)}{\sqrt{2}d} - \frac{2b\sqrt{e\cot(c+dx)}}{d} - \frac{(a+b)\sqrt{e}\log\left(\sqrt{e}+\sqrt{e}\cot(c+dx)-\sqrt{2}\sqrt{e\cot(c+dx)}\right)}{2\sqrt{2}d} + \frac{(a+b)\sqrt{e}\log\left(\sqrt{e}+\sqrt{e}\cot(c+dx)+\sqrt{2}\sqrt{e\cot(c+dx)}\right)}{2\sqrt{2}d}$$

```
[Out] 1/2*(a-b)*arctan(1-2^(1/2)*(e*cot(d*x+c))^(1/2)/e^(1/2))*e^(1/2)/d*2^(1/2)-
1/2*(a-b)*arctan(1+2^(1/2)*(e*cot(d*x+c))^(1/2)/e^(1/2))*e^(1/2)/d*2^(1/2)-
1/4*(a+b)*ln(e^(1/2)+cot(d*x+c)*e^(1/2)-2^(1/2)*(e*cot(d*x+c))^(1/2))*e^(1/
2)/d*2^(1/2)+1/4*(a+b)*ln(e^(1/2)+cot(d*x+c)*e^(1/2)+2^(1/2)*(e*cot(d*x+c))
^(1/2))*e^(1/2)/d*2^(1/2)-2*b*(e*cot(d*x+c))^(1/2)/d
```

Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 226, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.348, Rules used

$= \{3609, 3615, 1182, 1176, 631, 210, 1179, 642\}$

$$\int \sqrt{e\cot(c+dx)}(a+b\cot(c+dx))\,dx$$
$$= \frac{\sqrt{e}(a-b)\arctan\left(1-\frac{\sqrt{2}\sqrt{e\cot(c+dx)}}{\sqrt{e}}\right)}{\sqrt{2}d} - \frac{\sqrt{e}(a-b)\arctan\left(\frac{\sqrt{2}\sqrt{e\cot(c+dx)}}{\sqrt{e}}+1\right)}{\sqrt{2}d}$$
$$- \frac{\sqrt{e}(a+b)\log\left(\sqrt{e}\cot(c+dx)-\sqrt{2}\sqrt{e\cot(c+dx)}+\sqrt{e}\right)}{2\sqrt{2}d}$$
$$+ \frac{\sqrt{e}(a+b)\log\left(\sqrt{e}\cot(c+dx)+\sqrt{2}\sqrt{e\cot(c+dx)}+\sqrt{e}\right)}{2\sqrt{2}d} - \frac{2b\sqrt{e\cot(c+dx)}}{d}$$

```
[In] Int[Sqrt[e*Cot[c + d*x]]*(a + b*Cot[c + d*x]),x]
```

```
[Out] ((a - b)*Sqrt[e]*ArcTan[1 - (Sqrt[2]*Sqrt[e*Cot[c + d*x]])/Sqrt[e]])/(Sqrt[
2]*d) - ((a - b)*Sqrt[e]*ArcTan[1 + (Sqrt[2]*Sqrt[e*Cot[c + d*x]])/Sqrt[e]]
)/(Sqrt[2]*d) - (2*b*Sqrt[e*Cot[c + d*x]])/d - ((a + b)*Sqrt[e]*Log[Sqrt[e]
 + Sqrt[e]*Cot[c + d*x] - Sqrt[2]*Sqrt[e*Cot[c + d*x]]])/(2*Sqrt[2]*d) + ((
a + b)*Sqrt[e]*Log[Sqrt[e] + Sqrt[e]*Cot[c + d*x] + Sqrt[2]*Sqrt[e*Cot[c +
d*x]]])/(2*Sqrt[2]*d)
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
```

```
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1179

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 1182

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
a*c, 2]}, Dist[(d*q + a*e)/(2*a*c), Int[(q + c*x^2)/(a + c*x^4), x], x] + D
ist[(d*q - a*e)/(2*a*c), Int[(q - c*x^2)/(a + c*x^4), x], x]] /; FreeQ[{a,
c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0] && NeQ[c*d^2 - a*e^2, 0] && NegQ[(-a)
*c]
```

Rule 3609

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)]), x_Symbol] :> Simp[d*((a + b*Tan[e + f*x])^m/(f*m)), x] + Int
[(a + b*Tan[e + f*x])^(m - 1)*Simp[a*c - b*d + (b*c + a*d)*Tan[e + f*x], x]
, x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2,
 0] && GtQ[m, 0]
```

Rule 3615

```
Int[((c_) + (d_.)*tan[(e_.) + (f_.)*(x_)])/Sqrt[(b_.)*tan[(e_.) + (f_.)*(x_
)]], x_Symbol] :> Dist[2/f, Subst[Int[(b*c + d*x^2)/(b^2 + x^4), x], x, Sqr
t[b*Tan[e + f*x]]], x] /; FreeQ[{b, c, d, e, f}, x] && NeQ[c^2 - d^2, 0] &&
 NeQ[c^2 + d^2, 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\frac{2b\sqrt{e\cot(c+dx)}}{d}+\int\frac{-be+ae\cot(c+dx)}{\sqrt{e\cot(c+dx)}}\,dx\\
&= -\frac{2b\sqrt{e\cot(c+dx)}}{d}+\frac{2\mathrm{Subst}\left(\int\frac{be^2-aex^2}{e^2+x^4}\,dx,x,\sqrt{e\cot(c+dx)}\right)}{d}\\
&= -\frac{2b\sqrt{e\cot(c+dx)}}{d}-\frac{((a-b)e)\mathrm{Subst}\left(\int\frac{e+x^2}{e^2+x^4}\,dx,x,\sqrt{e\cot(c+dx)}\right)}{d}\\
&\quad+\frac{((a+b)e)\mathrm{Subst}\left(\int\frac{e-x^2}{e^2+x^4}\,dx,x,\sqrt{e\cot(c+dx)}\right)}{d}
\end{aligned}$$

$$= -\frac{2b\sqrt{e\cot(c+dx)}}{d} - \frac{\left((a+b)\sqrt{e}\right)\text{Subst}\left(\int \frac{\sqrt{2}\sqrt{e}+2x}{-e-\sqrt{2}\sqrt{e}x-x^2}\,dx, x, \sqrt{e\cot(c+dx)}\right)}{2\sqrt{2}d}$$
$$- \frac{\left((a+b)\sqrt{e}\right)\text{Subst}\left(\int \frac{\sqrt{2}\sqrt{e}-2x}{-e+\sqrt{2}\sqrt{e}x-x^2}\,dx, x, \sqrt{e\cot(c+dx)}\right)}{2\sqrt{2}d}$$
$$- \frac{((a-b)e)\text{Subst}\left(\int \frac{1}{e-\sqrt{2}\sqrt{e}x+x^2}\,dx, x, \sqrt{e\cot(c+dx)}\right)}{2d}$$
$$- \frac{((a-b)e)\text{Subst}\left(\int \frac{1}{e+\sqrt{2}\sqrt{e}x+x^2}\,dx, x, \sqrt{e\cot(c+dx)}\right)}{2d}$$

$$= -\frac{2b\sqrt{e\cot(c+dx)}}{d} - \frac{(a+b)\sqrt{e}\log\left(\sqrt{e}+\sqrt{e}\cot(c+dx)-\sqrt{2}\sqrt{e\cot(c+dx)}\right)}{2\sqrt{2}d}$$
$$+ \frac{(a+b)\sqrt{e}\log\left(\sqrt{e}+\sqrt{e}\cot(c+dx)+\sqrt{2}\sqrt{e\cot(c+dx)}\right)}{2\sqrt{2}d}$$
$$- \frac{\left((a-b)\sqrt{e}\right)\text{Subst}\left(\int \frac{1}{-1-x^2}\,dx, x, 1-\frac{\sqrt{2}\sqrt{e\cot(c+dx)}}{\sqrt{e}}\right)}{\sqrt{2}d}$$
$$+ \frac{\left((a-b)\sqrt{e}\right)\text{Subst}\left(\int \frac{1}{-1-x^2}\,dx, x, 1+\frac{\sqrt{2}\sqrt{e\cot(c+dx)}}{\sqrt{e}}\right)}{\sqrt{2}d}$$

$$= \frac{(a-b)\sqrt{e}\arctan\left(1-\frac{\sqrt{2}\sqrt{e\cot(c+dx)}}{\sqrt{e}}\right)}{\sqrt{2}d} - \frac{(a-b)\sqrt{e}\arctan\left(1+\frac{\sqrt{2}\sqrt{e\cot(c+dx)}}{\sqrt{e}}\right)}{\sqrt{2}d}$$
$$- \frac{2b\sqrt{e\cot(c+dx)}}{d} - \frac{(a+b)\sqrt{e}\log\left(\sqrt{e}+\sqrt{e}\cot(c+dx)-\sqrt{2}\sqrt{e\cot(c+dx)}\right)}{2\sqrt{2}d}$$
$$+ \frac{(a+b)\sqrt{e}\log\left(\sqrt{e}+\sqrt{e}\cot(c+dx)+\sqrt{2}\sqrt{e\cot(c+dx)}\right)}{2\sqrt{2}d}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 0.32 (sec) , antiderivative size = 155, normalized size of antiderivative = 0.69

$$\int \sqrt{e\cot(c+dx)}(a+b\cot(c+dx))\,dx =$$
$$-\frac{\sqrt{e\cot(c+dx)}\left(8b\,\text{Hypergeometric2F1}\left(-\frac{1}{4},1,\frac{3}{4},-\tan^2(c+dx)\right)+\sqrt{2}a\left(2\arctan\left(1-\sqrt{2}\sqrt{\tan(c+d\right.\right.\right.}{}$$

```
[In] Integrate[Sqrt[e*Cot[c + d*x]]*(a + b*Cot[c + d*x]),x]
```

[Out] -1/4*(Sqrt[e*Cot[c + d*x]]*(8*b*Hypergeometric2F1[-1/4, 1, 3/4, -Tan[c + d*x]^2] + Sqrt[2]*a*(2*ArcTan[1 - Sqrt[2]*Sqrt[Tan[c + d*x]]] - 2*ArcTan[1 + Sqrt[2]*Sqrt[Tan[c + d*x]]] + Log[1 - Sqrt[2]*Sqrt[Tan[c + d*x]] + Tan[c + d*x]] - Log[1 + Sqrt[2]*Sqrt[Tan[c + d*x]] + Tan[c + d*x]])*Sqrt[Tan[c + d*x]]))/d

Maple [A] (verified)

Time = 0.03 (sec) , antiderivative size = 287, normalized size of antiderivative = 1.27

method	result
parts	$-\frac{ae\sqrt{2}\left(\ln\left(\frac{e\cot(dx+c)-\left(e^{2}\right)^{\frac{1}{4}}\sqrt{e\cot(dx+c)}\,\sqrt{2}+\sqrt{e^{2}}}{e\cot(dx+c)+\left(e^{2}\right)^{\frac{1}{4}}\sqrt{e\cot(dx+c)}\,\sqrt{2}+\sqrt{e^{2}}}\right)+2\arctan\left(\frac{\sqrt{2}\,\sqrt{e\cot(dx+c)}}{\left(e^{2}\right)^{\frac{1}{4}}}+1\right)-2\arctan\left(-\frac{\sqrt{2}\,\sqrt{e\cot(dx+c)}}{\left(e^{2}\right)^{\frac{1}{4}}}\right.\right.}{4d\left(e^{2}\right)^{\frac{1}{4}}}$
derivativedivides	$\frac{-2\sqrt{e\cot(dx+c)}\,b-2e\left(-\frac{b\left(e^{2}\right)^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\frac{e\cot(dx+c)+\left(e^{2}\right)^{\frac{1}{4}}\sqrt{e\cot(dx+c)}\,\sqrt{2}+\sqrt{e^{2}}}{e\cot(dx+c)-\left(e^{2}\right)^{\frac{1}{4}}\sqrt{e\cot(dx+c)}\,\sqrt{2}+\sqrt{e^{2}}}\right)+2\arctan\left(\frac{\sqrt{2}\,\sqrt{e\cot(dx+c)}}{\left(e^{2}\right)^{\frac{1}{4}}}+1\right)-2\,\mathrm{ar}\right.}{8e}\right.}{}$
default	$\frac{-2\sqrt{e\cot(dx+c)}\,b-2e\left(-\frac{b\left(e^{2}\right)^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\frac{e\cot(dx+c)+\left(e^{2}\right)^{\frac{1}{4}}\sqrt{e\cot(dx+c)}\,\sqrt{2}+\sqrt{e^{2}}}{e\cot(dx+c)-\left(e^{2}\right)^{\frac{1}{4}}\sqrt{e\cot(dx+c)}\,\sqrt{2}+\sqrt{e^{2}}}\right)+2\arctan\left(\frac{\sqrt{2}\,\sqrt{e\cot(dx+c)}}{\left(e^{2}\right)^{\frac{1}{4}}}+1\right)-2\,\mathrm{ar}\right.}{8e}\right.}{}$

[In] int((e*cot(d*x+c))^(1/2)*(a+b*cot(d*x+c)),x,method=_RETURNVERBOSE)

[Out] -1/4*a/d*e/(e^2)^(1/4)*2^(1/2)*(ln((e*cot(d*x+c)-(e^2)^(1/4)*(e*cot(d*x+c))^(1/2)*2^(1/2)+(e^2)^(1/2))/(e*cot(d*x+c)+(e^2)^(1/4)*(e*cot(d*x+c))^(1/2)*2^(1/2)+(e^2)^(1/2)))+2*arctan(2^(1/2)/(e^2)^(1/4)*(e*cot(d*x+c))^(1/2)+1)-2*arctan(-2^(1/2)/(e^2)^(1/4)*(e*cot(d*x+c))^(1/2)+1))+b/d*(-2*(e*cot(d*x+c))^(1/2)+1/4*(e^2)^(1/4)*2^(1/2)*(ln((e*cot(d*x+c)+(e^2)^(1/4)*(e*cot(d*x+c))^(1/2)*2^(1/2)+(e^2)^(1/2))/(e*cot(d*x+c)-(e^2)^(1/4)*(e*cot(d*x+c))^(1/2)*2^(1/2)+(e^2)^(1/2)))+2*arctan(2^(1/2)/(e^2)^(1/4)*(e*cot(d*x+c))^(1/2)+1)-2*arctan(-2^(1/2)/(e^2)^(1/4)*(e*cot(d*x+c))^(1/2)+1)))

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 730 vs. $2(177) = 354$.

Time = 0.27 (sec) , antiderivative size = 730, normalized size of antiderivative = 3.23

$$\int \sqrt{e\cot(c+dx)}(a+b\cot(c+dx))\,dx = \frac{d\sqrt{\frac{2\,abe+d^2\sqrt{-\frac{(a^4-2\,a^2b^2+b^4)e^2}{d^4}}}{d^2}}\log\left(-(a^4-b^4)e\sqrt{\frac{e\cos(2\,dx+2\,c)+e}{\sin(2\,dx+2\,c)}}+\left(ad^3\sqrt{-\frac{(a^4-2\,a^2b^2+b^4)e^2}{d^4}}-(a^2b-b^3)de\right)\sqrt{\ldots}\right.}{\ldots}$$

```
[In] integrate((e*cot(d*x+c))^(1/2)*(a+b*cot(d*x+c)),x, algorithm="fricas")
```

```
[Out] 1/2*(d*sqrt((2*a*b*e + d^2*sqrt(-(a^4 - 2*a^2*b^2 + b^4)*e^2/d^4))/d^2)*log
(-(a^4 - b^4)*e*sqrt((e*cos(2*d*x + 2*c) + e)/sin(2*d*x + 2*c)) + (a*d^3*sq
rt(-(a^4 - 2*a^2*b^2 + b^4)*e^2/d^4) - (a^2*b - b^3)*d*e)*sqrt((2*a*b*e + d
^2*sqrt(-(a^4 - 2*a^2*b^2 + b^4)*e^2/d^4))/d^2)) - d*sqrt((2*a*b*e + d^2*sq
rt(-(a^4 - 2*a^2*b^2 + b^4)*e^2/d^4))/d^2)*log(-(a^4 - b^4)*e*sqrt((e*cos(2
*d*x + 2*c) + e)/sin(2*d*x + 2*c)) - (a*d^3*sqrt(-(a^4 - 2*a^2*b^2 + b^4)*e
^2/d^4) - (a^2*b - b^3)*d*e)*sqrt((2*a*b*e + d^2*sqrt(-(a^4 - 2*a^2*b^2 + b
^4)*e^2/d^4))/d^2)) - d*sqrt((2*a*b*e - d^2*sqrt(-(a^4 - 2*a^2*b^2 + b^4)*e
^2/d^4))/d^2)*log(-(a^4 - b^4)*e*sqrt((e*cos(2*d*x + 2*c) + e)/sin(2*d*x +
2*c)) + (a*d^3*sqrt(-(a^4 - 2*a^2*b^2 + b^4)*e^2/d^4) + (a^2*b - b^3)*d*e)*
sqrt((2*a*b*e - d^2*sqrt(-(a^4 - 2*a^2*b^2 + b^4)*e^2/d^4))/d^2)) + d*sqrt(
(2*a*b*e - d^2*sqrt(-(a^4 - 2*a^2*b^2 + b^4)*e^2/d^4))/d^2)*log(-(a^4 - b^4
)*e*sqrt((e*cos(2*d*x + 2*c) + e)/sin(2*d*x + 2*c)) - (a*d^3*sqrt(-(a^4 - 2
*a^2*b^2 + b^4)*e^2/d^4) + (a^2*b - b^3)*d*e)*sqrt((2*a*b*e - d^2*sqrt(-(a^
4 - 2*a^2*b^2 + b^4)*e^2/d^4))/d^2)) - 4*b*sqrt((e*cos(2*d*x + 2*c) + e)/si
n(2*d*x + 2*c)))/d
```

Sympy [F]

$$\int \sqrt{e\cot(c+dx)}(a+b\cot(c+dx))\,dx = \int \sqrt{e\cot{(c+dx)}}(a+b\cot{(c+dx)})\;dx$$

```
[In] integrate((e*cot(d*x+c))**(1/2)*(a+b*cot(d*x+c)),x)
```

```
[Out] Integral(sqrt(e*cot(c + d*x))*(a + b*cot(c + d*x)), x)
```

Maxima [F(-2)]

Exception generated.

$$\int \sqrt{e\cot(c+dx)}(a+b\cot(c+dx))\,dx = \text{Exception raised: ValueError}$$

```
[In] integrate((e*cot(d*x+c))^(1/2)*(a+b*cot(d*x+c)),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(e>0)', see `assume?` for more detai
ls)Is e
```

Giac [F]

$$\int \sqrt{e\cot(c+dx)}(a+b\cot(c+dx))\,dx = \int (b\cot(dx+c)+a)\sqrt{e\cot(dx+c)}\,dx$$

```
[In] integrate((e*cot(d*x+c))^(1/2)*(a+b*cot(d*x+c)),x, algorithm="giac")
```

```
[Out] integrate((b*cot(d*x + c) + a)*sqrt(e*cot(d*x + c)), x)
```

Mupad [B] (verification not implemented)

Time = 13.49 (sec) , antiderivative size = 128, normalized size of antiderivative = 0.57

$$\begin{aligned}
&\int \sqrt{e\cot(c+dx)}(a+b\cot(c+dx))\,dx \\
&= -\frac{2b\sqrt{e\cot(c+dx)}}{d} \\
&\quad - \frac{(-1)^{1/4}a\sqrt{e}\left(\operatorname{atan}\left(\frac{(-1)^{1/4}\sqrt{e\cot(c+dx)}}{\sqrt{e}}\right)-\operatorname{atanh}\left(\frac{(-1)^{1/4}\sqrt{e\cot(c+dx)}}{\sqrt{e}}\right)\right)}{d} \\
&\quad - \frac{(-1)^{1/4}b\sqrt{e}\operatorname{atan}\left(\frac{(-1)^{1/4}\sqrt{e\cot(c+dx)}}{\sqrt{e}}\right)1\mathrm{i}}{d} - \frac{(-1)^{1/4}b\sqrt{e}\operatorname{atanh}\left(\frac{(-1)^{1/4}\sqrt{e\cot(c+dx)}}{\sqrt{e}}\right)1\mathrm{i}}{d}
\end{aligned}$$

```
[In] int((e*cot(c + d*x))^(1/2)*(a + b*cot(c + d*x)),x)
```

```
[Out] - (2*b*(e*cot(c + d*x))^(1/2))/d - ((-1)^(1/4)*b*e^(1/2)*atan(((-1)^(1/4)*(
e*cot(c + d*x))^(1/2))/e^(1/2))*1i)/d - ((-1)^(1/4)*b*e^(1/2)*atanh(((-1)^(
1/4)*(e*cot(c + d*x))^(1/2))/e^(1/2))*1i)/d - ((-1)^(1/4)*a*e^(1/2)*(atan((
(-1)^(1/4)*(e*cot(c + d*x))^(1/2))/e^(1/2)) - atanh(((-1)^(1/4)*(e*cot(c +
d*x))^(1/2))/e^(1/2))))/d
```

3.53 $\int \frac{a+b\cot(c+dx)}{\sqrt{e\cot(c+dx)}}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 208

$$\int \frac{a+b\cot(c+dx)}{\sqrt{e\cot(c+dx)}}\,dx = \frac{(a+b)\arctan\left(1-\frac{\sqrt{2}\sqrt{e\cot(c+dx)}}{\sqrt{e}}\right)}{\sqrt{2}d\sqrt{e}} - \frac{(a+b)\arctan\left(1+\frac{\sqrt{2}\sqrt{e\cot(c+dx)}}{\sqrt{e}}\right)}{\sqrt{2}d\sqrt{e}} + \frac{(a-b)\log\left(\sqrt{e}+\sqrt{e}\cot(c+dx)-\sqrt{2}\sqrt{e\cot(c+dx)}\right)}{2\sqrt{2}d\sqrt{e}} - \frac{(a-b)\log\left(\sqrt{e}+\sqrt{e}\cot(c+dx)+\sqrt{2}\sqrt{e\cot(c+dx)}\right)}{2\sqrt{2}d\sqrt{e}}$$

```
[Out] 1/2*(a+b)*arctan(1-2^(1/2)*(e*cot(d*x+c))^(1/2)/e^(1/2))/d*2^(1/2)/e^(1/2)-
1/2*(a+b)*arctan(1+2^(1/2)*(e*cot(d*x+c))^(1/2)/e^(1/2))/d*2^(1/2)/e^(1/2)+
1/4*(a-b)*ln(e^(1/2)+cot(d*x+c)*e^(1/2)-2^(1/2)*(e*cot(d*x+c))^(1/2))/d*2^(
1/2)/e^(1/2)-1/4*(a-b)*ln(e^(1/2)+cot(d*x+c)*e^(1/2)+2^(1/2)*(e*cot(d*x+c))
^(1/2))/d*2^(1/2)/e^(1/2)
```

Rubi [A] (verified)

Time = 0.18 (sec) , antiderivative size = 208, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.304$, Rules used = {3615, 1182, 1176, 631, 210, 1179, 642}

$$\int \frac{a+b\cot(c+dx)}{\sqrt{e\cot(c+dx)}}\,dx = \frac{(a+b)\arctan\left(1-\frac{\sqrt{2}\sqrt{e\cot(c+dx)}}{\sqrt{e}}\right)}{\sqrt{2}d\sqrt{e}} - \frac{(a+b)\arctan\left(\frac{\sqrt{2}\sqrt{e\cot(c+dx)}}{\sqrt{e}}+1\right)}{\sqrt{2}d\sqrt{e}} + \frac{(a-b)\log\left(\sqrt{e}\cot(c+dx)-\sqrt{2}\sqrt{e\cot(c+dx)}+\sqrt{e}\right)}{2\sqrt{2}d\sqrt{e}} - \frac{(a-b)\log\left(\sqrt{e}\cot(c+dx)+\sqrt{2}\sqrt{e\cot(c+dx)}+\sqrt{e}\right)}{2\sqrt{2}d\sqrt{e}}$$

```
[In] Int[(a + b*Cot[c + d*x])/Sqrt[e*Cot[c + d*x]],x]
```

```
[Out] ((a + b)*ArcTan[1 - (Sqrt[2]*Sqrt[e*Cot[c + d*x]])/Sqrt[e]])/(Sqrt[2]*d*Sqr
t[e]) - ((a + b)*ArcTan[1 + (Sqrt[2]*Sqrt[e*Cot[c + d*x]])/Sqrt[e]])/(Sqrt[
2]*d*Sqrt[e]) + ((a - b)*Log[Sqrt[e] + Sqrt[e]*Cot[c + d*x] - Sqrt[2]*Sqrt[
e*Cot[c + d*x]]])/(2*Sqrt[2]*d*Sqrt[e]) - ((a - b)*Log[Sqrt[e] + Sqrt[e]*Co
t[c + d*x] + Sqrt[2]*Sqrt[e*Cot[c + d*x]]])/(2*Sqrt[2]*d*Sqrt[e])
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1179

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 1182

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
a*c, 2]}, Dist[(d*q + a*e)/(2*a*c), Int[(q + c*x^2)/(a + c*x^4), x], x] + D
ist[(d*q - a*e)/(2*a*c), Int[(q - c*x^2)/(a + c*x^4), x], x]] /; FreeQ[{a,
c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0] && NeQ[c*d^2 - a*e^2, 0] && NegQ[(-a)
*c]
```

Rule 3615

```
Int[((c_) + (d_.)*tan[(e_.) + (f_.)*(x_)])/Sqrt[(b_.)*tan[(e_.) + (f_.)*(x_
)]], x_Symbol] :> Dist[2/f, Subst[Int[(b*c + d*x^2)/(b^2 + x^4), x], x, Sqr
t[b*Tan[e + f*x]]], x] /; FreeQ[{b, c, d, e, f}, x] && NeQ[c^2 - d^2, 0] &&
 NeQ[c^2 + d^2, 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{2\text{Subst}\left(\int \frac{-ae-bx^2}{e^2+x^4}\,dx, x, \sqrt{e\cot(c+dx)}\right)}{d}\\
&= -\frac{(a-b)\text{Subst}\left(\int \frac{e-x^2}{e^2+x^4}\,dx, x, \sqrt{e\cot(c+dx)}\right)}{d}\\
&\quad -\frac{(a+b)\text{Subst}\left(\int \frac{e+x^2}{e^2+x^4}\,dx, x, \sqrt{e\cot(c+dx)}\right)}{d}
\end{aligned}$$

$$= -\frac{(a+b)\text{Subst}\left(\int \frac{1}{e-\sqrt{2}\sqrt{e}x+x^2}\,dx, x, \sqrt{e\cot(c+dx)}\right)}{2d} - \frac{(a+b)\text{Subst}\left(\int \frac{1}{e+\sqrt{2}\sqrt{e}x+x^2}\,dx, x, \sqrt{e\cot(c+dx)}\right)}{2d} + \frac{(a-b)\text{Subst}\left(\int \frac{\sqrt{2}\sqrt{e}+2x}{-e-\sqrt{2}\sqrt{e}x-x^2}\,dx, x, \sqrt{e\cot(c+dx)}\right)}{2\sqrt{2}d\sqrt{e}} + \frac{(a-b)\text{Subst}\left(\int \frac{\sqrt{2}\sqrt{e}-2x}{-e+\sqrt{2}\sqrt{e}x-x^2}\,dx, x, \sqrt{e\cot(c+dx)}\right)}{2\sqrt{2}d\sqrt{e}}$$

$$= \frac{(a-b)\log\left(\sqrt{e}+\sqrt{e}\cot(c+dx)-\sqrt{2}\sqrt{e\cot(c+dx)}\right)}{2\sqrt{2}d\sqrt{e}} - \frac{(a-b)\log\left(\sqrt{e}+\sqrt{e}\cot(c+dx)+\sqrt{2}\sqrt{e\cot(c+dx)}\right)}{2\sqrt{2}d\sqrt{e}} - \frac{(a+b)\text{Subst}\left(\int \frac{1}{-1-x^2}\,dx, x, 1-\frac{\sqrt{2}\sqrt{e\cot(c+dx)}}{\sqrt{e}}\right)}{\sqrt{2}d\sqrt{e}} + \frac{(a+b)\text{Subst}\left(\int \frac{1}{-1-x^2}\,dx, x, 1+\frac{\sqrt{2}\sqrt{e\cot(c+dx)}}{\sqrt{e}}\right)}{\sqrt{2}d\sqrt{e}}$$

$$= \frac{(a+b)\arctan\left(1-\frac{\sqrt{2}\sqrt{e\cot(c+dx)}}{\sqrt{e}}\right)}{\sqrt{2}d\sqrt{e}} - \frac{(a+b)\arctan\left(1+\frac{\sqrt{2}\sqrt{e\cot(c+dx)}}{\sqrt{e}}\right)}{\sqrt{2}d\sqrt{e}} + \frac{(a-b)\log\left(\sqrt{e}+\sqrt{e}\cot(c+dx)-\sqrt{2}\sqrt{e\cot(c+dx)}\right)}{2\sqrt{2}d\sqrt{e}} - \frac{(a-b)\log\left(\sqrt{e}+\sqrt{e}\cot(c+dx)+\sqrt{2}\sqrt{e\cot(c+dx)}\right)}{2\sqrt{2}d\sqrt{e}}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 0.24 (sec) , antiderivative size = 166, normalized size of antiderivative = 0.80

$$\int \frac{a+b\cot(c+dx)}{\sqrt{e\cot(c+dx)}}\,dx$$

$$= \frac{3\sqrt{2}b\left(-2\arctan\left(1-\sqrt{2}\sqrt{\tan(c+dx)}\right)+2\arctan\left(1+\sqrt{2}\sqrt{\tan(c+dx)}\right)-\log\left(1-\sqrt{2}\sqrt{\tan(c+dx)}\right.\right.}{12d\sqrt{}}$$

```
[In] Integrate[(a + b*Cot[c + d*x])/Sqrt[e*Cot[c + d*x]],x]
```

```
[Out] (3*Sqrt[2]*b*(-2*ArcTan[1 - Sqrt[2]*Sqrt[Tan[c + d*x]]] + 2*ArcTan[1 + Sqrt
[2]*Sqrt[Tan[c + d*x]]] - Log[1 - Sqrt[2]*Sqrt[Tan[c + d*x]] + Tan[c + d*x]
] + Log[1 + Sqrt[2]*Sqrt[Tan[c + d*x]] + Tan[c + d*x]]) + 8*a*Hypergeometri
c2F1[3/4, 1, 7/4, -Tan[c + d*x]^2]*Tan[c + d*x]^(3/2))/(12*d*Sqrt[e*Cot[c +
 d*x]]*Sqrt[Tan[c + d*x]])
```

Maple [A] (verified)

Time = 0.06 (sec) , antiderivative size = 273, normalized size of antiderivative = 1.31

method	result
derivativedivides	$-\frac{\frac{a(e^2)^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\frac{e\cot(dx+c)+(e^2)^{\frac{1}{4}}\sqrt{e\cot(dx+c)}\sqrt{2}+\sqrt{e^2}}{e\cot(dx+c)-(e^2)^{\frac{1}{4}}\sqrt{e\cot(dx+c)}\sqrt{2}+\sqrt{e^2}}\right)+2\arctan\left(\frac{\sqrt{2}\sqrt{e\cot(dx+c)}}{(e^2)^{\frac{1}{4}}}+1\right)-2\arctan\left(-\frac{\sqrt{2}\sqrt{e\cot(dx+c)}}{(e^2)^{\frac{1}{4}}}+1\right)\right.}{4e}}{d}$
default	$-\frac{\frac{a(e^2)^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\frac{e\cot(dx+c)+(e^2)^{\frac{1}{4}}\sqrt{e\cot(dx+c)}\sqrt{2}+\sqrt{e^2}}{e\cot(dx+c)-(e^2)^{\frac{1}{4}}\sqrt{e\cot(dx+c)}\sqrt{2}+\sqrt{e^2}}\right)+2\arctan\left(\frac{\sqrt{2}\sqrt{e\cot(dx+c)}}{(e^2)^{\frac{1}{4}}}+1\right)-2\arctan\left(-\frac{\sqrt{2}\sqrt{e\cot(dx+c)}}{(e^2)^{\frac{1}{4}}}+1\right)\right.}{4e}}{d}$
parts	$-\frac{a(e^2)^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\frac{e\cot(dx+c)+(e^2)^{\frac{1}{4}}\sqrt{e\cot(dx+c)}\sqrt{2}+\sqrt{e^2}}{e\cot(dx+c)-(e^2)^{\frac{1}{4}}\sqrt{e\cot(dx+c)}\sqrt{2}+\sqrt{e^2}}\right)+2\arctan\left(\frac{\sqrt{2}\sqrt{e\cot(dx+c)}}{(e^2)^{\frac{1}{4}}}+1\right)-2\arctan\left(-\frac{\sqrt{2}\sqrt{e\cot(dx}}{(e^2)^{\frac{1}{4}}}\right.\right.}{4de}$

```
[In] int((a+b*cot(d*x+c))/(e*cot(d*x+c))^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(-1/4*a/e*(e^2)^(1/4)*2^(1/2)*(ln((e*cot(d*x+c)+(e^2)^(1/4)*(e*cot(d*x+
c))^(1/2)*2^(1/2)+(e^2)^(1/2))/(e*cot(d*x+c)-(e^2)^(1/4)*(e*cot(d*x+c))^(1/
2)*2^(1/2)+(e^2)^(1/2)))+2*arctan(2^(1/2)/(e^2)^(1/4)*(e*cot(d*x+c))^(1/2)+
1)-2*arctan(-2^(1/2)/(e^2)^(1/4)*(e*cot(d*x+c))^(1/2)+1))-1/4*b/(e^2)^(1/4)
*2^(1/2)*(ln((e*cot(d*x+c)-(e^2)^(1/4)*(e*cot(d*x+c))^(1/2)*2^(1/2)+(e^2)^(
1/2))/(e*cot(d*x+c)+(e^2)^(1/4)*(e*cot(d*x+c))^(1/2)*2^(1/2)+(e^2)^(1/2)))+
2*arctan(2^(1/2)/(e^2)^(1/4)*(e*cot(d*x+c))^(1/2)+1)-2*arctan(-2^(1/2)/(e^2
)^(1/4)*(e*cot(d*x+c))^(1/2)+1)))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 721 vs. $2(161) = 322$.

Time = 0.27 (sec) , antiderivative size = 721, normalized size of antiderivative = 3.47

$$
\begin{aligned}
&\int \frac{a+b\cot(c+dx)}{\sqrt{e\cot(c+dx)}}\,dx \\
&= \frac{1}{2}\sqrt{-\frac{d^2e\sqrt{-\frac{a^4-2\,a^2b^2+b^4}{d^4e^2}}+2\,ab}{d^2e}}\log\left(-(a^4-b^4)\sqrt{\frac{e\cos\left(2\,dx+2\,c\right)+e}{\sin\left(2\,dx+2\,c\right)}}\right. \\
&\quad \left.+\left(bd^3e^2\sqrt{-\frac{a^4-2\,a^2b^2+b^4}{d^4e^2}}+(a^3-ab^2)de\right)\sqrt{-\frac{d^2e\sqrt{-\frac{a^4-2\,a^2b^2+b^4}{d^4e^2}}+2\,ab}{d^2e}}\right) \\
&\quad -\frac{1}{2}\sqrt{-\frac{d^2e\sqrt{-\frac{a^4-2\,a^2b^2+b^4}{d^4e^2}}+2\,ab}{d^2e}}\log\left(-(a^4-b^4)\sqrt{\frac{e\cos\left(2\,dx+2\,c\right)+e}{\sin\left(2\,dx+2\,c\right)}}\right. \\
&\quad \left.-\left(bd^3e^2\sqrt{-\frac{a^4-2\,a^2b^2+b^4}{d^4e^2}}+(a^3-ab^2)de\right)\sqrt{-\frac{d^2e\sqrt{-\frac{a^4-2\,a^2b^2+b^4}{d^4e^2}}+2\,ab}{d^2e}}\right) \\
&\quad -\frac{1}{2}\sqrt{\frac{d^2e\sqrt{-\frac{a^4-2\,a^2b^2+b^4}{d^4e^2}}-2\,ab}{d^2e}}\log\left(-(a^4-b^4)\sqrt{\frac{e\cos\left(2\,dx+2\,c\right)+e}{\sin\left(2\,dx+2\,c\right)}}\right. \\
&\quad \left.+\left(bd^3e^2\sqrt{-\frac{a^4-2\,a^2b^2+b^4}{d^4e^2}}-(a^3-ab^2)de\right)\sqrt{\frac{d^2e\sqrt{-\frac{a^4-2\,a^2b^2+b^4}{d^4e^2}}-2\,ab}{d^2e}}\right) \\
&\quad +\frac{1}{2}\sqrt{\frac{d^2e\sqrt{-\frac{a^4-2\,a^2b^2+b^4}{d^4e^2}}-2\,ab}{d^2e}}\log\left(-(a^4-b^4)\sqrt{\frac{e\cos\left(2\,dx+2\,c\right)+e}{\sin\left(2\,dx+2\,c\right)}}\right. \\
&\quad \left.-\left(bd^3e^2\sqrt{-\frac{a^4-2\,a^2b^2+b^4}{d^4e^2}}-(a^3-ab^2)de\right)\sqrt{\frac{d^2e\sqrt{-\frac{a^4-2\,a^2b^2+b^4}{d^4e^2}}-2\,ab}{d^2e}}\right)
\end{aligned}
$$

```
[In] integrate((a+b*cot(d*x+c))/(e*cot(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] 1/2*sqrt(-(d^2*e*sqrt(-(a^4 - 2*a^2*b^2 + b^4)/(d^4*e^2)) + 2*a*b)/(d^2*e))
*log(-(a^4 - b^4)*sqrt((e*cos(2*d*x + 2*c) + e)/sin(2*d*x + 2*c)) + (b*d^3*
e^2*sqrt(-(a^4 - 2*a^2*b^2 + b^4)/(d^4*e^2)) + (a^3 - a*b^2)*d*e)*sqrt(-(d^
2*e*sqrt(-(a^4 - 2*a^2*b^2 + b^4)/(d^4*e^2)) + 2*a*b)/(d^2*e))) - 1/2*sqrt(
-(d^2*e*sqrt(-(a^4 - 2*a^2*b^2 + b^4)/(d^4*e^2)) + 2*a*b)/(d^2*e))*log(-(a^
4 - b^4)*sqrt((e*cos(2*d*x + 2*c) + e)/sin(2*d*x + 2*c)) - (b*d^3*e^2*sqrt(
-(a^4 - 2*a^2*b^2 + b^4)/(d^4*e^2)) + (a^3 - a*b^2)*d*e)*sqrt(-(d^2*e*sqrt(
-(a^4 - 2*a^2*b^2 + b^4)/(d^4*e^2)) + 2*a*b)/(d^2*e))) - 1/2*sqrt((d^2*e*sq
rt(-(a^4 - 2*a^2*b^2 + b^4)/(d^4*e^2)) - 2*a*b)/(d^2*e))*log(-(a^4 - b^4)*s
qrt((e*cos(2*d*x + 2*c) + e)/sin(2*d*x + 2*c)) + (b*d^3*e^2*sqrt(-(a^4 - 2*
a^2*b^2 + b^4)/(d^4*e^2)) - (a^3 - a*b^2)*d*e)*sqrt((d^2*e*sqrt(-(a^4 - 2*a
^2*b^2 + b^4)/(d^4*e^2)) - 2*a*b)/(d^2*e))) + 1/2*sqrt((d^2*e*sqrt(-(a^4 -
2*a^2*b^2 + b^4)/(d^4*e^2)) - 2*a*b)/(d^2*e))*log(-(a^4 - b^4)*sqrt((e*cos(
2*d*x + 2*c) + e)/sin(2*d*x + 2*c)) - (b*d^3*e^2*sqrt(-(a^4 - 2*a^2*b^2 + b
^4)/(d^4*e^2)) - (a^3 - a*b^2)*d*e)*sqrt((d^2*e*sqrt(-(a^4 - 2*a^2*b^2 + b^
4)/(d^4*e^2)) - 2*a*b)/(d^2*e)))
```

Sympy [F]

$$\int \frac{a+b\cot(c+dx)}{\sqrt{e\cot(c+dx)}}\,dx=\int \frac{a+b\cot\left(c+dx\right)}{\sqrt{e\cot\left(c+dx\right)}}\,dx$$

```
[In] integrate((a+b*cot(d*x+c))/(e*cot(d*x+c))**(1/2),x)
```

```
[Out] Integral((a + b*cot(c + d*x))/sqrt(e*cot(c + d*x)), x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{a+b\cot(c+dx)}{\sqrt{e\cot(c+dx)}}\,dx=\text{Exception raised: ValueError}$$

```
[In] integrate((a+b*cot(d*x+c))/(e*cot(d*x+c))^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(e>0)', see `assume?` for more detai
ls)Is e
```

Giac [F]

$$\int \frac{a+b\cot(c+dx)}{\sqrt{e\cot(c+dx)}}\,dx=\int \frac{b\cot{(dx+c)}+a}{\sqrt{e\cot{(dx+c)}}}\,dx$$

```
[In] integrate((a+b*cot(d*x+c))/(e*cot(d*x+c))^(1/2),x, algorithm="giac")
[Out] integrate((b*cot(d*x + c) + a)/sqrt(e*cot(d*x + c)), x)
```

Mupad [B] (verification not implemented)

Time = 13.20 (sec) , antiderivative size = 118, normalized size of antiderivative = 0.57

$$\begin{aligned}\int \frac{a+b\cot(c+dx)}{\sqrt{e\cot(c+dx)}}\,dx=&\frac{(-1)^{1/4}\,b\,\mathrm{atanh}\left(\frac{(-1)^{1/4}\,\sqrt{e\cot(c+d\,x)}}{\sqrt{e}}\right)}{d\,\sqrt{e}}\\&-\frac{(-1)^{1/4}\,b\,\mathrm{atan}\left(\frac{(-1)^{1/4}\,\sqrt{e\cot(c+d\,x)}}{\sqrt{e}}\right)}{d\,\sqrt{e}}\\&+\frac{(-1)^{1/4}\,a\,\mathrm{atan}\left(\frac{(-1)^{1/4}\,\sqrt{e\cot(c+d\,x)}}{\sqrt{e}}\right)\,1\mathrm{i}}{d\,\sqrt{e}}\\&+\frac{(-1)^{1/4}\,a\,\mathrm{atanh}\left(\frac{(-1)^{1/4}\,\sqrt{e\cot(c+d\,x)}}{\sqrt{e}}\right)\,1\mathrm{i}}{d\,\sqrt{e}}\end{aligned}$$

```
[In] int((a + b*cot(c + d*x))/(e*cot(c + d*x))^(1/2),x)
[Out] ((-1)^(1/4)*a*atan(((-1)^(1/4)*(e*cot(c + d*x))^(1/2))/e^(1/2))*1i)/(d*e^(1
/2)) + ((-1)^(1/4)*a*atanh(((-1)^(1/4)*(e*cot(c + d*x))^(1/2))/e^(1/2))*1i)
/(d*e^(1/2)) - ((-1)^(1/4)*b*atan(((-1)^(1/4)*(e*cot(c + d*x))^(1/2))/e^(1/
2)))/(d*e^(1/2)) + ((-1)^(1/4)*b*atanh(((-1)^(1/4)*(e*cot(c + d*x))^(1/2))/
e^(1/2)))/(d*e^(1/2))
```

3.54 $\int \frac{a+b\cot(c+dx)}{(e\cot(c+dx))^{3/2}}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 229

$$\int \frac{a+b\cot(c+dx)}{(e\cot(c+dx))^{3/2}}\,dx=-\frac{(a-b)\arctan\left(1-\frac{\sqrt{2}\sqrt{e\cot(c+dx)}}{\sqrt{e}}\right)}{\sqrt{2}de^{3/2}}+\frac{(a-b)\arctan\left(1+\frac{\sqrt{2}\sqrt{e\cot(c+dx)}}{\sqrt{e}}\right)}{\sqrt{2}de^{3/2}}+\frac{2a}{de\sqrt{e\cot(c+dx)}}+\frac{(a+b)\log\left(\sqrt{e}+\sqrt{e}\cot(c+dx)-\sqrt{2}\sqrt{e\cot(c+dx)}\right)}{2\sqrt{2}de^{3/2}}-\frac{(a+b)\log\left(\sqrt{e}+\sqrt{e}\cot(c+dx)+\sqrt{2}\sqrt{e\cot(c+dx)}\right)}{2\sqrt{2}de^{3/2}}$$

```
[Out] -1/2*(a-b)*arctan(1-2^(1/2)*(e*cot(d*x+c))^(1/2)/e^(1/2))/d/e^(3/2)*2^(1/2)
+1/2*(a-b)*arctan(1+2^(1/2)*(e*cot(d*x+c))^(1/2)/e^(1/2))/d/e^(3/2)*2^(1/2)
+1/4*(a+b)*ln(e^(1/2)+cot(d*x+c)*e^(1/2)-2^(1/2)*(e*cot(d*x+c))^(1/2))/d/e^
(3/2)*2^(1/2)-1/4*(a+b)*ln(e^(1/2)+cot(d*x+c)*e^(1/2)+2^(1/2)*(e*cot(d*x+c)
)^(1/2))/d/e^(3/2)*2^(1/2)+2*a/d/e/(e*cot(d*x+c))^(1/2)
```

Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 229, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.348, Rules used = {3610, 3615, 1182, 1176, 631, 210, 1179, 642}

$$\int \frac{a+b\cot(c+dx)}{(e\cot(c+dx))^{3/2}}\,dx = -\frac{(a-b)\arctan\left(1-\frac{\sqrt{2}\sqrt{e\cot(c+dx)}}{\sqrt{e}}\right)}{\sqrt{2}de^{3/2}} + \frac{(a-b)\arctan\left(\frac{\sqrt{2}\sqrt{e\cot(c+dx)}}{\sqrt{e}}+1\right)}{\sqrt{2}de^{3/2}} + \frac{(a+b)\log\left(\sqrt{e}\cot(c+dx)-\sqrt{2}\sqrt{e\cot(c+dx)}+\sqrt{e}\right)}{2\sqrt{2}de^{3/2}} - \frac{(a+b)\log\left(\sqrt{e}\cot(c+dx)+\sqrt{2}\sqrt{e\cot(c+dx)}+\sqrt{e}\right)}{2\sqrt{2}de^{3/2}} + \frac{2a}{de\sqrt{e\cot(c+dx)}}$$

```
[In] Int[(a + b*Cot[c + d*x])/(e*Cot[c + d*x])^(3/2),x]
```

```
[Out] -(((a - b)*ArcTan[1 - (Sqrt[2]*Sqrt[e*Cot[c + d*x]])/Sqrt[e]])/(Sqrt[2]*d*e
^(3/2))) + ((a - b)*ArcTan[1 + (Sqrt[2]*Sqrt[e*Cot[c + d*x]])/Sqrt[e]])/(Sq
rt[2]*d*e^(3/2)) + (2*a)/(d*e*Sqrt[e*Cot[c + d*x]]) + ((a + b)*Log[Sqrt[e]
+ Sqrt[e]*Cot[c + d*x] - Sqrt[2]*Sqrt[e*Cot[c + d*x]]])/(2*Sqrt[2]*d*e^(3/2
)) - ((a + b)*Log[Sqrt[e] + Sqrt[e]*Cot[c + d*x] + Sqrt[2]*Sqrt[e*Cot[c + d
*x]]])/(2*Sqrt[2]*d*e^(3/2))
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1179

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 1182

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
a*c, 2]}, Dist[(d*q + a*e)/(2*a*c), Int[(q + c*x^2)/(a + c*x^4), x], x] + D
ist[(d*q - a*e)/(2*a*c), Int[(q - c*x^2)/(a + c*x^4), x], x]] /; FreeQ[{a,
c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0] && NeQ[c*d^2 - a*e^2, 0] && NegQ[(-a)
*c]
```

Rule 3610

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)]), x_Symbol] :> Simp[(b*c - a*d)*((a + b*Tan[e + f*x])^(m + 1)/
(f*(m + 1)*(a^2 + b^2))), x] + Dist[1/(a^2 + b^2), Int[(a + b*Tan[e + f*x])
^(m + 1)*Simp[a*c + b*d - (b*c - a*d)*Tan[e + f*x], x], x], x] /; FreeQ[{a,
 b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && LtQ[m, -1]
```

Rule 3615

```
Int[((c_) + (d_.)*tan[(e_.) + (f_.)*(x_)])/Sqrt[(b_.)*tan[(e_.) + (f_.)*(x_
)]], x_Symbol] :> Dist[2/f, Subst[Int[(b*c + d*x^2)/(b^2 + x^4), x], x, Sqr
t[b*Tan[e + f*x]]], x] /; FreeQ[{b, c, d, e, f}, x] && NeQ[c^2 - d^2, 0] &&
 NeQ[c^2 + d^2, 0]
```

Rubi steps

$$\text{integral} = \frac{2a}{de\sqrt{e\cot(c+dx)}} + \frac{\int \frac{be-ae\cot(c+dx)}{\sqrt{e\cot(c+dx)}}\,dx}{e^2}$$

$$= \frac{2a}{de\sqrt{e\cot(c+dx)}} + \frac{2\text{Subst}\left(\int \frac{-be^2+aex^2}{e^2+x^4}\,dx, x, \sqrt{e\cot(c+dx)}\right)}{de^2}$$

$$
\begin{aligned}
&= \frac{2a}{de\sqrt{e\cot(c+dx)}} + \frac{(a-b)\text{Subst}\left(\int \frac{e+x^2}{e^2+x^4}\,dx, x, \sqrt{e\cot(c+dx)}\right)}{de} \\
&\quad - \frac{(a+b)\text{Subst}\left(\int \frac{e-x^2}{e^2+x^4}\,dx, x, \sqrt{e\cot(c+dx)}\right)}{de} \\
&= \frac{2a}{de\sqrt{e\cot(c+dx)}} + \frac{(a+b)\text{Subst}\left(\int \frac{\sqrt{2}\sqrt{e}+2x}{-e-\sqrt{2}\sqrt{e}x-x^2}\,dx, x, \sqrt{e\cot(c+dx)}\right)}{2\sqrt{2}de^{3/2}} \\
&\quad + \frac{(a+b)\text{Subst}\left(\int \frac{\sqrt{2}\sqrt{e}-2x}{-e+\sqrt{2}\sqrt{e}x-x^2}\,dx, x, \sqrt{e\cot(c+dx)}\right)}{2\sqrt{2}de^{3/2}} \\
&\quad + \frac{(a-b)\text{Subst}\left(\int \frac{1}{e-\sqrt{2}\sqrt{e}x+x^2}\,dx, x, \sqrt{e\cot(c+dx)}\right)}{2de} \\
&\quad + \frac{(a-b)\text{Subst}\left(\int \frac{1}{e+\sqrt{2}\sqrt{e}x+x^2}\,dx, x, \sqrt{e\cot(c+dx)}\right)}{2de} \\
&= \frac{2a}{de\sqrt{e\cot(c+dx)}} + \frac{(a+b)\log\left(\sqrt{e}+\sqrt{e}\cot(c+dx)-\sqrt{2}\sqrt{e\cot(c+dx)}\right)}{2\sqrt{2}de^{3/2}} \\
&\quad - \frac{(a+b)\log\left(\sqrt{e}+\sqrt{e}\cot(c+dx)+\sqrt{2}\sqrt{e\cot(c+dx)}\right)}{2\sqrt{2}de^{3/2}} \\
&\quad + \frac{(a-b)\text{Subst}\left(\int \frac{1}{-1-x^2}\,dx, x, 1-\frac{\sqrt{2}\sqrt{e\cot(c+dx)}}{\sqrt{e}}\right)}{\sqrt{2}de^{3/2}} \\
&\quad - \frac{(a-b)\text{Subst}\left(\int \frac{1}{-1-x^2}\,dx, x, 1+\frac{\sqrt{2}\sqrt{e\cot(c+dx)}}{\sqrt{e}}\right)}{\sqrt{2}de^{3/2}} \\
&= -\frac{(a-b)\arctan\left(1-\frac{\sqrt{2}\sqrt{e\cot(c+dx)}}{\sqrt{e}}\right)}{\sqrt{2}de^{3/2}} + \frac{(a-b)\arctan\left(1+\frac{\sqrt{2}\sqrt{e\cot(c+dx)}}{\sqrt{e}}\right)}{\sqrt{2}de^{3/2}} \\
&\quad + \frac{2a}{de\sqrt{e\cot(c+dx)}} + \frac{(a+b)\log\left(\sqrt{e}+\sqrt{e}\cot(c+dx)-\sqrt{2}\sqrt{e\cot(c+dx)}\right)}{2\sqrt{2}de^{3/2}} \\
&\quad - \frac{(a+b)\log\left(\sqrt{e}+\sqrt{e}\cot(c+dx)+\sqrt{2}\sqrt{e\cot(c+dx)}\right)}{2\sqrt{2}de^{3/2}}
\end{aligned}
$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 0.38 (sec) , antiderivative size = 194, normalized size of antiderivative = 0.85

$$\int \frac{a+b\cot(c+dx)}{(e\cot(c+dx))^{3/2}}\,dx = \frac{3a\Big(2\sqrt{2}\arctan\left(1-\sqrt{2}\sqrt{\tan(c+dx)}\right)-2\sqrt{2}\arctan\left(1+\sqrt{2}\sqrt{\tan(c+dx)}\right)+\cdots}{\cdots}$$

```
[In] Integrate[(a + b*Cot[c + d*x])/(e*Cot[c + d*x])^(3/2),x]
```

```
[Out] (3*a*(2*Sqrt[2]*ArcTan[1 - Sqrt[2]*Sqrt[Tan[c + d*x]]] - 2*Sqrt[2]*ArcTan[1
 + Sqrt[2]*Sqrt[Tan[c + d*x]]] + Sqrt[2]*Log[1 - Sqrt[2]*Sqrt[Tan[c + d*x]]
 + Tan[c + d*x]] - Sqrt[2]*Log[1 + Sqrt[2]*Sqrt[Tan[c + d*x]] + Tan[c + d*x
]] + 8*Sqrt[Tan[c + d*x]]) + 8*b*Hypergeometric2F1[3/4, 1, 7/4, -Tan[c + d*
x]^2]*Tan[c + d*x]^(3/2))/(12*d*(e*Cot[c + d*x])^(3/2)*Tan[c + d*x]^(3/2))
```

Maple [A] (verified)

Time = 0.03 (sec) , antiderivative size = 295, normalized size of antiderivative = 1.29

method	result
derivativedivides	$-\frac{2\left(\frac{b\left(e^2\right)^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\frac{e\cot(dx+c)+\left(e^2\right)^{\frac{1}{4}}\sqrt{e\cot(dx+c)}\,\sqrt{2}+\sqrt{e^2}}{e\cot(dx+c)-\left(e^2\right)^{\frac{1}{4}}\sqrt{e\cot(dx+c)}\,\sqrt{2}+\sqrt{e^2}}\right)+2\arctan\left(\frac{\sqrt{2}\,\sqrt{e\cot(dx+c)}}{\left(e^2\right)^{\frac{1}{4}}}+1\right)-2\arctan\left(-\frac{\sqrt{2}\,\sqrt{e\cot(dx+c)}}{\left(e^2\right)^{\frac{1}{4}}}+\cdots\right.\right.}{8e}\cdots\right)}{\cdots}$
default	$-\frac{2\left(\frac{b\left(e^2\right)^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\frac{e\cot(dx+c)+\left(e^2\right)^{\frac{1}{4}}\sqrt{e\cot(dx+c)}\,\sqrt{2}+\sqrt{e^2}}{e\cot(dx+c)-\left(e^2\right)^{\frac{1}{4}}\sqrt{e\cot(dx+c)}\,\sqrt{2}+\sqrt{e^2}}\right)+2\arctan\left(\frac{\sqrt{2}\,\sqrt{e\cot(dx+c)}}{\left(e^2\right)^{\frac{1}{4}}}+1\right)-2\arctan\left(-\frac{\sqrt{2}\,\sqrt{e\cot(dx+c)}}{\left(e^2\right)^{\frac{1}{4}}}+\cdots\right.\right.}{8e}\cdots\right)}{\cdots}$
parts	$-\frac{2ae\left(-\frac{\sqrt{2}\left(\ln\left(\frac{e\cot(dx+c)-\left(e^2\right)^{\frac{1}{4}}\sqrt{e\cot(dx+c)}\,\sqrt{2}+\sqrt{e^2}}{e\cot(dx+c)+\left(e^2\right)^{\frac{1}{4}}\sqrt{e\cot(dx+c)}\,\sqrt{2}+\sqrt{e^2}}\right)+2\arctan\left(\frac{\sqrt{2}\,\sqrt{e\cot(dx+c)}}{\left(e^2\right)^{\frac{1}{4}}}+1\right)-2\arctan\left(-\frac{\sqrt{2}\,\sqrt{e\cot(dx+c)}}{\left(e^2\right)^{\frac{1}{4}}}+1\right)\cdots\right.}{8e^2\left(e^2\right)^{\frac{1}{4}}}\cdots\right)}{d}$

```
[In] int((a+b*cot(d*x+c))/(e*cot(d*x+c))^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(-2/e*(1/8*b/e*(e^2)^(1/4)*2^(1/2)*(ln((e*cot(d*x+c)+(e^2)^(1/4)*(e*cot
(d*x+c))^(1/2)*2^(1/2)+(e^2)^(1/2))/(e*cot(d*x+c)-(e^2)^(1/4)*(e*cot(d*x+c)
)^(1/2)*2^(1/2)+(e^2)^(1/2)))+2*arctan(2^(1/2)/(e^2)^(1/4)*(e*cot(d*x+c))^(
1/2)+1)-2*arctan(-2^(1/2)/(e^2)^(1/4)*(e*cot(d*x+c))^(1/2)+1))-1/8*a/(e^2)^
```

```
(1/4)*2^(1/2)*(ln((e*cot(d*x+c)-(e^2)^(1/4)*(e*cot(d*x+c))^(1/2)*2^(1/2)+(e
^2)^(1/2))/(e*cot(d*x+c)+(e^2)^(1/4)*(e*cot(d*x+c))^(1/2)*2^(1/2)+(e^2)^(1/
2)))+2*arctan(2^(1/2)/(e^2)^(1/4)*(e*cot(d*x+c))^(1/2)+1)-2*arctan(-2^(1/2)
/(e^2)^(1/4)*(e*cot(d*x+c))^(1/2)+1)))+2*a/e/(e*cot(d*x+c))^(1/2))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 890 vs. $2(180) = 360$.

Time = 0.29 (sec) , antiderivative size = 890, normalized size of antiderivative = 3.89

$$\int \frac{a+b\cot(c+dx)}{(e\cot(c+dx))^{3/2}}\,dx = -\frac{(de^2\cos(2\,dx+2\,c)+de^2)\sqrt{\frac{d^2e^3\sqrt{-\frac{a^4-2\,a^2b^2+b^4}{d^4e^6}}+2\,ab}{d^2e^3}}\log\left(-(a^4-b^4)\sqrt{\frac{e\cos(2\,dx+2\,c)+e}{\sin(2\,dx+2\,c)}}+\left(ad^3e^5\sqrt{-\frac{a^4-2\,a^2b^2-}{d^4e^6}}\right.\right.}{}$$

```
[In] integrate((a+b*cot(d*x+c))/(e*cot(d*x+c))^(3/2),x, algorithm="fricas")
```

```
[Out] -1/2*((d*e^2*cos(2*d*x + 2*c) + d*e^2)*sqrt((d^2*e^3*sqrt(-(a^4 - 2*a^2*b^2
 + b^4)/(d^4*e^6)) + 2*a*b)/(d^2*e^3))*log(-(a^4 - b^4)*sqrt((e*cos(2*d*x +
 2*c) + e)/sin(2*d*x + 2*c)) + (a*d^3*e^5*sqrt(-(a^4 - 2*a^2*b^2 + b^4)/(d^
4*e^6)) - (a^2*b - b^3)*d*e^2)*sqrt((d^2*e^3*sqrt(-(a^4 - 2*a^2*b^2 + b^4)/
(d^4*e^6)) + 2*a*b)/(d^2*e^3))) - (d*e^2*cos(2*d*x + 2*c) + d*e^2)*sqrt((d^
2*e^3*sqrt(-(a^4 - 2*a^2*b^2 + b^4)/(d^4*e^6)) + 2*a*b)/(d^2*e^3))*log(-(a^
4 - b^4)*sqrt((e*cos(2*d*x + 2*c) + e)/sin(2*d*x + 2*c)) - (a*d^3*e^5*sqrt(
-(a^4 - 2*a^2*b^2 + b^4)/(d^4*e^6)) - (a^2*b - b^3)*d*e^2)*sqrt((d^2*e^3*sq
rt(-(a^4 - 2*a^2*b^2 + b^4)/(d^4*e^6)) + 2*a*b)/(d^2*e^3))) - (d*e^2*cos(2*
d*x + 2*c) + d*e^2)*sqrt(-(d^2*e^3*sqrt(-(a^4 - 2*a^2*b^2 + b^4)/(d^4*e^6))
 - 2*a*b)/(d^2*e^3))*log(-(a^4 - b^4)*sqrt((e*cos(2*d*x + 2*c) + e)/sin(2*d
*x + 2*c)) + (a*d^3*e^5*sqrt(-(a^4 - 2*a^2*b^2 + b^4)/(d^4*e^6)) + (a^2*b -
 b^3)*d*e^2)*sqrt(-(d^2*e^3*sqrt(-(a^4 - 2*a^2*b^2 + b^4)/(d^4*e^6)) - 2*a*
b)/(d^2*e^3))) + (d*e^2*cos(2*d*x + 2*c) + d*e^2)*sqrt(-(d^2*e^3*sqrt(-(a^4
 - 2*a^2*b^2 + b^4)/(d^4*e^6)) - 2*a*b)/(d^2*e^3))*log(-(a^4 - b^4)*sqrt((e
*cos(2*d*x + 2*c) + e)/sin(2*d*x + 2*c)) - (a*d^3*e^5*sqrt(-(a^4 - 2*a^2*b^
2 + b^4)/(d^4*e^6)) + (a^2*b - b^3)*d*e^2)*sqrt(-(d^2*e^3*sqrt(-(a^4 - 2*a^
2*b^2 + b^4)/(d^4*e^6)) - 2*a*b)/(d^2*e^3))) - 4*a*sqrt((e*cos(2*d*x + 2*c)
 + e)/sin(2*d*x + 2*c))*sin(2*d*x + 2*c))/(d*e^2*cos(2*d*x + 2*c) + d*e^2)
```

Sympy [F]

$$\int \frac{a+b\cot(c+dx)}{(e\cot(c+dx))^{3/2}}\,dx = \int \frac{a+b\cot\left(c+dx\right)}{\left(e\cot\left(c+dx\right)\right)^{\frac{3}{2}}}\,dx$$

```
[In] integrate((a+b*cot(d*x+c))/(e*cot(d*x+c))**(3/2),x)
[Out] Integral((a + b*cot(c + d*x))/(e*cot(c + d*x))**(3/2), x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{a+b\cot(c+dx)}{(e\cot(c+dx))^{3/2}}\,dx = \text{Exception raised: ValueError}$$

```
[In] integrate((a+b*cot(d*x+c))/(e*cot(d*x+c))^(3/2),x, algorithm="maxima")
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(e>0)', see `assume?` for more detai
ls)Is e
```

Giac [F]

$$\int \frac{a+b\cot(c+dx)}{(e\cot(c+dx))^{3/2}}\,dx = \int \frac{b\cot\left(dx+c\right)+a}{\left(e\cot\left(dx+c\right)\right)^{\frac{3}{2}}}\,dx$$

```
[In] integrate((a+b*cot(d*x+c))/(e*cot(d*x+c))^(3/2),x, algorithm="giac")
[Out] integrate((b*cot(d*x + c) + a)/(e*cot(d*x + c))^(3/2), x)
```

Mupad [B] (verification not implemented)

Time = 13.31 (sec) , antiderivative size = 137, normalized size of antiderivative = 0.60

$$\int \frac{a+b\cot(c+dx)}{(e\cot(c+dx))^{3/2}}\,dx = \frac{2\,a}{d\,e\,\sqrt{e\cot\left(c+d\,x\right)}} + \frac{{\left(-1\right)}^{1/4}\,a\,\mathrm{atan}\left(\frac{{\left(-1\right)}^{1/4}\,\sqrt{e\cot(c+d\,x)}}{\sqrt{e}}\right)}{d\,e^{3/2}} - \frac{{\left(-1\right)}^{1/4}\,a\,\mathrm{atanh}\left(\frac{{\left(-1\right)}^{1/4}\,\sqrt{e\cot(c+d\,x)}}{\sqrt{e}}\right)}{d\,e^{3/2}} + \frac{{\left(-1\right)}^{1/4}\,b\,\mathrm{atan}\left(\frac{{\left(-1\right)}^{1/4}\,\sqrt{e\cot(c+d\,x)}}{\sqrt{e}}\right)\,1{}\mathrm{i}}{d\,e^{3/2}} + \frac{{\left(-1\right)}^{1/4}\,b\,\mathrm{atanh}\left(\frac{{\left(-1\right)}^{1/4}\,\sqrt{e\cot(c+d\,x)}}{\sqrt{e}}\right)\,1{}\mathrm{i}}{d\,e^{3/2}}$$

```
[In] int((a + b*cot(c + d*x))/(e*cot(c + d*x))^(3/2),x)
```

```
[Out] (2*a)/(d*e*(e*cot(c + d*x))^(1/2)) + ((-1)^(1/4)*a*atan(((-1)^(1/4)*(e*cot(
c + d*x))^(1/2))/e^(1/2)))/(d*e^(3/2)) - ((-1)^(1/4)*a*atanh(((-1)^(1/4)*(e
*cot(c + d*x))^(1/2))/e^(1/2)))/(d*e^(3/2)) + ((-1)^(1/4)*b*atan(((-1)^(1/4
)*(e*cot(c + d*x))^(1/2))/e^(1/2))*1i)/(d*e^(3/2)) + ((-1)^(1/4)*b*atanh(((
-1)^(1/4)*(e*cot(c + d*x))^(1/2))/e^(1/2))*1i)/(d*e^(3/2))
```

3.55 $\int \frac{a+b\cot(c+dx)}{(e\cot(c+dx))^{5/2}}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 252

$$\begin{aligned}\int \frac{a+b\cot(c+dx)}{(e\cot(c+dx))^{5/2}}\,dx = &-\frac{(a+b)\arctan\left(1-\frac{\sqrt{2}\sqrt{e\cot(c+dx)}}{\sqrt{e}}\right)}{\sqrt{2}de^{5/2}} \\ &+\frac{(a+b)\arctan\left(1+\frac{\sqrt{2}\sqrt{e\cot(c+dx)}}{\sqrt{e}}\right)}{\sqrt{2}de^{5/2}}+\frac{2a}{3de(e\cot(c+dx))^{3/2}} \\ &+\frac{2b}{de^2\sqrt{e\cot(c+dx)}}-\frac{(a-b)\log\left(\sqrt{e}+\sqrt{e}\cot(c+dx)-\sqrt{2}\sqrt{e\cot(c+dx)}\right)}{2\sqrt{2}de^{5/2}} \\ &+\frac{(a-b)\log\left(\sqrt{e}+\sqrt{e}\cot(c+dx)+\sqrt{2}\sqrt{e\cot(c+dx)}\right)}{2\sqrt{2}de^{5/2}}\end{aligned}$$

```
[Out] 2/3*a/d/e/(e*cot(d*x+c))^(3/2)-1/2*(a+b)*arctan(1-2^(1/2)*(e*cot(d*x+c))^(1
/2)/e^(1/2))/d/e^(5/2)*2^(1/2)+1/2*(a+b)*arctan(1+2^(1/2)*(e*cot(d*x+c))^(1
/2)/e^(1/2))/d/e^(5/2)*2^(1/2)-1/4*(a-b)*ln(e^(1/2)+cot(d*x+c)*e^(1/2)-2^(1
/2)*(e*cot(d*x+c))^(1/2))/d/e^(5/2)*2^(1/2)+1/4*(a-b)*ln(e^(1/2)+cot(d*x+c)
*e^(1/2)+2^(1/2)*(e*cot(d*x+c))^(1/2))/d/e^(5/2)*2^(1/2)+2*b/d/e^2/(e*cot(d
*x+c))^(1/2)
```

Rubi [A] (verified)

Time = 0.32 (sec) , antiderivative size = 252, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.348$, Rules used = {3610, 3615, 1182, 1176, 631, 210, 1179, 642}

$$\int \frac{a+b\cot(c+dx)}{(e\cot(c+dx))^{5/2}}\,dx = -\frac{(a+b)\arctan\left(1-\frac{\sqrt{2}\sqrt{e\cot(c+dx)}}{\sqrt{e}}\right)}{\sqrt{2}de^{5/2}} + \frac{(a+b)\arctan\left(\frac{\sqrt{2}\sqrt{e\cot(c+dx)}}{\sqrt{e}}+1\right)}{\sqrt{2}de^{5/2}} - \frac{(a-b)\log\left(\sqrt{e}\cot(c+dx)-\sqrt{2}\sqrt{e\cot(c+dx)}+\sqrt{e}\right)}{2\sqrt{2}de^{5/2}} + \frac{(a-b)\log\left(\sqrt{e}\cot(c+dx)+\sqrt{2}\sqrt{e\cot(c+dx)}+\sqrt{e}\right)}{2\sqrt{2}de^{5/2}} + \frac{2a}{3de(e\cot(c+dx))^{3/2}} + \frac{2b}{de^2\sqrt{e\cot(c+dx)}}$$

```
[In] Int[(a + b*Cot[c + d*x])/(e*Cot[c + d*x])^(5/2),x]
```

```
[Out] -(((a + b)*ArcTan[1 - (Sqrt[2]*Sqrt[e*Cot[c + d*x]])/Sqrt[e]])/(Sqrt[2]*d*e
^(5/2))) + ((a + b)*ArcTan[1 + (Sqrt[2]*Sqrt[e*Cot[c + d*x]])/Sqrt[e]])/(Sq
rt[2]*d*e^(5/2)) + (2*a)/(3*d*e*(e*Cot[c + d*x])^(3/2)) + (2*b)/(d*e^2*Sqrt
[e*Cot[c + d*x]]) - ((a - b)*Log[Sqrt[e] + Sqrt[e]*Cot[c + d*x] - Sqrt[2]*S
qrt[e*Cot[c + d*x]]])/(2*Sqrt[2]*d*e^(5/2)) + ((a - b)*Log[Sqrt[e] + Sqrt[e
]*Cot[c + d*x] + Sqrt[2]*Sqrt[e*Cot[c + d*x]]])/(2*Sqrt[2]*d*e^(5/2))
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
```

```
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1179

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 1182

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
a*c, 2]}, Dist[(d*q + a*e)/(2*a*c), Int[(q + c*x^2)/(a + c*x^4), x], x] + D
ist[(d*q - a*e)/(2*a*c), Int[(q - c*x^2)/(a + c*x^4), x], x]] /; FreeQ[{a,
c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0] && NeQ[c*d^2 - a*e^2, 0] && NegQ[(-a)
*c]
```

Rule 3610

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)]), x_Symbol] :> Simp[(b*c - a*d)*((a + b*Tan[e + f*x])^(m + 1)/
(f*(m + 1)*(a^2 + b^2))), x] + Dist[1/(a^2 + b^2), Int[(a + b*Tan[e + f*x])
^(m + 1)*Simp[a*c + b*d - (b*c - a*d)*Tan[e + f*x], x], x], x] /; FreeQ[{a,
 b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && LtQ[m, -1]
```

Rule 3615

```
Int[((c_) + (d_.)*tan[(e_.) + (f_.)*(x_)])/Sqrt[(b_.)*tan[(e_.) + (f_.)*(x_
)]], x_Symbol] :> Dist[2/f, Subst[Int[(b*c + d*x^2)/(b^2 + x^4), x], x, Sqr
t[b*Tan[e + f*x]]], x] /; FreeQ[{b, c, d, e, f}, x] && NeQ[c^2 - d^2, 0] &&
 NeQ[c^2 + d^2, 0]
```

Rubi steps

$$\text{integral} = \frac{2a}{3de(e\cot(c+dx))^{3/2}} + \frac{\int \frac{be-ae\cot(c+dx)}{(e\cot(c+dx))^{3/2}}\,dx}{e^2}$$

$$= \frac{2a}{3de(e\cot(c+dx))^{3/2}} + \frac{2b}{de^2\sqrt{e\cot(c+dx)}} + \frac{\int \frac{-ae^2-be^2\cot(c+dx)}{\sqrt{e\cot(c+dx)}}\,dx}{e^4}$$

$$= \frac{2a}{3de(e\cot(c+dx))^{3/2}} + \frac{2b}{de^2\sqrt{e\cot(c+dx)}} + \frac{2\text{Subst}\left(\int \frac{ae^3+be^2x^2}{e^2+x^4}\,dx, x, \sqrt{e\cot(c+dx)}\right)}{de^4}$$

$$= \frac{2a}{3de(e\cot(c+dx))^{3/2}} + \frac{2b}{de^2\sqrt{e\cot(c+dx)}} + \frac{(a-b)\text{Subst}\left(\int \frac{e-x^2}{e^2+x^4}\,dx, x, \sqrt{e\cot(c+dx)}\right)}{de^2} + \frac{(a+b)\text{Subst}\left(\int \frac{e+x^2}{e^2+x^4}\,dx, x, \sqrt{e\cot(c+dx)}\right)}{de^2}$$

$$= \frac{2a}{3de(e\cot(c+dx))^{3/2}} + \frac{2b}{de^2\sqrt{e\cot(c+dx)}} - \frac{(a-b)\text{Subst}\left(\int \frac{\sqrt{2}\sqrt{e}+2x}{-e-\sqrt{2}\sqrt{e}x-x^2}\,dx, x, \sqrt{e\cot(c+dx)}\right)}{2\sqrt{2}de^{5/2}} - \frac{(a-b)\text{Subst}\left(\int \frac{\sqrt{2}\sqrt{e}-2x}{-e+\sqrt{2}\sqrt{e}x-x^2}\,dx, x, \sqrt{e\cot(c+dx)}\right)}{2\sqrt{2}de^{5/2}} + \frac{(a+b)\text{Subst}\left(\int \frac{1}{e-\sqrt{2}\sqrt{e}x+x^2}\,dx, x, \sqrt{e\cot(c+dx)}\right)}{2de^2} + \frac{(a+b)\text{Subst}\left(\int \frac{1}{e+\sqrt{2}\sqrt{e}x+x^2}\,dx, x, \sqrt{e\cot(c+dx)}\right)}{2de^2}$$

$$= \frac{2a}{3de(e\cot(c+dx))^{3/2}} + \frac{2b}{de^2\sqrt{e\cot(c+dx)}} - \frac{(a-b)\log\left(\sqrt{e}+\sqrt{e}\cot(c+dx)-\sqrt{2}\sqrt{e\cot(c+dx)}\right)}{2\sqrt{2}de^{5/2}} + \frac{(a-b)\log\left(\sqrt{e}+\sqrt{e}\cot(c+dx)+\sqrt{2}\sqrt{e\cot(c+dx)}\right)}{2\sqrt{2}de^{5/2}} + \frac{(a+b)\text{Subst}\left(\int \frac{1}{-1-x^2}\,dx, x, 1-\frac{\sqrt{2}\sqrt{e\cot(c+dx)}}{\sqrt{e}}\right)}{\sqrt{2}de^{5/2}} - \frac{(a+b)\text{Subst}\left(\int \frac{1}{-1-x^2}\,dx, x, 1+\frac{\sqrt{2}\sqrt{e\cot(c+dx)}}{\sqrt{e}}\right)}{\sqrt{2}de^{5/2}}$$

$$= -\frac{(a+b)\arctan\left(1-\frac{\sqrt{2}\sqrt{e\cot(c+dx)}}{\sqrt{e}}\right)}{\sqrt{2}de^{5/2}} + \frac{(a+b)\arctan\left(1+\frac{\sqrt{2}\sqrt{e\cot(c+dx)}}{\sqrt{e}}\right)}{\sqrt{2}de^{5/2}} + \frac{2a}{3de(e\cot(c+dx))^{3/2}} + \frac{2b}{de^2\sqrt{e\cot(c+dx)}} - \frac{(a-b)\log\left(\sqrt{e}+\sqrt{e}\cot(c+dx)-\sqrt{2}\sqrt{e\cot(c+dx)}\right)}{2\sqrt{2}de^{5/2}} + \frac{(a-b)\log\left(\sqrt{e}+\sqrt{e}\cot(c+dx)+\sqrt{2}\sqrt{e\cot(c+dx)}\right)}{2\sqrt{2}de^{5/2}}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 0.75 (sec) , antiderivative size = 196, normalized size of antiderivative = 0.78

$$\int \frac{a+b\cot(c+dx)}{(e\cot(c+dx))^{5/2}}\,dx = \frac{3b\Big(2\sqrt{2}\arctan\left(1-\sqrt{2}\sqrt{\tan(c+dx)}\right)-2\sqrt{2}\arctan\left(1+\sqrt{2}\sqrt{\tan(c+dx)}\right)+}{}$$

```
[In] Integrate[(a + b*Cot[c + d*x])/(e*Cot[c + d*x])^(5/2),x]
```

```
[Out] (3*b*(2*Sqrt[2]*ArcTan[1 - Sqrt[2]*Sqrt[Tan[c + d*x]]] - 2*Sqrt[2]*ArcTan[1
 + Sqrt[2]*Sqrt[Tan[c + d*x]]] + Sqrt[2]*Log[1 - Sqrt[2]*Sqrt[Tan[c + d*x]]
 + Tan[c + d*x]] - Sqrt[2]*Log[1 + Sqrt[2]*Sqrt[Tan[c + d*x]] + Tan[c + d*x
]] + 8*Sqrt[Tan[c + d*x]]) - 8*a*(-1 + Hypergeometric2F1[3/4, 1, 7/4, -Tan[
c + d*x]^2])*Tan[c + d*x]^(3/2))/(12*d*(e*Cot[c + d*x])^(5/2)*Tan[c + d*x]^
(5/2))
```

Maple [A] (verified)

Time = 0.03 (sec) , antiderivative size = 311, normalized size of antiderivative = 1.23

method	result
derivativedivides	$-\frac{2\left(-\frac{a\left(e^{2}\right)^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\frac{e\cot(dx+c)+\left(e^{2}\right)^{\frac{1}{4}}\sqrt{e\cot(dx+c)}\sqrt{2}+\sqrt{e^{2}}}{e\cot(dx+c)-\left(e^{2}\right)^{\frac{1}{4}}\sqrt{e\cot(dx+c)}\sqrt{2}+\sqrt{e^{2}}}\right)+2\arctan\left(\frac{\sqrt{2}\sqrt{e\cot(dx+c)}}{\left(e^{2}\right)^{\frac{1}{4}}}+1\right)-2\arctan\left(-\frac{\sqrt{2}\sqrt{e\cot(dx-}}{\left(e^{2}\right)^{\frac{1}{4}}}\right.\right.}{8e}\right.}{}$
default	$-\frac{2\left(-\frac{a\left(e^{2}\right)^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\frac{e\cot(dx+c)+\left(e^{2}\right)^{\frac{1}{4}}\sqrt{e\cot(dx+c)}\sqrt{2}+\sqrt{e^{2}}}{e\cot(dx+c)-\left(e^{2}\right)^{\frac{1}{4}}\sqrt{e\cot(dx+c)}\sqrt{2}+\sqrt{e^{2}}}\right)+2\arctan\left(\frac{\sqrt{2}\sqrt{e\cot(dx+c)}}{\left(e^{2}\right)^{\frac{1}{4}}}+1\right)-2\arctan\left(-\frac{\sqrt{2}\sqrt{e\cot(dx-}}{\left(e^{2}\right)^{\frac{1}{4}}}\right.\right.}{8e}\right.}{}$
parts	$-\frac{2ae\left(-\frac{\left(e^{2}\right)^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\frac{e\cot(dx+c)+\left(e^{2}\right)^{\frac{1}{4}}\sqrt{e\cot(dx+c)}\sqrt{2}+\sqrt{e^{2}}}{e\cot(dx+c)-\left(e^{2}\right)^{\frac{1}{4}}\sqrt{e\cot(dx+c)}\sqrt{2}+\sqrt{e^{2}}}\right)+2\arctan\left(\frac{\sqrt{2}\sqrt{e\cot(dx+c)}}{\left(e^{2}\right)^{\frac{1}{4}}}+1\right)-2\arctan\left(-\frac{\sqrt{2}\sqrt{e\cot(}}{\left(e^{2}\right)^{\frac{1}{4}}}\right.\right.}{8e^{4}}\right.}{d}$

```
[In] int((a+b*cot(d*x+c))/(e*cot(d*x+c))^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(-2/e^2*(-1/8*a/e*(e^2)^(1/4)*2^(1/2)*(ln((e*cot(d*x+c)+(e^2)^(1/4)*(e*
cot(d*x+c))^(1/2)*2^(1/2)+(e^2)^(1/2))/(e*cot(d*x+c)-(e^2)^(1/4)*(e*cot(d*x
+c))^(1/2)*2^(1/2)+(e^2)^(1/2)))+2*arctan(2^(1/2)/(e^2)^(1/4)*(e*cot(d*x+c)
)^(1/2)+1)-2*arctan(-2^(1/2)/(e^2)^(1/4)*(e*cot(d*x+c))^(1/2)+1))-1/8*b/(e^
2)^(1/4)*2^(1/2)*(ln((e*cot(d*x+c)-(e^2)^(1/4)*(e*cot(d*x+c))^(1/2)*2^(1/2)
+(e^2)^(1/2))/(e*cot(d*x+c)+(e^2)^(1/4)*(e*cot(d*x+c))^(1/2)*2^(1/2)+(e^2)^
(1/2)))+2*arctan(2^(1/2)/(e^2)^(1/4)*(e*cot(d*x+c))^(1/2)+1)-2*arctan(-2^(1
/2)/(e^2)^(1/4)*(e*cot(d*x+c))^(1/2)+1)))+2/3*a/e/(e*cot(d*x+c))^(3/2)+2*b/
e^2/(e*cot(d*x+c))^(1/2))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 905 vs. $2(199) = 398$.

Time = 0.27 (sec) , antiderivative size = 905, normalized size of antiderivative = 3.59

$$\int \frac{a+b\cot(c+dx)}{(e\cot(c+dx))^{5/2}}\,dx =$$

$$-\frac{3\left(de^{3}\cos\left(2\,dx+2\,c\right)+de^{3}\right)\sqrt{-\frac{d^{2}e^{5}\sqrt{-\frac{a^{4}-2\,a^{2}b^{2}+b^{4}}{d^{4}e^{10}}}+2\,ab}{d^{2}e^{5}}}\log\left(-(a^{4}-b^{4})\sqrt{\frac{e\cos(2\,dx+2\,c)+e}{\sin(2\,dx+2\,c)}}+\left(bd^{3}e^{8}\sqrt{-\frac{a^{4}-2\,a}{d^{4}}}\right.\right.}{}$$

```
[In] integrate((a+b*cot(d*x+c))/(e*cot(d*x+c))^(5/2),x, algorithm="fricas")
```

```
[Out] -1/6*(3*(d*e^3*cos(2*d*x + 2*c) + d*e^3)*sqrt(-(d^2*e^5*sqrt(-(a^4 - 2*a^2*
b^2 + b^4)/(d^4*e^10)) + 2*a*b)/(d^2*e^5))*log(-(a^4 - b^4)*sqrt((e*cos(2*d
*x + 2*c) + e)/sin(2*d*x + 2*c)) + (b*d^3*e^8*sqrt(-(a^4 - 2*a^2*b^2 + b^4)
/(d^4*e^10)) + (a^3 - a*b^2)*d*e^3)*sqrt(-(d^2*e^5*sqrt(-(a^4 - 2*a^2*b^2 +
 b^4)/(d^4*e^10)) + 2*a*b)/(d^2*e^5))) - 3*(d*e^3*cos(2*d*x + 2*c) + d*e^3)
*sqrt(-(d^2*e^5*sqrt(-(a^4 - 2*a^2*b^2 + b^4)/(d^4*e^10)) + 2*a*b)/(d^2*e^5
))*log(-(a^4 - b^4)*sqrt((e*cos(2*d*x + 2*c) + e)/sin(2*d*x + 2*c)) - (b*d^
3*e^8*sqrt(-(a^4 - 2*a^2*b^2 + b^4)/(d^4*e^10)) + (a^3 - a*b^2)*d*e^3)*sqrt
(-(d^2*e^5*sqrt(-(a^4 - 2*a^2*b^2 + b^4)/(d^4*e^10)) + 2*a*b)/(d^2*e^5))) -
 3*(d*e^3*cos(2*d*x + 2*c) + d*e^3)*sqrt((d^2*e^5*sqrt(-(a^4 - 2*a^2*b^2 +
b^4)/(d^4*e^10)) - 2*a*b)/(d^2*e^5))*log(-(a^4 - b^4)*sqrt((e*cos(2*d*x + 2
*c) + e)/sin(2*d*x + 2*c)) + (b*d^3*e^8*sqrt(-(a^4 - 2*a^2*b^2 + b^4)/(d^4*
e^10)) - (a^3 - a*b^2)*d*e^3)*sqrt((d^2*e^5*sqrt(-(a^4 - 2*a^2*b^2 + b^4)/(
d^4*e^10)) - 2*a*b)/(d^2*e^5))) + 3*(d*e^3*cos(2*d*x + 2*c) + d*e^3)*sqrt((
d^2*e^5*sqrt(-(a^4 - 2*a^2*b^2 + b^4)/(d^4*e^10)) - 2*a*b)/(d^2*e^5))*log(-
(a^4 - b^4)*sqrt((e*cos(2*d*x + 2*c) + e)/sin(2*d*x + 2*c)) - (b*d^3*e^8*sq
rt(-(a^4 - 2*a^2*b^2 + b^4)/(d^4*e^10)) - (a^3 - a*b^2)*d*e^3)*sqrt((d^2*e^
5*sqrt(-(a^4 - 2*a^2*b^2 + b^4)/(d^4*e^10)) - 2*a*b)/(d^2*e^5))) + 4*(a*cos
(2*d*x + 2*c) - 3*b*sin(2*d*x + 2*c) - a)*sqrt((e*cos(2*d*x + 2*c) + e)/sin
(2*d*x + 2*c)))/(d*e^3*cos(2*d*x + 2*c) + d*e^3)
```

Sympy [F]

$$\int \frac{a + b\cot(c + dx)}{(e\cot(c + dx))^{5/2}}\,dx = \int \frac{a + b\cot\left(c + dx\right)}{\left(e\cot\left(c + dx\right)\right)^{\frac{5}{2}}}\,dx$$

```
[In] integrate((a+b*cot(d*x+c))/(e*cot(d*x+c))**(5/2),x)
```

```
[Out] Integral((a + b*cot(c + d*x))/(e*cot(c + d*x))**(5/2), x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{a + b\cot(c + dx)}{(e\cot(c + dx))^{5/2}}\,dx = \text{Exception raised: ValueError}$$

```
[In] integrate((a+b*cot(d*x+c))/(e*cot(d*x+c))^(5/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(e>0)', see `assume?` for more detai
ls)Is e
```

Giac [F]

$$\int \frac{a+b\cot(c+dx)}{(e\cot(c+dx))^{5/2}}\,dx=\int \frac{b\cot(dx+c)+a}{(e\cot(dx+c))^{\frac{5}{2}}}\,dx$$

```
[In] integrate((a+b*cot(d*x+c))/(e*cot(d*x+c))^(5/2),x, algorithm="giac")
[Out] integrate((b*cot(d*x + c) + a)/(e*cot(d*x + c))^(5/2), x)
```

Mupad [B] (verification not implemented)

Time = 13.33 (sec) , antiderivative size = 158, normalized size of antiderivative = 0.63

$$\int \frac{a+b\cot(c+dx)}{(e\cot(c+dx))^{5/2}}\,dx=\frac{2a}{3de\,(e\cot(c+dx))^{3/2}}+\frac{2b}{de^2\sqrt{e\cot(c+dx)}}+\frac{(-1)^{1/4}\,b\,\mathrm{atan}\left(\frac{(-1)^{1/4}\sqrt{e\cot(c+dx)}}{\sqrt{e}}\right)}{de^{5/2}}-\frac{(-1)^{1/4}\,b\,\mathrm{atanh}\left(\frac{(-1)^{1/4}\sqrt{e\cot(c+dx)}}{\sqrt{e}}\right)}{de^{5/2}}-\frac{(-1)^{1/4}\,a\,\mathrm{atan}\left(\frac{(-1)^{1/4}\sqrt{e\cot(c+dx)}}{\sqrt{e}}\right)\,1\mathrm{i}}{de^{5/2}}-\frac{(-1)^{1/4}\,a\,\mathrm{atanh}\left(\frac{(-1)^{1/4}\sqrt{e\cot(c+dx)}}{\sqrt{e}}\right)\,1\mathrm{i}}{de^{5/2}}$$

```
[In] int((a + b*cot(c + d*x))/(e*cot(c + d*x))^(5/2),x)
[Out] (2*a)/(3*d*e*(e*cot(c + d*x))^(3/2)) + (2*b)/(d*e^2*(e*cot(c + d*x))^(1/2))
 - ((-1)^(1/4)*a*atan(((-1)^(1/4)*(e*cot(c + d*x))^(1/2))/e^(1/2))*1i)/(d*e
^(5/2)) - ((-1)^(1/4)*a*atanh(((-1)^(1/4)*(e*cot(c + d*x))^(1/2))/e^(1/2))*
1i)/(d*e^(5/2)) + ((-1)^(1/4)*b*atan(((-1)^(1/4)*(e*cot(c + d*x))^(1/2))/e^
(1/2)))/(d*e^(5/2)) - ((-1)^(1/4)*b*atanh(((-1)^(1/4)*(e*cot(c + d*x))^(1/2
))/e^(1/2)))/(d*e^(5/2))
```

3.56 $\int (e\cot(c+dx))^{3/2}(a+b\cot(c+dx))^2\,dx$

Optimal result

Integrand size = 25, antiderivative size = 317

$$\begin{aligned}\int (e\cot(c+dx))^{3/2}(a+b\cot(c+dx))^2\,dx = & -\frac{(a^2+2ab-b^2)\,e^{3/2}\arctan\left(1-\frac{\sqrt{2}\sqrt{e\cot(c+dx)}}{\sqrt{e}}\right)}{\sqrt{2}d} \\ & +\frac{(a^2+2ab-b^2)\,e^{3/2}\arctan\left(1+\frac{\sqrt{2}\sqrt{e\cot(c+dx)}}{\sqrt{e}}\right)}{\sqrt{2}d} \\ & -\frac{2(a^2-b^2)\,e\sqrt{e\cot(c+dx)}}{d}-\frac{4ab(e\cot(c+dx))^{3/2}}{3d}-\frac{2b^2(e\cot(c+dx))^{5/2}}{5de} \\ & -\frac{(a^2-2ab-b^2)\,e^{3/2}\log\left(\sqrt{e}+\sqrt{e}\cot(c+dx)-\sqrt{2}\sqrt{e\cot(c+dx)}\right)}{2\sqrt{2}d} \\ & +\frac{(a^2-2ab-b^2)\,e^{3/2}\log\left(\sqrt{e}+\sqrt{e}\cot(c+dx)+\sqrt{2}\sqrt{e\cot(c+dx)}\right)}{2\sqrt{2}d}\end{aligned}$$

```
[Out] -4/3*a*b*(e*cot(d*x+c))^(3/2)/d-2/5*b^2*(e*cot(d*x+c))^(5/2)/d/e-1/2*(a^2+2
*a*b-b^2)*e^(3/2)*arctan(1-2^(1/2)*(e*cot(d*x+c))^(1/2)/e^(1/2))/d*2^(1/2)+
1/2*(a^2+2*a*b-b^2)*e^(3/2)*arctan(1+2^(1/2)*(e*cot(d*x+c))^(1/2)/e^(1/2))/
d*2^(1/2)-1/4*(a^2-2*a*b-b^2)*e^(3/2)*ln(e^(1/2)+cot(d*x+c)*e^(1/2)-2^(1/2)
*(e*cot(d*x+c))^(1/2))/d*2^(1/2)+1/4*(a^2-2*a*b-b^2)*e^(3/2)*ln(e^(1/2)+cot
(d*x+c)*e^(1/2)+2^(1/2)*(e*cot(d*x+c))^(1/2))/d*2^(1/2)-2*(a^2-b^2)*e*(e*co
t(d*x+c))^(1/2)/d
```

Rubi [A] (verified)

Time = 0.38 (sec) , antiderivative size = 317, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.360, Rules used = {3624, 3609, 3615, 1182, 1176, 631, 210, 1179, 642}

$$\int (e\cot(c+dx))^{3/2}(a+b\cot(c+dx))^2\,dx = -\frac{e^{3/2}(a^2+2ab-b^2)\arctan\left(1-\frac{\sqrt{2}\sqrt{e\cot(c+dx)}}{\sqrt{e}}\right)}{\sqrt{2}d} + \frac{e^{3/2}(a^2+2ab-b^2)\arctan\left(\frac{\sqrt{2}\sqrt{e\cot(c+dx)}}{\sqrt{e}}+1\right)}{\sqrt{2}d} - \frac{e^{3/2}(a^2-2ab-b^2)\log\left(\sqrt{e}\cot(c+dx)-\sqrt{2}\sqrt{e\cot(c+dx)}+\sqrt{e}\right)}{2\sqrt{2}d} + \frac{e^{3/2}(a^2-2ab-b^2)\log\left(\sqrt{e}\cot(c+dx)+\sqrt{2}\sqrt{e\cot(c+dx)}+\sqrt{e}\right)}{2\sqrt{2}d} - \frac{2e(a^2-b^2)\sqrt{e\cot(c+dx)}}{d} - \frac{4ab(e\cot(c+dx))^{3/2}}{3d} - \frac{2b^2(e\cot(c+dx))^{5/2}}{5de}$$

```
[In] Int[(e*Cot[c + d*x])^(3/2)*(a + b*Cot[c + d*x])^2,x]
```

```
[Out] -(((a^2 + 2*a*b - b^2)*e^(3/2)*ArcTan[1 - (Sqrt[2]*Sqrt[e*Cot[c + d*x]])/Sq
rt[e]])/(Sqrt[2]*d)) + ((a^2 + 2*a*b - b^2)*e^(3/2)*ArcTan[1 + (Sqrt[2]*Sqr
t[e*Cot[c + d*x]])/Sqrt[e]])/(Sqrt[2]*d) - (2*(a^2 - b^2)*e*Sqrt[e*Cot[c +
d*x]])/d - (4*a*b*(e*Cot[c + d*x])^(3/2))/(3*d) - (2*b^2*(e*Cot[c + d*x])^(
5/2))/(5*d*e) - ((a^2 - 2*a*b - b^2)*e^(3/2)*Log[Sqrt[e] + Sqrt[e]*Cot[c +
d*x] - Sqrt[2]*Sqrt[e*Cot[c + d*x]]])/(2*Sqrt[2]*d) + ((a^2 - 2*a*b - b^2)*
e^(3/2)*Log[Sqrt[e] + Sqrt[e]*Cot[c + d*x] + Sqrt[2]*Sqrt[e*Cot[c + d*x]]])
/(2*Sqrt[2]*d)
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1179

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 1182

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
a*c, 2]}, Dist[(d*q + a*e)/(2*a*c), Int[(q + c*x^2)/(a + c*x^4), x], x] + D
ist[(d*q - a*e)/(2*a*c), Int[(q - c*x^2)/(a + c*x^4), x], x]] /; FreeQ[{a,
c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0] && NeQ[c*d^2 - a*e^2, 0] && NegQ[(-a)
*c]
```

Rule 3609

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)]), x_Symbol] :> Simp[d*((a + b*Tan[e + f*x])^m/(f*m)), x] + Int
[(a + b*Tan[e + f*x])^(m - 1)*Simp[a*c - b*d + (b*c + a*d)*Tan[e + f*x], x]
, x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2,
 0] && GtQ[m, 0]
```

Rule 3615

```
Int[((c_) + (d_.)*tan[(e_.) + (f_.)*(x_)])/Sqrt[(b_.)*tan[(e_.) + (f_.)*(x_
)]], x_Symbol] :> Dist[2/f, Subst[Int[(b*c + d*x^2)/(b^2 + x^4), x], x, Sqr
t[b*Tan[e + f*x]]], x] /; FreeQ[{b, c, d, e, f}, x] && NeQ[c^2 - d^2, 0] &&
 NeQ[c^2 + d^2, 0]
```

Rule 3624

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)])^2, x_Symbol] :> Simp[d^2*((a + b*Tan[e + f*x])^(m + 1)/(b*f*(
```

```
m + 1))), x] + Int[(a + b*Tan[e + f*x])^m*Simp[c^2 - d^2 + 2*c*d*Tan[e + f*
x], x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b*c - a*d, 0] &&  !LeQ
[m, -1] &&  !(EqQ[m, 2] && EqQ[a, 0])
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\frac{2b^2(e\cot(c+dx))^{5/2}}{5de}+\int (e\cot(c+dx))^{3/2}\left(a^2-b^2+2ab\cot(c+dx)\right)\,dx\\
&= -\frac{4ab(e\cot(c+dx))^{3/2}}{3d}-\frac{2b^2(e\cot(c+dx))^{5/2}}{5de}\\
&\quad+\int \sqrt{e\cot(c+dx)}\left(-2abe+\left(a^2-b^2\right)e\cot(c+dx)\right)\,dx\\
&= -\frac{2\left(a^2-b^2\right)e\sqrt{e\cot(c+dx)}}{d}-\frac{4ab(e\cot(c+dx))^{3/2}}{3d}\\
&\quad-\frac{2b^2(e\cot(c+dx))^{5/2}}{5de}+\int \frac{-\left(\left(a^2-b^2\right)e^2\right)-2abe^2\cot(c+dx)}{\sqrt{e\cot(c+dx)}}\,dx\\
&= -\frac{2\left(a^2-b^2\right)e\sqrt{e\cot(c+dx)}}{d}-\frac{4ab(e\cot(c+dx))^{3/2}}{3d}\\
&\quad-\frac{2b^2(e\cot(c+dx))^{5/2}}{5de}+\frac{2\text{Subst}\left(\int \frac{\left(a^2-b^2\right)e^3+2abe^2x^2}{e^2+x^4}\,dx,x,\sqrt{e\cot(c+dx)}\right)}{d}\\
&= -\frac{2\left(a^2-b^2\right)e\sqrt{e\cot(c+dx)}}{d}-\frac{4ab(e\cot(c+dx))^{3/2}}{3d}-\frac{2b^2(e\cot(c+dx))^{5/2}}{5de}\\
&\quad+\frac{\left(\left(a^2-2ab-b^2\right)e^2\right)\text{Subst}\left(\int \frac{e-x^2}{e^2+x^4}\,dx,x,\sqrt{e\cot(c+dx)}\right)}{d}\\
&\quad+\frac{\left(\left(a^2+2ab-b^2\right)e^2\right)\text{Subst}\left(\int \frac{e+x^2}{e^2+x^4}\,dx,x,\sqrt{e\cot(c+dx)}\right)}{d}\\
&= -\frac{2\left(a^2-b^2\right)e\sqrt{e\cot(c+dx)}}{d}-\frac{4ab(e\cot(c+dx))^{3/2}}{3d}-\frac{2b^2(e\cot(c+dx))^{5/2}}{5de}\\
&\quad-\frac{\left(\left(a^2-2ab-b^2\right)e^{3/2}\right)\text{Subst}\left(\int \frac{\sqrt{2}\sqrt{e}+2x}{-e-\sqrt{2}\sqrt{e}x-x^2}\,dx,x,\sqrt{e\cot(c+dx)}\right)}{2\sqrt{2}d}\\
&\quad-\frac{\left(\left(a^2-2ab-b^2\right)e^{3/2}\right)\text{Subst}\left(\int \frac{\sqrt{2}\sqrt{e}-2x}{-e+\sqrt{2}\sqrt{e}x-x^2}\,dx,x,\sqrt{e\cot(c+dx)}\right)}{2\sqrt{2}d}\\
&\quad+\frac{\left(\left(a^2+2ab-b^2\right)e^2\right)\text{Subst}\left(\int \frac{1}{e-\sqrt{2}\sqrt{e}x+x^2}\,dx,x,\sqrt{e\cot(c+dx)}\right)}{2d}\\
&\quad+\frac{\left(\left(a^2+2ab-b^2\right)e^2\right)\text{Subst}\left(\int \frac{1}{e+\sqrt{2}\sqrt{e}x+x^2}\,dx,x,\sqrt{e\cot(c+dx)}\right)}{2d}
\end{aligned}$$

$$= -\frac{2(a^2-b^2)\,e\sqrt{e\cot(c+dx)}}{d} - \frac{4ab(e\cot(c+dx))^{3/2}}{3d} - \frac{2b^2(e\cot(c+dx))^{5/2}}{5de}$$
$$- \frac{(a^2-2ab-b^2)\,e^{3/2}\log\left(\sqrt{e}+\sqrt{e}\cot(c+dx)-\sqrt{2}\sqrt{e\cot(c+dx)}\right)}{2\sqrt{2}d}$$
$$+ \frac{(a^2-2ab-b^2)\,e^{3/2}\log\left(\sqrt{e}+\sqrt{e}\cot(c+dx)+\sqrt{2}\sqrt{e\cot(c+dx)}\right)}{2\sqrt{2}d}$$
$$+ \frac{\left((a^2+2ab-b^2)\,e^{3/2}\right)\mathrm{Subst}\left(\int \frac{1}{-1-x^2}\,dx, x, 1-\frac{\sqrt{2}\sqrt{e\cot(c+dx)}}{\sqrt{e}}\right)}{\sqrt{2}d}$$
$$- \frac{\left((a^2+2ab-b^2)\,e^{3/2}\right)\mathrm{Subst}\left(\int \frac{1}{-1-x^2}\,dx, x, 1+\frac{\sqrt{2}\sqrt{e\cot(c+dx)}}{\sqrt{e}}\right)}{\sqrt{2}d}$$
$$= -\frac{(a^2+2ab-b^2)\,e^{3/2}\arctan\left(1-\frac{\sqrt{2}\sqrt{e\cot(c+dx)}}{\sqrt{e}}\right)}{\sqrt{2}d}$$
$$+ \frac{(a^2+2ab-b^2)\,e^{3/2}\arctan\left(1+\frac{\sqrt{2}\sqrt{e\cot(c+dx)}}{\sqrt{e}}\right)}{\sqrt{2}d}$$
$$- \frac{2(a^2-b^2)\,e\sqrt{e\cot(c+dx)}}{d} - \frac{4ab(e\cot(c+dx))^{3/2}}{3d} - \frac{2b^2(e\cot(c+dx))^{5/2}}{5de}$$
$$- \frac{(a^2-2ab-b^2)\,e^{3/2}\log\left(\sqrt{e}+\sqrt{e}\cot(c+dx)-\sqrt{2}\sqrt{e\cot(c+dx)}\right)}{2\sqrt{2}d}$$
$$+ \frac{(a^2-2ab-b^2)\,e^{3/2}\log\left(\sqrt{e}+\sqrt{e}\cot(c+dx)+\sqrt{2}\sqrt{e\cot(c+dx)}\right)}{2\sqrt{2}d}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 2.17 (sec) , antiderivative size = 224, normalized size of antiderivative = 0.71

$$\int (e\cot(c+dx))^{3/2}(a+b\cot(c+dx))^2\,dx =$$
$$-\frac{(e\cot(c+dx))^{3/2}\left(\frac{2}{5}b^2\cot^{\frac{5}{2}}(c+dx)-\frac{4}{3}ab\cot^{\frac{3}{2}}(c+dx)\left(-1+\text{Hypergeometric2F1}\left(\frac{3}{4},1,\frac{7}{4},-\cot^2(c+dx)\right.\right.\right.}{}$$

```
[In] Integrate[(e*Cot[c + d*x])^(3/2)*(a + b*Cot[c + d*x])^2,x]
```

```
[Out] -(((e*Cot[c + d*x])^(3/2)*((2*b^2*Cot[c + d*x]^(5/2))/5 - (4*a*b*Cot[c + d*
x]^(3/2)*(-1 + Hypergeometric2F1[3/4, 1, 7/4, -Cot[c + d*x]^2]))/3 + ((a^2
- b^2)*(2*Sqrt[2]*ArcTan[1 - Sqrt[2]*Sqrt[Cot[c + d*x]]] - 2*Sqrt[2]*ArcTan
[1 + Sqrt[2]*Sqrt[Cot[c + d*x]]] + 8*Sqrt[Cot[c + d*x]] + Sqrt[2]*Log[1 - S
qrt[2]*Sqrt[Cot[c + d*x]] + Cot[c + d*x]] - Sqrt[2]*Log[1 + Sqrt[2]*Sqrt[Co
t[c + d*x]] + Cot[c + d*x]]))/4))/(d*Cot[c + d*x]^(3/2)))
```

Maple [A] (verified)

Time = 0.03 (sec) , antiderivative size = 360, normalized size of antiderivative = 1.14

method	result
derivativedivides	$-\frac{2\left(\frac{b^2(e\cot(dx+c))^{\frac{5}{2}}}{5}+\frac{2aeb(e\cot(dx+c))^{\frac{3}{2}}}{3}+a^2e^2\sqrt{e\cot(dx+c)}-\sqrt{e\cot(dx+c)}\,b^2e^2-e^3\left(\frac{(a^2e-b^2e)(e^2)^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\frac{e\cot\cdots}{e\cot\cdots}\right.\right.\cdots}{\cdots}\right.\cdots\right)}{\cdots}$
default	$-\frac{2\left(\frac{b^2(e\cot(dx+c))^{\frac{5}{2}}}{5}+\frac{2aeb(e\cot(dx+c))^{\frac{3}{2}}}{3}+a^2e^2\sqrt{e\cot(dx+c)}-\sqrt{e\cot(dx+c)}\,b^2e^2-e^3\left(\frac{(a^2e-b^2e)(e^2)^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\frac{e\cot\cdots}{e\cot\cdots}\right.\right.\cdots}{\cdots}\right.\cdots\right)}{\cdots}$
parts	$-\frac{2a^2e\left(\sqrt{e\cot(dx+c)}-\frac{(e^2)^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\frac{e\cot(dx+c)+(e^2)^{\frac{1}{4}}\sqrt{e\cot(dx+c)}\sqrt{2}+\sqrt{e^2}}{e\cot(dx+c)-(e^2)^{\frac{1}{4}}\sqrt{e\cot(dx+c)}\sqrt{2}+\sqrt{e^2}}\right)+2\arctan\left(\frac{\sqrt{2}\sqrt{e\cot(dx+c)}}{(e^2)^{\frac{1}{4}}}+1\right)-2\arctan\cdots\right.}{8}\cdots\right)}{d}$

```
[In] int((e*cot(d*x+c))^(3/2)*(a+b*cot(d*x+c))^2,x,method=_RETURNVERBOSE)
```

```
[Out] -2/d/e*(1/5*b^2*(e*cot(d*x+c))^(5/2)+2/3*a*e*b*(e*cot(d*x+c))^(3/2)+a^2*e^2
*(e*cot(d*x+c))^(1/2)-(e*cot(d*x+c))^(1/2)*b^2*e^2-e^3*(1/8*(a^2*e-b^2*e)*(
e^2)^(1/4)/e^2*2^(1/2)*(ln((e*cot(d*x+c)+(e^2)^(1/4)*(e*cot(d*x+c))^(1/2)*2
^(1/2)+(e^2)^(1/2))/(e*cot(d*x+c)-(e^2)^(1/4)*(e*cot(d*x+c))^(1/2)*2^(1/2)+
(e^2)^(1/2)))+2*arctan(2^(1/2)/(e^2)^(1/4)*(e*cot(d*x+c))^(1/2)+1)-2*arctan
(-2^(1/2)/(e^2)^(1/4)*(e*cot(d*x+c))^(1/2)+1))+1/4*a*b/(e^2)^(1/4)*2^(1/2)*
(ln((e*cot(d*x+c)-(e^2)^(1/4)*(e*cot(d*x+c))^(1/2)*2^(1/2)+(e^2)^(1/2))/(e*
cot(d*x+c)+(e^2)^(1/4)*(e*cot(d*x+c))^(1/2)*2^(1/2)+(e^2)^(1/2)))+2*arctan(
2^(1/2)/(e^2)^(1/4)*(e*cot(d*x+c))^(1/2)+1)-2*arctan(-2^(1/2)/(e^2)^(1/4)*(
e*cot(d*x+c))^(1/2)+1))))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1313 vs. 2(260) = 520.

Time = 0.30 (sec) , antiderivative size = 1313, normalized size of antiderivative = 4.14

$$\int (e\cot(c+dx))^{3/2}(a+b\cot(c+dx))^2\,dx = \text{Too large to display}$$

```
[In] integrate((e*cot(d*x+c))^(3/2)*(a+b*cot(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] 1/30*(15*(d*cos(2*d*x + 2*c) - d)*sqrt(-(4*(a^3*b - a*b^3)*e^3 + sqrt(-(a^8
 - 12*a^6*b^2 + 38*a^4*b^4 - 12*a^2*b^6 + b^8)*e^6/d^4)*d^2)/d^2)*log((a^8
```

```
- 4*a^6*b^2 - 10*a^4*b^4 - 4*a^2*b^6 + b^8)*e^4*sqrt((e*cos(2*d*x + 2*c) +
e)/sin(2*d*x + 2*c)) + (2*sqrt(-(a^8 - 12*a^6*b^2 + 38*a^4*b^4 - 12*a^2*b^6
 + b^8)*e^6/d^4)*a*b*d^3 + (a^6 - 7*a^4*b^2 + 7*a^2*b^4 - b^6)*d*e^3)*sqrt(
-(4*(a^3*b - a*b^3)*e^3 + sqrt(-(a^8 - 12*a^6*b^2 + 38*a^4*b^4 - 12*a^2*b^6
 + b^8)*e^6/d^4)*d^2)/d^2)) - 15*(d*cos(2*d*x + 2*c) - d)*sqrt(-(4*(a^3*b -
 a*b^3)*e^3 + sqrt(-(a^8 - 12*a^6*b^2 + 38*a^4*b^4 - 12*a^2*b^6 + b^8)*e^6/
d^4)*d^2)/d^2)*log((a^8 - 4*a^6*b^2 - 10*a^4*b^4 - 4*a^2*b^6 + b^8)*e^4*sqr
t((e*cos(2*d*x + 2*c) + e)/sin(2*d*x + 2*c)) - (2*sqrt(-(a^8 - 12*a^6*b^2 +
 38*a^4*b^4 - 12*a^2*b^6 + b^8)*e^6/d^4)*a*b*d^3 + (a^6 - 7*a^4*b^2 + 7*a^2
*b^4 - b^6)*d*e^3)*sqrt(-(4*(a^3*b - a*b^3)*e^3 + sqrt(-(a^8 - 12*a^6*b^2 +
 38*a^4*b^4 - 12*a^2*b^6 + b^8)*e^6/d^4)*d^2)/d^2)) - 15*(d*cos(2*d*x + 2*c
) - d)*sqrt(-(4*(a^3*b - a*b^3)*e^3 - sqrt(-(a^8 - 12*a^6*b^2 + 38*a^4*b^4
- 12*a^2*b^6 + b^8)*e^6/d^4)*d^2)/d^2)*log((a^8 - 4*a^6*b^2 - 10*a^4*b^4 -
4*a^2*b^6 + b^8)*e^4*sqrt((e*cos(2*d*x + 2*c) + e)/sin(2*d*x + 2*c)) + (2*s
qrt(-(a^8 - 12*a^6*b^2 + 38*a^4*b^4 - 12*a^2*b^6 + b^8)*e^6/d^4)*a*b*d^3 -
(a^6 - 7*a^4*b^2 + 7*a^2*b^4 - b^6)*d*e^3)*sqrt(-(4*(a^3*b - a*b^3)*e^3 - s
qrt(-(a^8 - 12*a^6*b^2 + 38*a^4*b^4 - 12*a^2*b^6 + b^8)*e^6/d^4)*d^2)/d^2))
 + 15*(d*cos(2*d*x + 2*c) - d)*sqrt(-(4*(a^3*b - a*b^3)*e^3 - sqrt(-(a^8 -
12*a^6*b^2 + 38*a^4*b^4 - 12*a^2*b^6 + b^8)*e^6/d^4)*d^2)/d^2)*log((a^8 - 4
*a^6*b^2 - 10*a^4*b^4 - 4*a^2*b^6 + b^8)*e^4*sqrt((e*cos(2*d*x + 2*c) + e)/
sin(2*d*x + 2*c)) - (2*sqrt(-(a^8 - 12*a^6*b^2 + 38*a^4*b^4 - 12*a^2*b^6 +
b^8)*e^6/d^4)*a*b*d^3 - (a^6 - 7*a^4*b^2 + 7*a^2*b^4 - b^6)*d*e^3)*sqrt(-(4
*(a^3*b - a*b^3)*e^3 - sqrt(-(a^8 - 12*a^6*b^2 + 38*a^4*b^4 - 12*a^2*b^6 +
b^8)*e^6/d^4)*d^2)/d^2)) + 4*(10*a*b*e*sin(2*d*x + 2*c) - 3*(5*a^2 - 6*b^2)
*e*cos(2*d*x + 2*c) + 3*(5*a^2 - 4*b^2)*e)*sqrt((e*cos(2*d*x + 2*c) + e)/si
n(2*d*x + 2*c)))/(d*cos(2*d*x + 2*c) - d)
```

Sympy [F]

$$\int (e\cot(c+dx))^{3/2}(a+b\cot(c+dx))^2\,dx = \int (e\cot(c+dx))^{\frac{3}{2}}\,(a+b\cot(c+dx))^2\,dx$$

```
[In] integrate((e*cot(d*x+c))**(3/2)*(a+b*cot(d*x+c))**2,x)
[Out] Integral((e*cot(c + d*x))**(3/2)*(a + b*cot(c + d*x))**2, x)
```

Maxima [F(-2)]

Exception generated.

$$\int (e\cot(c+dx))^{3/2}(a+b\cot(c+dx))^2\,dx = \text{Exception raised: ValueError}$$

```
[In] integrate((e*cot(d*x+c))^(3/2)*(a+b*cot(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(e>0)', see `assume?` for more detai
ls)Is e
```

Giac [F]

$$\int (e\cot(c+dx))^{3/2}(a+b\cot(c+dx))^2\,dx = \int (b\cot(dx+c)+a)^2(e\cot(dx+c))^{\frac{3}{2}}\,dx$$

```
[In] integrate((e*cot(d*x+c))^(3/2)*(a+b*cot(d*x+c))^2,x, algorithm="giac")
```

```
[Out] integrate((b*cot(d*x + c) + a)^2*(e*cot(d*x + c))^(3/2), x)
```

Mupad [B] (verification not implemented)

Time = 15.24 (sec) , antiderivative size = 1274, normalized size of antiderivative = 4.02

$$\int (e\cot(c+dx))^{3/2}(a+b\cot(c+dx))^2\,dx = \text{Too large to display}$$

```
[In] int((e*cot(c + d*x))^(3/2)*(a + b*cot(c + d*x))^2,x)
```

```
[Out] atan((a^4*e^6*(e*cot(c + d*x))^(1/2)*((a*b^3*e^3)/d^2 - (b^4*e^3*1i)/(4*d^2
) - (a^4*e^3*1i)/(4*d^2) - (a^3*b*e^3)/d^2 + (a^2*b^2*e^3*3i)/(2*d^2))^(1/2
)*32i)/((a^6*e^8*16i)/d - (b^6*e^8*16i)/d + (32*a*b^5*e^8)/d + (32*a^5*b*e^
8)/d + (a^2*b^4*e^8*112i)/d - (192*a^3*b^3*e^8)/d - (a^4*b^2*e^8*112i)/d) +
 (b^4*e^6*(e*cot(c + d*x))^(1/2)*((a*b^3*e^3)/d^2 - (b^4*e^3*1i)/(4*d^2) -
(a^4*e^3*1i)/(4*d^2) - (a^3*b*e^3)/d^2 + (a^2*b^2*e^3*3i)/(2*d^2))^(1/2)*32
i)/((a^6*e^8*16i)/d - (b^6*e^8*16i)/d + (32*a*b^5*e^8)/d + (32*a^5*b*e^8)/d
 + (a^2*b^4*e^8*112i)/d - (192*a^3*b^3*e^8)/d - (a^4*b^2*e^8*112i)/d) - (a^
2*b^2*e^6*(e*cot(c + d*x))^(1/2)*((a*b^3*e^3)/d^2 - (b^4*e^3*1i)/(4*d^2) -
(a^4*e^3*1i)/(4*d^2) - (a^3*b*e^3)/d^2 + (a^2*b^2*e^3*3i)/(2*d^2))^(1/2)*19
2i)/((a^6*e^8*16i)/d - (b^6*e^8*16i)/d + (32*a*b^5*e^8)/d + (32*a^5*b*e^8)/
d + (a^2*b^4*e^8*112i)/d - (192*a^3*b^3*e^8)/d - (a^4*b^2*e^8*112i)/d))*(-(
a^4*e^3*1i + b^4*e^3*1i - 4*a*b^3*e^3 + 4*a^3*b*e^3 - a^2*b^2*e^3*6i)/(4*d^
```

```
2))^(1/2)*2i + atan((a^4*e^6*(e*cot(c + d*x))^(1/2)*((a^4*e^3*1i)/(4*d^2) +
 (b^4*e^3*1i)/(4*d^2) + (a*b^3*e^3)/d^2 - (a^3*b*e^3)/d^2 - (a^2*b^2*e^3*3i
)/(2*d^2))^(1/2)*32i)/((b^6*e^8*16i)/d - (a^6*e^8*16i)/d + (32*a*b^5*e^8)/d
 + (32*a^5*b*e^8)/d - (a^2*b^4*e^8*112i)/d - (192*a^3*b^3*e^8)/d + (a^4*b^2
*e^8*112i)/d) + (b^4*e^6*(e*cot(c + d*x))^(1/2)*((a^4*e^3*1i)/(4*d^2) + (b^
4*e^3*1i)/(4*d^2) + (a*b^3*e^3)/d^2 - (a^3*b*e^3)/d^2 - (a^2*b^2*e^3*3i)/(2
*d^2))^(1/2)*32i)/((b^6*e^8*16i)/d - (a^6*e^8*16i)/d + (32*a*b^5*e^8)/d + (
32*a^5*b*e^8)/d - (a^2*b^4*e^8*112i)/d - (192*a^3*b^3*e^8)/d + (a^4*b^2*e^8
*112i)/d) - (a^2*b^2*e^6*(e*cot(c + d*x))^(1/2)*((a^4*e^3*1i)/(4*d^2) + (b^
4*e^3*1i)/(4*d^2) + (a*b^3*e^3)/d^2 - (a^3*b*e^3)/d^2 - (a^2*b^2*e^3*3i)/(2
*d^2))^(1/2)*192i)/((b^6*e^8*16i)/d - (a^6*e^8*16i)/d + (32*a*b^5*e^8)/d +
(32*a^5*b*e^8)/d - (a^2*b^4*e^8*112i)/d - (192*a^3*b^3*e^8)/d + (a^4*b^2*e^
8*112i)/d))*((a^4*e^3*1i + b^4*e^3*1i + 4*a*b^3*e^3 - 4*a^3*b*e^3 - a^2*b^2
*e^3*6i)/(4*d^2))^(1/2)*2i - (e*cot(c + d*x))^(1/2)*((2*a^2*e)/d - (2*b^2*e
)/d) - (2*b^2*(e*cot(c + d*x))^(5/2))/(5*d*e) - (4*a*b*(e*cot(c + d*x))^(3/
2))/(3*d)
```

3.57 $\int \sqrt{e\cot(c+dx)}(a+b\cot(c+dx))^2\,dx$

Optimal result

Integrand size = 25, antiderivative size = 288

$$\begin{aligned}\int \sqrt{e\cot(c+dx)}(a+b\cot(c+dx))^2\,dx &= \frac{(a^2-2ab-b^2)\sqrt{e}\arctan\left(1-\frac{\sqrt{2}\sqrt{e\cot(c+dx)}}{\sqrt{e}}\right)}{\sqrt{2}d} \\ &\quad-\frac{(a^2-2ab-b^2)\sqrt{e}\arctan\left(1+\frac{\sqrt{2}\sqrt{e\cot(c+dx)}}{\sqrt{e}}\right)}{\sqrt{2}d} \\ &\quad-\frac{4ab\sqrt{e\cot(c+dx)}}{d}-\frac{2b^2(e\cot(c+dx))^{3/2}}{3de} \\ &\quad-\frac{(a^2+2ab-b^2)\sqrt{e}\log\left(\sqrt{e}+\sqrt{e}\cot(c+dx)-\sqrt{2}\sqrt{e\cot(c+dx)}\right)}{2\sqrt{2}d} \\ &\quad+\frac{(a^2+2ab-b^2)\sqrt{e}\log\left(\sqrt{e}+\sqrt{e}\cot(c+dx)+\sqrt{2}\sqrt{e\cot(c+dx)}\right)}{2\sqrt{2}d}\end{aligned}$$

```
[Out] -2/3*b^2*(e*cot(d*x+c))^(3/2)/d/e+1/2*(a^2-2*a*b-b^2)*arctan(1-2^(1/2)*(e*c
ot(d*x+c))^(1/2)/e^(1/2))*e^(1/2)/d*2^(1/2)-1/2*(a^2-2*a*b-b^2)*arctan(1+2^
(1/2)*(e*cot(d*x+c))^(1/2)/e^(1/2))*e^(1/2)/d*2^(1/2)-1/4*(a^2+2*a*b-b^2)*l
n(e^(1/2)+cot(d*x+c)*e^(1/2)-2^(1/2)*(e*cot(d*x+c))^(1/2))*e^(1/2)/d*2^(1/2
)+1/4*(a^2+2*a*b-b^2)*ln(e^(1/2)+cot(d*x+c)*e^(1/2)+2^(1/2)*(e*cot(d*x+c))^
(1/2))*e^(1/2)/d*2^(1/2)-4*a*b*(e*cot(d*x+c))^(1/2)/d
```

Rubi [A] (verified)

Time = 0.33 (sec) , antiderivative size = 288, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.360, Rules used = {3624, 3609, 3615, 1182, 1176, 631, 210, 1179, 642}

$$\begin{aligned}&\int \sqrt{e\cot(c+dx)}(a+b\cot(c+dx))^2\,dx\\&=\frac{\sqrt{e}(a^2-2ab-b^2)\arctan\left(1-\frac{\sqrt{2}\sqrt{e\cot(c+dx)}}{\sqrt{e}}\right)}{\sqrt{2}d}\\&-\frac{\sqrt{e}(a^2-2ab-b^2)\arctan\left(\frac{\sqrt{2}\sqrt{e\cot(c+dx)}}{\sqrt{e}}+1\right)}{\sqrt{2}d}\\&-\frac{\sqrt{e}(a^2+2ab-b^2)\log\left(\sqrt{e}\cot(c+dx)-\sqrt{2}\sqrt{e\cot(c+dx)}+\sqrt{e}\right)}{2\sqrt{2}d}\\&+\frac{\sqrt{e}(a^2+2ab-b^2)\log\left(\sqrt{e}\cot(c+dx)+\sqrt{2}\sqrt{e\cot(c+dx)}+\sqrt{e}\right)}{2\sqrt{2}d}\\&-\frac{4ab\sqrt{e\cot(c+dx)}}{d}-\frac{2b^2(e\cot(c+dx))^{3/2}}{3de}\end{aligned}$$

```
[In] Int[Sqrt[e*Cot[c + d*x]]*(a + b*Cot[c + d*x])^2,x]
```

```
[Out] ((a^2 - 2*a*b - b^2)*Sqrt[e]*ArcTan[1 - (Sqrt[2]*Sqrt[e*Cot[c + d*x]])/Sqrt
[e]])/(Sqrt[2]*d) - ((a^2 - 2*a*b - b^2)*Sqrt[e]*ArcTan[1 + (Sqrt[2]*Sqrt[e
*Cot[c + d*x]])/Sqrt[e]])/(Sqrt[2]*d) - (4*a*b*Sqrt[e*Cot[c + d*x]])/d - (2
*b^2*(e*Cot[c + d*x])^(3/2))/(3*d*e) - ((a^2 + 2*a*b - b^2)*Sqrt[e]*Log[Sqr
t[e] + Sqrt[e]*Cot[c + d*x] - Sqrt[2]*Sqrt[e*Cot[c + d*x]]])/(2*Sqrt[2]*d)
+ ((a^2 + 2*a*b - b^2)*Sqrt[e]*Log[Sqrt[e] + Sqrt[e]*Cot[c + d*x] + Sqrt[2]
*Sqrt[e*Cot[c + d*x]]])/(2*Sqrt[2]*d)
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1179

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 1182

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
a*c, 2]}, Dist[(d*q + a*e)/(2*a*c), Int[(q + c*x^2)/(a + c*x^4), x], x] + D
ist[(d*q - a*e)/(2*a*c), Int[(q - c*x^2)/(a + c*x^4), x], x]] /; FreeQ[{a,
c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0] && NeQ[c*d^2 - a*e^2, 0] && NegQ[(-a)
*c]
```

Rule 3609

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)]), x_Symbol] :> Simp[d*((a + b*Tan[e + f*x])^m/(f*m)), x] + Int
[(a + b*Tan[e + f*x])^(m - 1)*Simp[a*c - b*d + (b*c + a*d)*Tan[e + f*x], x]
, x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2,
 0] && GtQ[m, 0]
```

Rule 3615

```
Int[((c_) + (d_.)*tan[(e_.) + (f_.)*(x_)])/Sqrt[(b_.)*tan[(e_.) + (f_.)*(x_
)]], x_Symbol] :> Dist[2/f, Subst[Int[(b*c + d*x^2)/(b^2 + x^4), x], x, Sqr
t[b*Tan[e + f*x]]], x] /; FreeQ[{b, c, d, e, f}, x] && NeQ[c^2 - d^2, 0] &&
 NeQ[c^2 + d^2, 0]
```

Rule 3624

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)])^2, x_Symbol] :> Simp[d^2*((a + b*Tan[e + f*x])^(m + 1)/(b*f*(
m + 1))), x] + Int[(a + b*Tan[e + f*x])^m*Simp[c^2 - d^2 + 2*c*d*Tan[e + f*
```

```
x], x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b*c - a*d, 0] && !LeQ
[m, -1] && !(EqQ[m, 2] && EqQ[a, 0])
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\frac{2b^2(e\cot(c+dx))^{3/2}}{3de}+\int \sqrt{e\cot(c+dx)}\left(a^2-b^2+2ab\cot(c+dx)\right)\,dx\\
&= -\frac{4ab\sqrt{e\cot(c+dx)}}{d}-\frac{2b^2(e\cot(c+dx))^{3/2}}{3de}+\int \frac{-2abe+\left(a^2-b^2\right)e\cot(c+dx)}{\sqrt{e\cot(c+dx)}}\,dx\\
&= -\frac{4ab\sqrt{e\cot(c+dx)}}{d}-\frac{2b^2(e\cot(c+dx))^{3/2}}{3de}\\
&\quad+\frac{2\text{Subst}\left(\int \frac{2abe^2-\left(a^2-b^2\right)ex^2}{e^2+x^4}\,dx,x,\sqrt{e\cot(c+dx)}\right)}{d}\\
&= -\frac{4ab\sqrt{e\cot(c+dx)}}{d}-\frac{2b^2(e\cot(c+dx))^{3/2}}{3de}\\
&\quad-\frac{\left(\left(a^2-2ab-b^2\right)e\right)\text{Subst}\left(\int \frac{e+x^2}{e^2+x^4}\,dx,x,\sqrt{e\cot(c+dx)}\right)}{d}\\
&\quad+\frac{\left(\left(a^2+2ab-b^2\right)e\right)\text{Subst}\left(\int \frac{e-x^2}{e^2+x^4}\,dx,x,\sqrt{e\cot(c+dx)}\right)}{d}\\
&= -\frac{4ab\sqrt{e\cot(c+dx)}}{d}-\frac{2b^2(e\cot(c+dx))^{3/2}}{3de}\\
&\quad-\frac{\left(\left(a^2+2ab-b^2\right)\sqrt{e}\right)\text{Subst}\left(\int \frac{\sqrt{2}\sqrt{e}+2x}{-e-\sqrt{2}\sqrt{e}x-x^2}\,dx,x,\sqrt{e\cot(c+dx)}\right)}{2\sqrt{2}d}\\
&\quad-\frac{\left(\left(a^2+2ab-b^2\right)\sqrt{e}\right)\text{Subst}\left(\int \frac{\sqrt{2}\sqrt{e}-2x}{-e+\sqrt{2}\sqrt{e}x-x^2}\,dx,x,\sqrt{e\cot(c+dx)}\right)}{2\sqrt{2}d}\\
&\quad-\frac{\left(\left(a^2-2ab-b^2\right)e\right)\text{Subst}\left(\int \frac{1}{e-\sqrt{2}\sqrt{e}x+x^2}\,dx,x,\sqrt{e\cot(c+dx)}\right)}{2d}\\
&\quad-\frac{\left(\left(a^2-2ab-b^2\right)e\right)\text{Subst}\left(\int \frac{1}{e+\sqrt{2}\sqrt{e}x+x^2}\,dx,x,\sqrt{e\cot(c+dx)}\right)}{2d}
\end{aligned}$$

$$= -\frac{4ab\sqrt{e\cot(c+dx)}}{d} - \frac{2b^2(e\cot(c+dx))^{3/2}}{3de} - \frac{(a^2+2ab-b^2)\sqrt{e}\log\left(\sqrt{e}+\sqrt{e}\cot(c+dx)-\sqrt{2}\sqrt{e\cot(c+dx)}\right)}{2\sqrt{2}d} + \frac{(a^2+2ab-b^2)\sqrt{e}\log\left(\sqrt{e}+\sqrt{e}\cot(c+dx)+\sqrt{2}\sqrt{e\cot(c+dx)}\right)}{2\sqrt{2}d} - \frac{\left((a^2-2ab-b^2)\sqrt{e}\right)\mathrm{Subst}\left(\int\frac{1}{-1-x^2}\,dx,x,1-\frac{\sqrt{2}\sqrt{e\cot(c+dx)}}{\sqrt{e}}\right)}{\sqrt{2}d} + \frac{\left((a^2-2ab-b^2)\sqrt{e}\right)\mathrm{Subst}\left(\int\frac{1}{-1-x^2}\,dx,x,1+\frac{\sqrt{2}\sqrt{e\cot(c+dx)}}{\sqrt{e}}\right)}{\sqrt{2}d}$$

$$= \frac{(a^2-2ab-b^2)\sqrt{e}\arctan\left(1-\frac{\sqrt{2}\sqrt{e\cot(c+dx)}}{\sqrt{e}}\right)}{\sqrt{2}d} - \frac{(a^2-2ab-b^2)\sqrt{e}\arctan\left(1+\frac{\sqrt{2}\sqrt{e\cot(c+dx)}}{\sqrt{e}}\right)}{\sqrt{2}d} - \frac{4ab\sqrt{e\cot(c+dx)}}{d} - \frac{2b^2(e\cot(c+dx))^{3/2}}{3de} - \frac{(a^2+2ab-b^2)\sqrt{e}\log\left(\sqrt{e}+\sqrt{e}\cot(c+dx)-\sqrt{2}\sqrt{e\cot(c+dx)}\right)}{2\sqrt{2}d} + \frac{(a^2+2ab-b^2)\sqrt{e}\log\left(\sqrt{e}+\sqrt{e}\cot(c+dx)+\sqrt{2}\sqrt{e\cot(c+dx)}\right)}{2\sqrt{2}d}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 0.61 (sec) , antiderivative size = 220, normalized size of antiderivative = 0.76

$$\int\sqrt{e\cot(c+dx)}(a+b\cot(c+dx))^2\,dx =$$

$$-\frac{\sqrt{e\cot(c+dx)}\Big(4(a^2-b^2)\cot^{\frac{3}{2}}(c+dx)\,\mathrm{Hypergeometric2F1}\left(\frac{3}{4},1,\frac{7}{4},-\cot^2(c+dx)\right)+b\Big(6\sqrt{2}a\arctan$$

```
[In] Integrate[Sqrt[e*Cot[c + d*x]]*(a + b*Cot[c + d*x])^2,x]
[Out] -1/6*(Sqrt[e*Cot[c + d*x]]*(4*(a^2 - b^2)*Cot[c + d*x]^(3/2)*Hypergeometric
2F1[3/4, 1, 7/4, -Cot[c + d*x]^2] + b*(6*Sqrt[2]*a*ArcTan[1 - Sqrt[2]*Sqrt[
Cot[c + d*x]]] - 6*Sqrt[2]*a*ArcTan[1 + Sqrt[2]*Sqrt[Cot[c + d*x]]] + 24*a*
Sqrt[Cot[c + d*x]] + 4*b*Cot[c + d*x]^(3/2) + 3*Sqrt[2]*a*Log[1 - Sqrt[2]*S
qrt[Cot[c + d*x]] + Cot[c + d*x]] - 3*Sqrt[2]*a*Log[1 + Sqrt[2]*Sqrt[Cot[c
+ d*x]] + Cot[c + d*x]])))/(d*Sqrt[Cot[c + d*x]])
```

Maple [A] (verified)

Time = 0.04 (sec) , antiderivative size = 321, normalized size of antiderivative = 1.11

method	result
derivativedivides	$-\frac{2\left(\frac{b^{2}(e\cot(dx+c))^{\frac{3}{2}}}{3}+2abe\sqrt{e\cot(dx+c)}+e^{2}\left(-\frac{ab\left(e^{2}\right)^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\frac{e\cot(dx+c)+\left(e^{2}\right)^{\frac{1}{4}}\sqrt{e\cot(dx+c)}\,\sqrt{2}+\sqrt{e^{2}}}{e\cot(dx+c)-\left(e^{2}\right)^{\frac{1}{4}}\sqrt{e\cot(dx+c)}\,\sqrt{2}+\sqrt{e^{2}}}\right)+2\arctan\left(\sqrt{\cdots}\right.\right.}{4e}\right.\right.}{\cdots}$
default	$-\frac{2\left(\frac{b^{2}(e\cot(dx+c))^{\frac{3}{2}}}{3}+2abe\sqrt{e\cot(dx+c)}+e^{2}\left(-\frac{ab\left(e^{2}\right)^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\frac{e\cot(dx+c)+\left(e^{2}\right)^{\frac{1}{4}}\sqrt{e\cot(dx+c)}\,\sqrt{2}+\sqrt{e^{2}}}{e\cot(dx+c)-\left(e^{2}\right)^{\frac{1}{4}}\sqrt{e\cot(dx+c)}\,\sqrt{2}+\sqrt{e^{2}}}\right)+2\arctan\left(\sqrt{\cdots}\right.\right.}{4e}\right.\right.}{\cdots}$
parts	$-\frac{a^{2}e\sqrt{2}\left(\ln\left(\frac{e\cot(dx+c)-\left(e^{2}\right)^{\frac{1}{4}}\sqrt{e\cot(dx+c)}\,\sqrt{2}+\sqrt{e^{2}}}{e\cot(dx+c)+\left(e^{2}\right)^{\frac{1}{4}}\sqrt{e\cot(dx+c)}\,\sqrt{2}+\sqrt{e^{2}}}\right)+2\arctan\left(\frac{\sqrt{2}\,\sqrt{e\cot(dx+c)}}{\left(e^{2}\right)^{\frac{1}{4}}}+1\right)-2\arctan\left(-\frac{\sqrt{2}\,\sqrt{e\cot(dx+c)}}{\left(e^{2}\right)^{\frac{1}{4}}}\cdots\right.\right.}{4d\left(e^{2}\right)^{\frac{1}{4}}}$

```
[In] int((e*cot(d*x+c))^(1/2)*(a+b*cot(d*x+c))^2,x,method=_RETURNVERBOSE)
```

```
[Out] -2/d/e*(1/3*b^2*(e*cot(d*x+c))^(3/2)+2*a*b*e*(e*cot(d*x+c))^(1/2)+e^2*(-1/4
*a/e*b*(e^2)^(1/4)*2^(1/2)*(ln((e*cot(d*x+c)+(e^2)^(1/4)*(e*cot(d*x+c))^(1/
2)*2^(1/2)+(e^2)^(1/2))/(e*cot(d*x+c)-(e^2)^(1/4)*(e*cot(d*x+c))^(1/2)*2^(1
/2)+(e^2)^(1/2)))+2*arctan(2^(1/2)/(e^2)^(1/4)*(e*cot(d*x+c))^(1/2)+1)-2*ar
ctan(-2^(1/2)/(e^2)^(1/4)*(e*cot(d*x+c))^(1/2)+1))+1/8*(a^2-b^2)/(e^2)^(1/4
)*2^(1/2)*(ln((e*cot(d*x+c)-(e^2)^(1/4)*(e*cot(d*x+c))^(1/2)*2^(1/2)+(e^2)^
(1/2))/(e*cot(d*x+c)+(e^2)^(1/4)*(e*cot(d*x+c))^(1/2)*2^(1/2)+(e^2)^(1/2)))
+2*arctan(2^(1/2)/(e^2)^(1/4)*(e*cot(d*x+c))^(1/2)+1)-2*arctan(-2^(1/2)/(e^
2)^(1/4)*(e*cot(d*x+c))^(1/2)+1))))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1230 vs. $2(235) = 470$.

Time = 0.29 (sec) , antiderivative size = 1230, normalized size of antiderivative = 4.27

$$\int \sqrt{e\cot(c+dx)}(a+b\cot(c+dx))^2\,dx = \text{Too large to display}$$

```
[In] integrate((e*cot(d*x+c))^(1/2)*(a+b*cot(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] -1/6*(3*d*sqrt((d^2*sqrt(-(a^8 - 12*a^6*b^2 + 38*a^4*b^4 - 12*a^2*b^6 + b^8
)*e^2/d^4) + 4*(a^3*b - a*b^3)*e)/d^2)*log((a^8 - 4*a^6*b^2 - 10*a^4*b^4 -
```

```
4*a^2*b^6 + b^8)*e*sqrt((e*cos(2*d*x + 2*c) + e)/sin(2*d*x + 2*c)) + ((a^2
- b^2)*d^3*sqrt(-(a^8 - 12*a^6*b^2 + 38*a^4*b^4 - 12*a^2*b^6 + b^8)*e^2/d^4
) - 2*(a^5*b - 6*a^3*b^3 + a*b^5)*d*e)*sqrt((d^2*sqrt(-(a^8 - 12*a^6*b^2 +
38*a^4*b^4 - 12*a^2*b^6 + b^8)*e^2/d^4) + 4*(a^3*b - a*b^3)*e)/d^2))*sin(2*
d*x + 2*c) - 3*d*sqrt((d^2*sqrt(-(a^8 - 12*a^6*b^2 + 38*a^4*b^4 - 12*a^2*b^
6 + b^8)*e^2/d^4) + 4*(a^3*b - a*b^3)*e)/d^2)*log((a^8 - 4*a^6*b^2 - 10*a^4
*b^4 - 4*a^2*b^6 + b^8)*e*sqrt((e*cos(2*d*x + 2*c) + e)/sin(2*d*x + 2*c)) -
 ((a^2 - b^2)*d^3*sqrt(-(a^8 - 12*a^6*b^2 + 38*a^4*b^4 - 12*a^2*b^6 + b^8)*
e^2/d^4) - 2*(a^5*b - 6*a^3*b^3 + a*b^5)*d*e)*sqrt((d^2*sqrt(-(a^8 - 12*a^6
*b^2 + 38*a^4*b^4 - 12*a^2*b^6 + b^8)*e^2/d^4) + 4*(a^3*b - a*b^3)*e)/d^2))
*sin(2*d*x + 2*c) - 3*d*sqrt(-(d^2*sqrt(-(a^8 - 12*a^6*b^2 + 38*a^4*b^4 - 1
2*a^2*b^6 + b^8)*e^2/d^4) - 4*(a^3*b - a*b^3)*e)/d^2)*log((a^8 - 4*a^6*b^2
- 10*a^4*b^4 - 4*a^2*b^6 + b^8)*e*sqrt((e*cos(2*d*x + 2*c) + e)/sin(2*d*x +
 2*c)) + ((a^2 - b^2)*d^3*sqrt(-(a^8 - 12*a^6*b^2 + 38*a^4*b^4 - 12*a^2*b^6
 + b^8)*e^2/d^4) + 2*(a^5*b - 6*a^3*b^3 + a*b^5)*d*e)*sqrt(-(d^2*sqrt(-(a^8
 - 12*a^6*b^2 + 38*a^4*b^4 - 12*a^2*b^6 + b^8)*e^2/d^4) - 4*(a^3*b - a*b^3)
*e)/d^2))*sin(2*d*x + 2*c) + 3*d*sqrt(-(d^2*sqrt(-(a^8 - 12*a^6*b^2 + 38*a^
4*b^4 - 12*a^2*b^6 + b^8)*e^2/d^4) - 4*(a^3*b - a*b^3)*e)/d^2)*log((a^8 - 4
*a^6*b^2 - 10*a^4*b^4 - 4*a^2*b^6 + b^8)*e*sqrt((e*cos(2*d*x + 2*c) + e)/si
n(2*d*x + 2*c)) - ((a^2 - b^2)*d^3*sqrt(-(a^8 - 12*a^6*b^2 + 38*a^4*b^4 - 1
2*a^2*b^6 + b^8)*e^2/d^4) + 2*(a^5*b - 6*a^3*b^3 + a*b^5)*d*e)*sqrt(-(d^2*s
qrt(-(a^8 - 12*a^6*b^2 + 38*a^4*b^4 - 12*a^2*b^6 + b^8)*e^2/d^4) - 4*(a^3*b
 - a*b^3)*e)/d^2))*sin(2*d*x + 2*c) + 4*(b^2*cos(2*d*x + 2*c) + 6*a*b*sin(2
*d*x + 2*c) + b^2)*sqrt((e*cos(2*d*x + 2*c) + e)/sin(2*d*x + 2*c)))/(d*sin(
2*d*x + 2*c))
```

Sympy [F]

$$\int \sqrt{e\cot(c+dx)}(a+b\cot(c+dx))^2\,dx = \int \sqrt{e\cot{(c+dx)}}(a+b\cot{(c+dx)})^2\,dx$$

```
[In] integrate((e*cot(d*x+c))**(1/2)*(a+b*cot(d*x+c))**2,x)
[Out] Integral(sqrt(e*cot(c + d*x))*(a + b*cot(c + d*x))**2, x)
```

Maxima [F(-2)]

Exception generated.

$$\int \sqrt{e\cot(c+dx)}(a+b\cot(c+dx))^2\,dx = \text{Exception raised: ValueError}$$

```
[In] integrate((e*cot(d*x+c))^(1/2)*(a+b*cot(d*x+c))^2,x, algorithm="maxima")
```

[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(e>0)', see `assume?` for more detai
ls)Is e

Giac [F]

$$\int \sqrt{e\cot(c+dx)}(a+b\cot(c+dx))^2\,dx = \int (b\cot(dx+c)+a)^2\sqrt{e\cot(dx+c)}\,dx$$

[In] integrate((e*cot(d*x+c))^(1/2)*(a+b*cot(d*x+c))^2,x, algorithm="giac")

[Out] integrate((b*cot(d*x + c) + a)^2*sqrt(e*cot(d*x + c)), x)

Mupad [B] (verification not implemented)

Time = 13.90 (sec) , antiderivative size = 1157, normalized size of antiderivative = 4.02

$$\int \sqrt{e\cot(c+dx)}(a+b\cot(c+dx))^2\,dx = \text{Too large to display}$$

[In] int((e*cot(c + d*x))^(1/2)*(a + b*cot(c + d*x))^2,x)

[Out] atan((a^4*e^4*(e*cot(c + d*x))^(1/2)*((a^4*e*1i)/(4*d^2) + (b^4*e*1i)/(4*d^
2) - (a^2*b^2*e*3i)/(2*d^2) - (a*b^3*e)/d^2 + (a^3*b*e)/d^2)^(1/2)*32i)/((1
6*b^6*e^5)/d - (16*a^6*e^5)/d + (a*b^5*e^5*32i)/d + (a^5*b*e^5*32i)/d - (11
2*a^2*b^4*e^5)/d - (a^3*b^3*e^5*192i)/d + (112*a^4*b^2*e^5)/d) + (b^4*e^4*(
e*cot(c + d*x))^(1/2)*((a^4*e*1i)/(4*d^2) + (b^4*e*1i)/(4*d^2) - (a^2*b^2*e
*3i)/(2*d^2) - (a*b^3*e)/d^2 + (a^3*b*e)/d^2)^(1/2)*32i)/((16*b^6*e^5)/d -
(16*a^6*e^5)/d + (a*b^5*e^5*32i)/d + (a^5*b*e^5*32i)/d - (112*a^2*b^4*e^5)/
d - (a^3*b^3*e^5*192i)/d + (112*a^4*b^2*e^5)/d) - (a^2*b^2*e^4*(e*cot(c + d
x))^(1/2)((a^4*e*1i)/(4*d^2) + (b^4*e*1i)/(4*d^2) - (a^2*b^2*e*3i)/(2*d^2
) - (a*b^3*e)/d^2 + (a^3*b*e)/d^2)^(1/2)*192i)/((16*b^6*e^5)/d - (16*a^6*e^
5)/d + (a*b^5*e^5*32i)/d + (a^5*b*e^5*32i)/d - (112*a^2*b^4*e^5)/d - (a^3*b
^3*e^5*192i)/d + (112*a^4*b^2*e^5)/d))*((a^4*e*1i + b^4*e*1i - a^2*b^2*e*6i
 - 4*a*b^3*e + 4*a^3*b*e)/(4*d^2))^(1/2)*2i - atan((a^4*e^4*(e*cot(c + d*x)
)^(1/2)*((a^2*b^2*e*3i)/(2*d^2) - (b^4*e*1i)/(4*d^2) - (a^4*e*1i)/(4*d^2) -
 (a*b^3*e)/d^2 + (a^3*b*e)/d^2)^(1/2)*32i)/((16*a^6*e^5)/d - (16*b^6*e^5)/d
 + (a*b^5*e^5*32i)/d + (a^5*b*e^5*32i)/d + (112*a^2*b^4*e^5)/d - (a^3*b^3*e
^5*192i)/d - (112*a^4*b^2*e^5)/d) + (b^4*e^4*(e*cot(c + d*x))^(1/2)*((a^2*b
^2*e*3i)/(2*d^2) - (b^4*e*1i)/(4*d^2) - (a^4*e*1i)/(4*d^2) - (a*b^3*e)/d^2
+ (a^3*b*e)/d^2)^(1/2)*32i)/((16*a^6*e^5)/d - (16*b^6*e^5)/d + (a*b^5*e^5*3
2i)/d + (a^5*b*e^5*32i)/d + (112*a^2*b^4*e^5)/d - (a^3*b^3*e^5*192i)/d - (1
12*a^4*b^2*e^5)/d) - (a^2*b^2*e^4*(e*cot(c + d*x))^(1/2)*((a^2*b^2*e*3i)/(2

```
*d^2) - (b^4*e*1i)/(4*d^2) - (a^4*e*1i)/(4*d^2) - (a*b^3*e)/d^2 + (a^3*b*e)
/d^2)^(1/2)*192i)/((16*a^6*e^5)/d - (16*b^6*e^5)/d + (a*b^5*e^5*32i)/d + (a
^5*b*e^5*32i)/d + (112*a^2*b^4*e^5)/d - (a^3*b^3*e^5*192i)/d - (112*a^4*b^2
*e^5)/d))*(-(a^4*e*1i + b^4*e*1i - a^2*b^2*e*6i + 4*a*b^3*e - 4*a^3*b*e)/(4
*d^2))^(1/2)*2i - (2*b^2*(e*cot(c + d*x))^(3/2))/(3*d*e) - (4*a*b*(e*cot(c
+ d*x))^(1/2))/d
```

3.58 $\int \frac{(a+b\cot(c+dx))^2}{\sqrt{e\cot(c+dx)}}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 267

$$\begin{aligned}
\int \frac{(a+b\cot(c+dx))^2}{\sqrt{e\cot(c+dx)}}\,dx
&= \frac{(a^2+2ab-b^2)\arctan\left(1-\frac{\sqrt{2}\sqrt{e\cot(c+dx)}}{\sqrt{e}}\right)}{\sqrt{2}d\sqrt{e}} - \frac{(a^2+2ab-b^2)\arctan\left(1+\frac{\sqrt{2}\sqrt{e\cot(c+dx)}}{\sqrt{e}}\right)}{\sqrt{2}d\sqrt{e}} \\
&\quad - \frac{2b^2\sqrt{e\cot(c+dx)}}{de} + \frac{(a^2-2ab-b^2)\log\left(\sqrt{e}+\sqrt{e}\cot(c+dx)-\sqrt{2}\sqrt{e\cot(c+dx)}\right)}{2\sqrt{2}d\sqrt{e}} \\
&\quad - \frac{(a^2-2ab-b^2)\log\left(\sqrt{e}+\sqrt{e}\cot(c+dx)+\sqrt{2}\sqrt{e\cot(c+dx)}\right)}{2\sqrt{2}d\sqrt{e}}
\end{aligned}$$

```
[Out] 1/2*(a^2+2*a*b-b^2)*arctan(1-2^(1/2)*(e*cot(d*x+c))^(1/2)/e^(1/2))/d*2^(1/2
)/e^(1/2)-1/2*(a^2+2*a*b-b^2)*arctan(1+2^(1/2)*(e*cot(d*x+c))^(1/2)/e^(1/2)
)/d*2^(1/2)/e^(1/2)+1/4*(a^2-2*a*b-b^2)*ln(e^(1/2)+cot(d*x+c)*e^(1/2)-2^(1/
2)*(e*cot(d*x+c))^(1/2))/d*2^(1/2)/e^(1/2)-1/4*(a^2-2*a*b-b^2)*ln(e^(1/2)+c
ot(d*x+c)*e^(1/2)+2^(1/2)*(e*cot(d*x+c))^(1/2))/d*2^(1/2)/e^(1/2)-2*b^2*(e*
cot(d*x+c))^(1/2)/d/e
```

Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 267, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.320, Rules used = {3624, 3615, 1182, 1176, 631, 210, 1179, 642}

$$\int \frac{(a+b\cot(c+dx))^2}{\sqrt{e\cot(c+dx)}}\,dx = \frac{(a^2+2ab-b^2)\arctan\left(1-\frac{\sqrt{2}\sqrt{e\cot(c+dx)}}{\sqrt{e}}\right)}{\sqrt{2}d\sqrt{e}} - \frac{(a^2+2ab-b^2)\arctan\left(\frac{\sqrt{2}\sqrt{e\cot(c+dx)}}{\sqrt{e}}+1\right)}{\sqrt{2}d\sqrt{e}} + \frac{(a^2-2ab-b^2)\log\left(\sqrt{e}\cot(c+dx)-\sqrt{2}\sqrt{e\cot(c+dx)}+\sqrt{e}\right)}{2\sqrt{2}d\sqrt{e}} - \frac{(a^2-2ab-b^2)\log\left(\sqrt{e}\cot(c+dx)+\sqrt{2}\sqrt{e\cot(c+dx)}+\sqrt{e}\right)}{2\sqrt{2}d\sqrt{e}} - \frac{2b^2\sqrt{e\cot(c+dx)}}{de}$$

```
[In] Int[(a + b*Cot[c + d*x])^2/Sqrt[e*Cot[c + d*x]],x]
```

```
[Out] ((a^2 + 2*a*b - b^2)*ArcTan[1 - (Sqrt[2]*Sqrt[e*Cot[c + d*x]])/Sqrt[e]])/(S
qrt[2]*d*Sqrt[e]) - ((a^2 + 2*a*b - b^2)*ArcTan[1 + (Sqrt[2]*Sqrt[e*Cot[c +
 d*x]])/Sqrt[e]])/(Sqrt[2]*d*Sqrt[e]) - (2*b^2*Sqrt[e*Cot[c + d*x]])/(d*e)
+ ((a^2 - 2*a*b - b^2)*Log[Sqrt[e] + Sqrt[e]*Cot[c + d*x] - Sqrt[2]*Sqrt[e*
Cot[c + d*x]]])/(2*Sqrt[2]*d*Sqrt[e]) - ((a^2 - 2*a*b - b^2)*Log[Sqrt[e] +
Sqrt[e]*Cot[c + d*x] + Sqrt[2]*Sqrt[e*Cot[c + d*x]]])/(2*Sqrt[2]*d*Sqrt[e])
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1179

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 1182

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
a*c, 2]}, Dist[(d*q + a*e)/(2*a*c), Int[(q + c*x^2)/(a + c*x^4), x], x] + D
ist[(d*q - a*e)/(2*a*c), Int[(q - c*x^2)/(a + c*x^4), x], x]] /; FreeQ[{a,
c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0] && NeQ[c*d^2 - a*e^2, 0] && NegQ[(-a)
*c]
```

Rule 3615

```
Int[((c_) + (d_.)*tan[(e_.) + (f_.)*(x_)])/Sqrt[(b_.)*tan[(e_.) + (f_.)*(x_
)]], x_Symbol] :> Dist[2/f, Subst[Int[(b*c + d*x^2)/(b^2 + x^4), x], x, Sqr
t[b*Tan[e + f*x]]], x] /; FreeQ[{b, c, d, e, f}, x] && NeQ[c^2 - d^2, 0] &&
 NeQ[c^2 + d^2, 0]
```

Rule 3624

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)])^2, x_Symbol] :> Simp[d^2*((a + b*Tan[e + f*x])^(m + 1)/(b*f*(
m + 1))), x] + Int[(a + b*Tan[e + f*x])^m*Simp[c^2 - d^2 + 2*c*d*Tan[e + f*
x], x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b*c - a*d, 0] &&  !LeQ
[m, -1] &&  !(EqQ[m, 2] && EqQ[a, 0])
```

Rubi steps

$$\text{integral} = -\frac{2b^2\sqrt{e\cot(c+dx)}}{de} + \int \frac{a^2-b^2+2ab\cot(c+dx)}{\sqrt{e\cot(c+dx)}}\,dx$$

$$= -\frac{2b^2\sqrt{e\cot(c+dx)}}{de} + \frac{2\text{Subst}\left(\int \frac{-((a^2-b^2)e)-2abx^2}{e^2+x^4}\,dx, x, \sqrt{e\cot(c+dx)}\right)}{d}$$

$$= -\frac{2b^2\sqrt{e\cot(c+dx)}}{de} - \frac{(a^2-2ab-b^2)\,\text{Subst}\left(\int \frac{e-x^2}{e^2+x^4}\,dx, x, \sqrt{e\cot(c+dx)}\right)}{d} - \frac{(a^2+2ab-b^2)\,\text{Subst}\left(\int \frac{e+x^2}{e^2+x^4}\,dx, x, \sqrt{e\cot(c+dx)}\right)}{d}$$

$$= -\frac{2b^2\sqrt{e\cot(c+dx)}}{de} - \frac{(a^2+2ab-b^2)\,\text{Subst}\left(\int \frac{1}{e-\sqrt{2}\sqrt{e}x+x^2}\,dx, x, \sqrt{e\cot(c+dx)}\right)}{2d} - \frac{(a^2+2ab-b^2)\,\text{Subst}\left(\int \frac{1}{e+\sqrt{2}\sqrt{e}x+x^2}\,dx, x, \sqrt{e\cot(c+dx)}\right)}{2d} + \frac{(a^2-2ab-b^2)\,\text{Subst}\left(\int \frac{\sqrt{2}\sqrt{e}+2x}{-e-\sqrt{2}\sqrt{e}x-x^2}\,dx, x, \sqrt{e\cot(c+dx)}\right)}{2\sqrt{2}d\sqrt{e}} + \frac{(a^2-2ab-b^2)\,\text{Subst}\left(\int \frac{\sqrt{2}\sqrt{e}-2x}{-e+\sqrt{2}\sqrt{e}x-x^2}\,dx, x, \sqrt{e\cot(c+dx)}\right)}{2\sqrt{2}d\sqrt{e}}$$

$$= -\frac{2b^2\sqrt{e\cot(c+dx)}}{de} + \frac{(a^2-2ab-b^2)\log\left(\sqrt{e}+\sqrt{e}\cot(c+dx)-\sqrt{2}\sqrt{e\cot(c+dx)}\right)}{2\sqrt{2}d\sqrt{e}} - \frac{(a^2-2ab-b^2)\log\left(\sqrt{e}+\sqrt{e}\cot(c+dx)+\sqrt{2}\sqrt{e\cot(c+dx)}\right)}{2\sqrt{2}d\sqrt{e}} - \frac{(a^2+2ab-b^2)\,\text{Subst}\left(\int \frac{1}{-1-x^2}\,dx, x, 1-\frac{\sqrt{2}\sqrt{e\cot(c+dx)}}{\sqrt{e}}\right)}{\sqrt{2}d\sqrt{e}} + \frac{(a^2+2ab-b^2)\,\text{Subst}\left(\int \frac{1}{-1-x^2}\,dx, x, 1+\frac{\sqrt{2}\sqrt{e\cot(c+dx)}}{\sqrt{e}}\right)}{\sqrt{2}d\sqrt{e}}$$

$$= \frac{(a^2+2ab-b^2)\arctan\left(1-\frac{\sqrt{2}\sqrt{e\cot(c+dx)}}{\sqrt{e}}\right)}{\sqrt{2}d\sqrt{e}} - \frac{(a^2+2ab-b^2)\arctan\left(1+\frac{\sqrt{2}\sqrt{e\cot(c+dx)}}{\sqrt{e}}\right)}{\sqrt{2}d\sqrt{e}} - \frac{2b^2\sqrt{e\cot(c+dx)}}{de} + \frac{(a^2-2ab-b^2)\log\left(\sqrt{e}+\sqrt{e}\cot(c+dx)-\sqrt{2}\sqrt{e\cot(c+dx)}\right)}{2\sqrt{2}d\sqrt{e}} - \frac{(a^2-2ab-b^2)\log\left(\sqrt{e}+\sqrt{e}\cot(c+dx)+\sqrt{2}\sqrt{e\cot(c+dx)}\right)}{2\sqrt{2}d\sqrt{e}}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 0.94 (sec) , antiderivative size = 192, normalized size of antiderivative = 0.72

$$\int \frac{(a+b\cot(c+dx))^2}{\sqrt{e\cot(c+dx)}}\,dx =$$

$$-\frac{\sqrt{\cot(c+dx)}\left(2b^2\sqrt{\cot(c+dx)}+\frac{4}{3}ab\cot^{\frac{3}{2}}(c+dx)\,\text{Hypergeometric2F1}\left(\frac{3}{4},1,\frac{7}{4},-\cot^2(c+dx)\right)-\frac{(a^2-\cdots}{\cdots}\right.}{d\sqrt{e\,\mathrm{co}\cdots}}$$

```
[In] Integrate[(a + b*Cot[c + d*x])^2/Sqrt[e*Cot[c + d*x]],x]
```

```
[Out] -((Sqrt[Cot[c + d*x]]*(2*b^2*Sqrt[Cot[c + d*x]] + (4*a*b*Cot[c + d*x]^(3/2)
*Hypergeometric2F1[3/4, 1, 7/4, -Cot[c + d*x]^2])/3 - ((a^2 - b^2)*(2*ArcTa
n[1 - Sqrt[2]*Sqrt[Cot[c + d*x]]] - 2*ArcTan[1 + Sqrt[2]*Sqrt[Cot[c + d*x]]
] + Log[1 - Sqrt[2]*Sqrt[Cot[c + d*x]] + Cot[c + d*x]] - Log[1 + Sqrt[2]*Sq
rt[Cot[c + d*x]] + Cot[c + d*x]]))/(2*Sqrt[2])))/(d*Sqrt[e*Cot[c + d*x]]))
```

Maple [A] (verified)

Time = 0.05 (sec) , antiderivative size = 306, normalized size of antiderivative = 1.15

method	result
derivativedivides	$-\frac{2\left(\sqrt{e\cot(dx+c)}\,b^2+e\left(\frac{(a^2e-b^2e)(e^2)^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\frac{e\cot(dx+c)+(e^2)^{\frac{1}{4}}\sqrt{e\cot(dx+c)}\sqrt{2}+\sqrt{e^2}}{e\cot(dx+c)-(e^2)^{\frac{1}{4}}\sqrt{e\cot(dx+c)}\sqrt{2}+\sqrt{e^2}}\right)+2\arctan\left(\frac{\sqrt{2}\sqrt{e\cot(dx+c)}}{(e^2)^{\frac{1}{4}}}+\cdots\right.\right.}{8e^2}\right.\right.}{\cdots}$
default	$-\frac{2\left(\sqrt{e\cot(dx+c)}\,b^2+e\left(\frac{(a^2e-b^2e)(e^2)^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\frac{e\cot(dx+c)+(e^2)^{\frac{1}{4}}\sqrt{e\cot(dx+c)}\sqrt{2}+\sqrt{e^2}}{e\cot(dx+c)-(e^2)^{\frac{1}{4}}\sqrt{e\cot(dx+c)}\sqrt{2}+\sqrt{e^2}}\right)+2\arctan\left(\frac{\sqrt{2}\sqrt{e\cot(dx+c)}}{(e^2)^{\frac{1}{4}}}+\cdots\right.\right.}{8e^2}\right.\right.}{\cdots}$
parts	$-\frac{a^2(e^2)^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\frac{e\cot(dx+c)+(e^2)^{\frac{1}{4}}\sqrt{e\cot(dx+c)}\sqrt{2}+\sqrt{e^2}}{e\cot(dx+c)-(e^2)^{\frac{1}{4}}\sqrt{e\cot(dx+c)}\sqrt{2}+\sqrt{e^2}}\right)+2\arctan\left(\frac{\sqrt{2}\sqrt{e\cot(dx+c)}}{(e^2)^{\frac{1}{4}}}+1\right)-2\arctan\left(-\frac{\sqrt{2}\sqrt{e\cot(d\cdots}}{(e^2)^{\frac{1}{4}}}\cdots\right.\right.}{4de}$

```
[In] int((a+b*cot(d*x+c))^2/(e*cot(d*x+c))^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -2/d/e*((e*cot(d*x+c))^(1/2)*b^2+e*(1/8*(a^2*e-b^2*e)*(e^2)^(1/4)/e^2*2^(1/
2)*(ln((e*cot(d*x+c)+(e^2)^(1/4)*(e*cot(d*x+c))^(1/2)*2^(1/2)+(e^2)^(1/2))/
```

```
(e*cot(d*x+c)-(e^2)^(1/4)*(e*cot(d*x+c))^(1/2)*2^(1/2)+(e^2)^(1/2)))+2*arct
an(2^(1/2)/(e^2)^(1/4)*(e*cot(d*x+c))^(1/2)+1)-2*arctan(-2^(1/2)/(e^2)^(1/4
)*(e*cot(d*x+c))^(1/2)+1))+1/4*a*b/(e^2)^(1/4)*2^(1/2)*(ln((e*cot(d*x+c)-(e
^2)^(1/4)*(e*cot(d*x+c))^(1/2)*2^(1/2)+(e^2)^(1/2))/(e*cot(d*x+c)+(e^2)^(1/
4)*(e*cot(d*x+c))^(1/2)*2^(1/2)+(e^2)^(1/2)))+2*arctan(2^(1/2)/(e^2)^(1/4)*
(e*cot(d*x+c))^(1/2)+1)-2*arctan(-2^(1/2)/(e^2)^(1/4)*(e*cot(d*x+c))^(1/2)+
1))))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1183 vs. $2(218) = 436$.

Time = 0.29 (sec) , antiderivative size = 1183, normalized size of antiderivative = 4.43

$$\int \frac{(a+b\cot(c+dx))^2}{\sqrt{e\cot(c+dx)}}\,dx = \text{Too large to display}$$

```
[In] integrate((a+b*cot(d*x+c))^2/(e*cot(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] -1/2*(d*e*sqrt(-(4*a^3*b - 4*a*b^3 + d^2*e*sqrt(-(a^8 - 12*a^6*b^2 + 38*a^4
*b^4 - 12*a^2*b^6 + b^8)/(d^4*e^2)))/(d^2*e))*log((a^8 - 4*a^6*b^2 - 10*a^4
*b^4 - 4*a^2*b^6 + b^8)*sqrt((e*cos(2*d*x + 2*c) + e)/sin(2*d*x + 2*c)) + (
2*a*b*d^3*e^2*sqrt(-(a^8 - 12*a^6*b^2 + 38*a^4*b^4 - 12*a^2*b^6 + b^8)/(d^4
*e^2)) + (a^6 - 7*a^4*b^2 + 7*a^2*b^4 - b^6)*d*e)*sqrt(-(4*a^3*b - 4*a*b^3
+ d^2*e*sqrt(-(a^8 - 12*a^6*b^2 + 38*a^4*b^4 - 12*a^2*b^6 + b^8)/(d^4*e^2))
)/(d^2*e))) - d*e*sqrt(-(4*a^3*b - 4*a*b^3 + d^2*e*sqrt(-(a^8 - 12*a^6*b^2
+ 38*a^4*b^4 - 12*a^2*b^6 + b^8)/(d^4*e^2)))/(d^2*e))*log((a^8 - 4*a^6*b^2
- 10*a^4*b^4 - 4*a^2*b^6 + b^8)*sqrt((e*cos(2*d*x + 2*c) + e)/sin(2*d*x + 2
*c)) - (2*a*b*d^3*e^2*sqrt(-(a^8 - 12*a^6*b^2 + 38*a^4*b^4 - 12*a^2*b^6 + b
^8)/(d^4*e^2)) + (a^6 - 7*a^4*b^2 + 7*a^2*b^4 - b^6)*d*e)*sqrt(-(4*a^3*b -
4*a*b^3 + d^2*e*sqrt(-(a^8 - 12*a^6*b^2 + 38*a^4*b^4 - 12*a^2*b^6 + b^8)/(d
^4*e^2)))/(d^2*e))) - d*e*sqrt(-(4*a^3*b - 4*a*b^3 - d^2*e*sqrt(-(a^8 - 12*
a^6*b^2 + 38*a^4*b^4 - 12*a^2*b^6 + b^8)/(d^4*e^2)))/(d^2*e))*log((a^8 - 4*
a^6*b^2 - 10*a^4*b^4 - 4*a^2*b^6 + b^8)*sqrt((e*cos(2*d*x + 2*c) + e)/sin(2
*d*x + 2*c)) + (2*a*b*d^3*e^2*sqrt(-(a^8 - 12*a^6*b^2 + 38*a^4*b^4 - 12*a^2
*b^6 + b^8)/(d^4*e^2)) - (a^6 - 7*a^4*b^2 + 7*a^2*b^4 - b^6)*d*e)*sqrt(-(4*
a^3*b - 4*a*b^3 - d^2*e*sqrt(-(a^8 - 12*a^6*b^2 + 38*a^4*b^4 - 12*a^2*b^6 +
 b^8)/(d^4*e^2)))/(d^2*e))) + d*e*sqrt(-(4*a^3*b - 4*a*b^3 - d^2*e*sqrt(-(a
^8 - 12*a^6*b^2 + 38*a^4*b^4 - 12*a^2*b^6 + b^8)/(d^4*e^2)))/(d^2*e))*log((
a^8 - 4*a^6*b^2 - 10*a^4*b^4 - 4*a^2*b^6 + b^8)*sqrt((e*cos(2*d*x + 2*c) +
e)/sin(2*d*x + 2*c)) - (2*a*b*d^3*e^2*sqrt(-(a^8 - 12*a^6*b^2 + 38*a^4*b^4
- 12*a^2*b^6 + b^8)/(d^4*e^2)) - (a^6 - 7*a^4*b^2 + 7*a^2*b^4 - b^6)*d*e)*s
qrt(-(4*a^3*b - 4*a*b^3 - d^2*e*sqrt(-(a^8 - 12*a^6*b^2 + 38*a^4*b^4 - 12*a
^2*b^6 + b^8)/(d^4*e^2)))/(d^2*e))) + 4*b^2*sqrt((e*cos(2*d*x + 2*c) + e)/s
in(2*d*x + 2*c)))/(d*e)
```

Sympy [F]

$$\int \frac{(a+b\cot(c+dx))^2}{\sqrt{e\cot(c+dx)}}\,dx = \int \frac{(a+b\cot{(c+dx)})^2}{\sqrt{e\cot{(c+dx)}}}\,dx$$

```
[In] integrate((a+b*cot(d*x+c))**2/(e*cot(d*x+c))**(1/2),x)
```

```
[Out] Integral((a + b*cot(c + d*x))**2/sqrt(e*cot(c + d*x)), x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{(a+b\cot(c+dx))^2}{\sqrt{e\cot(c+dx)}}\,dx = \text{Exception raised: ValueError}$$

```
[In] integrate((a+b*cot(d*x+c))^2/(e*cot(d*x+c))^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(e>0)', see `assume?` for more detai
ls)Is e
```

Giac [F]

$$\int \frac{(a+b\cot(c+dx))^2}{\sqrt{e\cot(c+dx)}}\,dx = \int \frac{(b\cot{(dx+c)}+a)^2}{\sqrt{e\cot{(dx+c)}}}\,dx$$

```
[In] integrate((a+b*cot(d*x+c))^2/(e*cot(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] integrate((b*cot(d*x + c) + a)^2/sqrt(e*cot(d*x + c)), x)
```

Mupad [B] (verification not implemented)

Time = 13.42 (sec) , antiderivative size = 1234, normalized size of antiderivative = 4.62

$$\int \frac{(a+b\cot(c+dx))^2}{\sqrt{e\cot(c+dx)}}\,dx = \text{Too large to display}$$

```
[In] int((a + b*cot(c + d*x))^2/(e*cot(c + d*x))^(1/2),x)
```

```
[Out] 2*atanh((32*a^4*e^2*(e*cot(c + d*x))^(1/2)*((a*b^3)/(d^2*e) - (b^4*1i)/(4*d
^2*e) - (a^4*1i)/(4*d^2*e) - (a^3*b)/(d^2*e) + (a^2*b^2*3i)/(2*d^2*e))^(1/2
```

```
))/((a^6*e^2*16i)/d - (b^6*e^2*16i)/d + (32*a*b^5*e^2)/d + (32*a^5*b*e^2)/d
 + (a^2*b^4*e^2*112i)/d - (192*a^3*b^3*e^2)/d - (a^4*b^2*e^2*112i)/d) + (32
*b^4*e^2*(e*cot(c + d*x))^(1/2)*((a*b^3)/(d^2*e) - (b^4*1i)/(4*d^2*e) - (a^
4*1i)/(4*d^2*e) - (a^3*b)/(d^2*e) + (a^2*b^2*3i)/(2*d^2*e))^(1/2))/((a^6*e^
2*16i)/d - (b^6*e^2*16i)/d + (32*a*b^5*e^2)/d + (32*a^5*b*e^2)/d + (a^2*b^4
*e^2*112i)/d - (192*a^3*b^3*e^2)/d - (a^4*b^2*e^2*112i)/d) - (192*a^2*b^2*e
^2*(e*cot(c + d*x))^(1/2)*((a*b^3)/(d^2*e) - (b^4*1i)/(4*d^2*e) - (a^4*1i)/
(4*d^2*e) - (a^3*b)/(d^2*e) + (a^2*b^2*3i)/(2*d^2*e))^(1/2))/((a^6*e^2*16i)
/d - (b^6*e^2*16i)/d + (32*a*b^5*e^2)/d + (32*a^5*b*e^2)/d + (a^2*b^4*e^2*1
12i)/d - (192*a^3*b^3*e^2)/d - (a^4*b^2*e^2*112i)/d))*((a*b^3)/(d^2*e) - (b
^4*1i)/(4*d^2*e) - (a^4*1i)/(4*d^2*e) - (a^3*b)/(d^2*e) + (a^2*b^2*3i)/(2*d
^2*e))^(1/2) + 2*atanh((32*a^4*e^2*(e*cot(c + d*x))^(1/2)*((a^4*1i)/(4*d^2*
e) + (b^4*1i)/(4*d^2*e) + (a*b^3)/(d^2*e) - (a^3*b)/(d^2*e) - (a^2*b^2*3i)/
(2*d^2*e))^(1/2))/((b^6*e^2*16i)/d - (a^6*e^2*16i)/d + (32*a*b^5*e^2)/d + (
32*a^5*b*e^2)/d - (a^2*b^4*e^2*112i)/d - (192*a^3*b^3*e^2)/d + (a^4*b^2*e^2
*112i)/d) + (32*b^4*e^2*(e*cot(c + d*x))^(1/2)*((a^4*1i)/(4*d^2*e) + (b^4*1
i)/(4*d^2*e) + (a*b^3)/(d^2*e) - (a^3*b)/(d^2*e) - (a^2*b^2*3i)/(2*d^2*e))^
(1/2))/((b^6*e^2*16i)/d - (a^6*e^2*16i)/d + (32*a*b^5*e^2)/d + (32*a^5*b*e^
2)/d - (a^2*b^4*e^2*112i)/d - (192*a^3*b^3*e^2)/d + (a^4*b^2*e^2*112i)/d) -
 (192*a^2*b^2*e^2*(e*cot(c + d*x))^(1/2)*((a^4*1i)/(4*d^2*e) + (b^4*1i)/(4*
d^2*e) + (a*b^3)/(d^2*e) - (a^3*b)/(d^2*e) - (a^2*b^2*3i)/(2*d^2*e))^(1/2))
/((b^6*e^2*16i)/d - (a^6*e^2*16i)/d + (32*a*b^5*e^2)/d + (32*a^5*b*e^2)/d -
 (a^2*b^4*e^2*112i)/d - (192*a^3*b^3*e^2)/d + (a^4*b^2*e^2*112i)/d))*((a^4*
1i)/(4*d^2*e) + (b^4*1i)/(4*d^2*e) + (a*b^3)/(d^2*e) - (a^3*b)/(d^2*e) - (a
^2*b^2*3i)/(2*d^2*e))^(1/2) - (2*b^2*(e*cot(c + d*x))^(1/2))/(d*e)
```

3.59 $\int \frac{(a+b\cot(c+dx))^2}{(e\cot(c+dx))^{3/2}}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 267

$$\begin{aligned}\int \frac{(a+b\cot(c+dx))^2}{(e\cot(c+dx))^{3/2}}\,dx &= -\frac{(a^2-2ab-b^2)\arctan\left(1-\frac{\sqrt{2}\sqrt{e\cot(c+dx)}}{\sqrt{e}}\right)}{\sqrt{2}de^{3/2}}\\ &+\frac{(a^2-2ab-b^2)\arctan\left(1+\frac{\sqrt{2}\sqrt{e\cot(c+dx)}}{\sqrt{e}}\right)}{\sqrt{2}de^{3/2}}+\frac{2a^2}{de\sqrt{e\cot(c+dx)}}\\ &+\frac{(a^2+2ab-b^2)\log\left(\sqrt{e}+\sqrt{e}\cot(c+dx)-\sqrt{2}\sqrt{e\cot(c+dx)}\right)}{2\sqrt{2}de^{3/2}}\\ &-\frac{(a^2+2ab-b^2)\log\left(\sqrt{e}+\sqrt{e}\cot(c+dx)+\sqrt{2}\sqrt{e\cot(c+dx)}\right)}{2\sqrt{2}de^{3/2}}\end{aligned}$$

```
[Out] -1/2*(a^2-2*a*b-b^2)*arctan(1-2^(1/2)*(e*cot(d*x+c))^(1/2)/e^(1/2))/d/e^(3/
2)*2^(1/2)+1/2*(a^2-2*a*b-b^2)*arctan(1+2^(1/2)*(e*cot(d*x+c))^(1/2)/e^(1/2
))/d/e^(3/2)*2^(1/2)+1/4*(a^2+2*a*b-b^2)*ln(e^(1/2)+cot(d*x+c)*e^(1/2)-2^(1
/2)*(e*cot(d*x+c))^(1/2))/d/e^(3/2)*2^(1/2)-1/4*(a^2+2*a*b-b^2)*ln(e^(1/2)+
cot(d*x+c)*e^(1/2)+2^(1/2)*(e*cot(d*x+c))^(1/2))/d/e^(3/2)*2^(1/2)+2*a^2/d/
e/(e*cot(d*x+c))^(1/2)
```

Rubi [A] (verified)

Time = 0.31 (sec) , antiderivative size = 267, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.320, Rules used = {3623, 3615, 1182, 1176, 631, 210, 1179, 642}

$$\int \frac{(a+b\cot(c+dx))^2}{(e\cot(c+dx))^{3/2}}\,dx = -\frac{(a^2-2ab-b^2)\arctan\left(1-\frac{\sqrt{2}\sqrt{e\cot(c+dx)}}{\sqrt{e}}\right)}{\sqrt{2}de^{3/2}} + \frac{(a^2-2ab-b^2)\arctan\left(\frac{\sqrt{2}\sqrt{e\cot(c+dx)}}{\sqrt{e}}+1\right)}{\sqrt{2}de^{3/2}} + \frac{(a^2+2ab-b^2)\log\left(\sqrt{e}\cot(c+dx)-\sqrt{2}\sqrt{e\cot(c+dx)}+\sqrt{e}\right)}{2\sqrt{2}de^{3/2}} - \frac{(a^2+2ab-b^2)\log\left(\sqrt{e}\cot(c+dx)+\sqrt{2}\sqrt{e\cot(c+dx)}+\sqrt{e}\right)}{2\sqrt{2}de^{3/2}} + \frac{2a^2}{de\sqrt{e\cot(c+dx)}}$$

```
[In] Int[(a + b*Cot[c + d*x])^2/(e*Cot[c + d*x])^(3/2),x]
```

```
[Out] -(((a^2 - 2*a*b - b^2)*ArcTan[1 - (Sqrt[2]*Sqrt[e*Cot[c + d*x]])/Sqrt[e]])/
(Sqrt[2]*d*e^(3/2))) + ((a^2 - 2*a*b - b^2)*ArcTan[1 + (Sqrt[2]*Sqrt[e*Cot[
c + d*x]])/Sqrt[e]])/(Sqrt[2]*d*e^(3/2)) + (2*a^2)/(d*e*Sqrt[e*Cot[c + d*x]
]) + ((a^2 + 2*a*b - b^2)*Log[Sqrt[e] + Sqrt[e]*Cot[c + d*x] - Sqrt[2]*Sqrt
[e*Cot[c + d*x]]])/(2*Sqrt[2]*d*e^(3/2)) - ((a^2 + 2*a*b - b^2)*Log[Sqrt[e]
 + Sqrt[e]*Cot[c + d*x] + Sqrt[2]*Sqrt[e*Cot[c + d*x]]])/(2*Sqrt[2]*d*e^(3/
2))
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
```

```
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1179

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 1182

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
a*c, 2]}, Dist[(d*q + a*e)/(2*a*c), Int[(q + c*x^2)/(a + c*x^4), x], x] + D
ist[(d*q - a*e)/(2*a*c), Int[(q - c*x^2)/(a + c*x^4), x], x]] /; FreeQ[{a,
c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0] && NeQ[c*d^2 - a*e^2, 0] && NegQ[(-a)
*c]
```

Rule 3615

```
Int[((c_) + (d_.)*tan[(e_.) + (f_.)*(x_)])/Sqrt[(b_.)*tan[(e_.) + (f_.)*(x_
)]], x_Symbol] :> Dist[2/f, Subst[Int[(b*c + d*x^2)/(b^2 + x^4), x], x, Sqr
t[b*Tan[e + f*x]]], x] /; FreeQ[{b, c, d, e, f}, x] && NeQ[c^2 - d^2, 0] &&
 NeQ[c^2 + d^2, 0]
```

Rule 3623

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)])^2, x_Symbol] :> Simp[(b*c - a*d)^2*((a + b*Tan[e + f*x])^(m +
 1)/(b*f*(m + 1)*(a^2 + b^2))), x] + Dist[1/(a^2 + b^2), Int[(a + b*Tan[e +
 f*x])^(m + 1)*Simp[a*c^2 + 2*b*c*d - a*d^2 - (b*c^2 - 2*a*c*d - b*d^2)*Tan
[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
&& LtQ[m, -1] && NeQ[a^2 + b^2, 0]
```

Rubi steps

$$\text{integral} = \frac{2a^2}{de\sqrt{e\cot(c+dx)}} + \frac{\int \frac{2abe-(a^2-b^2)e\cot(c+dx)}{\sqrt{e\cot(c+dx)}}\,dx}{e^2}$$

$$= \frac{2a^2}{de\sqrt{e\cot(c+dx)}} + \frac{2\text{Subst}\left(\int \frac{-2abe^2+(a^2-b^2)ex^2}{e^2+x^4}\,dx, x, \sqrt{e\cot(c+dx)}\right)}{de^2}$$

$$= \frac{2a^2}{de\sqrt{e\cot(c+dx)}} + \frac{(a^2-2ab-b^2)\,\text{Subst}\left(\int \frac{e+x^2}{e^2+x^4}\,dx, x, \sqrt{e\cot(c+dx)}\right)}{de} - \frac{(a^2+2ab-b^2)\,\text{Subst}\left(\int \frac{e-x^2}{e^2+x^4}\,dx, x, \sqrt{e\cot(c+dx)}\right)}{de}$$

$$= \frac{2a^2}{de\sqrt{e\cot(c+dx)}} + \frac{(a^2+2ab-b^2)\,\text{Subst}\left(\int \frac{\sqrt{2}\sqrt{e}+2x}{-e-\sqrt{2}\sqrt{e}x-x^2}\,dx, x, \sqrt{e\cot(c+dx)}\right)}{2\sqrt{2}de^{3/2}} + \frac{(a^2+2ab-b^2)\,\text{Subst}\left(\int \frac{\sqrt{2}\sqrt{e}-2x}{-e+\sqrt{2}\sqrt{e}x-x^2}\,dx, x, \sqrt{e\cot(c+dx)}\right)}{2\sqrt{2}de^{3/2}} + \frac{(a^2-2ab-b^2)\,\text{Subst}\left(\int \frac{1}{e-\sqrt{2}\sqrt{e}x+x^2}\,dx, x, \sqrt{e\cot(c+dx)}\right)}{2de} + \frac{(a^2-2ab-b^2)\,\text{Subst}\left(\int \frac{1}{e+\sqrt{2}\sqrt{e}x+x^2}\,dx, x, \sqrt{e\cot(c+dx)}\right)}{2de}$$

$$= \frac{2a^2}{de\sqrt{e\cot(c+dx)}} + \frac{(a^2+2ab-b^2)\log\left(\sqrt{e}+\sqrt{e}\cot(c+dx)-\sqrt{2}\sqrt{e\cot(c+dx)}\right)}{2\sqrt{2}de^{3/2}} - \frac{(a^2+2ab-b^2)\log\left(\sqrt{e}+\sqrt{e}\cot(c+dx)+\sqrt{2}\sqrt{e\cot(c+dx)}\right)}{2\sqrt{2}de^{3/2}} + \frac{(a^2-2ab-b^2)\,\text{Subst}\left(\int \frac{1}{-1-x^2}\,dx, x, 1-\frac{\sqrt{2}\sqrt{e\cot(c+dx)}}{\sqrt{e}}\right)}{\sqrt{2}de^{3/2}} - \frac{(a^2-2ab-b^2)\,\text{Subst}\left(\int \frac{1}{-1-x^2}\,dx, x, 1+\frac{\sqrt{2}\sqrt{e\cot(c+dx)}}{\sqrt{e}}\right)}{\sqrt{2}de^{3/2}}$$

$$= -\frac{(a^2-2ab-b^2)\arctan\left(1-\frac{\sqrt{2}\sqrt{e\cot(c+dx)}}{\sqrt{e}}\right)}{\sqrt{2}de^{3/2}} + \frac{(a^2-2ab-b^2)\arctan\left(1+\frac{\sqrt{2}\sqrt{e\cot(c+dx)}}{\sqrt{e}}\right)}{\sqrt{2}de^{3/2}} + \frac{2a^2}{de\sqrt{e\cot(c+dx)}} + \frac{(a^2+2ab-b^2)\log\left(\sqrt{e}+\sqrt{e}\cot(c+dx)-\sqrt{2}\sqrt{e\cot(c+dx)}\right)}{2\sqrt{2}de^{3/2}} - \frac{(a^2+2ab-b^2)\log\left(\sqrt{e}+\sqrt{e}\cot(c+dx)+\sqrt{2}\sqrt{e\cot(c+dx)}\right)}{2\sqrt{2}de^{3/2}}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 0.82 (sec) , antiderivative size = 188, normalized size of antiderivative = 0.70

$$\int \frac{(a+b\cot(c+dx))^2}{(e\cot(c+dx))^{3/2}}\,dx = -\frac{\cot^{\frac{3}{2}}(c+dx)\left(-\frac{2b^2}{\sqrt{\cot(c+dx)}}-\frac{2(a^2-b^2)\,\text{Hypergeometric2F1}\left(-\frac{1}{4},1,\frac{3}{4},-\cot^2(c+dx)\right)}{\sqrt{\cot(c+dx)}}-\frac{ab\left(2\arctan\left(1-\sqrt{2}\sqrt{\cot(c+dx)}\right)-2\arctan\left(1+\right.\right.}{}\right.}{d(e\cot(c+dx))^{3/2}}$$

```
[In] Integrate[(a + b*Cot[c + d*x])^2/(e*Cot[c + d*x])^(3/2),x]
```

```
[Out] -((Cot[c + d*x]^(3/2)*((-2*b^2)/Sqrt[Cot[c + d*x]] - (2*(a^2 - b^2)*Hyperge
ometric2F1[-1/4, 1, 3/4, -Cot[c + d*x]^2])/Sqrt[Cot[c + d*x]] - (a*b*(2*Arc
Tan[1 - Sqrt[2]*Sqrt[Cot[c + d*x]]] - 2*ArcTan[1 + Sqrt[2]*Sqrt[Cot[c + d*x
]]] + Log[1 - Sqrt[2]*Sqrt[Cot[c + d*x]] + Cot[c + d*x]] - Log[1 + Sqrt[2]*
Sqrt[Cot[c + d*x]] + Cot[c + d*x]]))/Sqrt[2]))/(d*(e*Cot[c + d*x])^(3/2)))
```

Maple [A] (verified)

Time = 0.04 (sec) , antiderivative size = 301, normalized size of antiderivative = 1.13

method	result
derivativedivides	$-\frac{2\left(\frac{ab\left(e^2\right)^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\frac{e\cot(dx+c)+\left(e^2\right)^{\frac{1}{4}}\sqrt{e\cot(dx+c)}\sqrt{2}+\sqrt{e^2}}{e\cot(dx+c)-\left(e^2\right)^{\frac{1}{4}}\sqrt{e\cot(dx+c)}\sqrt{2}+\sqrt{e^2}}\right)+2\arctan\left(\frac{\sqrt{2}\sqrt{e\cot(dx+c)}}{\left(e^2\right)^{\frac{1}{4}}}+1\right)-2\arctan\left(-\frac{\sqrt{2}\sqrt{e\cot(dx+c}}{\left(e^2\right)^{\frac{1}{4}}}\right.\right.}{4e}\right.}{}$
default	$-\frac{2\left(\frac{ab\left(e^2\right)^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\frac{e\cot(dx+c)+\left(e^2\right)^{\frac{1}{4}}\sqrt{e\cot(dx+c)}\sqrt{2}+\sqrt{e^2}}{e\cot(dx+c)-\left(e^2\right)^{\frac{1}{4}}\sqrt{e\cot(dx+c)}\sqrt{2}+\sqrt{e^2}}\right)+2\arctan\left(\frac{\sqrt{2}\sqrt{e\cot(dx+c)}}{\left(e^2\right)^{\frac{1}{4}}}+1\right)-2\arctan\left(-\frac{\sqrt{2}\sqrt{e\cot(dx+c}}{\left(e^2\right)^{\frac{1}{4}}}\right.\right.}{4e}\right.}{}$
parts	$-\frac{2a^2e\left(-\frac{\sqrt{2}\left(\ln\left(\frac{e\cot(dx+c)-\left(e^2\right)^{\frac{1}{4}}\sqrt{e\cot(dx+c)}\sqrt{2}+\sqrt{e^2}}{e\cot(dx+c)+\left(e^2\right)^{\frac{1}{4}}\sqrt{e\cot(dx+c)}\sqrt{2}+\sqrt{e^2}}\right)+2\arctan\left(\frac{\sqrt{2}\sqrt{e\cot(dx+c)}}{\left(e^2\right)^{\frac{1}{4}}}+1\right)-2\arctan\left(-\frac{\sqrt{2}\sqrt{e\cot(dx+c)}}{\left(e^2\right)^{\frac{1}{4}}}+\right.\right.}{8e^2\left(e^2\right)^{\frac{1}{4}}}\right.}{d}$

```
[In] int((a+b*cot(d*x+c))^2/(e*cot(d*x+c))^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] -2/d/e*(1/4*a/e*b*(e^2)^(1/4)*2^(1/2)*(ln((e*cot(d*x+c)+(e^2)^(1/4)*(e*cot(
d*x+c))^(1/2)*2^(1/2)+(e^2)^(1/2))/(e*cot(d*x+c)-(e^2)^(1/4)*(e*cot(d*x+c))
```

```
^(1/2)*2^(1/2)+(e^2)^(1/2)))+2*arctan(2^(1/2)/(e^2)^(1/4)*(e*cot(d*x+c))^(1
/2)+1)-2*arctan(-2^(1/2)/(e^2)^(1/4)*(e*cot(d*x+c))^(1/2)+1))+1/8*(-a^2+b^2
)/(e^2)^(1/4)*2^(1/2)*(ln((e*cot(d*x+c)-(e^2)^(1/4)*(e*cot(d*x+c))^(1/2)*2^
(1/2)+(e^2)^(1/2))/(e*cot(d*x+c)+(e^2)^(1/4)*(e*cot(d*x+c))^(1/2)*2^(1/2)+(
e^2)^(1/2)))+2*arctan(2^(1/2)/(e^2)^(1/4)*(e*cot(d*x+c))^(1/2)+1)-2*arctan(
-2^(1/2)/(e^2)^(1/4)*(e*cot(d*x+c))^(1/2)+1))-a^2/(e*cot(d*x+c))^(1/2))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1298 vs. $2(218) = 436$.

Time = 0.31 (sec) , antiderivative size = 1298, normalized size of antiderivative = 4.86

$$\int \frac{(a+b\cot(c+dx))^2}{(e\cot(c+dx))^{3/2}}\,dx = \text{Too large to display}$$

```
[In] integrate((a+b*cot(d*x+c))^2/(e*cot(d*x+c))^(3/2),x, algorithm="fricas")
```

```
[Out] 1/2*(4*a^2*sqrt((e*cos(2*d*x + 2*c) + e)/sin(2*d*x + 2*c))*sin(2*d*x + 2*c)
 + (d*e^2*cos(2*d*x + 2*c) + d*e^2)*sqrt((d^2*e^3*sqrt(-(a^8 - 12*a^6*b^2 +
 38*a^4*b^4 - 12*a^2*b^6 + b^8)/(d^4*e^6)) + 4*a^3*b - 4*a*b^3)/(d^2*e^3))*
log((a^8 - 4*a^6*b^2 - 10*a^4*b^4 - 4*a^2*b^6 + b^8)*sqrt((e*cos(2*d*x + 2*
c) + e)/sin(2*d*x + 2*c)) + ((a^2 - b^2)*d^3*e^5*sqrt(-(a^8 - 12*a^6*b^2 +
38*a^4*b^4 - 12*a^2*b^6 + b^8)/(d^4*e^6)) - 2*(a^5*b - 6*a^3*b^3 + a*b^5)*d
*e^2)*sqrt((d^2*e^3*sqrt(-(a^8 - 12*a^6*b^2 + 38*a^4*b^4 - 12*a^2*b^6 + b^8
)/(d^4*e^6)) + 4*a^3*b - 4*a*b^3)/(d^2*e^3))) - (d*e^2*cos(2*d*x + 2*c) + d
*e^2)*sqrt((d^2*e^3*sqrt(-(a^8 - 12*a^6*b^2 + 38*a^4*b^4 - 12*a^2*b^6 + b^8
)/(d^4*e^6)) + 4*a^3*b - 4*a*b^3)/(d^2*e^3))*log((a^8 - 4*a^6*b^2 - 10*a^4*
b^4 - 4*a^2*b^6 + b^8)*sqrt((e*cos(2*d*x + 2*c) + e)/sin(2*d*x + 2*c)) - ((
a^2 - b^2)*d^3*e^5*sqrt(-(a^8 - 12*a^6*b^2 + 38*a^4*b^4 - 12*a^2*b^6 + b^8)
/(d^4*e^6)) - 2*(a^5*b - 6*a^3*b^3 + a*b^5)*d*e^2)*sqrt((d^2*e^3*sqrt(-(a^8
 - 12*a^6*b^2 + 38*a^4*b^4 - 12*a^2*b^6 + b^8)/(d^4*e^6)) + 4*a^3*b - 4*a*b
^3)/(d^2*e^3))) - (d*e^2*cos(2*d*x + 2*c) + d*e^2)*sqrt(-(d^2*e^3*sqrt(-(a^
8 - 12*a^6*b^2 + 38*a^4*b^4 - 12*a^2*b^6 + b^8)/(d^4*e^6)) - 4*a^3*b + 4*a*
b^3)/(d^2*e^3))*log((a^8 - 4*a^6*b^2 - 10*a^4*b^4 - 4*a^2*b^6 + b^8)*sqrt((
e*cos(2*d*x + 2*c) + e)/sin(2*d*x + 2*c)) + ((a^2 - b^2)*d^3*e^5*sqrt(-(a^8
 - 12*a^6*b^2 + 38*a^4*b^4 - 12*a^2*b^6 + b^8)/(d^4*e^6)) + 2*(a^5*b - 6*a^
3*b^3 + a*b^5)*d*e^2)*sqrt(-(d^2*e^3*sqrt(-(a^8 - 12*a^6*b^2 + 38*a^4*b^4 -
 12*a^2*b^6 + b^8)/(d^4*e^6)) - 4*a^3*b + 4*a*b^3)/(d^2*e^3))) + (d*e^2*cos
(2*d*x + 2*c) + d*e^2)*sqrt(-(d^2*e^3*sqrt(-(a^8 - 12*a^6*b^2 + 38*a^4*b^4
- 12*a^2*b^6 + b^8)/(d^4*e^6)) - 4*a^3*b + 4*a*b^3)/(d^2*e^3))*log((a^8 - 4
*a^6*b^2 - 10*a^4*b^4 - 4*a^2*b^6 + b^8)*sqrt((e*cos(2*d*x + 2*c) + e)/sin(
2*d*x + 2*c)) - ((a^2 - b^2)*d^3*e^5*sqrt(-(a^8 - 12*a^6*b^2 + 38*a^4*b^4 -
 12*a^2*b^6 + b^8)/(d^4*e^6)) + 2*(a^5*b - 6*a^3*b^3 + a*b^5)*d*e^2)*sqrt(-
(d^2*e^3*sqrt(-(a^8 - 12*a^6*b^2 + 38*a^4*b^4 - 12*a^2*b^6 + b^8)/(d^4*e^6)
) - 4*a^3*b + 4*a*b^3)/(d^2*e^3))))/(d*e^2*cos(2*d*x + 2*c) + d*e^2)
```

Sympy [F]

$$\int \frac{(a+b\cot(c+dx))^2}{(e\cot(c+dx))^{3/2}}\,dx = \int \frac{(a+b\cot{(c+dx)})^2}{(e\cot{(c+dx)})^{\frac{3}{2}}}\,dx$$

```
[In] integrate((a+b*cot(d*x+c))**2/(e*cot(d*x+c))**(3/2),x)
[Out] Integral((a + b*cot(c + d*x))**2/(e*cot(c + d*x))**(3/2), x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{(a+b\cot(c+dx))^2}{(e\cot(c+dx))^{3/2}}\,dx = \text{Exception raised: ValueError}$$

```
[In] integrate((a+b*cot(d*x+c))^2/(e*cot(d*x+c))^(3/2),x, algorithm="maxima")
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(e>0)', see `assume?` for more detai
ls)Is e
```

Giac [F]

$$\int \frac{(a+b\cot(c+dx))^2}{(e\cot(c+dx))^{3/2}}\,dx = \int \frac{(b\cot{(dx+c)}+a)^2}{(e\cot{(dx+c)})^{\frac{3}{2}}}\,dx$$

```
[In] integrate((a+b*cot(d*x+c))^2/(e*cot(d*x+c))^(3/2),x, algorithm="giac")
[Out] integrate((b*cot(d*x + c) + a)^2/(e*cot(d*x + c))^(3/2), x)
```

Mupad [B] (verification not implemented)

Time = 12.92 (sec) , antiderivative size = 1196, normalized size of antiderivative = 4.48

$$\int \frac{(a+b\cot(c+dx))^2}{(e\cot(c+dx))^{3/2}}\,dx = \frac{2\,a^2}{d\,e\,\sqrt{e\cot\left(c+d\,x\right)}}$$

$$+2\,\mathrm{atanh}\left(\frac{32\,a^4\,d^3\,e^5\,\sqrt{e\cot\left(c+d\,x\right)}\,\sqrt{\frac{a^4\,1\mathrm{i}}{4\,d^2\,e^3}+\frac{b^4\,1\mathrm{i}}{4\,d^2\,e^3}-\frac{a\,b^3}{d^2\,e^3}+\frac{a^3\,b}{d^2\,e^3}-\frac{a^2\,b^2\,3\mathrm{i}}{2\,d^2\,e^3}}}{-16\,a^6\,d^2\,e^4+a^5\,b\,d^2\,e^4\,32\mathrm{i}+112\,a^4\,b^2\,d^2\,e^4-a^3\,b^3\,d^2\,e^4\,192\mathrm{i}-112\,a^2\,b^4\,d^2\,e^4+a\,b^5\,d^2\,e^4\,32\mathrm{i}+1}\right.$$

$$+\frac{32\,b^4\,d^3\,e^5\,\sqrt{e\cot\left(c+d\,x\right)}\,\sqrt{\frac{a^4\,1\mathrm{i}}{4\,d^2\,e^3}+\frac{b^4\,1\mathrm{i}}{4\,d^2\,e^3}-\frac{a\,b^3}{d^2\,e^3}+\frac{a^3\,b}{d^2\,e^3}-\frac{a^2\,b^2\,3\mathrm{i}}{2\,d^2\,e^3}}}{-16\,a^6\,d^2\,e^4+a^5\,b\,d^2\,e^4\,32\mathrm{i}+112\,a^4\,b^2\,d^2\,e^4-a^3\,b^3\,d^2\,e^4\,192\mathrm{i}-112\,a^2\,b^4\,d^2\,e^4+a\,b^5\,d^2\,e^4\,32\mathrm{i}+16\,b^6\,d^2\,e^4}$$

$$\left.-\frac{192\,a^2\,b^2\,d^3\,e^5\,\sqrt{e\cot\left(c+d\,x\right)}\,\sqrt{\frac{a^4\,1\mathrm{i}}{4\,d^2\,e^3}+\frac{b^4\,1\mathrm{i}}{4\,d^2\,e^3}-\frac{a\,b^3}{d^2\,e^3}+\frac{a^3\,b}{d^2\,e^3}-\frac{a^2\,b^2\,3\mathrm{i}}{2\,d^2\,e^3}}}{-16\,a^6\,d^2\,e^4+a^5\,b\,d^2\,e^4\,32\mathrm{i}+112\,a^4\,b^2\,d^2\,e^4-a^3\,b^3\,d^2\,e^4\,192\mathrm{i}-112\,a^2\,b^4\,d^2\,e^4+a\,b^5\,d^2\,e^4\,32\mathrm{i}+16\,b^6\,d^2\,e^4}\right)$$

$$-2\,\mathrm{atanh}\left(\frac{32\,a^4\,d^3\,e^5\,\sqrt{e\cot\left(c+d\,x\right)}\,\sqrt{\frac{a^3\,b}{d^2\,e^3}-\frac{b^4\,1\mathrm{i}}{4\,d^2\,e^3}-\frac{a\,b^3}{d^2\,e^3}-\frac{a^4\,1\mathrm{i}}{4\,d^2\,e^3}+\frac{a^2\,b^2\,3\mathrm{i}}{2\,d^2\,e^3}}}{16\,a^6\,d^2\,e^4+a^5\,b\,d^2\,e^4\,32\mathrm{i}-112\,a^4\,b^2\,d^2\,e^4-a^3\,b^3\,d^2\,e^4\,192\mathrm{i}+112\,a^2\,b^4\,d^2\,e^4+a\,b^5\,d^2\,e^4\,32\mathrm{i}-16}\right.$$

$$+\frac{32\,b^4\,d^3\,e^5\,\sqrt{e\cot\left(c+d\,x\right)}\,\sqrt{\frac{a^3\,b}{d^2\,e^3}-\frac{b^4\,1\mathrm{i}}{4\,d^2\,e^3}-\frac{a\,b^3}{d^2\,e^3}-\frac{a^4\,1\mathrm{i}}{4\,d^2\,e^3}+\frac{a^2\,b^2\,3\mathrm{i}}{2\,d^2\,e^3}}}{16\,a^6\,d^2\,e^4+a^5\,b\,d^2\,e^4\,32\mathrm{i}-112\,a^4\,b^2\,d^2\,e^4-a^3\,b^3\,d^2\,e^4\,192\mathrm{i}+112\,a^2\,b^4\,d^2\,e^4+a\,b^5\,d^2\,e^4\,32\mathrm{i}-16\,b^6\,d^2\,e^4}$$

$$\left.-\frac{192\,a^2\,b^2\,d^3\,e^5\,\sqrt{e\cot\left(c+d\,x\right)}\,\sqrt{\frac{a^3\,b}{d^2\,e^3}-\frac{b^4\,1\mathrm{i}}{4\,d^2\,e^3}-\frac{a\,b^3}{d^2\,e^3}-\frac{a^4\,1\mathrm{i}}{4\,d^2\,e^3}+\frac{a^2\,b^2\,3\mathrm{i}}{2\,d^2\,e^3}}}{16\,a^6\,d^2\,e^4+a^5\,b\,d^2\,e^4\,32\mathrm{i}-112\,a^4\,b^2\,d^2\,e^4-a^3\,b^3\,d^2\,e^4\,192\mathrm{i}+112\,a^2\,b^4\,d^2\,e^4+a\,b^5\,d^2\,e^4\,32\mathrm{i}-16\,b^6\,d^2\,e^4}\right)$$

```
[In] int((a + b*cot(c + d*x))^2/(e*cot(c + d*x))^(3/2),x)
```

```
[Out] 2*atanh((32*a^4*d^3*e^5*(e*cot(c + d*x))^(1/2)*((a^4*1i)/(4*d^2*e^3) + (b^4
*1i)/(4*d^2*e^3) - (a*b^3)/(d^2*e^3) + (a^3*b)/(d^2*e^3) - (a^2*b^2*3i)/(2*
d^2*e^3))^(1/2))/(16*b^6*d^2*e^4 - 16*a^6*d^2*e^4 + a*b^5*d^2*e^4*32i + a^5
*b*d^2*e^4*32i - 112*a^2*b^4*d^2*e^4 - a^3*b^3*d^2*e^4*192i + 112*a^4*b^2*d
^2*e^4) + (32*b^4*d^3*e^5*(e*cot(c + d*x))^(1/2)*((a^4*1i)/(4*d^2*e^3) + (b
^4*1i)/(4*d^2*e^3) - (a*b^3)/(d^2*e^3) + (a^3*b)/(d^2*e^3) - (a^2*b^2*3i)/(
2*d^2*e^3))^(1/2))/(16*b^6*d^2*e^4 - 16*a^6*d^2*e^4 + a*b^5*d^2*e^4*32i + a
^5*b*d^2*e^4*32i - 112*a^2*b^4*d^2*e^4 - a^3*b^3*d^2*e^4*192i + 112*a^4*b^2
*d^2*e^4) - (192*a^2*b^2*d^3*e^5*(e*cot(c + d*x))^(1/2)*((a^4*1i)/(4*d^2*e^
3) + (b^4*1i)/(4*d^2*e^3) - (a*b^3)/(d^2*e^3) + (a^3*b)/(d^2*e^3) - (a^2*b^
2*3i)/(2*d^2*e^3))^(1/2))/(16*b^6*d^2*e^4 - 16*a^6*d^2*e^4 + a*b^5*d^2*e^4*
32i + a^5*b*d^2*e^4*32i - 112*a^2*b^4*d^2*e^4 - a^3*b^3*d^2*e^4*192i + 112*
a^4*b^2*d^2*e^4))*(((a*b^3*4i - a^3*b*4i + a^4 + b^4 - 6*a^2*b^2)*1i)/(4*d^
2*e^3))^(1/2) - 2*atanh((32*a^4*d^3*e^5*(e*cot(c + d*x))^(1/2)*((a^3*b)/(d^
2*e^3) - (b^4*1i)/(4*d^2*e^3) - (a*b^3)/(d^2*e^3) - (a^4*1i)/(4*d^2*e^3) +
(a^2*b^2*3i)/(2*d^2*e^3))^(1/2))/(16*a^6*d^2*e^4 - 16*b^6*d^2*e^4 + a*b^5*d
^2*e^4*32i + a^5*b*d^2*e^4*32i + 112*a^2*b^4*d^2*e^4 - a^3*b^3*d^2*e^4*192i
```

- 112*a^4*b^2*d^2*e^4) + (32*b^4*d^3*e^5*(e*cot(c + d*x))^(1/2)*((a^3*b)/(d^2*e^3) - (b^4*1i)/(4*d^2*e^3) - (a*b^3)/(d^2*e^3) - (a^4*1i)/(4*d^2*e^3) + (a^2*b^2*3i)/(2*d^2*e^3))^(1/2))/(16*a^6*d^2*e^4 - 16*b^6*d^2*e^4 + a*b^5*d^2*e^4*32i + a^5*b*d^2*e^4*32i + 112*a^2*b^4*d^2*e^4 - a^3*b^3*d^2*e^4*192i - 112*a^4*b^2*d^2*e^4) - (192*a^2*b^2*d^3*e^5*(e*cot(c + d*x))^(1/2)*((a^3*b)/(d^2*e^3) - (b^4*1i)/(4*d^2*e^3) - (a*b^3)/(d^2*e^3) - (a^4*1i)/(4*d^2*e^3) + (a^2*b^2*3i)/(2*d^2*e^3))^(1/2))/(16*a^6*d^2*e^4 - 16*b^6*d^2*e^4 + a*b^5*d^2*e^4*32i + a^5*b*d^2*e^4*32i + 112*a^2*b^4*d^2*e^4 - a^3*b^3*d^2*e^4*192i - 112*a^4*b^2*d^2*e^4))*(-((a^3*b*4i - a*b^3*4i + a^4 + b^4 - 6*a^2*b^2)*1i)/(4*d^2*e^3))^(1/2) + (2*a^2)/(d*e*(e*cot(c + d*x))^(1/2))

3.60 $\int \frac{(a+b\cot(c+dx))^2}{(e\cot(c+dx))^{5/2}}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 291

$$\begin{aligned}\int \frac{(a+b\cot(c+dx))^2}{(e\cot(c+dx))^{5/2}}\,dx = &-\frac{(a^2+2ab-b^2)\arctan\left(1-\frac{\sqrt{2}\sqrt{e\cot(c+dx)}}{\sqrt{e}}\right)}{\sqrt{2}de^{5/2}} \\ &+\frac{(a^2+2ab-b^2)\arctan\left(1+\frac{\sqrt{2}\sqrt{e\cot(c+dx)}}{\sqrt{e}}\right)}{\sqrt{2}de^{5/2}} \\ &+\frac{2a^2}{3de(e\cot(c+dx))^{3/2}}+\frac{4ab}{de^2\sqrt{e\cot(c+dx)}} \\ &-\frac{(a^2-2ab-b^2)\log\left(\sqrt{e}+\sqrt{e}\cot(c+dx)-\sqrt{2}\sqrt{e\cot(c+dx)}\right)}{2\sqrt{2}de^{5/2}} \\ &+\frac{(a^2-2ab-b^2)\log\left(\sqrt{e}+\sqrt{e}\cot(c+dx)+\sqrt{2}\sqrt{e\cot(c+dx)}\right)}{2\sqrt{2}de^{5/2}}\end{aligned}$$

```
[Out] 2/3*a^2/d/e/(e*cot(d*x+c))^(3/2)-1/2*(a^2+2*a*b-b^2)*arctan(1-2^(1/2)*(e*co
t(d*x+c))^(1/2)/e^(1/2))/d/e^(5/2)*2^(1/2)+1/2*(a^2+2*a*b-b^2)*arctan(1+2^(
1/2)*(e*cot(d*x+c))^(1/2)/e^(1/2))/d/e^(5/2)*2^(1/2)-1/4*(a^2-2*a*b-b^2)*ln
(e^(1/2)+cot(d*x+c)*e^(1/2)-2^(1/2)*(e*cot(d*x+c))^(1/2))/d/e^(5/2)*2^(1/2)
+1/4*(a^2-2*a*b-b^2)*ln(e^(1/2)+cot(d*x+c)*e^(1/2)+2^(1/2)*(e*cot(d*x+c))^(
1/2))/d/e^(5/2)*2^(1/2)+4*a*b/d/e^2/(e*cot(d*x+c))^(1/2)
```

Rubi [A] (verified)

Time = 0.43 (sec) , antiderivative size = 291, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.360$, Rules used = {3623, 3610, 3615, 1182, 1176, 631, 210, 1179, 642}

$$\int \frac{(a+b\cot(c+dx))^2}{(e\cot(c+dx))^{5/2}}\,dx = -\frac{(a^2+2ab-b^2)\arctan\left(1-\frac{\sqrt{2}\sqrt{e\cot(c+dx)}}{\sqrt{e}}\right)}{\sqrt{2}de^{5/2}} + \frac{(a^2+2ab-b^2)\arctan\left(\frac{\sqrt{2}\sqrt{e\cot(c+dx)}}{\sqrt{e}}+1\right)}{\sqrt{2}de^{5/2}} - \frac{(a^2-2ab-b^2)\log\left(\sqrt{e}\cot(c+dx)-\sqrt{2}\sqrt{e\cot(c+dx)}+\sqrt{e}\right)}{2\sqrt{2}de^{5/2}} + \frac{(a^2-2ab-b^2)\log\left(\sqrt{e}\cot(c+dx)+\sqrt{2}\sqrt{e\cot(c+dx)}+\sqrt{e}\right)}{2\sqrt{2}de^{5/2}} + \frac{2a^2}{3de(e\cot(c+dx))^{3/2}} + \frac{4ab}{de^2\sqrt{e\cot(c+dx)}}$$

```
[In] Int[(a + b*Cot[c + d*x])^2/(e*Cot[c + d*x])^(5/2),x]
```

```
[Out] -(((a^2 + 2*a*b - b^2)*ArcTan[1 - (Sqrt[2]*Sqrt[e*Cot[c + d*x]])/Sqrt[e]])/
(Sqrt[2]*d*e^(5/2))) + ((a^2 + 2*a*b - b^2)*ArcTan[1 + (Sqrt[2]*Sqrt[e*Cot[
c + d*x]])/Sqrt[e]])/(Sqrt[2]*d*e^(5/2)) + (2*a^2)/(3*d*e*(e*Cot[c + d*x])^
(3/2)) + (4*a*b)/(d*e^2*Sqrt[e*Cot[c + d*x]]) - ((a^2 - 2*a*b - b^2)*Log[Sq
rt[e] + Sqrt[e]*Cot[c + d*x] - Sqrt[2]*Sqrt[e*Cot[c + d*x]]])/(2*Sqrt[2]*d*
e^(5/2)) + ((a^2 - 2*a*b - b^2)*Log[Sqrt[e] + Sqrt[e]*Cot[c + d*x] + Sqrt[2
]*Sqrt[e*Cot[c + d*x]]])/(2*Sqrt[2]*d*e^(5/2))
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
```

```
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1179

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 1182

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
a*c, 2]}, Dist[(d*q + a*e)/(2*a*c), Int[(q + c*x^2)/(a + c*x^4), x], x] + D
ist[(d*q - a*e)/(2*a*c), Int[(q - c*x^2)/(a + c*x^4), x], x]] /; FreeQ[{a,
c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0] && NeQ[c*d^2 - a*e^2, 0] && NegQ[(-a)
*c]
```

Rule 3610

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)]), x_Symbol] :> Simp[(b*c - a*d)*((a + b*Tan[e + f*x])^(m + 1)/
(f*(m + 1)*(a^2 + b^2))), x] + Dist[1/(a^2 + b^2), Int[(a + b*Tan[e + f*x])
^(m + 1)*Simp[a*c + b*d - (b*c - a*d)*Tan[e + f*x], x], x], x] /; FreeQ[{a,
 b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && LtQ[m, -1]
```

Rule 3615

```
Int[((c_) + (d_.)*tan[(e_.) + (f_.)*(x_)])/Sqrt[(b_.)*tan[(e_.) + (f_.)*(x_
)]], x_Symbol] :> Dist[2/f, Subst[Int[(b*c + d*x^2)/(b^2 + x^4), x], x, Sqr
t[b*Tan[e + f*x]]], x] /; FreeQ[{b, c, d, e, f}, x] && NeQ[c^2 - d^2, 0] &&
 NeQ[c^2 + d^2, 0]
```

Rule 3623

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)])^2, x_Symbol] :> Simp[(b*c - a*d)^2*((a + b*Tan[e + f*x])^(m +
 1)/(b*f*(m + 1)*(a^2 + b^2))), x] + Dist[1/(a^2 + b^2), Int[(a + b*Tan[e +
 f*x])^(m + 1)*Simp[a*c^2 + 2*b*c*d - a*d^2 - (b*c^2 - 2*a*c*d - b*d^2)*Tan
[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

```
&& LtQ[m, -1] && NeQ[a^2 + b^2, 0]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{2a^2}{3de(e\cot(c+dx))^{3/2}}+\frac{\int \frac{2abe-(a^2-b^2)e\cot(c+dx)}{(e\cot(c+dx))^{3/2}}\,dx}{e^2}\\
&= \frac{2a^2}{3de(e\cot(c+dx))^{3/2}}+\frac{4ab}{de^2\sqrt{e\cot(c+dx)}}+\frac{\int \frac{-((a^2-b^2)e^2)-2abe^2\cot(c+dx)}{\sqrt{e\cot(c+dx)}}\,dx}{e^4}\\
&= \frac{2a^2}{3de(e\cot(c+dx))^{3/2}}+\frac{4ab}{de^2\sqrt{e\cot(c+dx)}}\\
&\quad+\frac{2\text{Subst}\left(\int \frac{(a^2-b^2)e^3+2abe^2x^2}{e^2+x^4}\,dx,x,\sqrt{e\cot(c+dx)}\right)}{de^4}\\
&= \frac{2a^2}{3de(e\cot(c+dx))^{3/2}}+\frac{4ab}{de^2\sqrt{e\cot(c+dx)}}\\
&\quad+\frac{(a^2-2ab-b^2)\,\text{Subst}\left(\int \frac{e-x^2}{e^2+x^4}\,dx,x,\sqrt{e\cot(c+dx)}\right)}{de^2}\\
&\quad+\frac{(a^2+2ab-b^2)\,\text{Subst}\left(\int \frac{e+x^2}{e^2+x^4}\,dx,x,\sqrt{e\cot(c+dx)}\right)}{de^2}\\
&= \frac{2a^2}{3de(e\cot(c+dx))^{3/2}}+\frac{4ab}{de^2\sqrt{e\cot(c+dx)}}\\
&\quad-\frac{(a^2-2ab-b^2)\,\text{Subst}\left(\int \frac{\sqrt{2}\sqrt{e}+2x}{-e-\sqrt{2}\sqrt{e}x-x^2}\,dx,x,\sqrt{e\cot(c+dx)}\right)}{2\sqrt{2}de^{5/2}}\\
&\quad-\frac{(a^2-2ab-b^2)\,\text{Subst}\left(\int \frac{\sqrt{2}\sqrt{e}-2x}{-e+\sqrt{2}\sqrt{e}x-x^2}\,dx,x,\sqrt{e\cot(c+dx)}\right)}{2\sqrt{2}de^{5/2}}\\
&\quad+\frac{(a^2+2ab-b^2)\,\text{Subst}\left(\int \frac{1}{e-\sqrt{2}\sqrt{e}x+x^2}\,dx,x,\sqrt{e\cot(c+dx)}\right)}{2de^2}\\
&\quad+\frac{(a^2+2ab-b^2)\,\text{Subst}\left(\int \frac{1}{e+\sqrt{2}\sqrt{e}x+x^2}\,dx,x,\sqrt{e\cot(c+dx)}\right)}{2de^2}
\end{aligned}
$$

$$= \frac{2a^2}{3de(e\cot(c+dx))^{3/2}} + \frac{4ab}{de^2\sqrt{e\cot(c+dx)}}$$
$$- \frac{(a^2-2ab-b^2)\log\left(\sqrt{e}+\sqrt{e}\cot(c+dx)-\sqrt{2}\sqrt{e\cot(c+dx)}\right)}{2\sqrt{2}de^{5/2}}$$
$$+ \frac{(a^2-2ab-b^2)\log\left(\sqrt{e}+\sqrt{e}\cot(c+dx)+\sqrt{2}\sqrt{e\cot(c+dx)}\right)}{2\sqrt{2}de^{5/2}}$$
$$+ \frac{(a^2+2ab-b^2)\,\mathrm{Subst}\left(\int \frac{1}{-1-x^2}\,dx, x, 1-\frac{\sqrt{2}\sqrt{e\cot(c+dx)}}{\sqrt{e}}\right)}{\sqrt{2}de^{5/2}}$$
$$- \frac{(a^2+2ab-b^2)\,\mathrm{Subst}\left(\int \frac{1}{-1-x^2}\,dx, x, 1+\frac{\sqrt{2}\sqrt{e\cot(c+dx)}}{\sqrt{e}}\right)}{\sqrt{2}de^{5/2}}$$

$$= -\frac{(a^2+2ab-b^2)\arctan\left(1-\frac{\sqrt{2}\sqrt{e\cot(c+dx)}}{\sqrt{e}}\right)}{\sqrt{2}de^{5/2}}$$
$$+ \frac{(a^2+2ab-b^2)\arctan\left(1+\frac{\sqrt{2}\sqrt{e\cot(c+dx)}}{\sqrt{e}}\right)}{\sqrt{2}de^{5/2}}$$
$$+ \frac{2a^2}{3de(e\cot(c+dx))^{3/2}} + \frac{4ab}{de^2\sqrt{e\cot(c+dx)}}$$
$$- \frac{(a^2-2ab-b^2)\log\left(\sqrt{e}+\sqrt{e}\cot(c+dx)-\sqrt{2}\sqrt{e\cot(c+dx)}\right)}{2\sqrt{2}de^{5/2}}$$
$$+ \frac{(a^2-2ab-b^2)\log\left(\sqrt{e}+\sqrt{e}\cot(c+dx)+\sqrt{2}\sqrt{e\cot(c+dx)}\right)}{2\sqrt{2}de^{5/2}}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 0.32 (sec) , antiderivative size = 82, normalized size of antiderivative = 0.28

$$\int \frac{(a+b\cot(c+dx))^2}{(e\cot(c+dx))^{5/2}}\,dx = \frac{2\left((a^2-b^2)\,\mathrm{Hypergeometric2F1}\left(-\frac{3}{4},1,\frac{1}{4},-\cot^2(c+dx)\right)+b\left(b+6a\cot(c+\right.\right.}{3de(e\cot(c+dx))^{3/2}}$$

```
[In] Integrate[(a + b*Cot[c + d*x])^2/(e*Cot[c + d*x])^(5/2),x]
```

```
[Out] (2*((a^2 - b^2)*Hypergeometric2F1[-3/4, 1, 1/4, -Cot[c + d*x]^2] + b*(b + 6
*a*Cot[c + d*x]*Hypergeometric2F1[-1/4, 1, 3/4, -Cot[c + d*x]^2])))/(3*d*e*
(e*Cot[c + d*x])^(3/2))
```

Maple [A] (verified)

Time = 0.04 (sec) , antiderivative size = 326, normalized size of antiderivative = 1.12

method	result
derivativedivides	$-\frac{2\left(\frac{\left(-a^{2}e+b^{2}e\right)\left(e^{2}\right)^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\frac{e\cot(dx+c)+\left(e^{2}\right)^{\frac{1}{4}}\sqrt{e\cot(dx+c)}\,\sqrt{2}+\sqrt{e^{2}}}{e\cot(dx+c)-\left(e^{2}\right)^{\frac{1}{4}}\sqrt{e\cot(dx+c)}\,\sqrt{2}+\sqrt{e^{2}}}\right)+2\arctan\left(\frac{\sqrt{2}\,\sqrt{e\cot(dx+c)}}{\left(e^{2}\right)^{\frac{1}{4}}}+1\right)-2\arctan\left(-\frac{\sqrt{2}\,\sqrt{\cdots}}{\cdots}\right)\cdots\right)}{8e^{2}}\cdots\right)}{\cdots}$
default	$-\frac{2\left(\frac{\left(-a^{2}e+b^{2}e\right)\left(e^{2}\right)^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\frac{e\cot(dx+c)+\left(e^{2}\right)^{\frac{1}{4}}\sqrt{e\cot(dx+c)}\,\sqrt{2}+\sqrt{e^{2}}}{e\cot(dx+c)-\left(e^{2}\right)^{\frac{1}{4}}\sqrt{e\cot(dx+c)}\,\sqrt{2}+\sqrt{e^{2}}}\right)+2\arctan\left(\frac{\sqrt{2}\,\sqrt{e\cot(dx+c)}}{\left(e^{2}\right)^{\frac{1}{4}}}+1\right)-2\arctan\left(-\frac{\sqrt{2}\,\sqrt{\cdots}}{\cdots}\right)\cdots\right)}{8e^{2}}\cdots\right)}{\cdots}$
parts	$-\frac{2a^{2}e\left(-\frac{\left(e^{2}\right)^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\frac{e\cot(dx+c)+\left(e^{2}\right)^{\frac{1}{4}}\sqrt{e\cot(dx+c)}\,\sqrt{2}+\sqrt{e^{2}}}{e\cot(dx+c)-\left(e^{2}\right)^{\frac{1}{4}}\sqrt{e\cot(dx+c)}\,\sqrt{2}+\sqrt{e^{2}}}\right)+2\arctan\left(\frac{\sqrt{2}\,\sqrt{e\cot(dx+c)}}{\left(e^{2}\right)^{\frac{1}{4}}}+1\right)-2\arctan\left(-\frac{\sqrt{2}\,\sqrt{e\cot(d\cdots}}{\left(e^{2}\right)^{\frac{1}{4}}}\cdots\right)\cdots\right)}{8e^{4}}\cdots\right)\cdots}{d}$

```
[In] int((a+b*cot(d*x+c))^2/(e*cot(d*x+c))^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] -2/d/e*(1/e*(1/8*(-a^2*e+b^2*e)*(e^2)^(1/4)/e^2*2^(1/2)*(ln((e*cot(d*x+c)+(
e^2)^(1/4)*(e*cot(d*x+c))^(1/2)*2^(1/2)+(e^2)^(1/2))/(e*cot(d*x+c)-(e^2)^(1
/4)*(e*cot(d*x+c))^(1/2)*2^(1/2)+(e^2)^(1/2)))+2*arctan(2^(1/2)/(e^2)^(1/4)
*(e*cot(d*x+c))^(1/2)+1)-2*arctan(-2^(1/2)/(e^2)^(1/4)*(e*cot(d*x+c))^(1/2)
+1))-1/4*a*b/(e^2)^(1/4)*2^(1/2)*(ln((e*cot(d*x+c)-(e^2)^(1/4)*(e*cot(d*x+c
))^(1/2)*2^(1/2)+(e^2)^(1/2))/(e*cot(d*x+c)+(e^2)^(1/4)*(e*cot(d*x+c))^(1/2
)*2^(1/2)+(e^2)^(1/2)))+2*arctan(2^(1/2)/(e^2)^(1/4)*(e*cot(d*x+c))^(1/2)+1
)-2*arctan(-2^(1/2)/(e^2)^(1/4)*(e*cot(d*x+c))^(1/2)+1)))-1/3*a^2/(e*cot(d*
x+c))^(3/2)-2*a*b/e/(e*cot(d*x+c))^(1/2))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1318 vs. 2(238) = 476.

Time = 0.31 (sec) , antiderivative size = 1318, normalized size of antiderivative = 4.53

$$\int \frac{(a+b\cot(c+dx))^2}{(e\cot(c+dx))^{5/2}}\,dx = \text{Too large to display}$$

```
[In] integrate((a+b*cot(d*x+c))^2/(e*cot(d*x+c))^(5/2),x, algorithm="fricas")
```

```
[Out] 1/6*(3*(d*e^3*cos(2*d*x + 2*c) + d*e^3)*sqrt(-(d^2*e^5*sqrt(-(a^8 - 12*a^6*
b^2 + 38*a^4*b^4 - 12*a^2*b^6 + b^8)/(d^4*e^10)) + 4*a^3*b - 4*a*b^3)/(d^2*
e^5))*log((a^8 - 4*a^6*b^2 - 10*a^4*b^4 - 4*a^2*b^6 + b^8)*sqrt((e*cos(2*d*
x + 2*c) + e)/sin(2*d*x + 2*c)) + (2*a*b*d^3*e^8*sqrt(-(a^8 - 12*a^6*b^2 +
38*a^4*b^4 - 12*a^2*b^6 + b^8)/(d^4*e^10)) + (a^6 - 7*a^4*b^2 + 7*a^2*b^4 -
 b^6)*d*e^3)*sqrt(-(d^2*e^5*sqrt(-(a^8 - 12*a^6*b^2 + 38*a^4*b^4 - 12*a^2*b
^6 + b^8)/(d^4*e^10)) + 4*a^3*b - 4*a*b^3)/(d^2*e^5))) - 3*(d*e^3*cos(2*d*x
 + 2*c) + d*e^3)*sqrt(-(d^2*e^5*sqrt(-(a^8 - 12*a^6*b^2 + 38*a^4*b^4 - 12*a
^2*b^6 + b^8)/(d^4*e^10)) + 4*a^3*b - 4*a*b^3)/(d^2*e^5))*log((a^8 - 4*a^6*
b^2 - 10*a^4*b^4 - 4*a^2*b^6 + b^8)*sqrt((e*cos(2*d*x + 2*c) + e)/sin(2*d*x
 + 2*c)) - (2*a*b*d^3*e^8*sqrt(-(a^8 - 12*a^6*b^2 + 38*a^4*b^4 - 12*a^2*b^6
 + b^8)/(d^4*e^10)) + (a^6 - 7*a^4*b^2 + 7*a^2*b^4 - b^6)*d*e^3)*sqrt(-(d^2
*e^5*sqrt(-(a^8 - 12*a^6*b^2 + 38*a^4*b^4 - 12*a^2*b^6 + b^8)/(d^4*e^10)) +
 4*a^3*b - 4*a*b^3)/(d^2*e^5))) - 3*(d*e^3*cos(2*d*x + 2*c) + d*e^3)*sqrt((
d^2*e^5*sqrt(-(a^8 - 12*a^6*b^2 + 38*a^4*b^4 - 12*a^2*b^6 + b^8)/(d^4*e^10)
) - 4*a^3*b + 4*a*b^3)/(d^2*e^5))*log((a^8 - 4*a^6*b^2 - 10*a^4*b^4 - 4*a^2
*b^6 + b^8)*sqrt((e*cos(2*d*x + 2*c) + e)/sin(2*d*x + 2*c)) + (2*a*b*d^3*e^
8*sqrt(-(a^8 - 12*a^6*b^2 + 38*a^4*b^4 - 12*a^2*b^6 + b^8)/(d^4*e^10)) - (a
^6 - 7*a^4*b^2 + 7*a^2*b^4 - b^6)*d*e^3)*sqrt((d^2*e^5*sqrt(-(a^8 - 12*a^6*
b^2 + 38*a^4*b^4 - 12*a^2*b^6 + b^8)/(d^4*e^10)) - 4*a^3*b + 4*a*b^3)/(d^2*
e^5))) + 3*(d*e^3*cos(2*d*x + 2*c) + d*e^3)*sqrt((d^2*e^5*sqrt(-(a^8 - 12*a
^6*b^2 + 38*a^4*b^4 - 12*a^2*b^6 + b^8)/(d^4*e^10)) - 4*a^3*b + 4*a*b^3)/(d
^2*e^5))*log((a^8 - 4*a^6*b^2 - 10*a^4*b^4 - 4*a^2*b^6 + b^8)*sqrt((e*cos(2
*d*x + 2*c) + e)/sin(2*d*x + 2*c)) - (2*a*b*d^3*e^8*sqrt(-(a^8 - 12*a^6*b^2
 + 38*a^4*b^4 - 12*a^2*b^6 + b^8)/(d^4*e^10)) - (a^6 - 7*a^4*b^2 + 7*a^2*b^
4 - b^6)*d*e^3)*sqrt((d^2*e^5*sqrt(-(a^8 - 12*a^6*b^2 + 38*a^4*b^4 - 12*a^2
*b^6 + b^8)/(d^4*e^10)) - 4*a^3*b + 4*a*b^3)/(d^2*e^5))) - 4*(a^2*cos(2*d*x
 + 2*c) - 6*a*b*sin(2*d*x + 2*c) - a^2)*sqrt((e*cos(2*d*x + 2*c) + e)/sin(2
*d*x + 2*c)))/(d*e^3*cos(2*d*x + 2*c) + d*e^3)
```

Sympy [F]

$$\int \frac{(a+b\cot(c+dx))^2}{(e\cot(c+dx))^{5/2}}\,dx = \int \frac{(a+b\cot{(c+dx)})^2}{(e\cot{(c+dx)})^{\frac{5}{2}}}\,dx$$

```
[In] integrate((a+b*cot(d*x+c))**2/(e*cot(d*x+c))**(5/2),x)
[Out] Integral((a + b*cot(c + d*x))**2/(e*cot(c + d*x))**(5/2), x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{(a+b\cot(c+dx))^2}{(e\cot(c+dx))^{5/2}}\,dx = \text{Exception raised: ValueError}$$

```
[In] integrate((a+b*cot(d*x+c))^2/(e*cot(d*x+c))^(5/2),x, algorithm="maxima")
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(e>0)', see `assume?` for more detai
ls)Is e
```

Giac [F]

$$\int \frac{(a+b\cot(c+dx))^2}{(e\cot(c+dx))^{5/2}}\,dx = \int \frac{(b\cot{(dx+c)}+a)^2}{(e\cot{(dx+c)})^{\frac{5}{2}}}\,dx$$

```
[In] integrate((a+b*cot(d*x+c))^2/(e*cot(d*x+c))^(5/2),x, algorithm="giac")
[Out] integrate((b*cot(d*x + c) + a)^2/(e*cot(d*x + c))^(5/2), x)
```

Mupad [B] (verification not implemented)

Time = 14.23 (sec) , antiderivative size = 1214, normalized size of antiderivative = 4.17

$$\int \frac{(a+b\cot(c+dx))^2}{(e\cot(c+dx))^{5/2}}\,dx = \text{Too large to display}$$

```
[In] int((a + b*cot(c + d*x))^2/(e*cot(c + d*x))^(5/2),x)
[Out] ((2*a^2)/3 + 4*a*b*cot(c + d*x))/(d*e*(e*cot(c + d*x))^(3/2)) - 2*atanh((32
*a^4*d^3*e^8*(e*cot(c + d*x))^(1/2)*((a^4*1i)/(4*d^2*e^5) + (b^4*1i)/(4*d^2
```

```
*e^5) + (a*b^3)/(d^2*e^5) - (a^3*b)/(d^2*e^5) - (a^2*b^2*3i)/(2*d^2*e^5))^(
1/2))/(b^6*d^2*e^6*16i - a^6*d^2*e^6*16i + 32*a*b^5*d^2*e^6 + 32*a^5*b*d^2*
e^6 - a^2*b^4*d^2*e^6*112i - 192*a^3*b^3*d^2*e^6 + a^4*b^2*d^2*e^6*112i) +
(32*b^4*d^3*e^8*(e*cot(c + d*x))^(1/2)*((a^4*1i)/(4*d^2*e^5) + (b^4*1i)/(4*
d^2*e^5) + (a*b^3)/(d^2*e^5) - (a^3*b)/(d^2*e^5) - (a^2*b^2*3i)/(2*d^2*e^5)
)^(1/2))/(b^6*d^2*e^6*16i - a^6*d^2*e^6*16i + 32*a*b^5*d^2*e^6 + 32*a^5*b*d
^2*e^6 - a^2*b^4*d^2*e^6*112i - 192*a^3*b^3*d^2*e^6 + a^4*b^2*d^2*e^6*112i)
 - (192*a^2*b^2*d^3*e^8*(e*cot(c + d*x))^(1/2)*((a^4*1i)/(4*d^2*e^5) + (b^4
*1i)/(4*d^2*e^5) + (a*b^3)/(d^2*e^5) - (a^3*b)/(d^2*e^5) - (a^2*b^2*3i)/(2*
d^2*e^5))^(1/2))/(b^6*d^2*e^6*16i - a^6*d^2*e^6*16i + 32*a*b^5*d^2*e^6 + 32
*a^5*b*d^2*e^6 - a^2*b^4*d^2*e^6*112i - 192*a^3*b^3*d^2*e^6 + a^4*b^2*d^2*e
^6*112i))*(((a^3*b*4i - a*b^3*4i + a^4 + b^4 - 6*a^2*b^2)*1i)/(4*d^2*e^5))^
(1/2) - 2*atanh((32*a^4*d^3*e^8*(e*cot(c + d*x))^(1/2)*((a*b^3)/(d^2*e^5) -
 (b^4*1i)/(4*d^2*e^5) - (a^4*1i)/(4*d^2*e^5) - (a^3*b)/(d^2*e^5) + (a^2*b^2
*3i)/(2*d^2*e^5))^(1/2))/(a^6*d^2*e^6*16i - b^6*d^2*e^6*16i + 32*a*b^5*d^2*
e^6 + 32*a^5*b*d^2*e^6 + a^2*b^4*d^2*e^6*112i - 192*a^3*b^3*d^2*e^6 - a^4*b
^2*d^2*e^6*112i) + (32*b^4*d^3*e^8*(e*cot(c + d*x))^(1/2)*((a*b^3)/(d^2*e^5
) - (b^4*1i)/(4*d^2*e^5) - (a^4*1i)/(4*d^2*e^5) - (a^3*b)/(d^2*e^5) + (a^2*
b^2*3i)/(2*d^2*e^5))^(1/2))/(a^6*d^2*e^6*16i - b^6*d^2*e^6*16i + 32*a*b^5*d
^2*e^6 + 32*a^5*b*d^2*e^6 + a^2*b^4*d^2*e^6*112i - 192*a^3*b^3*d^2*e^6 - a^
4*b^2*d^2*e^6*112i) - (192*a^2*b^2*d^3*e^8*(e*cot(c + d*x))^(1/2)*((a*b^3)/
(d^2*e^5) - (b^4*1i)/(4*d^2*e^5) - (a^4*1i)/(4*d^2*e^5) - (a^3*b)/(d^2*e^5)
 + (a^2*b^2*3i)/(2*d^2*e^5))^(1/2))/(a^6*d^2*e^6*16i - b^6*d^2*e^6*16i + 32
*a*b^5*d^2*e^6 + 32*a^5*b*d^2*e^6 + a^2*b^4*d^2*e^6*112i - 192*a^3*b^3*d^2*
e^6 - a^4*b^2*d^2*e^6*112i))*(-((a*b^3*4i - a^3*b*4i + a^4 + b^4 - 6*a^2*b^
2)*1i)/(4*d^2*e^5))^(1/2)
```

3.61 $\int \frac{(a+b\cot(c+dx))^2}{(e\cot(c+dx))^{7/2}}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 322

$$\begin{aligned}\int \frac{(a+b\cot(c+dx))^2}{(e\cot(c+dx))^{7/2}}\,dx &= \frac{(a^2-2ab-b^2)\arctan\left(1-\frac{\sqrt{2}\sqrt{e\cot(c+dx)}}{\sqrt{e}}\right)}{\sqrt{2}de^{7/2}} \\ &- \frac{(a^2-2ab-b^2)\arctan\left(1+\frac{\sqrt{2}\sqrt{e\cot(c+dx)}}{\sqrt{e}}\right)}{\sqrt{2}de^{7/2}} + \frac{2a^2}{5de(e\cot(c+dx))^{5/2}} \\ &+ \frac{4ab}{3de^2(e\cot(c+dx))^{3/2}} - \frac{2(a^2-b^2)}{de^3\sqrt{e\cot(c+dx)}} \\ &- \frac{(a^2+2ab-b^2)\log\left(\sqrt{e}+\sqrt{e}\cot(c+dx)-\sqrt{2}\sqrt{e\cot(c+dx)}\right)}{2\sqrt{2}de^{7/2}} \\ &+ \frac{(a^2+2ab-b^2)\log\left(\sqrt{e}+\sqrt{e}\cot(c+dx)+\sqrt{2}\sqrt{e\cot(c+dx)}\right)}{2\sqrt{2}de^{7/2}}\end{aligned}$$

```
[Out] 2/5*a^2/d/e/(e*cot(d*x+c))^(5/2)+4/3*a*b/d/e^2/(e*cot(d*x+c))^(3/2)+1/2*(a^
2-2*a*b-b^2)*arctan(1-2^(1/2)*(e*cot(d*x+c))^(1/2)/e^(1/2))/d/e^(7/2)*2^(1/
2)-1/2*(a^2-2*a*b-b^2)*arctan(1+2^(1/2)*(e*cot(d*x+c))^(1/2)/e^(1/2))/d/e^(
7/2)*2^(1/2)-1/4*(a^2+2*a*b-b^2)*ln(e^(1/2)+cot(d*x+c)*e^(1/2)-2^(1/2)*(e*c
ot(d*x+c))^(1/2))/d/e^(7/2)*2^(1/2)+1/4*(a^2+2*a*b-b^2)*ln(e^(1/2)+cot(d*x+
c)*e^(1/2)+2^(1/2)*(e*cot(d*x+c))^(1/2))/d/e^(7/2)*2^(1/2)-2*(a^2-b^2)/d/e^
3/(e*cot(d*x+c))^(1/2)
```

Rubi [A] (verified)

Time = 0.52 (sec) , antiderivative size = 322, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.360$, Rules used = {3623, 3610, 3615, 1182, 1176, 631, 210, 1179, 642}

$$\int \frac{(a+b\cot(c+dx))^2}{(e\cot(c+dx))^{7/2}}\,dx = \frac{(a^2-2ab-b^2)\arctan\left(1-\frac{\sqrt{2}\sqrt{e\cot(c+dx)}}{\sqrt{e}}\right)}{\sqrt{2}de^{7/2}} - \frac{(a^2-2ab-b^2)\arctan\left(\frac{\sqrt{2}\sqrt{e\cot(c+dx)}}{\sqrt{e}}+1\right)}{\sqrt{2}de^{7/2}} - \frac{(a^2+2ab-b^2)\log\left(\sqrt{e}\cot(c+dx)-\sqrt{2}\sqrt{e\cot(c+dx)}+\sqrt{e}\right)}{2\sqrt{2}de^{7/2}} + \frac{(a^2+2ab-b^2)\log\left(\sqrt{e}\cot(c+dx)+\sqrt{2}\sqrt{e\cot(c+dx)}+\sqrt{e}\right)}{2\sqrt{2}de^{7/2}} - \frac{2(a^2-b^2)}{de^3\sqrt{e\cot(c+dx)}} + \frac{2a^2}{5de(e\cot(c+dx))^{5/2}} + \frac{4ab}{3de^2(e\cot(c+dx))^{3/2}}$$

```
[In] Int[(a + b*Cot[c + d*x])^2/(e*Cot[c + d*x])^(7/2),x]
```

```
[Out] ((a^2 - 2*a*b - b^2)*ArcTan[1 - (Sqrt[2]*Sqrt[e*Cot[c + d*x]])/Sqrt[e]])/(S
qrt[2]*d*e^(7/2)) - ((a^2 - 2*a*b - b^2)*ArcTan[1 + (Sqrt[2]*Sqrt[e*Cot[c +
 d*x]])/Sqrt[e]])/(Sqrt[2]*d*e^(7/2)) + (2*a^2)/(5*d*e*(e*Cot[c + d*x])^(5/
2)) + (4*a*b)/(3*d*e^2*(e*Cot[c + d*x])^(3/2)) - (2*(a^2 - b^2))/(d*e^3*Sqr
t[e*Cot[c + d*x]]) - ((a^2 + 2*a*b - b^2)*Log[Sqrt[e] + Sqrt[e]*Cot[c + d*x
] - Sqrt[2]*Sqrt[e*Cot[c + d*x]]])/(2*Sqrt[2]*d*e^(7/2)) + ((a^2 + 2*a*b -
b^2)*Log[Sqrt[e] + Sqrt[e]*Cot[c + d*x] + Sqrt[2]*Sqrt[e*Cot[c + d*x]]])/(2
*Sqrt[2]*d*e^(7/2))
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1179

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 1182

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
a*c, 2]}, Dist[(d*q + a*e)/(2*a*c), Int[(q + c*x^2)/(a + c*x^4), x], x] + D
ist[(d*q - a*e)/(2*a*c), Int[(q - c*x^2)/(a + c*x^4), x], x]] /; FreeQ[{a,
c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0] && NeQ[c*d^2 - a*e^2, 0] && NegQ[(-a)
*c]
```

Rule 3610

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)]), x_Symbol] :> Simp[(b*c - a*d)*((a + b*Tan[e + f*x])^(m + 1)/
(f*(m + 1)*(a^2 + b^2))), x] + Dist[1/(a^2 + b^2), Int[(a + b*Tan[e + f*x])
^(m + 1)*Simp[a*c + b*d - (b*c - a*d)*Tan[e + f*x], x], x], x] /; FreeQ[{a,
 b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && LtQ[m, -1]
```

Rule 3615

```
Int[((c_) + (d_.)*tan[(e_.) + (f_.)*(x_)])/Sqrt[(b_.)*tan[(e_.) + (f_.)*(x_
)]], x_Symbol] :> Dist[2/f, Subst[Int[(b*c + d*x^2)/(b^2 + x^4), x], x, Sqr
t[b*Tan[e + f*x]]], x] /; FreeQ[{b, c, d, e, f}, x] && NeQ[c^2 - d^2, 0] &&
 NeQ[c^2 + d^2, 0]
```

Rule 3623

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)])^2, x_Symbol] :> Simp[(b*c - a*d)^2*((a + b*Tan[e + f*x])^(m +
 1)/(b*f*(m + 1)*(a^2 + b^2))), x] + Dist[1/(a^2 + b^2), Int[(a + b*Tan[e +
```

```
f*x])^(m + 1)*Simp[a*c^2 + 2*b*c*d - a*d^2 - (b*c^2 - 2*a*c*d - b*d^2)*Tan
[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
&& LtQ[m, -1] && NeQ[a^2 + b^2, 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{2a^2}{5de(e\cot(c+dx))^{5/2}}+\frac{\int \frac{2abe-(a^2-b^2)e\cot(c+dx)}{(e\cot(c+dx))^{5/2}}\,dx}{e^2}\\
&= \frac{2a^2}{5de(e\cot(c+dx))^{5/2}}+\frac{4ab}{3de^2(e\cot(c+dx))^{3/2}}+\frac{\int \frac{-((a^2-b^2)e^2)-2abe^2\cot(c+dx)}{(e\cot(c+dx))^{3/2}}\,dx}{e^4}\\
&= \frac{2a^2}{5de(e\cot(c+dx))^{5/2}}+\frac{4ab}{3de^2(e\cot(c+dx))^{3/2}}\\
&\quad-\frac{2(a^2-b^2)}{de^3\sqrt{e\cot(c+dx)}}+\frac{\int \frac{-2abe^3+(a^2-b^2)e^3\cot(c+dx)}{\sqrt{e\cot(c+dx)}}\,dx}{e^6}\\
&= \frac{2a^2}{5de(e\cot(c+dx))^{5/2}}+\frac{4ab}{3de^2(e\cot(c+dx))^{3/2}}-\frac{2(a^2-b^2)}{de^3\sqrt{e\cot(c+dx)}}\\
&\quad+\frac{2\text{Subst}\left(\int \frac{2abe^4-(a^2-b^2)e^3x^2}{e^2+x^4}\,dx,x,\sqrt{e\cot(c+dx)}\right)}{de^6}\\
&= \frac{2a^2}{5de(e\cot(c+dx))^{5/2}}+\frac{4ab}{3de^2(e\cot(c+dx))^{3/2}}-\frac{2(a^2-b^2)}{de^3\sqrt{e\cot(c+dx)}}\\
&\quad-\frac{(a^2-2ab-b^2)\,\text{Subst}\left(\int \frac{e+x^2}{e^2+x^4}\,dx,x,\sqrt{e\cot(c+dx)}\right)}{de^3}\\
&\quad+\frac{(a^2+2ab-b^2)\,\text{Subst}\left(\int \frac{e-x^2}{e^2+x^4}\,dx,x,\sqrt{e\cot(c+dx)}\right)}{de^3}\\
&= \frac{2a^2}{5de(e\cot(c+dx))^{5/2}}+\frac{4ab}{3de^2(e\cot(c+dx))^{3/2}}-\frac{2(a^2-b^2)}{de^3\sqrt{e\cot(c+dx)}}\\
&\quad-\frac{(a^2+2ab-b^2)\,\text{Subst}\left(\int \frac{\sqrt{2}\sqrt{e}+2x}{-e-\sqrt{2}\sqrt{e}x-x^2}\,dx,x,\sqrt{e\cot(c+dx)}\right)}{2\sqrt{2}de^{7/2}}\\
&\quad-\frac{(a^2+2ab-b^2)\,\text{Subst}\left(\int \frac{\sqrt{2}\sqrt{e}-2x}{-e+\sqrt{2}\sqrt{e}x-x^2}\,dx,x,\sqrt{e\cot(c+dx)}\right)}{2\sqrt{2}de^{7/2}}\\
&\quad-\frac{(a^2-2ab-b^2)\,\text{Subst}\left(\int \frac{1}{e-\sqrt{2}\sqrt{e}x+x^2}\,dx,x,\sqrt{e\cot(c+dx)}\right)}{2de^3}\\
&\quad-\frac{(a^2-2ab-b^2)\,\text{Subst}\left(\int \frac{1}{e+\sqrt{2}\sqrt{e}x+x^2}\,dx,x,\sqrt{e\cot(c+dx)}\right)}{2de^3}
\end{aligned}$$

$$= \frac{2a^2}{5de(e\cot(c+dx))^{5/2}} + \frac{4ab}{3de^2(e\cot(c+dx))^{3/2}} - \frac{2(a^2-b^2)}{de^3\sqrt{e\cot(c+dx)}}$$
$$- \frac{(a^2+2ab-b^2)\log\left(\sqrt{e}+\sqrt{e}\cot(c+dx)-\sqrt{2}\sqrt{e\cot(c+dx)}\right)}{2\sqrt{2}de^{7/2}}$$
$$+ \frac{(a^2+2ab-b^2)\log\left(\sqrt{e}+\sqrt{e}\cot(c+dx)+\sqrt{2}\sqrt{e\cot(c+dx)}\right)}{2\sqrt{2}de^{7/2}}$$
$$- \frac{(a^2-2ab-b^2)\,\text{Subst}\left(\int \frac{1}{-1-x^2}\,dx, x, 1-\frac{\sqrt{2}\sqrt{e\cot(c+dx)}}{\sqrt{e}}\right)}{\sqrt{2}de^{7/2}}$$
$$+ \frac{(a^2-2ab-b^2)\,\text{Subst}\left(\int \frac{1}{-1-x^2}\,dx, x, 1+\frac{\sqrt{2}\sqrt{e\cot(c+dx)}}{\sqrt{e}}\right)}{\sqrt{2}de^{7/2}}$$

$$= \frac{(a^2-2ab-b^2)\arctan\left(1-\frac{\sqrt{2}\sqrt{e\cot(c+dx)}}{\sqrt{e}}\right)}{\sqrt{2}de^{7/2}}$$
$$- \frac{(a^2-2ab-b^2)\arctan\left(1+\frac{\sqrt{2}\sqrt{e\cot(c+dx)}}{\sqrt{e}}\right)}{\sqrt{2}de^{7/2}} + \frac{2a^2}{5de(e\cot(c+dx))^{5/2}}$$
$$+ \frac{4ab}{3de^2(e\cot(c+dx))^{3/2}} - \frac{2(a^2-b^2)}{de^3\sqrt{e\cot(c+dx)}}$$
$$- \frac{(a^2+2ab-b^2)\log\left(\sqrt{e}+\sqrt{e}\cot(c+dx)-\sqrt{2}\sqrt{e\cot(c+dx)}\right)}{2\sqrt{2}de^{7/2}}$$
$$+ \frac{(a^2+2ab-b^2)\log\left(\sqrt{e}+\sqrt{e}\cot(c+dx)+\sqrt{2}\sqrt{e\cot(c+dx)}\right)}{2\sqrt{2}de^{7/2}}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 0.41 (sec) , antiderivative size = 85, normalized size of antiderivative = 0.26

$$\int \frac{(a+b\cot(c+dx))^2}{(e\cot(c+dx))^{7/2}}\,dx = \frac{2\left(3(a^2-b^2)\,\text{Hypergeometric2F1}\left(-\frac{5}{4},1,-\frac{1}{4},-\cot^2(c+dx)\right)+b\left(3b+10a\cot(c\right.\right.}{15de(e\cot(c+dx))^{5/2}}$$

```
[In] Integrate[(a + b*Cot[c + d*x])^2/(e*Cot[c + d*x])^(7/2),x]
```

```
[Out] (2*(3*(a^2 - b^2)*Hypergeometric2F1[-5/4, 1, -1/4, -Cot[c + d*x]^2] + b*(3*
b + 10*a*Cot[c + d*x]*Hypergeometric2F1[-3/4, 1, 1/4, -Cot[c + d*x]^2])))/(
15*d*e*(e*Cot[c + d*x])^(5/2))
```

Maple [A] (verified)

Time = 0.04 (sec) , antiderivative size = 347, normalized size of antiderivative = 1.08

method	result
derivativedivides	$-\frac{2\left(-\frac{ab\left(e^{2}\right)^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\frac{e\cot(dx+c)+\left(e^{2}\right)^{\frac{1}{4}}\sqrt{e\cot(dx+c)}\sqrt{2}+\sqrt{e^{2}}}{e\cot(dx+c)-\left(e^{2}\right)^{\frac{1}{4}}\sqrt{e\cot(dx+c)}\sqrt{2}+\sqrt{e^{2}}}\right)+2\arctan\left(\frac{\sqrt{2}\sqrt{e\cot(dx+c)}}{\left(e^{2}\right)^{\frac{1}{4}}}+1\right)-2\arctan\left(-\frac{\sqrt{2}\sqrt{e\cot(d}}{\left(e^{2}\right)^{\frac{1}{4}}}\right.\right.}{4e}\right.}{}$
default	$-\frac{2\left(-\frac{ab\left(e^{2}\right)^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\frac{e\cot(dx+c)+\left(e^{2}\right)^{\frac{1}{4}}\sqrt{e\cot(dx+c)}\sqrt{2}+\sqrt{e^{2}}}{e\cot(dx+c)-\left(e^{2}\right)^{\frac{1}{4}}\sqrt{e\cot(dx+c)}\sqrt{2}+\sqrt{e^{2}}}\right)+2\arctan\left(\frac{\sqrt{2}\sqrt{e\cot(dx+c)}}{\left(e^{2}\right)^{\frac{1}{4}}}+1\right)-2\arctan\left(-\frac{\sqrt{2}\sqrt{e\cot(d}}{\left(e^{2}\right)^{\frac{1}{4}}}\right.\right.}{4e}\right.}{}$
parts	$-\frac{2a^{2}e\left(-\frac{1}{5e^{2}(e\cot(dx+c))^{\frac{5}{2}}}+\frac{1}{e^{4}\sqrt{e\cot(dx+c)}}+\frac{\sqrt{2}\left(\ln\left(\frac{e\cot(dx+c)-\left(e^{2}\right)^{\frac{1}{4}}\sqrt{e\cot(dx+c)}\sqrt{2}+\sqrt{e^{2}}}{e\cot(dx+c)+\left(e^{2}\right)^{\frac{1}{4}}\sqrt{e\cot(dx+c)}\sqrt{2}+\sqrt{e^{2}}}\right)+2\arctan\left(\frac{\sqrt{2}\sqrt{e\cot(d}}{\left(e^{2}\right)^{\frac{1}{4}}}\right.\right.}{8e^{4}\left(e^{2}\right)^{\frac{1}{4}}}\right.}{d}$

```
[In] int((a+b*cot(d*x+c))^2/(e*cot(d*x+c))^(7/2),x,method=_RETURNVERBOSE)
```

```
[Out] -2/d/e*(1/e^2*(-1/4*a/e*b*(e^2)^(1/4)*2^(1/2)*(ln((e*cot(d*x+c)+(e^2)^(1/4)
*(e*cot(d*x+c))^(1/2)*2^(1/2)+(e^2)^(1/2))/(e*cot(d*x+c)-(e^2)^(1/4)*(e*cot
(d*x+c))^(1/2)*2^(1/2)+(e^2)^(1/2)))+2*arctan(2^(1/2)/(e^2)^(1/4)*(e*cot(d*
x+c))^(1/2)+1)-2*arctan(-2^(1/2)/(e^2)^(1/4)*(e*cot(d*x+c))^(1/2)+1))+1/8*(
a^2-b^2)/(e^2)^(1/4)*2^(1/2)*(ln((e*cot(d*x+c)-(e^2)^(1/4)*(e*cot(d*x+c))^(
1/2)*2^(1/2)+(e^2)^(1/2))/(e*cot(d*x+c)+(e^2)^(1/4)*(e*cot(d*x+c))^(1/2)*2^
(1/2)+(e^2)^(1/2)))+2*arctan(2^(1/2)/(e^2)^(1/4)*(e*cot(d*x+c))^(1/2)+1)-2*
arctan(-2^(1/2)/(e^2)^(1/4)*(e*cot(d*x+c))^(1/2)+1)))-1/5*a^2/(e*cot(d*x+c)
)^(5/2)-(-a^2+b^2)/e^2/(e*cot(d*x+c))^(1/2)-2/3*a*b/e/(e*cot(d*x+c))^(3/2))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1436 vs. $2(265) = 530$.

Time = 0.32 (sec) , antiderivative size = 1436, normalized size of antiderivative = 4.46

$$\int \frac{(a+b\cot(c+dx))^2}{(e\cot(c+dx))^{7/2}}\,dx = \text{Too large to display}$$

```
[In] integrate((a+b*cot(d*x+c))^2/(e*cot(d*x+c))^(7/2),x, algorithm="fricas")
```

```
[Out] -1/30*(15*(d*e^4*cos(2*d*x + 2*c)^2 + 2*d*e^4*cos(2*d*x + 2*c) + d*e^4)*sqr
t((d^2*e^7*sqrt(-(a^8 - 12*a^6*b^2 + 38*a^4*b^4 - 12*a^2*b^6 + b^8)/(d^4*e^
14)) + 4*a^3*b - 4*a*b^3)/(d^2*e^7))*log((a^8 - 4*a^6*b^2 - 10*a^4*b^4 - 4*
a^2*b^6 + b^8)*sqrt((e*cos(2*d*x + 2*c) + e)/sin(2*d*x + 2*c)) + ((a^2 - b^
2)*d^3*e^11*sqrt(-(a^8 - 12*a^6*b^2 + 38*a^4*b^4 - 12*a^2*b^6 + b^8)/(d^4*e
^14)) - 2*(a^5*b - 6*a^3*b^3 + a*b^5)*d*e^4)*sqrt((d^2*e^7*sqrt(-(a^8 - 12*
a^6*b^2 + 38*a^4*b^4 - 12*a^2*b^6 + b^8)/(d^4*e^14)) + 4*a^3*b - 4*a*b^3)/(
d^2*e^7))) - 15*(d*e^4*cos(2*d*x + 2*c)^2 + 2*d*e^4*cos(2*d*x + 2*c) + d*e^
4)*sqrt((d^2*e^7*sqrt(-(a^8 - 12*a^6*b^2 + 38*a^4*b^4 - 12*a^2*b^6 + b^8)/(
d^4*e^14)) + 4*a^3*b - 4*a*b^3)/(d^2*e^7))*log((a^8 - 4*a^6*b^2 - 10*a^4*b^
4 - 4*a^2*b^6 + b^8)*sqrt((e*cos(2*d*x + 2*c) + e)/sin(2*d*x + 2*c)) - ((a^
2 - b^2)*d^3*e^11*sqrt(-(a^8 - 12*a^6*b^2 + 38*a^4*b^4 - 12*a^2*b^6 + b^8)/
(d^4*e^14)) - 2*(a^5*b - 6*a^3*b^3 + a*b^5)*d*e^4)*sqrt((d^2*e^7*sqrt(-(a^8
 - 12*a^6*b^2 + 38*a^4*b^4 - 12*a^2*b^6 + b^8)/(d^4*e^14)) + 4*a^3*b - 4*a*
b^3)/(d^2*e^7))) - 15*(d*e^4*cos(2*d*x + 2*c)^2 + 2*d*e^4*cos(2*d*x + 2*c)
+ d*e^4)*sqrt(-(d^2*e^7*sqrt(-(a^8 - 12*a^6*b^2 + 38*a^4*b^4 - 12*a^2*b^6 +
 b^8)/(d^4*e^14)) - 4*a^3*b + 4*a*b^3)/(d^2*e^7))*log((a^8 - 4*a^6*b^2 - 10
*a^4*b^4 - 4*a^2*b^6 + b^8)*sqrt((e*cos(2*d*x + 2*c) + e)/sin(2*d*x + 2*c))
 + ((a^2 - b^2)*d^3*e^11*sqrt(-(a^8 - 12*a^6*b^2 + 38*a^4*b^4 - 12*a^2*b^6
+ b^8)/(d^4*e^14)) + 2*(a^5*b - 6*a^3*b^3 + a*b^5)*d*e^4)*sqrt(-(d^2*e^7*sq
rt(-(a^8 - 12*a^6*b^2 + 38*a^4*b^4 - 12*a^2*b^6 + b^8)/(d^4*e^14)) - 4*a^3*
b + 4*a*b^3)/(d^2*e^7))) + 15*(d*e^4*cos(2*d*x + 2*c)^2 + 2*d*e^4*cos(2*d*x
 + 2*c) + d*e^4)*sqrt(-(d^2*e^7*sqrt(-(a^8 - 12*a^6*b^2 + 38*a^4*b^4 - 12*a
^2*b^6 + b^8)/(d^4*e^14)) - 4*a^3*b + 4*a*b^3)/(d^2*e^7))*log((a^8 - 4*a^6*
b^2 - 10*a^4*b^4 - 4*a^2*b^6 + b^8)*sqrt((e*cos(2*d*x + 2*c) + e)/sin(2*d*x
 + 2*c)) - ((a^2 - b^2)*d^3*e^11*sqrt(-(a^8 - 12*a^6*b^2 + 38*a^4*b^4 - 12*
a^2*b^6 + b^8)/(d^4*e^14)) + 2*(a^5*b - 6*a^3*b^3 + a*b^5)*d*e^4)*sqrt(-(d^
2*e^7*sqrt(-(a^8 - 12*a^6*b^2 + 38*a^4*b^4 - 12*a^2*b^6 + b^8)/(d^4*e^14))
- 4*a^3*b + 4*a*b^3)/(d^2*e^7))) + 4*(10*a*b*cos(2*d*x + 2*c)^2 - 10*a*b +
3*(4*a^2 - 5*b^2 + (6*a^2 - 5*b^2)*cos(2*d*x + 2*c))*sin(2*d*x + 2*c))*sqrt
((e*cos(2*d*x + 2*c) + e)/sin(2*d*x + 2*c)))/(d*e^4*cos(2*d*x + 2*c)^2 + 2*
d*e^4*cos(2*d*x + 2*c) + d*e^4)
```

Sympy [F]

$$\int \frac{(a+b\cot(c+dx))^2}{(e\cot(c+dx))^{7/2}}\,dx = \int \frac{(a+b\cot{(c+dx)})^2}{(e\cot{(c+dx)})^{\frac{7}{2}}}\,dx$$

```
[In] integrate((a+b*cot(d*x+c))**2/(e*cot(d*x+c))**(7/2),x)
[Out] Integral((a + b*cot(c + d*x))**2/(e*cot(c + d*x))**(7/2), x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{(a+b\cot(c+dx))^2}{(e\cot(c+dx))^{7/2}}\,dx = \text{Exception raised: ValueError}$$

```
[In] integrate((a+b*cot(d*x+c))^2/(e*cot(d*x+c))^(7/2),x, algorithm="maxima")
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(e>0)', see `assume?` for more detai
ls)Is e
```

Giac [F]

$$\int \frac{(a+b\cot(c+dx))^2}{(e\cot(c+dx))^{7/2}}\,dx = \int \frac{(b\cot{(dx+c)}+a)^2}{(e\cot{(dx+c)})^{\frac{7}{2}}}\,dx$$

```
[In] integrate((a+b*cot(d*x+c))^2/(e*cot(d*x+c))^(7/2),x, algorithm="giac")
[Out] integrate((b*cot(d*x + c) + a)^2/(e*cot(d*x + c))^(7/2), x)
```

Mupad [B] (verification not implemented)

Time = 15.09 (sec) , antiderivative size = 1227, normalized size of antiderivative = 3.81

$$\int \frac{(a+b\cot(c+dx))^2}{(e\cot(c+dx))^{7/2}}\,dx = \text{Too large to display}$$

```
[In] int((a + b*cot(c + d*x))^2/(e*cot(c + d*x))^(7/2),x)
[Out] 2*atanh((32*a^4*d^3*e^11*(e*cot(c + d*x))^(1/2)*((a^3*b)/(d^2*e^7) - (b^4*1
i)/(4*d^2*e^7) - (a*b^3)/(d^2*e^7) - (a^4*1i)/(4*d^2*e^7) + (a^2*b^2*3i)/(2
```

```
*d^2*e^7))^(1/2))/(16*a^6*d^2*e^8 - 16*b^6*d^2*e^8 + a*b^5*d^2*e^8*32i + a^
5*b*d^2*e^8*32i + 112*a^2*b^4*d^2*e^8 - a^3*b^3*d^2*e^8*192i - 112*a^4*b^2*
d^2*e^8) + (32*b^4*d^3*e^11*(e*cot(c + d*x))^(1/2)*((a^3*b)/(d^2*e^7) - (b^
4*1i)/(4*d^2*e^7) - (a*b^3)/(d^2*e^7) - (a^4*1i)/(4*d^2*e^7) + (a^2*b^2*3i)
/(2*d^2*e^7))^(1/2))/(16*a^6*d^2*e^8 - 16*b^6*d^2*e^8 + a*b^5*d^2*e^8*32i +
 a^5*b*d^2*e^8*32i + 112*a^2*b^4*d^2*e^8 - a^3*b^3*d^2*e^8*192i - 112*a^4*b
^2*d^2*e^8) - (192*a^2*b^2*d^3*e^11*(e*cot(c + d*x))^(1/2)*((a^3*b)/(d^2*e^
7) - (b^4*1i)/(4*d^2*e^7) - (a*b^3)/(d^2*e^7) - (a^4*1i)/(4*d^2*e^7) + (a^2
*b^2*3i)/(2*d^2*e^7))^(1/2))/(16*a^6*d^2*e^8 - 16*b^6*d^2*e^8 + a*b^5*d^2*e
^8*32i + a^5*b*d^2*e^8*32i + 112*a^2*b^4*d^2*e^8 - a^3*b^3*d^2*e^8*192i - 1
12*a^4*b^2*d^2*e^8))*(-((a^3*b*4i - a*b^3*4i + a^4 + b^4 - 6*a^2*b^2)*1i)/(
4*d^2*e^7))^(1/2) - 2*atanh((32*a^4*d^3*e^11*(e*cot(c + d*x))^(1/2)*((a^4*1
i)/(4*d^2*e^7) + (b^4*1i)/(4*d^2*e^7) - (a*b^3)/(d^2*e^7) + (a^3*b)/(d^2*e^
7) - (a^2*b^2*3i)/(2*d^2*e^7))^(1/2))/(16*b^6*d^2*e^8 - 16*a^6*d^2*e^8 + a*
b^5*d^2*e^8*32i + a^5*b*d^2*e^8*32i - 112*a^2*b^4*d^2*e^8 - a^3*b^3*d^2*e^8
*192i + 112*a^4*b^2*d^2*e^8) + (32*b^4*d^3*e^11*(e*cot(c + d*x))^(1/2)*((a^
4*1i)/(4*d^2*e^7) + (b^4*1i)/(4*d^2*e^7) - (a*b^3)/(d^2*e^7) + (a^3*b)/(d^2
*e^7) - (a^2*b^2*3i)/(2*d^2*e^7))^(1/2))/(16*b^6*d^2*e^8 - 16*a^6*d^2*e^8 +
 a*b^5*d^2*e^8*32i + a^5*b*d^2*e^8*32i - 112*a^2*b^4*d^2*e^8 - a^3*b^3*d^2*
e^8*192i + 112*a^4*b^2*d^2*e^8) - (192*a^2*b^2*d^3*e^11*(e*cot(c + d*x))^(1
/2)*((a^4*1i)/(4*d^2*e^7) + (b^4*1i)/(4*d^2*e^7) - (a*b^3)/(d^2*e^7) + (a^3
*b)/(d^2*e^7) - (a^2*b^2*3i)/(2*d^2*e^7))^(1/2))/(16*b^6*d^2*e^8 - 16*a^6*d
^2*e^8 + a*b^5*d^2*e^8*32i + a^5*b*d^2*e^8*32i - 112*a^2*b^4*d^2*e^8 - a^3*
b^3*d^2*e^8*192i + 112*a^4*b^2*d^2*e^8))*(((a*b^3*4i - a^3*b*4i + a^4 + b^4
 - 6*a^2*b^2)*1i)/(4*d^2*e^7))^(1/2) + ((2*a^2)/5 - 2*cot(c + d*x)^2*(a^2 -
 b^2) + (4*a*b*cot(c + d*x))/3)/(d*e*(e*cot(c + d*x))^(5/2))
```

3.62 $\int (e\cot(c+dx))^{3/2}(a+b\cot(c+dx))^3\,dx$

Optimal result

Integrand size = 25, antiderivative size = 372

$$\begin{aligned}
\int (e\cot(c+dx))^{3/2}(a+b\cot(c+dx))^3\,dx =
&-\frac{(a-b)\left(a^2+4ab+b^2\right)e^{3/2}\arctan\left(1-\frac{\sqrt{2}\sqrt{e\cot(c+dx)}}{\sqrt{e}}\right)}{\sqrt{2}d}\\
&+\frac{(a-b)\left(a^2+4ab+b^2\right)e^{3/2}\arctan\left(1+\frac{\sqrt{2}\sqrt{e\cot(c+dx)}}{\sqrt{e}}\right)}{\sqrt{2}d}\\
&-\frac{2a(a^2-3b^2)\,e\sqrt{e\cot(c+dx)}}{d}-\frac{2b(3a^2-b^2)\,(e\cot(c+dx))^{3/2}}{3d}\\
&-\frac{32ab^2(e\cot(c+dx))^{5/2}}{35de}-\frac{2b^2(e\cot(c+dx))^{5/2}(a+b\cot(c+dx))}{7de}\\
&-\frac{(a+b)\left(a^2-4ab+b^2\right)e^{3/2}\log\left(\sqrt{e}+\sqrt{e}\cot(c+dx)-\sqrt{2}\sqrt{e\cot(c+dx)}\right)}{2\sqrt{2}d}\\
&+\frac{(a+b)\left(a^2-4ab+b^2\right)e^{3/2}\log\left(\sqrt{e}+\sqrt{e}\cot(c+dx)+\sqrt{2}\sqrt{e\cot(c+dx)}\right)}{2\sqrt{2}d}
\end{aligned}$$

```
[Out] -2/3*b*(3*a^2-b^2)*(e*cot(d*x+c))^(3/2)/d-32/35*a*b^2*(e*cot(d*x+c))^(5/2)/
d/e-2/7*b^2*(e*cot(d*x+c))^(5/2)*(a+b*cot(d*x+c))/d/e-1/2*(a-b)*(a^2+4*a*b+
b^2)*e^(3/2)*arctan(1-2^(1/2)*(e*cot(d*x+c))^(1/2)/e^(1/2))/d*2^(1/2)+1/2*(
a-b)*(a^2+4*a*b+b^2)*e^(3/2)*arctan(1+2^(1/2)*(e*cot(d*x+c))^(1/2)/e^(1/2))
/d*2^(1/2)-1/4*(a+b)*(a^2-4*a*b+b^2)*e^(3/2)*ln(e^(1/2)+cot(d*x+c)*e^(1/2)-
2^(1/2)*(e*cot(d*x+c))^(1/2))/d*2^(1/2)+1/4*(a+b)*(a^2-4*a*b+b^2)*e^(3/2)*l
n(e^(1/2)+cot(d*x+c)*e^(1/2)+2^(1/2)*(e*cot(d*x+c))^(1/2))/d*2^(1/2)-2*a*(a
^2-3*b^2)*e*(e*cot(d*x+c))^(1/2)/d
```

Rubi [A] (verified)

Time = 0.65 (sec) , antiderivative size = 372, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.400, Rules used = {3647, 3711, 3609, 3615, 1182, 1176, 631, 210, 1179, 642}

$$\begin{aligned}\int (e\cot(c+dx))^{3/2}(a+b\cot(c+dx))^3\,dx =& -\frac{e^{3/2}(a-b)\left(a^2+4ab+b^2\right)\arctan\left(1-\frac{\sqrt{2}\sqrt{e\cot(c+dx)}}{\sqrt{e}}\right)}{\sqrt{2}d}\\&+\frac{e^{3/2}(a-b)\left(a^2+4ab+b^2\right)\arctan\left(\frac{\sqrt{2}\sqrt{e\cot(c+dx)}}{\sqrt{e}}+1\right)}{\sqrt{2}d}\\&-\frac{e^{3/2}(a+b)\left(a^2-4ab+b^2\right)\log\left(\sqrt{e}\cot(c+dx)-\sqrt{2}\sqrt{e\cot(c+dx)}+\sqrt{e}\right)}{2\sqrt{2}d}\\&+\frac{e^{3/2}(a+b)\left(a^2-4ab+b^2\right)\log\left(\sqrt{e}\cot(c+dx)+\sqrt{2}\sqrt{e\cot(c+dx)}+\sqrt{e}\right)}{2\sqrt{2}d}\\&-\frac{2b(3a^2-b^2)\left(e\cot(c+dx)\right)^{3/2}}{3d}-\frac{2ae(a^2-3b^2)\sqrt{e\cot(c+dx)}}{d}\\&-\frac{2b^2(e\cot(c+dx))^{5/2}(a+b\cot(c+dx))}{7de}-\frac{32ab^2(e\cot(c+dx))^{5/2}}{35de}\end{aligned}$$

```
[In] Int[(e*Cot[c + d*x])^(3/2)*(a + b*Cot[c + d*x])^3,x]
```

```
[Out] -(((a - b)*(a^2 + 4*a*b + b^2)*e^(3/2)*ArcTan[1 - (Sqrt[2]*Sqrt[e*Cot[c + d
*x]])/Sqrt[e]])/(Sqrt[2]*d)) + ((a - b)*(a^2 + 4*a*b + b^2)*e^(3/2)*ArcTan[
1 + (Sqrt[2]*Sqrt[e*Cot[c + d*x]])/Sqrt[e]])/(Sqrt[2]*d) - (2*a*(a^2 - 3*b^
2)*e*Sqrt[e*Cot[c + d*x]])/d - (2*b*(3*a^2 - b^2)*(e*Cot[c + d*x])^(3/2))/(
3*d) - (32*a*b^2*(e*Cot[c + d*x])^(5/2))/(35*d*e) - (2*b^2*(e*Cot[c + d*x])
^(5/2)*(a + b*Cot[c + d*x]))/(7*d*e) - ((a + b)*(a^2 - 4*a*b + b^2)*e^(3/2)
*Log[Sqrt[e] + Sqrt[e]*Cot[c + d*x] - Sqrt[2]*Sqrt[e*Cot[c + d*x]]])/(2*Sqr
t[2]*d) + ((a + b)*(a^2 - 4*a*b + b^2)*e^(3/2)*Log[Sqrt[e] + Sqrt[e]*Cot[c
+ d*x] + Sqrt[2]*Sqrt[e*Cot[c + d*x]]])/(2*Sqrt[2]*d)
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
```

```
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1179

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 1182

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
a*c, 2]}, Dist[(d*q + a*e)/(2*a*c), Int[(q + c*x^2)/(a + c*x^4), x], x] + D
ist[(d*q - a*e)/(2*a*c), Int[(q - c*x^2)/(a + c*x^4), x], x]] /; FreeQ[{a,
c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0] && NeQ[c*d^2 - a*e^2, 0] && NegQ[(-a)
*c]
```

Rule 3609

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)]), x_Symbol] :> Simp[d*((a + b*Tan[e + f*x])^m/(f*m)), x] + Int
[(a + b*Tan[e + f*x])^(m - 1)*Simp[a*c - b*d + (b*c + a*d)*Tan[e + f*x], x]
, x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2,
 0] && GtQ[m, 0]
```

Rule 3615

```
Int[((c_) + (d_.)*tan[(e_.) + (f_.)*(x_)])/Sqrt[(b_.)*tan[(e_.) + (f_.)*(x_
)]], x_Symbol] :> Dist[2/f, Subst[Int[(b*c + d*x^2)/(b^2 + x^4), x], x, Sqr
t[b*Tan[e + f*x]]], x] /; FreeQ[{b, c, d, e, f}, x] && NeQ[c^2 - d^2, 0] &&
 NeQ[c^2 + d^2, 0]
```

Rule 3647

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)])^(n_), x_Symbol] :> Simp[b^2*(a + b*Tan[e + f*x])^(m - 2)*((c
+ d*Tan[e + f*x])^(n + 1)/(d*f*(m + n - 1))), x] + Dist[1/(d*(m + n - 1)),
Int[(a + b*Tan[e + f*x])^(m - 3)*(c + d*Tan[e + f*x])^n*Simp[a^3*d*(m + n -
 1) - b^2*(b*c*(m - 2) + a*d*(1 + n)) + b*d*(m + n - 1)*(3*a^2 - b^2)*Tan[e
 + f*x] - b^2*(b*c*(m - 2) - a*d*(3*m + 2*n - 4))*Tan[e + f*x]^2, x], x], x
] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2,
 0] && NeQ[c^2 + d^2, 0] && IntegerQ[2*m] && GtQ[m, 2] && (GeQ[n, -1] || In
tegerQ[m]) &&  !(IGtQ[n, 2] && ( !IntegerQ[m] || (EqQ[c, 0] && NeQ[a, 0])))
```

Rule 3711

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*tan[(e_.)
+ (f_.)*(x_)] + (C_.)*tan[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[C*((a +
 b*Tan[e + f*x])^(m + 1)/(b*f*(m + 1))), x] + Int[(a + b*Tan[e + f*x])^m*Si
mp[A - C + B*Tan[e + f*x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] &&
NeQ[A*b^2 - a*b*B + a^2*C, 0] &&  !LeQ[m, -1]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\frac{2b^2(e\cot(c+dx))^{5/2}(a+b\cot(c+dx))}{7de} \\
&\quad -\frac{2\int (e\cot(c+dx))^{3/2}\left(-\frac{1}{2}a(7a^2-5b^2)e-\frac{7}{2}b(3a^2-b^2)e\cot(c+dx)-8ab^2e\cot^2(c+dx)\right)dx}{7e} \\
&= -\frac{32ab^2(e\cot(c+dx))^{5/2}}{35de}-\frac{2b^2(e\cot(c+dx))^{5/2}(a+b\cot(c+dx))}{7de} \\
&\quad -\frac{2\int (e\cot(c+dx))^{3/2}\left(-\frac{7}{2}a(a^2-3b^2)e-\frac{7}{2}b(3a^2-b^2)e\cot(c+dx)\right)dx}{7e} \\
&= -\frac{2b(3a^2-b^2)(e\cot(c+dx))^{3/2}}{3d}-\frac{32ab^2(e\cot(c+dx))^{5/2}}{35de} \\
&\quad -\frac{2b^2(e\cot(c+dx))^{5/2}(a+b\cot(c+dx))}{7de} \\
&\quad -\frac{2\int \sqrt{e\cot(c+dx)}\left(\frac{7}{2}b(3a^2-b^2)e^2-\frac{7}{2}a(a^2-3b^2)e^2\cot(c+dx)\right)dx}{7e} \\
&= -\frac{2a(a^2-3b^2)e\sqrt{e\cot(c+dx)}}{d}-\frac{2b(3a^2-b^2)(e\cot(c+dx))^{3/2}}{3d}-\frac{32ab^2(e\cot(c+dx))^{5/2}}{35de} \\
&\quad -\frac{2b^2(e\cot(c+dx))^{5/2}(a+b\cot(c+dx))}{7de}-\frac{2\int \frac{\frac{7}{2}a(a^2-3b^2)e^3+\frac{7}{2}b(3a^2-b^2)e^3\cot(c+dx)}{\sqrt{e\cot(c+dx)}}dx}{7e}
\end{aligned}$$

$$= -\frac{2a(a^2-3b^2)\,e\sqrt{e\cot(c+dx)}}{d} - \frac{2b(3a^2-b^2)\,(e\cot(c+dx))^{3/2}}{3d} - \frac{32ab^2(e\cot(c+dx))^{5/2}}{35de} - \frac{2b^2(e\cot(c+dx))^{5/2}(a+b\cot(c+dx))}{7de} - \frac{4\mathrm{Subst}\left(\int \frac{-\frac{7}{2}a(a^2-3b^2)e^4-\frac{7}{2}b(3a^2-b^2)e^3x^2}{e^2+x^4}\,dx, x, \sqrt{e\cot(c+dx)}\right)}{7de}$$

$$= -\frac{2a(a^2-3b^2)\,e\sqrt{e\cot(c+dx)}}{d} - \frac{2b(3a^2-b^2)\,(e\cot(c+dx))^{3/2}}{3d} - \frac{32ab^2(e\cot(c+dx))^{5/2}}{35de} - \frac{2b^2(e\cot(c+dx))^{5/2}(a+b\cot(c+dx))}{7de} + \frac{((a+b)\,(a^2-4ab+b^2)\,e^2)\,\mathrm{Subst}\left(\int \frac{e-x^2}{e^2+x^4}\,dx, x, \sqrt{e\cot(c+dx)}\right)}{d} + \frac{((a-b)\,(a^2+4ab+b^2)\,e^2)\,\mathrm{Subst}\left(\int \frac{e+x^2}{e^2+x^4}\,dx, x, \sqrt{e\cot(c+dx)}\right)}{d}$$

$$= -\frac{2a(a^2-3b^2)\,e\sqrt{e\cot(c+dx)}}{d} - \frac{2b(3a^2-b^2)\,(e\cot(c+dx))^{3/2}}{3d} - \frac{32ab^2(e\cot(c+dx))^{5/2}}{35de} - \frac{2b^2(e\cot(c+dx))^{5/2}(a+b\cot(c+dx))}{7de} - \frac{((a+b)\,(a^2-4ab+b^2)\,e^{3/2})\,\mathrm{Subst}\left(\int \frac{\sqrt{2}\sqrt{e}+2x}{-e-\sqrt{2}\sqrt{e}x-x^2}\,dx, x, \sqrt{e\cot(c+dx)}\right)}{2\sqrt{2}d} - \frac{((a+b)\,(a^2-4ab+b^2)\,e^{3/2})\,\mathrm{Subst}\left(\int \frac{\sqrt{2}\sqrt{e}-2x}{-e+\sqrt{2}\sqrt{e}x-x^2}\,dx, x, \sqrt{e\cot(c+dx)}\right)}{2\sqrt{2}d} + \frac{((a-b)\,(a^2+4ab+b^2)\,e^2)\,\mathrm{Subst}\left(\int \frac{1}{e-\sqrt{2}\sqrt{e}x+x^2}\,dx, x, \sqrt{e\cot(c+dx)}\right)}{2d} + \frac{((a-b)\,(a^2+4ab+b^2)\,e^2)\,\mathrm{Subst}\left(\int \frac{1}{e+\sqrt{2}\sqrt{e}x+x^2}\,dx, x, \sqrt{e\cot(c+dx)}\right)}{2d}$$

$$= -\frac{2a(a^2-3b^2)\,e\sqrt{e\cot(c+dx)}}{d} - \frac{2b(3a^2-b^2)\,(e\cot(c+dx))^{3/2}}{3d} - \frac{32ab^2(e\cot(c+dx))^{5/2}}{35de} - \frac{2b^2(e\cot(c+dx))^{5/2}(a+b\cot(c+dx))}{7de} - \frac{(a+b)\,(a^2-4ab+b^2)\,e^{3/2}\log\left(\sqrt{e}+\sqrt{e}\cot(c+dx)-\sqrt{2}\sqrt{e\cot(c+dx)}\right)}{2\sqrt{2}d} + \frac{(a+b)\,(a^2-4ab+b^2)\,e^{3/2}\log\left(\sqrt{e}+\sqrt{e}\cot(c+dx)+\sqrt{2}\sqrt{e\cot(c+dx)}\right)}{2\sqrt{2}d} + \frac{((a-b)\,(a^2+4ab+b^2)\,e^{3/2})\,\text{Subst}\left(\int\frac{1}{-1-x^2}\,dx,x,1-\frac{\sqrt{2}\sqrt{e\cot(c+dx)}}{\sqrt{e}}\right)}{\sqrt{2}d} - \frac{((a-b)\,(a^2+4ab+b^2)\,e^{3/2})\,\text{Subst}\left(\int\frac{1}{-1-x^2}\,dx,x,1+\frac{\sqrt{2}\sqrt{e\cot(c+dx)}}{\sqrt{e}}\right)}{\sqrt{2}d}$$

$$= -\frac{(a-b)\,(a^2+4ab+b^2)\,e^{3/2}\arctan\left(1-\frac{\sqrt{2}\sqrt{e\cot(c+dx)}}{\sqrt{e}}\right)}{\sqrt{2}d} + \frac{(a-b)\,(a^2+4ab+b^2)\,e^{3/2}\arctan\left(1+\frac{\sqrt{2}\sqrt{e\cot(c+dx)}}{\sqrt{e}}\right)}{\sqrt{2}d} - \frac{2a(a^2-3b^2)\,e\sqrt{e\cot(c+dx)}}{d} - \frac{2b(3a^2-b^2)\,(e\cot(c+dx))^{3/2}}{3d} - \frac{32ab^2(e\cot(c+dx))^{5/2}}{35de} - \frac{2b^2(e\cot(c+dx))^{5/2}(a+b\cot(c+dx))}{7de} - \frac{(a+b)\,(a^2-4ab+b^2)\,e^{3/2}\log\left(\sqrt{e}+\sqrt{e}\cot(c+dx)-\sqrt{2}\sqrt{e\cot(c+dx)}\right)}{2\sqrt{2}d} + \frac{(a+b)\,(a^2-4ab+b^2)\,e^{3/2}\log\left(\sqrt{e}+\sqrt{e}\cot(c+dx)+\sqrt{2}\sqrt{e\cot(c+dx)}\right)}{2\sqrt{2}d}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 2.98 (sec) , antiderivative size = 251, normalized size of antiderivative = 0.67

$$\int(e\cot(c+dx))^{3/2}(a+b\cot(c+dx))^3\,dx = -\frac{(e\cot(c+dx))^{3/2}\left(\frac{6}{5}ab^2\cot^{\frac{5}{2}}(c+dx)+\frac{2}{7}b^3\cot^{\frac{7}{2}}(c+dx)+\frac{2}{3}b(-3a^2+b^2)\cot^{\frac{3}{2}}(c+dx)\left(-1+\text{Hypergeome}\right.\right.}{}$$

```
[In] Integrate[(e*Cot[c + d*x])^(3/2)*(a + b*Cot[c + d*x])^3,x]
```

```
[Out] -(((e*Cot[c + d*x])^(3/2)*((6*a*b^2*Cot[c + d*x]^(5/2))/5 + (2*b^3*Cot[c +
d*x]^(7/2))/7 + (2*b*(-3*a^2 + b^2)*Cot[c + d*x]^(3/2)*(-1 + Hypergeometric
2F1[3/4, 1, 7/4, -Cot[c + d*x]^2]))/3 + (a*(a^2 - 3*b^2)*(2*Sqrt[2]*ArcTan[
1 - Sqrt[2]*Sqrt[Cot[c + d*x]]] - 2*Sqrt[2]*ArcTan[1 + Sqrt[2]*Sqrt[Cot[c +
 d*x]]] + 8*Sqrt[Cot[c + d*x]] + Sqrt[2]*Log[1 - Sqrt[2]*Sqrt[Cot[c + d*x]]
 + Cot[c + d*x]] - Sqrt[2]*Log[1 + Sqrt[2]*Sqrt[Cot[c + d*x]] + Cot[c + d*x
]]))/4))/(d*Cot[c + d*x]^(3/2)))
```

Maple [A] (verified)

Time = 0.04 (sec) , antiderivative size = 410, normalized size of antiderivative = 1.10

method	result
derivativedivides	$-\frac{2\left(\frac{b^3(e\cot(dx+c))^{\frac{7}{2}}}{7}+\frac{3ae\,b^2(e\cot(dx+c))^{\frac{5}{2}}}{5}+a^2e^2b(e\cot(dx+c))^{\frac{3}{2}}-\frac{b^3e^2(e\cot(dx+c))^{\frac{3}{2}}}{3}+a^3e^3\sqrt{e\cot(dx+c)}-3\sqrt{e\cot(}\right.}{}$
default	$-\frac{2\left(\frac{b^3(e\cot(dx+c))^{\frac{7}{2}}}{7}+\frac{3ae\,b^2(e\cot(dx+c))^{\frac{5}{2}}}{5}+a^2e^2b(e\cot(dx+c))^{\frac{3}{2}}-\frac{b^3e^2(e\cot(dx+c))^{\frac{3}{2}}}{3}+a^3e^3\sqrt{e\cot(dx+c)}-3\sqrt{e\cot(}\right.}{}$
parts	$-\frac{2a^3e\left(\sqrt{e\cot(dx+c)}-\frac{\left(e^2\right)^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\frac{e\cot(dx+c)+\left(e^2\right)^{\frac{1}{4}}\sqrt{e\cot(dx+c)}\sqrt{2}+\sqrt{e^2}}{e\cot(dx+c)-\left(e^2\right)^{\frac{1}{4}}\sqrt{e\cot(dx+c)}\sqrt{2}+\sqrt{e^2}}\right)+2\arctan\left(\frac{\sqrt{2}\sqrt{e\cot(dx+c)}}{\left(e^2\right)^{\frac{1}{4}}}+1\right)-2\arcta\right.}{8}\right.}{d}$

```
[In] int((e*cot(d*x+c))^(3/2)*(a+b*cot(d*x+c))^3,x,method=_RETURNVERBOSE)
```

```
[Out] -2/d/e^2*(1/7*b^3*(e*cot(d*x+c))^(7/2)+3/5*a*e*b^2*(e*cot(d*x+c))^(5/2)+a^2
*e^2*b*(e*cot(d*x+c))^(3/2)-1/3*b^3*e^2*(e*cot(d*x+c))^(3/2)+a^3*e^3*(e*cot
(d*x+c))^(1/2)-3*(e*cot(d*x+c))^(1/2)*a*b^2*e^3-e^4*(1/8*(a^3*e-3*a*b^2*e)*
(e^2)^(1/4)/e^2*2^(1/2)*(ln((e*cot(d*x+c)+(e^2)^(1/4)*(e*cot(d*x+c))^(1/2)*
2^(1/2)+(e^2)^(1/2))/(e*cot(d*x+c)-(e^2)^(1/4)*(e*cot(d*x+c))^(1/2)*2^(1/2)
+(e^2)^(1/2)))+2*arctan(2^(1/2)/(e^2)^(1/4)*(e*cot(d*x+c))^(1/2)+1)-2*arcta
n(-2^(1/2)/(e^2)^(1/4)*(e*cot(d*x+c))^(1/2)+1))+1/8*(3*a^2*b-b^3)/(e^2)^(1/
4)*2^(1/2)*(ln((e*cot(d*x+c)-(e^2)^(1/4)*(e*cot(d*x+c))^(1/2)*2^(1/2)+(e^2)
^(1/2))/(e*cot(d*x+c)+(e^2)^(1/4)*(e*cot(d*x+c))^(1/2)*2^(1/2)+(e^2)^(1/2))
)+2*arctan(2^(1/2)/(e^2)^(1/4)*(e*cot(d*x+c))^(1/2)+1)-2*arctan(-2^(1/2)/(e
^2)^(1/4)*(e*cot(d*x+c))^(1/2)+1))))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1795 vs. $2(311) = 622$.

Time = 0.34 (sec) , antiderivative size = 1795, normalized size of antiderivative = 4.83

$$\int (e\cot(c+dx))^{3/2}(a+b\cot(c+dx))^3\,dx = \text{Too large to display}$$

```
[In] integrate((e*cot(d*x+c))^(3/2)*(a+b*cot(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] -1/210*(105*(d*cos(2*d*x + 2*c) - d)*sqrt(-(2*(3*a^5*b - 10*a^3*b^3 + 3*a*b
^5)*e^3 + sqrt(-(a^12 - 30*a^10*b^2 + 255*a^8*b^4 - 452*a^6*b^6 + 255*a^4*b
^8 - 30*a^2*b^10 + b^12)*e^6/d^4)*d^2)/d^2)*log(-(a^12 - 12*a^10*b^2 - 27*a
^8*b^4 + 27*a^4*b^8 + 12*a^2*b^10 - b^12)*e^4*sqrt((e*cos(2*d*x + 2*c) + e)
/sin(2*d*x + 2*c)) + ((a^9 - 18*a^7*b^2 + 60*a^5*b^4 - 46*a^3*b^6 + 3*a*b^8
)*d*e^3 + (3*a^2*b - b^3)*sqrt(-(a^12 - 30*a^10*b^2 + 255*a^8*b^4 - 452*a^6
*b^6 + 255*a^4*b^8 - 30*a^2*b^10 + b^12)*e^6/d^4)*d^3)*sqrt(-(2*(3*a^5*b -
10*a^3*b^3 + 3*a*b^5)*e^3 + sqrt(-(a^12 - 30*a^10*b^2 + 255*a^8*b^4 - 452*a
^6*b^6 + 255*a^4*b^8 - 30*a^2*b^10 + b^12)*e^6/d^4)*d^2)/d^2))*sin(2*d*x +
2*c) - 105*(d*cos(2*d*x + 2*c) - d)*sqrt(-(2*(3*a^5*b - 10*a^3*b^3 + 3*a*b^
5)*e^3 + sqrt(-(a^12 - 30*a^10*b^2 + 255*a^8*b^4 - 452*a^6*b^6 + 255*a^4*b^
8 - 30*a^2*b^10 + b^12)*e^6/d^4)*d^2)/d^2)*log(-(a^12 - 12*a^10*b^2 - 27*a^
8*b^4 + 27*a^4*b^8 + 12*a^2*b^10 - b^12)*e^4*sqrt((e*cos(2*d*x + 2*c) + e)/
sin(2*d*x + 2*c)) - ((a^9 - 18*a^7*b^2 + 60*a^5*b^4 - 46*a^3*b^6 + 3*a*b^8)
*d*e^3 + (3*a^2*b - b^3)*sqrt(-(a^12 - 30*a^10*b^2 + 255*a^8*b^4 - 452*a^6*
b^6 + 255*a^4*b^8 - 30*a^2*b^10 + b^12)*e^6/d^4)*d^3)*sqrt(-(2*(3*a^5*b - 1
0*a^3*b^3 + 3*a*b^5)*e^3 + sqrt(-(a^12 - 30*a^10*b^2 + 255*a^8*b^4 - 452*a^
6*b^6 + 255*a^4*b^8 - 30*a^2*b^10 + b^12)*e^6/d^4)*d^2)/d^2))*sin(2*d*x + 2
*c) + 105*(d*cos(2*d*x + 2*c) - d)*sqrt(-(2*(3*a^5*b - 10*a^3*b^3 + 3*a*b^5
)*e^3 - sqrt(-(a^12 - 30*a^10*b^2 + 255*a^8*b^4 - 452*a^6*b^6 + 255*a^4*b^8
 - 30*a^2*b^10 + b^12)*e^6/d^4)*d^2)/d^2)*log(-(a^12 - 12*a^10*b^2 - 27*a^8
*b^4 + 27*a^4*b^8 + 12*a^2*b^10 - b^12)*e^4*sqrt((e*cos(2*d*x + 2*c) + e)/s
in(2*d*x + 2*c)) + ((a^9 - 18*a^7*b^2 + 60*a^5*b^4 - 46*a^3*b^6 + 3*a*b^8)*
d*e^3 - (3*a^2*b - b^3)*sqrt(-(a^12 - 30*a^10*b^2 + 255*a^8*b^4 - 452*a^6*b
^6 + 255*a^4*b^8 - 30*a^2*b^10 + b^12)*e^6/d^4)*d^3)*sqrt(-(2*(3*a^5*b - 10
*a^3*b^3 + 3*a*b^5)*e^3 - sqrt(-(a^12 - 30*a^10*b^2 + 255*a^8*b^4 - 452*a^6
*b^6 + 255*a^4*b^8 - 30*a^2*b^10 + b^12)*e^6/d^4)*d^2)/d^2))*sin(2*d*x + 2*
c) - 105*(d*cos(2*d*x + 2*c) - d)*sqrt(-(2*(3*a^5*b - 10*a^3*b^3 + 3*a*b^5)
*e^3 - sqrt(-(a^12 - 30*a^10*b^2 + 255*a^8*b^4 - 452*a^6*b^6 + 255*a^4*b^8
- 30*a^2*b^10 + b^12)*e^6/d^4)*d^2)/d^2)*log(-(a^12 - 12*a^10*b^2 - 27*a^8*
b^4 + 27*a^4*b^8 + 12*a^2*b^10 - b^12)*e^4*sqrt((e*cos(2*d*x + 2*c) + e)/si
n(2*d*x + 2*c)) - ((a^9 - 18*a^7*b^2 + 60*a^5*b^4 - 46*a^3*b^6 + 3*a*b^8)*d
*e^3 - (3*a^2*b - b^3)*sqrt(-(a^12 - 30*a^10*b^2 + 255*a^8*b^4 - 452*a^6*b^
6 + 255*a^4*b^8 - 30*a^2*b^10 + b^12)*e^6/d^4)*d^3)*sqrt(-(2*(3*a^5*b - 10*
a^3*b^3 + 3*a*b^5)*e^3 - sqrt(-(a^12 - 30*a^10*b^2 + 255*a^8*b^4 - 452*a^6*
```

```
b^6 + 255*a^4*b^8 - 30*a^2*b^10 + b^12)*e^6/d^4)*d^2)/d^2))*sin(2*d*x + 2*c
) - 4*(30*b^3*e*cos(2*d*x + 2*c) - 5*(21*a^2*b - 10*b^3)*e*cos(2*d*x + 2*c)
^2 + 5*(21*a^2*b - 4*b^3)*e - 21*((5*a^3 - 18*a*b^2)*e*cos(2*d*x + 2*c) - (
5*a^3 - 12*a*b^2)*e)*sin(2*d*x + 2*c))*sqrt((e*cos(2*d*x + 2*c) + e)/sin(2*
d*x + 2*c)))/((d*cos(2*d*x + 2*c) - d)*sin(2*d*x + 2*c))
```

Sympy [F]

$$\int (e\cot(c+dx))^{3/2}(a+b\cot(c+dx))^3\,dx = \int (e\cot(c+dx))^{\frac{3}{2}}(a+b\cot(c+dx))^3\,dx$$

```
[In] integrate((e*cot(d*x+c))**(3/2)*(a+b*cot(d*x+c))**3,x)
[Out] Integral((e*cot(c + d*x))**(3/2)*(a + b*cot(c + d*x))**3, x)
```

Maxima [F(-2)]

Exception generated.

$$\int (e\cot(c+dx))^{3/2}(a+b\cot(c+dx))^3\,dx = \text{Exception raised: ValueError}$$

```
[In] integrate((e*cot(d*x+c))^(3/2)*(a+b*cot(d*x+c))^3,x, algorithm="maxima")
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(e>0)', see `assume?` for more detai
ls)Is e
```

Giac [F]

$$\int (e\cot(c+dx))^{3/2}(a+b\cot(c+dx))^3\,dx = \int (b\cot(dx+c)+a)^3(e\cot(dx+c))^{\frac{3}{2}}\,dx$$

```
[In] integrate((e*cot(d*x+c))^(3/2)*(a+b*cot(d*x+c))^3,x, algorithm="giac")
[Out] integrate((b*cot(d*x + c) + a)^3*(e*cot(d*x + c))^(3/2), x)
```

Mupad [B] (verification not implemented)

Time = 18.06 (sec) , antiderivative size = 2317, normalized size of antiderivative = 6.23

$$\int (e\cot(c+dx))^{3/2}(a+b\cot(c+dx))^3\,dx = \text{Too large to display}$$

```
[In] int((e*cot(c + d*x))^(3/2)*(a + b*cot(c + d*x))^3,x)
```

```
[Out] (e*cot(c + d*x))^(3/2)*((2*b^3)/(3*d) - (2*a^2*b)/d) - (e*cot(c + d*x))^(1/
2)*((2*a^3*e)/d - (6*a*b^2*e)/d) - atan((((16*(e*cot(c + d*x))^(1/2)*(a^6*e
^6 - b^6*e^6 + 15*a^2*b^4*e^6 - 15*a^4*b^2*e^6))/d^2 - (8*(4*a^3*d^2*e^5 -
12*a*b^2*d^2*e^5)*(-(b^6*e^3*1i - a^6*e^3*1i + 6*a*b^5*e^3 + 6*a^5*b*e^3 -
a^2*b^4*e^3*15i - 20*a^3*b^3*e^3 + a^4*b^2*e^3*15i)/(4*d^2))^(1/2))/d^3)*(-
(b^6*e^3*1i - a^6*e^3*1i + 6*a*b^5*e^3 + 6*a^5*b*e^3 - a^2*b^4*e^3*15i - 20
*a^3*b^3*e^3 + a^4*b^2*e^3*15i)/(4*d^2))^(1/2)*1i + ((16*(e*cot(c + d*x))^(
1/2)*(a^6*e^6 - b^6*e^6 + 15*a^2*b^4*e^6 - 15*a^4*b^2*e^6))/d^2 + (8*(4*a^3
*d^2*e^5 - 12*a*b^2*d^2*e^5)*(-(b^6*e^3*1i - a^6*e^3*1i + 6*a*b^5*e^3 + 6*a
^5*b*e^3 - a^2*b^4*e^3*15i - 20*a^3*b^3*e^3 + a^4*b^2*e^3*15i)/(4*d^2))^(1/
2))/d^3)*(-(b^6*e^3*1i - a^6*e^3*1i + 6*a*b^5*e^3 + 6*a^5*b*e^3 - a^2*b^4*e
^3*15i - 20*a^3*b^3*e^3 + a^4*b^2*e^3*15i)/(4*d^2))^(1/2)*1i)/(((16*(e*cot(
c + d*x))^(1/2)*(a^6*e^6 - b^6*e^6 + 15*a^2*b^4*e^6 - 15*a^4*b^2*e^6))/d^2
+ (8*(4*a^3*d^2*e^5 - 12*a*b^2*d^2*e^5)*(-(b^6*e^3*1i - a^6*e^3*1i + 6*a*b^
5*e^3 + 6*a^5*b*e^3 - a^2*b^4*e^3*15i - 20*a^3*b^3*e^3 + a^4*b^2*e^3*15i)/(
4*d^2))^(1/2))/d^3)*(-(b^6*e^3*1i - a^6*e^3*1i + 6*a*b^5*e^3 + 6*a^5*b*e^3
- a^2*b^4*e^3*15i - 20*a^3*b^3*e^3 + a^4*b^2*e^3*15i)/(4*d^2))^(1/2) - ((16
*(e*cot(c + d*x))^(1/2)*(a^6*e^6 - b^6*e^6 + 15*a^2*b^4*e^6 - 15*a^4*b^2*e^
6))/d^2 - (8*(4*a^3*d^2*e^5 - 12*a*b^2*d^2*e^5)*(-(b^6*e^3*1i - a^6*e^3*1i
+ 6*a*b^5*e^3 + 6*a^5*b*e^3 - a^2*b^4*e^3*15i - 20*a^3*b^3*e^3 + a^4*b^2*e^
3*15i)/(4*d^2))^(1/2))/d^3)*(-(b^6*e^3*1i - a^6*e^3*1i + 6*a*b^5*e^3 + 6*a^
5*b*e^3 - a^2*b^4*e^3*15i - 20*a^3*b^3*e^3 + a^4*b^2*e^3*15i)/(4*d^2))^(1/2
) + (16*(3*a^8*b*e^8 - b^9*e^8 + 6*a^4*b^5*e^8 + 8*a^6*b^3*e^8))/d^3))*(-(b
^6*e^3*1i - a^6*e^3*1i + 6*a*b^5*e^3 + 6*a^5*b*e^3 - a^2*b^4*e^3*15i - 20*a
^3*b^3*e^3 + a^4*b^2*e^3*15i)/(4*d^2))^(1/2)*2i - atan((((16*(e*cot(c + d*x
))^(1/2)*(a^6*e^6 - b^6*e^6 + 15*a^2*b^4*e^6 - 15*a^4*b^2*e^6))/d^2 - (8*(4
*a^3*d^2*e^5 - 12*a*b^2*d^2*e^5)*(-(a^6*e^3*1i - b^6*e^3*1i + 6*a*b^5*e^3 +
 6*a^5*b*e^3 + a^2*b^4*e^3*15i - 20*a^3*b^3*e^3 - a^4*b^2*e^3*15i)/(4*d^2))
^(1/2))/d^3)*(-(a^6*e^3*1i - b^6*e^3*1i + 6*a*b^5*e^3 + 6*a^5*b*e^3 + a^2*b
^4*e^3*15i - 20*a^3*b^3*e^3 - a^4*b^2*e^3*15i)/(4*d^2))^(1/2)*1i + ((16*(e*
cot(c + d*x))^(1/2)*(a^6*e^6 - b^6*e^6 + 15*a^2*b^4*e^6 - 15*a^4*b^2*e^6))/
d^2 + (8*(4*a^3*d^2*e^5 - 12*a*b^2*d^2*e^5)*(-(a^6*e^3*1i - b^6*e^3*1i + 6*
a*b^5*e^3 + 6*a^5*b*e^3 + a^2*b^4*e^3*15i - 20*a^3*b^3*e^3 - a^4*b^2*e^3*15
i)/(4*d^2))^(1/2))/d^3)*(-(a^6*e^3*1i - b^6*e^3*1i + 6*a*b^5*e^3 + 6*a^5*b*
e^3 + a^2*b^4*e^3*15i - 20*a^3*b^3*e^3 - a^4*b^2*e^3*15i)/(4*d^2))^(1/2)*1i
)/(((16*(e*cot(c + d*x))^(1/2)*(a^6*e^6 - b^6*e^6 + 15*a^2*b^4*e^6 - 15*a^4
```

```
*b^2*e^6))/d^2 + (8*(4*a^3*d^2*e^5 - 12*a*b^2*d^2*e^5)*(-(a^6*e^3*1i - b^6*
e^3*1i + 6*a*b^5*e^3 + 6*a^5*b*e^3 + a^2*b^4*e^3*15i - 20*a^3*b^3*e^3 - a^4
*b^2*e^3*15i)/(4*d^2))^(1/2))/d^3)*(-(a^6*e^3*1i - b^6*e^3*1i + 6*a*b^5*e^3
 + 6*a^5*b*e^3 + a^2*b^4*e^3*15i - 20*a^3*b^3*e^3 - a^4*b^2*e^3*15i)/(4*d^2
))^(1/2) - ((16*(e*cot(c + d*x))^(1/2)*(a^6*e^6 - b^6*e^6 + 15*a^2*b^4*e^6
- 15*a^4*b^2*e^6))/d^2 - (8*(4*a^3*d^2*e^5 - 12*a*b^2*d^2*e^5)*(-(a^6*e^3*1
i - b^6*e^3*1i + 6*a*b^5*e^3 + 6*a^5*b*e^3 + a^2*b^4*e^3*15i - 20*a^3*b^3*e
^3 - a^4*b^2*e^3*15i)/(4*d^2))^(1/2))/d^3)*(-(a^6*e^3*1i - b^6*e^3*1i + 6*a
*b^5*e^3 + 6*a^5*b*e^3 + a^2*b^4*e^3*15i - 20*a^3*b^3*e^3 - a^4*b^2*e^3*15i
)/(4*d^2))^(1/2) + (16*(3*a^8*b*e^8 - b^9*e^8 + 6*a^4*b^5*e^8 + 8*a^6*b^3*e
^8))/d^3))*(-(a^6*e^3*1i - b^6*e^3*1i + 6*a*b^5*e^3 + 6*a^5*b*e^3 + a^2*b^4
*e^3*15i - 20*a^3*b^3*e^3 - a^4*b^2*e^3*15i)/(4*d^2))^(1/2)*2i - (2*b^3*(e*
cot(c + d*x))^(7/2))/(7*d*e^2) - (6*a*b^2*(e*cot(c + d*x))^(5/2))/(5*d*e)
```

3.63 $\int \sqrt{e \cot(c+dx)}(a+b\cot(c+dx))^3\,dx$

Optimal result

Integrand size = 25, antiderivative size = 342

$$\begin{aligned}\int \sqrt{e \cot(c+dx)}(a+b\cot(c+dx))^3\,dx &= \frac{(a+b)\left(a^2-4ab+b^2\right)\sqrt{e}\arctan\left(1-\frac{\sqrt{2}\sqrt{e\cot(c+dx)}}{\sqrt{e}}\right)}{\sqrt{2}d} \\ &\quad -\frac{(a+b)\left(a^2-4ab+b^2\right)\sqrt{e}\arctan\left(1+\frac{\sqrt{2}\sqrt{e\cot(c+dx)}}{\sqrt{e}}\right)}{\sqrt{2}d}-\frac{2b(3a^2-b^2)\sqrt{e\cot(c+dx)}}{d} \\ &\quad -\frac{8ab^2(e\cot(c+dx))^{3/2}}{5de}-\frac{2b^2(e\cot(c+dx))^{3/2}(a+b\cot(c+dx))}{5de} \\ &\quad -\frac{(a-b)\left(a^2+4ab+b^2\right)\sqrt{e}\log\left(\sqrt{e}+\sqrt{e}\cot(c+dx)-\sqrt{2}\sqrt{e\cot(c+dx)}\right)}{2\sqrt{2}d} \\ &\quad +\frac{(a-b)\left(a^2+4ab+b^2\right)\sqrt{e}\log\left(\sqrt{e}+\sqrt{e}\cot(c+dx)+\sqrt{2}\sqrt{e\cot(c+dx)}\right)}{2\sqrt{2}d}\end{aligned}$$

```
[Out] -8/5*a*b^2*(e*cot(d*x+c))^(3/2)/d/e-2/5*b^2*(e*cot(d*x+c))^(3/2)*(a+b*cot(d
*x+c))/d/e+1/2*(a+b)*(a^2-4*a*b+b^2)*arctan(1-2^(1/2)*(e*cot(d*x+c))^(1/2)/
e^(1/2))*e^(1/2)/d*2^(1/2)-1/2*(a+b)*(a^2-4*a*b+b^2)*arctan(1+2^(1/2)*(e*co
t(d*x+c))^(1/2)/e^(1/2))*e^(1/2)/d*2^(1/2)-1/4*(a-b)*(a^2+4*a*b+b^2)*ln(e^(
1/2)+cot(d*x+c)*e^(1/2)-2^(1/2)*(e*cot(d*x+c))^(1/2))*e^(1/2)/d*2^(1/2)+1/4
*(a-b)*(a^2+4*a*b+b^2)*ln(e^(1/2)+cot(d*x+c)*e^(1/2)+2^(1/2)*(e*cot(d*x+c))
^(1/2))*e^(1/2)/d*2^(1/2)-2*b*(3*a^2-b^2)*(e*cot(d*x+c))^(1/2)/d
```

Rubi [A] (verified)

Time = 0.54 (sec) , antiderivative size = 342, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.400, Rules used = {3647, 3711, 3609, 3615, 1182, 1176, 631, 210, 1179, 642}

$$\begin{aligned}
&\int \sqrt{e\cot(c+dx)}(a+b\cot(c+dx))^3\,dx \\
&= \frac{\sqrt{e}(a+b)\left(a^2-4ab+b^2\right)\arctan\left(1-\frac{\sqrt{2}\sqrt{e\cot(c+dx)}}{\sqrt{e}}\right)}{\sqrt{2}d} \\
&\quad -\frac{\sqrt{e}(a+b)\left(a^2-4ab+b^2\right)\arctan\left(\frac{\sqrt{2}\sqrt{e\cot(c+dx)}}{\sqrt{e}}+1\right)}{\sqrt{2}d}-\frac{2b\left(3a^2-b^2\right)\sqrt{e\cot(c+dx)}}{d} \\
&\quad -\frac{\sqrt{e}(a-b)\left(a^2+4ab+b^2\right)\log\left(\sqrt{e}\cot(c+dx)-\sqrt{2}\sqrt{e\cot(c+dx)}+\sqrt{e}\right)}{2\sqrt{2}d} \\
&\quad +\frac{\sqrt{e}(a-b)\left(a^2+4ab+b^2\right)\log\left(\sqrt{e}\cot(c+dx)+\sqrt{2}\sqrt{e\cot(c+dx)}+\sqrt{e}\right)}{2\sqrt{2}d} \\
&\quad -\frac{8ab^2(e\cot(c+dx))^{3/2}}{5de}-\frac{2b^2(e\cot(c+dx))^{3/2}(a+b\cot(c+dx))}{5de}
\end{aligned}$$

```
[In] Int[Sqrt[e*Cot[c + d*x]]*(a + b*Cot[c + d*x])^3,x]
```

```
[Out] ((a + b)*(a^2 - 4*a*b + b^2)*Sqrt[e]*ArcTan[1 - (Sqrt[2]*Sqrt[e*Cot[c + d*x
]])/Sqrt[e]])/(Sqrt[2]*d) - ((a + b)*(a^2 - 4*a*b + b^2)*Sqrt[e]*ArcTan[1 +
 (Sqrt[2]*Sqrt[e*Cot[c + d*x]])/Sqrt[e]])/(Sqrt[2]*d) - (2*b*(3*a^2 - b^2)*
Sqrt[e*Cot[c + d*x]])/d - (8*a*b^2*(e*Cot[c + d*x])^(3/2))/(5*d*e) - (2*b^2
*(e*Cot[c + d*x])^(3/2)*(a + b*Cot[c + d*x]))/(5*d*e) - ((a - b)*(a^2 + 4*a
*b + b^2)*Sqrt[e]*Log[Sqrt[e] + Sqrt[e]*Cot[c + d*x] - Sqrt[2]*Sqrt[e*Cot[c
 + d*x]]])/(2*Sqrt[2]*d) + ((a - b)*(a^2 + 4*a*b + b^2)*Sqrt[e]*Log[Sqrt[e]
 + Sqrt[e]*Cot[c + d*x] + Sqrt[2]*Sqrt[e*Cot[c + d*x]]])/(2*Sqrt[2]*d)
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1179

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 1182

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
a*c, 2]}, Dist[(d*q + a*e)/(2*a*c), Int[(q + c*x^2)/(a + c*x^4), x], x] + D
ist[(d*q - a*e)/(2*a*c), Int[(q - c*x^2)/(a + c*x^4), x], x]] /; FreeQ[{a,
c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0] && NeQ[c*d^2 - a*e^2, 0] && NegQ[(-a)
*c]
```

Rule 3609

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)]), x_Symbol] :> Simp[d*((a + b*Tan[e + f*x])^m/(f*m)), x] + Int
[(a + b*Tan[e + f*x])^(m - 1)*Simp[a*c - b*d + (b*c + a*d)*Tan[e + f*x], x]
, x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2,
 0] && GtQ[m, 0]
```

Rule 3615

```
Int[((c_) + (d_.)*tan[(e_.) + (f_.)*(x_)])/Sqrt[(b_.)*tan[(e_.) + (f_.)*(x_
)]], x_Symbol] :> Dist[2/f, Subst[Int[(b*c + d*x^2)/(b^2 + x^4), x], x, Sqr
t[b*Tan[e + f*x]]], x] /; FreeQ[{b, c, d, e, f}, x] && NeQ[c^2 - d^2, 0] &&
 NeQ[c^2 + d^2, 0]
```

Rule 3647

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)])^(n_), x_Symbol] :> Simp[b^2*(a + b*Tan[e + f*x])^(m - 2)*((c
```

```
+ d*Tan[e + f*x])^(n + 1)/(d*f*(m + n - 1))), x] + Dist[1/(d*(m + n - 1)),
Int[(a + b*Tan[e + f*x])^(m - 3)*(c + d*Tan[e + f*x])^n*Simp[a^3*d*(m + n -
 1) - b^2*(b*c*(m - 2) + a*d*(1 + n)) + b*d*(m + n - 1)*(3*a^2 - b^2)*Tan[e
 + f*x] - b^2*(b*c*(m - 2) - a*d*(3*m + 2*n - 4))*Tan[e + f*x]^2, x], x], x
] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2,
 0] && NeQ[c^2 + d^2, 0] && IntegerQ[2*m] && GtQ[m, 2] && (GeQ[n, -1] || In
tegerQ[m]) &&  !(IGtQ[n, 2] && ( !IntegerQ[m] || (EqQ[c, 0] && NeQ[a, 0])))
```

Rule 3711

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*tan[(e_.)
+ (f_.)*(x_)] + (C_.)*tan[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[C*((a +
 b*Tan[e + f*x])^(m + 1)/(b*f*(m + 1))), x] + Int[(a + b*Tan[e + f*x])^m*Si
mp[A - C + B*Tan[e + f*x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] &&
NeQ[A*b^2 - a*b*B + a^2*C, 0] &&  !LeQ[m, -1]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\frac{2b^2(e\cot(c+dx))^{3/2}(a+b\cot(c+dx))}{5de} \\
&\quad -\frac{2\int \sqrt{e\cot(c+dx)}\left(-\frac{1}{2}a(5a^2-3b^2)\,e-\frac{5}{2}b(3a^2-b^2)\,e\cot(c+dx)-6ab^2e\cot^2(c+dx)\right)\,dx}{5e} \\
&= -\frac{8ab^2(e\cot(c+dx))^{3/2}}{5de}-\frac{2b^2(e\cot(c+dx))^{3/2}(a+b\cot(c+dx))}{5de} \\
&\quad -\frac{2\int \sqrt{e\cot(c+dx)}\left(-\frac{5}{2}a(a^2-3b^2)\,e-\frac{5}{2}b(3a^2-b^2)\,e\cot(c+dx)\right)\,dx}{5e} \\
&= -\frac{2b(3a^2-b^2)\sqrt{e\cot(c+dx)}}{d}-\frac{8ab^2(e\cot(c+dx))^{3/2}}{5de} \\
&\quad -\frac{2b^2(e\cot(c+dx))^{3/2}(a+b\cot(c+dx))}{5de}-\frac{2\int \frac{\frac{5}{2}b(3a^2-b^2)e^2-\frac{5}{2}a(a^2-3b^2)e^2\cot(c+dx)}{\sqrt{e\cot(c+dx)}}\,dx}{5e} \\
&= -\frac{2b(3a^2-b^2)\sqrt{e\cot(c+dx)}}{d}-\frac{8ab^2(e\cot(c+dx))^{3/2}}{5de} \\
&\quad -\frac{2b^2(e\cot(c+dx))^{3/2}(a+b\cot(c+dx))}{5de} \\
&\quad -\frac{4\text{Subst}\left(\int \frac{-\frac{5}{2}b(3a^2-b^2)e^3+\frac{5}{2}a(a^2-3b^2)e^2x^2}{e^2+x^4}\,dx,x,\sqrt{e\cot(c+dx)}\right)}{5de}
\end{aligned}$$

$$
\begin{aligned}
&= -\frac{2b(3a^2-b^2)\sqrt{e\cot(c+dx)}}{d} - \frac{8ab^2(e\cot(c+dx))^{3/2}}{5de} \\
&\quad - \frac{2b^2(e\cot(c+dx))^{3/2}(a+b\cot(c+dx))}{5de} \\
&\quad - \frac{((a+b)(a^2-4ab+b^2)\,e)\operatorname{Subst}\left(\int \frac{e+x^2}{e^2+x^4}\,dx, x, \sqrt{e\cot(c+dx)}\right)}{d} \\
&\quad + \frac{((a-b)(a^2+4ab+b^2)\,e)\operatorname{Subst}\left(\int \frac{e-x^2}{e^2+x^4}\,dx, x, \sqrt{e\cot(c+dx)}\right)}{d} \\
&= -\frac{2b(3a^2-b^2)\sqrt{e\cot(c+dx)}}{d} - \frac{8ab^2(e\cot(c+dx))^{3/2}}{5de} \\
&\quad - \frac{2b^2(e\cot(c+dx))^{3/2}(a+b\cot(c+dx))}{5de} \\
&\quad - \frac{((a-b)(a^2+4ab+b^2)\sqrt{e})\operatorname{Subst}\left(\int \frac{\sqrt{2}\sqrt{e}+2x}{-e-\sqrt{2}\sqrt{e}x-x^2}\,dx, x, \sqrt{e\cot(c+dx)}\right)}{2\sqrt{2}d} \\
&\quad - \frac{((a-b)(a^2+4ab+b^2)\sqrt{e})\operatorname{Subst}\left(\int \frac{\sqrt{2}\sqrt{e}-2x}{-e+\sqrt{2}\sqrt{e}x-x^2}\,dx, x, \sqrt{e\cot(c+dx)}\right)}{2\sqrt{2}d} \\
&\quad - \frac{((a+b)(a^2-4ab+b^2)\,e)\operatorname{Subst}\left(\int \frac{1}{e-\sqrt{2}\sqrt{e}x+x^2}\,dx, x, \sqrt{e\cot(c+dx)}\right)}{2d} \\
&\quad - \frac{((a+b)(a^2-4ab+b^2)\,e)\operatorname{Subst}\left(\int \frac{1}{e+\sqrt{2}\sqrt{e}x+x^2}\,dx, x, \sqrt{e\cot(c+dx)}\right)}{2d} \\
&= -\frac{2b(3a^2-b^2)\sqrt{e\cot(c+dx)}}{d} - \frac{8ab^2(e\cot(c+dx))^{3/2}}{5de} \\
&\quad - \frac{2b^2(e\cot(c+dx))^{3/2}(a+b\cot(c+dx))}{5de} \\
&\quad - \frac{(a-b)(a^2+4ab+b^2)\sqrt{e}\log\left(\sqrt{e}+\sqrt{e}\cot(c+dx)-\sqrt{2}\sqrt{e\cot(c+dx)}\right)}{2\sqrt{2}d} \\
&\quad + \frac{(a-b)(a^2+4ab+b^2)\sqrt{e}\log\left(\sqrt{e}+\sqrt{e}\cot(c+dx)+\sqrt{2}\sqrt{e\cot(c+dx)}\right)}{2\sqrt{2}d} \\
&\quad - \frac{((a+b)(a^2-4ab+b^2)\sqrt{e})\operatorname{Subst}\left(\int \frac{1}{-1-x^2}\,dx, x, 1-\frac{\sqrt{2}\sqrt{e\cot(c+dx)}}{\sqrt{e}}\right)}{\sqrt{2}d} \\
&\quad + \frac{((a+b)(a^2-4ab+b^2)\sqrt{e})\operatorname{Subst}\left(\int \frac{1}{-1-x^2}\,dx, x, 1+\frac{\sqrt{2}\sqrt{e\cot(c+dx)}}{\sqrt{e}}\right)}{\sqrt{2}d}
\end{aligned}
$$

$$= \frac{(a+b)(a^2-4ab+b^2)\sqrt{e}\arctan\left(1-\frac{\sqrt{2}\sqrt{e\cot(c+dx)}}{\sqrt{e}}\right)}{\sqrt{2}d} - \frac{(a+b)(a^2-4ab+b^2)\sqrt{e}\arctan\left(1+\frac{\sqrt{2}\sqrt{e\cot(c+dx)}}{\sqrt{e}}\right)}{\sqrt{2}d} - \frac{2b(3a^2-b^2)\sqrt{e\cot(c+dx)}}{d} - \frac{8ab^2(e\cot(c+dx))^{3/2}}{5de} - \frac{2b^2(e\cot(c+dx))^{3/2}(a+b\cot(c+dx))}{5de} - \frac{(a-b)(a^2+4ab+b^2)\sqrt{e}\log\left(\sqrt{e}+\sqrt{e}\cot(c+dx)-\sqrt{2}\sqrt{e\cot(c+dx)}\right)}{2\sqrt{2}d} + \frac{(a-b)(a^2+4ab+b^2)\sqrt{e}\log\left(\sqrt{e}+\sqrt{e}\cot(c+dx)+\sqrt{2}\sqrt{e\cot(c+dx)}\right)}{2\sqrt{2}d}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 2.64 (sec) , antiderivative size = 247, normalized size of antiderivative = 0.72

$$\int \sqrt{e\cot(c+dx)}(a+b\cot(c+dx))^3\,dx = -\frac{\sqrt{e\cot(c+dx)}\left(2ab^2\cot^{\frac{3}{2}}(c+dx)+\frac{2}{5}b^3\cot^{\frac{5}{2}}(c+dx)+\frac{2}{3}a(a^2-3b^2)\cot^{\frac{3}{2}}(c+dx)\,\text{Hypergeometric2F1}\right.}{}$$

```
[In] Integrate[Sqrt[e*Cot[c + d*x]]*(a + b*Cot[c + d*x])^3,x]
```

```
[Out] -((Sqrt[e*Cot[c + d*x]]*(2*a*b^2*Cot[c + d*x]^(3/2) + (2*b^3*Cot[c + d*x]^(
5/2))/5 + (2*a*(a^2 - 3*b^2)*Cot[c + d*x]^(3/2)*Hypergeometric2F1[3/4, 1, 7
/4, -Cot[c + d*x]^2])/3 - (b*(-3*a^2 + b^2)*(2*Sqrt[2]*ArcTan[1 - Sqrt[2]*S
qrt[Cot[c + d*x]]] - 2*Sqrt[2]*ArcTan[1 + Sqrt[2]*Sqrt[Cot[c + d*x]]] + 8*S
qrt[Cot[c + d*x]] + Sqrt[2]*Log[1 - Sqrt[2]*Sqrt[Cot[c + d*x]] + Cot[c + d*
x]] - Sqrt[2]*Log[1 + Sqrt[2]*Sqrt[Cot[c + d*x]] + Cot[c + d*x]]))/4))/(d*S
qrt[Cot[c + d*x]]))
```

Maple [A] (verified)

Time = 0.04 (sec), antiderivative size = 371, normalized size of antiderivative = 1.08

method	result
derivativedivides	$-\frac{2\left(\frac{b^3(e\cot(dx+c))^{\frac{5}{2}}}{5}+ae\,b^2(e\cot(dx+c))^{\frac{3}{2}}+3a^2b\,e^2\sqrt{e\cot(dx+c)}-\sqrt{e\cot(dx+c)}\,b^3e^2+e^3\left(\frac{(-3a^2be+b^3e)(e^2)^{\frac{1}{4}}\sqrt{2}\left(\ln \cdots\right.}{\cdots}\right.\right.}{\cdots}$
default	$-\frac{2\left(\frac{b^3(e\cot(dx+c))^{\frac{5}{2}}}{5}+ae\,b^2(e\cot(dx+c))^{\frac{3}{2}}+3a^2b\,e^2\sqrt{e\cot(dx+c)}-\sqrt{e\cot(dx+c)}\,b^3e^2+e^3\left(\frac{(-3a^2be+b^3e)(e^2)^{\frac{1}{4}}\sqrt{2}\left(\ln \cdots\right.}{\cdots}\right.\right.}{\cdots}$
parts	$-\frac{a^3e\sqrt{2}\left(\ln\left(\frac{e\cot(dx+c)-(e^2)^{\frac{1}{4}}\sqrt{e\cot(dx+c)}\,\sqrt{2}+\sqrt{e^2}}{e\cot(dx+c)+(e^2)^{\frac{1}{4}}\sqrt{e\cot(dx+c)}\,\sqrt{2}+\sqrt{e^2}}\right)+2\arctan\left(\frac{\sqrt{2}\,\sqrt{e\cot(dx+c)}}{(e^2)^{\frac{1}{4}}}+1\right)-2\arctan\left(-\frac{\sqrt{2}\,\sqrt{e\cot(dx+c)}}{(e^2)^{\frac{1}{4}}}\cdots\right.\right.}{4d(e^2)^{\frac{1}{4}}}$

```
[In] int((e*cot(d*x+c))^(1/2)*(a+b*cot(d*x+c))^3,x,method=_RETURNVERBOSE)
```

```
[Out] -2/d/e^2*(1/5*b^3*(e*cot(d*x+c))^(5/2)+a*e*b^2*(e*cot(d*x+c))^(3/2)+3*a^2*b
*e^2*(e*cot(d*x+c))^(1/2)-(e*cot(d*x+c))^(1/2)*b^3*e^2+e^3*(1/8*(-3*a^2*b*e
+b^3*e)*(e^2)^(1/4)/e^2*2^(1/2)*(ln((e*cot(d*x+c)+(e^2)^(1/4)*(e*cot(d*x+c)
)^(1/2)*2^(1/2)+(e^2)^(1/2))/(e*cot(d*x+c)-(e^2)^(1/4)*(e*cot(d*x+c))^(1/2)
*2^(1/2)+(e^2)^(1/2)))+2*arctan(2^(1/2)/(e^2)^(1/4)*(e*cot(d*x+c))^(1/2)+1)
-2*arctan(-2^(1/2)/(e^2)^(1/4)*(e*cot(d*x+c))^(1/2)+1))+1/8*(a^3-3*a*b^2)/(
e^2)^(1/4)*2^(1/2)*(ln((e*cot(d*x+c)-(e^2)^(1/4)*(e*cot(d*x+c))^(1/2)*2^(1/
2)+(e^2)^(1/2))/(e*cot(d*x+c)+(e^2)^(1/4)*(e*cot(d*x+c))^(1/2)*2^(1/2)+(e^2
)^(1/2)))+2*arctan(2^(1/2)/(e^2)^(1/4)*(e*cot(d*x+c))^(1/2)+1)-2*arctan(-2^
(1/2)/(e^2)^(1/4)*(e*cot(d*x+c))^(1/2)+1))))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1643 vs. 2(285) = 570.

Time = 0.34 (sec), antiderivative size = 1643, normalized size of antiderivative = 4.80

$$\int \sqrt{e\cot(c+dx)}(a+b\cot(c+dx))^3\,dx = \text{Too large to display}$$

```
[In] integrate((e*cot(d*x+c))^(1/2)*(a+b*cot(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] 1/10*(5*(d*cos(2*d*x + 2*c) - d)*sqrt((d^2*sqrt(-(a^12 - 30*a^10*b^2 + 255*
a^8*b^4 - 452*a^6*b^6 + 255*a^4*b^8 - 30*a^2*b^10 + b^12)*e^2/d^4) + 2*(3*a
```

```
^5*b - 10*a^3*b^3 + 3*a*b^5)*e)/d^2)*log(-(a^12 - 12*a^10*b^2 - 27*a^8*b^4
+ 27*a^4*b^8 + 12*a^2*b^10 - b^12)*e*sqrt((e*cos(2*d*x + 2*c) + e)/sin(2*d*
x + 2*c)) + ((a^3 - 3*a*b^2)*d^3*sqrt(-(a^12 - 30*a^10*b^2 + 255*a^8*b^4 -
452*a^6*b^6 + 255*a^4*b^8 - 30*a^2*b^10 + b^12)*e^2/d^4) - (3*a^8*b - 46*a^
6*b^3 + 60*a^4*b^5 - 18*a^2*b^7 + b^9)*d*e)*sqrt((d^2*sqrt(-(a^12 - 30*a^10
*b^2 + 255*a^8*b^4 - 452*a^6*b^6 + 255*a^4*b^8 - 30*a^2*b^10 + b^12)*e^2/d^
4) + 2*(3*a^5*b - 10*a^3*b^3 + 3*a*b^5)*e)/d^2)) - 5*(d*cos(2*d*x + 2*c) -
d)*sqrt((d^2*sqrt(-(a^12 - 30*a^10*b^2 + 255*a^8*b^4 - 452*a^6*b^6 + 255*a^
4*b^8 - 30*a^2*b^10 + b^12)*e^2/d^4) + 2*(3*a^5*b - 10*a^3*b^3 + 3*a*b^5)*e
)/d^2)*log(-(a^12 - 12*a^10*b^2 - 27*a^8*b^4 + 27*a^4*b^8 + 12*a^2*b^10 - b
^12)*e*sqrt((e*cos(2*d*x + 2*c) + e)/sin(2*d*x + 2*c)) - ((a^3 - 3*a*b^2)*d
^3*sqrt(-(a^12 - 30*a^10*b^2 + 255*a^8*b^4 - 452*a^6*b^6 + 255*a^4*b^8 - 30
*a^2*b^10 + b^12)*e^2/d^4) - (3*a^8*b - 46*a^6*b^3 + 60*a^4*b^5 - 18*a^2*b^
7 + b^9)*d*e)*sqrt((d^2*sqrt(-(a^12 - 30*a^10*b^2 + 255*a^8*b^4 - 452*a^6*b
^6 + 255*a^4*b^8 - 30*a^2*b^10 + b^12)*e^2/d^4) + 2*(3*a^5*b - 10*a^3*b^3 +
 3*a*b^5)*e)/d^2)) - 5*(d*cos(2*d*x + 2*c) - d)*sqrt(-(d^2*sqrt(-(a^12 - 30
*a^10*b^2 + 255*a^8*b^4 - 452*a^6*b^6 + 255*a^4*b^8 - 30*a^2*b^10 + b^12)*e
^2/d^4) - 2*(3*a^5*b - 10*a^3*b^3 + 3*a*b^5)*e)/d^2)*log(-(a^12 - 12*a^10*b
^2 - 27*a^8*b^4 + 27*a^4*b^8 + 12*a^2*b^10 - b^12)*e*sqrt((e*cos(2*d*x + 2*
c) + e)/sin(2*d*x + 2*c)) + ((a^3 - 3*a*b^2)*d^3*sqrt(-(a^12 - 30*a^10*b^2
+ 255*a^8*b^4 - 452*a^6*b^6 + 255*a^4*b^8 - 30*a^2*b^10 + b^12)*e^2/d^4) +
(3*a^8*b - 46*a^6*b^3 + 60*a^4*b^5 - 18*a^2*b^7 + b^9)*d*e)*sqrt(-(d^2*sqrt
(-(a^12 - 30*a^10*b^2 + 255*a^8*b^4 - 452*a^6*b^6 + 255*a^4*b^8 - 30*a^2*b^
10 + b^12)*e^2/d^4) - 2*(3*a^5*b - 10*a^3*b^3 + 3*a*b^5)*e)/d^2)) + 5*(d*co
s(2*d*x + 2*c) - d)*sqrt(-(d^2*sqrt(-(a^12 - 30*a^10*b^2 + 255*a^8*b^4 - 45
2*a^6*b^6 + 255*a^4*b^8 - 30*a^2*b^10 + b^12)*e^2/d^4) - 2*(3*a^5*b - 10*a^
3*b^3 + 3*a*b^5)*e)/d^2)*log(-(a^12 - 12*a^10*b^2 - 27*a^8*b^4 + 27*a^4*b^8
 + 12*a^2*b^10 - b^12)*e*sqrt((e*cos(2*d*x + 2*c) + e)/sin(2*d*x + 2*c)) -
((a^3 - 3*a*b^2)*d^3*sqrt(-(a^12 - 30*a^10*b^2 + 255*a^8*b^4 - 452*a^6*b^6
+ 255*a^4*b^8 - 30*a^2*b^10 + b^12)*e^2/d^4) + (3*a^8*b - 46*a^6*b^3 + 60*a
^4*b^5 - 18*a^2*b^7 + b^9)*d*e)*sqrt(-(d^2*sqrt(-(a^12 - 30*a^10*b^2 + 255*
a^8*b^4 - 452*a^6*b^6 + 255*a^4*b^8 - 30*a^2*b^10 + b^12)*e^2/d^4) - 2*(3*a
^5*b - 10*a^3*b^3 + 3*a*b^5)*e)/d^2)) + 4*(5*a*b^2*sin(2*d*x + 2*c) + 15*a^
2*b - 4*b^3 - 3*(5*a^2*b - 2*b^3)*cos(2*d*x + 2*c))*sqrt((e*cos(2*d*x + 2*c
) + e)/sin(2*d*x + 2*c)))/(d*cos(2*d*x + 2*c) - d)
```

Sympy [F]

$$\int \sqrt{e\cot(c+dx)}(a+b\cot(c+dx))^3\,dx = \int \sqrt{e\cot{(c+dx)}}(a+b\cot{(c+dx)})^3\,dx$$

```
[In] integrate((e*cot(d*x+c))**(1/2)*(a+b*cot(d*x+c))**3,x)
[Out] Integral(sqrt(e*cot(c + d*x))*(a + b*cot(c + d*x))**3, x)
```

Maxima [F(-2)]

Exception generated.

$$\int \sqrt{e\cot(c+dx)}(a+b\cot(c+dx))^3\,dx = \text{Exception raised: ValueError}$$

```
[In] integrate((e*cot(d*x+c))^(1/2)*(a+b*cot(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(e>0)', see `assume?` for more detai
ls)Is e
```

Giac [F]

$$\int \sqrt{e\cot(c+dx)}(a+b\cot(c+dx))^3\,dx = \int (b\cot(dx+c)+a)^3\sqrt{e\cot(dx+c)}\,dx$$

```
[In] integrate((e*cot(d*x+c))^(1/2)*(a+b*cot(d*x+c))^3,x, algorithm="giac")
```

```
[Out] integrate((b*cot(d*x + c) + a)^3*sqrt(e*cot(d*x + c)), x)
```

Mupad [B] (verification not implemented)

Time = 15.06 (sec) , antiderivative size = 2071, normalized size of antiderivative = 6.06

$$\int \sqrt{e\cot(c+dx)}(a+b\cot(c+dx))^3\,dx = \text{Too large to display}$$

```
[In] int((e*cot(c + d*x))^(1/2)*(a + b*cot(c + d*x))^3,x)
```

```
[Out] (e*cot(c + d*x))^(1/2)*((2*b^3)/d - (6*a^2*b)/d) + atan((((16*(e*cot(c + d*
x))^(1/2)*(a^6*e^4 - b^6*e^4 + 15*a^2*b^4*e^4 - 15*a^4*b^2*e^4))/d^2 - (8*(
4*b^3*d^2*e^4 - 12*a^2*b*d^2*e^4)*((b^6*e*1i - a^6*e*1i - a^2*b^4*e*15i - 2
0*a^3*b^3*e + a^4*b^2*e*15i + 6*a*b^5*e + 6*a^5*b*e)/(4*d^2))^(1/2))/d^3)*(
(b^6*e*1i - a^6*e*1i - a^2*b^4*e*15i - 20*a^3*b^3*e + a^4*b^2*e*15i + 6*a*b
^5*e + 6*a^5*b*e)/(4*d^2))^(1/2)*1i + ((16*(e*cot(c + d*x))^(1/2)*(a^6*e^4
- b^6*e^4 + 15*a^2*b^4*e^4 - 15*a^4*b^2*e^4))/d^2 + (8*(4*b^3*d^2*e^4 - 12*
a^2*b*d^2*e^4)*((b^6*e*1i - a^6*e*1i - a^2*b^4*e*15i - 20*a^3*b^3*e + a^4*b
^2*e*15i + 6*a*b^5*e + 6*a^5*b*e)/(4*d^2))^(1/2))/d^3)*((b^6*e*1i - a^6*e*1
i - a^2*b^4*e*15i - 20*a^3*b^3*e + a^4*b^2*e*15i + 6*a*b^5*e + 6*a^5*b*e)/(
4*d^2))^(1/2)*1i)/(((16*(e*cot(c + d*x))^(1/2)*(a^6*e^4 - b^6*e^4 + 15*a^2*
b^4*e^4 - 15*a^4*b^2*e^4))/d^2 - (8*(4*b^3*d^2*e^4 - 12*a^2*b*d^2*e^4)*((b^
6*e*1i - a^6*e*1i - a^2*b^4*e*15i - 20*a^3*b^3*e + a^4*b^2*e*15i + 6*a*b^5*
```

```
e + 6*a^5*b*e)/(4*d^2))^(1/2))/d^3)*((b^6*e*1i - a^6*e*1i - a^2*b^4*e*15i -
 20*a^3*b^3*e + a^4*b^2*e*15i + 6*a*b^5*e + 6*a^5*b*e)/(4*d^2))^(1/2) - ((1
6*(e*cot(c + d*x))^(1/2)*(a^6*e^4 - b^6*e^4 + 15*a^2*b^4*e^4 - 15*a^4*b^2*e
^4))/d^2 + (8*(4*b^3*d^2*e^4 - 12*a^2*b*d^2*e^4)*((b^6*e*1i - a^6*e*1i - a^
2*b^4*e*15i - 20*a^3*b^3*e + a^4*b^2*e*15i + 6*a*b^5*e + 6*a^5*b*e)/(4*d^2)
)^(1/2))/d^3)*((b^6*e*1i - a^6*e*1i - a^2*b^4*e*15i - 20*a^3*b^3*e + a^4*b^
2*e*15i + 6*a*b^5*e + 6*a^5*b*e)/(4*d^2))^(1/2) + (16*(3*a*b^8*e^5 - a^9*e^
5 + 8*a^3*b^6*e^5 + 6*a^5*b^4*e^5))/d^3))*((b^6*e*1i - a^6*e*1i - a^2*b^4*e
*15i - 20*a^3*b^3*e + a^4*b^2*e*15i + 6*a*b^5*e + 6*a^5*b*e)/(4*d^2))^(1/2)
*2i + atan((((16*(e*cot(c + d*x))^(1/2)*(a^6*e^4 - b^6*e^4 + 15*a^2*b^4*e^4
 - 15*a^4*b^2*e^4))/d^2 - (8*(4*b^3*d^2*e^4 - 12*a^2*b*d^2*e^4)*((a^6*e*1i
- b^6*e*1i + a^2*b^4*e*15i - 20*a^3*b^3*e - a^4*b^2*e*15i + 6*a*b^5*e + 6*a
^5*b*e)/(4*d^2))^(1/2))/d^3)*((a^6*e*1i - b^6*e*1i + a^2*b^4*e*15i - 20*a^3
*b^3*e - a^4*b^2*e*15i + 6*a*b^5*e + 6*a^5*b*e)/(4*d^2))^(1/2)*1i + ((16*(e
*cot(c + d*x))^(1/2)*(a^6*e^4 - b^6*e^4 + 15*a^2*b^4*e^4 - 15*a^4*b^2*e^4))
/d^2 + (8*(4*b^3*d^2*e^4 - 12*a^2*b*d^2*e^4)*((a^6*e*1i - b^6*e*1i + a^2*b^
4*e*15i - 20*a^3*b^3*e - a^4*b^2*e*15i + 6*a*b^5*e + 6*a^5*b*e)/(4*d^2))^(1
/2))/d^3)*((a^6*e*1i - b^6*e*1i + a^2*b^4*e*15i - 20*a^3*b^3*e - a^4*b^2*e*
15i + 6*a*b^5*e + 6*a^5*b*e)/(4*d^2))^(1/2)*1i)/((((16*(e*cot(c + d*x))^(1/2
)*(a^6*e^4 - b^6*e^4 + 15*a^2*b^4*e^4 - 15*a^4*b^2*e^4))/d^2 - (8*(4*b^3*d^
2*e^4 - 12*a^2*b*d^2*e^4)*((a^6*e*1i - b^6*e*1i + a^2*b^4*e*15i - 20*a^3*b^
3*e - a^4*b^2*e*15i + 6*a*b^5*e + 6*a^5*b*e)/(4*d^2))^(1/2))/d^3)*((a^6*e*1
i - b^6*e*1i + a^2*b^4*e*15i - 20*a^3*b^3*e - a^4*b^2*e*15i + 6*a*b^5*e + 6
*a^5*b*e)/(4*d^2))^(1/2) - ((16*(e*cot(c + d*x))^(1/2)*(a^6*e^4 - b^6*e^4 +
 15*a^2*b^4*e^4 - 15*a^4*b^2*e^4))/d^2 + (8*(4*b^3*d^2*e^4 - 12*a^2*b*d^2*e
^4)*((a^6*e*1i - b^6*e*1i + a^2*b^4*e*15i - 20*a^3*b^3*e - a^4*b^2*e*15i +
6*a*b^5*e + 6*a^5*b*e)/(4*d^2))^(1/2))/d^3)*((a^6*e*1i - b^6*e*1i + a^2*b^4
*e*15i - 20*a^3*b^3*e - a^4*b^2*e*15i + 6*a*b^5*e + 6*a^5*b*e)/(4*d^2))^(1/
2) + (16*(3*a*b^8*e^5 - a^9*e^5 + 8*a^3*b^6*e^5 + 6*a^5*b^4*e^5))/d^3))*((a
^6*e*1i - b^6*e*1i + a^2*b^4*e*15i - 20*a^3*b^3*e - a^4*b^2*e*15i + 6*a*b^5
*e + 6*a^5*b*e)/(4*d^2))^(1/2)*2i - (2*b^3*(e*cot(c + d*x))^(5/2))/(5*d*e^2
) - (2*a*b^2*(e*cot(c + d*x))^(3/2))/(d*e)
```

3.64 $\int \frac{(a+b\cot(c+dx))^3}{\sqrt{e\cot(c+dx)}}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 313

$$\begin{aligned}\int \frac{(a+b\cot(c+dx))^3}{\sqrt{e\cot(c+dx)}}\,dx &= \frac{(a-b)\left(a^2+4ab+b^2\right)\arctan\left(1-\frac{\sqrt{2}\sqrt{e\cot(c+dx)}}{\sqrt{e}}\right)}{\sqrt{2}d\sqrt{e}} \\ &\quad - \frac{(a-b)\left(a^2+4ab+b^2\right)\arctan\left(1+\frac{\sqrt{2}\sqrt{e\cot(c+dx)}}{\sqrt{e}}\right)}{\sqrt{2}d\sqrt{e}} \\ &\quad - \frac{16ab^2\sqrt{e\cot(c+dx)}}{3de} - \frac{2b^2\sqrt{e\cot(c+dx)}(a+b\cot(c+dx))}{3de} \\ &\quad + \frac{(a+b)\left(a^2-4ab+b^2\right)\log\left(\sqrt{e}+\sqrt{e}\cot(c+dx)-\sqrt{2}\sqrt{e\cot(c+dx)}\right)}{2\sqrt{2}d\sqrt{e}} \\ &\quad - \frac{(a+b)\left(a^2-4ab+b^2\right)\log\left(\sqrt{e}+\sqrt{e}\cot(c+dx)+\sqrt{2}\sqrt{e\cot(c+dx)}\right)}{2\sqrt{2}d\sqrt{e}}\end{aligned}$$

```
[Out] 1/2*(a-b)*(a^2+4*a*b+b^2)*arctan(1-2^(1/2)*(e*cot(d*x+c))^(1/2)/e^(1/2))/d*
2^(1/2)/e^(1/2)-1/2*(a-b)*(a^2+4*a*b+b^2)*arctan(1+2^(1/2)*(e*cot(d*x+c))^(
1/2)/e^(1/2))/d*2^(1/2)/e^(1/2)+1/4*(a+b)*(a^2-4*a*b+b^2)*ln(e^(1/2)+cot(d*
x+c)*e^(1/2)-2^(1/2)*(e*cot(d*x+c))^(1/2))/d*2^(1/2)/e^(1/2)-1/4*(a+b)*(a^2
-4*a*b+b^2)*ln(e^(1/2)+cot(d*x+c)*e^(1/2)+2^(1/2)*(e*cot(d*x+c))^(1/2))/d*2
^(1/2)/e^(1/2)-16/3*a*b^2*(e*cot(d*x+c))^(1/2)/d/e-2/3*b^2*(a+b*cot(d*x+c))
*(e*cot(d*x+c))^(1/2)/d/e
```

Rubi [A] (verified)

Time = 0.48 (sec) , antiderivative size = 313, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.360$, Rules used = {3647, 3711, 3615, 1182, 1176, 631, 210, 1179, 642}

$$\int \frac{(a+b\cot(c+dx))^3}{\sqrt{e\cot(c+dx)}}\,dx$$
$$= \frac{(a-b)\left(a^2+4ab+b^2\right)\arctan\left(1-\frac{\sqrt{2}\sqrt{e\cot(c+dx)}}{\sqrt{e}}\right)}{\sqrt{2}d\sqrt{e}} - \frac{(a-b)\left(a^2+4ab+b^2\right)\arctan\left(\frac{\sqrt{2}\sqrt{e\cot(c+dx)}}{\sqrt{e}}+1\right)}{\sqrt{2}d\sqrt{e}} + \frac{(a+b)\left(a^2-4ab+b^2\right)\log\left(\sqrt{e}\cot(c+dx)-\sqrt{2}\sqrt{e\cot(c+dx)}+\sqrt{e}\right)}{2\sqrt{2}d\sqrt{e}} - \frac{(a+b)\left(a^2-4ab+b^2\right)\log\left(\sqrt{e}\cot(c+dx)+\sqrt{2}\sqrt{e\cot(c+dx)}+\sqrt{e}\right)}{2\sqrt{2}d\sqrt{e}} - \frac{2b^2\sqrt{e\cot(c+dx)}(a+b\cot(c+dx))}{3de} - \frac{16ab^2\sqrt{e\cot(c+dx)}}{3de}$$

```
[In] Int[(a + b*Cot[c + d*x])^3/Sqrt[e*Cot[c + d*x]],x]
```

```
[Out] ((a - b)*(a^2 + 4*a*b + b^2)*ArcTan[1 - (Sqrt[2]*Sqrt[e*Cot[c + d*x]])/Sqrt
[e]])/(Sqrt[2]*d*Sqrt[e]) - ((a - b)*(a^2 + 4*a*b + b^2)*ArcTan[1 + (Sqrt[2
]*Sqrt[e*Cot[c + d*x]])/Sqrt[e]])/(Sqrt[2]*d*Sqrt[e]) - (16*a*b^2*Sqrt[e*Co
t[c + d*x]])/(3*d*e) - (2*b^2*Sqrt[e*Cot[c + d*x]]*(a + b*Cot[c + d*x]))/(3
*d*e) + ((a + b)*(a^2 - 4*a*b + b^2)*Log[Sqrt[e] + Sqrt[e]*Cot[c + d*x] - S
qrt[2]*Sqrt[e*Cot[c + d*x]]])/(2*Sqrt[2]*d*Sqrt[e]) - ((a + b)*(a^2 - 4*a*b
 + b^2)*Log[Sqrt[e] + Sqrt[e]*Cot[c + d*x] + Sqrt[2]*Sqrt[e*Cot[c + d*x]]])
/(2*Sqrt[2]*d*Sqrt[e])
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1179

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 1182

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
a*c, 2]}, Dist[(d*q + a*e)/(2*a*c), Int[(q + c*x^2)/(a + c*x^4), x], x] + D
ist[(d*q - a*e)/(2*a*c), Int[(q - c*x^2)/(a + c*x^4), x], x]] /; FreeQ[{a,
c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0] && NeQ[c*d^2 - a*e^2, 0] && NegQ[(-a)
*c]
```

Rule 3615

```
Int[((c_) + (d_.)*tan[(e_.) + (f_.)*(x_)])/Sqrt[(b_.)*tan[(e_.) + (f_.)*(x_
)]], x_Symbol] :> Dist[2/f, Subst[Int[(b*c + d*x^2)/(b^2 + x^4), x], x, Sqr
t[b*Tan[e + f*x]]], x] /; FreeQ[{b, c, d, e, f}, x] && NeQ[c^2 - d^2, 0] &&
 NeQ[c^2 + d^2, 0]
```

Rule 3647

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)])^(n_), x_Symbol] :> Simp[b^2*(a + b*Tan[e + f*x])^(m - 2)*((c
+ d*Tan[e + f*x])^(n + 1)/(d*f*(m + n - 1))), x] + Dist[1/(d*(m + n - 1)),
Int[(a + b*Tan[e + f*x])^(m - 3)*(c + d*Tan[e + f*x])^n*Simp[a^3*d*(m + n -
 1) - b^2*(b*c*(m - 2) + a*d*(1 + n)) + b*d*(m + n - 1)*(3*a^2 - b^2)*Tan[e
 + f*x] - b^2*(b*c*(m - 2) - a*d*(3*m + 2*n - 4))*Tan[e + f*x]^2, x], x], x
] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2,
 0] && NeQ[c^2 + d^2, 0] && IntegerQ[2*m] && GtQ[m, 2] && (GeQ[n, -1] || In
tegerQ[m]) &&  !(IGtQ[n, 2] && ( !IntegerQ[m] || (EqQ[c, 0] && NeQ[a, 0])))
```

Rule 3711

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*tan[(e_.)
+ (f_.)*(x_)] + (C_.)*tan[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[C*((a +
 b*Tan[e + f*x])^(m + 1)/(b*f*(m + 1))), x] + Int[(a + b*Tan[e + f*x])^m*Si
mp[A - C + B*Tan[e + f*x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] &&
NeQ[A*b^2 - a*b*B + a^2*C, 0] &&  !LeQ[m, -1]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= -\frac{2b^2\sqrt{e\cot(c+dx)}(a+b\cot(c+dx))}{3de} - \frac{2\int \frac{-\frac{1}{2}a(3a^2-b^2)e-\frac{3}{2}b(3a^2-b^2)e\cot(c+dx)-4ab^2e\cot^2(c+dx)}{\sqrt{e\cot(c+dx)}}\,dx}{3e}\\
&= -\frac{16ab^2\sqrt{e\cot(c+dx)}}{3de} - \frac{2b^2\sqrt{e\cot(c+dx)}(a+b\cot(c+dx))}{3de} - \frac{2\int \frac{-\frac{3}{2}a(a^2-3b^2)e-\frac{3}{2}b(3a^2-b^2)e\cot(c+dx)}{\sqrt{e\cot(c+dx)}}\,dx}{3e}\\
&= -\frac{16ab^2\sqrt{e\cot(c+dx)}}{3de} - \frac{2b^2\sqrt{e\cot(c+dx)}(a+b\cot(c+dx))}{3de} - \frac{4\text{Subst}\left(\int \frac{\frac{3}{2}a(a^2-3b^2)e^2+\frac{3}{2}b(3a^2-b^2)ex^2}{e^2+x^4}\,dx, x, \sqrt{e\cot(c+dx)}\right)}{3de}\\
&= -\frac{16ab^2\sqrt{e\cot(c+dx)}}{3de} - \frac{2b^2\sqrt{e\cot(c+dx)}(a+b\cot(c+dx))}{3de} - \frac{((a+b)(a^2-4ab+b^2))\,\text{Subst}\left(\int \frac{e-x^2}{e^2+x^4}\,dx, x, \sqrt{e\cot(c+dx)}\right)}{d}\\
&\quad - \frac{((a-b)(a^2+4ab+b^2))\,\text{Subst}\left(\int \frac{e+x^2}{e^2+x^4}\,dx, x, \sqrt{e\cot(c+dx)}\right)}{d}\\
&= -\frac{16ab^2\sqrt{e\cot(c+dx)}}{3de} - \frac{2b^2\sqrt{e\cot(c+dx)}(a+b\cot(c+dx))}{3de}\\
&\quad - \frac{((a-b)(a^2+4ab+b^2))\,\text{Subst}\left(\int \frac{1}{e-\sqrt{2}\sqrt{e}x+x^2}\,dx, x, \sqrt{e\cot(c+dx)}\right)}{2d}\\
&\quad - \frac{((a-b)(a^2+4ab+b^2))\,\text{Subst}\left(\int \frac{1}{e+\sqrt{2}\sqrt{e}x+x^2}\,dx, x, \sqrt{e\cot(c+dx)}\right)}{2d}\\
&\quad + \frac{((a+b)(a^2-4ab+b^2))\,\text{Subst}\left(\int \frac{\sqrt{2}\sqrt{e}+2x}{-e-\sqrt{2}\sqrt{e}x-x^2}\,dx, x, \sqrt{e\cot(c+dx)}\right)}{2\sqrt{2}d\sqrt{e}}\\
&\quad + \frac{((a+b)(a^2-4ab+b^2))\,\text{Subst}\left(\int \frac{\sqrt{2}\sqrt{e}-2x}{-e+\sqrt{2}\sqrt{e}x-x^2}\,dx, x, \sqrt{e\cot(c+dx)}\right)}{2\sqrt{2}d\sqrt{e}}
\end{aligned}
$$

$$= -\frac{16ab^2\sqrt{e\cot(c+dx)}}{3de} - \frac{2b^2\sqrt{e\cot(c+dx)}(a+b\cot(c+dx))}{3de} + \frac{(a+b)\left(a^2-4ab+b^2\right)\log\left(\sqrt{e}+\sqrt{e}\cot(c+dx)-\sqrt{2}\sqrt{e\cot(c+dx)}\right)}{2\sqrt{2}d\sqrt{e}} - \frac{(a+b)\left(a^2-4ab+b^2\right)\log\left(\sqrt{e}+\sqrt{e}\cot(c+dx)+\sqrt{2}\sqrt{e\cot(c+dx)}\right)}{2\sqrt{2}d\sqrt{e}} - \frac{\left((a-b)\left(a^2+4ab+b^2\right)\right)\text{Subst}\left(\int\frac{1}{-1-x^2}\,dx,x,1-\frac{\sqrt{2}\sqrt{e\cot(c+dx)}}{\sqrt{e}}\right)}{\sqrt{2}d\sqrt{e}} + \frac{\left((a-b)\left(a^2+4ab+b^2\right)\right)\text{Subst}\left(\int\frac{1}{-1-x^2}\,dx,x,1+\frac{\sqrt{2}\sqrt{e\cot(c+dx)}}{\sqrt{e}}\right)}{\sqrt{2}d\sqrt{e}}$$

$$= \frac{(a-b)\left(a^2+4ab+b^2\right)\arctan\left(1-\frac{\sqrt{2}\sqrt{e\cot(c+dx)}}{\sqrt{e}}\right)}{\sqrt{2}d\sqrt{e}} - \frac{(a-b)\left(a^2+4ab+b^2\right)\arctan\left(1+\frac{\sqrt{2}\sqrt{e\cot(c+dx)}}{\sqrt{e}}\right)}{\sqrt{2}d\sqrt{e}} - \frac{16ab^2\sqrt{e\cot(c+dx)}}{3de} - \frac{2b^2\sqrt{e\cot(c+dx)}(a+b\cot(c+dx))}{3de} + \frac{(a+b)\left(a^2-4ab+b^2\right)\log\left(\sqrt{e}+\sqrt{e}\cot(c+dx)-\sqrt{2}\sqrt{e\cot(c+dx)}\right)}{2\sqrt{2}d\sqrt{e}} - \frac{(a+b)\left(a^2-4ab+b^2\right)\log\left(\sqrt{e}+\sqrt{e}\cot(c+dx)+\sqrt{2}\sqrt{e\cot(c+dx)}\right)}{2\sqrt{2}d\sqrt{e}}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 1.07 (sec) , antiderivative size = 215, normalized size of antiderivative = 0.69

$$\int \frac{(a+b\cot(c+dx))^3}{\sqrt{e\cot(c+dx)}}\,dx = -\frac{\sqrt{\cot(c+dx)}\left(72ab^2\sqrt{\cot(c+dx)}+8b^3\cot^{\frac{3}{2}}(c+dx)-8b(-3a^2+b^2)\cot^{\frac{3}{2}}(c+dx)\,\text{Hypergeometric2F}\right.}{}$$

```
[In] Integrate[(a + b*Cot[c + d*x])^3/Sqrt[e*Cot[c + d*x]],x]
```

```
[Out] -1/12*(Sqrt[Cot[c + d*x]]*(72*a*b^2*Sqrt[Cot[c + d*x]] + 8*b^3*Cot[c + d*x]
^(3/2) - 8*b*(-3*a^2 + b^2)*Cot[c + d*x]^(3/2)*Hypergeometric2F1[3/4, 1, 7/
4, -Cot[c + d*x]^2] - 3*Sqrt[2]*a*(a^2 - 3*b^2)*(2*ArcTan[1 - Sqrt[2]*Sqrt[
```

```
Cot[c + d*x]]] - 2*ArcTan[1 + Sqrt[2]*Sqrt[Cot[c + d*x]]] + Log[1 - Sqrt[2]
*Sqrt[Cot[c + d*x]] + Cot[c + d*x]] - Log[1 + Sqrt[2]*Sqrt[Cot[c + d*x]] +
Cot[c + d*x]])))/(d*Sqrt[e*Cot[c + d*x]])
```

Maple [A] (verified)

Time = 0.04 (sec) , antiderivative size = 337, normalized size of antiderivative = 1.08

method	result
derivativedivides	$-\frac{2\left(\frac{b^{3}(e\cot(dx+c))^{\frac{3}{2}}}{3}+3\sqrt{e\cot(dx+c)}\,a\,b^{2}e+e^{2}\left(\frac{\left(a^{3}e-3ae\,b^{2}\right)\left(e^{2}\right)^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\frac{e\cot(dx+c)+\left(e^{2}\right)^{\frac{1}{4}}\sqrt{e\cot(dx+c)}\,\sqrt{2}+\sqrt{e^{2}}}{e\cot(dx+c)-\left(e^{2}\right)^{\frac{1}{4}}\sqrt{e\cot(dx+c)}\,\sqrt{2}+\sqrt{e^{2}}}\right)\cdots\right)}{8e^{2}}\cdots\right)\cdots\right)}{\cdots}$
default	$-\frac{2\left(\frac{b^{3}(e\cot(dx+c))^{\frac{3}{2}}}{3}+3\sqrt{e\cot(dx+c)}\,a\,b^{2}e+e^{2}\left(\frac{\left(a^{3}e-3ae\,b^{2}\right)\left(e^{2}\right)^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\frac{e\cot(dx+c)+\left(e^{2}\right)^{\frac{1}{4}}\sqrt{e\cot(dx+c)}\,\sqrt{2}+\sqrt{e^{2}}}{e\cot(dx+c)-\left(e^{2}\right)^{\frac{1}{4}}\sqrt{e\cot(dx+c)}\,\sqrt{2}+\sqrt{e^{2}}}\right)\cdots\right)}{8e^{2}}\cdots\right)\cdots\right)}{\cdots}$
parts	$-\frac{a^{3}\left(e^{2}\right)^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\frac{e\cot(dx+c)+\left(e^{2}\right)^{\frac{1}{4}}\sqrt{e\cot(dx+c)}\,\sqrt{2}+\sqrt{e^{2}}}{e\cot(dx+c)-\left(e^{2}\right)^{\frac{1}{4}}\sqrt{e\cot(dx+c)}\,\sqrt{2}+\sqrt{e^{2}}}\right)+2\arctan\left(\frac{\sqrt{2}\sqrt{e\cot(dx+c)}}{\left(e^{2}\right)^{\frac{1}{4}}}+1\right)-2\arctan\left(-\frac{\sqrt{2}\sqrt{e\cot\cdots}}{\left(e^{2}\right)\cdots}\cdots\right)\cdots\right)}{4de}\cdots$

```
[In] int((a+b*cot(d*x+c))^3/(e*cot(d*x+c))^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -2/d/e^2*(1/3*b^3*(e*cot(d*x+c))^(3/2)+3*(e*cot(d*x+c))^(1/2)*a*b^2*e+e^2*(
1/8*(a^3*e-3*a*b^2*e)*(e^2)^(1/4)/e^2*2^(1/2)*(ln((e*cot(d*x+c)+(e^2)^(1/4)
*(e*cot(d*x+c))^(1/2)*2^(1/2)+(e^2)^(1/2))/(e*cot(d*x+c)-(e^2)^(1/4)*(e*cot
(d*x+c))^(1/2)*2^(1/2)+(e^2)^(1/2)))+2*arctan(2^(1/2)/(e^2)^(1/4)*(e*cot(d*
x+c))^(1/2)+1)-2*arctan(-2^(1/2)/(e^2)^(1/4)*(e*cot(d*x+c))^(1/2)+1))+1/8*(
3*a^2*b-b^3)/(e^2)^(1/4)*2^(1/2)*(ln((e*cot(d*x+c)-(e^2)^(1/4)*(e*cot(d*x+c
))^(1/2)*2^(1/2)+(e^2)^(1/2))/(e*cot(d*x+c)+(e^2)^(1/4)*(e*cot(d*x+c))^(1/2
)*2^(1/2)+(e^2)^(1/2)))+2*arctan(2^(1/2)/(e^2)^(1/4)*(e*cot(d*x+c))^(1/2)+1
)-2*arctan(-2^(1/2)/(e^2)^(1/4)*(e*cot(d*x+c))^(1/2)+1))))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1633 vs. 2(258) = 516.

Time = 0.41 (sec) , antiderivative size = 1633, normalized size of antiderivative = 5.22

$$\int \frac{(a + b\cot(c + dx))^3}{\sqrt{e\cot(c + dx)}}\,dx = \text{Too large to display}$$

```
[In] integrate((a+b*cot(d*x+c))^3/(e*cot(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] 1/6*(3*d*e*sqrt(-(6*a^5*b - 20*a^3*b^3 + 6*a*b^5 + d^2*e*sqrt(-(a^12 - 30*a
^10*b^2 + 255*a^8*b^4 - 452*a^6*b^6 + 255*a^4*b^8 - 30*a^2*b^10 + b^12)/(d^
4*e^2)))/(d^2*e))*log(-(a^12 - 12*a^10*b^2 - 27*a^8*b^4 + 27*a^4*b^8 + 12*a
^2*b^10 - b^12)*sqrt((e*cos(2*d*x + 2*c) + e)/sin(2*d*x + 2*c)) + ((3*a^2*b
 - b^3)*d^3*e^2*sqrt(-(a^12 - 30*a^10*b^2 + 255*a^8*b^4 - 452*a^6*b^6 + 255
*a^4*b^8 - 30*a^2*b^10 + b^12)/(d^4*e^2)) + (a^9 - 18*a^7*b^2 + 60*a^5*b^4
- 46*a^3*b^6 + 3*a*b^8)*d*e)*sqrt(-(6*a^5*b - 20*a^3*b^3 + 6*a*b^5 + d^2*e*
sqrt(-(a^12 - 30*a^10*b^2 + 255*a^8*b^4 - 452*a^6*b^6 + 255*a^4*b^8 - 30*a^
2*b^10 + b^12)/(d^4*e^2)))/(d^2*e)))*sin(2*d*x + 2*c) - 3*d*e*sqrt(-(6*a^5*
b - 20*a^3*b^3 + 6*a*b^5 + d^2*e*sqrt(-(a^12 - 30*a^10*b^2 + 255*a^8*b^4 -
452*a^6*b^6 + 255*a^4*b^8 - 30*a^2*b^10 + b^12)/(d^4*e^2)))/(d^2*e))*log(-(
a^12 - 12*a^10*b^2 - 27*a^8*b^4 + 27*a^4*b^8 + 12*a^2*b^10 - b^12)*sqrt((e*
cos(2*d*x + 2*c) + e)/sin(2*d*x + 2*c)) - ((3*a^2*b - b^3)*d^3*e^2*sqrt(-(a
^12 - 30*a^10*b^2 + 255*a^8*b^4 - 452*a^6*b^6 + 255*a^4*b^8 - 30*a^2*b^10 +
 b^12)/(d^4*e^2)) + (a^9 - 18*a^7*b^2 + 60*a^5*b^4 - 46*a^3*b^6 + 3*a*b^8)*
d*e)*sqrt(-(6*a^5*b - 20*a^3*b^3 + 6*a*b^5 + d^2*e*sqrt(-(a^12 - 30*a^10*b^
2 + 255*a^8*b^4 - 452*a^6*b^6 + 255*a^4*b^8 - 30*a^2*b^10 + b^12)/(d^4*e^2)
))/(d^2*e)))*sin(2*d*x + 2*c) - 3*d*e*sqrt(-(6*a^5*b - 20*a^3*b^3 + 6*a*b^5
 - d^2*e*sqrt(-(a^12 - 30*a^10*b^2 + 255*a^8*b^4 - 452*a^6*b^6 + 255*a^4*b^
8 - 30*a^2*b^10 + b^12)/(d^4*e^2)))/(d^2*e))*log(-(a^12 - 12*a^10*b^2 - 27*
a^8*b^4 + 27*a^4*b^8 + 12*a^2*b^10 - b^12)*sqrt((e*cos(2*d*x + 2*c) + e)/si
n(2*d*x + 2*c)) + ((3*a^2*b - b^3)*d^3*e^2*sqrt(-(a^12 - 30*a^10*b^2 + 255*
a^8*b^4 - 452*a^6*b^6 + 255*a^4*b^8 - 30*a^2*b^10 + b^12)/(d^4*e^2)) - (a^9
 - 18*a^7*b^2 + 60*a^5*b^4 - 46*a^3*b^6 + 3*a*b^8)*d*e)*sqrt(-(6*a^5*b - 20
*a^3*b^3 + 6*a*b^5 - d^2*e*sqrt(-(a^12 - 30*a^10*b^2 + 255*a^8*b^4 - 452*a^
6*b^6 + 255*a^4*b^8 - 30*a^2*b^10 + b^12)/(d^4*e^2)))/(d^2*e)))*sin(2*d*x +
 2*c) + 3*d*e*sqrt(-(6*a^5*b - 20*a^3*b^3 + 6*a*b^5 - d^2*e*sqrt(-(a^12 - 3
0*a^10*b^2 + 255*a^8*b^4 - 452*a^6*b^6 + 255*a^4*b^8 - 30*a^2*b^10 + b^12)/
(d^4*e^2)))/(d^2*e))*log(-(a^12 - 12*a^10*b^2 - 27*a^8*b^4 + 27*a^4*b^8 + 1
2*a^2*b^10 - b^12)*sqrt((e*cos(2*d*x + 2*c) + e)/sin(2*d*x + 2*c)) - ((3*a^
2*b - b^3)*d^3*e^2*sqrt(-(a^12 - 30*a^10*b^2 + 255*a^8*b^4 - 452*a^6*b^6 +
255*a^4*b^8 - 30*a^2*b^10 + b^12)/(d^4*e^2)) - (a^9 - 18*a^7*b^2 + 60*a^5*b
^4 - 46*a^3*b^6 + 3*a*b^8)*d*e)*sqrt(-(6*a^5*b - 20*a^3*b^3 + 6*a*b^5 - d^2
*e*sqrt(-(a^12 - 30*a^10*b^2 + 255*a^8*b^4 - 452*a^6*b^6 + 255*a^4*b^8 - 30
*a^2*b^10 + b^12)/(d^4*e^2)))/(d^2*e)))*sin(2*d*x + 2*c) - 4*(b^3*cos(2*d*x
```

```
 + 2*c) + 9*a*b^2*sin(2*d*x + 2*c) + b^3)*sqrt((e*cos(2*d*x + 2*c) + e)/sin
(2*d*x + 2*c)))/(d*e*sin(2*d*x + 2*c))
```

Sympy [F]

$$\int \frac{(a+b\cot(c+dx))^3}{\sqrt{e\cot(c+dx)}}\,dx=\int \frac{(a+b\cot{(c+dx)})^3}{\sqrt{e\cot{(c+dx)}}}\,dx$$

```
[In] integrate((a+b*cot(d*x+c))**3/(e*cot(d*x+c))**(1/2),x)
[Out] Integral((a + b*cot(c + d*x))**3/sqrt(e*cot(c + d*x)), x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{(a+b\cot(c+dx))^3}{\sqrt{e\cot(c+dx)}}\,dx=\text{Exception raised: ValueError}$$

```
[In] integrate((a+b*cot(d*x+c))^3/(e*cot(d*x+c))^(1/2),x, algorithm="maxima")
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(e>0)', see `assume?` for more detai
ls)Is e
```

Giac [F]

$$\int \frac{(a+b\cot(c+dx))^3}{\sqrt{e\cot(c+dx)}}\,dx=\int \frac{(b\cot{(dx+c)}+a)^3}{\sqrt{e\cot{(dx+c)}}}\,dx$$

```
[In] integrate((a+b*cot(d*x+c))^3/(e*cot(d*x+c))^(1/2),x, algorithm="giac")
[Out] integrate((b*cot(d*x + c) + a)^3/sqrt(e*cot(d*x + c)), x)
```

Mupad [B] (verification not implemented)

Time = 13.69 (sec) , antiderivative size = 1896, normalized size of antiderivative = 6.06

$$\int \frac{(a+b\cot(c+dx))^3}{\sqrt{e\cot(c+dx)}}\,dx = \text{Too large to display}$$

[In] int((a + b*cot(c + d*x))^3/(e*cot(c + d*x))^(1/2),x)

[Out] atan((((16*(e*cot(c + d*x))^(1/2)*(a^6*e^2 - b^6*e^2 + 15*a^2*b^4*e^2 - 15*a^4*b^2*e^2))/d^2 - (8*(4*a^3*d^2*e^3 - 12*a*b^2*d^2*e^3)*(((a*b^5*6i + a^5*b*6i - a^6 + b^6 - 15*a^2*b^4 - a^3*b^3*20i + 15*a^4*b^2)*1i)/(4*d^2*e))^(1/2))/d^3)*(((a*b^5*6i + a^5*b*6i - a^6 + b^6 - 15*a^2*b^4 - a^3*b^3*20i + 15*a^4*b^2)*1i)/(4*d^2*e))^(1/2)*1i + ((16*(e*cot(c + d*x))^(1/2)*(a^6*e^2 - b^6*e^2 + 15*a^2*b^4*e^2 - 15*a^4*b^2*e^2))/d^2 + (8*(4*a^3*d^2*e^3 - 12*a*b^2*d^2*e^3)*(((a*b^5*6i + a^5*b*6i - a^6 + b^6 - 15*a^2*b^4 - a^3*b^3*20i + 15*a^4*b^2)*1i)/(4*d^2*e))^(1/2))/d^3)*(((a*b^5*6i + a^5*b*6i - a^6 + b^6 - 15*a^2*b^4 - a^3*b^3*20i + 15*a^4*b^2)*1i)/(4*d^2*e))^(1/2)*1i)/(((16*(e*cot(c + d*x))^(1/2)*(a^6*e^2 - b^6*e^2 + 15*a^2*b^4*e^2 - 15*a^4*b^2*e^2))/d^2 + (8*(4*a^3*d^2*e^3 - 12*a*b^2*d^2*e^3)*(((a*b^5*6i + a^5*b*6i - a^6 + b^6 - 15*a^2*b^4 - a^3*b^3*20i + 15*a^4*b^2)*1i)/(4*d^2*e))^(1/2))/d^3)*(((a*b^5*6i + a^5*b*6i - a^6 + b^6 - 15*a^2*b^4 - a^3*b^3*20i + 15*a^4*b^2)*1i)/(4*d^2*e))^(1/2) - ((16*(e*cot(c + d*x))^(1/2)*(a^6*e^2 - b^6*e^2 + 15*a^2*b^4*e^2 - 15*a^4*b^2*e^2))/d^2 - (8*(4*a^3*d^2*e^3 - 12*a*b^2*d^2*e^3)*(((a*b^5*6i + a^5*b*6i - a^6 + b^6 - 15*a^2*b^4 - a^3*b^3*20i + 15*a^4*b^2)*1i)/(4*d^2*e))^(1/2))/d^3)*(((a*b^5*6i + a^5*b*6i - a^6 + b^6 - 15*a^2*b^4 - a^3*b^3*20i + 15*a^4*b^2)*1i)/(4*d^2*e))^(1/2) + (16*(3*a^8*b*e^2 - b^9*e^2 + 6*a^4*b^5*e^2 + 8*a^6*b^3*e^2))/d^3))*(((a*b^5*6i + a^5*b*6i - a^6 + b^6 - 15*a^2*b^4 - a^3*b^3*20i + 15*a^4*b^2)*1i)/(4*d^2*e))^(1/2)*2i + atan((((16*(e*cot(c + d*x))^(1/2)*(a^6*e^2 - b^6*e^2 + 15*a^2*b^4*e^2 - 15*a^4*b^2*e^2))/d^2 - (8*(4*a^3*d^2*e^3 - 12*a*b^2*d^2*e^3)*(((a*b^5*6i + a^5*b*6i + a^6 - b^6 + 15*a^2*b^4 - a^3*b^3*20i - 15*a^4*b^2)*1i)/(4*d^2*e))^(1/2))/d^3)*(((a*b^5*6i + a^5*b*6i + a^6 - b^6 + 15*a^2*b^4 - a^3*b^3*20i - 15*a^4*b^2)*1i)/(4*d^2*e))^(1/2)*1i + ((16*(e*cot(c + d*x))^(1/2)*(a^6*e^2 - b^6*e^2 + 15*a^2*b^4*e^2 - 15*a^4*b^2*e^2))/d^2 + (8*(4*a^3*d^2*e^3 - 12*a*b^2*d^2*e^3)*(((a*b^5*6i + a^5*b*6i + a^6 - b^6 + 15*a^2*b^4 - a^3*b^3*20i - 15*a^4*b^2)*1i)/(4*d^2*e))^(1/2))/d^3)*(((a*b^5*6i + a^5*b*6i + a^6 - b^6 + 15*a^2*b^4 - a^3*b^3*20i - 15*a^4*b^2)*1i)/(4*d^2*e))^(1/2)*1i)/(((16*(e*cot(c + d*x))^(1/2)*(a^6*e^2 - b^6*e^2 + 15*a^2*b^4*e^2 - 15*a^4*b^2*e^2))/d^2 + (8*(4*a^3*d^2*e^3 - 12*a*b^2*d^2*e^3)*(((a*b^5*6i + a^5*b*6i + a^6 - b^6 + 15*a^2*b^4 - a^3*b^3*20i - 15*a^4*b^2)*1i)/(4*d^2*e))^(1/2))/d^3)*(((a*b^5*6i + a^5*b*6i + a^6 - b^6 + 15*a^2*b^4 - a^3*b^3*20i - 15*a^4*b^2)*1i)/(4*d^2*e))^(1/2) - ((16*(e*cot(c + d*x))^(1/2)*(a^6*e^2 - b^6*e^2 + 15*a^2*b^4*e^2 - 15*a^4*b^2*e^2))/d^2 - (8*(4*a^3*d^2*e^3 - 12*a*b^2*d^2*e^3)*(((a*b^5*6i + a^5*b*6i + a^6 - b^6 + 15*a^2*b^4 - a^3*b^3*20i - 15*a^4*b^2)*1

```
i)/(4*d^2*e))^(1/2))/d^3)*(((a*b^5*6i + a^5*b*6i + a^6 - b^6 + 15*a^2*b^4 -
 a^3*b^3*20i - 15*a^4*b^2)*1i)/(4*d^2*e))^(1/2) + (16*(3*a^8*b*e^2 - b^9*e^
2 + 6*a^4*b^5*e^2 + 8*a^6*b^3*e^2))/d^3))*(((a*b^5*6i + a^5*b*6i + a^6 - b^
6 + 15*a^2*b^4 - a^3*b^3*20i - 15*a^4*b^2)*1i)/(4*d^2*e))^(1/2)*2i - (2*b^3
*(e*cot(c + d*x))^(3/2))/(3*d*e^2) - (6*a*b^2*(e*cot(c + d*x))^(1/2))/(d*e)
```

3.65 $\int \frac{(a+b\cot(c+dx))^3}{(e\cot(c+dx))^{3/2}}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 313

$$\begin{aligned}\int \frac{(a+b\cot(c+dx))^3}{(e\cot(c+dx))^{3/2}}\,dx =& -\frac{(a+b)\left(a^2-4ab+b^2\right)\arctan\left(1-\frac{\sqrt{2}\sqrt{e\cot(c+dx)}}{\sqrt{e}}\right)}{\sqrt{2}de^{3/2}} \\ &+\frac{(a+b)\left(a^2-4ab+b^2\right)\arctan\left(1+\frac{\sqrt{2}\sqrt{e\cot(c+dx)}}{\sqrt{e}}\right)}{\sqrt{2}de^{3/2}} \\ &-\frac{2b(a^2+b^2)\sqrt{e\cot(c+dx)}}{de^2}+\frac{2a^2(a+b\cot(c+dx))}{de\sqrt{e\cot(c+dx)}} \\ &+\frac{(a-b)\left(a^2+4ab+b^2\right)\log\left(\sqrt{e}+\sqrt{e}\cot(c+dx)-\sqrt{2}\sqrt{e\cot(c+dx)}\right)}{2\sqrt{2}de^{3/2}} \\ &-\frac{(a-b)\left(a^2+4ab+b^2\right)\log\left(\sqrt{e}+\sqrt{e}\cot(c+dx)+\sqrt{2}\sqrt{e\cot(c+dx)}\right)}{2\sqrt{2}de^{3/2}}\end{aligned}$$

```
[Out] -1/2*(a+b)*(a^2-4*a*b+b^2)*arctan(1-2^(1/2)*(e*cot(d*x+c))^(1/2)/e^(1/2))/d
/e^(3/2)*2^(1/2)+1/2*(a+b)*(a^2-4*a*b+b^2)*arctan(1+2^(1/2)*(e*cot(d*x+c))^
(1/2)/e^(1/2))/d/e^(3/2)*2^(1/2)+1/4*(a-b)*(a^2+4*a*b+b^2)*ln(e^(1/2)+cot(d
*x+c)*e^(1/2)-2^(1/2)*(e*cot(d*x+c))^(1/2))/d/e^(3/2)*2^(1/2)-1/4*(a-b)*(a^
2+4*a*b+b^2)*ln(e^(1/2)+cot(d*x+c)*e^(1/2)+2^(1/2)*(e*cot(d*x+c))^(1/2))/d/
e^(3/2)*2^(1/2)+2*a^2*(a+b*cot(d*x+c))/d/e/(e*cot(d*x+c))^(1/2)-2*b*(a^2+b^
2)*(e*cot(d*x+c))^(1/2)/d/e^2
```

Rubi [A] (verified)

Time = 0.48 (sec) , antiderivative size = 313, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.360$, Rules used = {3646, 3711, 3615, 1182, 1176, 631, 210, 1179, 642}

$$\int \frac{(a+b\cot(c+dx))^3}{(e\cot(c+dx))^{3/2}}\,dx = -\frac{(a+b)\left(a^2-4ab+b^2\right)\arctan\left(1-\frac{\sqrt{2}\sqrt{e\cot(c+dx)}}{\sqrt{e}}\right)}{\sqrt{2}de^{3/2}} + \frac{(a+b)\left(a^2-4ab+b^2\right)\arctan\left(\frac{\sqrt{2}\sqrt{e\cot(c+dx)}}{\sqrt{e}}+1\right)}{\sqrt{2}de^{3/2}} + \frac{(a-b)\left(a^2+4ab+b^2\right)\log\left(\sqrt{e}\cot(c+dx)-\sqrt{2}\sqrt{e\cot(c+dx)}+\sqrt{e}\right)}{2\sqrt{2}de^{3/2}} - \frac{(a-b)\left(a^2+4ab+b^2\right)\log\left(\sqrt{e}\cot(c+dx)+\sqrt{2}\sqrt{e\cot(c+dx)}+\sqrt{e}\right)}{2\sqrt{2}de^{3/2}} - \frac{2b\left(a^2+b^2\right)\sqrt{e\cot(c+dx)}}{de^2} + \frac{2a^2(a+b\cot(c+dx))}{de\sqrt{e\cot(c+dx)}}$$

```
[In] Int[(a + b*Cot[c + d*x])^3/(e*Cot[c + d*x])^(3/2),x]
```

```
[Out] -(((a + b)*(a^2 - 4*a*b + b^2)*ArcTan[1 - (Sqrt[2]*Sqrt[e*Cot[c + d*x]])/Sq
rt[e]])/(Sqrt[2]*d*e^(3/2))) + ((a + b)*(a^2 - 4*a*b + b^2)*ArcTan[1 + (Sqr
t[2]*Sqrt[e*Cot[c + d*x]])/Sqrt[e]])/(Sqrt[2]*d*e^(3/2)) - (2*b*(a^2 + b^2)
*Sqrt[e*Cot[c + d*x]])/(d*e^2) + (2*a^2*(a + b*Cot[c + d*x]))/(d*e*Sqrt[e*C
ot[c + d*x]]) + ((a - b)*(a^2 + 4*a*b + b^2)*Log[Sqrt[e] + Sqrt[e]*Cot[c +
d*x] - Sqrt[2]*Sqrt[e*Cot[c + d*x]]])/(2*Sqrt[2]*d*e^(3/2)) - ((a - b)*(a^2
 + 4*a*b + b^2)*Log[Sqrt[e] + Sqrt[e]*Cot[c + d*x] + Sqrt[2]*Sqrt[e*Cot[c +
 d*x]]])/(2*Sqrt[2]*d*e^(3/2))
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1179

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 1182

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
a*c, 2]}, Dist[(d*q + a*e)/(2*a*c), Int[(q + c*x^2)/(a + c*x^4), x], x] + D
ist[(d*q - a*e)/(2*a*c), Int[(q - c*x^2)/(a + c*x^4), x], x]] /; FreeQ[{a,
c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0] && NeQ[c*d^2 - a*e^2, 0] && NegQ[(-a)
*c]
```

Rule 3615

```
Int[((c_) + (d_.)*tan[(e_.) + (f_.)*(x_)])/Sqrt[(b_.)*tan[(e_.) + (f_.)*(x_
)]], x_Symbol] :> Dist[2/f, Subst[Int[(b*c + d*x^2)/(b^2 + x^4), x], x, Sqr
t[b*Tan[e + f*x]]], x] /; FreeQ[{b, c, d, e, f}, x] && NeQ[c^2 - d^2, 0] &&
 NeQ[c^2 + d^2, 0]
```

Rule 3646

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*c - a*d)^2*(a + b*Tan[e + f*x])^(m
 - 2)*((c + d*Tan[e + f*x])^(n + 1)/(d*f*(n + 1)*(c^2 + d^2))), x] - Dist[1
/(d*(n + 1)*(c^2 + d^2)), Int[(a + b*Tan[e + f*x])^(m - 3)*(c + d*Tan[e + f
*x])^(n + 1)*Simp[a^2*d*(b*d*(m - 2) - a*c*(n + 1)) + b*(b*c - 2*a*d)*(b*c*
(m - 2) + a*d*(n + 1)) - d*(n + 1)*(3*a^2*b*c - b^3*c - a^3*d + 3*a*b^2*d)*
Tan[e + f*x] - b*(a*d*(2*b*c - a*d)*(m + n - 1) - b^2*(c^2*(m - 2) - d^2*(n
 + 1)))*Tan[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b
*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] && GtQ[m, 2] && LtQ[
n, -1] && IntegerQ[2*m]
```

Rule 3711

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*tan[(e_.)
+ (f_.)*(x_)] + (C_.)*tan[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[C*((a +
 b*Tan[e + f*x])^(m + 1)/(b*f*(m + 1))), x] + Int[(a + b*Tan[e + f*x])^m*Si
mp[A - C + B*Tan[e + f*x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] &&
NeQ[A*b^2 - a*b*B + a^2*C, 0] &&  !LeQ[m, -1]
```

Rubi steps

$$\text{integral} = \frac{2a^2(a+b\cot(c+dx))}{de\sqrt{e\cot(c+dx)}} - \frac{2\int \frac{-2a^2be^2+\frac{1}{2}a(a^2-3b^2)e^2\cot(c+dx)-\frac{1}{2}b(a^2+b^2)e^2\cot^2(c+dx)}{\sqrt{e\cot(c+dx)}}\,dx}{e^3}$$

$$= -\frac{2b(a^2+b^2)\sqrt{e\cot(c+dx)}}{de^2} + \frac{2a^2(a+b\cot(c+dx))}{de\sqrt{e\cot(c+dx)}} - \frac{2\int \frac{-\frac{1}{2}b(3a^2-b^2)e^2+\frac{1}{2}a(a^2-3b^2)e^2\cot(c+dx)}{\sqrt{e\cot(c+dx)}}\,dx}{e^3}$$

$$= -\frac{2b(a^2+b^2)\sqrt{e\cot(c+dx)}}{de^2} + \frac{2a^2(a+b\cot(c+dx))}{de\sqrt{e\cot(c+dx)}} - \frac{4\text{Subst}\left(\int \frac{\frac{1}{2}b(3a^2-b^2)e^3-\frac{1}{2}a(a^2-3b^2)e^2x^2}{e^2+x^4}\,dx, x, \sqrt{e\cot(c+dx)}\right)}{de^3}$$

$$= -\frac{2b(a^2+b^2)\sqrt{e\cot(c+dx)}}{de^2} + \frac{2a^2(a+b\cot(c+dx))}{de\sqrt{e\cot(c+dx)}} + \frac{((a+b)(a^2-4ab+b^2))\,\text{Subst}\left(\int \frac{e+x^2}{e^2+x^4}\,dx, x, \sqrt{e\cot(c+dx)}\right)}{de} - \frac{((a-b)(a^2+4ab+b^2))\,\text{Subst}\left(\int \frac{e-x^2}{e^2+x^4}\,dx, x, \sqrt{e\cot(c+dx)}\right)}{de}$$

$$= -\frac{2b(a^2+b^2)\sqrt{e\cot(c+dx)}}{de^2} + \frac{2a^2(a+b\cot(c+dx))}{de\sqrt{e\cot(c+dx)}} + \frac{((a-b)(a^2+4ab+b^2))\,\text{Subst}\left(\int \frac{\sqrt{2}\sqrt{e}+2x}{-e-\sqrt{2}\sqrt{e}x-x^2}\,dx, x, \sqrt{e\cot(c+dx)}\right)}{2\sqrt{2}de^{3/2}} + \frac{((a-b)(a^2+4ab+b^2))\,\text{Subst}\left(\int \frac{\sqrt{2}\sqrt{e}-2x}{-e+\sqrt{2}\sqrt{e}x-x^2}\,dx, x, \sqrt{e\cot(c+dx)}\right)}{2\sqrt{2}de^{3/2}} + \frac{((a+b)(a^2-4ab+b^2))\,\text{Subst}\left(\int \frac{1}{e-\sqrt{2}\sqrt{e}x+x^2}\,dx, x, \sqrt{e\cot(c+dx)}\right)}{2de} + \frac{((a+b)(a^2-4ab+b^2))\,\text{Subst}\left(\int \frac{1}{e+\sqrt{2}\sqrt{e}x+x^2}\,dx, x, \sqrt{e\cot(c+dx)}\right)}{2de}$$

$$= -\frac{2b(a^2+b^2)\sqrt{e\cot(c+dx)}}{de^2} + \frac{2a^2(a+b\cot(c+dx))}{de\sqrt{e\cot(c+dx)}}$$
$$+ \frac{(a-b)(a^2+4ab+b^2)\log\left(\sqrt{e}+\sqrt{e}\cot(c+dx)-\sqrt{2}\sqrt{e\cot(c+dx)}\right)}{2\sqrt{2}de^{3/2}}$$
$$- \frac{(a-b)(a^2+4ab+b^2)\log\left(\sqrt{e}+\sqrt{e}\cot(c+dx)+\sqrt{2}\sqrt{e\cot(c+dx)}\right)}{2\sqrt{2}de^{3/2}}$$
$$+ \frac{((a+b)(a^2-4ab+b^2))\,\mathrm{Subst}\left(\int \frac{1}{-1-x^2}\,dx, x, 1-\frac{\sqrt{2}\sqrt{e\cot(c+dx)}}{\sqrt{e}}\right)}{\sqrt{2}de^{3/2}}$$
$$- \frac{((a+b)(a^2-4ab+b^2))\,\mathrm{Subst}\left(\int \frac{1}{-1-x^2}\,dx, x, 1+\frac{\sqrt{2}\sqrt{e\cot(c+dx)}}{\sqrt{e}}\right)}{\sqrt{2}de^{3/2}}$$
$$= -\frac{(a+b)(a^2-4ab+b^2)\arctan\left(1-\frac{\sqrt{2}\sqrt{e\cot(c+dx)}}{\sqrt{e}}\right)}{\sqrt{2}de^{3/2}}$$
$$+ \frac{(a+b)(a^2-4ab+b^2)\arctan\left(1+\frac{\sqrt{2}\sqrt{e\cot(c+dx)}}{\sqrt{e}}\right)}{\sqrt{2}de^{3/2}}$$
$$- \frac{2b(a^2+b^2)\sqrt{e\cot(c+dx)}}{de^2} + \frac{2a^2(a+b\cot(c+dx))}{de\sqrt{e\cot(c+dx)}}$$
$$+ \frac{(a-b)(a^2+4ab+b^2)\log\left(\sqrt{e}+\sqrt{e}\cot(c+dx)-\sqrt{2}\sqrt{e\cot(c+dx)}\right)}{2\sqrt{2}de^{3/2}}$$
$$- \frac{(a-b)(a^2+4ab+b^2)\log\left(\sqrt{e}+\sqrt{e}\cot(c+dx)+\sqrt{2}\sqrt{e\cot(c+dx)}\right)}{2\sqrt{2}de^{3/2}}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 3.55 (sec) , antiderivative size = 193, normalized size of antiderivative = 0.62

$$\int \frac{(a+b\cot(c+dx))^3}{(e\cot(c+dx))^{3/2}}\,dx =$$
$$-\frac{-24ab^2+8b^3\cot(c+dx)-8a(a^2-3b^2)\,\mathrm{Hypergeometric2F1}\left(-\frac{1}{4},1,\frac{3}{4},-\cot^2(c+dx)\right)+\sqrt{2}b(-3a^2+b^2)}{}$$

```
[In] Integrate[(a + b*Cot[c + d*x])^3/(e*Cot[c + d*x])^(3/2),x]
[Out] -1/4*(-24*a*b^2 + 8*b^3*Cot[c + d*x] - 8*a*(a^2 - 3*b^2)*Hypergeometric2F1[
-1/4, 1, 3/4, -Cot[c + d*x]^2] + Sqrt[2]*b*(-3*a^2 + b^2)*Sqrt[Cot[c + d*x]
]*(2*ArcTan[1 - Sqrt[2]*Sqrt[Cot[c + d*x]]] - 2*ArcTan[1 + Sqrt[2]*Sqrt[Cot
[c + d*x]]] + Log[1 - Sqrt[2]*Sqrt[Cot[c + d*x]] + Cot[c + d*x]] - Log[1 +
Sqrt[2]*Sqrt[Cot[c + d*x]] + Cot[c + d*x]]))/(d*e*Sqrt[e*Cot[c + d*x]])
```

Maple [A] (verified)

Time = 0.04 (sec) , antiderivative size = 332, normalized size of antiderivative = 1.06

method	result
derivativedivides	$-\frac{2\left(\sqrt{e\cot(dx+c)}\,b^{3}-e\left(\frac{\left(-3a^{2}be+b^{3}e\right)\left(e^{2}\right)^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\frac{e\cot(dx+c)+\left(e^{2}\right)^{\frac{1}{4}}\sqrt{e\cot(dx+c)}\,\sqrt{2}+\sqrt{e^{2}}}{e\cot(dx+c)-\left(e^{2}\right)^{\frac{1}{4}}\sqrt{e\cot(dx+c)}\,\sqrt{2}+\sqrt{e^{2}}}\right)+2\arctan\left(\frac{\sqrt{2}\,\sqrt{e\cot(d}}{\left(e^{2}\right)^{\frac{1}{4}}}\cdots\right.\right.}{8e^{2}}\cdots\right.\right.}{\cdots}$
default	$-\frac{2\left(\sqrt{e\cot(dx+c)}\,b^{3}-e\left(\frac{\left(-3a^{2}be+b^{3}e\right)\left(e^{2}\right)^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\frac{e\cot(dx+c)+\left(e^{2}\right)^{\frac{1}{4}}\sqrt{e\cot(dx+c)}\,\sqrt{2}+\sqrt{e^{2}}}{e\cot(dx+c)-\left(e^{2}\right)^{\frac{1}{4}}\sqrt{e\cot(dx+c)}\,\sqrt{2}+\sqrt{e^{2}}}\right)+2\arctan\left(\frac{\sqrt{2}\,\sqrt{e\cot(d}}{\left(e^{2}\right)^{\frac{1}{4}}}\cdots\right.\right.}{8e^{2}}\cdots\right.\right.}{\cdots}$
parts	$-\frac{2a^{3}e\left(-\frac{\sqrt{2}\left(\ln\left(\frac{e\cot(dx+c)-\left(e^{2}\right)^{\frac{1}{4}}\sqrt{e\cot(dx+c)}\,\sqrt{2}+\sqrt{e^{2}}}{e\cot(dx+c)+\left(e^{2}\right)^{\frac{1}{4}}\sqrt{e\cot(dx+c)}\,\sqrt{2}+\sqrt{e^{2}}}\right)+2\arctan\left(\frac{\sqrt{2}\,\sqrt{e\cot(dx+c)}}{\left(e^{2}\right)^{\frac{1}{4}}}+1\right)-2\arctan\left(-\frac{\sqrt{2}\,\sqrt{e\cot(dx+c)}}{\left(e^{2}\right)^{\frac{1}{4}}}\cdots\right.\right.}{8e^{2}\left(e^{2}\right)^{\frac{1}{4}}}\cdots\right.}{d}$

```
[In] int((a+b*cot(d*x+c))^3/(e*cot(d*x+c))^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] -2/d/e^2*((e*cot(d*x+c))^(1/2)*b^3-e*(1/8*(-3*a^2*b*e+b^3*e)*(e^2)^(1/4)/e^
2*2^(1/2)*(ln((e*cot(d*x+c)+(e^2)^(1/4)*(e*cot(d*x+c))^(1/2)*2^(1/2)+(e^2)^
(1/2))/(e*cot(d*x+c)-(e^2)^(1/4)*(e*cot(d*x+c))^(1/2)*2^(1/2)+(e^2)^(1/2)))
+2*arctan(2^(1/2)/(e^2)^(1/4)*(e*cot(d*x+c))^(1/2)+1)-2*arctan(-2^(1/2)/(e^
2)^(1/4)*(e*cot(d*x+c))^(1/2)+1))+1/8*(a^3-3*a*b^2)/(e^2)^(1/4)*2^(1/2)*(ln
((e*cot(d*x+c)-(e^2)^(1/4)*(e*cot(d*x+c))^(1/2)*2^(1/2)+(e^2)^(1/2))/(e*cot
(d*x+c)+(e^2)^(1/4)*(e*cot(d*x+c))^(1/2)*2^(1/2)+(e^2)^(1/2)))+2*arctan(2^(
1/2)/(e^2)^(1/4)*(e*cot(d*x+c))^(1/2)+1)-2*arctan(-2^(1/2)/(e^2)^(1/4)*(e*c
ot(d*x+c))^(1/2)+1)))-a^3*e/(e*cot(d*x+c))^(1/2))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1679 vs. 2(262) = 524.

Time = 0.38 (sec) , antiderivative size = 1679, normalized size of antiderivative = 5.36

$$\int \frac{(a+b\cot(c+dx))^3}{(e\cot(c+dx))^{3/2}}\,dx = \text{Too large to display}$$

```
[In] integrate((a+b*cot(d*x+c))^3/(e*cot(d*x+c))^(3/2),x, algorithm="fricas")
```

```
[Out] -1/2*((d*e^2*cos(2*d*x + 2*c) + d*e^2)*sqrt((6*a^5*b - 20*a^3*b^3 + 6*a*b^5
 + d^2*e^3*sqrt(-(a^12 - 30*a^10*b^2 + 255*a^8*b^4 - 452*a^6*b^6 + 255*a^4*
```

b^8 - 30*a^2*b^10 + b^12)/(d^4*e^6)))/(d^2*e^3))*log(-(a^12 - 12*a^10*b^2 - 27*a^8*b^4 + 27*a^4*b^8 + 12*a^2*b^10 - b^12)*sqrt((e*cos(2*d*x + 2*c) + e)/sin(2*d*x + 2*c)) + ((a^3 - 3*a*b^2)*d^3*e^5*sqrt(-(a^12 - 30*a^10*b^2 + 255*a^8*b^4 - 452*a^6*b^6 + 255*a^4*b^8 - 30*a^2*b^10 + b^12)/(d^4*e^6)) - (3*a^8*b - 46*a^6*b^3 + 60*a^4*b^5 - 18*a^2*b^7 + b^9)*d*e^2)*sqrt((6*a^5*b - 20*a^3*b^3 + 6*a*b^5 + d^2*e^3*sqrt(-(a^12 - 30*a^10*b^2 + 255*a^8*b^4 - 452*a^6*b^6 + 255*a^4*b^8 - 30*a^2*b^10 + b^12)/(d^4*e^6)))/(d^2*e^3))) - (d*e^2*cos(2*d*x + 2*c) + d*e^2)*sqrt((6*a^5*b - 20*a^3*b^3 + 6*a*b^5 + d^2 *e^3*sqrt(-(a^12 - 30*a^10*b^2 + 255*a^8*b^4 - 452*a^6*b^6 + 255*a^4*b^8 - 30*a^2*b^10 + b^12)/(d^4*e^6)))/(d^2*e^3))*log(-(a^12 - 12*a^10*b^2 - 27*a^8*b^4 + 27*a^4*b^8 + 12*a^2*b^10 - b^12)*sqrt((e*cos(2*d*x + 2*c) + e)/sin(2*d*x + 2*c)) - ((a^3 - 3*a*b^2)*d^3*e^5*sqrt(-(a^12 - 30*a^10*b^2 + 255*a^8*b^4 - 452*a^6*b^6 + 255*a^4*b^8 - 30*a^2*b^10 + b^12)/(d^4*e^6)) - (3*a^8 *b - 46*a^6*b^3 + 60*a^4*b^5 - 18*a^2*b^7 + b^9)*d*e^2)*sqrt((6*a^5*b - 20* a^3*b^3 + 6*a*b^5 + d^2*e^3*sqrt(-(a^12 - 30*a^10*b^2 + 255*a^8*b^4 - 452*a ^6*b^6 + 255*a^4*b^8 - 30*a^2*b^10 + b^12)/(d^4*e^6)))/(d^2*e^3))) - (d*e^2 *cos(2*d*x + 2*c) + d*e^2)*sqrt((6*a^5*b - 20*a^3*b^3 + 6*a*b^5 - d^2*e^3*s qrt(-(a^12 - 30*a^10*b^2 + 255*a^8*b^4 - 452*a^6*b^6 + 255*a^4*b^8 - 30*a^2 *b^10 + b^12)/(d^4*e^6)))/(d^2*e^3))*log(-(a^12 - 12*a^10*b^2 - 27*a^8*b^4 + 27*a^4*b^8 + 12*a^2*b^10 - b^12)*sqrt((e*cos(2*d*x + 2*c) + e)/sin(2*d*x + 2*c)) + ((a^3 - 3*a*b^2)*d^3*e^5*sqrt(-(a^12 - 30*a^10*b^2 + 255*a^8*b^4 - 452*a^6*b^6 + 255*a^4*b^8 - 30*a^2*b^10 + b^12)/(d^4*e^6)) + (3*a^8*b - 4 6*a^6*b^3 + 60*a^4*b^5 - 18*a^2*b^7 + b^9)*d*e^2)*sqrt((6*a^5*b - 20*a^3*b^ 3 + 6*a*b^5 - d^2*e^3*sqrt(-(a^12 - 30*a^10*b^2 + 255*a^8*b^4 - 452*a^6*b^6 + 255*a^4*b^8 - 30*a^2*b^10 + b^12)/(d^4*e^6)))/(d^2*e^3))) + (d*e^2*cos(2 *d*x + 2*c) + d*e^2)*sqrt((6*a^5*b - 20*a^3*b^3 + 6*a*b^5 - d^2*e^3*sqrt(-(a^12 - 30*a^10*b^2 + 255*a^8*b^4 - 452*a^6*b^6 + 255*a^4*b^8 - 30*a^2*b^10 + b^12)/(d^4*e^6)))/(d^2*e^3))*log(-(a^12 - 12*a^10*b^2 - 27*a^8*b^4 + 27*a ^4*b^8 + 12*a^2*b^10 - b^12)*sqrt((e*cos(2*d*x + 2*c) + e)/sin(2*d*x + 2*c)) - ((a^3 - 3*a*b^2)*d^3*e^5*sqrt(-(a^12 - 30*a^10*b^2 + 255*a^8*b^4 - 452* a^6*b^6 + 255*a^4*b^8 - 30*a^2*b^10 + b^12)/(d^4*e^6)) + (3*a^8*b - 46*a^6* b^3 + 60*a^4*b^5 - 18*a^2*b^7 + b^9)*d*e^2)*sqrt((6*a^5*b - 20*a^3*b^3 + 6* a*b^5 - d^2*e^3*sqrt(-(a^12 - 30*a^10*b^2 + 255*a^8*b^4 - 452*a^6*b^6 + 255 *a^4*b^8 - 30*a^2*b^10 + b^12)/(d^4*e^6)))/(d^2*e^3))) + 4*(b^3*cos(2*d*x + 2*c) - a^3*sin(2*d*x + 2*c) + b^3)*sqrt((e*cos(2*d*x + 2*c) + e)/sin(2*d*x + 2*c)))/(d*e^2*cos(2*d*x + 2*c) + d*e^2)

Sympy [F]

$$\int \frac{(a+b\cot(c+dx))^3}{(e\cot(c+dx))^{3/2}}\,dx = \int \frac{(a+b\cot{(c+dx)})^3}{(e\cot{(c+dx)})^{\frac{3}{2}}}\,dx$$

```
[In] integrate((a+b*cot(d*x+c))**3/(e*cot(d*x+c))**(3/2),x)
[Out] Integral((a + b*cot(c + d*x))**3/(e*cot(c + d*x))**(3/2), x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{(a+b\cot(c+dx))^3}{(e\cot(c+dx))^{3/2}}\,dx = \text{Exception raised: ValueError}$$

```
[In] integrate((a+b*cot(d*x+c))^3/(e*cot(d*x+c))^(3/2),x, algorithm="maxima")
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(e>0)', see `assume?` for more detai
ls)Is e
```

Giac [F]

$$\int \frac{(a+b\cot(c+dx))^3}{(e\cot(c+dx))^{3/2}}\,dx = \int \frac{(b\cot{(dx+c)}+a)^3}{(e\cot{(dx+c)})^{\frac{3}{2}}}\,dx$$

```
[In] integrate((a+b*cot(d*x+c))^3/(e*cot(d*x+c))^(3/2),x, algorithm="giac")
[Out] integrate((b*cot(d*x + c) + a)^3/(e*cot(d*x + c))^(3/2), x)
```

Mupad [B] (verification not implemented)

Time = 13.43 (sec) , antiderivative size = 1951, normalized size of antiderivative = 6.23

$$\int \frac{(a+b\cot(c+dx))^3}{(e\cot(c+dx))^{3/2}}\,dx = \text{Too large to display}$$

```
[In] int((a + b*cot(c + d*x))^3/(e*cot(c + d*x))^(3/2),x)
[Out] (2*a^3)/(d*e*(e*cot(c + d*x))^(1/2)) - atan((((e*cot(c + d*x))^(1/2)*(16*a^
6*d^3*e^5 - 16*b^6*d^3*e^5 + 240*a^2*b^4*d^3*e^5 - 240*a^4*b^2*d^3*e^5) + (
```

```
32*b^3*d^4*e^7 - 96*a^2*b*d^4*e^7)*(-((a*b^5*6i + a^5*b*6i + a^6 - b^6 + 15
*a^2*b^4 - a^3*b^3*20i - 15*a^4*b^2)*1i)/(4*d^2*e^3))^(1/2))*(-((a*b^5*6i +
 a^5*b*6i + a^6 - b^6 + 15*a^2*b^4 - a^3*b^3*20i - 15*a^4*b^2)*1i)/(4*d^2*e
^3))^(1/2)*1i + ((e*cot(c + d*x))^(1/2)*(16*a^6*d^3*e^5 - 16*b^6*d^3*e^5 +
240*a^2*b^4*d^3*e^5 - 240*a^4*b^2*d^3*e^5) - (32*b^3*d^4*e^7 - 96*a^2*b*d^4
*e^7)*(-((a*b^5*6i + a^5*b*6i + a^6 - b^6 + 15*a^2*b^4 - a^3*b^3*20i - 15*a
^4*b^2)*1i)/(4*d^2*e^3))^(1/2))*(-((a*b^5*6i + a^5*b*6i + a^6 - b^6 + 15*a^
2*b^4 - a^3*b^3*20i - 15*a^4*b^2)*1i)/(4*d^2*e^3))^(1/2)*1i)/(((e*cot(c + d
*x))^(1/2)*(16*a^6*d^3*e^5 - 16*b^6*d^3*e^5 + 240*a^2*b^4*d^3*e^5 - 240*a^4
*b^2*d^3*e^5) - (32*b^3*d^4*e^7 - 96*a^2*b*d^4*e^7)*(-((a*b^5*6i + a^5*b*6i
 + a^6 - b^6 + 15*a^2*b^4 - a^3*b^3*20i - 15*a^4*b^2)*1i)/(4*d^2*e^3))^(1/2
))*(-((a*b^5*6i + a^5*b*6i + a^6 - b^6 + 15*a^2*b^4 - a^3*b^3*20i - 15*a^4*
b^2)*1i)/(4*d^2*e^3))^(1/2) - ((e*cot(c + d*x))^(1/2)*(16*a^6*d^3*e^5 - 16*
b^6*d^3*e^5 + 240*a^2*b^4*d^3*e^5 - 240*a^4*b^2*d^3*e^5) + (32*b^3*d^4*e^7
- 96*a^2*b*d^4*e^7)*(-((a*b^5*6i + a^5*b*6i + a^6 - b^6 + 15*a^2*b^4 - a^3*
b^3*20i - 15*a^4*b^2)*1i)/(4*d^2*e^3))^(1/2))*(-((a*b^5*6i + a^5*b*6i + a^6
 - b^6 + 15*a^2*b^4 - a^3*b^3*20i - 15*a^4*b^2)*1i)/(4*d^2*e^3))^(1/2) - 16
*a^9*d^2*e^4 + 48*a*b^8*d^2*e^4 + 128*a^3*b^6*d^2*e^4 + 96*a^5*b^4*d^2*e^4)
)*(-((a*b^5*6i + a^5*b*6i + a^6 - b^6 + 15*a^2*b^4 - a^3*b^3*20i - 15*a^4*b
^2)*1i)/(4*d^2*e^3))^(1/2)*2i - atan((((e*cot(c + d*x))^(1/2)*(16*a^6*d^3*e
^5 - 16*b^6*d^3*e^5 + 240*a^2*b^4*d^3*e^5 - 240*a^4*b^2*d^3*e^5) + (32*b^3*
d^4*e^7 - 96*a^2*b*d^4*e^7)*(-((a*b^5*6i + a^5*b*6i - a^6 + b^6 - 15*a^2*b^
4 - a^3*b^3*20i + 15*a^4*b^2)*1i)/(4*d^2*e^3))^(1/2))*(-((a*b^5*6i + a^5*b*
6i - a^6 + b^6 - 15*a^2*b^4 - a^3*b^3*20i + 15*a^4*b^2)*1i)/(4*d^2*e^3))^(1
/2)*1i + ((e*cot(c + d*x))^(1/2)*(16*a^6*d^3*e^5 - 16*b^6*d^3*e^5 + 240*a^2
*b^4*d^3*e^5 - 240*a^4*b^2*d^3*e^5) - (32*b^3*d^4*e^7 - 96*a^2*b*d^4*e^7)*(
-((a*b^5*6i + a^5*b*6i - a^6 + b^6 - 15*a^2*b^4 - a^3*b^3*20i + 15*a^4*b^2)
*1i)/(4*d^2*e^3))^(1/2))*(-((a*b^5*6i + a^5*b*6i - a^6 + b^6 - 15*a^2*b^4 -
 a^3*b^3*20i + 15*a^4*b^2)*1i)/(4*d^2*e^3))^(1/2)*1i)/(((e*cot(c + d*x))^(1
/2)*(16*a^6*d^3*e^5 - 16*b^6*d^3*e^5 + 240*a^2*b^4*d^3*e^5 - 240*a^4*b^2*d^
3*e^5) - (32*b^3*d^4*e^7 - 96*a^2*b*d^4*e^7)*(-((a*b^5*6i + a^5*b*6i - a^6
+ b^6 - 15*a^2*b^4 - a^3*b^3*20i + 15*a^4*b^2)*1i)/(4*d^2*e^3))^(1/2))*(-((
a*b^5*6i + a^5*b*6i - a^6 + b^6 - 15*a^2*b^4 - a^3*b^3*20i + 15*a^4*b^2)*1i
)/(4*d^2*e^3))^(1/2) - ((e*cot(c + d*x))^(1/2)*(16*a^6*d^3*e^5 - 16*b^6*d^3
*e^5 + 240*a^2*b^4*d^3*e^5 - 240*a^4*b^2*d^3*e^5) + (32*b^3*d^4*e^7 - 96*a^
2*b*d^4*e^7)*(-((a*b^5*6i + a^5*b*6i - a^6 + b^6 - 15*a^2*b^4 - a^3*b^3*20i
 + 15*a^4*b^2)*1i)/(4*d^2*e^3))^(1/2))*(-((a*b^5*6i + a^5*b*6i - a^6 + b^6
- 15*a^2*b^4 - a^3*b^3*20i + 15*a^4*b^2)*1i)/(4*d^2*e^3))^(1/2) - 16*a^9*d^
2*e^4 + 48*a*b^8*d^2*e^4 + 128*a^3*b^6*d^2*e^4 + 96*a^5*b^4*d^2*e^4))*(-((a
*b^5*6i + a^5*b*6i - a^6 + b^6 - 15*a^2*b^4 - a^3*b^3*20i + 15*a^4*b^2)*1i)
/(4*d^2*e^3))^(1/2)*2i - (2*b^3*(e*cot(c + d*x))^(1/2))/(d*e^2)
```

3.66 $\int \frac{(a+b\cot(c+dx))^3}{(e\cot(c+dx))^{5/2}}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 313

$$\int \frac{(a+b\cot(c+dx))^3}{(e\cot(c+dx))^{5/2}}\,dx = -\frac{(a-b)\left(a^2+4ab+b^2\right)\arctan\left(1-\frac{\sqrt{2}\sqrt{e\cot(c+dx)}}{\sqrt{e}}\right)}{\sqrt{2}de^{5/2}}$$
$$+\frac{(a-b)\left(a^2+4ab+b^2\right)\arctan\left(1+\frac{\sqrt{2}\sqrt{e\cot(c+dx)}}{\sqrt{e}}\right)}{\sqrt{2}de^{5/2}}$$
$$+\frac{16a^2b}{3de^2\sqrt{e\cot(c+dx)}}+\frac{2a^2(a+b\cot(c+dx))}{3de(e\cot(c+dx))^{3/2}}$$
$$-\frac{(a+b)\left(a^2-4ab+b^2\right)\log\left(\sqrt{e}+\sqrt{e}\cot(c+dx)-\sqrt{2}\sqrt{e\cot(c+dx)}\right)}{2\sqrt{2}de^{5/2}}$$
$$+\frac{(a+b)\left(a^2-4ab+b^2\right)\log\left(\sqrt{e}+\sqrt{e}\cot(c+dx)+\sqrt{2}\sqrt{e\cot(c+dx)}\right)}{2\sqrt{2}de^{5/2}}$$

```
[Out] 2/3*a^2*(a+b*cot(d*x+c))/d/e/(e*cot(d*x+c))^(3/2)-1/2*(a-b)*(a^2+4*a*b+b^2)
*arctan(1-2^(1/2)*(e*cot(d*x+c))^(1/2)/e^(1/2))/d/e^(5/2)*2^(1/2)+1/2*(a-b)
*(a^2+4*a*b+b^2)*arctan(1+2^(1/2)*(e*cot(d*x+c))^(1/2)/e^(1/2))/d/e^(5/2)*2
^(1/2)-1/4*(a+b)*(a^2-4*a*b+b^2)*ln(e^(1/2)+cot(d*x+c)*e^(1/2)-2^(1/2)*(e*c
ot(d*x+c))^(1/2))/d/e^(5/2)*2^(1/2)+1/4*(a+b)*(a^2-4*a*b+b^2)*ln(e^(1/2)+co
t(d*x+c)*e^(1/2)+2^(1/2)*(e*cot(d*x+c))^(1/2))/d/e^(5/2)*2^(1/2)+16/3*a^2*b
/d/e^2/(e*cot(d*x+c))^(1/2)
```

Rubi [A] (verified)

Time = 0.54 (sec) , antiderivative size = 313, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.360, Rules used = {3646, 3709, 3615, 1182, 1176, 631, 210, 1179, 642}

$$\int \frac{(a+b\cot(c+dx))^3}{(e\cot(c+dx))^{5/2}}\,dx = -\frac{(a-b)\left(a^2+4ab+b^2\right)\arctan\left(1-\frac{\sqrt{2}\sqrt{e\cot(c+dx)}}{\sqrt{e}}\right)}{\sqrt{2}de^{5/2}} + \frac{(a-b)\left(a^2+4ab+b^2\right)\arctan\left(\frac{\sqrt{2}\sqrt{e\cot(c+dx)}}{\sqrt{e}}+1\right)}{\sqrt{2}de^{5/2}} - \frac{(a+b)\left(a^2-4ab+b^2\right)\log\left(\sqrt{e}\cot(c+dx)-\sqrt{2}\sqrt{e\cot(c+dx)}+\sqrt{e}\right)}{2\sqrt{2}de^{5/2}} + \frac{(a+b)\left(a^2-4ab+b^2\right)\log\left(\sqrt{e}\cot(c+dx)+\sqrt{2}\sqrt{e\cot(c+dx)}+\sqrt{e}\right)}{2\sqrt{2}de^{5/2}} + \frac{16a^2b}{3de^2\sqrt{e\cot(c+dx)}} + \frac{2a^2(a+b\cot(c+dx))}{3de(e\cot(c+dx))^{3/2}}$$

```
[In] Int[(a + b*Cot[c + d*x])^3/(e*Cot[c + d*x])^(5/2),x]
```

```
[Out] -(((a - b)*(a^2 + 4*a*b + b^2)*ArcTan[1 - (Sqrt[2]*Sqrt[e*Cot[c + d*x]])/Sq
rt[e]])/(Sqrt[2]*d*e^(5/2))) + ((a - b)*(a^2 + 4*a*b + b^2)*ArcTan[1 + (Sqr
t[2]*Sqrt[e*Cot[c + d*x]])/Sqrt[e]])/(Sqrt[2]*d*e^(5/2)) + (16*a^2*b)/(3*d*
e^2*Sqrt[e*Cot[c + d*x]]) + (2*a^2*(a + b*Cot[c + d*x]))/(3*d*e*(e*Cot[c +
d*x])^(3/2)) - ((a + b)*(a^2 - 4*a*b + b^2)*Log[Sqrt[e] + Sqrt[e]*Cot[c + d
*x] - Sqrt[2]*Sqrt[e*Cot[c + d*x]]])/(2*Sqrt[2]*d*e^(5/2)) + ((a + b)*(a^2
- 4*a*b + b^2)*Log[Sqrt[e] + Sqrt[e]*Cot[c + d*x] + Sqrt[2]*Sqrt[e*Cot[c +
d*x]]])/(2*Sqrt[2]*d*e^(5/2))
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1179

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 1182

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
a*c, 2]}, Dist[(d*q + a*e)/(2*a*c), Int[(q + c*x^2)/(a + c*x^4), x], x] + D
ist[(d*q - a*e)/(2*a*c), Int[(q - c*x^2)/(a + c*x^4), x], x]] /; FreeQ[{a,
c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0] && NeQ[c*d^2 - a*e^2, 0] && NegQ[(-a)
*c]
```

Rule 3615

```
Int[((c_) + (d_.)*tan[(e_.) + (f_.)*(x_)])/Sqrt[(b_.)*tan[(e_.) + (f_.)*(x_
)]], x_Symbol] :> Dist[2/f, Subst[Int[(b*c + d*x^2)/(b^2 + x^4), x], x, Sqr
t[b*Tan[e + f*x]]], x] /; FreeQ[{b, c, d, e, f}, x] && NeQ[c^2 - d^2, 0] &&
 NeQ[c^2 + d^2, 0]
```

Rule 3646

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*c - a*d)^2*(a + b*Tan[e + f*x])^(m
 - 2)*((c + d*Tan[e + f*x])^(n + 1)/(d*f*(n + 1)*(c^2 + d^2))), x] - Dist[1
/(d*(n + 1)*(c^2 + d^2)), Int[(a + b*Tan[e + f*x])^(m - 3)*(c + d*Tan[e + f
*x])^(n + 1)*Simp[a^2*d*(b*d*(m - 2) - a*c*(n + 1)) + b*(b*c - 2*a*d)*(b*c*
(m - 2) + a*d*(n + 1)) - d*(n + 1)*(3*a^2*b*c - b^3*c - a^3*d + 3*a*b^2*d)*
Tan[e + f*x] - b*(a*d*(2*b*c - a*d)*(m + n - 1) - b^2*(c^2*(m - 2) - d^2*(n
 + 1)))*Tan[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b
*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] && GtQ[m, 2] && LtQ[
n, -1] && IntegerQ[2*m]
```

Rule 3709

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
 (f_.)*(x_)] + (C_.)*tan[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(A*b^2 -
 a*b*B + a^2*C)*((a + b*Tan[e + f*x])^(m + 1)/(b*f*(m + 1)*(a^2 + b^2))), x
] + Dist[1/(a^2 + b^2), Int[(a + b*Tan[e + f*x])^(m + 1)*Simp[b*B + a*(A -
C) - (A*b - a*B - b*C)*Tan[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B,
 C}, x] && NeQ[A*b^2 - a*b*B + a^2*C, 0] && LtQ[m, -1] && NeQ[a^2 + b^2, 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{2a^2(a+b\cot(c+dx))}{3de(e\cot(c+dx))^{3/2}} - \frac{2\int \frac{-4a^2be^2+\frac{3}{2}a(a^2-3b^2)e^2\cot(c+dx)+\frac{1}{2}b(a^2-3b^2)e^2\cot^2(c+dx)}{(e\cot(c+dx))^{3/2}}\,dx}{3e^3} \\
&= \frac{16a^2b}{3de^2\sqrt{e\cot(c+dx)}} + \frac{2a^2(a+b\cot(c+dx))}{3de(e\cot(c+dx))^{3/2}} - \frac{2\int \frac{\frac{3}{2}a(a^2-3b^2)e^3+\frac{3}{2}b(3a^2-b^2)e^3\cot(c+dx)}{\sqrt{e\cot(c+dx)}}\,dx}{3e^5} \\
&= \frac{16a^2b}{3de^2\sqrt{e\cot(c+dx)}} + \frac{2a^2(a+b\cot(c+dx))}{3de(e\cot(c+dx))^{3/2}} \\
&\quad - \frac{4\text{Subst}\left(\int \frac{-\frac{3}{2}a(a^2-3b^2)e^4-\frac{3}{2}b(3a^2-b^2)e^3x^2}{e^2+x^4}\,dx, x, \sqrt{e\cot(c+dx)}\right)}{3de^5} \\
&= \frac{16a^2b}{3de^2\sqrt{e\cot(c+dx)}} + \frac{2a^2(a+b\cot(c+dx))}{3de(e\cot(c+dx))^{3/2}} \\
&\quad + \frac{((a+b)(a^2-4ab+b^2))\,\text{Subst}\left(\int \frac{e-x^2}{e^2+x^4}\,dx, x, \sqrt{e\cot(c+dx)}\right)}{de^2} \\
&\quad + \frac{((a-b)(a^2+4ab+b^2))\,\text{Subst}\left(\int \frac{e+x^2}{e^2+x^4}\,dx, x, \sqrt{e\cot(c+dx)}\right)}{de^2} \\
&= \frac{16a^2b}{3de^2\sqrt{e\cot(c+dx)}} + \frac{2a^2(a+b\cot(c+dx))}{3de(e\cot(c+dx))^{3/2}} \\
&\quad - \frac{((a+b)(a^2-4ab+b^2))\,\text{Subst}\left(\int \frac{\sqrt{2}\sqrt{e}+2x}{-e-\sqrt{2}\sqrt{e}x-x^2}\,dx, x, \sqrt{e\cot(c+dx)}\right)}{2\sqrt{2}de^{5/2}} \\
&\quad - \frac{((a+b)(a^2-4ab+b^2))\,\text{Subst}\left(\int \frac{\sqrt{2}\sqrt{e}-2x}{-e+\sqrt{2}\sqrt{e}x-x^2}\,dx, x, \sqrt{e\cot(c+dx)}\right)}{2\sqrt{2}de^{5/2}} \\
&\quad + \frac{((a-b)(a^2+4ab+b^2))\,\text{Subst}\left(\int \frac{1}{e-\sqrt{2}\sqrt{e}x+x^2}\,dx, x, \sqrt{e\cot(c+dx)}\right)}{2de^2} \\
&\quad + \frac{((a-b)(a^2+4ab+b^2))\,\text{Subst}\left(\int \frac{1}{e+\sqrt{2}\sqrt{e}x+x^2}\,dx, x, \sqrt{e\cot(c+dx)}\right)}{2de^2}
\end{aligned}$$

$$= \frac{16a^2b}{3de^2\sqrt{e\cot(c+dx)}} + \frac{2a^2(a+b\cot(c+dx))}{3de(e\cot(c+dx))^{3/2}} - \frac{(a+b)\left(a^2-4ab+b^2\right)\log\left(\sqrt{e}+\sqrt{e}\cot(c+dx)-\sqrt{2}\sqrt{e\cot(c+dx)}\right)}{2\sqrt{2}de^{5/2}} + \frac{(a+b)\left(a^2-4ab+b^2\right)\log\left(\sqrt{e}+\sqrt{e}\cot(c+dx)+\sqrt{2}\sqrt{e\cot(c+dx)}\right)}{2\sqrt{2}de^{5/2}} + \frac{\left((a-b)\left(a^2+4ab+b^2\right)\right)\mathrm{Subst}\left(\int\frac{1}{-1-x^2}\,dx,x,1-\frac{\sqrt{2}\sqrt{e\cot(c+dx)}}{\sqrt{e}}\right)}{\sqrt{2}de^{5/2}} - \frac{\left((a-b)\left(a^2+4ab+b^2\right)\right)\mathrm{Subst}\left(\int\frac{1}{-1-x^2}\,dx,x,1+\frac{\sqrt{2}\sqrt{e\cot(c+dx)}}{\sqrt{e}}\right)}{\sqrt{2}de^{5/2}}$$

$$= -\frac{(a-b)\left(a^2+4ab+b^2\right)\arctan\left(1-\frac{\sqrt{2}\sqrt{e\cot(c+dx)}}{\sqrt{e}}\right)}{\sqrt{2}de^{5/2}} + \frac{(a-b)\left(a^2+4ab+b^2\right)\arctan\left(1+\frac{\sqrt{2}\sqrt{e\cot(c+dx)}}{\sqrt{e}}\right)}{\sqrt{2}de^{5/2}} + \frac{16a^2b}{3de^2\sqrt{e\cot(c+dx)}} + \frac{2a^2(a+b\cot(c+dx))}{3de(e\cot(c+dx))^{3/2}} - \frac{(a+b)\left(a^2-4ab+b^2\right)\log\left(\sqrt{e}+\sqrt{e}\cot(c+dx)-\sqrt{2}\sqrt{e\cot(c+dx)}\right)}{2\sqrt{2}de^{5/2}} + \frac{(a+b)\left(a^2-4ab+b^2\right)\log\left(\sqrt{e}+\sqrt{e}\cot(c+dx)+\sqrt{2}\sqrt{e\cot(c+dx)}\right)}{2\sqrt{2}de^{5/2}}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 0.42 (sec) , antiderivative size = 104, normalized size of antiderivative = 0.33

$$\int \frac{(a+b\cot(c+dx))^3}{(e\cot(c+dx))^{5/2}}\,dx = \frac{-6b(-3a^2+b^2)\,\mathrm{Hypergeometric2F1}\left(-\frac{1}{4},1,\frac{3}{4},-\cot^2(c+dx)\right)+2a(a^2-3b^2)}{3de^2\sqrt{e}}$$

```
[In] Integrate[(a + b*Cot[c + d*x])^3/(e*Cot[c + d*x])^(5/2),x]
```

```
[Out] (-6*b*(-3*a^2 + b^2)*Hypergeometric2F1[-1/4, 1, 3/4, -Cot[c + d*x]^2] + 2*a
*(a^2 - 3*b^2)*Hypergeometric2F1[-3/4, 1, 1/4, -Cot[c + d*x]^2]*Tan[c + d*x
] + 6*b^2*(b + a*Tan[c + d*x]))/(3*d*e^2*Sqrt[e*Cot[c + d*x]])
```

Maple [A] (verified)

Time = 0.03 (sec) , antiderivative size = 331, normalized size of antiderivative = 1.06

method	result
derivativedivides	$-\frac{2\left(\frac{\left(-a^3e+3aeb^2\right)\left(e^2\right)^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\frac{e\cot(dx+c)+\left(e^2\right)^{\frac{1}{4}}\sqrt{e\cot(dx+c)}\sqrt{2}+\sqrt{e^2}}{e\cot(dx+c)-\left(e^2\right)^{\frac{1}{4}}\sqrt{e\cot(dx+c)}\sqrt{2}+\sqrt{e^2}}\right)+2\arctan\left(\frac{\sqrt{2}\sqrt{e\cot(dx+c)}}{\left(e^2\right)^{\frac{1}{4}}}+1\right)-2\arctan\left(-\frac{\sqrt{2}\cdots}{\cdots}\right)\right)}{8e^2}\cdots\right)}{\cdots}$
default	$-\frac{2\left(\frac{\left(-a^3e+3aeb^2\right)\left(e^2\right)^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\frac{e\cot(dx+c)+\left(e^2\right)^{\frac{1}{4}}\sqrt{e\cot(dx+c)}\sqrt{2}+\sqrt{e^2}}{e\cot(dx+c)-\left(e^2\right)^{\frac{1}{4}}\sqrt{e\cot(dx+c)}\sqrt{2}+\sqrt{e^2}}\right)+2\arctan\left(\frac{\sqrt{2}\sqrt{e\cot(dx+c)}}{\left(e^2\right)^{\frac{1}{4}}}+1\right)-2\arctan\left(-\frac{\sqrt{2}\cdots}{\cdots}\right)\right)}{8e^2}\cdots\right)}{\cdots}$
parts	$-\frac{2a^3e\left(-\frac{\left(e^2\right)^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\frac{e\cot(dx+c)+\left(e^2\right)^{\frac{1}{4}}\sqrt{e\cot(dx+c)}\sqrt{2}+\sqrt{e^2}}{e\cot(dx+c)-\left(e^2\right)^{\frac{1}{4}}\sqrt{e\cot(dx+c)}\sqrt{2}+\sqrt{e^2}}\right)+2\arctan\left(\frac{\sqrt{2}\sqrt{e\cot(dx+c)}}{\left(e^2\right)^{\frac{1}{4}}}+1\right)-2\arctan\left(-\frac{\sqrt{2}\sqrt{e\cot(d\cdots}}{\left(e^2\right)^{\frac{1}{4}}}\cdots\right)\right)}{8e^4}\cdots\right)}{d}$

```
[In] int((a+b*cot(d*x+c))^3/(e*cot(d*x+c))^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] -2/d/e^2*(1/8*(-a^3*e+3*a*b^2*e)*(e^2)^(1/4)/e^2*2^(1/2)*(ln((e*cot(d*x+c)+
(e^2)^(1/4)*(e*cot(d*x+c))^(1/2)*2^(1/2)+(e^2)^(1/2))/(e*cot(d*x+c)-(e^2)^(
1/4)*(e*cot(d*x+c))^(1/2)*2^(1/2)+(e^2)^(1/2)))+2*arctan(2^(1/2)/(e^2)^(1/4
)*(e*cot(d*x+c))^(1/2)+1)-2*arctan(-2^(1/2)/(e^2)^(1/4)*(e*cot(d*x+c))^(1/2
)+1))+1/8*(-3*a^2*b+b^3)/(e^2)^(1/4)*2^(1/2)*(ln((e*cot(d*x+c)-(e^2)^(1/4)*
(e*cot(d*x+c))^(1/2)*2^(1/2)+(e^2)^(1/2))/(e*cot(d*x+c)+(e^2)^(1/4)*(e*cot(
d*x+c))^(1/2)*2^(1/2)+(e^2)^(1/2)))+2*arctan(2^(1/2)/(e^2)^(1/4)*(e*cot(d*x
+c))^(1/2)+1)-2*arctan(-2^(1/2)/(e^2)^(1/4)*(e*cot(d*x+c))^(1/2)+1))-1/3*a^
3*e/(e*cot(d*x+c))^(3/2)-3*a^2*b/(e*cot(d*x+c))^(1/2))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1692 vs. 2(258) = 516.

Time = 0.37 (sec) , antiderivative size = 1692, normalized size of antiderivative = 5.41

$$\int \frac{(a+b\cot(c+dx))^3}{(e\cot(c+dx))^{5/2}}\,dx = \text{Too large to display}$$

```
[In] integrate((a+b*cot(d*x+c))^3/(e*cot(d*x+c))^(5/2),x, algorithm="fricas")
```

```
[Out] -1/6*(3*(d*e^3*cos(2*d*x + 2*c) + d*e^3)*sqrt(-(d^2*e^5*sqrt(-(a^12 - 30*a^
10*b^2 + 255*a^8*b^4 - 452*a^6*b^6 + 255*a^4*b^8 - 30*a^2*b^10 + b^12)/(d^4
```

```
*e^10)) + 6*a^5*b - 20*a^3*b^3 + 6*a*b^5)/(d^2*e^5))*log(-(a^12 - 12*a^10*b
^2 - 27*a^8*b^4 + 27*a^4*b^8 + 12*a^2*b^10 - b^12)*sqrt((e*cos(2*d*x + 2*c)
 + e)/sin(2*d*x + 2*c)) + ((3*a^2*b - b^3)*d^3*e^8*sqrt(-(a^12 - 30*a^10*b^
2 + 255*a^8*b^4 - 452*a^6*b^6 + 255*a^4*b^8 - 30*a^2*b^10 + b^12)/(d^4*e^10
)) + (a^9 - 18*a^7*b^2 + 60*a^5*b^4 - 46*a^3*b^6 + 3*a*b^8)*d*e^3)*sqrt(-(d
^2*e^5*sqrt(-(a^12 - 30*a^10*b^2 + 255*a^8*b^4 - 452*a^6*b^6 + 255*a^4*b^8
- 30*a^2*b^10 + b^12)/(d^4*e^10)) + 6*a^5*b - 20*a^3*b^3 + 6*a*b^5)/(d^2*e^
5))) - 3*(d*e^3*cos(2*d*x + 2*c) + d*e^3)*sqrt(-(d^2*e^5*sqrt(-(a^12 - 30*a
^10*b^2 + 255*a^8*b^4 - 452*a^6*b^6 + 255*a^4*b^8 - 30*a^2*b^10 + b^12)/(d^
4*e^10)) + 6*a^5*b - 20*a^3*b^3 + 6*a*b^5)/(d^2*e^5))*log(-(a^12 - 12*a^10*
b^2 - 27*a^8*b^4 + 27*a^4*b^8 + 12*a^2*b^10 - b^12)*sqrt((e*cos(2*d*x + 2*c
) + e)/sin(2*d*x + 2*c)) - ((3*a^2*b - b^3)*d^3*e^8*sqrt(-(a^12 - 30*a^10*b
^2 + 255*a^8*b^4 - 452*a^6*b^6 + 255*a^4*b^8 - 30*a^2*b^10 + b^12)/(d^4*e^1
0)) + (a^9 - 18*a^7*b^2 + 60*a^5*b^4 - 46*a^3*b^6 + 3*a*b^8)*d*e^3)*sqrt(-(
d^2*e^5*sqrt(-(a^12 - 30*a^10*b^2 + 255*a^8*b^4 - 452*a^6*b^6 + 255*a^4*b^8
 - 30*a^2*b^10 + b^12)/(d^4*e^10)) + 6*a^5*b - 20*a^3*b^3 + 6*a*b^5)/(d^2*e
^5))) - 3*(d*e^3*cos(2*d*x + 2*c) + d*e^3)*sqrt((d^2*e^5*sqrt(-(a^12 - 30*a
^10*b^2 + 255*a^8*b^4 - 452*a^6*b^6 + 255*a^4*b^8 - 30*a^2*b^10 + b^12)/(d^
4*e^10)) - 6*a^5*b + 20*a^3*b^3 - 6*a*b^5)/(d^2*e^5))*log(-(a^12 - 12*a^10*
b^2 - 27*a^8*b^4 + 27*a^4*b^8 + 12*a^2*b^10 - b^12)*sqrt((e*cos(2*d*x + 2*c
) + e)/sin(2*d*x + 2*c)) + ((3*a^2*b - b^3)*d^3*e^8*sqrt(-(a^12 - 30*a^10*b
^2 + 255*a^8*b^4 - 452*a^6*b^6 + 255*a^4*b^8 - 30*a^2*b^10 + b^12)/(d^4*e^1
0)) - (a^9 - 18*a^7*b^2 + 60*a^5*b^4 - 46*a^3*b^6 + 3*a*b^8)*d*e^3)*sqrt((d
^2*e^5*sqrt(-(a^12 - 30*a^10*b^2 + 255*a^8*b^4 - 452*a^6*b^6 + 255*a^4*b^8
- 30*a^2*b^10 + b^12)/(d^4*e^10)) - 6*a^5*b + 20*a^3*b^3 - 6*a*b^5)/(d^2*e^
5))) + 3*(d*e^3*cos(2*d*x + 2*c) + d*e^3)*sqrt((d^2*e^5*sqrt(-(a^12 - 30*a^
10*b^2 + 255*a^8*b^4 - 452*a^6*b^6 + 255*a^4*b^8 - 30*a^2*b^10 + b^12)/(d^4
*e^10)) - 6*a^5*b + 20*a^3*b^3 - 6*a*b^5)/(d^2*e^5))*log(-(a^12 - 12*a^10*b
^2 - 27*a^8*b^4 + 27*a^4*b^8 + 12*a^2*b^10 - b^12)*sqrt((e*cos(2*d*x + 2*c)
 + e)/sin(2*d*x + 2*c)) - ((3*a^2*b - b^3)*d^3*e^8*sqrt(-(a^12 - 30*a^10*b^
2 + 255*a^8*b^4 - 452*a^6*b^6 + 255*a^4*b^8 - 30*a^2*b^10 + b^12)/(d^4*e^10
)) - (a^9 - 18*a^7*b^2 + 60*a^5*b^4 - 46*a^3*b^6 + 3*a*b^8)*d*e^3)*sqrt((d^
2*e^5*sqrt(-(a^12 - 30*a^10*b^2 + 255*a^8*b^4 - 452*a^6*b^6 + 255*a^4*b^8 -
 30*a^2*b^10 + b^12)/(d^4*e^10)) - 6*a^5*b + 20*a^3*b^3 - 6*a*b^5)/(d^2*e^5
))) + 4*(a^3*cos(2*d*x + 2*c) - 9*a^2*b*sin(2*d*x + 2*c) - a^3)*sqrt((e*cos
(2*d*x + 2*c) + e)/sin(2*d*x + 2*c)))/(d*e^3*cos(2*d*x + 2*c) + d*e^3)
```

Sympy [F]

$$\int \frac{(a+b\cot(c+dx))^3}{(e\cot(c+dx))^{5/2}}\,dx = \int \frac{(a+b\cot{(c+dx)})^3}{(e\cot{(c+dx)})^{\frac{5}{2}}}\,dx$$

```
[In] integrate((a+b*cot(d*x+c))**3/(e*cot(d*x+c))**(5/2),x)
```

```
[Out] Integral((a + b*cot(c + d*x))**3/(e*cot(c + d*x))**(5/2), x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{(a+b\cot(c+dx))^3}{(e\cot(c+dx))^{5/2}}\,dx = \text{Exception raised: ValueError}$$

```
[In] integrate((a+b*cot(d*x+c))^3/(e*cot(d*x+c))^(5/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(e>0)', see `assume?` for more detai
ls)Is e
```

Giac [F]

$$\int \frac{(a+b\cot(c+dx))^3}{(e\cot(c+dx))^{5/2}}\,dx = \int \frac{(b\cot{(dx+c)}+a)^3}{(e\cot{(dx+c)})^{\frac{5}{2}}}\,dx$$

```
[In] integrate((a+b*cot(d*x+c))^3/(e*cot(d*x+c))^(5/2),x, algorithm="giac")
```

```
[Out] integrate((b*cot(d*x + c) + a)^3/(e*cot(d*x + c))^(5/2), x)
```

Mupad [B] (verification not implemented)

Time = 14.34 (sec) , antiderivative size = 1946, normalized size of antiderivative = 6.22

$$\int \frac{(a+b\cot(c+dx))^3}{(e\cot(c+dx))^{5/2}}\,dx = \text{Too large to display}$$

```
[In] int((a + b*cot(c + d*x))^3/(e*cot(c + d*x))^(5/2),x)
```

```
[Out] ((2*a^3*e)/3 + 6*a^2*b*e*cot(c + d*x))/(d*e^2*(e*cot(c + d*x))^(3/2)) - ata
n((((e*cot(c + d*x))^(1/2)*(16*a^6*d^3*e^8 - 16*b^6*d^3*e^8 + 240*a^2*b^4*d
```

^3*e^8 - 240*a^4*b^2*d^3*e^8) + (32*a^3*d^4*e^11 - 96*a*b^2*d^4*e^11)*(((a*b^5*6i + a^5*b*6i + a^6 - b^6 + 15*a^2*b^4 - a^3*b^3*20i - 15*a^4*b^2)*1i)/(4*d^2*e^5))^(1/2))*(((a*b^5*6i + a^5*b*6i + a^6 - b^6 + 15*a^2*b^4 - a^3*b^3*20i - 15*a^4*b^2)*1i)/(4*d^2*e^5))^(1/2)*1i + ((e*cot(c + d*x))^(1/2)*(16*a^6*d^3*e^8 - 16*b^6*d^3*e^8 + 240*a^2*b^4*d^3*e^8 - 240*a^4*b^2*d^3*e^8) - (32*a^3*d^4*e^11 - 96*a*b^2*d^4*e^11)*(((a*b^5*6i + a^5*b*6i + a^6 - b^6 + 15*a^2*b^4 - a^3*b^3*20i - 15*a^4*b^2)*1i)/(4*d^2*e^5))^(1/2))*(((a*b^5*6i + a^5*b*6i + a^6 - b^6 + 15*a^2*b^4 - a^3*b^3*20i - 15*a^4*b^2)*1i)/(4*d^2*e^5))^(1/2)*1i)/(((e*cot(c + d*x))^(1/2)*(16*a^6*d^3*e^8 - 16*b^6*d^3*e^8 + 240*a^2*b^4*d^3*e^8 - 240*a^4*b^2*d^3*e^8) + (32*a^3*d^4*e^11 - 96*a*b^2*d^4*e^11)*(((a*b^5*6i + a^5*b*6i + a^6 - b^6 + 15*a^2*b^4 - a^3*b^3*20i - 15*a^4*b^2)*1i)/(4*d^2*e^5))^(1/2))*(((a*b^5*6i + a^5*b*6i + a^6 - b^6 + 15*a^2*b^4 - a^3*b^3*20i - 15*a^4*b^2)*1i)/(4*d^2*e^5))^(1/2) - ((e*cot(c + d*x))^(1/2)*(16*a^6*d^3*e^8 - 16*b^6*d^3*e^8 + 240*a^2*b^4*d^3*e^8 - 240*a^4*b^2*d^3*e^8) - (32*a^3*d^4*e^11 - 96*a*b^2*d^4*e^11)*(((a*b^5*6i + a^5*b*6i + a^6 - b^6 + 15*a^2*b^4 - a^3*b^3*20i - 15*a^4*b^2)*1i)/(4*d^2*e^5))^(1/2))*(((a*b^5*6i + a^5*b*6i + a^6 - b^6 + 15*a^2*b^4 - a^3*b^3*20i - 15*a^4*b^2)*1i)/(4*d^2*e^5))^(1/2) - 16*b^9*d^2*e^6 + 48*a^8*b*d^2*e^6 + 96*a^4*b^5*d^2*e^6 + 128*a^6*b^3*d^2*e^6))*(((a*b^5*6i + a^5*b*6i + a^6 - b^6 + 15*a^2*b^4 - a^3*b^3*20i - 15*a^4*b^2)*1i)/(4*d^2*e^5))^(1/2)*2i - atan((((e*cot(c + d*x))^(1/2)*(16*a^6*d^3*e^8 - 16*b^6*d^3*e^8 + 240*a^2*b^4*d^3*e^8 - 240*a^4*b^2*d^3*e^8) + (32*a^3*d^4*e^11 - 96*a*b^2*d^4*e^11)*(((a*b^5*6i + a^5*b*6i - a^6 + b^6 - 15*a^2*b^4 - a^3*b^3*20i + 15*a^4*b^2)*1i)/(4*d^2*e^5))^(1/2))*(((a*b^5*6i + a^5*b*6i - a^6 + b^6 - 15*a^2*b^4 - a^3*b^3*20i + 15*a^4*b^2)*1i)/(4*d^2*e^5))^(1/2)*1i + ((e*cot(c + d*x))^(1/2)*(16*a^6*d^3*e^8 - 16*b^6*d^3*e^8 + 240*a^2*b^4*d^3*e^8 - 240*a^4*b^2*d^3*e^8) - (32*a^3*d^4*e^11 - 96*a*b^2*d^4*e^11)*(((a*b^5*6i + a^5*b*6i - a^6 + b^6 - 15*a^2*b^4 - a^3*b^3*20i + 15*a^4*b^2)*1i)/(4*d^2*e^5))^(1/2))*(((a*b^5*6i + a^5*b*6i - a^6 + b^6 - 15*a^2*b^4 - a^3*b^3*20i + 15*a^4*b^2)*1i)/(4*d^2*e^5))^(1/2)*1i)/(((e*cot(c + d*x))^(1/2)*(16*a^6*d^3*e^8 - 16*b^6*d^3*e^8 + 240*a^2*b^4*d^3*e^8 - 240*a^4*b^2*d^3*e^8) + (32*a^3*d^4*e^11 - 96*a*b^2*d^4*e^11)*(((a*b^5*6i + a^5*b*6i - a^6 + b^6 - 15*a^2*b^4 - a^3*b^3*20i + 15*a^4*b^2)*1i)/(4*d^2*e^5))^(1/2))*(((a*b^5*6i + a^5*b*6i - a^6 + b^6 - 15*a^2*b^4 - a^3*b^3*20i + 15*a^4*b^2)*1i)/(4*d^2*e^5))^(1/2) - ((e*cot(c + d*x))^(1/2)*(16*a^6*d^3*e^8 - 16*b^6*d^3*e^8 + 240*a^2*b^4*d^3*e^8 - 240*a^4*b^2*d^3*e^8) - (32*a^3*d^4*e^11 - 96*a*b^2*d^4*e^11)*(((a*b^5*6i + a^5*b*6i - a^6 + b^6 - 15*a^2*b^4 - a^3*b^3*20i + 15*a^4*b^2)*1i)/(4*d^2*e^5))^(1/2))*(((a*b^5*6i + a^5*b*6i - a^6 + b^6 - 15*a^2*b^4 - a^3*b^3*20i + 15*a^4*b^2)*1i)/(4*d^2*e^5))^(1/2) - 16*b^9*d^2*e^6 + 48*a^8*b*d^2*e^6 + 96*a^4*b^5*d^2*e^6 + 128*a^6*b^3*d^2*e^6))*(((a*b^5*6i + a^5*b*6i - a^6 + b^6 - 15*a^2*b^4 - a^3*b^3*20i + 15*a^4*b^2)*1i)/(4*d^2*e^5))^(1/2)*2i

3.67 $\int \frac{(a+b\cot(c+dx))^3}{(e\cot(c+dx))^{7/2}}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 343

$$\begin{aligned}\int \frac{(a+b\cot(c+dx))^3}{(e\cot(c+dx))^{7/2}}\,dx &= \frac{(a+b)\left(a^2-4ab+b^2\right)\arctan\left(1-\frac{\sqrt{2}\sqrt{e\cot(c+dx)}}{\sqrt{e}}\right)}{\sqrt{2}de^{7/2}} \\ &- \frac{(a+b)\left(a^2-4ab+b^2\right)\arctan\left(1+\frac{\sqrt{2}\sqrt{e\cot(c+dx)}}{\sqrt{e}}\right)}{\sqrt{2}de^{7/2}} \\ &+ \frac{8a^2b}{5de^2(e\cot(c+dx))^{3/2}} - \frac{2a(a^2-3b^2)}{de^3\sqrt{e\cot(c+dx)}} + \frac{2a^2(a+b\cot(c+dx))}{5de(e\cot(c+dx))^{5/2}} \\ &- \frac{(a-b)\left(a^2+4ab+b^2\right)\log\left(\sqrt{e}+\sqrt{e}\cot(c+dx)-\sqrt{2}\sqrt{e\cot(c+dx)}\right)}{2\sqrt{2}de^{7/2}} \\ &+ \frac{(a-b)\left(a^2+4ab+b^2\right)\log\left(\sqrt{e}+\sqrt{e}\cot(c+dx)+\sqrt{2}\sqrt{e\cot(c+dx)}\right)}{2\sqrt{2}de^{7/2}}\end{aligned}$$

```
[Out] 8/5*a^2*b/d/e^2/(e*cot(d*x+c))^(3/2)+2/5*a^2*(a+b*cot(d*x+c))/d/e/(e*cot(d*
x+c))^(5/2)+1/2*(a+b)*(a^2-4*a*b+b^2)*arctan(1-2^(1/2)*(e*cot(d*x+c))^(1/2)
/e^(1/2))/d/e^(7/2)*2^(1/2)-1/2*(a+b)*(a^2-4*a*b+b^2)*arctan(1+2^(1/2)*(e*c
ot(d*x+c))^(1/2)/e^(1/2))/d/e^(7/2)*2^(1/2)-1/4*(a-b)*(a^2+4*a*b+b^2)*ln(e^
(1/2)+cot(d*x+c)*e^(1/2)-2^(1/2)*(e*cot(d*x+c))^(1/2))/d/e^(7/2)*2^(1/2)+1/
4*(a-b)*(a^2+4*a*b+b^2)*ln(e^(1/2)+cot(d*x+c)*e^(1/2)+2^(1/2)*(e*cot(d*x+c)
)^(1/2))/d/e^(7/2)*2^(1/2)-2*a*(a^2-3*b^2)/d/e^3/(e*cot(d*x+c))^(1/2)
```

Rubi [A] (verified)

Time = 0.66 (sec) , antiderivative size = 343, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.400, Rules used = {3646, 3709, 3610, 3615, 1182, 1176, 631, 210, 1179, 642}

$$\int \frac{(a+b\cot(c+dx))^3}{(e\cot(c+dx))^{7/2}}\,dx = \frac{(a+b)\left(a^2-4ab+b^2\right)\arctan\left(1-\frac{\sqrt{2}\sqrt{e\cot(c+dx)}}{\sqrt{e}}\right)}{\sqrt{2}de^{7/2}} - \frac{(a+b)\left(a^2-4ab+b^2\right)\arctan\left(\frac{\sqrt{2}\sqrt{e\cot(c+dx)}}{\sqrt{e}}+1\right)}{\sqrt{2}de^{7/2}} - \frac{(a-b)\left(a^2+4ab+b^2\right)\log\left(\sqrt{e}\cot(c+dx)-\sqrt{2}\sqrt{e\cot(c+dx)}+\sqrt{e}\right)}{2\sqrt{2}de^{7/2}} + \frac{(a-b)\left(a^2+4ab+b^2\right)\log\left(\sqrt{e}\cot(c+dx)+\sqrt{2}\sqrt{e\cot(c+dx)}+\sqrt{e}\right)}{2\sqrt{2}de^{7/2}} - \frac{2a\left(a^2-3b^2\right)}{de^3\sqrt{e\cot(c+dx)}} + \frac{8a^2b}{5de^2(e\cot(c+dx))^{3/2}} + \frac{2a^2(a+b\cot(c+dx))}{5de(e\cot(c+dx))^{5/2}}$$

```
[In] Int[(a + b*Cot[c + d*x])^3/(e*Cot[c + d*x])^(7/2),x]
```

```
[Out] ((a + b)*(a^2 - 4*a*b + b^2)*ArcTan[1 - (Sqrt[2]*Sqrt[e*Cot[c + d*x]])/Sqrt
[e]])/(Sqrt[2]*d*e^(7/2)) - ((a + b)*(a^2 - 4*a*b + b^2)*ArcTan[1 + (Sqrt[2
]*Sqrt[e*Cot[c + d*x]])/Sqrt[e]])/(Sqrt[2]*d*e^(7/2)) + (8*a^2*b)/(5*d*e^2*
(e*Cot[c + d*x])^(3/2)) - (2*a*(a^2 - 3*b^2))/(d*e^3*Sqrt[e*Cot[c + d*x]])
+ (2*a^2*(a + b*Cot[c + d*x]))/(5*d*e*(e*Cot[c + d*x])^(5/2)) - ((a - b)*(a
^2 + 4*a*b + b^2)*Log[Sqrt[e] + Sqrt[e]*Cot[c + d*x] - Sqrt[2]*Sqrt[e*Cot[c
 + d*x]]])/(2*Sqrt[2]*d*e^(7/2)) + ((a - b)*(a^2 + 4*a*b + b^2)*Log[Sqrt[e]
 + Sqrt[e]*Cot[c + d*x] + Sqrt[2]*Sqrt[e*Cot[c + d*x]]])/(2*Sqrt[2]*d*e^(7/
2))
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1179

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 1182

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
a*c, 2]}, Dist[(d*q + a*e)/(2*a*c), Int[(q + c*x^2)/(a + c*x^4), x], x] + D
ist[(d*q - a*e)/(2*a*c), Int[(q - c*x^2)/(a + c*x^4), x], x]] /; FreeQ[{a,
c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0] && NeQ[c*d^2 - a*e^2, 0] && NegQ[(-a)
*c]
```

Rule 3610

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)]), x_Symbol] :> Simp[(b*c - a*d)*((a + b*Tan[e + f*x])^(m + 1)/
(f*(m + 1)*(a^2 + b^2))), x] + Dist[1/(a^2 + b^2), Int[(a + b*Tan[e + f*x])
^(m + 1)*Simp[a*c + b*d - (b*c - a*d)*Tan[e + f*x], x], x], x] /; FreeQ[{a,
 b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && LtQ[m, -1]
```

Rule 3615

```
Int[((c_) + (d_.)*tan[(e_.) + (f_.)*(x_)])/Sqrt[(b_.)*tan[(e_.) + (f_.)*(x_
)]], x_Symbol] :> Dist[2/f, Subst[Int[(b*c + d*x^2)/(b^2 + x^4), x], x, Sqr
t[b*Tan[e + f*x]]], x] /; FreeQ[{b, c, d, e, f}, x] && NeQ[c^2 - d^2, 0] &&
 NeQ[c^2 + d^2, 0]
```

Rule 3646

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*c - a*d)^2*(a + b*Tan[e + f*x])^(m
 - 2)*((c + d*Tan[e + f*x])^(n + 1)/(d*f*(n + 1)*(c^2 + d^2))), x] - Dist[1
```

```
/(d*(n + 1)*(c^2 + d^2)), Int[(a + b*Tan[e + f*x])^(m - 3)*(c + d*Tan[e + f
*x])^(n + 1)*Simp[a^2*d*(b*d*(m - 2) - a*c*(n + 1)) + b*(b*c - 2*a*d)*(b*c*
(m - 2) + a*d*(n + 1)) - d*(n + 1)*(3*a^2*b*c - b^3*c - a^3*d + 3*a*b^2*d)*
Tan[e + f*x] - b*(a*d*(2*b*c - a*d)*(m + n - 1) - b^2*(c^2*(m - 2) - d^2*(n
 + 1)))*Tan[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b
*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] && GtQ[m, 2] && LtQ[
n, -1] && IntegerQ[2*m]
```

Rule 3709

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
 (f_.)*(x_)] + (C_.)*tan[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(A*b^2 -
 a*b*B + a^2*C)*((a + b*Tan[e + f*x])^(m + 1)/(b*f*(m + 1)*(a^2 + b^2))), x
] + Dist[1/(a^2 + b^2), Int[(a + b*Tan[e + f*x])^(m + 1)*Simp[b*B + a*(A -
C) - (A*b - a*B - b*C)*Tan[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B,
 C}, x] && NeQ[A*b^2 - a*b*B + a^2*C, 0] && LtQ[m, -1] && NeQ[a^2 + b^2, 0]
```

Rubi steps

$$\text{integral} = \frac{2a^2(a+b\cot(c+dx))}{5de(e\cot(c+dx))^{5/2}} - \frac{2\int \frac{-6a^2be^2+\frac{5}{2}a(a^2-3b^2)e^2\cot(c+dx)+\frac{1}{2}b(3a^2-5b^2)e^2\cot^2(c+dx)}{(e\cot(c+dx))^{5/2}}\,dx}{5e^3}$$

$$= \frac{8a^2b}{5de^2(e\cot(c+dx))^{3/2}} + \frac{2a^2(a+b\cot(c+dx))}{5de(e\cot(c+dx))^{5/2}} - \frac{2\int \frac{\frac{5}{2}a(a^2-3b^2)e^3+\frac{5}{2}b(3a^2-b^2)e^3\cot(c+dx)}{(e\cot(c+dx))^{3/2}}\,dx}{5e^5}$$

$$= \frac{8a^2b}{5de^2(e\cot(c+dx))^{3/2}} - \frac{2a(a^2-3b^2)}{de^3\sqrt{e\cot(c+dx)}} + \frac{2a^2(a+b\cot(c+dx))}{5de(e\cot(c+dx))^{5/2}} - \frac{2\int \frac{\frac{5}{2}b(3a^2-b^2)e^4-\frac{5}{2}a(a^2-3b^2)e^4\cot(c+dx)}{\sqrt{e\cot(c+dx)}}\,dx}{5e^7}$$

$$= \frac{8a^2b}{5de^2(e\cot(c+dx))^{3/2}} - \frac{2a(a^2-3b^2)}{de^3\sqrt{e\cot(c+dx)}} + \frac{2a^2(a+b\cot(c+dx))}{5de(e\cot(c+dx))^{5/2}} - \frac{4\text{Subst}\left(\int \frac{-\frac{5}{2}b(3a^2-b^2)e^5+\frac{5}{2}a(a^2-3b^2)e^4x^2}{e^2+x^4}\,dx, x, \sqrt{e\cot(c+dx)}\right)}{5de^7}$$

$$= \frac{8a^2b}{5de^2(e\cot(c+dx))^{3/2}} - \frac{2a(a^2-3b^2)}{de^3\sqrt{e\cot(c+dx)}} + \frac{2a^2(a+b\cot(c+dx))}{5de(e\cot(c+dx))^{5/2}} - \frac{((a+b)(a^2-4ab+b^2))\,\text{Subst}\left(\int \frac{e+x^2}{e^2+x^4}\,dx, x, \sqrt{e\cot(c+dx)}\right)}{de^3} + \frac{((a-b)(a^2+4ab+b^2))\,\text{Subst}\left(\int \frac{e-x^2}{e^2+x^4}\,dx, x, \sqrt{e\cot(c+dx)}\right)}{de^3}$$

$$= \frac{8a^2b}{5de^2(e\cot(c+dx))^{3/2}} - \frac{2a(a^2-3b^2)}{de^3\sqrt{e\cot(c+dx)}} + \frac{2a^2(a+b\cot(c+dx))}{5de(e\cot(c+dx))^{5/2}}$$
$$- \frac{((a-b)(a^2+4ab+b^2))\,\text{Subst}\left(\int \frac{\sqrt{2}\sqrt{e}+2x}{-e-\sqrt{2}\sqrt{e}x-x^2}\,dx, x, \sqrt{e\cot(c+dx)}\right)}{2\sqrt{2}de^{7/2}}$$
$$- \frac{((a-b)(a^2+4ab+b^2))\,\text{Subst}\left(\int \frac{\sqrt{2}\sqrt{e}-2x}{-e+\sqrt{2}\sqrt{e}x-x^2}\,dx, x, \sqrt{e\cot(c+dx)}\right)}{2\sqrt{2}de^{7/2}}$$
$$- \frac{((a+b)(a^2-4ab+b^2))\,\text{Subst}\left(\int \frac{1}{e-\sqrt{2}\sqrt{e}x+x^2}\,dx, x, \sqrt{e\cot(c+dx)}\right)}{2de^3}$$
$$- \frac{((a+b)(a^2-4ab+b^2))\,\text{Subst}\left(\int \frac{1}{e+\sqrt{2}\sqrt{e}x+x^2}\,dx, x, \sqrt{e\cot(c+dx)}\right)}{2de^3}$$

$$= \frac{8a^2b}{5de^2(e\cot(c+dx))^{3/2}} - \frac{2a(a^2-3b^2)}{de^3\sqrt{e\cot(c+dx)}} + \frac{2a^2(a+b\cot(c+dx))}{5de(e\cot(c+dx))^{5/2}}$$
$$- \frac{(a-b)(a^2+4ab+b^2)\log\left(\sqrt{e}+\sqrt{e}\cot(c+dx)-\sqrt{2}\sqrt{e\cot(c+dx)}\right)}{2\sqrt{2}de^{7/2}}$$
$$+ \frac{(a-b)(a^2+4ab+b^2)\log\left(\sqrt{e}+\sqrt{e}\cot(c+dx)+\sqrt{2}\sqrt{e\cot(c+dx)}\right)}{2\sqrt{2}de^{7/2}}$$
$$- \frac{((a+b)(a^2-4ab+b^2))\,\text{Subst}\left(\int \frac{1}{-1-x^2}\,dx, x, 1-\frac{\sqrt{2}\sqrt{e\cot(c+dx)}}{\sqrt{e}}\right)}{\sqrt{2}de^{7/2}}$$
$$+ \frac{((a+b)(a^2-4ab+b^2))\,\text{Subst}\left(\int \frac{1}{-1-x^2}\,dx, x, 1+\frac{\sqrt{2}\sqrt{e\cot(c+dx)}}{\sqrt{e}}\right)}{\sqrt{2}de^{7/2}}$$

$$= \frac{(a+b)(a^2-4ab+b^2)\arctan\left(1-\frac{\sqrt{2}\sqrt{e\cot(c+dx)}}{\sqrt{e}}\right)}{\sqrt{2}de^{7/2}}$$
$$- \frac{(a+b)(a^2-4ab+b^2)\arctan\left(1+\frac{\sqrt{2}\sqrt{e\cot(c+dx)}}{\sqrt{e}}\right)}{\sqrt{2}de^{7/2}}$$
$$+ \frac{8a^2b}{5de^2(e\cot(c+dx))^{3/2}} - \frac{2a(a^2-3b^2)}{de^3\sqrt{e\cot(c+dx)}} + \frac{2a^2(a+b\cot(c+dx))}{5de(e\cot(c+dx))^{5/2}}$$
$$- \frac{(a-b)(a^2+4ab+b^2)\log\left(\sqrt{e}+\sqrt{e}\cot(c+dx)-\sqrt{2}\sqrt{e\cot(c+dx)}\right)}{2\sqrt{2}de^{7/2}}$$
$$+ \frac{(a-b)(a^2+4ab+b^2)\log\left(\sqrt{e}+\sqrt{e}\cot(c+dx)+\sqrt{2}\sqrt{e\cot(c+dx)}\right)}{2\sqrt{2}de^{7/2}}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 0.68 (sec) , antiderivative size = 108, normalized size of antiderivative = 0.31

$$\int \frac{(a+b\cot(c+dx))^3}{(e\cot(c+dx))^{7/2}}\,dx = \frac{2\left(3a(a^2-3b^2)\,\text{Hypergeometric2F1}\left(-\frac{5}{4},1,-\frac{1}{4},-\cot^2(c+dx)\right)+b\left(b(9a+5b\ldots\right.\right.}{15de(e\ldots}$$

[In] Integrate[(a + b*Cot[c + d*x])^3/(e*Cot[c + d*x])^(7/2),x]

[Out] (2*(3*a*(a^2 - 3*b^2)*Hypergeometric2F1[-5/4, 1, -1/4, -Cot[c + d*x]^2] + b
(b(9*a + 5*b*Cot[c + d*x]) + 5*(3*a^2 - b^2)*Cot[c + d*x]*Hypergeometric2
F1[-3/4, 1, 1/4, -Cot[c + d*x]^2])))/(15*d*e*(e*Cot[c + d*x])^(5/2))

Maple [A] (verified)

Time = 0.04 (sec) , antiderivative size = 359, normalized size of antiderivative = 1.05

method	result
derivativedivides	$-\frac{2\left(\frac{\left(-3a^2be+b^3e\right)\left(e^2\right)^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\frac{e\cot(dx+c)+\left(e^2\right)^{\frac{1}{4}}\sqrt{e\cot(dx+c)}\sqrt{2}+\sqrt{e^2}}{e\cot(dx+c)-\left(e^2\right)^{\frac{1}{4}}\sqrt{e\cot(dx+c)}\sqrt{2}+\sqrt{e^2}}\right)+2\arctan\left(\frac{\sqrt{2}\sqrt{e\cot(dx+c)}}{\left(e^2\right)^{\frac{1}{4}}}+1\right)-2\arctan\left(-\ldots\right.\right.}{8e^2}\ldots\right.}{\ldots}$
default	$-\frac{2\left(\frac{\left(-3a^2be+b^3e\right)\left(e^2\right)^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\frac{e\cot(dx+c)+\left(e^2\right)^{\frac{1}{4}}\sqrt{e\cot(dx+c)}\sqrt{2}+\sqrt{e^2}}{e\cot(dx+c)-\left(e^2\right)^{\frac{1}{4}}\sqrt{e\cot(dx+c)}\sqrt{2}+\sqrt{e^2}}\right)+2\arctan\left(\frac{\sqrt{2}\sqrt{e\cot(dx+c)}}{\left(e^2\right)^{\frac{1}{4}}}+1\right)-2\arctan\left(-\ldots\right.\right.}{8e^2}\ldots\right.}{\ldots}$
parts	$-\frac{2a^3e\left(-\frac{1}{5e^2(e\cot(dx+c))^{\frac{5}{2}}}+\frac{1}{e^4\sqrt{e\cot(dx+c)}}+\frac{\sqrt{2}\left(\ln\left(\frac{e\cot(dx+c)-\left(e^2\right)^{\frac{1}{4}}\sqrt{e\cot(dx+c)}\sqrt{2}+\sqrt{e^2}}{e\cot(dx+c)+\left(e^2\right)^{\frac{1}{4}}\sqrt{e\cot(dx+c)}\sqrt{2}+\sqrt{e^2}}\right)+2\arctan\left(\frac{\sqrt{2}\sqrt{e\cot(d\ldots}}{\left(e^2\right)^{\frac{1}{4}}}\ldots\right.\right.}{8e^4\left(e^2\right)^{\frac{1}{4}}}\ldots\right.}{d}$

[In] int((a+b*cot(d*x+c))^3/(e*cot(d*x+c))^(7/2),x,method=_RETURNVERBOSE)

[Out] -2/d/e^2*(1/e*(1/8*(-3*a^2*b*e+b^3*e)*(e^2)^(1/4)/e^2*2^(1/2)*(ln((e*cot(d*
x+c)+(e^2)^(1/4)*(e*cot(d*x+c))^(1/2)*2^(1/2)+(e^2)^(1/2))/(e*cot(d*x+c)-(e
^2)^(1/4)*(e*cot(d*x+c))^(1/2)*2^(1/2)+(e^2)^(1/2)))+2*arctan(2^(1/2)/(e^2)

```
^(1/4)*(e*cot(d*x+c))^(1/2)+1)-2*arctan(-2^(1/2)/(e^2)^(1/4)*(e*cot(d*x+c))
^(1/2)+1))+1/8*(a^3-3*a*b^2)/(e^2)^(1/4)*2^(1/2)*(ln((e*cot(d*x+c)-(e^2)^(1
/4)*(e*cot(d*x+c))^(1/2)*2^(1/2)+(e^2)^(1/2))/(e*cot(d*x+c)+(e^2)^(1/4)*(e*
cot(d*x+c))^(1/2)*2^(1/2)+(e^2)^(1/2)))+2*arctan(2^(1/2)/(e^2)^(1/4)*(e*cot
(d*x+c))^(1/2)+1)-2*arctan(-2^(1/2)/(e^2)^(1/4)*(e*cot(d*x+c))^(1/2)+1)))-1
/5*a^3*e/(e*cot(d*x+c))^(5/2)-a^2*b/(e*cot(d*x+c))^(3/2)+a/e*(a^2-3*b^2)/(e
*cot(d*x+c))^(1/2))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1804 vs. $2(286) = 572$.

Time = 0.36 (sec) , antiderivative size = 1804, normalized size of antiderivative = 5.26

$$\int \frac{(a+b\cot(c+dx))^3}{(e\cot(c+dx))^{7/2}}\,dx = \text{Too large to display}$$

```
[In] integrate((a+b*cot(d*x+c))^3/(e*cot(d*x+c))^(7/2),x, algorithm="fricas")
```

```
[Out] 1/10*(5*(d*e^4*cos(2*d*x + 2*c)^2 + 2*d*e^4*cos(2*d*x + 2*c) + d*e^4)*sqrt(
(d^2*e^7*sqrt(-(a^12 - 30*a^10*b^2 + 255*a^8*b^4 - 452*a^6*b^6 + 255*a^4*b^
8 - 30*a^2*b^10 + b^12)/(d^4*e^14)) + 6*a^5*b - 20*a^3*b^3 + 6*a*b^5)/(d^2*
e^7))*log(-(a^12 - 12*a^10*b^2 - 27*a^8*b^4 + 27*a^4*b^8 + 12*a^2*b^10 - b^
12)*sqrt((e*cos(2*d*x + 2*c) + e)/sin(2*d*x + 2*c)) + ((a^3 - 3*a*b^2)*d^3*
e^11*sqrt(-(a^12 - 30*a^10*b^2 + 255*a^8*b^4 - 452*a^6*b^6 + 255*a^4*b^8 -
30*a^2*b^10 + b^12)/(d^4*e^14)) - (3*a^8*b - 46*a^6*b^3 + 60*a^4*b^5 - 18*a
^2*b^7 + b^9)*d*e^4)*sqrt((d^2*e^7*sqrt(-(a^12 - 30*a^10*b^2 + 255*a^8*b^4
- 452*a^6*b^6 + 255*a^4*b^8 - 30*a^2*b^10 + b^12)/(d^4*e^14)) + 6*a^5*b - 2
0*a^3*b^3 + 6*a*b^5)/(d^2*e^7))) - 5*(d*e^4*cos(2*d*x + 2*c)^2 + 2*d*e^4*co
s(2*d*x + 2*c) + d*e^4)*sqrt((d^2*e^7*sqrt(-(a^12 - 30*a^10*b^2 + 255*a^8*b
^4 - 452*a^6*b^6 + 255*a^4*b^8 - 30*a^2*b^10 + b^12)/(d^4*e^14)) + 6*a^5*b
- 20*a^3*b^3 + 6*a*b^5)/(d^2*e^7))*log(-(a^12 - 12*a^10*b^2 - 27*a^8*b^4 +
27*a^4*b^8 + 12*a^2*b^10 - b^12)*sqrt((e*cos(2*d*x + 2*c) + e)/sin(2*d*x +
2*c)) - ((a^3 - 3*a*b^2)*d^3*e^11*sqrt(-(a^12 - 30*a^10*b^2 + 255*a^8*b^4 -
 452*a^6*b^6 + 255*a^4*b^8 - 30*a^2*b^10 + b^12)/(d^4*e^14)) - (3*a^8*b - 4
6*a^6*b^3 + 60*a^4*b^5 - 18*a^2*b^7 + b^9)*d*e^4)*sqrt((d^2*e^7*sqrt(-(a^12
 - 30*a^10*b^2 + 255*a^8*b^4 - 452*a^6*b^6 + 255*a^4*b^8 - 30*a^2*b^10 + b^
12)/(d^4*e^14)) + 6*a^5*b - 20*a^3*b^3 + 6*a*b^5)/(d^2*e^7))) - 5*(d*e^4*co
s(2*d*x + 2*c)^2 + 2*d*e^4*cos(2*d*x + 2*c) + d*e^4)*sqrt(-(d^2*e^7*sqrt(-(
a^12 - 30*a^10*b^2 + 255*a^8*b^4 - 452*a^6*b^6 + 255*a^4*b^8 - 30*a^2*b^10
+ b^12)/(d^4*e^14)) - 6*a^5*b + 20*a^3*b^3 - 6*a*b^5)/(d^2*e^7))*log(-(a^12
 - 12*a^10*b^2 - 27*a^8*b^4 + 27*a^4*b^8 + 12*a^2*b^10 - b^12)*sqrt((e*cos(
2*d*x + 2*c) + e)/sin(2*d*x + 2*c)) + ((a^3 - 3*a*b^2)*d^3*e^11*sqrt(-(a^12
 - 30*a^10*b^2 + 255*a^8*b^4 - 452*a^6*b^6 + 255*a^4*b^8 - 30*a^2*b^10 + b^
12)/(d^4*e^14)) + (3*a^8*b - 46*a^6*b^3 + 60*a^4*b^5 - 18*a^2*b^7 + b^9)*d*
e^4)*sqrt(-(d^2*e^7*sqrt(-(a^12 - 30*a^10*b^2 + 255*a^8*b^4 - 452*a^6*b^6 +
```

```
255*a^4*b^8 - 30*a^2*b^10 + b^12)/(d^4*e^14)) - 6*a^5*b + 20*a^3*b^3 - 6*a
*b^5)/(d^2*e^7))) + 5*(d*e^4*cos(2*d*x + 2*c)^2 + 2*d*e^4*cos(2*d*x + 2*c)
+ d*e^4)*sqrt(-(d^2*e^7*sqrt(-(a^12 - 30*a^10*b^2 + 255*a^8*b^4 - 452*a^6*b
^6 + 255*a^4*b^8 - 30*a^2*b^10 + b^12)/(d^4*e^14)) - 6*a^5*b + 20*a^3*b^3 -
 6*a*b^5)/(d^2*e^7))*log(-(a^12 - 12*a^10*b^2 - 27*a^8*b^4 + 27*a^4*b^8 + 1
2*a^2*b^10 - b^12)*sqrt((e*cos(2*d*x + 2*c) + e)/sin(2*d*x + 2*c)) - ((a^3
- 3*a*b^2)*d^3*e^11*sqrt(-(a^12 - 30*a^10*b^2 + 255*a^8*b^4 - 452*a^6*b^6 +
 255*a^4*b^8 - 30*a^2*b^10 + b^12)/(d^4*e^14)) + (3*a^8*b - 46*a^6*b^3 + 60
*a^4*b^5 - 18*a^2*b^7 + b^9)*d*e^4)*sqrt(-(d^2*e^7*sqrt(-(a^12 - 30*a^10*b^
2 + 255*a^8*b^4 - 452*a^6*b^6 + 255*a^4*b^8 - 30*a^2*b^10 + b^12)/(d^4*e^14
)) - 6*a^5*b + 20*a^3*b^3 - 6*a*b^5)/(d^2*e^7))) - 4*(5*a^2*b*cos(2*d*x + 2
*c)^2 - 5*a^2*b + (4*a^3 - 15*a*b^2 + 3*(2*a^3 - 5*a*b^2)*cos(2*d*x + 2*c))
*sin(2*d*x + 2*c))*sqrt((e*cos(2*d*x + 2*c) + e)/sin(2*d*x + 2*c)))/(d*e^4*
cos(2*d*x + 2*c)^2 + 2*d*e^4*cos(2*d*x + 2*c) + d*e^4)
```

Sympy [F]

$$\int \frac{(a+b\cot(c+dx))^3}{(e\cot(c+dx))^{7/2}}\,dx = \int \frac{(a+b\cot(c+dx))^3}{(e\cot(c+dx))^{\frac{7}{2}}}\,dx$$

```
[In] integrate((a+b*cot(d*x+c))**3/(e*cot(d*x+c))**(7/2),x)
[Out] Integral((a + b*cot(c + d*x))**3/(e*cot(c + d*x))**(7/2), x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{(a+b\cot(c+dx))^3}{(e\cot(c+dx))^{7/2}}\,dx = \text{Exception raised: ValueError}$$

```
[In] integrate((a+b*cot(d*x+c))^3/(e*cot(d*x+c))^(7/2),x, algorithm="maxima")
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(e>0)', see `assume?` for more detai
ls)Is e
```

Giac [F(-1)]

Timed out.

$$\int \frac{(a+b\cot(c+dx))^3}{(e\cot(c+dx))^{7/2}}\,dx = \text{Timed out}$$

```
[In] integrate((a+b*cot(d*x+c))^3/(e*cot(d*x+c))^(7/2),x, algorithm="giac")
[Out] Timed out
```

Mupad [B] (verification not implemented)

Time = 15.67 (sec) , antiderivative size = 1969, normalized size of antiderivative = 5.74

$$\int \frac{(a+b\cot(c+dx))^3}{(e\cot(c+dx))^{7/2}}\,dx = \text{Too large to display}$$

```
[In] int((a + b*cot(c + d*x))^3/(e*cot(c + d*x))^(7/2),x)
[Out] atan((((e*cot(c + d*x))^(1/2)*(16*a^6*d^3*e^11 - 16*b^6*d^3*e^11 + 240*a^2*
b^4*d^3*e^11 - 240*a^4*b^2*d^3*e^11) + (32*b^3*d^4*e^15 - 96*a^2*b*d^4*e^15
)*(-((a*b^5*6i + a^5*b*6i - a^6 + b^6 - 15*a^2*b^4 - a^3*b^3*20i + 15*a^4*b
^2)*1i)/(4*d^2*e^7))^(1/2))*(-((a*b^5*6i + a^5*b*6i - a^6 + b^6 - 15*a^2*b^
4 - a^3*b^3*20i + 15*a^4*b^2)*1i)/(4*d^2*e^7))^(1/2)*1i + ((e*cot(c + d*x))
^(1/2)*(16*a^6*d^3*e^11 - 16*b^6*d^3*e^11 + 240*a^2*b^4*d^3*e^11 - 240*a^4*
b^2*d^3*e^11) - (32*b^3*d^4*e^15 - 96*a^2*b*d^4*e^15)*(-((a*b^5*6i + a^5*b*
6i - a^6 + b^6 - 15*a^2*b^4 - a^3*b^3*20i + 15*a^4*b^2)*1i)/(4*d^2*e^7))^(1
/2))*(-((a*b^5*6i + a^5*b*6i - a^6 + b^6 - 15*a^2*b^4 - a^3*b^3*20i + 15*a^
4*b^2)*1i)/(4*d^2*e^7))^(1/2)*1i)/(((e*cot(c + d*x))^(1/2)*(16*a^6*d^3*e^11
 - 16*b^6*d^3*e^11 + 240*a^2*b^4*d^3*e^11 - 240*a^4*b^2*d^3*e^11) - (32*b^3
*d^4*e^15 - 96*a^2*b*d^4*e^15)*(-((a*b^5*6i + a^5*b*6i - a^6 + b^6 - 15*a^2
*b^4 - a^3*b^3*20i + 15*a^4*b^2)*1i)/(4*d^2*e^7))^(1/2))*(-((a*b^5*6i + a^5
*b*6i - a^6 + b^6 - 15*a^2*b^4 - a^3*b^3*20i + 15*a^4*b^2)*1i)/(4*d^2*e^7))
^(1/2) - ((e*cot(c + d*x))^(1/2)*(16*a^6*d^3*e^11 - 16*b^6*d^3*e^11 + 240*a
^2*b^4*d^3*e^11 - 240*a^4*b^2*d^3*e^11) + (32*b^3*d^4*e^15 - 96*a^2*b*d^4*e
^15)*(-((a*b^5*6i + a^5*b*6i - a^6 + b^6 - 15*a^2*b^4 - a^3*b^3*20i + 15*a^
4*b^2)*1i)/(4*d^2*e^7))^(1/2))*(-((a*b^5*6i + a^5*b*6i - a^6 + b^6 - 15*a^2
*b^4 - a^3*b^3*20i + 15*a^4*b^2)*1i)/(4*d^2*e^7))^(1/2) - 16*a^9*d^2*e^8 +
48*a*b^8*d^2*e^8 + 128*a^3*b^6*d^2*e^8 + 96*a^5*b^4*d^2*e^8))*(-((a*b^5*6i
+ a^5*b*6i - a^6 + b^6 - 15*a^2*b^4 - a^3*b^3*20i + 15*a^4*b^2)*1i)/(4*d^2*
e^7))^(1/2)*2i + atan((((e*cot(c + d*x))^(1/2)*(16*a^6*d^3*e^11 - 16*b^6*d^
3*e^11 + 240*a^2*b^4*d^3*e^11 - 240*a^4*b^2*d^3*e^11) + (32*b^3*d^4*e^15 -
96*a^2*b*d^4*e^15)*(-((a*b^5*6i + a^5*b*6i + a^6 - b^6 + 15*a^2*b^4 - a^3*b
^3*20i - 15*a^4*b^2)*1i)/(4*d^2*e^7))^(1/2))*(-((a*b^5*6i + a^5*b*6i + a^6
- b^6 + 15*a^2*b^4 - a^3*b^3*20i - 15*a^4*b^2)*1i)/(4*d^2*e^7))^(1/2)*1i +
```

```
((e*cot(c + d*x))^(1/2)*(16*a^6*d^3*e^11 - 16*b^6*d^3*e^11 + 240*a^2*b^4*d^
3*e^11 - 240*a^4*b^2*d^3*e^11) - (32*b^3*d^4*e^15 - 96*a^2*b*d^4*e^15)*(-((
a*b^5*6i + a^5*b*6i + a^6 - b^6 + 15*a^2*b^4 - a^3*b^3*20i - 15*a^4*b^2)*1i
)/(4*d^2*e^7))^(1/2))*(-((a*b^5*6i + a^5*b*6i + a^6 - b^6 + 15*a^2*b^4 - a^
3*b^3*20i - 15*a^4*b^2)*1i)/(4*d^2*e^7))^(1/2)*1i)/(((e*cot(c + d*x))^(1/2)
*(16*a^6*d^3*e^11 - 16*b^6*d^3*e^11 + 240*a^2*b^4*d^3*e^11 - 240*a^4*b^2*d^
3*e^11) - (32*b^3*d^4*e^15 - 96*a^2*b*d^4*e^15)*(-((a*b^5*6i + a^5*b*6i + a
^6 - b^6 + 15*a^2*b^4 - a^3*b^3*20i - 15*a^4*b^2)*1i)/(4*d^2*e^7))^(1/2))*(
-((a*b^5*6i + a^5*b*6i + a^6 - b^6 + 15*a^2*b^4 - a^3*b^3*20i - 15*a^4*b^2)
*1i)/(4*d^2*e^7))^(1/2) - ((e*cot(c + d*x))^(1/2)*(16*a^6*d^3*e^11 - 16*b^6
*d^3*e^11 + 240*a^2*b^4*d^3*e^11 - 240*a^4*b^2*d^3*e^11) + (32*b^3*d^4*e^15
 - 96*a^2*b*d^4*e^15)*(-((a*b^5*6i + a^5*b*6i + a^6 - b^6 + 15*a^2*b^4 - a^
3*b^3*20i - 15*a^4*b^2)*1i)/(4*d^2*e^7))^(1/2))*(-((a*b^5*6i + a^5*b*6i + a
^6 - b^6 + 15*a^2*b^4 - a^3*b^3*20i - 15*a^4*b^2)*1i)/(4*d^2*e^7))^(1/2) -
16*a^9*d^2*e^8 + 48*a*b^8*d^2*e^8 + 128*a^3*b^6*d^2*e^8 + 96*a^5*b^4*d^2*e^
8))*(-((a*b^5*6i + a^5*b*6i + a^6 - b^6 + 15*a^2*b^4 - a^3*b^3*20i - 15*a^4
*b^2)*1i)/(4*d^2*e^7))^(1/2)*2i + ((2*a^3*e)/5 + 2*e*cot(c + d*x)^2*(3*a*b^
2 - a^3) + 2*a^2*b*e*cot(c + d*x))/(d*e^2*(e*cot(c + d*x))^(5/2))
```

3.68 $\int \frac{(a+b\cot(c+dx))^3}{(e\cot(c+dx))^{9/2}}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 377

$$\begin{aligned}\int \frac{(a+b\cot(c+dx))^3}{(e\cot(c+dx))^{9/2}}\,dx &= \frac{(a-b)\left(a^2+4ab+b^2\right)\arctan\left(1-\frac{\sqrt{2}\sqrt{e\cot(c+dx)}}{\sqrt{e}}\right)}{\sqrt{2}de^{9/2}} \\ &- \frac{(a-b)\left(a^2+4ab+b^2\right)\arctan\left(1+\frac{\sqrt{2}\sqrt{e\cot(c+dx)}}{\sqrt{e}}\right)}{\sqrt{2}de^{9/2}} + \frac{32a^2b}{35de^2(e\cot(c+dx))^{5/2}} \\ &- \frac{2a(a^2-3b^2)}{3de^3(e\cot(c+dx))^{3/2}} - \frac{2b(3a^2-b^2)}{de^4\sqrt{e\cot(c+dx)}} + \frac{2a^2(a+b\cot(c+dx))}{7de(e\cot(c+dx))^{7/2}} \\ &+ \frac{(a+b)\left(a^2-4ab+b^2\right)\log\left(\sqrt{e}+\sqrt{e}\cot(c+dx)-\sqrt{2}\sqrt{e\cot(c+dx)}\right)}{2\sqrt{2}de^{9/2}} \\ &- \frac{(a+b)\left(a^2-4ab+b^2\right)\log\left(\sqrt{e}+\sqrt{e}\cot(c+dx)+\sqrt{2}\sqrt{e\cot(c+dx)}\right)}{2\sqrt{2}de^{9/2}}\end{aligned}$$

```
[Out] 32/35*a^2*b/d/e^2/(e*cot(d*x+c))^(5/2)-2/3*a*(a^2-3*b^2)/d/e^3/(e*cot(d*x+c
))^(3/2)+2/7*a^2*(a+b*cot(d*x+c))/d/e/(e*cot(d*x+c))^(7/2)+1/2*(a-b)*(a^2+4
*a*b+b^2)*arctan(1-2^(1/2)*(e*cot(d*x+c))^(1/2)/e^(1/2))/d/e^(9/2)*2^(1/2)-
1/2*(a-b)*(a^2+4*a*b+b^2)*arctan(1+2^(1/2)*(e*cot(d*x+c))^(1/2)/e^(1/2))/d/
e^(9/2)*2^(1/2)+1/4*(a+b)*(a^2-4*a*b+b^2)*ln(e^(1/2)+cot(d*x+c)*e^(1/2)-2^(
1/2)*(e*cot(d*x+c))^(1/2))/d/e^(9/2)*2^(1/2)-1/4*(a+b)*(a^2-4*a*b+b^2)*ln(e
^(1/2)+cot(d*x+c)*e^(1/2)+2^(1/2)*(e*cot(d*x+c))^(1/2))/d/e^(9/2)*2^(1/2)-2
*b*(3*a^2-b^2)/d/e^4/(e*cot(d*x+c))^(1/2)
```

Rubi [A] (verified)

Time = 0.80 (sec) , antiderivative size = 377, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.400, Rules used = {3646, 3709, 3610, 3615, 1182, 1176, 631, 210, 1179, 642}

$$\begin{aligned}\int \frac{(a+b\cot(c+dx))^3}{(e\cot(c+dx))^{9/2}}\,dx &= \frac{(a-b)\left(a^2+4ab+b^2\right)\arctan\left(1-\frac{\sqrt{2}\sqrt{e\cot(c+dx)}}{\sqrt{e}}\right)}{\sqrt{2}de^{9/2}}\\ &-\frac{(a-b)\left(a^2+4ab+b^2\right)\arctan\left(\frac{\sqrt{2}\sqrt{e\cot(c+dx)}}{\sqrt{e}}+1\right)}{\sqrt{2}de^{9/2}}\\ &+\frac{(a+b)\left(a^2-4ab+b^2\right)\log\left(\sqrt{e}\cot(c+dx)-\sqrt{2}\sqrt{e\cot(c+dx)}+\sqrt{e}\right)}{2\sqrt{2}de^{9/2}}\\ &-\frac{(a+b)\left(a^2-4ab+b^2\right)\log\left(\sqrt{e}\cot(c+dx)+\sqrt{2}\sqrt{e\cot(c+dx)}+\sqrt{e}\right)}{2\sqrt{2}de^{9/2}}\\ &-\frac{2b(3a^2-b^2)}{de^4\sqrt{e\cot(c+dx)}}-\frac{2a(a^2-3b^2)}{3de^3(e\cot(c+dx))^{3/2}}\\ &+\frac{32a^2b}{35de^2(e\cot(c+dx))^{5/2}}+\frac{2a^2(a+b\cot(c+dx))}{7de(e\cot(c+dx))^{7/2}}\end{aligned}$$

```
[In] Int[(a + b*Cot[c + d*x])^3/(e*Cot[c + d*x])^(9/2),x]
```

```
[Out] ((a - b)*(a^2 + 4*a*b + b^2)*ArcTan[1 - (Sqrt[2]*Sqrt[e*Cot[c + d*x]])/Sqrt
[e]])/(Sqrt[2]*d*e^(9/2)) - ((a - b)*(a^2 + 4*a*b + b^2)*ArcTan[1 + (Sqrt[2
]*Sqrt[e*Cot[c + d*x]])/Sqrt[e]])/(Sqrt[2]*d*e^(9/2)) + (32*a^2*b)/(35*d*e^
2*(e*Cot[c + d*x])^(5/2)) - (2*a*(a^2 - 3*b^2))/(3*d*e^3*(e*Cot[c + d*x])^(
3/2)) - (2*b*(3*a^2 - b^2))/(d*e^4*Sqrt[e*Cot[c + d*x]]) + (2*a^2*(a + b*Co
t[c + d*x]))/(7*d*e*(e*Cot[c + d*x])^(7/2)) + ((a + b)*(a^2 - 4*a*b + b^2)*
Log[Sqrt[e] + Sqrt[e]*Cot[c + d*x] - Sqrt[2]*Sqrt[e*Cot[c + d*x]]])/(2*Sqrt
[2]*d*e^(9/2)) - ((a + b)*(a^2 - 4*a*b + b^2)*Log[Sqrt[e] + Sqrt[e]*Cot[c +
 d*x] + Sqrt[2]*Sqrt[e*Cot[c + d*x]]])/(2*Sqrt[2]*d*e^(9/2))
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1179

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 1182

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
a*c, 2]}, Dist[(d*q + a*e)/(2*a*c), Int[(q + c*x^2)/(a + c*x^4), x], x] + D
ist[(d*q - a*e)/(2*a*c), Int[(q - c*x^2)/(a + c*x^4), x], x]] /; FreeQ[{a,
c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0] && NeQ[c*d^2 - a*e^2, 0] && NegQ[(-a)
*c]
```

Rule 3610

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)]), x_Symbol] :> Simp[(b*c - a*d)*((a + b*Tan[e + f*x])^(m + 1)/
(f*(m + 1)*(a^2 + b^2))), x] + Dist[1/(a^2 + b^2), Int[(a + b*Tan[e + f*x])
^(m + 1)*Simp[a*c + b*d - (b*c - a*d)*Tan[e + f*x], x], x], x] /; FreeQ[{a,
 b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && LtQ[m, -1]
```

Rule 3615

```
Int[((c_) + (d_.)*tan[(e_.) + (f_.)*(x_)])/Sqrt[(b_.)*tan[(e_.) + (f_.)*(x_
)]], x_Symbol] :> Dist[2/f, Subst[Int[(b*c + d*x^2)/(b^2 + x^4), x], x, Sqr
t[b*Tan[e + f*x]]], x] /; FreeQ[{b, c, d, e, f}, x] && NeQ[c^2 - d^2, 0] &&
 NeQ[c^2 + d^2, 0]
```

Rule 3646

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*c - a*d)^2*(a + b*Tan[e + f*x])^(m
```

```
 - 2)*((c + d*Tan[e + f*x])^(n + 1)/(d*f*(n + 1)*(c^2 + d^2))), x] - Dist[1
/(d*(n + 1)*(c^2 + d^2)), Int[(a + b*Tan[e + f*x])^(m - 3)*(c + d*Tan[e + f
*x])^(n + 1)*Simp[a^2*d*(b*d*(m - 2) - a*c*(n + 1)) + b*(b*c - 2*a*d)*(b*c*
(m - 2) + a*d*(n + 1)) - d*(n + 1)*(3*a^2*b*c - b^3*c - a^3*d + 3*a*b^2*d)*
Tan[e + f*x] - b*(a*d*(2*b*c - a*d)*(m + n - 1) - b^2*(c^2*(m - 2) - d^2*(n
 + 1)))*Tan[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b
*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] && GtQ[m, 2] && LtQ[
n, -1] && IntegerQ[2*m]
```

Rule 3709

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
 (f_.)*(x_)] + (C_.)*tan[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(A*b^2 -
 a*b*B + a^2*C)*((a + b*Tan[e + f*x])^(m + 1)/(b*f*(m + 1)*(a^2 + b^2))), x
] + Dist[1/(a^2 + b^2), Int[(a + b*Tan[e + f*x])^(m + 1)*Simp[b*B + a*(A -
C) - (A*b - a*B - b*C)*Tan[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B,
 C}, x] && NeQ[A*b^2 - a*b*B + a^2*C, 0] && LtQ[m, -1] && NeQ[a^2 + b^2, 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{2a^2(a+b\cot(c+dx))}{7de(e\cot(c+dx))^{7/2}} - \frac{2\int \frac{-8a^2be^2+\frac{7}{2}a(a^2-3b^2)e^2\cot(c+dx)+\frac{1}{2}b(5a^2-7b^2)e^2\cot^2(c+dx)}{(e\cot(c+dx))^{7/2}}\,dx}{7e^3} \\
&= \frac{32a^2b}{35de^2(e\cot(c+dx))^{5/2}} + \frac{2a^2(a+b\cot(c+dx))}{7de(e\cot(c+dx))^{7/2}} - \frac{2\int \frac{\frac{7}{2}a(a^2-3b^2)e^3+\frac{7}{2}b(3a^2-b^2)e^3\cot(c+dx)}{(e\cot(c+dx))^{5/2}}\,dx}{7e^5} \\
&= \frac{32a^2b}{35de^2(e\cot(c+dx))^{5/2}} - \frac{2a(a^2-3b^2)}{3de^3(e\cot(c+dx))^{3/2}} \\
&\quad + \frac{2a^2(a+b\cot(c+dx))}{7de(e\cot(c+dx))^{7/2}} - \frac{2\int \frac{\frac{7}{2}b(3a^2-b^2)e^4-\frac{7}{2}a(a^2-3b^2)e^4\cot(c+dx)}{(e\cot(c+dx))^{3/2}}\,dx}{7e^7} \\
&= \frac{32a^2b}{35de^2(e\cot(c+dx))^{5/2}} - \frac{2a(a^2-3b^2)}{3de^3(e\cot(c+dx))^{3/2}} - \frac{2b(3a^2-b^2)}{de^4\sqrt{e\cot(c+dx)}} \\
&\quad + \frac{2a^2(a+b\cot(c+dx))}{7de(e\cot(c+dx))^{7/2}} - \frac{2\int \frac{-\frac{7}{2}a(a^2-3b^2)e^5-\frac{7}{2}b(3a^2-b^2)e^5\cot(c+dx)}{\sqrt{e\cot(c+dx)}}\,dx}{7e^9} \\
&= \frac{32a^2b}{35de^2(e\cot(c+dx))^{5/2}} - \frac{2a(a^2-3b^2)}{3de^3(e\cot(c+dx))^{3/2}} \\
&\quad - \frac{2b(3a^2-b^2)}{de^4\sqrt{e\cot(c+dx)}} + \frac{2a^2(a+b\cot(c+dx))}{7de(e\cot(c+dx))^{7/2}} \\
&\quad - \frac{4\text{Subst}\left(\int \frac{\frac{7}{2}a(a^2-3b^2)e^6+\frac{7}{2}b(3a^2-b^2)e^5x^2}{e^2+x^4}\,dx, x, \sqrt{e\cot(c+dx)}\right)}{7de^9}
\end{aligned}$$

$$
\begin{aligned}
&=\frac{32a^2b}{35de^2(e\cot(c+dx))^{5/2}}-\frac{2a(a^2-3b^2)}{3de^3(e\cot(c+dx))^{3/2}}\\
&\quad-\frac{2b(3a^2-b^2)}{de^4\sqrt{e\cot(c+dx)}}+\frac{2a^2(a+b\cot(c+dx))}{7de(e\cot(c+dx))^{7/2}}\\
&\quad-\frac{((a+b)\,(a^2-4ab+b^2))\,\text{Subst}\left(\int\frac{e-x^2}{e^2+x^4}\,dx,x,\sqrt{e\cot(c+dx)}\right)}{de^4}\\
&\quad-\frac{((a-b)\,(a^2+4ab+b^2))\,\text{Subst}\left(\int\frac{e+x^2}{e^2+x^4}\,dx,x,\sqrt{e\cot(c+dx)}\right)}{de^4}
\end{aligned}
$$

$$
\begin{aligned}
&=\frac{32a^2b}{35de^2(e\cot(c+dx))^{5/2}}-\frac{2a(a^2-3b^2)}{3de^3(e\cot(c+dx))^{3/2}}\\
&\quad-\frac{2b(3a^2-b^2)}{de^4\sqrt{e\cot(c+dx)}}+\frac{2a^2(a+b\cot(c+dx))}{7de(e\cot(c+dx))^{7/2}}\\
&\quad+\frac{((a+b)\,(a^2-4ab+b^2))\,\text{Subst}\left(\int\frac{\sqrt{2}\sqrt{e}+2x}{-e-\sqrt{2}\sqrt{e}x-x^2}\,dx,x,\sqrt{e\cot(c+dx)}\right)}{2\sqrt{2}de^{9/2}}\\
&\quad+\frac{((a+b)\,(a^2-4ab+b^2))\,\text{Subst}\left(\int\frac{\sqrt{2}\sqrt{e}-2x}{-e+\sqrt{2}\sqrt{e}x-x^2}\,dx,x,\sqrt{e\cot(c+dx)}\right)}{2\sqrt{2}de^{9/2}}\\
&\quad-\frac{((a-b)\,(a^2+4ab+b^2))\,\text{Subst}\left(\int\frac{1}{e-\sqrt{2}\sqrt{e}x+x^2}\,dx,x,\sqrt{e\cot(c+dx)}\right)}{2de^4}\\
&\quad-\frac{((a-b)\,(a^2+4ab+b^2))\,\text{Subst}\left(\int\frac{1}{e+\sqrt{2}\sqrt{e}x+x^2}\,dx,x,\sqrt{e\cot(c+dx)}\right)}{2de^4}
\end{aligned}
$$

$$
\begin{aligned}
&=\frac{32a^2b}{35de^2(e\cot(c+dx))^{5/2}}-\frac{2a(a^2-3b^2)}{3de^3(e\cot(c+dx))^{3/2}}\\
&\quad-\frac{2b(3a^2-b^2)}{de^4\sqrt{e\cot(c+dx)}}+\frac{2a^2(a+b\cot(c+dx))}{7de(e\cot(c+dx))^{7/2}}\\
&\quad+\frac{(a+b)\,(a^2-4ab+b^2)\log\left(\sqrt{e}+\sqrt{e}\cot(c+dx)-\sqrt{2}\sqrt{e\cot(c+dx)}\right)}{2\sqrt{2}de^{9/2}}\\
&\quad-\frac{(a+b)\,(a^2-4ab+b^2)\log\left(\sqrt{e}+\sqrt{e}\cot(c+dx)+\sqrt{2}\sqrt{e\cot(c+dx)}\right)}{2\sqrt{2}de^{9/2}}\\
&\quad-\frac{((a-b)\,(a^2+4ab+b^2))\,\text{Subst}\left(\int\frac{1}{-1-x^2}\,dx,x,1-\frac{\sqrt{2}\sqrt{e\cot(c+dx)}}{\sqrt{e}}\right)}{\sqrt{2}de^{9/2}}\\
&\quad+\frac{((a-b)\,(a^2+4ab+b^2))\,\text{Subst}\left(\int\frac{1}{-1-x^2}\,dx,x,1+\frac{\sqrt{2}\sqrt{e\cot(c+dx)}}{\sqrt{e}}\right)}{\sqrt{2}de^{9/2}}
\end{aligned}
$$

$$= \frac{(a-b)\left(a^2+4ab+b^2\right)\arctan\left(1-\frac{\sqrt{2}\sqrt{e\cot(c+dx)}}{\sqrt{e}}\right)}{\sqrt{2}de^{9/2}} - \frac{(a-b)\left(a^2+4ab+b^2\right)\arctan\left(1+\frac{\sqrt{2}\sqrt{e\cot(c+dx)}}{\sqrt{e}}\right)}{\sqrt{2}de^{9/2}} + \frac{32a^2b}{35de^2(e\cot(c+dx))^{5/2}} - \frac{2a(a^2-3b^2)}{3de^3(e\cot(c+dx))^{3/2}} - \frac{2b(3a^2-b^2)}{de^4\sqrt{e\cot(c+dx)}} + \frac{2a^2(a+b\cot(c+dx))}{7de(e\cot(c+dx))^{7/2}} + \frac{(a+b)\left(a^2-4ab+b^2\right)\log\left(\sqrt{e}+\sqrt{e}\cot(c+dx)-\sqrt{2}\sqrt{e\cot(c+dx)}\right)}{2\sqrt{2}de^{9/2}} - \frac{(a+b)\left(a^2-4ab+b^2\right)\log\left(\sqrt{e}+\sqrt{e}\cot(c+dx)+\sqrt{2}\sqrt{e\cot(c+dx)}\right)}{2\sqrt{2}de^{9/2}}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 0.80 (sec) , antiderivative size = 116, normalized size of antiderivative = 0.31

$$\int \frac{(a+b\cot(c+dx))^3}{(e\cot(c+dx))^{9/2}}\,dx = \frac{2\sqrt{e\cot(c+dx)}\left(5a(a^2-3b^2)\,\text{Hypergeometric2F1}\left(-\frac{7}{4},1,-\frac{3}{4},-\cot^2(c+dx)\right)\right.}{}$$

```
[In] Integrate[(a + b*Cot[c + d*x])^3/(e*Cot[c + d*x])^(9/2),x]
```

```
[Out] (2*Sqrt[e*Cot[c + d*x]]*(5*a*(a^2 - 3*b^2)*Hypergeometric2F1[-7/4, 1, -3/4,
 -Cot[c + d*x]^2] + b*(b*(15*a + 7*b*Cot[c + d*x]) + 7*(3*a^2 - b^2)*Cot[c
+ d*x]*Hypergeometric2F1[-5/4, 1, -1/4, -Cot[c + d*x]^2]))*Tan[c + d*x]^4)/
(35*d*e^5)
```

Maple [A] (verified)

Time = 0.04 (sec) , antiderivative size = 388, normalized size of antiderivative = 1.03

method	result
derivativedivides	$-\frac{2\left(\frac{\left(a^3e-3aeb^2\right)\left(e^2\right)^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\frac{e\cot(dx+c)+\left(e^2\right)^{\frac{1}{4}}\sqrt{e\cot(dx+c)}\,\sqrt{2}+\sqrt{e^2}}{e\cot(dx+c)-\left(e^2\right)^{\frac{1}{4}}\sqrt{e\cot(dx+c)}\,\sqrt{2}+\sqrt{e^2}}\right)+2\arctan\left(\frac{\sqrt{2}\,\sqrt{e\cot(dx+c)}}{\left(e^2\right)^{\frac{1}{4}}}+1\right)-2\arctan\left(-\frac{\sqrt{2}}{}\right.\right.}{8e^2}}{}\right.}{}$
default	$-\frac{2\left(\frac{\left(a^3e-3aeb^2\right)\left(e^2\right)^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\frac{e\cot(dx+c)+\left(e^2\right)^{\frac{1}{4}}\sqrt{e\cot(dx+c)}\,\sqrt{2}+\sqrt{e^2}}{e\cot(dx+c)-\left(e^2\right)^{\frac{1}{4}}\sqrt{e\cot(dx+c)}\,\sqrt{2}+\sqrt{e^2}}\right)+2\arctan\left(\frac{\sqrt{2}\,\sqrt{e\cot(dx+c)}}{\left(e^2\right)^{\frac{1}{4}}}+1\right)-2\arctan\left(-\frac{\sqrt{2}}{}\right.\right.}{8e^2}}{}\right.}{}$
parts	$-\frac{2a^3e\left(-\frac{1}{7e^2(e\cot(dx+c))^{\frac{7}{2}}}+\frac{1}{3e^4(e\cot(dx+c))^{\frac{3}{2}}}+\frac{\left(e^2\right)^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\frac{e\cot(dx+c)+\left(e^2\right)^{\frac{1}{4}}\sqrt{e\cot(dx+c)}\,\sqrt{2}+\sqrt{e^2}}{e\cot(dx+c)-\left(e^2\right)^{\frac{1}{4}}\sqrt{e\cot(dx+c)}\,\sqrt{2}+\sqrt{e^2}}\right)+2\arctan\left(\frac{\sqrt{2}\,\sqrt{}}{}\right.\right.}{8e^6}\right.}{d}$

```
[In] int((a+b*cot(d*x+c))^3/(e*cot(d*x+c))^(9/2),x,method=_RETURNVERBOSE)
```

```
[Out] -2/d/e^2*(1/e^2*(1/8*(a^3*e-3*a*b^2*e)*(e^2)^(1/4)/e^2*2^(1/2)*(ln((e*cot(d
*x+c)+(e^2)^(1/4)*(e*cot(d*x+c))^(1/2)*2^(1/2)+(e^2)^(1/2))/(e*cot(d*x+c)-(
e^2)^(1/4)*(e*cot(d*x+c))^(1/2)*2^(1/2)+(e^2)^(1/2)))+2*arctan(2^(1/2)/(e^2
)^(1/4)*(e*cot(d*x+c))^(1/2)+1)-2*arctan(-2^(1/2)/(e^2)^(1/4)*(e*cot(d*x+c)
)^(1/2)+1))+1/8*(3*a^2*b-b^3)/(e^2)^(1/4)*2^(1/2)*(ln((e*cot(d*x+c)-(e^2)^(
1/4)*(e*cot(d*x+c))^(1/2)*2^(1/2)+(e^2)^(1/2))/(e*cot(d*x+c)+(e^2)^(1/4)*(e
*cot(d*x+c))^(1/2)*2^(1/2)+(e^2)^(1/2)))+2*arctan(2^(1/2)/(e^2)^(1/4)*(e*co
t(d*x+c))^(1/2)+1)-2*arctan(-2^(1/2)/(e^2)^(1/4)*(e*cot(d*x+c))^(1/2)+1)))-
1/7*a^3*e/(e*cot(d*x+c))^(7/2)-3/5*a^2*b/(e*cot(d*x+c))^(5/2)+1/3*a/e*(a^2-
3*b^2)/(e*cot(d*x+c))^(3/2)+b*(3*a^2-b^2)/e^2/(e*cot(d*x+c))^(1/2))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1839 vs. 2(316) = 632.

Time = 0.34 (sec) , antiderivative size = 1839, normalized size of antiderivative = 4.88

$$\int \frac{(a+b\cot(c+dx))^3}{(e\cot(c+dx))^{9/2}}\,dx = \text{Too large to display}$$

```
[In] integrate((a+b*cot(d*x+c))^3/(e*cot(d*x+c))^(9/2),x, algorithm="fricas")
[Out] 1/210*(105*(d*e^5*cos(2*d*x + 2*c)^2 + 2*d*e^5*cos(2*d*x + 2*c) + d*e^5)*sq
rt(-(d^2*e^9*sqrt(-(a^12 - 30*a^10*b^2 + 255*a^8*b^4 - 452*a^6*b^6 + 255*a^
4*b^8 - 30*a^2*b^10 + b^12)/(d^4*e^18)) + 6*a^5*b - 20*a^3*b^3 + 6*a*b^5)/(
d^2*e^9))*log(-(a^12 - 12*a^10*b^2 - 27*a^8*b^4 + 27*a^4*b^8 + 12*a^2*b^10
- b^12)*sqrt((e*cos(2*d*x + 2*c) + e)/sin(2*d*x + 2*c)) + ((3*a^2*b - b^3)*
d^3*e^14*sqrt(-(a^12 - 30*a^10*b^2 + 255*a^8*b^4 - 452*a^6*b^6 + 255*a^4*b^
8 - 30*a^2*b^10 + b^12)/(d^4*e^18)) + (a^9 - 18*a^7*b^2 + 60*a^5*b^4 - 46*a
^3*b^6 + 3*a*b^8)*d*e^5)*sqrt(-(d^2*e^9*sqrt(-(a^12 - 30*a^10*b^2 + 255*a^8
*b^4 - 452*a^6*b^6 + 255*a^4*b^8 - 30*a^2*b^10 + b^12)/(d^4*e^18)) + 6*a^5*
b - 20*a^3*b^3 + 6*a*b^5)/(d^2*e^9))) - 105*(d*e^5*cos(2*d*x + 2*c)^2 + 2*d
*e^5*cos(2*d*x + 2*c) + d*e^5)*sqrt(-(d^2*e^9*sqrt(-(a^12 - 30*a^10*b^2 + 2
55*a^8*b^4 - 452*a^6*b^6 + 255*a^4*b^8 - 30*a^2*b^10 + b^12)/(d^4*e^18)) +
6*a^5*b - 20*a^3*b^3 + 6*a*b^5)/(d^2*e^9))*log(-(a^12 - 12*a^10*b^2 - 27*a^
8*b^4 + 27*a^4*b^8 + 12*a^2*b^10 - b^12)*sqrt((e*cos(2*d*x + 2*c) + e)/sin(
2*d*x + 2*c)) - ((3*a^2*b - b^3)*d^3*e^14*sqrt(-(a^12 - 30*a^10*b^2 + 255*a
^8*b^4 - 452*a^6*b^6 + 255*a^4*b^8 - 30*a^2*b^10 + b^12)/(d^4*e^18)) + (a^9
 - 18*a^7*b^2 + 60*a^5*b^4 - 46*a^3*b^6 + 3*a*b^8)*d*e^5)*sqrt(-(d^2*e^9*sq
rt(-(a^12 - 30*a^10*b^2 + 255*a^8*b^4 - 452*a^6*b^6 + 255*a^4*b^8 - 30*a^2*
b^10 + b^12)/(d^4*e^18)) + 6*a^5*b - 20*a^3*b^3 + 6*a*b^5)/(d^2*e^9))) - 10
5*(d*e^5*cos(2*d*x + 2*c)^2 + 2*d*e^5*cos(2*d*x + 2*c) + d*e^5)*sqrt((d^2*e
^9*sqrt(-(a^12 - 30*a^10*b^2 + 255*a^8*b^4 - 452*a^6*b^6 + 255*a^4*b^8 - 30
*a^2*b^10 + b^12)/(d^4*e^18)) - 6*a^5*b + 20*a^3*b^3 - 6*a*b^5)/(d^2*e^9))*
log(-(a^12 - 12*a^10*b^2 - 27*a^8*b^4 + 27*a^4*b^8 + 12*a^2*b^10 - b^12)*sq
rt((e*cos(2*d*x + 2*c) + e)/sin(2*d*x + 2*c)) + ((3*a^2*b - b^3)*d^3*e^14*s
qrt(-(a^12 - 30*a^10*b^2 + 255*a^8*b^4 - 452*a^6*b^6 + 255*a^4*b^8 - 30*a^2
*b^10 + b^12)/(d^4*e^18)) - (a^9 - 18*a^7*b^2 + 60*a^5*b^4 - 46*a^3*b^6 + 3
*a*b^8)*d*e^5)*sqrt((d^2*e^9*sqrt(-(a^12 - 30*a^10*b^2 + 255*a^8*b^4 - 452*
a^6*b^6 + 255*a^4*b^8 - 30*a^2*b^10 + b^12)/(d^4*e^18)) - 6*a^5*b + 20*a^3*
b^3 - 6*a*b^5)/(d^2*e^9))) + 105*(d*e^5*cos(2*d*x + 2*c)^2 + 2*d*e^5*cos(2*
d*x + 2*c) + d*e^5)*sqrt((d^2*e^9*sqrt(-(a^12 - 30*a^10*b^2 + 255*a^8*b^4 -
 452*a^6*b^6 + 255*a^4*b^8 - 30*a^2*b^10 + b^12)/(d^4*e^18)) - 6*a^5*b + 20
*a^3*b^3 - 6*a*b^5)/(d^2*e^9))*log(-(a^12 - 12*a^10*b^2 - 27*a^8*b^4 + 27*a
^4*b^8 + 12*a^2*b^10 - b^12)*sqrt((e*cos(2*d*x + 2*c) + e)/sin(2*d*x + 2*c)
) - ((3*a^2*b - b^3)*d^3*e^14*sqrt(-(a^12 - 30*a^10*b^2 + 255*a^8*b^4 - 452
*a^6*b^6 + 255*a^4*b^8 - 30*a^2*b^10 + b^12)/(d^4*e^18)) - (a^9 - 18*a^7*b^
```

```
2 + 60*a^5*b^4 - 46*a^3*b^6 + 3*a*b^8)*d*e^5)*sqrt((d^2*e^9*sqrt(-(a^12 - 3
0*a^10*b^2 + 255*a^8*b^4 - 452*a^6*b^6 + 255*a^4*b^8 - 30*a^2*b^10 + b^12)/
(d^4*e^18)) - 6*a^5*b + 20*a^3*b^3 - 6*a*b^5)/(d^2*e^9))) - 4*(30*a^3*cos(2
*d*x + 2*c) + 20*a^3 - 105*a*b^2 - 5*(10*a^3 - 21*a*b^2)*cos(2*d*x + 2*c)^2
 + 21*(12*a^2*b - 5*b^3 + (18*a^2*b - 5*b^3)*cos(2*d*x + 2*c))*sin(2*d*x +
2*c))*sqrt((e*cos(2*d*x + 2*c) + e)/sin(2*d*x + 2*c)))/(d*e^5*cos(2*d*x + 2
*c)^2 + 2*d*e^5*cos(2*d*x + 2*c) + d*e^5)
```

Sympy [F]

$$\int \frac{(a+b\cot(c+dx))^3}{(e\cot(c+dx))^{9/2}}\,dx = \int \frac{(a+b\cot{(c+dx)})^3}{(e\cot{(c+dx)})^{\frac{9}{2}}}\,dx$$

```
[In] integrate((a+b*cot(d*x+c))**3/(e*cot(d*x+c))**(9/2),x)
[Out] Integral((a + b*cot(c + d*x))**3/(e*cot(c + d*x))**(9/2), x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{(a+b\cot(c+dx))^3}{(e\cot(c+dx))^{9/2}}\,dx = \text{Exception raised: ValueError}$$

```
[In] integrate((a+b*cot(d*x+c))^3/(e*cot(d*x+c))^(9/2),x, algorithm="maxima")
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(e>0)', see `assume?` for more detai
ls)Is e
```

Giac [F(-1)]

Timed out.

$$\int \frac{(a+b\cot(c+dx))^3}{(e\cot(c+dx))^{9/2}}\,dx = \text{Timed out}$$

```
[In] integrate((a+b*cot(d*x+c))^3/(e*cot(d*x+c))^(9/2),x, algorithm="giac")
[Out] Timed out
```

Mupad [B] (verification not implemented)

Time = 18.02 (sec) , antiderivative size = 1992, normalized size of antiderivative = 5.28

$$\int \frac{(a+b\cot(c+dx))^3}{(e\cot(c+dx))^{9/2}}\,dx = \text{Too large to display}$$

[In] int((a + b*cot(c + d*x))^3/(e*cot(c + d*x))^(9/2),x)

[Out] atan((((e*cot(c + d*x))^(1/2)*(16*a^6*d^3*e^14 - 16*b^6*d^3*e^14 + 240*a^2*b^4*d^3*e^14 - 240*a^4*b^2*d^3*e^14) + (32*a^3*d^4*e^19 - 96*a*b^2*d^4*e^19)*(((a*b^5*6i + a^5*b*6i - a^6 + b^6 - 15*a^2*b^4 - a^3*b^3*20i + 15*a^4*b^2)*1i)/(4*d^2*e^9))^(1/2))*(((a*b^5*6i + a^5*b*6i - a^6 + b^6 - 15*a^2*b^4 - a^3*b^3*20i + 15*a^4*b^2)*1i)/(4*d^2*e^9))^(1/2)*1i + ((e*cot(c + d*x))^(1/2)*(16*a^6*d^3*e^14 - 16*b^6*d^3*e^14 + 240*a^2*b^4*d^3*e^14 - 240*a^4*b^2*d^3*e^14) - (32*a^3*d^4*e^19 - 96*a*b^2*d^4*e^19)*(((a*b^5*6i + a^5*b*6i - a^6 + b^6 - 15*a^2*b^4 - a^3*b^3*20i + 15*a^4*b^2)*1i)/(4*d^2*e^9))^(1/2))*(((a*b^5*6i + a^5*b*6i - a^6 + b^6 - 15*a^2*b^4 - a^3*b^3*20i + 15*a^4*b^2)*1i)/(4*d^2*e^9))^(1/2)*1i)/(((e*cot(c + d*x))^(1/2)*(16*a^6*d^3*e^14 - 16*b^6*d^3*e^14 + 240*a^2*b^4*d^3*e^14 - 240*a^4*b^2*d^3*e^14) + (32*a^3*d^4*e^19 - 96*a*b^2*d^4*e^19)*(((a*b^5*6i + a^5*b*6i - a^6 + b^6 - 15*a^2*b^4 - a^3*b^3*20i + 15*a^4*b^2)*1i)/(4*d^2*e^9))^(1/2))*(((a*b^5*6i + a^5*b*6i - a^6 + b^6 - 15*a^2*b^4 - a^3*b^3*20i + 15*a^4*b^2)*1i)/(4*d^2*e^9))^(1/2) - ((e*cot(c + d*x))^(1/2)*(16*a^6*d^3*e^14 - 16*b^6*d^3*e^14 + 240*a^2*b^4*d^3*e^14 - 240*a^4*b^2*d^3*e^14) - (32*a^3*d^4*e^19 - 96*a*b^2*d^4*e^19)*(((a*b^5*6i + a^5*b*6i - a^6 + b^6 - 15*a^2*b^4 - a^3*b^3*20i + 15*a^4*b^2)*1i)/(4*d^2*e^9))^(1/2))*(((a*b^5*6i + a^5*b*6i - a^6 + b^6 - 15*a^2*b^4 - a^3*b^3*20i + 15*a^4*b^2)*1i)/(4*d^2*e^9))^(1/2) - 16*b^9*d^2*e^10 + 48*a^8*b*d^2*e^10 + 96*a^4*b^5*d^2*e^10 + 128*a^6*b^3*d^2*e^10))*(((a*b^5*6i + a^5*b*6i - a^6 + b^6 - 15*a^2*b^4 - a^3*b^3*20i + 15*a^4*b^2)*1i)/(4*d^2*e^9))^(1/2)*2i + atan((((e*cot(c + d*x))^(1/2)*(16*a^6*d^3*e^14 - 16*b^6*d^3*e^14 + 240*a^2*b^4*d^3*e^14 - 240*a^4*b^2*d^3*e^14) + (32*a^3*d^4*e^19 - 96*a*b^2*d^4*e^19)*(((a*b^5*6i + a^5*b*6i + a^6 - b^6 + 15*a^2*b^4 - a^3*b^3*20i - 15*a^4*b^2)*1i)/(4*d^2*e^9))^(1/2))*(((a*b^5*6i + a^5*b*6i + a^6 - b^6 + 15*a^2*b^4 - a^3*b^3*20i - 15*a^4*b^2)*1i)/(4*d^2*e^9))^(1/2)*1i + ((e*cot(c + d*x))^(1/2)*(16*a^6*d^3*e^14 - 16*b^6*d^3*e^14 + 240*a^2*b^4*d^3*e^14 - 240*a^4*b^2*d^3*e^14) - (32*a^3*d^4*e^19 - 96*a*b^2*d^4*e^19)*(((a*b^5*6i + a^5*b*6i + a^6 - b^6 + 15*a^2*b^4 - a^3*b^3*20i - 15*a^4*b^2)*1i)/(4*d^2*e^9))^(1/2))*(((a*b^5*6i + a^5*b*6i + a^6 - b^6 + 15*a^2*b^4 - a^3*b^3*20i - 15*a^4*b^2)*1i)/(4*d^2*e^9))^(1/2)*1i)/(((e*cot(c + d*x))^(1/2)*(16*a^6*d^3*e^14 - 16*b^6*d^3*e^14 + 240*a^2*b^4*d^3*e^14 - 240*a^4*b^2*d^3*e^14) + (32*a^3*d^4*e^19 - 96*a*b^2*d^4*e^19)*(((a*b^5*6i + a^5*b*6i + a^6 - b^6 + 15*a^2*b^4 - a^3*b^3*20i - 15*a^4*b^2)*1i)/(4*d^2*e^9))^(1/2))*(((a*b^5*6i + a^5*b*6i + a^6 - b^6 + 15*a^2*b^4 - a^3*b^3*20i - 15*a^4*b^2)*1i)/(4*d^2*e^9))^(1/2) - ((e*cot(c + d*x))^(1/2)*(16*a^6*d^3*e^14 - 16*b^6*d^3*e^14 +

```
 240*a^2*b^4*d^3*e^14 - 240*a^4*b^2*d^3*e^14) - (32*a^3*d^4*e^19 - 96*a*b^2
*d^4*e^19)*(((a*b^5*6i + a^5*b*6i + a^6 - b^6 + 15*a^2*b^4 - a^3*b^3*20i -
15*a^4*b^2)*1i)/(4*d^2*e^9))^(1/2))*(((a*b^5*6i + a^5*b*6i + a^6 - b^6 + 15
*a^2*b^4 - a^3*b^3*20i - 15*a^4*b^2)*1i)/(4*d^2*e^9))^(1/2) - 16*b^9*d^2*e^
10 + 48*a^8*b*d^2*e^10 + 96*a^4*b^5*d^2*e^10 + 128*a^6*b^3*d^2*e^10))*(((a*
b^5*6i + a^5*b*6i + a^6 - b^6 + 15*a^2*b^4 - a^3*b^3*20i - 15*a^4*b^2)*1i)/
(4*d^2*e^9))^(1/2)*2i + ((2*a^3*e)/7 + (2*e*cot(c + d*x)^2*(3*a*b^2 - a^3))
/3 - 2*e*cot(c + d*x)^3*(3*a^2*b - b^3) + (6*a^2*b*e*cot(c + d*x))/5)/(d*e^
2*(e*cot(c + d*x))^(7/2))
```

3.69 $\int \frac{(e\cot(c+dx))^{5/2}}{a+b\cot(c+dx)}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 325

$$\int \frac{(e\cot(c+dx))^{5/2}}{a+b\cot(c+dx)}\,dx = \frac{2a^{5/2}e^{5/2}\arctan\left(\frac{\sqrt{b}\sqrt{e\cot(c+dx)}}{\sqrt{a}\sqrt{e}}\right)}{b^{3/2}(a^2+b^2)\,d} - \frac{(a+b)e^{5/2}\arctan\left(1-\frac{\sqrt{2}\sqrt{e\cot(c+dx)}}{\sqrt{e}}\right)}{\sqrt{2}\,(a^2+b^2)\,d} + \frac{(a+b)e^{5/2}\arctan\left(1+\frac{\sqrt{2}\sqrt{e\cot(c+dx)}}{\sqrt{e}}\right)}{\sqrt{2}\,(a^2+b^2)\,d} - \frac{2e^2\sqrt{e\cot(c+dx)}}{bd} + \frac{(a-b)e^{5/2}\log\left(\sqrt{e}+\sqrt{e}\cot(c+dx)-\sqrt{2}\sqrt{e\cot(c+dx)}\right)}{2\sqrt{2}\,(a^2+b^2)\,d} - \frac{(a-b)e^{5/2}\log\left(\sqrt{e}+\sqrt{e}\cot(c+dx)+\sqrt{2}\sqrt{e\cot(c+dx)}\right)}{2\sqrt{2}\,(a^2+b^2)\,d}$$

```
[Out] 2*a^(5/2)*e^(5/2)*arctan(b^(1/2)*(e*cot(d*x+c))^(1/2)/a^(1/2)/e^(1/2))/b^(3
/2)/(a^2+b^2)/d-1/2*(a+b)*e^(5/2)*arctan(1-2^(1/2)*(e*cot(d*x+c))^(1/2)/e^(
1/2))/(a^2+b^2)/d*2^(1/2)+1/2*(a+b)*e^(5/2)*arctan(1+2^(1/2)*(e*cot(d*x+c))
^(1/2)/e^(1/2))/(a^2+b^2)/d*2^(1/2)+1/4*(a-b)*e^(5/2)*ln(e^(1/2)+cot(d*x+c)
*e^(1/2)-2^(1/2)*(e*cot(d*x+c))^(1/2))/(a^2+b^2)/d*2^(1/2)-1/4*(a-b)*e^(5/2
)*ln(e^(1/2)+cot(d*x+c)*e^(1/2)+2^(1/2)*(e*cot(d*x+c))^(1/2))/(a^2+b^2)/d*2
^(1/2)-2*e^2*(e*cot(d*x+c))^(1/2)/b/d
```

Rubi [A] (verified)

Time = 0.80 (sec) , antiderivative size = 325, normalized size of antiderivative = 1.00, number of steps used = 15, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.480, Rules used = {3647, 3734, 3615, 1182, 1176, 631, 210, 1179, 642, 3715, 65, 211}

$$\int \frac{(e\cot(c+dx))^{5/2}}{a+b\cot(c+dx)}\,dx=-\frac{e^{5/2}(a+b)\arctan\left(1-\frac{\sqrt{2}\sqrt{e\cot(c+dx)}}{\sqrt{e}}\right)}{\sqrt{2}d\,(a^2+b^2)}+\frac{e^{5/2}(a+b)\arctan\left(\frac{\sqrt{2}\sqrt{e\cot(c+dx)}}{\sqrt{e}}+1\right)}{\sqrt{2}d\,(a^2+b^2)}+\frac{e^{5/2}(a-b)\log\left(\sqrt{e}\cot(c+dx)-\sqrt{2}\sqrt{e\cot(c+dx)}+\sqrt{e}\right)}{2\sqrt{2}d\,(a^2+b^2)}-\frac{e^{5/2}(a-b)\log\left(\sqrt{e}\cot(c+dx)+\sqrt{2}\sqrt{e\cot(c+dx)}+\sqrt{e}\right)}{2\sqrt{2}d\,(a^2+b^2)}+\frac{2a^{5/2}e^{5/2}\arctan\left(\frac{\sqrt{b}\sqrt{e\cot(c+dx)}}{\sqrt{a}\sqrt{e}}\right)}{b^{3/2}d\,(a^2+b^2)}-\frac{2e^2\sqrt{e\cot(c+dx)}}{bd}$$

```
[In] Int[(e*Cot[c + d*x])^(5/2)/(a + b*Cot[c + d*x]),x]
```

```
[Out] (2*a^(5/2)*e^(5/2)*ArcTan[(Sqrt[b]*Sqrt[e*Cot[c + d*x]])/(Sqrt[a]*Sqrt[e])]
)/(b^(3/2)*(a^2 + b^2)*d) - ((a + b)*e^(5/2)*ArcTan[1 - (Sqrt[2]*Sqrt[e*Cot
[c + d*x]])/Sqrt[e]])/(Sqrt[2]*(a^2 + b^2)*d) + ((a + b)*e^(5/2)*ArcTan[1 +
 (Sqrt[2]*Sqrt[e*Cot[c + d*x]])/Sqrt[e]])/(Sqrt[2]*(a^2 + b^2)*d) - (2*e^2*
Sqrt[e*Cot[c + d*x]])/(b*d) + ((a - b)*e^(5/2)*Log[Sqrt[e] + Sqrt[e]*Cot[c
+ d*x] - Sqrt[2]*Sqrt[e*Cot[c + d*x]]])/(2*Sqrt[2]*(a^2 + b^2)*d) - ((a - b
)*e^(5/2)*Log[Sqrt[e] + Sqrt[e]*Cot[c + d*x] + Sqrt[2]*Sqrt[e*Cot[c + d*x]]
])/(2*Sqrt[2]*(a^2 + b^2)*d)
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1179

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 1182

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
a*c, 2]}, Dist[(d*q + a*e)/(2*a*c), Int[(q + c*x^2)/(a + c*x^4), x], x] + D
ist[(d*q - a*e)/(2*a*c), Int[(q - c*x^2)/(a + c*x^4), x], x]] /; FreeQ[{a,
c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0] && NeQ[c*d^2 - a*e^2, 0] && NegQ[(-a)
*c]
```

Rule 3615

```
Int[((c_) + (d_.)*tan[(e_.) + (f_.)*(x_)])/Sqrt[(b_.)*tan[(e_.) + (f_.)*(x_
)]], x_Symbol] :> Dist[2/f, Subst[Int[(b*c + d*x^2)/(b^2 + x^4), x], x, Sqr
t[b*Tan[e + f*x]]], x] /; FreeQ[{b, c, d, e, f}, x] && NeQ[c^2 - d^2, 0] &&
 NeQ[c^2 + d^2, 0]
```

Rule 3647

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)])^(n_), x_Symbol] :> Simp[b^2*(a + b*Tan[e + f*x])^(m - 2)*((c
+ d*Tan[e + f*x])^(n + 1)/(d*f*(m + n - 1))), x] + Dist[1/(d*(m + n - 1)),
Int[(a + b*Tan[e + f*x])^(m - 3)*(c + d*Tan[e + f*x])^n*Simp[a^3*d*(m + n -
 1) - b^2*(b*c*(m - 2) + a*d*(1 + n)) + b*d*(m + n - 1)*(3*a^2 - b^2)*Tan[e
 + f*x] - b^2*(b*c*(m - 2) - a*d*(3*m + 2*n - 4))*Tan[e + f*x]^2, x], x], x
] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2,
 0] && NeQ[c^2 + d^2, 0] && IntegerQ[2*m] && GtQ[m, 2] && (GeQ[n, -1] || In
tegerQ[m]) &&  !(IGtQ[n, 2] && ( !IntegerQ[m] || (EqQ[c, 0] && NeQ[a, 0])))
```

Rule 3715

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*tan[(e_.)
+ (f_.)*(x_)])^(n_.)*((A_) + (C_.)*tan[(e_.) + (f_.)*(x_)]^2), x_Symbol] :>
 Dist[A/f, Subst[Int[(a + b*x)^m*(c + d*x)^n, x], x, Tan[e + f*x]], x] /; F
reeQ[{a, b, c, d, e, f, A, C, m, n}, x] && EqQ[A, C]
```

Rule 3734

```
Int[(((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_)*((A_.) + (B_.)*tan[(e_.)
+ (f_.)*(x_)] + (C_.)*tan[(e_.) + (f_.)*(x_)]^2))/((a_.) + (b_.)*tan[(e_.)
+ (f_.)*(x_)]), x_Symbol] :> Dist[1/(a^2 + b^2), Int[(c + d*Tan[e + f*x])^n
*Simp[b*B + a*(A - C) + (a*B - b*(A - C))*Tan[e + f*x], x], x], x] + Dist[(
A*b^2 - a*b*B + a^2*C)/(a^2 + b^2), Int[(c + d*Tan[e + f*x])^n*((1 + Tan[e
+ f*x]^2)/(a + b*Tan[e + f*x])), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C
, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] &&
 !GtQ[n, 0] &&  !LeQ[n, -1]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\frac{2e^2\sqrt{e\cot(c+dx)}}{bd} - \frac{2\int \frac{\frac{ae^3}{2}+\frac{1}{2}be^3\cot(c+dx)+\frac{1}{2}ae^3\cot^2(c+dx)}{\sqrt{e\cot(c+dx)}(a+b\cot(c+dx))}\,dx}{b} \\
&= -\frac{2e^2\sqrt{e\cot(c+dx)}}{bd} - \frac{2\int \frac{\frac{b^2e^3}{2}+\frac{1}{2}abe^3\cot(c+dx)}{\sqrt{e\cot(c+dx)}}\,dx}{b(a^2+b^2)} - \frac{(a^3e^3)\int \frac{1+\cot^2(c+dx)}{\sqrt{e\cot(c+dx)}(a+b\cot(c+dx))}\,dx}{b(a^2+b^2)} \\
&= -\frac{2e^2\sqrt{e\cot(c+dx)}}{bd} - \frac{4\text{Subst}\left(\int \frac{-\frac{1}{2}b^2e^4-\frac{1}{2}abe^3x^2}{e^2+x^4}\,dx, x, \sqrt{e\cot(c+dx)}\right)}{b(a^2+b^2)d} \\
&\quad - \frac{(a^3e^3)\,\text{Subst}\left(\int \frac{1}{\sqrt{-ex}(a-bx)}\,dx, x, -\cot(c+dx)\right)}{b(a^2+b^2)d}
\end{aligned}$$

$$= -\frac{2e^2\sqrt{e\cot(c+dx)}}{bd} + \frac{(2a^3e^2)\,\text{Subst}\left(\int \frac{1}{a+\frac{bx^2}{e}}\,dx, x, \sqrt{e\cot(c+dx)}\right)}{b\,(a^2+b^2)\,d}$$
$$- \frac{((a-b)e^3)\,\text{Subst}\left(\int \frac{e-x^2}{e^2+x^4}\,dx, x, \sqrt{e\cot(c+dx)}\right)}{(a^2+b^2)\,d}$$
$$+ \frac{((a+b)e^3)\,\text{Subst}\left(\int \frac{e+x^2}{e^2+x^4}\,dx, x, \sqrt{e\cot(c+dx)}\right)}{(a^2+b^2)\,d}$$

$$= \frac{2a^{5/2}e^{5/2}\arctan\left(\frac{\sqrt{b}\sqrt{e\cot(c+dx)}}{\sqrt{a}\sqrt{e}}\right)}{b^{3/2}\,(a^2+b^2)\,d} - \frac{2e^2\sqrt{e\cot(c+dx)}}{bd}$$
$$+ \frac{((a-b)e^{5/2})\,\text{Subst}\left(\int \frac{\sqrt{2}\sqrt{e}+2x}{-e-\sqrt{2}\sqrt{e}x-x^2}\,dx, x, \sqrt{e\cot(c+dx)}\right)}{2\sqrt{2}\,(a^2+b^2)\,d}$$
$$+ \frac{((a-b)e^{5/2})\,\text{Subst}\left(\int \frac{\sqrt{2}\sqrt{e}-2x}{-e+\sqrt{2}\sqrt{e}x-x^2}\,dx, x, \sqrt{e\cot(c+dx)}\right)}{2\sqrt{2}\,(a^2+b^2)\,d}$$
$$+ \frac{((a+b)e^3)\,\text{Subst}\left(\int \frac{1}{e-\sqrt{2}\sqrt{e}x+x^2}\,dx, x, \sqrt{e\cot(c+dx)}\right)}{2\,(a^2+b^2)\,d}$$
$$+ \frac{((a+b)e^3)\,\text{Subst}\left(\int \frac{1}{e+\sqrt{2}\sqrt{e}x+x^2}\,dx, x, \sqrt{e\cot(c+dx)}\right)}{2\,(a^2+b^2)\,d}$$

$$= \frac{2a^{5/2}e^{5/2}\arctan\left(\frac{\sqrt{b}\sqrt{e\cot(c+dx)}}{\sqrt{a}\sqrt{e}}\right)}{b^{3/2}\,(a^2+b^2)\,d} - \frac{2e^2\sqrt{e\cot(c+dx)}}{bd}$$
$$+ \frac{(a-b)e^{5/2}\log\left(\sqrt{e}+\sqrt{e}\cot(c+dx)-\sqrt{2}\sqrt{e\cot(c+dx)}\right)}{2\sqrt{2}\,(a^2+b^2)\,d}$$
$$- \frac{(a-b)e^{5/2}\log\left(\sqrt{e}+\sqrt{e}\cot(c+dx)+\sqrt{2}\sqrt{e\cot(c+dx)}\right)}{2\sqrt{2}\,(a^2+b^2)\,d}$$
$$+ \frac{((a+b)e^{5/2})\,\text{Subst}\left(\int \frac{1}{-1-x^2}\,dx, x, 1-\frac{\sqrt{2}\sqrt{e\cot(c+dx)}}{\sqrt{e}}\right)}{\sqrt{2}\,(a^2+b^2)\,d}$$
$$- \frac{((a+b)e^{5/2})\,\text{Subst}\left(\int \frac{1}{-1-x^2}\,dx, x, 1+\frac{\sqrt{2}\sqrt{e\cot(c+dx)}}{\sqrt{e}}\right)}{\sqrt{2}\,(a^2+b^2)\,d}$$

$$= \frac{2a^{5/2}e^{5/2}\arctan\left(\frac{\sqrt{b}\sqrt{e\cot(c+dx)}}{\sqrt{a}\sqrt{e}}\right)}{b^{3/2}\left(a^2+b^2\right)d} - \frac{(a+b)e^{5/2}\arctan\left(1-\frac{\sqrt{2}\sqrt{e\cot(c+dx)}}{\sqrt{e}}\right)}{\sqrt{2}\left(a^2+b^2\right)d}$$
$$+ \frac{(a+b)e^{5/2}\arctan\left(1+\frac{\sqrt{2}\sqrt{e\cot(c+dx)}}{\sqrt{e}}\right)}{\sqrt{2}\left(a^2+b^2\right)d} - \frac{2e^2\sqrt{e\cot(c+dx)}}{bd}$$
$$+ \frac{(a-b)e^{5/2}\log\left(\sqrt{e}+\sqrt{e}\cot(c+dx)-\sqrt{2}\sqrt{e\cot(c+dx)}\right)}{2\sqrt{2}\left(a^2+b^2\right)d}$$
$$- \frac{(a-b)e^{5/2}\log\left(\sqrt{e}+\sqrt{e}\cot(c+dx)+\sqrt{2}\sqrt{e\cot(c+dx)}\right)}{2\sqrt{2}\left(a^2+b^2\right)d}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 0.82 (sec) , antiderivative size = 286, normalized size of antiderivative = 0.88

$$\int \frac{(e\cot(c+dx))^{5/2}}{a+b\cot(c+dx)}\,dx = \frac{(e\cot(c+dx))^{5/2}\left(8ab^{3/2}\cot^{\frac{3}{2}}(c+dx)\,\text{Hypergeometric2F1}\left(\frac{3}{4},1,\frac{7}{4},-\cot^2(c+dx)\right)\right.}{}$$

```
[In] Integrate[(e*Cot[c + d*x])^(5/2)/(a + b*Cot[c + d*x]),x]
```

```
[Out] ((e*Cot[c + d*x])^(5/2)*(8*a*b^(3/2)*Cot[c + d*x]^(3/2)*Hypergeometric2F1[3
/4, 1, 7/4, -Cot[c + d*x]^2] - 3*(2*Sqrt[2]*b^(5/2)*ArcTan[1 - Sqrt[2]*Sqrt
[Cot[c + d*x]]] - 2*Sqrt[2]*b^(5/2)*ArcTan[1 + Sqrt[2]*Sqrt[Cot[c + d*x]]]
- 8*a^(5/2)*ArcTan[(Sqrt[b]*Sqrt[Cot[c + d*x]])/Sqrt[a]] + 8*a^2*Sqrt[b]*Sq
rt[Cot[c + d*x]] + 8*b^(5/2)*Sqrt[Cot[c + d*x]] + Sqrt[2]*b^(5/2)*Log[1 - S
qrt[2]*Sqrt[Cot[c + d*x]] + Cot[c + d*x]] - Sqrt[2]*b^(5/2)*Log[1 + Sqrt[2]
*Sqrt[Cot[c + d*x]] + Cot[c + d*x]])))/(12*b^(3/2)*(a^2 + b^2)*d*Cot[c + d*
x]^(5/2))
```

Maple [A] (verified)

Time = 0.11 (sec) , antiderivative size = 347, normalized size of antiderivative = 1.07

method	result
derivativedivides	$-\frac{2e^2\left(\frac{\sqrt{e\cot(dx+c)}}{b}-\frac{a^3 e\arctan\left(\frac{\sqrt{e\cot(dx+c)}\,b}{\sqrt{aeb}}\right)}{b(a^2+b^2)\sqrt{aeb}}-\frac{e\left(\frac{b(e^2)^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\frac{e\cot(dx+c)+(e^2)^{\frac{1}{4}}\sqrt{e\cot(dx+c)}\sqrt{2}+\sqrt{e^2}}{e\cot(dx+c)-(e^2)^{\frac{1}{4}}\sqrt{e\cot(dx+c)}\sqrt{2}+\sqrt{e^2}}\right)+2\,\text{arcta}\right.}{8e}\right.}{}\right.}{}$
default	$-\frac{2e^2\left(\frac{\sqrt{e\cot(dx+c)}}{b}-\frac{a^3 e\arctan\left(\frac{\sqrt{e\cot(dx+c)}\,b}{\sqrt{aeb}}\right)}{b(a^2+b^2)\sqrt{aeb}}-\frac{e\left(\frac{b(e^2)^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\frac{e\cot(dx+c)+(e^2)^{\frac{1}{4}}\sqrt{e\cot(dx+c)}\sqrt{2}+\sqrt{e^2}}{e\cot(dx+c)-(e^2)^{\frac{1}{4}}\sqrt{e\cot(dx+c)}\sqrt{2}+\sqrt{e^2}}\right)+2\,\text{arcta}\right.}{8e}\right.}{}\right.}{}$

```
[In] int((e*cot(d*x+c))^(5/2)/(a+b*cot(d*x+c)),x,method=_RETURNVERBOSE)
[Out] -2/d*e^2*((e*cot(d*x+c))^(1/2)/b-1/b*a^3*e/(a^2+b^2)/(a*e*b)^(1/2)*arctan((
e*cot(d*x+c))^(1/2)*b/(a*e*b)^(1/2))-e/(a^2+b^2)*(1/8*b/e*(e^2)^(1/4)*2^(1/
2)*(ln((e*cot(d*x+c)+(e^2)^(1/4)*(e*cot(d*x+c))^(1/2)*2^(1/2)+(e^2)^(1/2))/
(e*cot(d*x+c)-(e^2)^(1/4)*(e*cot(d*x+c))^(1/2)*2^(1/2)+(e^2)^(1/2)))+2*arct
an(2^(1/2)/(e^2)^(1/4)*(e*cot(d*x+c))^(1/2)+1)-2*arctan(-2^(1/2)/(e^2)^(1/4
)*(e*cot(d*x+c))^(1/2)+1))+1/8*a/(e^2)^(1/4)*2^(1/2)*(ln((e*cot(d*x+c)-(e^2
)^(1/4)*(e*cot(d*x+c))^(1/2)*2^(1/2)+(e^2)^(1/2))/(e*cot(d*x+c)+(e^2)^(1/4)
*(e*cot(d*x+c))^(1/2)*2^(1/2)+(e^2)^(1/2)))+2*arctan(2^(1/2)/(e^2)^(1/4)*(e
*cot(d*x+c))^(1/2)+1)-2*arctan(-2^(1/2)/(e^2)^(1/4)*(e*cot(d*x+c))^(1/2)+1)
)))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1602 vs. $2(262) = 524$.

Time = 0.37 (sec) , antiderivative size = 3267, normalized size of antiderivative = 10.05

$$\int \frac{(e\cot(c+dx))^{5/2}}{a+b\cot(c+dx)}\,dx = \text{Too large to display}$$

```
[In] integrate((e*cot(d*x+c))^(5/2)/(a+b*cot(d*x+c)),x, algorithm="fricas")
[Out] [1/2*(2*a^2*sqrt(-a*e/b)*e^2*log((b*e*cos(2*d*x + 2*c) - a*e*sin(2*d*x + 2*
c) + 2*b*sqrt(-a*e/b)*sqrt((e*cos(2*d*x + 2*c) + e)/sin(2*d*x + 2*c))*sin(2
```

*d*x + 2*c) + b*e)/(b*cos(2*d*x + 2*c) + a*sin(2*d*x + 2*c) + b)) - 4*(a^2 + b^2)*e^2*sqrt((e*cos(2*d*x + 2*c) + e)/sin(2*d*x + 2*c)) - (a^2*b + b^3)*d*sqrt(-(2*a*b*e^5 + sqrt(-(a^4 - 2*a^2*b^2 + b^4)*e^10/((a^8 + 4*a^6*b^2 + 6*a^4*b^4 + 4*a^2*b^6 + b^8)*d^4))*(a^4 + 2*a^2*b^2 + b^4)*d^2)/((a^4 + 2*a^2*b^2 + b^4)*d^2))*log(-(a^2 - b^2)*e^7*sqrt((e*cos(2*d*x + 2*c) + e)/sin(2*d*x + 2*c)) + ((a^2*b - b^3)*d*e^5 - sqrt(-(a^4 - 2*a^2*b^2 + b^4)*e^10/((a^8 + 4*a^6*b^2 + 6*a^4*b^4 + 4*a^2*b^6 + b^8)*d^4))*(a^5 + 2*a^3*b^2 + a*b^4)*d^3)*sqrt(-(2*a*b*e^5 + sqrt(-(a^4 - 2*a^2*b^2 + b^4)*e^10/((a^8 + 4*a^6*b^2 + 6*a^4*b^4 + 4*a^2*b^6 + b^8)*d^4))*(a^4 + 2*a^2*b^2 + b^4)*d^2)/((a^4 + 2*a^2*b^2 + b^4)*d^2))) + (a^2*b + b^3)*d*sqrt(-(2*a*b*e^5 + sqrt(-(a^4 - 2*a^2*b^2 + b^4)*e^10/((a^8 + 4*a^6*b^2 + 6*a^4*b^4 + 4*a^2*b^6 + b^8)*d^4))*(a^4 + 2*a^2*b^2 + b^4)*d^2)/((a^4 + 2*a^2*b^2 + b^4)*d^2))*log(-(a^2 - b^2)*e^7*sqrt((e*cos(2*d*x + 2*c) + e)/sin(2*d*x + 2*c)) - ((a^2*b - b^3)*d*e^5 - sqrt(-(a^4 - 2*a^2*b^2 + b^4)*e^10/((a^8 + 4*a^6*b^2 + 6*a^4*b^4 + 4*a^2*b^6 + b^8)*d^4))*(a^5 + 2*a^3*b^2 + a*b^4)*d^3)*sqrt(-(2*a*b*e^5 + sqrt(-(a^4 - 2*a^2*b^2 + b^4)*e^10/((a^8 + 4*a^6*b^2 + 6*a^4*b^4 + 4*a^2*b^6 + b^8)*d^4))*(a^4 + 2*a^2*b^2 + b^4)*d^2)/((a^4 + 2*a^2*b^2 + b^4)*d^2))) - (a^2*b + b^3)*d*sqrt(-(2*a*b*e^5 - sqrt(-(a^4 - 2*a^2*b^2 + b^4)*e^10/((a^8 + 4*a^6*b^2 + 6*a^4*b^4 + 4*a^2*b^6 + b^8)*d^4))*(a^4 + 2*a^2*b^2 + b^4)*d^2)/((a^4 + 2*a^2*b^2 + b^4)*d^2))*log(-(a^2 - b^2)*e^7*sqrt((e*cos(2*d*x + 2*c) + e)/sin(2*d*x + 2*c)) + ((a^2*b - b^3)*d*e^5 + sqrt(-(a^4 - 2*a^2*b^2 + b^4)*e^10/((a^8 + 4*a^6*b^2 + 6*a^4*b^4 + 4*a^2*b^6 + b^8)*d^4))*(a^5 + 2*a^3*b^2 + a*b^4)*d^3)*sqrt(-(2*a*b*e^5 - sqrt(-(a^4 - 2*a^2*b^2 + b^4)*e^10/((a^8 + 4*a^6*b^2 + 6*a^4*b^4 + 4*a^2*b^6 + b^8)*d^4))*(a^4 + 2*a^2*b^2 + b^4)*d^2)/((a^4 + 2*a^2*b^2 + b^4)*d^2))) + (a^2*b + b^3)*d*sqrt(-(2*a*b*e^5 - sqrt(-(a^4 - 2*a^2*b^2 + b^4)*e^10/((a^8 + 4*a^6*b^2 + 6*a^4*b^4 + 4*a^2*b^6 + b^8)*d^4))*(a^4 + 2*a^2*b^2 + b^4)*d^2)/((a^4 + 2*a^2*b^2 + b^4)*d^2))*log(-(a^2 - b^2)*e^7*sqrt((e*cos(2*d*x + 2*c) + e)/sin(2*d*x + 2*c)) - ((a^2*b - b^3)*d*e^5 + sqrt(-(a^4 - 2*a^2*b^2 + b^4)*e^10/((a^8 + 4*a^6*b^2 + 6*a^4*b^4 + 4*a^2*b^6 + b^8)*d^4))*(a^5 + 2*a^3*b^2 + a*b^4)*d^3)*sqrt(-(2*a*b*e^5 - sqrt(-(a^4 - 2*a^2*b^2 + b^4)*e^10/((a^8 + 4*a^6*b^2 + 6*a^4*b^4 + 4*a^2*b^6 + b^8)*d^4))*(a^4 + 2*a^2*b^2 + b^4)*d^2)/((a^4 + 2*a^2*b^2 + b^4)*d^2))))/((a^2*b + b^3)*d), 1/2*(4*a^2*sqrt(a*e/b)*e^2*arctan(b*sqrt(a*e/b)*sqrt((e*cos(2*d*x + 2*c) + e)/sin(2*d*x + 2*c))/(a*e)) - 4*(a^2 + b^2)*e^2*sqrt((e*cos(2*d*x + 2*c) + e)/sin(2*d*x + 2*c)) - (a^2*b + b^3)*d*sqrt(-(2*a*b*e^5 + sqrt(-(a^4 - 2*a^2*b^2 + b^4)*e^10/((a^8 + 4*a^6*b^2 + 6*a^4*b^4 + 4*a^2*b^6 + b^8)*d^4))*(a^4 + 2*a^2*b^2 + b^4)*d^2)/((a^4 + 2*a^2*b^2 + b^4)*d^2))*log(-(a^2 - b^2)*e^7*sqrt((e*cos(2*d*x + 2*c) + e)/sin(2*d*x + 2*c)) + ((a^2*b - b^3)*d*e^5 - sqrt(-(a^4 - 2*a^2*b^2 + b^4)*e^10/((a^8 + 4*a^6*b^2 + 6*a^4*b^4 + 4*a^2*b^6 + b^8)*d^4))*(a^5 + 2*a^3*b^2 + a*b^4)*d^3)*sqrt(-(2*a*b*e^5 + sqrt(-(a^4 - 2*a^2*b^2 + b^4)*e^10/((a^8 + 4*a^6*b^2 + 6*a^4*b^4 + 4*a^2*b^6 + b^8)*d^4))*(a^4 + 2*a^2*b^2 + b^4)*d^2)/((a^4 + 2*a^2*b^2 + b^4)*d^2))) + (a^2*b + b^3)*d*sqrt(-(2*a*b*e^5 + sqrt(-(a^4 - 2*a^2*b^2 + b^4)*e^10/((a^8 + 4*a^6*b^2 + 6*a^4*b^4 + 4*a^2*b^6 + b^8)*d^4))*(a^4 + 2*a^2*b^2 + b^4)*d^2)/((a^4 + 2*a^2*b^2 + b^4)*d^2))*log

```
(-(a^2 - b^2)*e^7*sqrt((e*cos(2*d*x + 2*c) + e)/sin(2*d*x + 2*c)) - ((a^2*b
 - b^3)*d*e^5 - sqrt(-(a^4 - 2*a^2*b^2 + b^4)*e^10/((a^8 + 4*a^6*b^2 + 6*a^
4*b^4 + 4*a^2*b^6 + b^8)*d^4))*(a^5 + 2*a^3*b^2 + a*b^4)*d^3)*sqrt(-(2*a*b*
e^5 + sqrt(-(a^4 - 2*a^2*b^2 + b^4)*e^10/((a^8 + 4*a^6*b^2 + 6*a^4*b^4 + 4*
a^2*b^6 + b^8)*d^4))*(a^4 + 2*a^2*b^2 + b^4)*d^2)/((a^4 + 2*a^2*b^2 + b^4)*
d^2))) - (a^2*b + b^3)*d*sqrt(-(2*a*b*e^5 - sqrt(-(a^4 - 2*a^2*b^2 + b^4)*e
^10/((a^8 + 4*a^6*b^2 + 6*a^4*b^4 + 4*a^2*b^6 + b^8)*d^4))*(a^4 + 2*a^2*b^2
 + b^4)*d^2)/((a^4 + 2*a^2*b^2 + b^4)*d^2))*log(-(a^2 - b^2)*e^7*sqrt((e*co
s(2*d*x + 2*c) + e)/sin(2*d*x + 2*c)) + ((a^2*b - b^3)*d*e^5 + sqrt(-(a^4 -
 2*a^2*b^2 + b^4)*e^10/((a^8 + 4*a^6*b^2 + 6*a^4*b^4 + 4*a^2*b^6 + b^8)*d^4
))*(a^5 + 2*a^3*b^2 + a*b^4)*d^3)*sqrt(-(2*a*b*e^5 - sqrt(-(a^4 - 2*a^2*b^2
 + b^4)*e^10/((a^8 + 4*a^6*b^2 + 6*a^4*b^4 + 4*a^2*b^6 + b^8)*d^4))*(a^4 +
2*a^2*b^2 + b^4)*d^2)/((a^4 + 2*a^2*b^2 + b^4)*d^2))) + (a^2*b + b^3)*d*sqr
t(-(2*a*b*e^5 - sqrt(-(a^4 - 2*a^2*b^2 + b^4)*e^10/((a^8 + 4*a^6*b^2 + 6*a^
4*b^4 + 4*a^2*b^6 + b^8)*d^4))*(a^4 + 2*a^2*b^2 + b^4)*d^2)/((a^4 + 2*a^2*b
^2 + b^4)*d^2))*log(-(a^2 - b^2)*e^7*sqrt((e*cos(2*d*x + 2*c) + e)/sin(2*d*
x + 2*c)) - ((a^2*b - b^3)*d*e^5 + sqrt(-(a^4 - 2*a^2*b^2 + b^4)*e^10/((a^8
 + 4*a^6*b^2 + 6*a^4*b^4 + 4*a^2*b^6 + b^8)*d^4))*(a^5 + 2*a^3*b^2 + a*b^4)
*d^3)*sqrt(-(2*a*b*e^5 - sqrt(-(a^4 - 2*a^2*b^2 + b^4)*e^10/((a^8 + 4*a^6*b
^2 + 6*a^4*b^4 + 4*a^2*b^6 + b^8)*d^4))*(a^4 + 2*a^2*b^2 + b^4)*d^2)/((a^4
+ 2*a^2*b^2 + b^4)*d^2))))/((a^2*b + b^3)*d)]
```

Sympy [F]

$$\int \frac{(e\cot(c+dx))^{5/2}}{a+b\cot(c+dx)}\,dx = \int \frac{(e\cot(c+dx))^{\frac{5}{2}}}{a+b\cot(c+dx)}\,dx$$

```
[In] integrate((e*cot(d*x+c))**(5/2)/(a+b*cot(d*x+c)),x)
[Out] Integral((e*cot(c + d*x))**(5/2)/(a + b*cot(c + d*x)), x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{(e\cot(c+dx))^{5/2}}{a+b\cot(c+dx)}\,dx = \text{Exception raised: ValueError}$$

```
[In] integrate((e*cot(d*x+c))^(5/2)/(a+b*cot(d*x+c)),x, algorithm="maxima")
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(e>0)', see `assume?` for more detai
ls)Is e
```

Giac [F]

$$\int \frac{(e\cot(c+dx))^{5/2}}{a+b\cot(c+dx)}\,dx=\int \frac{(e\cot(dx+c))^{\frac{5}{2}}}{b\cot(dx+c)+a}\,dx$$

```
[In] integrate((e*cot(d*x+c))^(5/2)/(a+b*cot(d*x+c)),x, algorithm="giac")
[Out] integrate((e*cot(d*x + c))^(5/2)/(b*cot(d*x + c) + a), x)
```

Mupad [B] (verification not implemented)

Time = 14.68 (sec) , antiderivative size = 5579, normalized size of antiderivative = 17.17

$$\int \frac{(e\cot(c+dx))^{5/2}}{a+b\cot(c+dx)}\,dx=\text{Too large to display}$$

```
[In] int((e*cot(c + d*x))^(5/2)/(a + b*cot(c + d*x)),x)
[Out] (atan(((((32*(e*cot(c + d*x))^(1/2)*(2*a^6*e^20 - b^6*e^20))/(b*d^4) + (((3
2*(12*a^6*b*d^2*e^18 + a^2*b^5*d^2*e^18 - 15*a^4*b^3*d^2*e^18))/(b*d^5) + (
((32*(e*cot(c + d*x))^(1/2)*(16*a^7*b*d^2*e^15 - 14*a*b^7*d^2*e^15 + 4*a^3*
b^5*d^2*e^15 + 2*a^5*b^3*d^2*e^15))/(b*d^4) - (((32*(4*a*b^8*d^4*e^13 + 8*a
^3*b^6*d^4*e^13 + 4*a^5*b^4*d^4*e^13))/(b*d^5) + (32*(e*cot(c + d*x))^(1/2)
*(-a^5*b^3*e^5)^(1/2)*(16*b^10*d^4*e^10 + 16*a^2*b^8*d^4*e^10 - 16*a^4*b^6*
d^4*e^10 - 16*a^6*b^4*d^4*e^10))/(b^4*d^5*(a^2 + b^2)))*(-a^5*b^3*e^5)^(1/2
))/(b^3*d*(a^2 + b^2)))*(-a^5*b^3*e^5)^(1/2))/(b^3*d*(a^2 + b^2)))*(-a^5*b^
3*e^5)^(1/2))/(b^3*d*(a^2 + b^2)))*(-a^5*b^3*e^5)^(1/2)*1i)/(b^3*d*(a^2 + b
^2)) + (((32*(e*cot(c + d*x))^(1/2)*(2*a^6*e^20 - b^6*e^20))/(b*d^4) - (((3
2*(12*a^6*b*d^2*e^18 + a^2*b^5*d^2*e^18 - 15*a^4*b^3*d^2*e^18))/(b*d^5) - (
((32*(e*cot(c + d*x))^(1/2)*(16*a^7*b*d^2*e^15 - 14*a*b^7*d^2*e^15 + 4*a^3*
b^5*d^2*e^15 + 2*a^5*b^3*d^2*e^15))/(b*d^4) + (((32*(4*a*b^8*d^4*e^13 + 8*a
^3*b^6*d^4*e^13 + 4*a^5*b^4*d^4*e^13))/(b*d^5) - (32*(e*cot(c + d*x))^(1/2)
*(-a^5*b^3*e^5)^(1/2)*(16*b^10*d^4*e^10 + 16*a^2*b^8*d^4*e^10 - 16*a^4*b^6*
d^4*e^10 - 16*a^6*b^4*d^4*e^10))/(b^4*d^5*(a^2 + b^2)))*(-a^5*b^3*e^5)^(1/2
))/(b^3*d*(a^2 + b^2)))*(-a^5*b^3*e^5)^(1/2))/(b^3*d*(a^2 + b^2)))*(-a^5*b^
3*e^5)^(1/2))/(b^3*d*(a^2 + b^2)))*(-a^5*b^3*e^5)^(1/2)*1i)/(b^3*d*(a^2 + b
^2)))/((64*(a^5*e^23 - a^3*b^2*e^23))/(b*d^5) + (((32*(e*cot(c + d*x))^(1/2
)*(2*a^6*e^20 - b^6*e^20))/(b*d^4) + (((32*(12*a^6*b*d^2*e^18 + a^2*b^5*d^2
*e^18 - 15*a^4*b^3*d^2*e^18))/(b*d^5) + (((32*(e*cot(c + d*x))^(1/2)*(16*a^
7*b*d^2*e^15 - 14*a*b^7*d^2*e^15 + 4*a^3*b^5*d^2*e^15 + 2*a^5*b^3*d^2*e^15)
)/(b*d^4) - (((32*(4*a*b^8*d^4*e^13 + 8*a^3*b^6*d^4*e^13 + 4*a^5*b^4*d^4*e^
13))/(b*d^5) + (32*(e*cot(c + d*x))^(1/2)*(-a^5*b^3*e^5)^(1/2)*(16*b^10*d^4
*e^10 + 16*a^2*b^8*d^4*e^10 - 16*a^4*b^6*d^4*e^10 - 16*a^6*b^4*d^4*e^10))/(
b^4*d^5*(a^2 + b^2)))*(-a^5*b^3*e^5)^(1/2))/(b^3*d*(a^2 + b^2)))*(-a^5*b^3*
```

```
e^5)^(1/2))/(b^3*d*(a^2 + b^2)))*(-a^5*b^3*e^5)^(1/2))/(b^3*d*(a^2 + b^2)))
*(-a^5*b^3*e^5)^(1/2))/(b^3*d*(a^2 + b^2)) - (((32*(e*cot(c + d*x))^(1/2)*(
2*a^6*e^20 - b^6*e^20))/(b*d^4) - (((32*(12*a^6*b*d^2*e^18 + a^2*b^5*d^2*e^
18 - 15*a^4*b^3*d^2*e^18))/(b*d^5) - (((32*(e*cot(c + d*x))^(1/2)*(16*a^7*b
*d^2*e^15 - 14*a*b^7*d^2*e^15 + 4*a^3*b^5*d^2*e^15 + 2*a^5*b^3*d^2*e^15))/(
b*d^4) + (((32*(4*a*b^8*d^4*e^13 + 8*a^3*b^6*d^4*e^13 + 4*a^5*b^4*d^4*e^13)
)/(b*d^5) - (32*(e*cot(c + d*x))^(1/2)*(-a^5*b^3*e^5)^(1/2)*(16*b^10*d^4*e^
10 + 16*a^2*b^8*d^4*e^10 - 16*a^4*b^6*d^4*e^10 - 16*a^6*b^4*d^4*e^10))/(b^4
*d^5*(a^2 + b^2)))*(-a^5*b^3*e^5)^(1/2))/(b^3*d*(a^2 + b^2)))*(-a^5*b^3*e^5
)^(1/2))/(b^3*d*(a^2 + b^2)))*(-a^5*b^3*e^5)^(1/2))/(b^3*d*(a^2 + b^2)))*(-
a^5*b^3*e^5)^(1/2))/(b^3*d*(a^2 + b^2))))*(-a^5*b^3*e^5)^(1/2)*2i)/(b^3*d*(
a^2 + b^2)) - atan((((((((32*(4*a*b^8*d^4*e^13 + 8*a^3*b^6*d^4*e^13 + 4*a^5*
b^4*d^4*e^13))/(b*d^5) - (32*(e*cot(c + d*x))^(1/2)*(-e^5/(4*(b^2*d^2*1i -
a^2*d^2*1i + 2*a*b*d^2)))^(1/2)*(16*b^10*d^4*e^10 + 16*a^2*b^8*d^4*e^10 - 1
6*a^4*b^6*d^4*e^10 - 16*a^6*b^4*d^4*e^10))/(b*d^4))*(-e^5/(4*(b^2*d^2*1i -
a^2*d^2*1i + 2*a*b*d^2)))^(1/2) + (32*(e*cot(c + d*x))^(1/2)*(16*a^7*b*d^2*
e^15 - 14*a*b^7*d^2*e^15 + 4*a^3*b^5*d^2*e^15 + 2*a^5*b^3*d^2*e^15))/(b*d^4
))*(-e^5/(4*(b^2*d^2*1i - a^2*d^2*1i + 2*a*b*d^2)))^(1/2) - (32*(12*a^6*b*d
^2*e^18 + a^2*b^5*d^2*e^18 - 15*a^4*b^3*d^2*e^18))/(b*d^5))*(-e^5/(4*(b^2*d
^2*1i - a^2*d^2*1i + 2*a*b*d^2)))^(1/2) + (32*(e*cot(c + d*x))^(1/2)*(2*a^6
*e^20 - b^6*e^20))/(b*d^4))*(-e^5/(4*(b^2*d^2*1i - a^2*d^2*1i + 2*a*b*d^2))
)^(1/2)*1i - (((((32*(4*a*b^8*d^4*e^13 + 8*a^3*b^6*d^4*e^13 + 4*a^5*b^4*d^4
*e^13))/(b*d^5) + (32*(e*cot(c + d*x))^(1/2)*(-e^5/(4*(b^2*d^2*1i - a^2*d^2
*1i + 2*a*b*d^2)))^(1/2)*(16*b^10*d^4*e^10 + 16*a^2*b^8*d^4*e^10 - 16*a^4*b
^6*d^4*e^10 - 16*a^6*b^4*d^4*e^10))/(b*d^4))*(-e^5/(4*(b^2*d^2*1i - a^2*d^2
*1i + 2*a*b*d^2)))^(1/2) - (32*(e*cot(c + d*x))^(1/2)*(16*a^7*b*d^2*e^15 -
14*a*b^7*d^2*e^15 + 4*a^3*b^5*d^2*e^15 + 2*a^5*b^3*d^2*e^15))/(b*d^4))*(-e^
5/(4*(b^2*d^2*1i - a^2*d^2*1i + 2*a*b*d^2)))^(1/2) - (32*(12*a^6*b*d^2*e^18
 + a^2*b^5*d^2*e^18 - 15*a^4*b^3*d^2*e^18))/(b*d^5))*(-e^5/(4*(b^2*d^2*1i -
 a^2*d^2*1i + 2*a*b*d^2)))^(1/2) - (32*(e*cot(c + d*x))^(1/2)*(2*a^6*e^20 -
 b^6*e^20))/(b*d^4))*(-e^5/(4*(b^2*d^2*1i - a^2*d^2*1i + 2*a*b*d^2)))^(1/2)
*1i)/((((((32*(4*a*b^8*d^4*e^13 + 8*a^3*b^6*d^4*e^13 + 4*a^5*b^4*d^4*e^13))
/(b*d^5) - (32*(e*cot(c + d*x))^(1/2)*(-e^5/(4*(b^2*d^2*1i - a^2*d^2*1i + 2
*a*b*d^2)))^(1/2)*(16*b^10*d^4*e^10 + 16*a^2*b^8*d^4*e^10 - 16*a^4*b^6*d^4*
e^10 - 16*a^6*b^4*d^4*e^10))/(b*d^4))*(-e^5/(4*(b^2*d^2*1i - a^2*d^2*1i + 2
*a*b*d^2)))^(1/2) + (32*(e*cot(c + d*x))^(1/2)*(16*a^7*b*d^2*e^15 - 14*a*b^
7*d^2*e^15 + 4*a^3*b^5*d^2*e^15 + 2*a^5*b^3*d^2*e^15))/(b*d^4))*(-e^5/(4*(b
^2*d^2*1i - a^2*d^2*1i + 2*a*b*d^2)))^(1/2) - (32*(12*a^6*b*d^2*e^18 + a^2*
b^5*d^2*e^18 - 15*a^4*b^3*d^2*e^18))/(b*d^5))*(-e^5/(4*(b^2*d^2*1i - a^2*d^
2*1i + 2*a*b*d^2)))^(1/2) + (32*(e*cot(c + d*x))^(1/2)*(2*a^6*e^20 - b^6*e^
20))/(b*d^4))*(-e^5/(4*(b^2*d^2*1i - a^2*d^2*1i + 2*a*b*d^2)))^(1/2) + ((((
(32*(4*a*b^8*d^4*e^13 + 8*a^3*b^6*d^4*e^13 + 4*a^5*b^4*d^4*e^13))/(b*d^5) +
 (32*(e*cot(c + d*x))^(1/2)*(-e^5/(4*(b^2*d^2*1i - a^2*d^2*1i + 2*a*b*d^2))
)^(1/2)*(16*b^10*d^4*e^10 + 16*a^2*b^8*d^4*e^10 - 16*a^4*b^6*d^4*e^10 - 16*
a^6*b^4*d^4*e^10))/(b*d^4))*(-e^5/(4*(b^2*d^2*1i - a^2*d^2*1i + 2*a*b*d^2))
```

)^(1/2) - (32*(e*cot(c + d*x))^(1/2)*(16*a^7*b*d^2*e^15 - 14*a*b^7*d^2*e^15
 + 4*a^3*b^5*d^2*e^15 + 2*a^5*b^3*d^2*e^15))/(b*d^4))*(-e^5/(4*(b^2*d^2*1i
- a^2*d^2*1i + 2*a*b*d^2)))^(1/2) - (32*(12*a^6*b*d^2*e^18 + a^2*b^5*d^2*e^
18 - 15*a^4*b^3*d^2*e^18))/(b*d^5))*(-e^5/(4*(b^2*d^2*1i - a^2*d^2*1i + 2*a
*b*d^2)))^(1/2) - (32*(e*cot(c + d*x))^(1/2)*(2*a^6*e^20 - b^6*e^20))/(b*d^
4))*(-e^5/(4*(b^2*d^2*1i - a^2*d^2*1i + 2*a*b*d^2)))^(1/2) - (64*(a^5*e^23
- a^3*b^2*e^23))/(b*d^5))*(-e^5/(4*(b^2*d^2*1i - a^2*d^2*1i + 2*a*b*d^2)))
^(1/2)*2i - (2*e^2*(e*cot(c + d*x))^(1/2))/(b*d) - atan((((((32*(4*a*b^8*d
^4*e^13 + 8*a^3*b^6*d^4*e^13 + 4*a^5*b^4*d^4*e^13))/(b*d^5) - (32*(e*cot(c
+ d*x))^(1/2)*(-(e^5*1i)/(4*(b^2*d^2 - a^2*d^2 + a*b*d^2*2i)))^(1/2)*(16*b^
10*d^4*e^10 + 16*a^2*b^8*d^4*e^10 - 16*a^4*b^6*d^4*e^10 - 16*a^6*b^4*d^4*e^
10))/(b*d^4))*(-(e^5*1i)/(4*(b^2*d^2 - a^2*d^2 + a*b*d^2*2i)))^(1/2) + (32*
(e*cot(c + d*x))^(1/2)*(16*a^7*b*d^2*e^15 - 14*a*b^7*d^2*e^15 + 4*a^3*b^5*d
^2*e^15 + 2*a^5*b^3*d^2*e^15))/(b*d^4))*(-(e^5*1i)/(4*(b^2*d^2 - a^2*d^2 +
a*b*d^2*2i)))^(1/2) - (32*(12*a^6*b*d^2*e^18 + a^2*b^5*d^2*e^18 - 15*a^4*b^
3*d^2*e^18))/(b*d^5))*(-(e^5*1i)/(4*(b^2*d^2 - a^2*d^2 + a*b*d^2*2i)))^(1/2
) + (32*(e*cot(c + d*x))^(1/2)*(2*a^6*e^20 - b^6*e^20))/(b*d^4))*(-(e^5*1i)
/(4*(b^2*d^2 - a^2*d^2 + a*b*d^2*2i)))^(1/2)*1i - (((((32*(4*a*b^8*d^4*e^13
 + 8*a^3*b^6*d^4*e^13 + 4*a^5*b^4*d^4*e^13))/(b*d^5) + (32*(e*cot(c + d*x))
^(1/2)*(-(e^5*1i)/(4*(b^2*d^2 - a^2*d^2 + a*b*d^2*2i)))^(1/2)*(16*b^10*d^4*
e^10 + 16*a^2*b^8*d^4*e^10 - 16*a^4*b^6*d^4*e^10 - 16*a^6*b^4*d^4*e^10))/(b
d^4))(-(e^5*1i)/(4*(b^2*d^2 - a^2*d^2 + a*b*d^2*2i)))^(1/2) - (32*(e*cot(
c + d*x))^(1/2)*(16*a^7*b*d^2*e^15 - 14*a*b^7*d^2*e^15 + 4*a^3*b^5*d^2*e^15
 + 2*a^5*b^3*d^2*e^15))/(b*d^4))*(-(e^5*1i)/(4*(b^2*d^2 - a^2*d^2 + a*b*d^2
2i)))^(1/2) - (32(12*a^6*b*d^2*e^18 + a^2*b^5*d^2*e^18 - 15*a^4*b^3*d^2*e
^18))/(b*d^5))*(-(e^5*1i)/(4*(b^2*d^2 - a^2*d^2 + a*b*d^2*2i)))^(1/2) - (32
*(e*cot(c + d*x))^(1/2)*(2*a^6*e^20 - b^6*e^20))/(b*d^4))*(-(e^5*1i)/(4*(b^
2*d^2 - a^2*d^2 + a*b*d^2*2i)))^(1/2)*1i)/((((((32*(4*a*b^8*d^4*e^13 + 8*a^
3*b^6*d^4*e^13 + 4*a^5*b^4*d^4*e^13))/(b*d^5) - (32*(e*cot(c + d*x))^(1/2)*
(-(e^5*1i)/(4*(b^2*d^2 - a^2*d^2 + a*b*d^2*2i)))^(1/2)*(16*b^10*d^4*e^10 +
16*a^2*b^8*d^4*e^10 - 16*a^4*b^6*d^4*e^10 - 16*a^6*b^4*d^4*e^10))/(b*d^4))*
(-(e^5*1i)/(4*(b^2*d^2 - a^2*d^2 + a*b*d^2*2i)))^(1/2) + (32*(e*cot(c + d*x
))^(1/2)*(16*a^7*b*d^2*e^15 - 14*a*b^7*d^2*e^15 + 4*a^3*b^5*d^2*e^15 + 2*a^
5*b^3*d^2*e^15))/(b*d^4))*(-(e^5*1i)/(4*(b^2*d^2 - a^2*d^2 + a*b*d^2*2i)))^
(1/2) - (32*(12*a^6*b*d^2*e^18 + a^2*b^5*d^2*e^18 - 15*a^4*b^3*d^2*e^18))/(
b*d^5))*(-(e^5*1i)/(4*(b^2*d^2 - a^2*d^2 + a*b*d^2*2i)))^(1/2) + (32*(e*cot
(c + d*x))^(1/2)*(2*a^6*e^20 - b^6*e^20))/(b*d^4))*(-(e^5*1i)/(4*(b^2*d^2 -
 a^2*d^2 + a*b*d^2*2i)))^(1/2) + (((((32*(4*a*b^8*d^4*e^13 + 8*a^3*b^6*d^4*
e^13 + 4*a^5*b^4*d^4*e^13))/(b*d^5) + (32*(e*cot(c + d*x))^(1/2)*(-(e^5*1i)
/(4*(b^2*d^2 - a^2*d^2 + a*b*d^2*2i)))^(1/2)*(16*b^10*d^4*e^10 + 16*a^2*b^8
*d^4*e^10 - 16*a^4*b^6*d^4*e^10 - 16*a^6*b^4*d^4*e^10))/(b*d^4))*(-(e^5*1i)
/(4*(b^2*d^2 - a^2*d^2 + a*b*d^2*2i)))^(1/2) - (32*(e*cot(c + d*x))^(1/2)*(
16*a^7*b*d^2*e^15 - 14*a*b^7*d^2*e^15 + 4*a^3*b^5*d^2*e^15 + 2*a^5*b^3*d^2*
e^15))/(b*d^4))*(-(e^5*1i)/(4*(b^2*d^2 - a^2*d^2 + a*b*d^2*2i)))^(1/2) - (3
2*(12*a^6*b*d^2*e^18 + a^2*b^5*d^2*e^18 - 15*a^4*b^3*d^2*e^18))/(b*d^5))*(-

```
(e^5*1i)/(4*(b^2*d^2 - a^2*d^2 + a*b*d^2*2i)))^(1/2) - (32*(e*cot(c + d*x))
^(1/2)*(2*a^6*e^20 - b^6*e^20))/(b*d^4))*(-(e^5*1i)/(4*(b^2*d^2 - a^2*d^2 +
 a*b*d^2*2i)))^(1/2) - (64*(a^5*e^23 - a^3*b^2*e^23))/(b*d^5)))*(-(e^5*1i)/
(4*(b^2*d^2 - a^2*d^2 + a*b*d^2*2i)))^(1/2)*2i
```

3.70 $\int \frac{(e\cot(c+dx))^{3/2}}{a+b\cot(c+dx)}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 302

$$\int \frac{(e\cot(c+dx))^{3/2}}{a+b\cot(c+dx)}\,dx = -\frac{2a^{3/2}e^{3/2}\arctan\left(\frac{\sqrt{b}\sqrt{e\cot(c+dx)}}{\sqrt{a}\sqrt{e}}\right)}{\sqrt{b}\,(a^2+b^2)\,d} - \frac{(a-b)e^{3/2}\arctan\left(1-\frac{\sqrt{2}\sqrt{e\cot(c+dx)}}{\sqrt{e}}\right)}{\sqrt{2}\,(a^2+b^2)\,d} + \frac{(a-b)e^{3/2}\arctan\left(1+\frac{\sqrt{2}\sqrt{e\cot(c+dx)}}{\sqrt{e}}\right)}{\sqrt{2}\,(a^2+b^2)\,d} - \frac{(a+b)e^{3/2}\log\left(\sqrt{e}+\sqrt{e}\cot(c+dx)-\sqrt{2}\sqrt{e\cot(c+dx)}\right)}{2\sqrt{2}\,(a^2+b^2)\,d} + \frac{(a+b)e^{3/2}\log\left(\sqrt{e}+\sqrt{e}\cot(c+dx)+\sqrt{2}\sqrt{e\cot(c+dx)}\right)}{2\sqrt{2}\,(a^2+b^2)\,d}$$

```
[Out] -1/2*(a-b)*e^(3/2)*arctan(1-2^(1/2)*(e*cot(d*x+c))^(1/2)/e^(1/2))/(a^2+b^2)
/d*2^(1/2)+1/2*(a-b)*e^(3/2)*arctan(1+2^(1/2)*(e*cot(d*x+c))^(1/2)/e^(1/2))
/(a^2+b^2)/d*2^(1/2)-1/4*(a+b)*e^(3/2)*ln(e^(1/2)+cot(d*x+c)*e^(1/2)-2^(1/2
)*(e*cot(d*x+c))^(1/2))/(a^2+b^2)/d*2^(1/2)+1/4*(a+b)*e^(3/2)*ln(e^(1/2)+co
t(d*x+c)*e^(1/2)+2^(1/2)*(e*cot(d*x+c))^(1/2))/(a^2+b^2)/d*2^(1/2)-2*a^(3/2
)*e^(3/2)*arctan(b^(1/2)*(e*cot(d*x+c))^(1/2)/a^(1/2)/e^(1/2))/(a^2+b^2)/d/
b^(1/2)
```

Rubi [A] (verified)

Time = 0.45 (sec) , antiderivative size = 302, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.440, Rules used = {3654, 3615, 1182, 1176, 631, 210, 1179, 642, 3715, 65, 211}

$$\int \frac{(e\cot(c+dx))^{3/2}}{a+b\cot(c+dx)}\,dx = -\frac{e^{3/2}(a-b)\arctan\left(1-\frac{\sqrt{2}\sqrt{e\cot(c+dx)}}{\sqrt{e}}\right)}{\sqrt{2}d\left(a^2+b^2\right)} + \frac{e^{3/2}(a-b)\arctan\left(\frac{\sqrt{2}\sqrt{e\cot(c+dx)}}{\sqrt{e}}+1\right)}{\sqrt{2}d\left(a^2+b^2\right)} - \frac{e^{3/2}(a+b)\log\left(\sqrt{e}\cot(c+dx)-\sqrt{2}\sqrt{e\cot(c+dx)}+\sqrt{e}\right)}{2\sqrt{2}d\left(a^2+b^2\right)} + \frac{e^{3/2}(a+b)\log\left(\sqrt{e}\cot(c+dx)+\sqrt{2}\sqrt{e\cot(c+dx)}+\sqrt{e}\right)}{2\sqrt{2}d\left(a^2+b^2\right)} - \frac{2a^{3/2}e^{3/2}\arctan\left(\frac{\sqrt{b}\sqrt{e\cot(c+dx)}}{\sqrt{a}\sqrt{e}}\right)}{\sqrt{b}d\left(a^2+b^2\right)}$$

```
[In] Int[(e*Cot[c + d*x])^(3/2)/(a + b*Cot[c + d*x]),x]
```

```
[Out] (-2*a^(3/2)*e^(3/2)*ArcTan[(Sqrt[b]*Sqrt[e*Cot[c + d*x]])/(Sqrt[a]*Sqrt[e])
])/(Sqrt[b]*(a^2 + b^2)*d) - ((a - b)*e^(3/2)*ArcTan[1 - (Sqrt[2]*Sqrt[e*Co
t[c + d*x]])/Sqrt[e]])/(Sqrt[2]*(a^2 + b^2)*d) + ((a - b)*e^(3/2)*ArcTan[1
+ (Sqrt[2]*Sqrt[e*Cot[c + d*x]])/Sqrt[e]])/(Sqrt[2]*(a^2 + b^2)*d) - ((a +
b)*e^(3/2)*Log[Sqrt[e] + Sqrt[e]*Cot[c + d*x] - Sqrt[2]*Sqrt[e*Cot[c + d*x]
]])/(2*Sqrt[2]*(a^2 + b^2)*d) + ((a + b)*e^(3/2)*Log[Sqrt[e] + Sqrt[e]*Cot[
c + d*x] + Sqrt[2]*Sqrt[e*Cot[c + d*x]]])/(2*Sqrt[2]*(a^2 + b^2)*d)
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1179

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 1182

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
a*c, 2]}, Dist[(d*q + a*e)/(2*a*c), Int[(q + c*x^2)/(a + c*x^4), x], x] + D
ist[(d*q - a*e)/(2*a*c), Int[(q - c*x^2)/(a + c*x^4), x], x]] /; FreeQ[{a,
c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0] && NeQ[c*d^2 - a*e^2, 0] && NegQ[(-a)
*c]
```

Rule 3615

```
Int[((c_) + (d_.)*tan[(e_.) + (f_.)*(x_)])/Sqrt[(b_.)*tan[(e_.) + (f_.)*(x_
)]], x_Symbol] :> Dist[2/f, Subst[Int[(b*c + d*x^2)/(b^2 + x^4), x], x, Sqr
t[b*Tan[e + f*x]]], x] /; FreeQ[{b, c, d, e, f}, x] && NeQ[c^2 - d^2, 0] &&
 NeQ[c^2 + d^2, 0]
```

Rule 3654

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(3/2)/((c_.) + (d_.)*tan[(e_.)
+ (f_.)*(x_)]), x_Symbol] :> Dist[1/(c^2 + d^2), Int[Simp[a^2*c - b^2*c + 2
*a*b*d + (2*a*b*c - a^2*d + b^2*d)*Tan[e + f*x], x]/Sqrt[a + b*Tan[e + f*x]
], x], x] + Dist[(b*c - a*d)^2/(c^2 + d^2), Int[(1 + Tan[e + f*x]^2)/(Sqrt[
a + b*Tan[e + f*x]]*(c + d*Tan[e + f*x])), x], x] /; FreeQ[{a, b, c, d, e,
f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0]
```

Rule 3715

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*tan[(e_.)
+ (f_.)*(x_)])^(n_.)*((A_) + (C_.)*tan[(e_.) + (f_.)*(x_)]^2), x_Symbol] :>
 Dist[A/f, Subst[Int[(a + b*x)^m*(c + d*x)^n, x], x, Tan[e + f*x]], x] /; F
reeQ[{a, b, c, d, e, f, A, C, m, n}, x] && EqQ[A, C]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{\int \frac{-ae^2+be^2\cot(c+dx)}{\sqrt{e\cot(c+dx)}}\,dx}{a^2+b^2}+\frac{(a^2e^2)\int \frac{1+\cot^2(c+dx)}{\sqrt{e\cot(c+dx)}(a+b\cot(c+dx))}\,dx}{a^2+b^2}\\
&= \frac{2\text{Subst}\left(\int \frac{ae^3-be^2x^2}{e^2+x^4}\,dx,x,\sqrt{e\cot(c+dx)}\right)}{(a^2+b^2)\,d}+\frac{(a^2e^2)\,\text{Subst}\left(\int \frac{1}{\sqrt{-ex}(a-bx)}\,dx,x,-\cot(c+dx)\right)}{(a^2+b^2)\,d}\\
&= -\frac{(2a^2e)\,\text{Subst}\left(\int \frac{1}{a+\frac{bx^2}{e}}\,dx,x,\sqrt{e\cot(c+dx)}\right)}{(a^2+b^2)\,d}\\
&\quad+\frac{((a-b)e^2)\,\text{Subst}\left(\int \frac{e+x^2}{e^2+x^4}\,dx,x,\sqrt{e\cot(c+dx)}\right)}{(a^2+b^2)\,d}\\
&\quad+\frac{((a+b)e^2)\,\text{Subst}\left(\int \frac{e-x^2}{e^2+x^4}\,dx,x,\sqrt{e\cot(c+dx)}\right)}{(a^2+b^2)\,d}\\
&= -\frac{2a^{3/2}e^{3/2}\arctan\left(\frac{\sqrt{b}\sqrt{e\cot(c+dx)}}{\sqrt{a}\sqrt{e}}\right)}{\sqrt{b}\,(a^2+b^2)\,d}\\
&\quad-\frac{((a+b)e^{3/2})\,\text{Subst}\left(\int \frac{\sqrt{2}\sqrt{e}+2x}{-e-\sqrt{2}\sqrt{e}x-x^2}\,dx,x,\sqrt{e\cot(c+dx)}\right)}{2\sqrt{2}\,(a^2+b^2)\,d}\\
&\quad-\frac{((a+b)e^{3/2})\,\text{Subst}\left(\int \frac{\sqrt{2}\sqrt{e}-2x}{-e+\sqrt{2}\sqrt{e}x-x^2}\,dx,x,\sqrt{e\cot(c+dx)}\right)}{2\sqrt{2}\,(a^2+b^2)\,d}\\
&\quad+\frac{((a-b)e^2)\,\text{Subst}\left(\int \frac{1}{e-\sqrt{2}\sqrt{e}x+x^2}\,dx,x,\sqrt{e\cot(c+dx)}\right)}{2\,(a^2+b^2)\,d}\\
&\quad+\frac{((a-b)e^2)\,\text{Subst}\left(\int \frac{1}{e+\sqrt{2}\sqrt{e}x+x^2}\,dx,x,\sqrt{e\cot(c+dx)}\right)}{2\,(a^2+b^2)\,d}
\end{aligned}$$

$$= -\frac{2a^{3/2}e^{3/2}\arctan\left(\frac{\sqrt{b}\sqrt{e\cot(c+dx)}}{\sqrt{a}\sqrt{e}}\right)}{\sqrt{b}\,(a^2+b^2)\,d} - \frac{(a+b)e^{3/2}\log\left(\sqrt{e}+\sqrt{e}\cot(c+dx)-\sqrt{2}\sqrt{e\cot(c+dx)}\right)}{2\sqrt{2}\,(a^2+b^2)\,d} + \frac{(a+b)e^{3/2}\log\left(\sqrt{e}+\sqrt{e}\cot(c+dx)+\sqrt{2}\sqrt{e\cot(c+dx)}\right)}{2\sqrt{2}\,(a^2+b^2)\,d} + \frac{\left((a-b)e^{3/2}\right)\text{Subst}\left(\int\frac{1}{-1-x^2}\,dx,x,1-\frac{\sqrt{2}\sqrt{e\cot(c+dx)}}{\sqrt{e}}\right)}{\sqrt{2}\,(a^2+b^2)\,d} - \frac{\left((a-b)e^{3/2}\right)\text{Subst}\left(\int\frac{1}{-1-x^2}\,dx,x,1+\frac{\sqrt{2}\sqrt{e\cot(c+dx)}}{\sqrt{e}}\right)}{\sqrt{2}\,(a^2+b^2)\,d}$$

$$= -\frac{2a^{3/2}e^{3/2}\arctan\left(\frac{\sqrt{b}\sqrt{e\cot(c+dx)}}{\sqrt{a}\sqrt{e}}\right)}{\sqrt{b}\,(a^2+b^2)\,d} - \frac{(a-b)e^{3/2}\arctan\left(1-\frac{\sqrt{2}\sqrt{e\cot(c+dx)}}{\sqrt{e}}\right)}{\sqrt{2}\,(a^2+b^2)\,d} + \frac{(a-b)e^{3/2}\arctan\left(1+\frac{\sqrt{2}\sqrt{e\cot(c+dx)}}{\sqrt{e}}\right)}{\sqrt{2}\,(a^2+b^2)\,d} - \frac{(a+b)e^{3/2}\log\left(\sqrt{e}+\sqrt{e}\cot(c+dx)-\sqrt{2}\sqrt{e\cot(c+dx)}\right)}{2\sqrt{2}\,(a^2+b^2)\,d} + \frac{(a+b)e^{3/2}\log\left(\sqrt{e}+\sqrt{e}\cot(c+dx)+\sqrt{2}\sqrt{e\cot(c+dx)}\right)}{2\sqrt{2}\,(a^2+b^2)\,d}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 0.51 (sec) , antiderivative size = 249, normalized size of antiderivative = 0.82

$$\int\frac{(e\cot(c+dx))^{3/2}}{a+b\cot(c+dx)}\,dx = -\frac{(e\cot(c+dx))^{3/2}\left(8b^{3/2}\cot^{\frac{3}{2}}(c+dx)\,\text{Hypergeometric2F1}\left(\frac{3}{4},1,\frac{7}{4},-\cot^2(c+dx)\right)+3a\left(2\sqrt{2}\sqrt{b}\arctan\left(1\right.\right.\right.}{}$$

```
[In] Integrate[(e*Cot[c + d*x])^(3/2)/(a + b*Cot[c + d*x]),x]
```

```
[Out] -1/12*((e*Cot[c + d*x])^(3/2)*(8*b^(3/2)*Cot[c + d*x]^(3/2)*Hypergeometric2
F1[3/4, 1, 7/4, -Cot[c + d*x]^2] + 3*a*(2*Sqrt[2]*Sqrt[b]*ArcTan[1 - Sqrt[2
]*Sqrt[Cot[c + d*x]]] - 2*Sqrt[2]*Sqrt[b]*ArcTan[1 + Sqrt[2]*Sqrt[Cot[c + d
*x]]] + 8*Sqrt[a]*ArcTan[(Sqrt[b]*Sqrt[Cot[c + d*x]])/Sqrt[a]] + Sqrt[2]*Sq
```

```
rt[b]*Log[1 - Sqrt[2]*Sqrt[Cot[c + d*x]] + Cot[c + d*x]] - Sqrt[2]*Sqrt[b]*
Log[1 + Sqrt[2]*Sqrt[Cot[c + d*x]] + Cot[c + d*x]])))/(Sqrt[b]*(a^2 + b^2)*
d*Cot[c + d*x]^(3/2))
```

Maple [A] (verified)

Time = 0.04 (sec) , antiderivative size = 326, normalized size of antiderivative = 1.08

method	result
derivativedivides	$-\frac{2e^2\left(\frac{a^2\arctan\left(\frac{\sqrt{e\cot(dx+c)}\,b}{\sqrt{aeb}}\right)}{\left(a^2+b^2\right)\sqrt{aeb}}+\frac{-\frac{a\left(e^2\right)^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\frac{e\cot(dx+c)+\left(e^2\right)^{\frac{1}{4}}\sqrt{e\cot(dx+c)}\,\sqrt{2}+\sqrt{e^2}}{e\cot(dx+c)-\left(e^2\right)^{\frac{1}{4}}\sqrt{e\cot(dx+c)}\,\sqrt{2}+\sqrt{e^2}}\right)+2\arctan\left(\frac{\sqrt{2}\,\sqrt{e\cot(dx+}}{\left(e^2\right)^{\frac{1}{4}}}\right.\right.}{8e}}{}\right.}{}$
default	$-\frac{2e^2\left(\frac{a^2\arctan\left(\frac{\sqrt{e\cot(dx+c)}\,b}{\sqrt{aeb}}\right)}{\left(a^2+b^2\right)\sqrt{aeb}}+\frac{-\frac{a\left(e^2\right)^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\frac{e\cot(dx+c)+\left(e^2\right)^{\frac{1}{4}}\sqrt{e\cot(dx+c)}\,\sqrt{2}+\sqrt{e^2}}{e\cot(dx+c)-\left(e^2\right)^{\frac{1}{4}}\sqrt{e\cot(dx+c)}\,\sqrt{2}+\sqrt{e^2}}\right)+2\arctan\left(\frac{\sqrt{2}\,\sqrt{e\cot(dx+}}{\left(e^2\right)^{\frac{1}{4}}}\right.\right.}{8e}}{}\right.}{}$

```
[In] int((e*cot(d*x+c))^(3/2)/(a+b*cot(d*x+c)),x,method=_RETURNVERBOSE)

[Out] -2/d*e^2*(a^2/(a^2+b^2)/(a*e*b)^(1/2)*arctan((e*cot(d*x+c))^(1/2)*b/(a*e*b)
^(1/2))+1/(a^2+b^2)*(-1/8*a/e*(e^2)^(1/4)*2^(1/2)*(ln((e*cot(d*x+c)+(e^2)^(
1/4)*(e*cot(d*x+c))^(1/2)*2^(1/2)+(e^2)^(1/2))/(e*cot(d*x+c)-(e^2)^(1/4)*(e
*cot(d*x+c))^(1/2)*2^(1/2)+(e^2)^(1/2)))+2*arctan(2^(1/2)/(e^2)^(1/4)*(e*co
t(d*x+c))^(1/2)+1)-2*arctan(-2^(1/2)/(e^2)^(1/4)*(e*cot(d*x+c))^(1/2)+1))+1
/8*b/(e^2)^(1/4)*2^(1/2)*(ln((e*cot(d*x+c)-(e^2)^(1/4)*(e*cot(d*x+c))^(1/2)
*2^(1/2)+(e^2)^(1/2))/(e*cot(d*x+c)+(e^2)^(1/4)*(e*cot(d*x+c))^(1/2)*2^(1/2
)+(e^2)^(1/2)))+2*arctan(2^(1/2)/(e^2)^(1/4)*(e*cot(d*x+c))^(1/2)+1)-2*arct
an(-2^(1/2)/(e^2)^(1/4)*(e*cot(d*x+c))^(1/2)+1))))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1537 vs. $2(241) = 482$.

Time = 0.38 (sec) , antiderivative size = 3137, normalized size of antiderivative = 10.39

$$\int \frac{(e\cot(c+dx))^{3/2}}{a+b\cot(c+dx)}\,dx = \text{Too large to display}$$

```
[In] integrate((e*cot(d*x+c))^(3/2)/(a+b*cot(d*x+c)),x, algorithm="fricas")
```

```
[Out] [-1/2*((a^2 + b^2)*d*sqrt((2*a*b*e^3 + (a^4 + 2*a^2*b^2 + b^4)*sqrt(-(a^4 -
 2*a^2*b^2 + b^4)*e^6/((a^8 + 4*a^6*b^2 + 6*a^4*b^4 + 4*a^2*b^6 + b^8)*d^4)
)*d^2)/((a^4 + 2*a^2*b^2 + b^4)*d^2))*log(-(a^2 - b^2)*e^4*sqrt((e*cos(2*d*
x + 2*c) + e)/sin(2*d*x + 2*c)) + ((a^3 - a*b^2)*d*e^3 + (a^4*b + 2*a^2*b^3
 + b^5)*sqrt(-(a^4 - 2*a^2*b^2 + b^4)*e^6/((a^8 + 4*a^6*b^2 + 6*a^4*b^4 + 4
*a^2*b^6 + b^8)*d^4))*d^3)*sqrt((2*a*b*e^3 + (a^4 + 2*a^2*b^2 + b^4)*sqrt(-
(a^4 - 2*a^2*b^2 + b^4)*e^6/((a^8 + 4*a^6*b^2 + 6*a^4*b^4 + 4*a^2*b^6 + b^8
)*d^4))*d^2)/((a^4 + 2*a^2*b^2 + b^4)*d^2))) - (a^2 + b^2)*d*sqrt((2*a*b*e^
3 + (a^4 + 2*a^2*b^2 + b^4)*sqrt(-(a^4 - 2*a^2*b^2 + b^4)*e^6/((a^8 + 4*a^6
*b^2 + 6*a^4*b^4 + 4*a^2*b^6 + b^8)*d^4))*d^2)/((a^4 + 2*a^2*b^2 + b^4)*d^2
))*log(-(a^2 - b^2)*e^4*sqrt((e*cos(2*d*x + 2*c) + e)/sin(2*d*x + 2*c)) - (
(a^3 - a*b^2)*d*e^3 + (a^4*b + 2*a^2*b^3 + b^5)*sqrt(-(a^4 - 2*a^2*b^2 + b^
4)*e^6/((a^8 + 4*a^6*b^2 + 6*a^4*b^4 + 4*a^2*b^6 + b^8)*d^4))*d^3)*sqrt((2*
a*b*e^3 + (a^4 + 2*a^2*b^2 + b^4)*sqrt(-(a^4 - 2*a^2*b^2 + b^4)*e^6/((a^8 +
 4*a^6*b^2 + 6*a^4*b^4 + 4*a^2*b^6 + b^8)*d^4))*d^2)/((a^4 + 2*a^2*b^2 + b^
4)*d^2))) + (a^2 + b^2)*d*sqrt((2*a*b*e^3 - (a^4 + 2*a^2*b^2 + b^4)*sqrt(-(
a^4 - 2*a^2*b^2 + b^4)*e^6/((a^8 + 4*a^6*b^2 + 6*a^4*b^4 + 4*a^2*b^6 + b^8)
*d^4))*d^2)/((a^4 + 2*a^2*b^2 + b^4)*d^2))*log(-(a^2 - b^2)*e^4*sqrt((e*cos
(2*d*x + 2*c) + e)/sin(2*d*x + 2*c)) + ((a^3 - a*b^2)*d*e^3 - (a^4*b + 2*a^
2*b^3 + b^5)*sqrt(-(a^4 - 2*a^2*b^2 + b^4)*e^6/((a^8 + 4*a^6*b^2 + 6*a^4*b^
4 + 4*a^2*b^6 + b^8)*d^4))*d^3)*sqrt((2*a*b*e^3 - (a^4 + 2*a^2*b^2 + b^4)*s
qrt(-(a^4 - 2*a^2*b^2 + b^4)*e^6/((a^8 + 4*a^6*b^2 + 6*a^4*b^4 + 4*a^2*b^6
+ b^8)*d^4))*d^2)/((a^4 + 2*a^2*b^2 + b^4)*d^2))) - (a^2 + b^2)*d*sqrt((2*a
*b*e^3 - (a^4 + 2*a^2*b^2 + b^4)*sqrt(-(a^4 - 2*a^2*b^2 + b^4)*e^6/((a^8 +
4*a^6*b^2 + 6*a^4*b^4 + 4*a^2*b^6 + b^8)*d^4))*d^2)/((a^4 + 2*a^2*b^2 + b^4
)*d^2))*log(-(a^2 - b^2)*e^4*sqrt((e*cos(2*d*x + 2*c) + e)/sin(2*d*x + 2*c)
) - ((a^3 - a*b^2)*d*e^3 - (a^4*b + 2*a^2*b^3 + b^5)*sqrt(-(a^4 - 2*a^2*b^2
 + b^4)*e^6/((a^8 + 4*a^6*b^2 + 6*a^4*b^4 + 4*a^2*b^6 + b^8)*d^4))*d^3)*sqr
t((2*a*b*e^3 - (a^4 + 2*a^2*b^2 + b^4)*sqrt(-(a^4 - 2*a^2*b^2 + b^4)*e^6/((
a^8 + 4*a^6*b^2 + 6*a^4*b^4 + 4*a^2*b^6 + b^8)*d^4))*d^2)/((a^4 + 2*a^2*b^2
 + b^4)*d^2))) - 2*a*sqrt(-a*e/b)*e*log((b*e*cos(2*d*x + 2*c) - a*e*sin(2*d
*x + 2*c) - 2*b*sqrt(-a*e/b)*sqrt((e*cos(2*d*x + 2*c) + e)/sin(2*d*x + 2*c)
)*sin(2*d*x + 2*c) + b*e)/(b*cos(2*d*x + 2*c) + a*sin(2*d*x + 2*c) + b)))/(
(a^2 + b^2)*d), -1/2*(4*a*sqrt(a*e/b)*e*arctan(b*sqrt(a*e/b)*sqrt((e*cos(2*
d*x + 2*c) + e)/sin(2*d*x + 2*c))/(a*e)) + (a^2 + b^2)*d*sqrt((2*a*b*e^3 +
```

```
(a^4 + 2*a^2*b^2 + b^4)*sqrt(-(a^4 - 2*a^2*b^2 + b^4)*e^6/((a^8 + 4*a^6*b^2
 + 6*a^4*b^4 + 4*a^2*b^6 + b^8)*d^4))*d^2)/((a^4 + 2*a^2*b^2 + b^4)*d^2))*l
og(-(a^2 - b^2)*e^4*sqrt((e*cos(2*d*x + 2*c) + e)/sin(2*d*x + 2*c)) + ((a^3
 - a*b^2)*d*e^3 + (a^4*b + 2*a^2*b^3 + b^5)*sqrt(-(a^4 - 2*a^2*b^2 + b^4)*e
^6/((a^8 + 4*a^6*b^2 + 6*a^4*b^4 + 4*a^2*b^6 + b^8)*d^4))*d^3)*sqrt((2*a*b*
e^3 + (a^4 + 2*a^2*b^2 + b^4)*sqrt(-(a^4 - 2*a^2*b^2 + b^4)*e^6/((a^8 + 4*a
^6*b^2 + 6*a^4*b^4 + 4*a^2*b^6 + b^8)*d^4))*d^2)/((a^4 + 2*a^2*b^2 + b^4)*d
^2))) - (a^2 + b^2)*d*sqrt((2*a*b*e^3 + (a^4 + 2*a^2*b^2 + b^4)*sqrt(-(a^4
- 2*a^2*b^2 + b^4)*e^6/((a^8 + 4*a^6*b^2 + 6*a^4*b^4 + 4*a^2*b^6 + b^8)*d^4
))*d^2)/((a^4 + 2*a^2*b^2 + b^4)*d^2))*log(-(a^2 - b^2)*e^4*sqrt((e*cos(2*d
*x + 2*c) + e)/sin(2*d*x + 2*c)) - ((a^3 - a*b^2)*d*e^3 + (a^4*b + 2*a^2*b^
3 + b^5)*sqrt(-(a^4 - 2*a^2*b^2 + b^4)*e^6/((a^8 + 4*a^6*b^2 + 6*a^4*b^4 +
4*a^2*b^6 + b^8)*d^4))*d^3)*sqrt((2*a*b*e^3 + (a^4 + 2*a^2*b^2 + b^4)*sqrt(
-(a^4 - 2*a^2*b^2 + b^4)*e^6/((a^8 + 4*a^6*b^2 + 6*a^4*b^4 + 4*a^2*b^6 + b^
8)*d^4))*d^2)/((a^4 + 2*a^2*b^2 + b^4)*d^2))) + (a^2 + b^2)*d*sqrt((2*a*b*e
^3 - (a^4 + 2*a^2*b^2 + b^4)*sqrt(-(a^4 - 2*a^2*b^2 + b^4)*e^6/((a^8 + 4*a^
6*b^2 + 6*a^4*b^4 + 4*a^2*b^6 + b^8)*d^4))*d^2)/((a^4 + 2*a^2*b^2 + b^4)*d^
2))*log(-(a^2 - b^2)*e^4*sqrt((e*cos(2*d*x + 2*c) + e)/sin(2*d*x + 2*c)) +
((a^3 - a*b^2)*d*e^3 - (a^4*b + 2*a^2*b^3 + b^5)*sqrt(-(a^4 - 2*a^2*b^2 + b
^4)*e^6/((a^8 + 4*a^6*b^2 + 6*a^4*b^4 + 4*a^2*b^6 + b^8)*d^4))*d^3)*sqrt((2
*a*b*e^3 - (a^4 + 2*a^2*b^2 + b^4)*sqrt(-(a^4 - 2*a^2*b^2 + b^4)*e^6/((a^8
+ 4*a^6*b^2 + 6*a^4*b^4 + 4*a^2*b^6 + b^8)*d^4))*d^2)/((a^4 + 2*a^2*b^2 + b
^4)*d^2))) - (a^2 + b^2)*d*sqrt((2*a*b*e^3 - (a^4 + 2*a^2*b^2 + b^4)*sqrt(-
(a^4 - 2*a^2*b^2 + b^4)*e^6/((a^8 + 4*a^6*b^2 + 6*a^4*b^4 + 4*a^2*b^6 + b^8
)*d^4))*d^2)/((a^4 + 2*a^2*b^2 + b^4)*d^2))*log(-(a^2 - b^2)*e^4*sqrt((e*co
s(2*d*x + 2*c) + e)/sin(2*d*x + 2*c)) - ((a^3 - a*b^2)*d*e^3 - (a^4*b + 2*a
^2*b^3 + b^5)*sqrt(-(a^4 - 2*a^2*b^2 + b^4)*e^6/((a^8 + 4*a^6*b^2 + 6*a^4*b
^4 + 4*a^2*b^6 + b^8)*d^4))*d^3)*sqrt((2*a*b*e^3 - (a^4 + 2*a^2*b^2 + b^4)*
sqrt(-(a^4 - 2*a^2*b^2 + b^4)*e^6/((a^8 + 4*a^6*b^2 + 6*a^4*b^4 + 4*a^2*b^6
 + b^8)*d^4))*d^2)/((a^4 + 2*a^2*b^2 + b^4)*d^2))))/((a^2 + b^2)*d)]
```

Sympy [F]

$$\int \frac{(e\cot(c+dx))^{3/2}}{a+b\cot(c+dx)}\,dx=\int \frac{(e\cot{(c+dx)})^{\frac{3}{2}}}{a+b\cot{(c+dx)}}\,dx$$

```
[In] integrate((e*cot(d*x+c))**(3/2)/(a+b*cot(d*x+c)),x)
[Out] Integral((e*cot(c + d*x))**(3/2)/(a + b*cot(c + d*x)), x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{(e\cot(c+dx))^{3/2}}{a+b\cot(c+dx)}\,dx = \text{Exception raised: ValueError}$$

```
[In] integrate((e*cot(d*x+c))^(3/2)/(a+b*cot(d*x+c)),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(e>0)', see `assume?` for more detai
ls)Is e
```

Giac [F]

$$\int \frac{(e\cot(c+dx))^{3/2}}{a+b\cot(c+dx)}\,dx = \int \frac{(e\cot(dx+c))^{\frac{3}{2}}}{b\cot(dx+c)+a}\,dx$$

```
[In] integrate((e*cot(d*x+c))^(3/2)/(a+b*cot(d*x+c)),x, algorithm="giac")
```

```
[Out] integrate((e*cot(d*x + c))^(3/2)/(b*cot(d*x + c) + a), x)
```

Mupad [B] (verification not implemented)

Time = 14.51 (sec) , antiderivative size = 5129, normalized size of antiderivative = 16.98

$$\int \frac{(e\cot(c+dx))^{3/2}}{a+b\cot(c+dx)}\,dx = \text{Too large to display}$$

```
[In] int((e*cot(c + d*x))^(3/2)/(a + b*cot(c + d*x)),x)
```

```
[Out] atan(((((((32*(4*a^2*b^6*d^4*e^12 + 8*a^4*b^4*d^4*e^12 + 4*a^6*b^2*d^4*e^12
))/d^5 - (32*(e*cot(c + d*x))^(1/2)*((e^3*1i)/(4*(b^2*d^2 - a^2*d^2 + a*b*d
^2*2i)))^(1/2)*(16*b^9*d^4*e^10 + 16*a^2*b^7*d^4*e^10 - 16*a^4*b^5*d^4*e^10
 - 16*a^6*b^3*d^4*e^10))/d^4)*((e^3*1i)/(4*(b^2*d^2 - a^2*d^2 + a*b*d^2*2i)
))^(1/2) + (32*(e*cot(c + d*x))^(1/2)*(14*a*b^6*d^2*e^13 - 4*a^3*b^4*d^2*e^
13 + 14*a^5*b^2*d^2*e^13))/d^4)*((e^3*1i)/(4*(b^2*d^2 - a^2*d^2 + a*b*d^2*2
i)))^(1/2) + (32*(a*b^5*d^2*e^15 + 4*a^5*b*d^2*e^15 - 15*a^3*b^3*d^2*e^15))
/d^5)*((e^3*1i)/(4*(b^2*d^2 - a^2*d^2 + a*b*d^2*2i)))^(1/2) - (32*(e*cot(c
+ d*x))^(1/2)*(b^5*e^16 + 2*a^4*b*e^16))/d^4)*((e^3*1i)/(4*(b^2*d^2 - a^2*d
^2 + a*b*d^2*2i)))^(1/2)*1i - (((((32*(4*a^2*b^6*d^4*e^12 + 8*a^4*b^4*d^4*e
^12 + 4*a^6*b^2*d^4*e^12))/d^5 + (32*(e*cot(c + d*x))^(1/2)*((e^3*1i)/(4*(b
^2*d^2 - a^2*d^2 + a*b*d^2*2i)))^(1/2)*(16*b^9*d^4*e^10 + 16*a^2*b^7*d^4*e^
```

```
10 - 16*a^4*b^5*d^4*e^10 - 16*a^6*b^3*d^4*e^10))/d^4)*((e^3*1i)/(4*(b^2*d^2
 - a^2*d^2 + a*b*d^2*2i)))^(1/2) - (32*(e*cot(c + d*x))^(1/2)*(14*a*b^6*d^2
*e^13 - 4*a^3*b^4*d^2*e^13 + 14*a^5*b^2*d^2*e^13))/d^4)*((e^3*1i)/(4*(b^2*d
^2 - a^2*d^2 + a*b*d^2*2i)))^(1/2) + (32*(a*b^5*d^2*e^15 + 4*a^5*b*d^2*e^15
 - 15*a^3*b^3*d^2*e^15))/d^5)*((e^3*1i)/(4*(b^2*d^2 - a^2*d^2 + a*b*d^2*2i)
))^(1/2) + (32*(e*cot(c + d*x))^(1/2)*(b^5*e^16 + 2*a^4*b*e^16))/d^4)*((e^3
*1i)/(4*(b^2*d^2 - a^2*d^2 + a*b*d^2*2i)))^(1/2)*1i)/((((((32*(4*a^2*b^6*d^
4*e^12 + 8*a^4*b^4*d^4*e^12 + 4*a^6*b^2*d^4*e^12))/d^5 - (32*(e*cot(c + d*x
))^(1/2)*((e^3*1i)/(4*(b^2*d^2 - a^2*d^2 + a*b*d^2*2i)))^(1/2)*(16*b^9*d^4*
e^10 + 16*a^2*b^7*d^4*e^10 - 16*a^4*b^5*d^4*e^10 - 16*a^6*b^3*d^4*e^10))/d^
4)*((e^3*1i)/(4*(b^2*d^2 - a^2*d^2 + a*b*d^2*2i)))^(1/2) + (32*(e*cot(c + d
*x))^(1/2)*(14*a*b^6*d^2*e^13 - 4*a^3*b^4*d^2*e^13 + 14*a^5*b^2*d^2*e^13))/
d^4)*((e^3*1i)/(4*(b^2*d^2 - a^2*d^2 + a*b*d^2*2i)))^(1/2) + (32*(a*b^5*d^2
*e^15 + 4*a^5*b*d^2*e^15 - 15*a^3*b^3*d^2*e^15))/d^5)*((e^3*1i)/(4*(b^2*d^2
 - a^2*d^2 + a*b*d^2*2i)))^(1/2) - (32*(e*cot(c + d*x))^(1/2)*(b^5*e^16 + 2
*a^4*b*e^16))/d^4)*((e^3*1i)/(4*(b^2*d^2 - a^2*d^2 + a*b*d^2*2i)))^(1/2) +
(((((32*(4*a^2*b^6*d^4*e^12 + 8*a^4*b^4*d^4*e^12 + 4*a^6*b^2*d^4*e^12))/d^5
 + (32*(e*cot(c + d*x))^(1/2)*((e^3*1i)/(4*(b^2*d^2 - a^2*d^2 + a*b*d^2*2i)
))^(1/2)*(16*b^9*d^4*e^10 + 16*a^2*b^7*d^4*e^10 - 16*a^4*b^5*d^4*e^10 - 16*
a^6*b^3*d^4*e^10))/d^4)*((e^3*1i)/(4*(b^2*d^2 - a^2*d^2 + a*b*d^2*2i)))^(1/
2) - (32*(e*cot(c + d*x))^(1/2)*(14*a*b^6*d^2*e^13 - 4*a^3*b^4*d^2*e^13 + 1
4*a^5*b^2*d^2*e^13))/d^4)*((e^3*1i)/(4*(b^2*d^2 - a^2*d^2 + a*b*d^2*2i)))^(
1/2) + (32*(a*b^5*d^2*e^15 + 4*a^5*b*d^2*e^15 - 15*a^3*b^3*d^2*e^15))/d^5)*
((e^3*1i)/(4*(b^2*d^2 - a^2*d^2 + a*b*d^2*2i)))^(1/2) + (32*(e*cot(c + d*x)
)^(1/2)*(b^5*e^16 + 2*a^4*b*e^16))/d^4)*((e^3*1i)/(4*(b^2*d^2 - a^2*d^2 + a
*b*d^2*2i)))^(1/2) + (64*a^2*b^2*e^18)/d^5))*((e^3*1i)/(4*(b^2*d^2 - a^2*d^
2 + a*b*d^2*2i)))^(1/2)*2i + atan(((((((32*(4*a^2*b^6*d^4*e^12 + 8*a^4*b^4*
d^4*e^12 + 4*a^6*b^2*d^4*e^12))/d^5 - (32*(e*cot(c + d*x))^(1/2)*(e^3/(4*(b
^2*d^2*1i - a^2*d^2*1i + 2*a*b*d^2)))^(1/2)*(16*b^9*d^4*e^10 + 16*a^2*b^7*d
^4*e^10 - 16*a^4*b^5*d^4*e^10 - 16*a^6*b^3*d^4*e^10))/d^4)*(e^3/(4*(b^2*d^2
*1i - a^2*d^2*1i + 2*a*b*d^2)))^(1/2) + (32*(e*cot(c + d*x))^(1/2)*(14*a*b^
6*d^2*e^13 - 4*a^3*b^4*d^2*e^13 + 14*a^5*b^2*d^2*e^13))/d^4)*(e^3/(4*(b^2*d
^2*1i - a^2*d^2*1i + 2*a*b*d^2)))^(1/2) + (32*(a*b^5*d^2*e^15 + 4*a^5*b*d^2
*e^15 - 15*a^3*b^3*d^2*e^15))/d^5)*(e^3/(4*(b^2*d^2*1i - a^2*d^2*1i + 2*a*b
*d^2)))^(1/2) - (32*(e*cot(c + d*x))^(1/2)*(b^5*e^16 + 2*a^4*b*e^16))/d^4)*
(e^3/(4*(b^2*d^2*1i - a^2*d^2*1i + 2*a*b*d^2)))^(1/2)*1i - (((((32*(4*a^2*b
^6*d^4*e^12 + 8*a^4*b^4*d^4*e^12 + 4*a^6*b^2*d^4*e^12))/d^5 + (32*(e*cot(c
+ d*x))^(1/2)*(e^3/(4*(b^2*d^2*1i - a^2*d^2*1i + 2*a*b*d^2)))^(1/2)*(16*b^9
*d^4*e^10 + 16*a^2*b^7*d^4*e^10 - 16*a^4*b^5*d^4*e^10 - 16*a^6*b^3*d^4*e^10
))/d^4)*(e^3/(4*(b^2*d^2*1i - a^2*d^2*1i + 2*a*b*d^2)))^(1/2) - (32*(e*cot(
c + d*x))^(1/2)*(14*a*b^6*d^2*e^13 - 4*a^3*b^4*d^2*e^13 + 14*a^5*b^2*d^2*e^
13))/d^4)*(e^3/(4*(b^2*d^2*1i - a^2*d^2*1i + 2*a*b*d^2)))^(1/2) + (32*(a*b^
5*d^2*e^15 + 4*a^5*b*d^2*e^15 - 15*a^3*b^3*d^2*e^15))/d^5)*(e^3/(4*(b^2*d^2
*1i - a^2*d^2*1i + 2*a*b*d^2)))^(1/2) + (32*(e*cot(c + d*x))^(1/2)*(b^5*e^1
6 + 2*a^4*b*e^16))/d^4)*(e^3/(4*(b^2*d^2*1i - a^2*d^2*1i + 2*a*b*d^2)))^(1/
```

```
2)*1i)/((((((32*(4*a^2*b^6*d^4*e^12 + 8*a^4*b^4*d^4*e^12 + 4*a^6*b^2*d^4*e^
12))/d^5 - (32*(e*cot(c + d*x))^(1/2)*(e^3/(4*(b^2*d^2*1i - a^2*d^2*1i + 2*
a*b*d^2)))^(1/2)*(16*b^9*d^4*e^10 + 16*a^2*b^7*d^4*e^10 - 16*a^4*b^5*d^4*e^
10 - 16*a^6*b^3*d^4*e^10))/d^4)*(e^3/(4*(b^2*d^2*1i - a^2*d^2*1i + 2*a*b*d^
2)))^(1/2) + (32*(e*cot(c + d*x))^(1/2)*(14*a*b^6*d^2*e^13 - 4*a^3*b^4*d^2*
e^13 + 14*a^5*b^2*d^2*e^13))/d^4)*(e^3/(4*(b^2*d^2*1i - a^2*d^2*1i + 2*a*b*
d^2)))^(1/2) + (32*(a*b^5*d^2*e^15 + 4*a^5*b*d^2*e^15 - 15*a^3*b^3*d^2*e^15
))/d^5)*(e^3/(4*(b^2*d^2*1i - a^2*d^2*1i + 2*a*b*d^2)))^(1/2) - (32*(e*cot(
c + d*x))^(1/2)*(b^5*e^16 + 2*a^4*b*e^16))/d^4)*(e^3/(4*(b^2*d^2*1i - a^2*d
^2*1i + 2*a*b*d^2)))^(1/2) + (((((32*(4*a^2*b^6*d^4*e^12 + 8*a^4*b^4*d^4*e^
12 + 4*a^6*b^2*d^4*e^12))/d^5 + (32*(e*cot(c + d*x))^(1/2)*(e^3/(4*(b^2*d^2
*1i - a^2*d^2*1i + 2*a*b*d^2)))^(1/2)*(16*b^9*d^4*e^10 + 16*a^2*b^7*d^4*e^1
0 - 16*a^4*b^5*d^4*e^10 - 16*a^6*b^3*d^4*e^10))/d^4)*(e^3/(4*(b^2*d^2*1i -
a^2*d^2*1i + 2*a*b*d^2)))^(1/2) - (32*(e*cot(c + d*x))^(1/2)*(14*a*b^6*d^2*
e^13 - 4*a^3*b^4*d^2*e^13 + 14*a^5*b^2*d^2*e^13))/d^4)*(e^3/(4*(b^2*d^2*1i
- a^2*d^2*1i + 2*a*b*d^2)))^(1/2) + (32*(a*b^5*d^2*e^15 + 4*a^5*b*d^2*e^15
- 15*a^3*b^3*d^2*e^15))/d^5)*(e^3/(4*(b^2*d^2*1i - a^2*d^2*1i + 2*a*b*d^2))
)^(1/2) + (32*(e*cot(c + d*x))^(1/2)*(b^5*e^16 + 2*a^4*b*e^16))/d^4)*(e^3/(
4*(b^2*d^2*1i - a^2*d^2*1i + 2*a*b*d^2)))^(1/2) + (64*a^2*b^2*e^18)/d^5))*(
e^3/(4*(b^2*d^2*1i - a^2*d^2*1i + 2*a*b*d^2)))^(1/2)*2i - (atan(-((((((32*(
a*b^5*d^2*e^15 + 4*a^5*b*d^2*e^15 - 15*a^3*b^3*d^2*e^15))/d^5 + (((((32*(4*
a^2*b^6*d^4*e^12 + 8*a^4*b^4*d^4*e^12 + 4*a^6*b^2*d^4*e^12))/d^5 - (32*(e*c
ot(c + d*x))^(1/2)*(-a^3*b*e^3)^(1/2)*(16*b^9*d^4*e^10 + 16*a^2*b^7*d^4*e^1
0 - 16*a^4*b^5*d^4*e^10 - 16*a^6*b^3*d^4*e^10))/(d^4*(b^3*d + a^2*b*d)))*(-
a^3*b*e^3)^(1/2))/(b^3*d + a^2*b*d) + (32*(e*cot(c + d*x))^(1/2)*(14*a*b^6*
d^2*e^13 - 4*a^3*b^4*d^2*e^13 + 14*a^5*b^2*d^2*e^13))/d^4)*(-a^3*b*e^3)^(1/
2))/(b^3*d + a^2*b*d))*(-a^3*b*e^3)^(1/2))/(b^3*d + a^2*b*d) - (32*(e*cot(c
 + d*x))^(1/2)*(b^5*e^16 + 2*a^4*b*e^16))/d^4)*(-a^3*b*e^3)^(1/2)*1i)/(b^3*
d + a^2*b*d) - (((((32*(a*b^5*d^2*e^15 + 4*a^5*b*d^2*e^15 - 15*a^3*b^3*d^2*
e^15))/d^5 + (((((32*(4*a^2*b^6*d^4*e^12 + 8*a^4*b^4*d^4*e^12 + 4*a^6*b^2*d
^4*e^12))/d^5 + (32*(e*cot(c + d*x))^(1/2)*(-a^3*b*e^3)^(1/2)*(16*b^9*d^4*e
^10 + 16*a^2*b^7*d^4*e^10 - 16*a^4*b^5*d^4*e^10 - 16*a^6*b^3*d^4*e^10))/(d^
4*(b^3*d + a^2*b*d)))*(-a^3*b*e^3)^(1/2))/(b^3*d + a^2*b*d) - (32*(e*cot(c
+ d*x))^(1/2)*(14*a*b^6*d^2*e^13 - 4*a^3*b^4*d^2*e^13 + 14*a^5*b^2*d^2*e^13
))/d^4)*(-a^3*b*e^3)^(1/2))/(b^3*d + a^2*b*d))*(-a^3*b*e^3)^(1/2))/(b^3*d +
 a^2*b*d) + (32*(e*cot(c + d*x))^(1/2)*(b^5*e^16 + 2*a^4*b*e^16))/d^4)*(-a^
3*b*e^3)^(1/2)*1i)/(b^3*d + a^2*b*d))/((((((32*(a*b^5*d^2*e^15 + 4*a^5*b*d^
2*e^15 - 15*a^3*b^3*d^2*e^15))/d^5 + (((((32*(4*a^2*b^6*d^4*e^12 + 8*a^4*b^
4*d^4*e^12 + 4*a^6*b^2*d^4*e^12))/d^5 - (32*(e*cot(c + d*x))^(1/2)*(-a^3*b*
e^3)^(1/2)*(16*b^9*d^4*e^10 + 16*a^2*b^7*d^4*e^10 - 16*a^4*b^5*d^4*e^10 - 1
6*a^6*b^3*d^4*e^10))/(d^4*(b^3*d + a^2*b*d)))*(-a^3*b*e^3)^(1/2))/(b^3*d +
a^2*b*d) + (32*(e*cot(c + d*x))^(1/2)*(14*a*b^6*d^2*e^13 - 4*a^3*b^4*d^2*e^
13 + 14*a^5*b^2*d^2*e^13))/d^4)*(-a^3*b*e^3)^(1/2))/(b^3*d + a^2*b*d))*(-a^
3*b*e^3)^(1/2))/(b^3*d + a^2*b*d) - (32*(e*cot(c + d*x))^(1/2)*(b^5*e^16 +
2*a^4*b*e^16))/d^4)*(-a^3*b*e^3)^(1/2))/(b^3*d + a^2*b*d) + (((((32*(a*b^5*
```

```
d^2*e^15 + 4*a^5*b*d^2*e^15 - 15*a^3*b^3*d^2*e^15))/d^5 + (((((32*(4*a^2*b^
6*d^4*e^12 + 8*a^4*b^4*d^4*e^12 + 4*a^6*b^2*d^4*e^12))/d^5 + (32*(e*cot(c +
 d*x))^(1/2)*(-a^3*b*e^3)^(1/2)*(16*b^9*d^4*e^10 + 16*a^2*b^7*d^4*e^10 - 16
*a^4*b^5*d^4*e^10 - 16*a^6*b^3*d^4*e^10))/(d^4*(b^3*d + a^2*b*d)))*(-a^3*b*
e^3)^(1/2))/(b^3*d + a^2*b*d) - (32*(e*cot(c + d*x))^(1/2)*(14*a*b^6*d^2*e^
13 - 4*a^3*b^4*d^2*e^13 + 14*a^5*b^2*d^2*e^13))/d^4)*(-a^3*b*e^3)^(1/2))/(b
^3*d + a^2*b*d))*(-a^3*b*e^3)^(1/2))/(b^3*d + a^2*b*d) + (32*(e*cot(c + d*x
))^(1/2)*(b^5*e^16 + 2*a^4*b*e^16))/d^4)*(-a^3*b*e^3)^(1/2))/(b^3*d + a^2*b
*d) + (64*a^2*b^2*e^18)/d^5))*(-a^3*b*e^3)^(1/2)*2i)/(b^3*d + a^2*b*d)
```

3.71 $\int \frac{\sqrt{e\cot(c+dx)}}{a+b\cot(c+dx)}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 302

$$\begin{aligned}\int \frac{\sqrt{e\cot(c+dx)}}{a+b\cot(c+dx)}\,dx &= \frac{2\sqrt{a}\sqrt{b}\sqrt{e}\arctan\left(\frac{\sqrt{b}\sqrt{e\cot(c+dx)}}{\sqrt{a}\sqrt{e}}\right)}{(a^2+b^2)\,d} \\ &\quad+\frac{(a+b)\sqrt{e}\arctan\left(1-\frac{\sqrt{2}\sqrt{e\cot(c+dx)}}{\sqrt{e}}\right)}{\sqrt{2}\,(a^2+b^2)\,d} \\ &\quad-\frac{(a+b)\sqrt{e}\arctan\left(1+\frac{\sqrt{2}\sqrt{e\cot(c+dx)}}{\sqrt{e}}\right)}{\sqrt{2}\,(a^2+b^2)\,d} \\ &\quad-\frac{(a-b)\sqrt{e}\log\left(\sqrt{e}+\sqrt{e}\cot(c+dx)-\sqrt{2}\sqrt{e\cot(c+dx)}\right)}{2\sqrt{2}\,(a^2+b^2)\,d} \\ &\quad+\frac{(a-b)\sqrt{e}\log\left(\sqrt{e}+\sqrt{e}\cot(c+dx)+\sqrt{2}\sqrt{e\cot(c+dx)}\right)}{2\sqrt{2}\,(a^2+b^2)\,d}\end{aligned}$$

```
[Out] 1/2*(a+b)*arctan(1-2^(1/2)*(e*cot(d*x+c))^(1/2)/e^(1/2))*e^(1/2)/(a^2+b^2)/
d*2^(1/2)-1/2*(a+b)*arctan(1+2^(1/2)*(e*cot(d*x+c))^(1/2)/e^(1/2))*e^(1/2)/
(a^2+b^2)/d*2^(1/2)-1/4*(a-b)*ln(e^(1/2)+cot(d*x+c)*e^(1/2)-2^(1/2)*(e*cot(
d*x+c))^(1/2))*e^(1/2)/(a^2+b^2)/d*2^(1/2)+1/4*(a-b)*ln(e^(1/2)+cot(d*x+c)*
e^(1/2)+2^(1/2)*(e*cot(d*x+c))^(1/2))*e^(1/2)/(a^2+b^2)/d*2^(1/2)+2*arctan(
b^(1/2)*(e*cot(d*x+c))^(1/2)/a^(1/2)/e^(1/2))*a^(1/2)*b^(1/2)*e^(1/2)/(a^2+
b^2)/d
```

Rubi [A] (verified)

Time = 0.45 (sec) , antiderivative size = 302, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.440$, Rules used = {3653, 3615, 1182, 1176, 631, 210, 1179, 642, 3715, 65, 211}

$$\int \frac{\sqrt{e\cot(c+dx)}}{a+b\cot(c+dx)}\,dx = \frac{2\sqrt{a}\sqrt{b}\sqrt{e}\arctan\left(\frac{\sqrt{b}\sqrt{e\cot(c+dx)}}{\sqrt{a}\sqrt{e}}\right)}{d\left(a^2+b^2\right)} + \frac{\sqrt{e}(a+b)\arctan\left(1-\frac{\sqrt{2}\sqrt{e\cot(c+dx)}}{\sqrt{e}}\right)}{\sqrt{2}d\left(a^2+b^2\right)} - \frac{\sqrt{e}(a+b)\arctan\left(\frac{\sqrt{2}\sqrt{e\cot(c+dx)}}{\sqrt{e}}+1\right)}{\sqrt{2}d\left(a^2+b^2\right)} - \frac{\sqrt{e}(a-b)\log\left(\sqrt{e}\cot(c+dx)-\sqrt{2}\sqrt{e\cot(c+dx)}+\sqrt{e}\right)}{2\sqrt{2}d\left(a^2+b^2\right)} + \frac{\sqrt{e}(a-b)\log\left(\sqrt{e}\cot(c+dx)+\sqrt{2}\sqrt{e\cot(c+dx)}+\sqrt{e}\right)}{2\sqrt{2}d\left(a^2+b^2\right)}$$

```
[In] Int[Sqrt[e*Cot[c + d*x]]/(a + b*Cot[c + d*x]),x]
```

```
[Out] (2*Sqrt[a]*Sqrt[b]*Sqrt[e]*ArcTan[(Sqrt[b]*Sqrt[e*Cot[c + d*x]])/(Sqrt[a]*S
qrt[e])])/((a^2 + b^2)*d) + ((a + b)*Sqrt[e]*ArcTan[1 - (Sqrt[2]*Sqrt[e*Cot
[c + d*x]])/Sqrt[e]])/(Sqrt[2]*(a^2 + b^2)*d) - ((a + b)*Sqrt[e]*ArcTan[1 +
 (Sqrt[2]*Sqrt[e*Cot[c + d*x]])/Sqrt[e]])/(Sqrt[2]*(a^2 + b^2)*d) - ((a - b
)*Sqrt[e]*Log[Sqrt[e] + Sqrt[e]*Cot[c + d*x] - Sqrt[2]*Sqrt[e*Cot[c + d*x]]
])/(2*Sqrt[2]*(a^2 + b^2)*d) + ((a - b)*Sqrt[e]*Log[Sqrt[e] + Sqrt[e]*Cot[c
 + d*x] + Sqrt[2]*Sqrt[e*Cot[c + d*x]]])/(2*Sqrt[2]*(a^2 + b^2)*d)
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1179

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 1182

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
a*c, 2]}, Dist[(d*q + a*e)/(2*a*c), Int[(q + c*x^2)/(a + c*x^4), x], x] + D
ist[(d*q - a*e)/(2*a*c), Int[(q - c*x^2)/(a + c*x^4), x], x]] /; FreeQ[{a,
c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0] && NeQ[c*d^2 - a*e^2, 0] && NegQ[(-a)
*c]
```

Rule 3615

```
Int[((c_) + (d_.)*tan[(e_.) + (f_.)*(x_)])/Sqrt[(b_.)*tan[(e_.) + (f_.)*(x_
)]], x_Symbol] :> Dist[2/f, Subst[Int[(b*c + d*x^2)/(b^2 + x^4), x], x, Sqr
t[b*Tan[e + f*x]]], x] /; FreeQ[{b, c, d, e, f}, x] && NeQ[c^2 - d^2, 0] &&
 NeQ[c^2 + d^2, 0]
```

Rule 3653

```
Int[Sqrt[(a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)]]/((c_.) + (d_.)*tan[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Dist[1/(c^2 + d^2), Int[Simp[a*c + b*d + (b*c -
a*d)*Tan[e + f*x], x]/Sqrt[a + b*Tan[e + f*x]], x], x] - Dist[d*((b*c - a*d
)/(c^2 + d^2)), Int[(1 + Tan[e + f*x]^2)/(Sqrt[a + b*Tan[e + f*x]]*(c + d*T
an[e + f*x])), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && NeQ
[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0]
```

Rule 3715

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*tan[(e_.)
+ (f_.)*(x_)])^(n_.)*((A_) + (C_.)*tan[(e_.) + (f_.)*(x_)]^2), x_Symbol] :>
 Dist[A/f, Subst[Int[(a + b*x)^m*(c + d*x)^n, x], x, Tan[e + f*x]], x] /; F
reeQ[{a, b, c, d, e, f, A, C, m, n}, x] && EqQ[A, C]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{\int \frac{be+ae\cot(c+dx)}{\sqrt{e\cot(c+dx)}}\,dx}{a^2+b^2} - \frac{(abe)\int \frac{1+\cot^2(c+dx)}{\sqrt{e\cot(c+dx)}(a+b\cot(c+dx))}\,dx}{a^2+b^2} \\
&= \frac{2\text{Subst}\left(\int \frac{-be^2-aex^2}{e^2+x^4}\,dx, x, \sqrt{e\cot(c+dx)}\right)}{(a^2+b^2)\,d} - \frac{(abe)\text{Subst}\left(\int \frac{1}{\sqrt{-ex}(a-bx)}\,dx, x, -\cot(c+dx)\right)}{(a^2+b^2)\,d} \\
&= \frac{(2ab)\text{Subst}\left(\int \frac{1}{a+\frac{bx^2}{e}}\,dx, x, \sqrt{e\cot(c+dx)}\right)}{(a^2+b^2)\,d} \\
&\quad + \frac{((a-b)e)\text{Subst}\left(\int \frac{e-x^2}{e^2+x^4}\,dx, x, \sqrt{e\cot(c+dx)}\right)}{(a^2+b^2)\,d} \\
&\quad - \frac{((a+b)e)\text{Subst}\left(\int \frac{e+x^2}{e^2+x^4}\,dx, x, \sqrt{e\cot(c+dx)}\right)}{(a^2+b^2)\,d} \\
&= \frac{2\sqrt{a}\sqrt{b}\sqrt{e}\arctan\left(\frac{\sqrt{b}\sqrt{e\cot(c+dx)}}{\sqrt{a}\sqrt{e}}\right)}{(a^2+b^2)\,d} \\
&\quad - \frac{\left((a-b)\sqrt{e}\right)\text{Subst}\left(\int \frac{\sqrt{2}\sqrt{e}+2x}{-e-\sqrt{2}\sqrt{e}x-x^2}\,dx, x, \sqrt{e\cot(c+dx)}\right)}{2\sqrt{2}\,(a^2+b^2)\,d} \\
&\quad - \frac{\left((a-b)\sqrt{e}\right)\text{Subst}\left(\int \frac{\sqrt{2}\sqrt{e}-2x}{-e+\sqrt{2}\sqrt{e}x-x^2}\,dx, x, \sqrt{e\cot(c+dx)}\right)}{2\sqrt{2}\,(a^2+b^2)\,d} \\
&\quad - \frac{((a+b)e)\text{Subst}\left(\int \frac{1}{e-\sqrt{2}\sqrt{e}x+x^2}\,dx, x, \sqrt{e\cot(c+dx)}\right)}{2\,(a^2+b^2)\,d} \\
&\quad - \frac{((a+b)e)\text{Subst}\left(\int \frac{1}{e+\sqrt{2}\sqrt{e}x+x^2}\,dx, x, \sqrt{e\cot(c+dx)}\right)}{2\,(a^2+b^2)\,d}
\end{aligned}$$

$$= \frac{2\sqrt{a}\sqrt{b}\sqrt{e}\arctan\left(\frac{\sqrt{b}\sqrt{e\cot(c+dx)}}{\sqrt{a}\sqrt{e}}\right)}{(a^2+b^2)\,d} - \frac{(a-b)\sqrt{e}\log\left(\sqrt{e}+\sqrt{e}\cot(c+dx)-\sqrt{2}\sqrt{e\cot(c+dx)}\right)}{2\sqrt{2}\,(a^2+b^2)\,d} + \frac{(a-b)\sqrt{e}\log\left(\sqrt{e}+\sqrt{e}\cot(c+dx)+\sqrt{2}\sqrt{e\cot(c+dx)}\right)}{2\sqrt{2}\,(a^2+b^2)\,d} - \frac{\left((a+b)\sqrt{e}\right)\text{Subst}\left(\int\frac{1}{-1-x^2}\,dx,x,1-\frac{\sqrt{2}\sqrt{e\cot(c+dx)}}{\sqrt{e}}\right)}{\sqrt{2}\,(a^2+b^2)\,d} + \frac{\left((a+b)\sqrt{e}\right)\text{Subst}\left(\int\frac{1}{-1-x^2}\,dx,x,1+\frac{\sqrt{2}\sqrt{e\cot(c+dx)}}{\sqrt{e}}\right)}{\sqrt{2}\,(a^2+b^2)\,d}$$

$$= \frac{2\sqrt{a}\sqrt{b}\sqrt{e}\arctan\left(\frac{\sqrt{b}\sqrt{e\cot(c+dx)}}{\sqrt{a}\sqrt{e}}\right)}{(a^2+b^2)\,d} + \frac{(a+b)\sqrt{e}\arctan\left(1-\frac{\sqrt{2}\sqrt{e\cot(c+dx)}}{\sqrt{e}}\right)}{\sqrt{2}\,(a^2+b^2)\,d} - \frac{(a+b)\sqrt{e}\arctan\left(1+\frac{\sqrt{2}\sqrt{e\cot(c+dx)}}{\sqrt{e}}\right)}{\sqrt{2}\,(a^2+b^2)\,d} - \frac{(a-b)\sqrt{e}\log\left(\sqrt{e}+\sqrt{e}\cot(c+dx)-\sqrt{2}\sqrt{e\cot(c+dx)}\right)}{2\sqrt{2}\,(a^2+b^2)\,d} + \frac{(a-b)\sqrt{e}\log\left(\sqrt{e}+\sqrt{e}\cot(c+dx)+\sqrt{2}\sqrt{e\cot(c+dx)}\right)}{2\sqrt{2}\,(a^2+b^2)\,d}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 0.27 (sec) , antiderivative size = 226, normalized size of antiderivative = 0.75

$$\int\frac{\sqrt{e\cot(c+dx)}}{a+b\cot(c+dx)}\,dx = \frac{\sqrt{e\cot(c+dx)}\left(6\sqrt{2}b\arctan\left(1-\sqrt{2}\sqrt{\cot(c+dx)}\right)-6\sqrt{2}b\arctan\left(1+\sqrt{2}\sqrt{\cot(c+dx)}\right)+24\sqrt{a}\sqrt{b}\,\mathrm{a}\right.}{}$$

```
[In] Integrate[Sqrt[e*Cot[c + d*x]]/(a + b*Cot[c + d*x]),x]
```

```
[Out] (Sqrt[e*Cot[c + d*x]]*(6*Sqrt[2]*b*ArcTan[1 - Sqrt[2]*Sqrt[Cot[c + d*x]]] -
 6*Sqrt[2]*b*ArcTan[1 + Sqrt[2]*Sqrt[Cot[c + d*x]]] + 24*Sqrt[a]*Sqrt[b]*Ar
cTan[(Sqrt[b]*Sqrt[Cot[c + d*x]])/Sqrt[a]] - 8*a*Cot[c + d*x]^(3/2)*Hyperge
ometric2F1[3/4, 1, 7/4, -Cot[c + d*x]^2] + 3*Sqrt[2]*b*Log[1 - Sqrt[2]*Sqrt
[Cot[c + d*x]] + Cot[c + d*x]] - 3*Sqrt[2]*b*Log[1 + Sqrt[2]*Sqrt[Cot[c + d
*x]] + Cot[c + d*x]]))/(12*(a^2 + b^2)*d*Sqrt[Cot[c + d*x]])
```

Maple [A] (verified)

Time = 0.04 (sec) , antiderivative size = 332, normalized size of antiderivative = 1.10

method	result
derivativedivides	$-\frac{2e^2\left(-\frac{ab\arctan\left(\frac{\sqrt{e\cot(dx+c)}\,b}{\sqrt{aeb}}\right)}{e(a^2+b^2)\sqrt{aeb}}+\frac{\frac{b(e^2)^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\frac{e\cot(dx+c)+(e^2)^{\frac{1}{4}}\sqrt{e\cot(dx+c)}\sqrt{2}+\sqrt{e^2}}{e\cot(dx+c)-(e^2)^{\frac{1}{4}}\sqrt{e\cot(dx+c)}\sqrt{2}+\sqrt{e^2}}\right)+2\arctan\left(\frac{\sqrt{2}\sqrt{e\cot(dx+}}{(e^2)^{\frac{1}{4}}}\right.\right.}{8e}}{}\right.}{}$
default	$-\frac{2e^2\left(-\frac{ab\arctan\left(\frac{\sqrt{e\cot(dx+c)}\,b}{\sqrt{aeb}}\right)}{e(a^2+b^2)\sqrt{aeb}}+\frac{\frac{b(e^2)^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\frac{e\cot(dx+c)+(e^2)^{\frac{1}{4}}\sqrt{e\cot(dx+c)}\sqrt{2}+\sqrt{e^2}}{e\cot(dx+c)-(e^2)^{\frac{1}{4}}\sqrt{e\cot(dx+c)}\sqrt{2}+\sqrt{e^2}}\right)+2\arctan\left(\frac{\sqrt{2}\sqrt{e\cot(dx+}}{(e^2)^{\frac{1}{4}}}\right.\right.}{8e}}{}\right.}{}$

```
[In] int((e*cot(d*x+c))^(1/2)/(a+b*cot(d*x+c)),x,method=_RETURNVERBOSE)
```

```
[Out] -2/d*e^2*(-a/e*b/(a^2+b^2)/(a*e*b)^(1/2)*arctan((e*cot(d*x+c))^(1/2)*b/(a*e
*b)^(1/2))+1/e/(a^2+b^2)*(1/8*b/e*(e^2)^(1/4)*2^(1/2)*(ln((e*cot(d*x+c)+(e^
2)^(1/4)*(e*cot(d*x+c))^(1/2)*2^(1/2)+(e^2)^(1/2))/(e*cot(d*x+c)-(e^2)^(1/4
)*(e*cot(d*x+c))^(1/2)*2^(1/2)+(e^2)^(1/2)))+2*arctan(2^(1/2)/(e^2)^(1/4)*(
e*cot(d*x+c))^(1/2)+1)-2*arctan(-2^(1/2)/(e^2)^(1/4)*(e*cot(d*x+c))^(1/2)+1
))+1/8*a/(e^2)^(1/4)*2^(1/2)*(ln((e*cot(d*x+c)-(e^2)^(1/4)*(e*cot(d*x+c))^(
1/2)*2^(1/2)+(e^2)^(1/2))/(e*cot(d*x+c)+(e^2)^(1/4)*(e*cot(d*x+c))^(1/2)*2^
(1/2)+(e^2)^(1/2)))+2*arctan(2^(1/2)/(e^2)^(1/4)*(e*cot(d*x+c))^(1/2)+1)-2*
arctan(-2^(1/2)/(e^2)^(1/4)*(e*cot(d*x+c))^(1/2)+1))))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1523 vs. 2(241) = 482.

Time = 0.36 (sec) , antiderivative size = 3088, normalized size of antiderivative = 10.23

$$\int \frac{\sqrt{e\cot(c+dx)}}{a+b\cot(c+dx)}\,dx = \text{Too large to display}$$

```
[In] integrate((e*cot(d*x+c))^(1/2)/(a+b*cot(d*x+c)),x, algorithm="fricas")
```

```
[Out] [-1/2*((a^2 + b^2)*d*sqrt(-((a^4 + 2*a^2*b^2 + b^4)*d^2*sqrt(-(a^4 - 2*a^2*
b^2 + b^4)*e^2/((a^8 + 4*a^6*b^2 + 6*a^4*b^4 + 4*a^2*b^6 + b^8)*d^4)) + 2*a
*b*e)/((a^4 + 2*a^2*b^2 + b^4)*d^2))*log(-(a^2 - b^2)*e*sqrt((e*cos(2*d*x +
```

```
 2*c) + e)/sin(2*d*x + 2*c)) + ((a^5 + 2*a^3*b^2 + a*b^4)*d^3*sqrt(-(a^4 -
2*a^2*b^2 + b^4)*e^2/((a^8 + 4*a^6*b^2 + 6*a^4*b^4 + 4*a^2*b^6 + b^8)*d^4))
 - (a^2*b - b^3)*d*e)*sqrt(-((a^4 + 2*a^2*b^2 + b^4)*d^2*sqrt(-(a^4 - 2*a^2
*b^2 + b^4)*e^2/((a^8 + 4*a^6*b^2 + 6*a^4*b^4 + 4*a^2*b^6 + b^8)*d^4)) + 2*
a*b*e)/((a^4 + 2*a^2*b^2 + b^4)*d^2))) - (a^2 + b^2)*d*sqrt(-((a^4 + 2*a^2*
b^2 + b^4)*d^2*sqrt(-(a^4 - 2*a^2*b^2 + b^4)*e^2/((a^8 + 4*a^6*b^2 + 6*a^4*
b^4 + 4*a^2*b^6 + b^8)*d^4)) + 2*a*b*e)/((a^4 + 2*a^2*b^2 + b^4)*d^2))*log(
-(a^2 - b^2)*e*sqrt((e*cos(2*d*x + 2*c) + e)/sin(2*d*x + 2*c)) - ((a^5 + 2*
a^3*b^2 + a*b^4)*d^3*sqrt(-(a^4 - 2*a^2*b^2 + b^4)*e^2/((a^8 + 4*a^6*b^2 +
6*a^4*b^4 + 4*a^2*b^6 + b^8)*d^4)) - (a^2*b - b^3)*d*e)*sqrt(-((a^4 + 2*a^2
*b^2 + b^4)*d^2*sqrt(-(a^4 - 2*a^2*b^2 + b^4)*e^2/((a^8 + 4*a^6*b^2 + 6*a^4
*b^4 + 4*a^2*b^6 + b^8)*d^4)) + 2*a*b*e)/((a^4 + 2*a^2*b^2 + b^4)*d^2))) -
(a^2 + b^2)*d*sqrt(((a^4 + 2*a^2*b^2 + b^4)*d^2*sqrt(-(a^4 - 2*a^2*b^2 + b^
4)*e^2/((a^8 + 4*a^6*b^2 + 6*a^4*b^4 + 4*a^2*b^6 + b^8)*d^4)) - 2*a*b*e)/((
a^4 + 2*a^2*b^2 + b^4)*d^2))*log(-(a^2 - b^2)*e*sqrt((e*cos(2*d*x + 2*c) +
e)/sin(2*d*x + 2*c)) + ((a^5 + 2*a^3*b^2 + a*b^4)*d^3*sqrt(-(a^4 - 2*a^2*b^
2 + b^4)*e^2/((a^8 + 4*a^6*b^2 + 6*a^4*b^4 + 4*a^2*b^6 + b^8)*d^4)) + (a^2*
b - b^3)*d*e)*sqrt(((a^4 + 2*a^2*b^2 + b^4)*d^2*sqrt(-(a^4 - 2*a^2*b^2 + b^
4)*e^2/((a^8 + 4*a^6*b^2 + 6*a^4*b^4 + 4*a^2*b^6 + b^8)*d^4)) - 2*a*b*e)/((
a^4 + 2*a^2*b^2 + b^4)*d^2))) + (a^2 + b^2)*d*sqrt(((a^4 + 2*a^2*b^2 + b^4)
*d^2*sqrt(-(a^4 - 2*a^2*b^2 + b^4)*e^2/((a^8 + 4*a^6*b^2 + 6*a^4*b^4 + 4*a^
2*b^6 + b^8)*d^4)) - 2*a*b*e)/((a^4 + 2*a^2*b^2 + b^4)*d^2))*log(-(a^2 - b^
2)*e*sqrt((e*cos(2*d*x + 2*c) + e)/sin(2*d*x + 2*c)) - ((a^5 + 2*a^3*b^2 +
a*b^4)*d^3*sqrt(-(a^4 - 2*a^2*b^2 + b^4)*e^2/((a^8 + 4*a^6*b^2 + 6*a^4*b^4
+ 4*a^2*b^6 + b^8)*d^4)) + (a^2*b - b^3)*d*e)*sqrt(((a^4 + 2*a^2*b^2 + b^4)
*d^2*sqrt(-(a^4 - 2*a^2*b^2 + b^4)*e^2/((a^8 + 4*a^6*b^2 + 6*a^4*b^4 + 4*a^
2*b^6 + b^8)*d^4)) - 2*a*b*e)/((a^4 + 2*a^2*b^2 + b^4)*d^2))) - 2*sqrt(-a*b
*e)*log((b*e*cos(2*d*x + 2*c) - a*e*sin(2*d*x + 2*c) + b*e + 2*sqrt(-a*b*e)
*sqrt((e*cos(2*d*x + 2*c) + e)/sin(2*d*x + 2*c))*sin(2*d*x + 2*c))/(b*cos(2
*d*x + 2*c) + a*sin(2*d*x + 2*c) + b)))/((a^2 + b^2)*d), -1/2*((a^2 + b^2)*
d*sqrt(-((a^4 + 2*a^2*b^2 + b^4)*d^2*sqrt(-(a^4 - 2*a^2*b^2 + b^4)*e^2/((a^
8 + 4*a^6*b^2 + 6*a^4*b^4 + 4*a^2*b^6 + b^8)*d^4)) + 2*a*b*e)/((a^4 + 2*a^2
*b^2 + b^4)*d^2))*log(-(a^2 - b^2)*e*sqrt((e*cos(2*d*x + 2*c) + e)/sin(2*d*
x + 2*c)) + ((a^5 + 2*a^3*b^2 + a*b^4)*d^3*sqrt(-(a^4 - 2*a^2*b^2 + b^4)*e^
2/((a^8 + 4*a^6*b^2 + 6*a^4*b^4 + 4*a^2*b^6 + b^8)*d^4)) - (a^2*b - b^3)*d*
e)*sqrt(-((a^4 + 2*a^2*b^2 + b^4)*d^2*sqrt(-(a^4 - 2*a^2*b^2 + b^4)*e^2/((a
^8 + 4*a^6*b^2 + 6*a^4*b^4 + 4*a^2*b^6 + b^8)*d^4)) + 2*a*b*e)/((a^4 + 2*a^
2*b^2 + b^4)*d^2))) - (a^2 + b^2)*d*sqrt(-((a^4 + 2*a^2*b^2 + b^4)*d^2*sqrt
(-(a^4 - 2*a^2*b^2 + b^4)*e^2/((a^8 + 4*a^6*b^2 + 6*a^4*b^4 + 4*a^2*b^6 + b
^8)*d^4)) + 2*a*b*e)/((a^4 + 2*a^2*b^2 + b^4)*d^2))*log(-(a^2 - b^2)*e*sqrt
((e*cos(2*d*x + 2*c) + e)/sin(2*d*x + 2*c)) - ((a^5 + 2*a^3*b^2 + a*b^4)*d^
3*sqrt(-(a^4 - 2*a^2*b^2 + b^4)*e^2/((a^8 + 4*a^6*b^2 + 6*a^4*b^4 + 4*a^2*b
^6 + b^8)*d^4)) - (a^2*b - b^3)*d*e)*sqrt(-((a^4 + 2*a^2*b^2 + b^4)*d^2*sqr
t(-(a^4 - 2*a^2*b^2 + b^4)*e^2/((a^8 + 4*a^6*b^2 + 6*a^4*b^4 + 4*a^2*b^6 +
b^8)*d^4)) + 2*a*b*e)/((a^4 + 2*a^2*b^2 + b^4)*d^2))) - (a^2 + b^2)*d*sqrt(
```

```
((a^4 + 2*a^2*b^2 + b^4)*d^2*sqrt(-(a^4 - 2*a^2*b^2 + b^4)*e^2/((a^8 + 4*a^
6*b^2 + 6*a^4*b^4 + 4*a^2*b^6 + b^8)*d^4)) - 2*a*b*e)/((a^4 + 2*a^2*b^2 + b
^4)*d^2))*log(-(a^2 - b^2)*e*sqrt((e*cos(2*d*x + 2*c) + e)/sin(2*d*x + 2*c)
) + ((a^5 + 2*a^3*b^2 + a*b^4)*d^3*sqrt(-(a^4 - 2*a^2*b^2 + b^4)*e^2/((a^8
+ 4*a^6*b^2 + 6*a^4*b^4 + 4*a^2*b^6 + b^8)*d^4)) + (a^2*b - b^3)*d*e)*sqrt(
((a^4 + 2*a^2*b^2 + b^4)*d^2*sqrt(-(a^4 - 2*a^2*b^2 + b^4)*e^2/((a^8 + 4*a^
6*b^2 + 6*a^4*b^4 + 4*a^2*b^6 + b^8)*d^4)) - 2*a*b*e)/((a^4 + 2*a^2*b^2 + b
^4)*d^2))) + (a^2 + b^2)*d*sqrt(((a^4 + 2*a^2*b^2 + b^4)*d^2*sqrt(-(a^4 - 2
*a^2*b^2 + b^4)*e^2/((a^8 + 4*a^6*b^2 + 6*a^4*b^4 + 4*a^2*b^6 + b^8)*d^4))
- 2*a*b*e)/((a^4 + 2*a^2*b^2 + b^4)*d^2))*log(-(a^2 - b^2)*e*sqrt((e*cos(2*
d*x + 2*c) + e)/sin(2*d*x + 2*c)) - ((a^5 + 2*a^3*b^2 + a*b^4)*d^3*sqrt(-(a
^4 - 2*a^2*b^2 + b^4)*e^2/((a^8 + 4*a^6*b^2 + 6*a^4*b^4 + 4*a^2*b^6 + b^8)*
d^4)) + (a^2*b - b^3)*d*e)*sqrt(((a^4 + 2*a^2*b^2 + b^4)*d^2*sqrt(-(a^4 - 2
*a^2*b^2 + b^4)*e^2/((a^8 + 4*a^6*b^2 + 6*a^4*b^4 + 4*a^2*b^6 + b^8)*d^4))
- 2*a*b*e)/((a^4 + 2*a^2*b^2 + b^4)*d^2))) + 4*sqrt(a*b*e)*arctan(sqrt(a*b*
e)*sqrt((e*cos(2*d*x + 2*c) + e)/sin(2*d*x + 2*c))*sin(2*d*x + 2*c)/(b*e*co
s(2*d*x + 2*c) + b*e)))/((a^2 + b^2)*d)]
```

Sympy [F]

$$\int \frac{\sqrt{e\cot(c+dx)}}{a+b\cot(c+dx)}\,dx = \int \frac{\sqrt{e\cot\left(c+dx\right)}}{a+b\cot\left(c+dx\right)}\,dx$$

```
[In] integrate((e*cot(d*x+c))**(1/2)/(a+b*cot(d*x+c)),x)
```

```
[Out] Integral(sqrt(e*cot(c + d*x))/(a + b*cot(c + d*x)), x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{\sqrt{e\cot(c+dx)}}{a+b\cot(c+dx)}\,dx = \text{Exception raised: ValueError}$$

```
[In] integrate((e*cot(d*x+c))^(1/2)/(a+b*cot(d*x+c)),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(e>0)', see `assume?` for more detai
ls)Is e
```

Giac [F]

$$\int \frac{\sqrt{e\cot(c+dx)}}{a+b\cot(c+dx)}\,dx = \int \frac{\sqrt{e\cot(dx+c)}}{b\cot(dx+c)+a}\,dx$$

```
[In] integrate((e*cot(d*x+c))^(1/2)/(a+b*cot(d*x+c)),x, algorithm="giac")
[Out] integrate(sqrt(e*cot(d*x + c))/(b*cot(d*x + c) + a), x)
```

Mupad [B] (verification not implemented)

Time = 13.90 (sec) , antiderivative size = 4808, normalized size of antiderivative = 15.92

$$\int \frac{\sqrt{e\cot(c+dx)}}{a+b\cot(c+dx)}\,dx = \text{Too large to display}$$

```
[In] int((e*cot(c + d*x))^(1/2)/(a + b*cot(c + d*x)),x)
[Out] (atan(((((32*(e*cot(c + d*x))^(1/2)*(b^5*e^12 - 2*a^2*b^3*e^12))/d^4 - (((3
2*(13*a^2*b^4*d^2*e^12 + a^4*b^2*d^2*e^12))/d^5 + (((32*(e*cot(c + d*x))^(1
/2)*(20*a^3*b^4*d^2*e^11 - 14*a*b^6*d^2*e^11 + 2*a^5*b^2*d^2*e^11))/d^4 + (
((32*(12*a*b^7*d^4*e^11 + 24*a^3*b^5*d^4*e^11 + 12*a^5*b^3*d^4*e^11))/d^5 -
 (32*(e*cot(c + d*x))^(1/2)*(-a*b*e)^(1/2)*(16*b^9*d^4*e^10 + 16*a^2*b^7*d^
4*e^10 - 16*a^4*b^5*d^4*e^10 - 16*a^6*b^3*d^4*e^10))/(d^5*(a^2 + b^2)))*(-a
*b*e)^(1/2))/(d*(a^2 + b^2)))*(-a*b*e)^(1/2))/(d*(a^2 + b^2)))*(-a*b*e)^(1/
2))/(d*(a^2 + b^2)))*(-a*b*e)^(1/2)*1i)/(d*(a^2 + b^2)) + (((32*(e*cot(c +
d*x))^(1/2)*(b^5*e^12 - 2*a^2*b^3*e^12))/d^4 + (((32*(13*a^2*b^4*d^2*e^12 +
 a^4*b^2*d^2*e^12))/d^5 - (((32*(e*cot(c + d*x))^(1/2)*(20*a^3*b^4*d^2*e^11
 - 14*a*b^6*d^2*e^11 + 2*a^5*b^2*d^2*e^11))/d^4 - (((32*(12*a*b^7*d^4*e^11
+ 24*a^3*b^5*d^4*e^11 + 12*a^5*b^3*d^4*e^11))/d^5 + (32*(e*cot(c + d*x))^(1
/2)*(-a*b*e)^(1/2)*(16*b^9*d^4*e^10 + 16*a^2*b^7*d^4*e^10 - 16*a^4*b^5*d^4*
e^10 - 16*a^6*b^3*d^4*e^10))/(d^5*(a^2 + b^2)))*(-a*b*e)^(1/2))/(d*(a^2 + b
^2)))*(-a*b*e)^(1/2))/(d*(a^2 + b^2)))*(-a*b*e)^(1/2))/(d*(a^2 + b^2)))*(-a
*b*e)^(1/2)*1i)/(d*(a^2 + b^2)))/((64*a*b^3*e^13)/d^5 - (((32*(e*cot(c + d*
x))^(1/2)*(b^5*e^12 - 2*a^2*b^3*e^12))/d^4 - (((32*(13*a^2*b^4*d^2*e^12 + a
^4*b^2*d^2*e^12))/d^5 + (((32*(e*cot(c + d*x))^(1/2)*(20*a^3*b^4*d^2*e^11 -
 14*a*b^6*d^2*e^11 + 2*a^5*b^2*d^2*e^11))/d^4 + (((32*(12*a*b^7*d^4*e^11 +
24*a^3*b^5*d^4*e^11 + 12*a^5*b^3*d^4*e^11))/d^5 - (32*(e*cot(c + d*x))^(1/2
)*(-a*b*e)^(1/2)*(16*b^9*d^4*e^10 + 16*a^2*b^7*d^4*e^10 - 16*a^4*b^5*d^4*e^
10 - 16*a^6*b^3*d^4*e^10))/(d^5*(a^2 + b^2)))*(-a*b*e)^(1/2))/(d*(a^2 + b^2
)))*(-a*b*e)^(1/2))/(d*(a^2 + b^2)))*(-a*b*e)^(1/2))/(d*(a^2 + b^2)))*(-a*b
*e)^(1/2))/(d*(a^2 + b^2)) + (((32*(e*cot(c + d*x))^(1/2)*(b^5*e^12 - 2*a^2
*b^3*e^12))/d^4 + (((32*(13*a^2*b^4*d^2*e^12 + a^4*b^2*d^2*e^12))/d^5 - (((
32*(e*cot(c + d*x))^(1/2)*(20*a^3*b^4*d^2*e^11 - 14*a*b^6*d^2*e^11 + 2*a^5*
```

b^2*d^2*e^11))/d^4 - (((32*(12*a*b^7*d^4*e^11 + 24*a^3*b^5*d^4*e^11 + 12*a^
5*b^3*d^4*e^11))/d^5 + (32*(e*cot(c + d*x))^(1/2)*(-a*b*e)^(1/2)*(16*b^9*d^
4*e^10 + 16*a^2*b^7*d^4*e^10 - 16*a^4*b^5*d^4*e^10 - 16*a^6*b^3*d^4*e^10))/
(d^5*(a^2 + b^2)))*(-a*b*e)^(1/2))/(d*(a^2 + b^2)))*(-a*b*e)^(1/2))/(d*(a^2
 + b^2)))*(-a*b*e)^(1/2))/(d*(a^2 + b^2)))*(-a*b*e)^(1/2))/(d*(a^2 + b^2)))
)*(-a*b*e)^(1/2)*2i)/(d*(a^2 + b^2)) - atan(((((32*(13*a^2*b^4*d^2*e^12 + a
^4*b^2*d^2*e^12))/d^5 + (((32*(12*a*b^7*d^4*e^11 + 24*a^3*b^5*d^4*e^11 + 12
*a^5*b^3*d^4*e^11))/d^5 - (32*(e*cot(c + d*x))^(1/2)*(-e/(4*(b^2*d^2*1i - a
^2*d^2*1i + 2*a*b*d^2)))^(1/2)*(16*b^9*d^4*e^10 + 16*a^2*b^7*d^4*e^10 - 16*
a^4*b^5*d^4*e^10 - 16*a^6*b^3*d^4*e^10))/d^4)*(-e/(4*(b^2*d^2*1i - a^2*d^2*
1i + 2*a*b*d^2)))^(1/2) + (32*(e*cot(c + d*x))^(1/2)*(20*a^3*b^4*d^2*e^11 -
 14*a*b^6*d^2*e^11 + 2*a^5*b^2*d^2*e^11))/d^4)*(-e/(4*(b^2*d^2*1i - a^2*d^2
*1i + 2*a*b*d^2)))^(1/2))*(-e/(4*(b^2*d^2*1i - a^2*d^2*1i + 2*a*b*d^2)))^(1
/2) - (32*(e*cot(c + d*x))^(1/2)*(b^5*e^12 - 2*a^2*b^3*e^12))/d^4)*(-e/(4*(
b^2*d^2*1i - a^2*d^2*1i + 2*a*b*d^2)))^(1/2)*1i - (((32*(13*a^2*b^4*d^2*e^1
2 + a^4*b^2*d^2*e^12))/d^5 + (((32*(12*a*b^7*d^4*e^11 + 24*a^3*b^5*d^4*e^11
 + 12*a^5*b^3*d^4*e^11))/d^5 + (32*(e*cot(c + d*x))^(1/2)*(-e/(4*(b^2*d^2*1
i - a^2*d^2*1i + 2*a*b*d^2)))^(1/2)*(16*b^9*d^4*e^10 + 16*a^2*b^7*d^4*e^10
- 16*a^4*b^5*d^4*e^10 - 16*a^6*b^3*d^4*e^10))/d^4)*(-e/(4*(b^2*d^2*1i - a^2
*d^2*1i + 2*a*b*d^2)))^(1/2) - (32*(e*cot(c + d*x))^(1/2)*(20*a^3*b^4*d^2*e
^11 - 14*a*b^6*d^2*e^11 + 2*a^5*b^2*d^2*e^11))/d^4)*(-e/(4*(b^2*d^2*1i - a^
2*d^2*1i + 2*a*b*d^2)))^(1/2))*(-e/(4*(b^2*d^2*1i - a^2*d^2*1i + 2*a*b*d^2)
))^(1/2) + (32*(e*cot(c + d*x))^(1/2)*(b^5*e^12 - 2*a^2*b^3*e^12))/d^4)*(-e
/(4*(b^2*d^2*1i - a^2*d^2*1i + 2*a*b*d^2)))^(1/2)*1i)/((((32*(13*a^2*b^4*d^
2*e^12 + a^4*b^2*d^2*e^12))/d^5 + (((32*(12*a*b^7*d^4*e^11 + 24*a^3*b^5*d^4
*e^11 + 12*a^5*b^3*d^4*e^11))/d^5 - (32*(e*cot(c + d*x))^(1/2)*(-e/(4*(b^2*
d^2*1i - a^2*d^2*1i + 2*a*b*d^2)))^(1/2)*(16*b^9*d^4*e^10 + 16*a^2*b^7*d^4*
e^10 - 16*a^4*b^5*d^4*e^10 - 16*a^6*b^3*d^4*e^10))/d^4)*(-e/(4*(b^2*d^2*1i
- a^2*d^2*1i + 2*a*b*d^2)))^(1/2) + (32*(e*cot(c + d*x))^(1/2)*(20*a^3*b^4*
d^2*e^11 - 14*a*b^6*d^2*e^11 + 2*a^5*b^2*d^2*e^11))/d^4)*(-e/(4*(b^2*d^2*1i
 - a^2*d^2*1i + 2*a*b*d^2)))^(1/2))*(-e/(4*(b^2*d^2*1i - a^2*d^2*1i + 2*a*b
d^2)))^(1/2) - (32(e*cot(c + d*x))^(1/2)*(b^5*e^12 - 2*a^2*b^3*e^12))/d^4
)*(-e/(4*(b^2*d^2*1i - a^2*d^2*1i + 2*a*b*d^2)))^(1/2) + (((32*(13*a^2*b^4*
d^2*e^12 + a^4*b^2*d^2*e^12))/d^5 + (((32*(12*a*b^7*d^4*e^11 + 24*a^3*b^5*d
^4*e^11 + 12*a^5*b^3*d^4*e^11))/d^5 + (32*(e*cot(c + d*x))^(1/2)*(-e/(4*(b^
2*d^2*1i - a^2*d^2*1i + 2*a*b*d^2)))^(1/2)*(16*b^9*d^4*e^10 + 16*a^2*b^7*d^
4*e^10 - 16*a^4*b^5*d^4*e^10 - 16*a^6*b^3*d^4*e^10))/d^4)*(-e/(4*(b^2*d^2*1
i - a^2*d^2*1i + 2*a*b*d^2)))^(1/2) - (32*(e*cot(c + d*x))^(1/2)*(20*a^3*b^
4*d^2*e^11 - 14*a*b^6*d^2*e^11 + 2*a^5*b^2*d^2*e^11))/d^4)*(-e/(4*(b^2*d^2*
1i - a^2*d^2*1i + 2*a*b*d^2)))^(1/2))*(-e/(4*(b^2*d^2*1i - a^2*d^2*1i + 2*a
*b*d^2)))^(1/2) + (32*(e*cot(c + d*x))^(1/2)*(b^5*e^12 - 2*a^2*b^3*e^12))/d
^4)*(-e/(4*(b^2*d^2*1i - a^2*d^2*1i + 2*a*b*d^2)))^(1/2) + (64*a*b^3*e^13)/
d^5))*(-e/(4*(b^2*d^2*1i - a^2*d^2*1i + 2*a*b*d^2)))^(1/2)*2i - atan(((((32
*(13*a^2*b^4*d^2*e^12 + a^4*b^2*d^2*e^12))/d^5 + (((32*(12*a*b^7*d^4*e^11 +
 24*a^3*b^5*d^4*e^11 + 12*a^5*b^3*d^4*e^11))/d^5 - (32*(e*cot(c + d*x))^(1/

```
2)*(-(e*1i)/(4*(b^2*d^2 - a^2*d^2 + a*b*d^2*2i)))^(1/2)*(16*b^9*d^4*e^10 +
16*a^2*b^7*d^4*e^10 - 16*a^4*b^5*d^4*e^10 - 16*a^6*b^3*d^4*e^10))/d^4)*(-(e
*1i)/(4*(b^2*d^2 - a^2*d^2 + a*b*d^2*2i)))^(1/2) + (32*(e*cot(c + d*x))^(1/
2)*(20*a^3*b^4*d^2*e^11 - 14*a*b^6*d^2*e^11 + 2*a^5*b^2*d^2*e^11))/d^4)*(-(
e*1i)/(4*(b^2*d^2 - a^2*d^2 + a*b*d^2*2i)))^(1/2))*(-(e*1i)/(4*(b^2*d^2 - a
^2*d^2 + a*b*d^2*2i)))^(1/2) - (32*(e*cot(c + d*x))^(1/2)*(b^5*e^12 - 2*a^2
*b^3*e^12))/d^4)*(-(e*1i)/(4*(b^2*d^2 - a^2*d^2 + a*b*d^2*2i)))^(1/2)*1i -
(((32*(13*a^2*b^4*d^2*e^12 + a^4*b^2*d^2*e^12))/d^5 + (((32*(12*a*b^7*d^4*e
^11 + 24*a^3*b^5*d^4*e^11 + 12*a^5*b^3*d^4*e^11))/d^5 + (32*(e*cot(c + d*x)
)^(1/2)*(-(e*1i)/(4*(b^2*d^2 - a^2*d^2 + a*b*d^2*2i)))^(1/2)*(16*b^9*d^4*e^
10 + 16*a^2*b^7*d^4*e^10 - 16*a^4*b^5*d^4*e^10 - 16*a^6*b^3*d^4*e^10))/d^4)
*(-(e*1i)/(4*(b^2*d^2 - a^2*d^2 + a*b*d^2*2i)))^(1/2) - (32*(e*cot(c + d*x)
)^(1/2)*(20*a^3*b^4*d^2*e^11 - 14*a*b^6*d^2*e^11 + 2*a^5*b^2*d^2*e^11))/d^4
)*(-(e*1i)/(4*(b^2*d^2 - a^2*d^2 + a*b*d^2*2i)))^(1/2))*(-(e*1i)/(4*(b^2*d^
2 - a^2*d^2 + a*b*d^2*2i)))^(1/2) + (32*(e*cot(c + d*x))^(1/2)*(b^5*e^12 -
2*a^2*b^3*e^12))/d^4)*(-(e*1i)/(4*(b^2*d^2 - a^2*d^2 + a*b*d^2*2i)))^(1/2)*
1i)/((((32*(13*a^2*b^4*d^2*e^12 + a^4*b^2*d^2*e^12))/d^5 + (((32*(12*a*b^7*
d^4*e^11 + 24*a^3*b^5*d^4*e^11 + 12*a^5*b^3*d^4*e^11))/d^5 - (32*(e*cot(c +
 d*x))^(1/2)*(-(e*1i)/(4*(b^2*d^2 - a^2*d^2 + a*b*d^2*2i)))^(1/2)*(16*b^9*d
^4*e^10 + 16*a^2*b^7*d^4*e^10 - 16*a^4*b^5*d^4*e^10 - 16*a^6*b^3*d^4*e^10))
/d^4)*(-(e*1i)/(4*(b^2*d^2 - a^2*d^2 + a*b*d^2*2i)))^(1/2) + (32*(e*cot(c +
 d*x))^(1/2)*(20*a^3*b^4*d^2*e^11 - 14*a*b^6*d^2*e^11 + 2*a^5*b^2*d^2*e^11)
)/d^4)*(-(e*1i)/(4*(b^2*d^2 - a^2*d^2 + a*b*d^2*2i)))^(1/2))*(-(e*1i)/(4*(b
^2*d^2 - a^2*d^2 + a*b*d^2*2i)))^(1/2) - (32*(e*cot(c + d*x))^(1/2)*(b^5*e^
12 - 2*a^2*b^3*e^12))/d^4)*(-(e*1i)/(4*(b^2*d^2 - a^2*d^2 + a*b*d^2*2i)))^(
1/2) + (((32*(13*a^2*b^4*d^2*e^12 + a^4*b^2*d^2*e^12))/d^5 + (((32*(12*a*b^
7*d^4*e^11 + 24*a^3*b^5*d^4*e^11 + 12*a^5*b^3*d^4*e^11))/d^5 + (32*(e*cot(c
 + d*x))^(1/2)*(-(e*1i)/(4*(b^2*d^2 - a^2*d^2 + a*b*d^2*2i)))^(1/2)*(16*b^9
*d^4*e^10 + 16*a^2*b^7*d^4*e^10 - 16*a^4*b^5*d^4*e^10 - 16*a^6*b^3*d^4*e^10
))/d^4)*(-(e*1i)/(4*(b^2*d^2 - a^2*d^2 + a*b*d^2*2i)))^(1/2) - (32*(e*cot(c
 + d*x))^(1/2)*(20*a^3*b^4*d^2*e^11 - 14*a*b^6*d^2*e^11 + 2*a^5*b^2*d^2*e^1
1))/d^4)*(-(e*1i)/(4*(b^2*d^2 - a^2*d^2 + a*b*d^2*2i)))^(1/2))*(-(e*1i)/(4*
(b^2*d^2 - a^2*d^2 + a*b*d^2*2i)))^(1/2) + (32*(e*cot(c + d*x))^(1/2)*(b^5*
e^12 - 2*a^2*b^3*e^12))/d^4)*(-(e*1i)/(4*(b^2*d^2 - a^2*d^2 + a*b*d^2*2i)))
^(1/2) + (64*a*b^3*e^13)/d^5))*(-(e*1i)/(4*(b^2*d^2 - a^2*d^2 + a*b*d^2*2i)
))^(1/2)*2i
```

3.72 $\int \frac{1}{\sqrt{e\cot(c+dx)}(a+b\cot(c+dx))}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 302

$$\begin{aligned}\int \frac{1}{\sqrt{e\cot(c+dx)}(a+b\cot(c+dx))}\,dx &= -\frac{2b^{3/2}\arctan\left(\frac{\sqrt{b}\sqrt{e\cot(c+dx)}}{\sqrt{a}\sqrt{e}}\right)}{\sqrt{a}\,(a^2+b^2)\,d\sqrt{e}}+\frac{(a-b)\arctan\left(1-\frac{\sqrt{2}\sqrt{e\cot(c+dx)}}{\sqrt{e}}\right)}{\sqrt{2}\,(a^2+b^2)\,d\sqrt{e}}\\ &\quad-\frac{(a-b)\arctan\left(1+\frac{\sqrt{2}\sqrt{e\cot(c+dx)}}{\sqrt{e}}\right)}{\sqrt{2}\,(a^2+b^2)\,d\sqrt{e}}\\ &\quad+\frac{(a+b)\log\left(\sqrt{e}+\sqrt{e}\cot(c+dx)-\sqrt{2}\sqrt{e\cot(c+dx)}\right)}{2\sqrt{2}\,(a^2+b^2)\,d\sqrt{e}}\\ &\quad-\frac{(a+b)\log\left(\sqrt{e}+\sqrt{e}\cot(c+dx)+\sqrt{2}\sqrt{e\cot(c+dx)}\right)}{2\sqrt{2}\,(a^2+b^2)\,d\sqrt{e}}\end{aligned}$$

```
[Out] 1/2*(a-b)*arctan(1-2^(1/2)*(e*cot(d*x+c))^(1/2)/e^(1/2))/(a^2+b^2)/d*2^(1/2
)/e^(1/2)-1/2*(a-b)*arctan(1+2^(1/2)*(e*cot(d*x+c))^(1/2)/e^(1/2))/(a^2+b^2
)/d*2^(1/2)/e^(1/2)+1/4*(a+b)*ln(e^(1/2)+cot(d*x+c)*e^(1/2)-2^(1/2)*(e*cot(
d*x+c))^(1/2))/(a^2+b^2)/d*2^(1/2)/e^(1/2)-1/4*(a+b)*ln(e^(1/2)+cot(d*x+c)*
e^(1/2)+2^(1/2)*(e*cot(d*x+c))^(1/2))/(a^2+b^2)/d*2^(1/2)/e^(1/2)-2*b^(3/2)
*arctan(b^(1/2)*(e*cot(d*x+c))^(1/2)/a^(1/2)/e^(1/2))/(a^2+b^2)/d/a^(1/2)/e
^(1/2)
```

Rubi [A] (verified)

Time = 0.45 (sec) , antiderivative size = 302, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.440, Rules used = {3655, 3615, 1182, 1176, 631, 210, 1179, 642, 3715, 65, 211}

$$\int \frac{1}{\sqrt{e\cot(c+dx)}(a+b\cot(c+dx))}\,dx$$
$$= \frac{(a-b)\arctan\left(1-\frac{\sqrt{2}\sqrt{e\cot(c+dx)}}{\sqrt{e}}\right)}{\sqrt{2}d\sqrt{e}\,(a^2+b^2)} - \frac{(a-b)\arctan\left(\frac{\sqrt{2}\sqrt{e\cot(c+dx)}}{\sqrt{e}}+1\right)}{\sqrt{2}d\sqrt{e}\,(a^2+b^2)} - \frac{2b^{3/2}\arctan\left(\frac{\sqrt{b}\sqrt{e\cot(c+dx)}}{\sqrt{a}\sqrt{e}}\right)}{\sqrt{a}d\sqrt{e}\,(a^2+b^2)}$$
$$+ \frac{(a+b)\log\left(\sqrt{e}\cot(c+dx)-\sqrt{2}\sqrt{e\cot(c+dx)}+\sqrt{e}\right)}{2\sqrt{2}d\sqrt{e}\,(a^2+b^2)}$$
$$- \frac{(a+b)\log\left(\sqrt{e}\cot(c+dx)+\sqrt{2}\sqrt{e\cot(c+dx)}+\sqrt{e}\right)}{2\sqrt{2}d\sqrt{e}\,(a^2+b^2)}$$

```
[In] Int[1/(Sqrt[e*Cot[c + d*x]]*(a + b*Cot[c + d*x])),x]
```

```
[Out] (-2*b^(3/2)*ArcTan[(Sqrt[b]*Sqrt[e*Cot[c + d*x]])/(Sqrt[a]*Sqrt[e])])/(Sqrt
[a]*(a^2 + b^2)*d*Sqrt[e]) + ((a - b)*ArcTan[1 - (Sqrt[2]*Sqrt[e*Cot[c + d*
x]])/Sqrt[e]])/(Sqrt[2]*(a^2 + b^2)*d*Sqrt[e]) - ((a - b)*ArcTan[1 + (Sqrt[
2]*Sqrt[e*Cot[c + d*x]])/Sqrt[e]])/(Sqrt[2]*(a^2 + b^2)*d*Sqrt[e]) + ((a +
b)*Log[Sqrt[e] + Sqrt[e]*Cot[c + d*x] - Sqrt[2]*Sqrt[e*Cot[c + d*x]]])/(2*S
qrt[2]*(a^2 + b^2)*d*Sqrt[e]) - ((a + b)*Log[Sqrt[e] + Sqrt[e]*Cot[c + d*x]
 + Sqrt[2]*Sqrt[e*Cot[c + d*x]]])/(2*Sqrt[2]*(a^2 + b^2)*d*Sqrt[e])
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1179

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 1182

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
a*c, 2]}, Dist[(d*q + a*e)/(2*a*c), Int[(q + c*x^2)/(a + c*x^4), x], x] + D
ist[(d*q - a*e)/(2*a*c), Int[(q - c*x^2)/(a + c*x^4), x], x]] /; FreeQ[{a,
c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0] && NeQ[c*d^2 - a*e^2, 0] && NegQ[(-a)
*c]
```

Rule 3615

```
Int[((c_) + (d_.)*tan[(e_.) + (f_.)*(x_)])/Sqrt[(b_.)*tan[(e_.) + (f_.)*(x_
)]], x_Symbol] :> Dist[2/f, Subst[Int[(b*c + d*x^2)/(b^2 + x^4), x], x, Sqr
t[b*Tan[e + f*x]]], x] /; FreeQ[{b, c, d, e, f}, x] && NeQ[c^2 - d^2, 0] &&
 NeQ[c^2 + d^2, 0]
```

Rule 3655

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)/((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)]), x_Symbol] :> Dist[1/(c^2 + d^2), Int[(a + b*Tan[e + f*x])^m*
(c - d*Tan[e + f*x]), x], x] + Dist[d^2/(c^2 + d^2), Int[(a + b*Tan[e + f*x
])^m*((1 + Tan[e + f*x]^2)/(c + d*Tan[e + f*x])), x], x] /; FreeQ[{a, b, c,
 d, e, f, m}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2
, 0] &&  !IntegerQ[m]
```

Rule 3715

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*tan[(e_.)
+ (f_.)*(x_)])^(n_.)*((A_) + (C_.)*tan[(e_.) + (f_.)*(x_)]^2), x_Symbol] :>
 Dist[A/f, Subst[Int[(a + b*x)^m*(c + d*x)^n, x], x, Tan[e + f*x]], x] /; F
reeQ[{a, b, c, d, e, f, A, C, m, n}, x] && EqQ[A, C]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{\int \frac{a-b\cot(c+dx)}{\sqrt{e\cot(c+dx)}}\,dx}{a^2+b^2}+\frac{b^2\int \frac{1+\cot^2(c+dx)}{\sqrt{e\cot(c+dx)}(a+b\cot(c+dx))}\,dx}{a^2+b^2}\\
&= \frac{2\text{Subst}\left(\int \frac{-ae+bx^2}{e^2+x^4}\,dx,x,\sqrt{e\cot(c+dx)}\right)}{(a^2+b^2)\,d}+\frac{b^2\text{Subst}\left(\int \frac{1}{\sqrt{-ex}(a-bx)}\,dx,x,-\cot(c+dx)\right)}{(a^2+b^2)\,d}\\
&= -\frac{(a-b)\text{Subst}\left(\int \frac{e+x^2}{e^2+x^4}\,dx,x,\sqrt{e\cot(c+dx)}\right)}{(a^2+b^2)\,d}\\
&\quad -\frac{(a+b)\text{Subst}\left(\int \frac{e-x^2}{e^2+x^4}\,dx,x,\sqrt{e\cot(c+dx)}\right)}{(a^2+b^2)\,d}\\
&\quad -\frac{(2b^2)\,\text{Subst}\left(\int \frac{1}{a+\frac{bx^2}{e}}\,dx,x,\sqrt{e\cot(c+dx)}\right)}{(a^2+b^2)\,de}\\
&= -\frac{2b^{3/2}\arctan\left(\frac{\sqrt{b}\sqrt{e\cot(c+dx)}}{\sqrt{a}\sqrt{e}}\right)}{\sqrt{a}\,(a^2+b^2)\,d\sqrt{e}}-\frac{(a-b)\text{Subst}\left(\int \frac{1}{e-\sqrt{2}\sqrt{e}x+x^2}\,dx,x,\sqrt{e\cot(c+dx)}\right)}{2\,(a^2+b^2)\,d}\\
&\quad -\frac{(a-b)\text{Subst}\left(\int \frac{1}{e+\sqrt{2}\sqrt{e}x+x^2}\,dx,x,\sqrt{e\cot(c+dx)}\right)}{2\,(a^2+b^2)\,d}\\
&\quad +\frac{(a+b)\text{Subst}\left(\int \frac{\sqrt{2}\sqrt{e}+2x}{-e-\sqrt{2}\sqrt{e}x-x^2}\,dx,x,\sqrt{e\cot(c+dx)}\right)}{2\sqrt{2}\,(a^2+b^2)\,d\sqrt{e}}\\
&\quad +\frac{(a+b)\text{Subst}\left(\int \frac{\sqrt{2}\sqrt{e}-2x}{-e+\sqrt{2}\sqrt{e}x-x^2}\,dx,x,\sqrt{e\cot(c+dx)}\right)}{2\sqrt{2}\,(a^2+b^2)\,d\sqrt{e}}
\end{aligned}$$

$$= -\frac{2b^{3/2}\arctan\left(\frac{\sqrt{b}\sqrt{e\cot(c+dx)}}{\sqrt{a}\sqrt{e}}\right)}{\sqrt{a}\,(a^2+b^2)\,d\sqrt{e}} + \frac{(a+b)\log\left(\sqrt{e}+\sqrt{e}\cot(c+dx)-\sqrt{2}\sqrt{e\cot(c+dx)}\right)}{2\sqrt{2}\,(a^2+b^2)\,d\sqrt{e}} - \frac{(a+b)\log\left(\sqrt{e}+\sqrt{e}\cot(c+dx)+\sqrt{2}\sqrt{e\cot(c+dx)}\right)}{2\sqrt{2}\,(a^2+b^2)\,d\sqrt{e}} - \frac{(a-b)\text{Subst}\left(\int\frac{1}{-1-x^2}\,dx,x,1-\frac{\sqrt{2}\sqrt{e\cot(c+dx)}}{\sqrt{e}}\right)}{\sqrt{2}\,(a^2+b^2)\,d\sqrt{e}} + \frac{(a-b)\text{Subst}\left(\int\frac{1}{-1-x^2}\,dx,x,1+\frac{\sqrt{2}\sqrt{e\cot(c+dx)}}{\sqrt{e}}\right)}{\sqrt{2}\,(a^2+b^2)\,d\sqrt{e}}$$

$$= -\frac{2b^{3/2}\arctan\left(\frac{\sqrt{b}\sqrt{e\cot(c+dx)}}{\sqrt{a}\sqrt{e}}\right)}{\sqrt{a}\,(a^2+b^2)\,d\sqrt{e}} + \frac{(a-b)\arctan\left(1-\frac{\sqrt{2}\sqrt{e\cot(c+dx)}}{\sqrt{e}}\right)}{\sqrt{2}\,(a^2+b^2)\,d\sqrt{e}} - \frac{(a-b)\arctan\left(1+\frac{\sqrt{2}\sqrt{e\cot(c+dx)}}{\sqrt{e}}\right)}{\sqrt{2}\,(a^2+b^2)\,d\sqrt{e}} + \frac{(a+b)\log\left(\sqrt{e}+\sqrt{e}\cot(c+dx)-\sqrt{2}\sqrt{e\cot(c+dx)}\right)}{2\sqrt{2}\,(a^2+b^2)\,d\sqrt{e}} - \frac{(a+b)\log\left(\sqrt{e}+\sqrt{e}\cot(c+dx)+\sqrt{2}\sqrt{e\cot(c+dx)}\right)}{2\sqrt{2}\,(a^2+b^2)\,d\sqrt{e}}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 0.31 (sec) , antiderivative size = 245, normalized size of antiderivative = 0.81

$$\int \frac{1}{\sqrt{e\cot(c+dx)}(a+b\cot(c+dx))}\,dx = -\frac{\sqrt{\cot(c+dx)}\left(\frac{2b^{3/2}\arctan\left(\frac{\sqrt{b}\sqrt{\cot(c+dx)}}{\sqrt{a}}\right)}{\sqrt{a}(a^2+b^2)} - \frac{2b\cot^{\frac{3}{2}}(c+dx)\,\text{Hypergeometric2F1}\left(\frac{3}{4},1,\frac{7}{4},-\cot^2(c+dx)\right)}{3(a^2+b^2)} - \frac{a\left(2\sqrt{2}\arctan\left(1-\sqrt{2}\right.\right.}{}\right.}{d\sqrt{e\cot(}}$$

```
[In] Integrate[1/(Sqrt[e*Cot[c + d*x]]*(a + b*Cot[c + d*x])),x]
```

```
[Out] -((Sqrt[Cot[c + d*x]]*((2*b^(3/2)*ArcTan[(Sqrt[b]*Sqrt[Cot[c + d*x]])/Sqrt[
a]])/(Sqrt[a]*(a^2 + b^2)) - (2*b*Cot[c + d*x]^(3/2)*Hypergeometric2F1[3/4,
 1, 7/4, -Cot[c + d*x]^2])/(3*(a^2 + b^2)) - (a*(2*Sqrt[2]*ArcTan[1 - Sqrt[
```

```
2]*Sqrt[Cot[c + d*x]]] - 2*Sqrt[2]*ArcTan[1 + Sqrt[2]*Sqrt[Cot[c + d*x]]] +
 Sqrt[2]*Log[1 - Sqrt[2]*Sqrt[Cot[c + d*x]] + Cot[c + d*x]] - Sqrt[2]*Log[1
 + Sqrt[2]*Sqrt[Cot[c + d*x]] + Cot[c + d*x]]))/(4*(a^2 + b^2))))/(d*Sqrt[e
*Cot[c + d*x]]))
```

Maple [A] (verified)

Time = 0.05 (sec) , antiderivative size = 332, normalized size of antiderivative = 1.10

method	result
derivativedivides	$-\frac{2e^2\left(\frac{b^2\arctan\left(\frac{\sqrt{e\cot(dx+c)}\,b}{\sqrt{aeb}}\right)}{e^2(a^2+b^2)\sqrt{aeb}}+\frac{\frac{a(e^2)^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\frac{e\cot(dx+c)+(e^2)^{\frac{1}{4}}\sqrt{e\cot(dx+c)}\,\sqrt{2}+\sqrt{e^2}}{e\cot(dx+c)-(e^2)^{\frac{1}{4}}\sqrt{e\cot(dx+c)}\,\sqrt{2}+\sqrt{e^2}}\right)+2\arctan\left(\frac{\sqrt{2}\sqrt{e\cot(dx+c)}}{(e^2)^{\frac{1}{4}}}+\right.\right.}{8e}}{}\right.}{}$
default	$-\frac{2e^2\left(\frac{b^2\arctan\left(\frac{\sqrt{e\cot(dx+c)}\,b}{\sqrt{aeb}}\right)}{e^2(a^2+b^2)\sqrt{aeb}}+\frac{\frac{a(e^2)^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\frac{e\cot(dx+c)+(e^2)^{\frac{1}{4}}\sqrt{e\cot(dx+c)}\,\sqrt{2}+\sqrt{e^2}}{e\cot(dx+c)-(e^2)^{\frac{1}{4}}\sqrt{e\cot(dx+c)}\,\sqrt{2}+\sqrt{e^2}}\right)+2\arctan\left(\frac{\sqrt{2}\sqrt{e\cot(dx+c)}}{(e^2)^{\frac{1}{4}}}+\right.\right.}{8e}}{}\right.}{}$

```
[In] int(1/(e*cot(d*x+c))^(1/2)/(a+b*cot(d*x+c)),x,method=_RETURNVERBOSE)
```

```
[Out] -2/d*e^2*(b^2/e^2/(a^2+b^2)/(a*e*b)^(1/2)*arctan((e*cot(d*x+c))^(1/2)*b/(a*
e*b)^(1/2))+1/e^2/(a^2+b^2)*(1/8*a/e*(e^2)^(1/4)*2^(1/2)*(ln((e*cot(d*x+c)+
(e^2)^(1/4)*(e*cot(d*x+c))^(1/2)*2^(1/2)+(e^2)^(1/2))/(e*cot(d*x+c)-(e^2)^(
1/4)*(e*cot(d*x+c))^(1/2)*2^(1/2)+(e^2)^(1/2)))+2*arctan(2^(1/2)/(e^2)^(1/4
)*(e*cot(d*x+c))^(1/2)+1)-2*arctan(-2^(1/2)/(e^2)^(1/4)*(e*cot(d*x+c))^(1/2
)+1))-1/8*b/(e^2)^(1/4)*2^(1/2)*(ln((e*cot(d*x+c)-(e^2)^(1/4)*(e*cot(d*x+c)
)^(1/2)*2^(1/2)+(e^2)^(1/2))/(e*cot(d*x+c)+(e^2)^(1/4)*(e*cot(d*x+c))^(1/2)
*2^(1/2)+(e^2)^(1/2)))+2*arctan(2^(1/2)/(e^2)^(1/4)*(e*cot(d*x+c))^(1/2)+1)
-2*arctan(-2^(1/2)/(e^2)^(1/4)*(e*cot(d*x+c))^(1/2)+1))))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1558 vs. 2(241) = 482.

Time = 0.38 (sec) , antiderivative size = 3160, normalized size of antiderivative = 10.46

$$\int \frac{1}{\sqrt{e\cot(c+dx)}(a+b\cot(c+dx))}\,dx = \text{Too large to display}$$

```
[In] integrate(1/(e*cot(d*x+c))^(1/2)/(a+b*cot(d*x+c)),x, algorithm="fricas")
```

```
[Out] [1/2*((a^2 + b^2)*d*sqrt(((a^4 + 2*a^2*b^2 + b^4)*d^2*e*sqrt(-(a^4 - 2*a^2*
b^2 + b^4)/((a^8 + 4*a^6*b^2 + 6*a^4*b^4 + 4*a^2*b^6 + b^8)*d^4*e^2)) + 2*a
*b)/((a^4 + 2*a^2*b^2 + b^4)*d^2*e))*log(-(a^2 - b^2)*sqrt((e*cos(2*d*x + 2
*c) + e)/sin(2*d*x + 2*c)) + ((a^4*b + 2*a^2*b^3 + b^5)*d^3*e^2*sqrt(-(a^4
- 2*a^2*b^2 + b^4)/((a^8 + 4*a^6*b^2 + 6*a^4*b^4 + 4*a^2*b^6 + b^8)*d^4*e^2
)) + (a^3 - a*b^2)*d*e)*sqrt(((a^4 + 2*a^2*b^2 + b^4)*d^2*e*sqrt(-(a^4 - 2*
a^2*b^2 + b^4)/((a^8 + 4*a^6*b^2 + 6*a^4*b^4 + 4*a^2*b^6 + b^8)*d^4*e^2)) +
 2*a*b)/((a^4 + 2*a^2*b^2 + b^4)*d^2*e))) - (a^2 + b^2)*d*sqrt(((a^4 + 2*a^
2*b^2 + b^4)*d^2*e*sqrt(-(a^4 - 2*a^2*b^2 + b^4)/((a^8 + 4*a^6*b^2 + 6*a^4*
b^4 + 4*a^2*b^6 + b^8)*d^4*e^2)) + 2*a*b)/((a^4 + 2*a^2*b^2 + b^4)*d^2*e))*
log(-(a^2 - b^2)*sqrt((e*cos(2*d*x + 2*c) + e)/sin(2*d*x + 2*c)) - ((a^4*b
+ 2*a^2*b^3 + b^5)*d^3*e^2*sqrt(-(a^4 - 2*a^2*b^2 + b^4)/((a^8 + 4*a^6*b^2
+ 6*a^4*b^4 + 4*a^2*b^6 + b^8)*d^4*e^2)) + (a^3 - a*b^2)*d*e)*sqrt(((a^4 +
2*a^2*b^2 + b^4)*d^2*e*sqrt(-(a^4 - 2*a^2*b^2 + b^4)/((a^8 + 4*a^6*b^2 + 6*
a^4*b^4 + 4*a^2*b^6 + b^8)*d^4*e^2)) + 2*a*b)/((a^4 + 2*a^2*b^2 + b^4)*d^2*
e))) - (a^2 + b^2)*d*sqrt(-((a^4 + 2*a^2*b^2 + b^4)*d^2*e*sqrt(-(a^4 - 2*a^
2*b^2 + b^4)/((a^8 + 4*a^6*b^2 + 6*a^4*b^4 + 4*a^2*b^6 + b^8)*d^4*e^2)) - 2
*a*b)/((a^4 + 2*a^2*b^2 + b^4)*d^2*e))*log(-(a^2 - b^2)*sqrt((e*cos(2*d*x +
 2*c) + e)/sin(2*d*x + 2*c)) + ((a^4*b + 2*a^2*b^3 + b^5)*d^3*e^2*sqrt(-(a^
4 - 2*a^2*b^2 + b^4)/((a^8 + 4*a^6*b^2 + 6*a^4*b^4 + 4*a^2*b^6 + b^8)*d^4*e
^2)) - (a^3 - a*b^2)*d*e)*sqrt(-((a^4 + 2*a^2*b^2 + b^4)*d^2*e*sqrt(-(a^4 -
 2*a^2*b^2 + b^4)/((a^8 + 4*a^6*b^2 + 6*a^4*b^4 + 4*a^2*b^6 + b^8)*d^4*e^2)
) - 2*a*b)/((a^4 + 2*a^2*b^2 + b^4)*d^2*e))) + (a^2 + b^2)*d*sqrt(-((a^4 +
2*a^2*b^2 + b^4)*d^2*e*sqrt(-(a^4 - 2*a^2*b^2 + b^4)/((a^8 + 4*a^6*b^2 + 6*
a^4*b^4 + 4*a^2*b^6 + b^8)*d^4*e^2)) - 2*a*b)/((a^4 + 2*a^2*b^2 + b^4)*d^2*
e))*log(-(a^2 - b^2)*sqrt((e*cos(2*d*x + 2*c) + e)/sin(2*d*x + 2*c)) - ((a^
4*b + 2*a^2*b^3 + b^5)*d^3*e^2*sqrt(-(a^4 - 2*a^2*b^2 + b^4)/((a^8 + 4*a^6*
b^2 + 6*a^4*b^4 + 4*a^2*b^6 + b^8)*d^4*e^2)) - (a^3 - a*b^2)*d*e)*sqrt(-((a
^4 + 2*a^2*b^2 + b^4)*d^2*e*sqrt(-(a^4 - 2*a^2*b^2 + b^4)/((a^8 + 4*a^6*b^2
 + 6*a^4*b^4 + 4*a^2*b^6 + b^8)*d^4*e^2)) - 2*a*b)/((a^4 + 2*a^2*b^2 + b^4)
*d^2*e))) + 2*b*sqrt(-b/(a*e))*log(-(2*a*sqrt((e*cos(2*d*x + 2*c) + e)/sin(
2*d*x + 2*c))*sqrt(-b/(a*e))*sin(2*d*x + 2*c) - b*cos(2*d*x + 2*c) + a*sin(
2*d*x + 2*c) - b)/(b*cos(2*d*x + 2*c) + a*sin(2*d*x + 2*c) + b)))/((a^2 + b
^2)*d), 1/2*((a^2 + b^2)*d*sqrt(((a^4 + 2*a^2*b^2 + b^4)*d^2*e*sqrt(-(a^4 -
 2*a^2*b^2 + b^4)/((a^8 + 4*a^6*b^2 + 6*a^4*b^4 + 4*a^2*b^6 + b^8)*d^4*e^2)
```

```
) + 2*a*b)/((a^4 + 2*a^2*b^2 + b^4)*d^2*e))*log(-(a^2 - b^2)*sqrt((e*cos(2*
d*x + 2*c) + e)/sin(2*d*x + 2*c)) + ((a^4*b + 2*a^2*b^3 + b^5)*d^3*e^2*sqrt
(-(a^4 - 2*a^2*b^2 + b^4)/((a^8 + 4*a^6*b^2 + 6*a^4*b^4 + 4*a^2*b^6 + b^8)*
d^4*e^2)) + (a^3 - a*b^2)*d*e)*sqrt(((a^4 + 2*a^2*b^2 + b^4)*d^2*e*sqrt(-(a
^4 - 2*a^2*b^2 + b^4)/((a^8 + 4*a^6*b^2 + 6*a^4*b^4 + 4*a^2*b^6 + b^8)*d^4*
e^2)) + 2*a*b)/((a^4 + 2*a^2*b^2 + b^4)*d^2*e))) - (a^2 + b^2)*d*sqrt(((a^4
 + 2*a^2*b^2 + b^4)*d^2*e*sqrt(-(a^4 - 2*a^2*b^2 + b^4)/((a^8 + 4*a^6*b^2 +
 6*a^4*b^4 + 4*a^2*b^6 + b^8)*d^4*e^2)) + 2*a*b)/((a^4 + 2*a^2*b^2 + b^4)*d
^2*e))*log(-(a^2 - b^2)*sqrt((e*cos(2*d*x + 2*c) + e)/sin(2*d*x + 2*c)) - (
(a^4*b + 2*a^2*b^3 + b^5)*d^3*e^2*sqrt(-(a^4 - 2*a^2*b^2 + b^4)/((a^8 + 4*a
^6*b^2 + 6*a^4*b^4 + 4*a^2*b^6 + b^8)*d^4*e^2)) + (a^3 - a*b^2)*d*e)*sqrt((
(a^4 + 2*a^2*b^2 + b^4)*d^2*e*sqrt(-(a^4 - 2*a^2*b^2 + b^4)/((a^8 + 4*a^6*b
^2 + 6*a^4*b^4 + 4*a^2*b^6 + b^8)*d^4*e^2)) + 2*a*b)/((a^4 + 2*a^2*b^2 + b^
4)*d^2*e))) - (a^2 + b^2)*d*sqrt(-((a^4 + 2*a^2*b^2 + b^4)*d^2*e*sqrt(-(a^4
 - 2*a^2*b^2 + b^4)/((a^8 + 4*a^6*b^2 + 6*a^4*b^4 + 4*a^2*b^6 + b^8)*d^4*e^
2)) - 2*a*b)/((a^4 + 2*a^2*b^2 + b^4)*d^2*e))*log(-(a^2 - b^2)*sqrt((e*cos(
2*d*x + 2*c) + e)/sin(2*d*x + 2*c)) + ((a^4*b + 2*a^2*b^3 + b^5)*d^3*e^2*sq
rt(-(a^4 - 2*a^2*b^2 + b^4)/((a^8 + 4*a^6*b^2 + 6*a^4*b^4 + 4*a^2*b^6 + b^8
)*d^4*e^2)) - (a^3 - a*b^2)*d*e)*sqrt(-((a^4 + 2*a^2*b^2 + b^4)*d^2*e*sqrt(
-(a^4 - 2*a^2*b^2 + b^4)/((a^8 + 4*a^6*b^2 + 6*a^4*b^4 + 4*a^2*b^6 + b^8)*d
^4*e^2)) - 2*a*b)/((a^4 + 2*a^2*b^2 + b^4)*d^2*e))) + (a^2 + b^2)*d*sqrt(-(
(a^4 + 2*a^2*b^2 + b^4)*d^2*e*sqrt(-(a^4 - 2*a^2*b^2 + b^4)/((a^8 + 4*a^6*b
^2 + 6*a^4*b^4 + 4*a^2*b^6 + b^8)*d^4*e^2)) - 2*a*b)/((a^4 + 2*a^2*b^2 + b^
4)*d^2*e))*log(-(a^2 - b^2)*sqrt((e*cos(2*d*x + 2*c) + e)/sin(2*d*x + 2*c))
 - ((a^4*b + 2*a^2*b^3 + b^5)*d^3*e^2*sqrt(-(a^4 - 2*a^2*b^2 + b^4)/((a^8 +
 4*a^6*b^2 + 6*a^4*b^4 + 4*a^2*b^6 + b^8)*d^4*e^2)) - (a^3 - a*b^2)*d*e)*sq
rt(-((a^4 + 2*a^2*b^2 + b^4)*d^2*e*sqrt(-(a^4 - 2*a^2*b^2 + b^4)/((a^8 + 4*
a^6*b^2 + 6*a^4*b^4 + 4*a^2*b^6 + b^8)*d^4*e^2)) - 2*a*b)/((a^4 + 2*a^2*b^2
 + b^4)*d^2*e))) + 4*b*sqrt(b/(a*e))*arctan(a*sqrt((e*cos(2*d*x + 2*c) + e)
/sin(2*d*x + 2*c))*sqrt(b/(a*e))*sin(2*d*x + 2*c)/(b*cos(2*d*x + 2*c) + b))
)/((a^2 + b^2)*d)]
```

Sympy [F]

$$\int \frac{1}{\sqrt{e\cot(c+dx)}(a+b\cot(c+dx))}\,dx=\int \frac{1}{\sqrt{e\cot\left(c+dx\right)}\left(a+b\cot\left(c+dx\right)\right)}\,dx$$

```
[In] integrate(1/(e*cot(d*x+c))**(1/2)/(a+b*cot(d*x+c)),x)
```

```
[Out] Integral(1/(sqrt(e*cot(c + d*x))*(a + b*cot(c + d*x))), x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{1}{\sqrt{e\cot(c+dx)}(a+b\cot(c+dx))}\,dx = \text{Exception raised: ValueError}$$

```
[In] integrate(1/(e*cot(d*x+c))^(1/2)/(a+b*cot(d*x+c)),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(e>0)', see `assume?` for more detai
ls)Is e
```

Giac [F]

$$\int \frac{1}{\sqrt{e\cot(c+dx)}(a+b\cot(c+dx))}\,dx = \int \frac{1}{(b\cot(dx+c)+a)\sqrt{e\cot(dx+c)}}\,dx$$

```
[In] integrate(1/(e*cot(d*x+c))^(1/2)/(a+b*cot(d*x+c)),x, algorithm="giac")
```

```
[Out] integrate(1/((b*cot(d*x + c) + a)*sqrt(e*cot(d*x + c))), x)
```

Mupad [B] (verification not implemented)

Time = 13.88 (sec) , antiderivative size = 4871, normalized size of antiderivative = 16.13

$$\int \frac{1}{\sqrt{e\cot(c+dx)}(a+b\cot(c+dx))}\,dx = \text{Too large to display}$$

```
[In] int(1/((e*cot(c + d*x))^(1/2)*(a + b*cot(c + d*x))),x)
```

```
[Out] atan((((((((32*(5*a*b^5*e^9 + a^3*b^3*e^9))/d^3 - ((((1/(b^2*d^2*e*1i - a^2*
d^2*e*1i + 2*a*b*d^2*e))^(1/2)*((32*(16*b^8*d^2*e^10 + 28*a^2*b^6*d^2*e^10
+ 8*a^4*b^4*d^2*e^10 - 4*a^6*b^2*d^2*e^10))/d^3 - (16*(e*cot(c + d*x))^(1/2
)*(1/(b^2*d^2*e*1i - a^2*d^2*e*1i + 2*a*b*d^2*e))^(1/2)*(16*b^9*d^4*e^10 +
16*a^2*b^7*d^4*e^10 - 16*a^4*b^5*d^4*e^10 - 16*a^6*b^3*d^4*e^10))/d^4))/2 -
 (32*(e*cot(c + d*x))^(1/2)*(4*a^3*b^4*d^2*e^9 - 30*a*b^6*d^2*e^9 + 2*a^5*b
^2*d^2*e^9))/d^4)*(1/(b^2*d^2*e*1i - a^2*d^2*e*1i + 2*a*b*d^2*e))^(1/2))/2)
*(1/(b^2*d^2*e*1i - a^2*d^2*e*1i + 2*a*b*d^2*e))^(1/2))/2 + (96*b^5*e^8*(e*
cot(c + d*x))^(1/2))/d^4)*(1/(b^2*d^2*e*1i - a^2*d^2*e*1i + 2*a*b*d^2*e))^(
1/2)*1i)/2 - (((((32*(5*a*b^5*e^9 + a^3*b^3*e^9))/d^3 - ((((1/(b^2*d^2*e*1i
 - a^2*d^2*e*1i + 2*a*b*d^2*e))^(1/2)*((32*(16*b^8*d^2*e^10 + 28*a^2*b^6*d^
2*e^10 + 8*a^4*b^4*d^2*e^10 - 4*a^6*b^2*d^2*e^10))/d^3 + (16*(e*cot(c + d*x
```

))^(1/2)*(1/(b^2*d^2*e*1i - a^2*d^2*e*1i + 2*a*b*d^2*e))^(1/2)*(16*b^9*d^4*
e^10 + 16*a^2*b^7*d^4*e^10 - 16*a^4*b^5*d^4*e^10 - 16*a^6*b^3*d^4*e^10))/d^
4))/2 + (32*(e*cot(c + d*x))^(1/2)*(4*a^3*b^4*d^2*e^9 - 30*a*b^6*d^2*e^9 +
2*a^5*b^2*d^2*e^9))/d^4)*(1/(b^2*d^2*e*1i - a^2*d^2*e*1i + 2*a*b*d^2*e))^(1
/2))/2)*(1/(b^2*d^2*e*1i - a^2*d^2*e*1i + 2*a*b*d^2*e))^(1/2))/2 - (96*b^5*
e^8*(e*cot(c + d*x))^(1/2))/d^4)*(1/(b^2*d^2*e*1i - a^2*d^2*e*1i + 2*a*b*d^
2*e))^(1/2)*1i)/2)/((((((32*(5*a*b^5*e^9 + a^3*b^3*e^9))/d^3 - ((((1/(b^2*d
^2*e*1i - a^2*d^2*e*1i + 2*a*b*d^2*e))^(1/2)*((32*(16*b^8*d^2*e^10 + 28*a^2
*b^6*d^2*e^10 + 8*a^4*b^4*d^2*e^10 - 4*a^6*b^2*d^2*e^10))/d^3 - (16*(e*cot(
c + d*x))^(1/2)*(1/(b^2*d^2*e*1i - a^2*d^2*e*1i + 2*a*b*d^2*e))^(1/2)*(16*b
^9*d^4*e^10 + 16*a^2*b^7*d^4*e^10 - 16*a^4*b^5*d^4*e^10 - 16*a^6*b^3*d^4*e^
10))/d^4))/2 - (32*(e*cot(c + d*x))^(1/2)*(4*a^3*b^4*d^2*e^9 - 30*a*b^6*d^2
*e^9 + 2*a^5*b^2*d^2*e^9))/d^4)*(1/(b^2*d^2*e*1i - a^2*d^2*e*1i + 2*a*b*d^2
e))^(1/2))/2)(1/(b^2*d^2*e*1i - a^2*d^2*e*1i + 2*a*b*d^2*e))^(1/2))/2 + (
96*b^5*e^8*(e*cot(c + d*x))^(1/2))/d^4)*(1/(b^2*d^2*e*1i - a^2*d^2*e*1i + 2
*a*b*d^2*e))^(1/2))/2 + (((((32*(5*a*b^5*e^9 + a^3*b^3*e^9))/d^3 - ((((1/(b
^2*d^2*e*1i - a^2*d^2*e*1i + 2*a*b*d^2*e))^(1/2)*((32*(16*b^8*d^2*e^10 + 28
*a^2*b^6*d^2*e^10 + 8*a^4*b^4*d^2*e^10 - 4*a^6*b^2*d^2*e^10))/d^3 + (16*(e*
cot(c + d*x))^(1/2)*(1/(b^2*d^2*e*1i - a^2*d^2*e*1i + 2*a*b*d^2*e))^(1/2)*(
16*b^9*d^4*e^10 + 16*a^2*b^7*d^4*e^10 - 16*a^4*b^5*d^4*e^10 - 16*a^6*b^3*d^
4*e^10))/d^4))/2 + (32*(e*cot(c + d*x))^(1/2)*(4*a^3*b^4*d^2*e^9 - 30*a*b^6
*d^2*e^9 + 2*a^5*b^2*d^2*e^9))/d^4)*(1/(b^2*d^2*e*1i - a^2*d^2*e*1i + 2*a*b
*d^2*e))^(1/2))/2)*(1/(b^2*d^2*e*1i - a^2*d^2*e*1i + 2*a*b*d^2*e))^(1/2))/2
 - (96*b^5*e^8*(e*cot(c + d*x))^(1/2))/d^4)*(1/(b^2*d^2*e*1i - a^2*d^2*e*1i
 + 2*a*b*d^2*e))^(1/2))/2))*(1/(b^2*d^2*e*1i - a^2*d^2*e*1i + 2*a*b*d^2*e))
^(1/2)*1i + atan(((((32*(5*a*b^5*e^9 + a^3*b^3*e^9))/d^3 - (((32*(16*b^8*d^
2*e^10 + 28*a^2*b^6*d^2*e^10 + 8*a^4*b^4*d^2*e^10 - 4*a^6*b^2*d^2*e^10))/d^
3 - (32*(e*cot(c + d*x))^(1/2)*(1i/(4*(b^2*d^2*e - a^2*d^2*e + a*b*d^2*e*2i
)))^(1/2)*(16*b^9*d^4*e^10 + 16*a^2*b^7*d^4*e^10 - 16*a^4*b^5*d^4*e^10 - 16
*a^6*b^3*d^4*e^10))/d^4)*(1i/(4*(b^2*d^2*e - a^2*d^2*e + a*b*d^2*e*2i)))^(1
/2) - (32*(e*cot(c + d*x))^(1/2)*(4*a^3*b^4*d^2*e^9 - 30*a*b^6*d^2*e^9 + 2*
a^5*b^2*d^2*e^9))/d^4)*(1i/(4*(b^2*d^2*e - a^2*d^2*e + a*b*d^2*e*2i)))^(1/2
))*(1i/(4*(b^2*d^2*e - a^2*d^2*e + a*b*d^2*e*2i)))^(1/2) + (96*b^5*e^8*(e*c
ot(c + d*x))^(1/2))/d^4)*(1i/(4*(b^2*d^2*e - a^2*d^2*e + a*b*d^2*e*2i)))^(1
/2)*1i - (((32*(5*a*b^5*e^9 + a^3*b^3*e^9))/d^3 - (((32*(16*b^8*d^2*e^10 +
28*a^2*b^6*d^2*e^10 + 8*a^4*b^4*d^2*e^10 - 4*a^6*b^2*d^2*e^10))/d^3 + (32*(
e*cot(c + d*x))^(1/2)*(1i/(4*(b^2*d^2*e - a^2*d^2*e + a*b*d^2*e*2i)))^(1/2)
*(16*b^9*d^4*e^10 + 16*a^2*b^7*d^4*e^10 - 16*a^4*b^5*d^4*e^10 - 16*a^6*b^3*
d^4*e^10))/d^4)*(1i/(4*(b^2*d^2*e - a^2*d^2*e + a*b*d^2*e*2i)))^(1/2) + (32
*(e*cot(c + d*x))^(1/2)*(4*a^3*b^4*d^2*e^9 - 30*a*b^6*d^2*e^9 + 2*a^5*b^2*d
^2*e^9))/d^4)*(1i/(4*(b^2*d^2*e - a^2*d^2*e + a*b*d^2*e*2i)))^(1/2))*(1i/(4
*(b^2*d^2*e - a^2*d^2*e + a*b*d^2*e*2i)))^(1/2) - (96*b^5*e^8*(e*cot(c + d*
x))^(1/2))/d^4)*(1i/(4*(b^2*d^2*e - a^2*d^2*e + a*b*d^2*e*2i)))^(1/2)*1i)/(
(((32*(5*a*b^5*e^9 + a^3*b^3*e^9))/d^3 - (((32*(16*b^8*d^2*e^10 + 28*a^2*b^
6*d^2*e^10 + 8*a^4*b^4*d^2*e^10 - 4*a^6*b^2*d^2*e^10))/d^3 - (32*(e*cot(c +

d*x))^(1/2)*(1i/(4*(b^2*d^2*e - a^2*d^2*e + a*b*d^2*e*2i)))^(1/2)*(16*b^9*
d^4*e^10 + 16*a^2*b^7*d^4*e^10 - 16*a^4*b^5*d^4*e^10 - 16*a^6*b^3*d^4*e^10)
)/d^4)*(1i/(4*(b^2*d^2*e - a^2*d^2*e + a*b*d^2*e*2i)))^(1/2) - (32*(e*cot(c
 + d*x))^(1/2)*(4*a^3*b^4*d^2*e^9 - 30*a*b^6*d^2*e^9 + 2*a^5*b^2*d^2*e^9))/
d^4)*(1i/(4*(b^2*d^2*e - a^2*d^2*e + a*b*d^2*e*2i)))^(1/2))*(1i/(4*(b^2*d^2
*e - a^2*d^2*e + a*b*d^2*e*2i)))^(1/2) + (96*b^5*e^8*(e*cot(c + d*x))^(1/2)
)/d^4)*(1i/(4*(b^2*d^2*e - a^2*d^2*e + a*b*d^2*e*2i)))^(1/2) + (((32*(5*a*b
^5*e^9 + a^3*b^3*e^9))/d^3 - (((32*(16*b^8*d^2*e^10 + 28*a^2*b^6*d^2*e^10 +
 8*a^4*b^4*d^2*e^10 - 4*a^6*b^2*d^2*e^10))/d^3 + (32*(e*cot(c + d*x))^(1/2)
(1i/(4(b^2*d^2*e - a^2*d^2*e + a*b*d^2*e*2i)))^(1/2)*(16*b^9*d^4*e^10 + 1
6*a^2*b^7*d^4*e^10 - 16*a^4*b^5*d^4*e^10 - 16*a^6*b^3*d^4*e^10))/d^4)*(1i/(
4*(b^2*d^2*e - a^2*d^2*e + a*b*d^2*e*2i)))^(1/2) + (32*(e*cot(c + d*x))^(1/
2)*(4*a^3*b^4*d^2*e^9 - 30*a*b^6*d^2*e^9 + 2*a^5*b^2*d^2*e^9))/d^4)*(1i/(4*
(b^2*d^2*e - a^2*d^2*e + a*b*d^2*e*2i)))^(1/2))*(1i/(4*(b^2*d^2*e - a^2*d^2
*e + a*b*d^2*e*2i)))^(1/2) - (96*b^5*e^8*(e*cot(c + d*x))^(1/2))/d^4)*(1i/(
4*(b^2*d^2*e - a^2*d^2*e + a*b*d^2*e*2i)))^(1/2)))*(1i/(4*(b^2*d^2*e - a^2*
d^2*e + a*b*d^2*e*2i)))^(1/2)*2i + (atan((((((32*(5*a*b^5*e^9 + a^3*b^3*e^
9))/d^3 - (((((32*(16*b^8*d^2*e^10 + 28*a^2*b^6*d^2*e^10 + 8*a^4*b^4*d^2*e^
10 - 4*a^6*b^2*d^2*e^10))/d^3 - (32*(e*cot(c + d*x))^(1/2)*(-a*b^3*e)^(1/2)
*(16*b^9*d^4*e^10 + 16*a^2*b^7*d^4*e^10 - 16*a^4*b^5*d^4*e^10 - 16*a^6*b^3*
d^4*e^10))/(d^4*(a^3*d*e + a*b^2*d*e)))*(-a*b^3*e)^(1/2))/(a^3*d*e + a*b^2*
d*e) - (32*(e*cot(c + d*x))^(1/2)*(4*a^3*b^4*d^2*e^9 - 30*a*b^6*d^2*e^9 + 2
*a^5*b^2*d^2*e^9))/d^4)*(-a*b^3*e)^(1/2))/(a^3*d*e + a*b^2*d*e))*(-a*b^3*e)
^(1/2))/(a^3*d*e + a*b^2*d*e) + (96*b^5*e^8*(e*cot(c + d*x))^(1/2))/d^4)*(-
a*b^3*e)^(1/2)*1i)/(a^3*d*e + a*b^2*d*e) - (((((32*(5*a*b^5*e^9 + a^3*b^3*e
^9))/d^3 - (((((32*(16*b^8*d^2*e^10 + 28*a^2*b^6*d^2*e^10 + 8*a^4*b^4*d^2*e
^10 - 4*a^6*b^2*d^2*e^10))/d^3 + (32*(e*cot(c + d*x))^(1/2)*(-a*b^3*e)^(1/2
)*(16*b^9*d^4*e^10 + 16*a^2*b^7*d^4*e^10 - 16*a^4*b^5*d^4*e^10 - 16*a^6*b^3
*d^4*e^10))/(d^4*(a^3*d*e + a*b^2*d*e)))*(-a*b^3*e)^(1/2))/(a^3*d*e + a*b^2
*d*e) + (32*(e*cot(c + d*x))^(1/2)*(4*a^3*b^4*d^2*e^9 - 30*a*b^6*d^2*e^9 +
2*a^5*b^2*d^2*e^9))/d^4)*(-a*b^3*e)^(1/2))/(a^3*d*e + a*b^2*d*e))*(-a*b^3*e
)^(1/2))/(a^3*d*e + a*b^2*d*e) - (96*b^5*e^8*(e*cot(c + d*x))^(1/2))/d^4)*(
-a*b^3*e)^(1/2)*1i)/(a^3*d*e + a*b^2*d*e))/((((((32*(5*a*b^5*e^9 + a^3*b^3*
e^9))/d^3 - (((((32*(16*b^8*d^2*e^10 + 28*a^2*b^6*d^2*e^10 + 8*a^4*b^4*d^2*
e^10 - 4*a^6*b^2*d^2*e^10))/d^3 - (32*(e*cot(c + d*x))^(1/2)*(-a*b^3*e)^(1/
2)*(16*b^9*d^4*e^10 + 16*a^2*b^7*d^4*e^10 - 16*a^4*b^5*d^4*e^10 - 16*a^6*b^
3*d^4*e^10))/(d^4*(a^3*d*e + a*b^2*d*e)))*(-a*b^3*e)^(1/2))/(a^3*d*e + a*b^
2*d*e) - (32*(e*cot(c + d*x))^(1/2)*(4*a^3*b^4*d^2*e^9 - 30*a*b^6*d^2*e^9 +
 2*a^5*b^2*d^2*e^9))/d^4)*(-a*b^3*e)^(1/2))/(a^3*d*e + a*b^2*d*e))*(-a*b^3*
e)^(1/2))/(a^3*d*e + a*b^2*d*e) + (96*b^5*e^8*(e*cot(c + d*x))^(1/2))/d^4)*
(-a*b^3*e)^(1/2))/(a^3*d*e + a*b^2*d*e) + (((((32*(5*a*b^5*e^9 + a^3*b^3*e^
9))/d^3 - (((((32*(16*b^8*d^2*e^10 + 28*a^2*b^6*d^2*e^10 + 8*a^4*b^4*d^2*e^
10 - 4*a^6*b^2*d^2*e^10))/d^3 + (32*(e*cot(c + d*x))^(1/2)*(-a*b^3*e)^(1/2)
*(16*b^9*d^4*e^10 + 16*a^2*b^7*d^4*e^10 - 16*a^4*b^5*d^4*e^10 - 16*a^6*b^3*
d^4*e^10))/(d^4*(a^3*d*e + a*b^2*d*e)))*(-a*b^3*e)^(1/2))/(a^3*d*e + a*b^2*

```
d*e) + (32*(e*cot(c + d*x))^(1/2)*(4*a^3*b^4*d^2*e^9 - 30*a*b^6*d^2*e^9 + 2
*a^5*b^2*d^2*e^9))/d^4)*(-a*b^3*e)^(1/2))/(a^3*d*e + a*b^2*d*e))*(-a*b^3*e)
^(1/2))/(a^3*d*e + a*b^2*d*e) - (96*b^5*e^8*(e*cot(c + d*x))^(1/2))/d^4)*(-
a*b^3*e)^(1/2))/(a^3*d*e + a*b^2*d*e)))*(-a*b^3*e)^(1/2)*2i)/(a^3*d*e + a*b
^2*d*e)
```

3.73 $\int \frac{1}{(e\cot(c+dx))^{3/2}(a+b\cot(c+dx))}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 325

$$\int \frac{1}{(e\cot(c+dx))^{3/2}(a+b\cot(c+dx))}\,dx = \frac{2b^{5/2}\arctan\left(\frac{\sqrt{b}\sqrt{e\cot(c+dx)}}{\sqrt{a}\sqrt{e}}\right)}{a^{3/2}\,(a^2+b^2)\,de^{3/2}} - \frac{(a+b)\arctan\left(1-\frac{\sqrt{2}\sqrt{e\cot(c+dx)}}{\sqrt{e}}\right)}{\sqrt{2}\,(a^2+b^2)\,de^{3/2}} + \frac{(a+b)\arctan\left(1+\frac{\sqrt{2}\sqrt{e\cot(c+dx)}}{\sqrt{e}}\right)}{\sqrt{2}\,(a^2+b^2)\,de^{3/2}} + \frac{2}{ade\sqrt{e\cot(c+dx)}} + \frac{(a-b)\log\left(\sqrt{e}+\sqrt{e}\cot(c+dx)-\sqrt{2}\sqrt{e\cot(c+dx)}\right)}{2\sqrt{2}\,(a^2+b^2)\,de^{3/2}} - \frac{(a-b)\log\left(\sqrt{e}+\sqrt{e}\cot(c+dx)+\sqrt{2}\sqrt{e\cot(c+dx)}\right)}{2\sqrt{2}\,(a^2+b^2)\,de^{3/2}}$$

```
[Out] 2*b^(5/2)*arctan(b^(1/2)*(e*cot(d*x+c))^(1/2)/a^(1/2)/e^(1/2))/a^(3/2)/(a^2
+b^2)/d/e^(3/2)-1/2*(a+b)*arctan(1-2^(1/2)*(e*cot(d*x+c))^(1/2)/e^(1/2))/(a
^2+b^2)/d/e^(3/2)*2^(1/2)+1/2*(a+b)*arctan(1+2^(1/2)*(e*cot(d*x+c))^(1/2)/e
^(1/2))/(a^2+b^2)/d/e^(3/2)*2^(1/2)+1/4*(a-b)*ln(e^(1/2)+cot(d*x+c)*e^(1/2)
-2^(1/2)*(e*cot(d*x+c))^(1/2))/(a^2+b^2)/d/e^(3/2)*2^(1/2)-1/4*(a-b)*ln(e^(
1/2)+cot(d*x+c)*e^(1/2)+2^(1/2)*(e*cot(d*x+c))^(1/2))/(a^2+b^2)/d/e^(3/2)*2
^(1/2)+2/a/d/e/(e*cot(d*x+c))^(1/2)
```

Rubi [A] (verified)

Time = 0.75 (sec) , antiderivative size = 325, normalized size of antiderivative = 1.00, number of steps used = 15, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.480, Rules used = {3650, 3734, 3615, 1182, 1176, 631, 210, 1179, 642, 3715, 65, 211}

$$\int \frac{1}{(e\cot(c+dx))^{3/2}(a+b\cot(c+dx))}\,dx = -\frac{(a+b)\arctan\left(1-\frac{\sqrt{2}\sqrt{e\cot(c+dx)}}{\sqrt{e}}\right)}{\sqrt{2}de^{3/2}(a^2+b^2)}+\frac{(a+b)\arctan\left(\frac{\sqrt{2}\sqrt{e\cot(c+dx)}}{\sqrt{e}}+1\right)}{\sqrt{2}de^{3/2}(a^2+b^2)}+\frac{(a-b)\log\left(\sqrt{e}\cot(c+dx)-\sqrt{2}\sqrt{e\cot(c+dx)}+\sqrt{e}\right)}{2\sqrt{2}de^{3/2}(a^2+b^2)}-\frac{(a-b)\log\left(\sqrt{e}\cot(c+dx)+\sqrt{2}\sqrt{e\cot(c+dx)}+\sqrt{e}\right)}{2\sqrt{2}de^{3/2}(a^2+b^2)}+\frac{2b^{5/2}\arctan\left(\frac{\sqrt{b}\sqrt{e\cot(c+dx)}}{\sqrt{a}\sqrt{e}}\right)}{a^{3/2}de^{3/2}(a^2+b^2)}+\frac{2}{ade\sqrt{e\cot(c+dx)}}$$

```
[In] Int[1/((e*Cot[c + d*x])^(3/2)*(a + b*Cot[c + d*x])),x]
```

```
[Out] (2*b^(5/2)*ArcTan[(Sqrt[b]*Sqrt[e*Cot[c + d*x]])/(Sqrt[a]*Sqrt[e])])/(a^(3/
2)*(a^2 + b^2)*d*e^(3/2)) - ((a + b)*ArcTan[1 - (Sqrt[2]*Sqrt[e*Cot[c + d*x
]])/Sqrt[e]])/(Sqrt[2]*(a^2 + b^2)*d*e^(3/2)) + ((a + b)*ArcTan[1 + (Sqrt[2
]*Sqrt[e*Cot[c + d*x]])/Sqrt[e]])/(Sqrt[2]*(a^2 + b^2)*d*e^(3/2)) + 2/(a*d*
e*Sqrt[e*Cot[c + d*x]]) + ((a - b)*Log[Sqrt[e] + Sqrt[e]*Cot[c + d*x] - Sqr
t[2]*Sqrt[e*Cot[c + d*x]]])/(2*Sqrt[2]*(a^2 + b^2)*d*e^(3/2)) - ((a - b)*Lo
g[Sqrt[e] + Sqrt[e]*Cot[c + d*x] + Sqrt[2]*Sqrt[e*Cot[c + d*x]]])/(2*Sqrt[2
]*(a^2 + b^2)*d*e^(3/2))
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1179

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 1182

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
a*c, 2]}, Dist[(d*q + a*e)/(2*a*c), Int[(q + c*x^2)/(a + c*x^4), x], x] + D
ist[(d*q - a*e)/(2*a*c), Int[(q - c*x^2)/(a + c*x^4), x], x]] /; FreeQ[{a,
c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0] && NeQ[c*d^2 - a*e^2, 0] && NegQ[(-a)
*c]
```

Rule 3615

```
Int[((c_) + (d_.)*tan[(e_.) + (f_.)*(x_)])/Sqrt[(b_.)*tan[(e_.) + (f_.)*(x_
)]], x_Symbol] :> Dist[2/f, Subst[Int[(b*c + d*x^2)/(b^2 + x^4), x], x, Sqr
t[b*Tan[e + f*x]]], x] /; FreeQ[{b, c, d, e, f}, x] && NeQ[c^2 - d^2, 0] &&
 NeQ[c^2 + d^2, 0]
```

Rule 3650

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)])^(n_), x_Symbol] :> Simp[b^2*(a + b*Tan[e + f*x])^(m + 1)*((c
+ d*Tan[e + f*x])^(n + 1)/(f*(m + 1)*(a^2 + b^2)*(b*c - a*d))), x] + Dist[1
/((m + 1)*(a^2 + b^2)*(b*c - a*d)), Int[(a + b*Tan[e + f*x])^(m + 1)*(c + d
*Tan[e + f*x])^n*Simp[a*(b*c - a*d)*(m + 1) - b^2*d*(m + n + 2) - b*(b*c -
a*d)*(m + 1)*Tan[e + f*x] - b^2*d*(m + n + 2)*Tan[e + f*x]^2, x], x], x] /;
 FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0]
&& NeQ[c^2 + d^2, 0] && IntegerQ[2*m] && LtQ[m, -1] && (LtQ[n, 0] || Intege
rQ[m]) &&  !(ILtQ[n, -1] && ( !IntegerQ[m] || (EqQ[c, 0] && NeQ[a, 0])))
```

Rule 3715

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*tan[(e_.)
+ (f_.)*(x_)])^(n_.)*((A_) + (C_.)*tan[(e_.) + (f_.)*(x_)]^2), x_Symbol] :>
 Dist[A/f, Subst[Int[(a + b*x)^m*(c + d*x)^n, x], x, Tan[e + f*x]], x] /; F
reeQ[{a, b, c, d, e, f, A, C, m, n}, x] && EqQ[A, C]
```

Rule 3734

```
Int[(((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_)*((A_.) + (B_.)*tan[(e_.)
+ (f_.)*(x_)] + (C_.)*tan[(e_.) + (f_.)*(x_)]^2))/((a_.) + (b_.)*tan[(e_.)
+ (f_.)*(x_)]), x_Symbol] :> Dist[1/(a^2 + b^2), Int[(c + d*Tan[e + f*x])^n
*Simp[b*B + a*(A - C) + (a*B - b*(A - C))*Tan[e + f*x], x], x], x] + Dist[(
A*b^2 - a*b*B + a^2*C)/(a^2 + b^2), Int[(c + d*Tan[e + f*x])^n*((1 + Tan[e
+ f*x]^2)/(a + b*Tan[e + f*x])), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C
, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] &&
 !GtQ[n, 0] &&  !LeQ[n, -1]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{2}{ade\sqrt{e\cot(c+dx)}} + \frac{2\int \frac{-\frac{be^2}{2}-\frac{1}{2}ae^2\cot(c+dx)-\frac{1}{2}be^2\cot^2(c+dx)}{\sqrt{e\cot(c+dx)}(a+b\cot(c+dx))}\,dx}{ae^3} \\
&= \frac{2}{ade\sqrt{e\cot(c+dx)}} + \frac{2\int \frac{-\frac{1}{2}abe^2-\frac{1}{2}a^2e^2\cot(c+dx)}{\sqrt{e\cot(c+dx)}}\,dx}{a(a^2+b^2)e^3} - \frac{b^3\int \frac{1+\cot^2(c+dx)}{\sqrt{e\cot(c+dx)}(a+b\cot(c+dx))}\,dx}{a(a^2+b^2)e} \\
&= \frac{2}{ade\sqrt{e\cot(c+dx)}} + \frac{4\text{Subst}\left(\int \frac{\frac{1}{2}abe^3+\frac{1}{2}a^2e^2x^2}{e^2+x^4}\,dx, x, \sqrt{e\cot(c+dx)}\right)}{a(a^2+b^2)de^3} \\
&\quad - \frac{b^3\text{Subst}\left(\int \frac{1}{\sqrt{-ex}(a-bx)}\,dx, x, -\cot(c+dx)\right)}{a(a^2+b^2)de}
\end{aligned}$$

$$= \frac{2}{ade\sqrt{e\cot(c+dx)}} + \frac{(2b^3)\,\text{Subst}\left(\int \frac{1}{a+\frac{bx^2}{e}}\,dx, x, \sqrt{e\cot(c+dx)}\right)}{a\,(a^2+b^2)\,de^2}$$
$$- \frac{(a-b)\text{Subst}\left(\int \frac{e-x^2}{e^2+x^4}\,dx, x, \sqrt{e\cot(c+dx)}\right)}{(a^2+b^2)\,de}$$
$$+ \frac{(a+b)\text{Subst}\left(\int \frac{e+x^2}{e^2+x^4}\,dx, x, \sqrt{e\cot(c+dx)}\right)}{(a^2+b^2)\,de}$$

$$= \frac{2b^{5/2}\arctan\left(\frac{\sqrt{b}\sqrt{e\cot(c+dx)}}{\sqrt{a}\sqrt{e}}\right)}{a^{3/2}\,(a^2+b^2)\,de^{3/2}} + \frac{2}{ade\sqrt{e\cot(c+dx)}}$$
$$+ \frac{(a-b)\text{Subst}\left(\int \frac{\sqrt{2}\sqrt{e}+2x}{-e-\sqrt{2}\sqrt{e}x-x^2}\,dx, x, \sqrt{e\cot(c+dx)}\right)}{2\sqrt{2}\,(a^2+b^2)\,de^{3/2}}$$
$$+ \frac{(a-b)\text{Subst}\left(\int \frac{\sqrt{2}\sqrt{e}-2x}{-e+\sqrt{2}\sqrt{e}x-x^2}\,dx, x, \sqrt{e\cot(c+dx)}\right)}{2\sqrt{2}\,(a^2+b^2)\,de^{3/2}}$$
$$+ \frac{(a+b)\text{Subst}\left(\int \frac{1}{e-\sqrt{2}\sqrt{e}x+x^2}\,dx, x, \sqrt{e\cot(c+dx)}\right)}{2\,(a^2+b^2)\,de}$$
$$+ \frac{(a+b)\text{Subst}\left(\int \frac{1}{e+\sqrt{2}\sqrt{e}x+x^2}\,dx, x, \sqrt{e\cot(c+dx)}\right)}{2\,(a^2+b^2)\,de}$$

$$= \frac{2b^{5/2}\arctan\left(\frac{\sqrt{b}\sqrt{e\cot(c+dx)}}{\sqrt{a}\sqrt{e}}\right)}{a^{3/2}\,(a^2+b^2)\,de^{3/2}} + \frac{2}{ade\sqrt{e\cot(c+dx)}}$$
$$+ \frac{(a-b)\log\left(\sqrt{e}+\sqrt{e}\cot(c+dx)-\sqrt{2}\sqrt{e\cot(c+dx)}\right)}{2\sqrt{2}\,(a^2+b^2)\,de^{3/2}}$$
$$- \frac{(a-b)\log\left(\sqrt{e}+\sqrt{e}\cot(c+dx)+\sqrt{2}\sqrt{e\cot(c+dx)}\right)}{2\sqrt{2}\,(a^2+b^2)\,de^{3/2}}$$
$$+ \frac{(a+b)\text{Subst}\left(\int \frac{1}{-1-x^2}\,dx, x, 1-\frac{\sqrt{2}\sqrt{e\cot(c+dx)}}{\sqrt{e}}\right)}{\sqrt{2}\,(a^2+b^2)\,de^{3/2}}$$
$$- \frac{(a+b)\text{Subst}\left(\int \frac{1}{-1-x^2}\,dx, x, 1+\frac{\sqrt{2}\sqrt{e\cot(c+dx)}}{\sqrt{e}}\right)}{\sqrt{2}\,(a^2+b^2)\,de^{3/2}}$$

$$= \frac{2b^{5/2}\arctan\left(\frac{\sqrt{b}\sqrt{e\cot(c+dx)}}{\sqrt{a}\sqrt{e}}\right)}{a^{3/2}(a^2+b^2)\,de^{3/2}} - \frac{(a+b)\arctan\left(1-\frac{\sqrt{2}\sqrt{e\cot(c+dx)}}{\sqrt{e}}\right)}{\sqrt{2}\,(a^2+b^2)\,de^{3/2}} + \frac{(a+b)\arctan\left(1+\frac{\sqrt{2}\sqrt{e\cot(c+dx)}}{\sqrt{e}}\right)}{\sqrt{2}\,(a^2+b^2)\,de^{3/2}} + \frac{2}{ade\sqrt{e\cot(c+dx)}} + \frac{(a-b)\log\left(\sqrt{e}+\sqrt{e}\cot(c+dx)-\sqrt{2}\sqrt{e\cot(c+dx)}\right)}{2\sqrt{2}\,(a^2+b^2)\,de^{3/2}} - \frac{(a-b)\log\left(\sqrt{e}+\sqrt{e}\cot(c+dx)+\sqrt{2}\sqrt{e\cot(c+dx)}\right)}{2\sqrt{2}\,(a^2+b^2)\,de^{3/2}}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 0.46 (sec) , antiderivative size = 198, normalized size of antiderivative = 0.61

$$\int \frac{1}{(e\cot(c+dx))^{3/2}(a+b\cot(c+dx))}\,dx = \frac{8b^2\,\text{Hypergeometric2F1}\left(-\frac{1}{2},1,\frac{1}{2},-\frac{b\cot(c+dx)}{a}\right)+a\Big(8a\,\text{Hyperge}}{}$$

```
[In] Integrate[1/((e*Cot[c + d*x])^(3/2)*(a + b*Cot[c + d*x])),x]
```

```
[Out] (8*b^2*Hypergeometric2F1[-1/2, 1, 1/2, -((b*Cot[c + d*x])/a)] + a*(8*a*Hype
rgeometric2F1[-1/4, 1, 3/4, -Cot[c + d*x]^2] + Sqrt[2]*b*Sqrt[Cot[c + d*x]]
*(-2*ArcTan[1 - Sqrt[2]*Sqrt[Cot[c + d*x]]] + 2*ArcTan[1 + Sqrt[2]*Sqrt[Cot
[c + d*x]]] - Log[1 - Sqrt[2]*Sqrt[Cot[c + d*x]] + Cot[c + d*x]] + Log[1 +
Sqrt[2]*Sqrt[Cot[c + d*x]] + Cot[c + d*x]])))/(4*a*(a^2 + b^2)*d*e*Sqrt[e*C
ot[c + d*x]])
```

Maple [A] (verified)

Time = 0.04 (sec) , antiderivative size = 354, normalized size of antiderivative = 1.09

method	result
derivativedivides	$-\frac{2e^2\left(-\frac{b^3\arctan\left(\frac{\sqrt{e\cot(dx+c)}\,b}{\sqrt{aeb}}\right)}{a\,e^3\left(a^2+b^2\right)\sqrt{aeb}}+\frac{-\frac{b\left(e^2\right)^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\frac{e\cot(dx+c)+\left(e^2\right)^{\frac{1}{4}}\sqrt{e\cot(dx+c)}\,\sqrt{2}+\sqrt{e^2}}{e\cot(dx+c)-\left(e^2\right)^{\frac{1}{4}}\sqrt{e\cot(dx+c)}\,\sqrt{2}+\sqrt{e^2}}\right)+2\arctan\left(\frac{\sqrt{2}\,\sqrt{e\cot(dx}{\left(e^2\right)^{\frac{1}{4}}}\right.\right.}{8e}}{}\right.}{}$
default	$-\frac{2e^2\left(-\frac{b^3\arctan\left(\frac{\sqrt{e\cot(dx+c)}\,b}{\sqrt{aeb}}\right)}{a\,e^3\left(a^2+b^2\right)\sqrt{aeb}}+\frac{-\frac{b\left(e^2\right)^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\frac{e\cot(dx+c)+\left(e^2\right)^{\frac{1}{4}}\sqrt{e\cot(dx+c)}\,\sqrt{2}+\sqrt{e^2}}{e\cot(dx+c)-\left(e^2\right)^{\frac{1}{4}}\sqrt{e\cot(dx+c)}\,\sqrt{2}+\sqrt{e^2}}\right)+2\arctan\left(\frac{\sqrt{2}\,\sqrt{e\cot(dx}{\left(e^2\right)^{\frac{1}{4}}}\right.\right.}{8e}}{}\right.}{}$

```
[In] int(1/(e*cot(d*x+c))^(3/2)/(a+b*cot(d*x+c)),x,method=_RETURNVERBOSE)
```

```
[Out] -2/d*e^2*(-1/a/e^3*b^3/(a^2+b^2)/(a*e*b)^(1/2)*arctan((e*cot(d*x+c))^(1/2)*
b/(a*e*b)^(1/2))+1/(a^2+b^2)/e^3*(-1/8*b/e*(e^2)^(1/4)*2^(1/2)*(ln((e*cot(d
*x+c)+(e^2)^(1/4)*(e*cot(d*x+c))^(1/2)*2^(1/2)+(e^2)^(1/2))/(e*cot(d*x+c)-(
e^2)^(1/4)*(e*cot(d*x+c))^(1/2)*2^(1/2)+(e^2)^(1/2)))+2*arctan(2^(1/2)/(e^2
)^(1/4)*(e*cot(d*x+c))^(1/2)+1)-2*arctan(-2^(1/2)/(e^2)^(1/4)*(e*cot(d*x+c)
)^(1/2)+1))-1/8*a/(e^2)^(1/4)*2^(1/2)*(ln((e*cot(d*x+c)-(e^2)^(1/4)*(e*cot(
d*x+c))^(1/2)*2^(1/2)+(e^2)^(1/2))/(e*cot(d*x+c)+(e^2)^(1/4)*(e*cot(d*x+c))
^(1/2)*2^(1/2)+(e^2)^(1/2)))+2*arctan(2^(1/2)/(e^2)^(1/4)*(e*cot(d*x+c))^(1
/2)+1)-2*arctan(-2^(1/2)/(e^2)^(1/4)*(e*cot(d*x+c))^(1/2)+1)))-1/a/e^3/(e*c
ot(d*x+c))^(1/2))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1798 vs. $2(262) = 524$.

Time = 0.41 (sec) , antiderivative size = 3637, normalized size of antiderivative = 11.19

$$\int \frac{1}{(e\cot(c+dx))^{3/2}(a+b\cot(c+dx))}\,dx = \text{Too large to display}$$

```
[In] integrate(1/(e*cot(d*x+c))^(3/2)/(a+b*cot(d*x+c)),x, algorithm="fricas")
```

```
[Out] [1/2*(((a^3 + a*b^2)*d*e^2*cos(2*d*x + 2*c) + (a^3 + a*b^2)*d*e^2)*sqrt(-((
a^4 + 2*a^2*b^2 + b^4)*d^2*e^3*sqrt(-(a^4 - 2*a^2*b^2 + b^4)/((a^8 + 4*a^6*
```

b^2 + 6*a^4*b^4 + 4*a^2*b^6 + b^8)*d^4*e^6)) + 2*a*b)/((a^4 + 2*a^2*b^2 + b
^4)*d^2*e^3))*log(-(a^2 - b^2)*sqrt((e*cos(2*d*x + 2*c) + e)/sin(2*d*x + 2*
c)) + ((a^5 + 2*a^3*b^2 + a*b^4)*d^3*e^5*sqrt(-(a^4 - 2*a^2*b^2 + b^4)/((a^
8 + 4*a^6*b^2 + 6*a^4*b^4 + 4*a^2*b^6 + b^8)*d^4*e^6)) - (a^2*b - b^3)*d*e^
2)*sqrt(-((a^4 + 2*a^2*b^2 + b^4)*d^2*e^3*sqrt(-(a^4 - 2*a^2*b^2 + b^4)/((a
^8 + 4*a^6*b^2 + 6*a^4*b^4 + 4*a^2*b^6 + b^8)*d^4*e^6)) + 2*a*b)/((a^4 + 2*
a^2*b^2 + b^4)*d^2*e^3))) - ((a^3 + a*b^2)*d*e^2*cos(2*d*x + 2*c) + (a^3 +
a*b^2)*d*e^2)*sqrt(-((a^4 + 2*a^2*b^2 + b^4)*d^2*e^3*sqrt(-(a^4 - 2*a^2*b^2
 + b^4)/((a^8 + 4*a^6*b^2 + 6*a^4*b^4 + 4*a^2*b^6 + b^8)*d^4*e^6)) + 2*a*b)
/((a^4 + 2*a^2*b^2 + b^4)*d^2*e^3))*log(-(a^2 - b^2)*sqrt((e*cos(2*d*x + 2*
c) + e)/sin(2*d*x + 2*c)) - ((a^5 + 2*a^3*b^2 + a*b^4)*d^3*e^5*sqrt(-(a^4 -
 2*a^2*b^2 + b^4)/((a^8 + 4*a^6*b^2 + 6*a^4*b^4 + 4*a^2*b^6 + b^8)*d^4*e^6)
) - (a^2*b - b^3)*d*e^2)*sqrt(-((a^4 + 2*a^2*b^2 + b^4)*d^2*e^3*sqrt(-(a^4
- 2*a^2*b^2 + b^4)/((a^8 + 4*a^6*b^2 + 6*a^4*b^4 + 4*a^2*b^6 + b^8)*d^4*e^6
)) + 2*a*b)/((a^4 + 2*a^2*b^2 + b^4)*d^2*e^3))) - ((a^3 + a*b^2)*d*e^2*cos(
2*d*x + 2*c) + (a^3 + a*b^2)*d*e^2)*sqrt(((a^4 + 2*a^2*b^2 + b^4)*d^2*e^3*s
qrt(-(a^4 - 2*a^2*b^2 + b^4)/((a^8 + 4*a^6*b^2 + 6*a^4*b^4 + 4*a^2*b^6 + b^
8)*d^4*e^6)) - 2*a*b)/((a^4 + 2*a^2*b^2 + b^4)*d^2*e^3))*log(-(a^2 - b^2)*s
qrt((e*cos(2*d*x + 2*c) + e)/sin(2*d*x + 2*c)) + ((a^5 + 2*a^3*b^2 + a*b^4)
*d^3*e^5*sqrt(-(a^4 - 2*a^2*b^2 + b^4)/((a^8 + 4*a^6*b^2 + 6*a^4*b^4 + 4*a^
2*b^6 + b^8)*d^4*e^6)) + (a^2*b - b^3)*d*e^2)*sqrt(((a^4 + 2*a^2*b^2 + b^4)
*d^2*e^3*sqrt(-(a^4 - 2*a^2*b^2 + b^4)/((a^8 + 4*a^6*b^2 + 6*a^4*b^4 + 4*a^
2*b^6 + b^8)*d^4*e^6)) - 2*a*b)/((a^4 + 2*a^2*b^2 + b^4)*d^2*e^3))) + ((a^3
 + a*b^2)*d*e^2*cos(2*d*x + 2*c) + (a^3 + a*b^2)*d*e^2)*sqrt(((a^4 + 2*a^2*
b^2 + b^4)*d^2*e^3*sqrt(-(a^4 - 2*a^2*b^2 + b^4)/((a^8 + 4*a^6*b^2 + 6*a^4*
b^4 + 4*a^2*b^6 + b^8)*d^4*e^6)) - 2*a*b)/((a^4 + 2*a^2*b^2 + b^4)*d^2*e^3)
)*log(-(a^2 - b^2)*sqrt((e*cos(2*d*x + 2*c) + e)/sin(2*d*x + 2*c)) - ((a^5
+ 2*a^3*b^2 + a*b^4)*d^3*e^5*sqrt(-(a^4 - 2*a^2*b^2 + b^4)/((a^8 + 4*a^6*b^
2 + 6*a^4*b^4 + 4*a^2*b^6 + b^8)*d^4*e^6)) + (a^2*b - b^3)*d*e^2)*sqrt(((a^
4 + 2*a^2*b^2 + b^4)*d^2*e^3*sqrt(-(a^4 - 2*a^2*b^2 + b^4)/((a^8 + 4*a^6*b^
2 + 6*a^4*b^4 + 4*a^2*b^6 + b^8)*d^4*e^6)) - 2*a*b)/((a^4 + 2*a^2*b^2 + b^4
)*d^2*e^3))) + 2*(b^2*e*cos(2*d*x + 2*c) + b^2*e)*sqrt(-b/(a*e))*log((2*a*s
qrt((e*cos(2*d*x + 2*c) + e)/sin(2*d*x + 2*c))*sqrt(-b/(a*e))*sin(2*d*x + 2
*c) + b*cos(2*d*x + 2*c) - a*sin(2*d*x + 2*c) + b)/(b*cos(2*d*x + 2*c) + a*
sin(2*d*x + 2*c) + b)) + 4*(a^2 + b^2)*sqrt((e*cos(2*d*x + 2*c) + e)/sin(2*
d*x + 2*c))*sin(2*d*x + 2*c))/((a^3 + a*b^2)*d*e^2*cos(2*d*x + 2*c) + (a^3
+ a*b^2)*d*e^2), -1/2*(4*(b^2*e*cos(2*d*x + 2*c) + b^2*e)*sqrt(b/(a*e))*arc
tan(a*sqrt((e*cos(2*d*x + 2*c) + e)/sin(2*d*x + 2*c))*sqrt(b/(a*e))*sin(2*d
*x + 2*c)/(b*cos(2*d*x + 2*c) + b)) - ((a^3 + a*b^2)*d*e^2*cos(2*d*x + 2*c)
 + (a^3 + a*b^2)*d*e^2)*sqrt(-((a^4 + 2*a^2*b^2 + b^4)*d^2*e^3*sqrt(-(a^4 -
 2*a^2*b^2 + b^4)/((a^8 + 4*a^6*b^2 + 6*a^4*b^4 + 4*a^2*b^6 + b^8)*d^4*e^6)
) + 2*a*b)/((a^4 + 2*a^2*b^2 + b^4)*d^2*e^3))*log(-(a^2 - b^2)*sqrt((e*cos(
2*d*x + 2*c) + e)/sin(2*d*x + 2*c)) + ((a^5 + 2*a^3*b^2 + a*b^4)*d^3*e^5*sq
rt(-(a^4 - 2*a^2*b^2 + b^4)/((a^8 + 4*a^6*b^2 + 6*a^4*b^4 + 4*a^2*b^6 + b^8
)*d^4*e^6)) - (a^2*b - b^3)*d*e^2)*sqrt(-((a^4 + 2*a^2*b^2 + b^4)*d^2*e^3*s

```
qrt(-(a^4 - 2*a^2*b^2 + b^4)/((a^8 + 4*a^6*b^2 + 6*a^4*b^4 + 4*a^2*b^6 + b^
8)*d^4*e^6)) + 2*a*b)/((a^4 + 2*a^2*b^2 + b^4)*d^2*e^3))) + ((a^3 + a*b^2)*
d*e^2*cos(2*d*x + 2*c) + (a^3 + a*b^2)*d*e^2)*sqrt(-((a^4 + 2*a^2*b^2 + b^4
)*d^2*e^3*sqrt(-(a^4 - 2*a^2*b^2 + b^4)/((a^8 + 4*a^6*b^2 + 6*a^4*b^4 + 4*a
^2*b^6 + b^8)*d^4*e^6)) + 2*a*b)/((a^4 + 2*a^2*b^2 + b^4)*d^2*e^3))*log(-(a
^2 - b^2)*sqrt((e*cos(2*d*x + 2*c) + e)/sin(2*d*x + 2*c)) - ((a^5 + 2*a^3*b
^2 + a*b^4)*d^3*e^5*sqrt(-(a^4 - 2*a^2*b^2 + b^4)/((a^8 + 4*a^6*b^2 + 6*a^4
*b^4 + 4*a^2*b^6 + b^8)*d^4*e^6)) - (a^2*b - b^3)*d*e^2)*sqrt(-((a^4 + 2*a^
2*b^2 + b^4)*d^2*e^3*sqrt(-(a^4 - 2*a^2*b^2 + b^4)/((a^8 + 4*a^6*b^2 + 6*a^
4*b^4 + 4*a^2*b^6 + b^8)*d^4*e^6)) + 2*a*b)/((a^4 + 2*a^2*b^2 + b^4)*d^2*e^
3))) + ((a^3 + a*b^2)*d*e^2*cos(2*d*x + 2*c) + (a^3 + a*b^2)*d*e^2)*sqrt(((
a^4 + 2*a^2*b^2 + b^4)*d^2*e^3*sqrt(-(a^4 - 2*a^2*b^2 + b^4)/((a^8 + 4*a^6*
b^2 + 6*a^4*b^4 + 4*a^2*b^6 + b^8)*d^4*e^6)) - 2*a*b)/((a^4 + 2*a^2*b^2 + b
^4)*d^2*e^3))*log(-(a^2 - b^2)*sqrt((e*cos(2*d*x + 2*c) + e)/sin(2*d*x + 2*
c)) + ((a^5 + 2*a^3*b^2 + a*b^4)*d^3*e^5*sqrt(-(a^4 - 2*a^2*b^2 + b^4)/((a^
8 + 4*a^6*b^2 + 6*a^4*b^4 + 4*a^2*b^6 + b^8)*d^4*e^6)) + (a^2*b - b^3)*d*e^
2)*sqrt(((a^4 + 2*a^2*b^2 + b^4)*d^2*e^3*sqrt(-(a^4 - 2*a^2*b^2 + b^4)/((a^
8 + 4*a^6*b^2 + 6*a^4*b^4 + 4*a^2*b^6 + b^8)*d^4*e^6)) - 2*a*b)/((a^4 + 2*a
^2*b^2 + b^4)*d^2*e^3))) - ((a^3 + a*b^2)*d*e^2*cos(2*d*x + 2*c) + (a^3 + a
*b^2)*d*e^2)*sqrt(((a^4 + 2*a^2*b^2 + b^4)*d^2*e^3*sqrt(-(a^4 - 2*a^2*b^2 +
 b^4)/((a^8 + 4*a^6*b^2 + 6*a^4*b^4 + 4*a^2*b^6 + b^8)*d^4*e^6)) - 2*a*b)/(
(a^4 + 2*a^2*b^2 + b^4)*d^2*e^3))*log(-(a^2 - b^2)*sqrt((e*cos(2*d*x + 2*c)
 + e)/sin(2*d*x + 2*c)) - ((a^5 + 2*a^3*b^2 + a*b^4)*d^3*e^5*sqrt(-(a^4 - 2
*a^2*b^2 + b^4)/((a^8 + 4*a^6*b^2 + 6*a^4*b^4 + 4*a^2*b^6 + b^8)*d^4*e^6))
+ (a^2*b - b^3)*d*e^2)*sqrt(((a^4 + 2*a^2*b^2 + b^4)*d^2*e^3*sqrt(-(a^4 - 2
*a^2*b^2 + b^4)/((a^8 + 4*a^6*b^2 + 6*a^4*b^4 + 4*a^2*b^6 + b^8)*d^4*e^6))
- 2*a*b)/((a^4 + 2*a^2*b^2 + b^4)*d^2*e^3))) - 4*(a^2 + b^2)*sqrt((e*cos(2*
d*x + 2*c) + e)/sin(2*d*x + 2*c))*sin(2*d*x + 2*c))/((a^3 + a*b^2)*d*e^2*co
s(2*d*x + 2*c) + (a^3 + a*b^2)*d*e^2)]
```

Sympy [F]

$$\int \frac{1}{(e\cot(c+dx))^{3/2}(a+b\cot(c+dx))}\,dx = \int \frac{1}{(e\cot{(c+dx)})^{\frac{3}{2}}\,(a+b\cot{(c+dx)})}\,dx$$

```
[In] integrate(1/(e*cot(d*x+c))**(3/2)/(a+b*cot(d*x+c)),x)
```

```
[Out] Integral(1/((e*cot(c + d*x))**(3/2)*(a + b*cot(c + d*x))), x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{1}{(e\cot(c+dx))^{3/2}(a+b\cot(c+dx))}\,dx = \text{Exception raised: ValueError}$$

```
[In] integrate(1/(e*cot(d*x+c))^(3/2)/(a+b*cot(d*x+c)),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(e>0)', see `assume?` for more detai
ls)Is e
```

Giac [F]

$$\int \frac{1}{(e\cot(c+dx))^{3/2}(a+b\cot(c+dx))}\,dx = \int \frac{1}{(b\cot(dx+c)+a)(e\cot(dx+c))^{\frac{3}{2}}}\,dx$$

```
[In] integrate(1/(e*cot(d*x+c))^(3/2)/(a+b*cot(d*x+c)),x, algorithm="giac")
```

```
[Out] integrate(1/((b*cot(d*x + c) + a)*(e*cot(d*x + c))^(3/2)), x)
```

Mupad [B] (verification not implemented)

Time = 13.93 (sec) , antiderivative size = 4899, normalized size of antiderivative = 15.07

$$\int \frac{1}{(e\cot(c+dx))^{3/2}(a+b\cot(c+dx))}\,dx = \text{Too large to display}$$

```
[In] int(1/((e*cot(c + d*x))^(3/2)*(a + b*cot(c + d*x))),x)
```

```
[Out] (log((e*cot(c + d*x))^(1/2)*(64*a^7*b^7*d^5*e^13 - 32*a^9*b^5*d^5*e^13) - (
(-1/(b^2*d^2*e^3*1i - a^2*d^2*e^3*1i + 2*a*b*d^2*e^3))^(1/2)*(((((-1/(b^2*d
^2*e^3*1i - a^2*d^2*e^3*1i + 2*a*b*d^2*e^3))^(1/2)*(((e*cot(c + d*x))^(1/2)
*(-1/(b^2*d^2*e^3*1i - a^2*d^2*e^3*1i + 2*a*b*d^2*e^3))^(1/2)*(512*a^9*b^9*
d^9*e^19 + 512*a^11*b^7*d^9*e^19 - 512*a^13*b^5*d^9*e^19 - 512*a^15*b^3*d^9
*e^19))/2 - 512*a^8*b^9*d^8*e^18 - 640*a^10*b^7*d^8*e^18 + 256*a^12*b^5*d^8
*e^18 + 384*a^14*b^3*d^8*e^18))/2 - (e*cot(c + d*x))^(1/2)*(512*a^8*b^8*d^7
*e^16 - 448*a^10*b^6*d^7*e^16 + 128*a^12*b^4*d^7*e^16 + 64*a^14*b^2*d^7*e^1
6))*(-1/(b^2*d^2*e^3*1i - a^2*d^2*e^3*1i + 2*a*b*d^2*e^3))^(1/2))/2 - 128*a
^7*b^8*d^6*e^15 + 32*a^11*b^4*d^6*e^15 + 32*a^13*b^2*d^6*e^15))/2)*(-1/(b^2
*d^2*e^3*1i - a^2*d^2*e^3*1i + 2*a*b*d^2*e^3))^(1/2))/2 - atan(((-1i/(4*(b^
2*d^2*e^3 - a^2*d^2*e^3 + a*b*d^2*e^3*2i)))^(1/2)*((e*cot(c + d*x))^(1/2)*(
```

64*a^7*b^7*d^5*e^13 - 32*a^9*b^5*d^5*e^13) - (-1i/(4*(b^2*d^2*e^3 - a^2*d^2
*e^3 + a*b*d^2*e^3*2i)))^(1/2)*((-1i/(4*(b^2*d^2*e^3 - a^2*d^2*e^3 + a*b*d^
2*e^3*2i)))^(1/2)*((-1i/(4*(b^2*d^2*e^3 - a^2*d^2*e^3 + a*b*d^2*e^3*2i)))^(
1/2)*((-1i/(4*(b^2*d^2*e^3 - a^2*d^2*e^3 + a*b*d^2*e^3*2i)))^(1/2)*(e*cot(c
 + d*x))^(1/2)*(512*a^9*b^9*d^9*e^19 + 512*a^11*b^7*d^9*e^19 - 512*a^13*b^5
*d^9*e^19 - 512*a^15*b^3*d^9*e^19) - 512*a^8*b^9*d^8*e^18 - 640*a^10*b^7*d^
8*e^18 + 256*a^12*b^5*d^8*e^18 + 384*a^14*b^3*d^8*e^18) - (e*cot(c + d*x))^
(1/2)*(512*a^8*b^8*d^7*e^16 - 448*a^10*b^6*d^7*e^16 + 128*a^12*b^4*d^7*e^16
 + 64*a^14*b^2*d^7*e^16)) - 128*a^7*b^8*d^6*e^15 + 32*a^11*b^4*d^6*e^15 + 3
2*a^13*b^2*d^6*e^15))*1i + (-1i/(4*(b^2*d^2*e^3 - a^2*d^2*e^3 + a*b*d^2*e^3
2i)))^(1/2)((e*cot(c + d*x))^(1/2)*(64*a^7*b^7*d^5*e^13 - 32*a^9*b^5*d^5*
e^13) - (-1i/(4*(b^2*d^2*e^3 - a^2*d^2*e^3 + a*b*d^2*e^3*2i)))^(1/2)*((-1i/
(4*(b^2*d^2*e^3 - a^2*d^2*e^3 + a*b*d^2*e^3*2i)))^(1/2)*((-1i/(4*(b^2*d^2*e
^3 - a^2*d^2*e^3 + a*b*d^2*e^3*2i)))^(1/2)*((-1i/(4*(b^2*d^2*e^3 - a^2*d^2*
e^3 + a*b*d^2*e^3*2i)))^(1/2)*(e*cot(c + d*x))^(1/2)*(512*a^9*b^9*d^9*e^19
+ 512*a^11*b^7*d^9*e^19 - 512*a^13*b^5*d^9*e^19 - 512*a^15*b^3*d^9*e^19) +
512*a^8*b^9*d^8*e^18 + 640*a^10*b^7*d^8*e^18 - 256*a^12*b^5*d^8*e^18 - 384*
a^14*b^3*d^8*e^18) - (e*cot(c + d*x))^(1/2)*(512*a^8*b^8*d^7*e^16 - 448*a^1
0*b^6*d^7*e^16 + 128*a^12*b^4*d^7*e^16 + 64*a^14*b^2*d^7*e^16)) + 128*a^7*b
^8*d^6*e^15 - 32*a^11*b^4*d^6*e^15 - 32*a^13*b^2*d^6*e^15))*1i)/((-1i/(4*(b
^2*d^2*e^3 - a^2*d^2*e^3 + a*b*d^2*e^3*2i)))^(1/2)*((e*cot(c + d*x))^(1/2)*
(64*a^7*b^7*d^5*e^13 - 32*a^9*b^5*d^5*e^13) - (-1i/(4*(b^2*d^2*e^3 - a^2*d^
2*e^3 + a*b*d^2*e^3*2i)))^(1/2)*((-1i/(4*(b^2*d^2*e^3 - a^2*d^2*e^3 + a*b*d
^2*e^3*2i)))^(1/2)*((-1i/(4*(b^2*d^2*e^3 - a^2*d^2*e^3 + a*b*d^2*e^3*2i)))^
(1/2)*((-1i/(4*(b^2*d^2*e^3 - a^2*d^2*e^3 + a*b*d^2*e^3*2i)))^(1/2)*(e*cot(
c + d*x))^(1/2)*(512*a^9*b^9*d^9*e^19 + 512*a^11*b^7*d^9*e^19 - 512*a^13*b^
5*d^9*e^19 - 512*a^15*b^3*d^9*e^19) - 512*a^8*b^9*d^8*e^18 - 640*a^10*b^7*d
^8*e^18 + 256*a^12*b^5*d^8*e^18 + 384*a^14*b^3*d^8*e^18) - (e*cot(c + d*x))
^(1/2)*(512*a^8*b^8*d^7*e^16 - 448*a^10*b^6*d^7*e^16 + 128*a^12*b^4*d^7*e^1
6 + 64*a^14*b^2*d^7*e^16)) - 128*a^7*b^8*d^6*e^15 + 32*a^11*b^4*d^6*e^15 +
32*a^13*b^2*d^6*e^15)) - (-1i/(4*(b^2*d^2*e^3 - a^2*d^2*e^3 + a*b*d^2*e^3*2
i)))^(1/2)*((e*cot(c + d*x))^(1/2)*(64*a^7*b^7*d^5*e^13 - 32*a^9*b^5*d^5*e^
13) - (-1i/(4*(b^2*d^2*e^3 - a^2*d^2*e^3 + a*b*d^2*e^3*2i)))^(1/2)*((-1i/(4
*(b^2*d^2*e^3 - a^2*d^2*e^3 + a*b*d^2*e^3*2i)))^(1/2)*((-1i/(4*(b^2*d^2*e^3
 - a^2*d^2*e^3 + a*b*d^2*e^3*2i)))^(1/2)*((-1i/(4*(b^2*d^2*e^3 - a^2*d^2*e^
3 + a*b*d^2*e^3*2i)))^(1/2)*(e*cot(c + d*x))^(1/2)*(512*a^9*b^9*d^9*e^19 +
512*a^11*b^7*d^9*e^19 - 512*a^13*b^5*d^9*e^19 - 512*a^15*b^3*d^9*e^19) + 51
2*a^8*b^9*d^8*e^18 + 640*a^10*b^7*d^8*e^18 - 256*a^12*b^5*d^8*e^18 - 384*a^
14*b^3*d^8*e^18) - (e*cot(c + d*x))^(1/2)*(512*a^8*b^8*d^7*e^16 - 448*a^10*
b^6*d^7*e^16 + 128*a^12*b^4*d^7*e^16 + 64*a^14*b^2*d^7*e^16)) + 128*a^7*b^8
*d^6*e^15 - 32*a^11*b^4*d^6*e^15 - 32*a^13*b^2*d^6*e^15))))*(-1i/(4*(b^2*d^
2*e^3 - a^2*d^2*e^3 + a*b*d^2*e^3*2i)))^(1/2)*2i - log((e*cot(c + d*x))^(1/
2)*(64*a^7*b^7*d^5*e^13 - 32*a^9*b^5*d^5*e^13) - (-1/(4*(b^2*d^2*e^3*1i - a
^2*d^2*e^3*1i + 2*a*b*d^2*e^3)))^(1/2)*((-1/(4*(b^2*d^2*e^3*1i - a^2*d^2*e^
3*1i + 2*a*b*d^2*e^3)))^(1/2)*((-1/(4*(b^2*d^2*e^3*1i - a^2*d^2*e^3*1i + 2*

a*b*d^2*e^3)))^(1/2)*((-1/(4*(b^2*d^2*e^3*1i - a^2*d^2*e^3*1i + 2*a*b*d^2*e
^3)))^(1/2)*(e*cot(c + d*x))^(1/2)*(512*a^9*b^9*d^9*e^19 + 512*a^11*b^7*d^9
*e^19 - 512*a^13*b^5*d^9*e^19 - 512*a^15*b^3*d^9*e^19) + 512*a^8*b^9*d^8*e^
18 + 640*a^10*b^7*d^8*e^18 - 256*a^12*b^5*d^8*e^18 - 384*a^14*b^3*d^8*e^18)
 - (e*cot(c + d*x))^(1/2)*(512*a^8*b^8*d^7*e^16 - 448*a^10*b^6*d^7*e^16 + 1
28*a^12*b^4*d^7*e^16 + 64*a^14*b^2*d^7*e^16)) + 128*a^7*b^8*d^6*e^15 - 32*a
^11*b^4*d^6*e^15 - 32*a^13*b^2*d^6*e^15))*(-1/(4*(b^2*d^2*e^3*1i - a^2*d^2*
e^3*1i + 2*a*b*d^2*e^3)))^(1/2) - (atan(((((e*cot(c + d*x))^(1/2)*(64*a^7*b
^7*d^5*e^13 - 32*a^9*b^5*d^5*e^13) + ((-a^3*b^5*e^3)^(1/2)*((((e*cot(c + d*
x))^(1/2)*(512*a^8*b^8*d^7*e^16 - 448*a^10*b^6*d^7*e^16 + 128*a^12*b^4*d^7*
e^16 + 64*a^14*b^2*d^7*e^16) - ((-a^3*b^5*e^3)^(1/2)*(256*a^12*b^5*d^8*e^18
 - 640*a^10*b^7*d^8*e^18 - 512*a^8*b^9*d^8*e^18 + 384*a^14*b^3*d^8*e^18 + (
(e*cot(c + d*x))^(1/2)*(-a^3*b^5*e^3)^(1/2)*(512*a^9*b^9*d^9*e^19 + 512*a^1
1*b^7*d^9*e^19 - 512*a^13*b^5*d^9*e^19 - 512*a^15*b^3*d^9*e^19))/(a^5*d*e^3
 + a^3*b^2*d*e^3)))/(a^5*d*e^3 + a^3*b^2*d*e^3))*(-a^3*b^5*e^3)^(1/2))/(a^5
*d*e^3 + a^3*b^2*d*e^3) + 128*a^7*b^8*d^6*e^15 - 32*a^11*b^4*d^6*e^15 - 32*
a^13*b^2*d^6*e^15))/(a^5*d*e^3 + a^3*b^2*d*e^3))*(-a^3*b^5*e^3)^(1/2)*1i)/(
a^5*d*e^3 + a^3*b^2*d*e^3) + (((e*cot(c + d*x))^(1/2)*(64*a^7*b^7*d^5*e^13
- 32*a^9*b^5*d^5*e^13) + ((-a^3*b^5*e^3)^(1/2)*((((e*cot(c + d*x))^(1/2)*(5
12*a^8*b^8*d^7*e^16 - 448*a^10*b^6*d^7*e^16 + 128*a^12*b^4*d^7*e^16 + 64*a^
14*b^2*d^7*e^16) - ((-a^3*b^5*e^3)^(1/2)*(512*a^8*b^9*d^8*e^18 + 640*a^10*b
^7*d^8*e^18 - 256*a^12*b^5*d^8*e^18 - 384*a^14*b^3*d^8*e^18 + ((e*cot(c + d
x))^(1/2)(-a^3*b^5*e^3)^(1/2)*(512*a^9*b^9*d^9*e^19 + 512*a^11*b^7*d^9*e^
19 - 512*a^13*b^5*d^9*e^19 - 512*a^15*b^3*d^9*e^19))/(a^5*d*e^3 + a^3*b^2*d
*e^3)))/(a^5*d*e^3 + a^3*b^2*d*e^3))*(-a^3*b^5*e^3)^(1/2))/(a^5*d*e^3 + a^3
*b^2*d*e^3) - 128*a^7*b^8*d^6*e^15 + 32*a^11*b^4*d^6*e^15 + 32*a^13*b^2*d^6
*e^15))/(a^5*d*e^3 + a^3*b^2*d*e^3))*(-a^3*b^5*e^3)^(1/2)*1i)/(a^5*d*e^3 +
a^3*b^2*d*e^3))/((((e*cot(c + d*x))^(1/2)*(64*a^7*b^7*d^5*e^13 - 32*a^9*b^5
*d^5*e^13) + ((-a^3*b^5*e^3)^(1/2)*((((e*cot(c + d*x))^(1/2)*(512*a^8*b^8*d
^7*e^16 - 448*a^10*b^6*d^7*e^16 + 128*a^12*b^4*d^7*e^16 + 64*a^14*b^2*d^7*e
^16) - ((-a^3*b^5*e^3)^(1/2)*(256*a^12*b^5*d^8*e^18 - 640*a^10*b^7*d^8*e^18
 - 512*a^8*b^9*d^8*e^18 + 384*a^14*b^3*d^8*e^18 + ((e*cot(c + d*x))^(1/2)*(
-a^3*b^5*e^3)^(1/2)*(512*a^9*b^9*d^9*e^19 + 512*a^11*b^7*d^9*e^19 - 512*a^1
3*b^5*d^9*e^19 - 512*a^15*b^3*d^9*e^19))/(a^5*d*e^3 + a^3*b^2*d*e^3)))/(a^5
*d*e^3 + a^3*b^2*d*e^3))*(-a^3*b^5*e^3)^(1/2))/(a^5*d*e^3 + a^3*b^2*d*e^3)
+ 128*a^7*b^8*d^6*e^15 - 32*a^11*b^4*d^6*e^15 - 32*a^13*b^2*d^6*e^15))/(a^5
*d*e^3 + a^3*b^2*d*e^3))*(-a^3*b^5*e^3)^(1/2))/(a^5*d*e^3 + a^3*b^2*d*e^3)
- (((e*cot(c + d*x))^(1/2)*(64*a^7*b^7*d^5*e^13 - 32*a^9*b^5*d^5*e^13) + ((
-a^3*b^5*e^3)^(1/2)*((((e*cot(c + d*x))^(1/2)*(512*a^8*b^8*d^7*e^16 - 448*a
^10*b^6*d^7*e^16 + 128*a^12*b^4*d^7*e^16 + 64*a^14*b^2*d^7*e^16) - ((-a^3*b
^5*e^3)^(1/2)*(512*a^8*b^9*d^8*e^18 + 640*a^10*b^7*d^8*e^18 - 256*a^12*b^5*
d^8*e^18 - 384*a^14*b^3*d^8*e^18 + ((e*cot(c + d*x))^(1/2)*(-a^3*b^5*e^3)^(
1/2)*(512*a^9*b^9*d^9*e^19 + 512*a^11*b^7*d^9*e^19 - 512*a^13*b^5*d^9*e^19
- 512*a^15*b^3*d^9*e^19))/(a^5*d*e^3 + a^3*b^2*d*e^3)))/(a^5*d*e^3 + a^3*b^
2*d*e^3))*(-a^3*b^5*e^3)^(1/2))/(a^5*d*e^3 + a^3*b^2*d*e^3) - 128*a^7*b^8*d

```
^6*e^15 + 32*a^11*b^4*d^6*e^15 + 32*a^13*b^2*d^6*e^15))/(a^5*d*e^3 + a^3*b^
2*d*e^3))*(-a^3*b^5*e^3)^(1/2))/(a^5*d*e^3 + a^3*b^2*d*e^3)))*(-a^3*b^5*e^3
)^(1/2)*2i)/(a^5*d*e^3 + a^3*b^2*d*e^3) + 2/(a*d*e*(e*cot(c + d*x))^(1/2))
```

3.74 $\int \frac{1}{(e\cot(c+dx))^{5/2}(a+b\cot(c+dx))}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 351

$$
\begin{aligned}
\int \frac{1}{(e\cot(c+dx))^{5/2}(a+b\cot(c+dx))}\,dx &= -\frac{2b^{7/2}\arctan\left(\frac{\sqrt{b}\sqrt{e\cot(c+dx)}}{\sqrt{a}\sqrt{e}}\right)}{a^{5/2}\left(a^2+b^2\right)de^{5/2}} \\
&- \frac{(a-b)\arctan\left(1-\frac{\sqrt{2}\sqrt{e\cot(c+dx)}}{\sqrt{e}}\right)}{\sqrt{2}\left(a^2+b^2\right)de^{5/2}} + \frac{(a-b)\arctan\left(1+\frac{\sqrt{2}\sqrt{e\cot(c+dx)}}{\sqrt{e}}\right)}{\sqrt{2}\left(a^2+b^2\right)de^{5/2}} \\
&+ \frac{2}{3ade(e\cot(c+dx))^{3/2}} - \frac{2b}{a^2de^2\sqrt{e\cot(c+dx)}} \\
&- \frac{(a+b)\log\left(\sqrt{e}+\sqrt{e}\cot(c+dx)-\sqrt{2}\sqrt{e\cot(c+dx)}\right)}{2\sqrt{2}\left(a^2+b^2\right)de^{5/2}} \\
&+ \frac{(a+b)\log\left(\sqrt{e}+\sqrt{e}\cot(c+dx)+\sqrt{2}\sqrt{e\cot(c+dx)}\right)}{2\sqrt{2}\left(a^2+b^2\right)de^{5/2}}
\end{aligned}
$$

```
[Out] -2*b^(7/2)*arctan(b^(1/2)*(e*cot(d*x+c))^(1/2)/a^(1/2)/e^(1/2))/a^(5/2)/(a^
2+b^2)/d/e^(5/2)+2/3/a/d/e/(e*cot(d*x+c))^(3/2)-1/2*(a-b)*arctan(1-2^(1/2)*
(e*cot(d*x+c))^(1/2)/e^(1/2))/(a^2+b^2)/d/e^(5/2)*2^(1/2)+1/2*(a-b)*arctan(
1+2^(1/2)*(e*cot(d*x+c))^(1/2)/e^(1/2))/(a^2+b^2)/d/e^(5/2)*2^(1/2)-1/4*(a+
b)*ln(e^(1/2)+cot(d*x+c)*e^(1/2)-2^(1/2)*(e*cot(d*x+c))^(1/2))/(a^2+b^2)/d/
e^(5/2)*2^(1/2)+1/4*(a+b)*ln(e^(1/2)+cot(d*x+c)*e^(1/2)+2^(1/2)*(e*cot(d*x+
c))^(1/2))/(a^2+b^2)/d/e^(5/2)*2^(1/2)-2*b/a^2/d/e^2/(e*cot(d*x+c))^(1/2)
```

Rubi [A] (verified)

Time = 1.00 (sec) , antiderivative size = 351, normalized size of antiderivative = 1.00, number of steps used = 16, number of rules used = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.520$, Rules used = {3650, 3730, 3735, 3615, 1182, 1176, 631, 210, 1179, 642, 3715, 65, 211}

$$\int \frac{1}{(e\cot(c+dx))^{5/2}(a+b\cot(c+dx))}\,dx = -\frac{(a-b)\arctan\left(1-\frac{\sqrt{2}\sqrt{e\cot(c+dx)}}{\sqrt{e}}\right)}{\sqrt{2}de^{5/2}\left(a^2+b^2\right)}+\frac{(a-b)\arctan\left(\frac{\sqrt{2}\sqrt{e\cot(c+dx)}}{\sqrt{e}}+1\right)}{\sqrt{2}de^{5/2}\left(a^2+b^2\right)} - \frac{(a+b)\log\left(\sqrt{e}\cot(c+dx)-\sqrt{2}\sqrt{e\cot(c+dx)}+\sqrt{e}\right)}{2\sqrt{2}de^{5/2}\left(a^2+b^2\right)} + \frac{(a+b)\log\left(\sqrt{e}\cot(c+dx)+\sqrt{2}\sqrt{e\cot(c+dx)}+\sqrt{e}\right)}{2\sqrt{2}de^{5/2}\left(a^2+b^2\right)} - \frac{2b}{a^2de^2\sqrt{e\cot(c+dx)}} - \frac{2b^{7/2}\arctan\left(\frac{\sqrt{b}\sqrt{e\cot(c+dx)}}{\sqrt{a}\sqrt{e}}\right)}{a^{5/2}de^{5/2}\left(a^2+b^2\right)} + \frac{2}{3ade(e\cot(c+dx))^{3/2}}$$

```
[In] Int[1/((e*Cot[c + d*x])^(5/2)*(a + b*Cot[c + d*x])),x]
```

```
[Out] (-2*b^(7/2)*ArcTan[(Sqrt[b]*Sqrt[e*Cot[c + d*x]])/(Sqrt[a]*Sqrt[e])])/(a^(5
/2)*(a^2 + b^2)*d*e^(5/2)) - ((a - b)*ArcTan[1 - (Sqrt[2]*Sqrt[e*Cot[c + d*
x]])/Sqrt[e]])/(Sqrt[2]*(a^2 + b^2)*d*e^(5/2)) + ((a - b)*ArcTan[1 + (Sqrt[
2]*Sqrt[e*Cot[c + d*x]])/Sqrt[e]])/(Sqrt[2]*(a^2 + b^2)*d*e^(5/2)) + 2/(3*a
*d*e*(e*Cot[c + d*x])^(3/2)) - (2*b)/(a^2*d*e^2*Sqrt[e*Cot[c + d*x]]) - ((a
 + b)*Log[Sqrt[e] + Sqrt[e]*Cot[c + d*x] - Sqrt[2]*Sqrt[e*Cot[c + d*x]]])/(
2*Sqrt[2]*(a^2 + b^2)*d*e^(5/2)) + ((a + b)*Log[Sqrt[e] + Sqrt[e]*Cot[c + d
*x] + Sqrt[2]*Sqrt[e*Cot[c + d*x]]])/(2*Sqrt[2]*(a^2 + b^2)*d*e^(5/2))
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1179

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 1182

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
a*c, 2]}, Dist[(d*q + a*e)/(2*a*c), Int[(q + c*x^2)/(a + c*x^4), x], x] + D
ist[(d*q - a*e)/(2*a*c), Int[(q - c*x^2)/(a + c*x^4), x], x]] /; FreeQ[{a,
c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0] && NeQ[c*d^2 - a*e^2, 0] && NegQ[(-a)
*c]
```

Rule 3615

```
Int[((c_) + (d_.)*tan[(e_.) + (f_.)*(x_)])/Sqrt[(b_.)*tan[(e_.) + (f_.)*(x_
)]], x_Symbol] :> Dist[2/f, Subst[Int[(b*c + d*x^2)/(b^2 + x^4), x], x, Sqr
t[b*Tan[e + f*x]]], x] /; FreeQ[{b, c, d, e, f}, x] && NeQ[c^2 - d^2, 0] &&
 NeQ[c^2 + d^2, 0]
```

Rule 3650

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)])^(n_), x_Symbol] :> Simp[b^2*(a + b*Tan[e + f*x])^(m + 1)*((c
+ d*Tan[e + f*x])^(n + 1)/(f*(m + 1)*(a^2 + b^2)*(b*c - a*d))), x] + Dist[1
/((m + 1)*(a^2 + b^2)*(b*c - a*d)), Int[(a + b*Tan[e + f*x])^(m + 1)*(c + d
*Tan[e + f*x])^n*Simp[a*(b*c - a*d)*(m + 1) - b^2*d*(m + n + 2) - b*(b*c -
a*d)*(m + 1)*Tan[e + f*x] - b^2*d*(m + n + 2)*Tan[e + f*x]^2, x], x], x] /;
 FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0]
&& NeQ[c^2 + d^2, 0] && IntegerQ[2*m] && LtQ[m, -1] && (LtQ[n, 0] || Intege
rQ[m]) &&  !(ILtQ[n, -1] && ( !IntegerQ[m] || (EqQ[c, 0] && NeQ[a, 0])))
```

Rule 3715

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*tan[(e_.)
+ (f_.)*(x_)])^(n_.)*((A_) + (C_.)*tan[(e_.) + (f_.)*(x_)]^2), x_Symbol] :>
 Dist[A/f, Subst[Int[(a + b*x)^m*(c + d*x)^n, x], x, Tan[e + f*x]], x] /; F
reeQ[{a, b, c, d, e, f, A, C, m, n}, x] && EqQ[A, C]
```

Rule 3730

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*tan[(e_.) + (f_.)*(x_)] + (C_.)*tan[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> Simp[(A*b^2 - a*(b*B - a*C))*(a + b*Tan[e +
 f*x])^(m + 1)*((c + d*Tan[e + f*x])^(n + 1)/(f*(m + 1)*(b*c - a*d)*(a^2 +
b^2))), x] + Dist[1/((m + 1)*(b*c - a*d)*(a^2 + b^2)), Int[(a + b*Tan[e + f
*x])^(m + 1)*(c + d*Tan[e + f*x])^n*Simp[A*(a*(b*c - a*d)*(m + 1) - b^2*d*(
m + n + 2)) + (b*B - a*C)*(b*c*(m + 1) + a*d*(n + 1)) - (m + 1)*(b*c - a*d)
*(A*b - a*B - b*C)*Tan[e + f*x] - d*(A*b^2 - a*(b*B - a*C))*(m + n + 2)*Tan
[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, n}, x] && NeQ[
b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] && LtQ[m, -1] &&  !
(ILtQ[n, -1] && ( !IntegerQ[m] || (EqQ[c, 0] && NeQ[a, 0])))
```

Rule 3735

```
Int[(((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_)*((A_.) + (C_.)*tan[(e_.)
+ (f_.)*(x_)]^2))/((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dis
t[1/(a^2 + b^2), Int[(c + d*Tan[e + f*x])^n*Simp[a*(A - C) - (A*b - b*C)*Ta
n[e + f*x], x], x], x] + Dist[(A*b^2 + a^2*C)/(a^2 + b^2), Int[(c + d*Tan[e
 + f*x])^n*((1 + Tan[e + f*x]^2)/(a + b*Tan[e + f*x])), x], x] /; FreeQ[{a,
 b, c, d, e, f, A, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && N
eQ[c^2 + d^2, 0] &&  !GtQ[n, 0] &&  !LeQ[n, -1]
```

Rubi steps

$$\text{integral} = \frac{2}{3ade(e\cot(c+dx))^{3/2}} + \frac{2\int \frac{-\frac{3be^2}{2}-\frac{3}{2}ae^2\cot(c+dx)-\frac{3}{2}be^2\cot^2(c+dx)}{(e\cot(c+dx))^{3/2}(a+b\cot(c+dx))}\,dx}{3ae^3}$$

$$= \frac{2}{3ade(e\cot(c+dx))^{3/2}} - \frac{2b}{a^2de^2\sqrt{e\cot(c+dx)}} + \frac{4\int \frac{-\frac{3}{4}(a^2-b^2)e^4+\frac{3}{4}b^2e^4\cot^2(c+dx)}{\sqrt{e\cot(c+dx)}(a+b\cot(c+dx))}dx}{3a^2e^6}$$

$$= \frac{2}{3ade(e\cot(c+dx))^{3/2}} - \frac{2b}{a^2de^2\sqrt{e\cot(c+dx)}} + \frac{4\int \frac{-\frac{3}{4}a^3e^4+\frac{3}{4}a^2be^4\cot(c+dx)}{\sqrt{e\cot(c+dx)}}dx}{3a^2(a^2+b^2)e^6} + \frac{b^4\int \frac{1+\cot^2(c+dx)}{\sqrt{e\cot(c+dx)}(a+b\cot(c+dx))}dx}{a^2(a^2+b^2)e^2}$$

$$= \frac{2}{3ade(e\cot(c+dx))^{3/2}} - \frac{2b}{a^2de^2\sqrt{e\cot(c+dx)}} + \frac{8\text{Subst}\left(\int \frac{\frac{3a^3e^5}{4}-\frac{3}{4}a^2be^4x^2}{e^2+x^4}dx, x, \sqrt{e\cot(c+dx)}\right)}{3a^2(a^2+b^2)de^6} + \frac{b^4\text{Subst}\left(\int \frac{1}{\sqrt{-ex}(a-bx)}dx, x, -\cot(c+dx)\right)}{a^2(a^2+b^2)de^2}$$

$$= \frac{2}{3ade(e\cot(c+dx))^{3/2}} - \frac{2b}{a^2de^2\sqrt{e\cot(c+dx)}} - \frac{(2b^4)\text{Subst}\left(\int \frac{1}{a+\frac{bx^2}{e}}dx, x, \sqrt{e\cot(c+dx)}\right)}{a^2(a^2+b^2)de^3} + \frac{(a-b)\text{Subst}\left(\int \frac{e+x^2}{e^2+x^4}dx, x, \sqrt{e\cot(c+dx)}\right)}{(a^2+b^2)de^2} + \frac{(a+b)\text{Subst}\left(\int \frac{e-x^2}{e^2+x^4}dx, x, \sqrt{e\cot(c+dx)}\right)}{(a^2+b^2)de^2}$$

$$= -\frac{2b^{7/2}\arctan\left(\frac{\sqrt{b}\sqrt{e\cot(c+dx)}}{\sqrt{a}\sqrt{e}}\right)}{a^{5/2}(a^2+b^2)de^{5/2}} + \frac{2}{3ade(e\cot(c+dx))^{3/2}} - \frac{2b}{a^2de^2\sqrt{e\cot(c+dx)}} - \frac{(a+b)\text{Subst}\left(\int \frac{\sqrt{2}\sqrt{e}+2x}{-e-\sqrt{2}\sqrt{e}x-x^2}dx, x, \sqrt{e\cot(c+dx)}\right)}{2\sqrt{2}(a^2+b^2)de^{5/2}} - \frac{(a+b)\text{Subst}\left(\int \frac{\sqrt{2}\sqrt{e}-2x}{-e+\sqrt{2}\sqrt{e}x-x^2}dx, x, \sqrt{e\cot(c+dx)}\right)}{2\sqrt{2}(a^2+b^2)de^{5/2}} + \frac{(a-b)\text{Subst}\left(\int \frac{1}{e-\sqrt{2}\sqrt{e}x+x^2}dx, x, \sqrt{e\cot(c+dx)}\right)}{2(a^2+b^2)de^2} + \frac{(a-b)\text{Subst}\left(\int \frac{1}{e+\sqrt{2}\sqrt{e}x+x^2}dx, x, \sqrt{e\cot(c+dx)}\right)}{2(a^2+b^2)de^2}$$

$$= -\frac{2b^{7/2}\arctan\left(\frac{\sqrt{b}\sqrt{e\cot(c+dx)}}{\sqrt{a}\sqrt{e}}\right)}{a^{5/2}(a^2+b^2)de^{5/2}} + \frac{2}{3ade(e\cot(c+dx))^{3/2}} - \frac{2b}{a^2de^2\sqrt{e\cot(c+dx)}}$$
$$- \frac{(a+b)\log\left(\sqrt{e}+\sqrt{e}\cot(c+dx)-\sqrt{2}\sqrt{e\cot(c+dx)}\right)}{2\sqrt{2}(a^2+b^2)de^{5/2}}$$
$$+ \frac{(a+b)\log\left(\sqrt{e}+\sqrt{e}\cot(c+dx)+\sqrt{2}\sqrt{e\cot(c+dx)}\right)}{2\sqrt{2}(a^2+b^2)de^{5/2}}$$
$$+ \frac{(a-b)\text{Subst}\left(\int \frac{1}{-1-x^2}\,dx, x, 1-\frac{\sqrt{2}\sqrt{e\cot(c+dx)}}{\sqrt{e}}\right)}{\sqrt{2}(a^2+b^2)de^{5/2}}$$
$$- \frac{(a-b)\text{Subst}\left(\int \frac{1}{-1-x^2}\,dx, x, 1+\frac{\sqrt{2}\sqrt{e\cot(c+dx)}}{\sqrt{e}}\right)}{\sqrt{2}(a^2+b^2)de^{5/2}}$$

$$= -\frac{2b^{7/2}\arctan\left(\frac{\sqrt{b}\sqrt{e\cot(c+dx)}}{\sqrt{a}\sqrt{e}}\right)}{a^{5/2}(a^2+b^2)de^{5/2}} - \frac{(a-b)\arctan\left(1-\frac{\sqrt{2}\sqrt{e\cot(c+dx)}}{\sqrt{e}}\right)}{\sqrt{2}(a^2+b^2)de^{5/2}}$$
$$+ \frac{(a-b)\arctan\left(1+\frac{\sqrt{2}\sqrt{e\cot(c+dx)}}{\sqrt{e}}\right)}{\sqrt{2}(a^2+b^2)de^{5/2}} + \frac{2}{3ade(e\cot(c+dx))^{3/2}}$$
$$- \frac{2b}{a^2de^2\sqrt{e\cot(c+dx)}} - \frac{(a+b)\log\left(\sqrt{e}+\sqrt{e}\cot(c+dx)-\sqrt{2}\sqrt{e\cot(c+dx)}\right)}{2\sqrt{2}(a^2+b^2)de^{5/2}}$$
$$+ \frac{(a+b)\log\left(\sqrt{e}+\sqrt{e}\cot(c+dx)+\sqrt{2}\sqrt{e\cot(c+dx)}\right)}{2\sqrt{2}(a^2+b^2)de^{5/2}}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 0.29 (sec) , antiderivative size = 109, normalized size of antiderivative = 0.31

$$\int \frac{1}{(e\cot(c+dx))^{5/2}(a+b\cot(c+dx))}\,dx = \frac{2\left(b^2\,\text{Hypergeometric2F1}\left(-\frac{3}{2},1,-\frac{1}{2},-\frac{b\cot(c+dx)}{a}\right)+a\left(a\,\text{Hype}\right.\right.}{}$$

```
[In] Integrate[1/((e*Cot[c + d*x])^(5/2)*(a + b*Cot[c + d*x])),x]
```

```
[Out] (2*(b^2*Hypergeometric2F1[-3/2, 1, -1/2, -((b*Cot[c + d*x])/a)] + a*(a*Hype
rgeometric2F1[-3/4, 1, 1/4, -Cot[c + d*x]^2] - 3*b*Cot[c + d*x]*Hypergeomet
ric2F1[-1/4, 1, 3/4, -Cot[c + d*x]^2])))/(3*a*(a^2 + b^2)*d*e*(e*Cot[c + d*
x])^(3/2))
```

Maple [A] (verified)

Time = 0.04 (sec) , antiderivative size = 371, normalized size of antiderivative = 1.06

method	result
derivativedivides	$-\frac{2e^2\left(\frac{b^4\arctan\left(\frac{\sqrt{e\cot(dx+c)}\,b}{\sqrt{aeb}}\right)}{a^2e^4\left(a^2+b^2\right)\sqrt{aeb}}+\frac{-\frac{a\left(e^2\right)^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\frac{e\cot(dx+c)+\left(e^2\right)^{\frac{1}{4}}\sqrt{e\cot(dx+c)}\,\sqrt{2}+\sqrt{e^2}}{e\cot(dx+c)-\left(e^2\right)^{\frac{1}{4}}\sqrt{e\cot(dx+c)}\,\sqrt{2}+\sqrt{e^2}}\right)+2\arctan\left(\frac{\sqrt{2}\,\sqrt{e\cot(dx+c)}}{\left(e^2\right)^{\frac{1}{4}}}\right.\right.}{8e}}{}\right.}{}$
default	$-\frac{2e^2\left(\frac{b^4\arctan\left(\frac{\sqrt{e\cot(dx+c)}\,b}{\sqrt{aeb}}\right)}{a^2e^4\left(a^2+b^2\right)\sqrt{aeb}}+\frac{-\frac{a\left(e^2\right)^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\frac{e\cot(dx+c)+\left(e^2\right)^{\frac{1}{4}}\sqrt{e\cot(dx+c)}\,\sqrt{2}+\sqrt{e^2}}{e\cot(dx+c)-\left(e^2\right)^{\frac{1}{4}}\sqrt{e\cot(dx+c)}\,\sqrt{2}+\sqrt{e^2}}\right)+2\arctan\left(\frac{\sqrt{2}\,\sqrt{e\cot(dx+c)}}{\left(e^2\right)^{\frac{1}{4}}}\right.\right.}{8e}}{}\right.}{}$

```
[In] int(1/(e*cot(d*x+c))^(5/2)/(a+b*cot(d*x+c)),x,method=_RETURNVERBOSE)
```

```
[Out] -2/d*e^2*(1/a^2/e^4*b^4/(a^2+b^2)/(a*e*b)^(1/2)*arctan((e*cot(d*x+c))^(1/2)
*b/(a*e*b)^(1/2))+1/(a^2+b^2)/e^4*(-1/8*a/e*(e^2)^(1/4)*2^(1/2)*(ln((e*cot(
d*x+c)+(e^2)^(1/4)*(e*cot(d*x+c))^(1/2)*2^(1/2)+(e^2)^(1/2))/(e*cot(d*x+c)-
(e^2)^(1/4)*(e*cot(d*x+c))^(1/2)*2^(1/2)+(e^2)^(1/2)))+2*arctan(2^(1/2)/(e^
2)^(1/4)*(e*cot(d*x+c))^(1/2)+1)-2*arctan(-2^(1/2)/(e^2)^(1/4)*(e*cot(d*x+c
))^(1/2)+1))+1/8*b/(e^2)^(1/4)*2^(1/2)*(ln((e*cot(d*x+c)-(e^2)^(1/4)*(e*cot
(d*x+c))^(1/2)*2^(1/2)+(e^2)^(1/2))/(e*cot(d*x+c)+(e^2)^(1/4)*(e*cot(d*x+c)
)^(1/2)*2^(1/2)+(e^2)^(1/2)))+2*arctan(2^(1/2)/(e^2)^(1/4)*(e*cot(d*x+c))^(
1/2)+1)-2*arctan(-2^(1/2)/(e^2)^(1/4)*(e*cot(d*x+c))^(1/2)+1)))-1/3/a/e^3/(
e*cot(d*x+c))^(3/2)+1/a^2/e^4*b/(e*cot(d*x+c))^(1/2))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1849 vs. 2(284) = 568.

Time = 0.47 (sec) , antiderivative size = 3742, normalized size of antiderivative = 10.66

$$\int \frac{1}{(e\cot(c+dx))^{5/2}(a+b\cot(c+dx))}\,dx = \text{Too large to display}$$

```
[In] integrate(1/(e*cot(d*x+c))^(5/2)/(a+b*cot(d*x+c)),x, algorithm="fricas")
```

```
[Out] [-1/6*(3*((a^4 + a^2*b^2)*d*e^3*cos(2*d*x + 2*c) + (a^4 + a^2*b^2)*d*e^3)*s
qrt(((a^4 + 2*a^2*b^2 + b^4)*d^2*e^5*sqrt(-(a^4 - 2*a^2*b^2 + b^4)/((a^8 +
```

4*a^6*b^2 + 6*a^4*b^4 + 4*a^2*b^6 + b^8)*d^4*e^10)) + 2*a*b)/((a^4 + 2*a^2*
b^2 + b^4)*d^2*e^5))*log(-(a^2 - b^2)*sqrt((e*cos(2*d*x + 2*c) + e)/sin(2*d
*x + 2*c)) + ((a^4*b + 2*a^2*b^3 + b^5)*d^3*e^8*sqrt(-(a^4 - 2*a^2*b^2 + b^
4)/((a^8 + 4*a^6*b^2 + 6*a^4*b^4 + 4*a^2*b^6 + b^8)*d^4*e^10)) + (a^3 - a*b
^2)*d*e^3)*sqrt(((a^4 + 2*a^2*b^2 + b^4)*d^2*e^5*sqrt(-(a^4 - 2*a^2*b^2 + b
^4)/((a^8 + 4*a^6*b^2 + 6*a^4*b^4 + 4*a^2*b^6 + b^8)*d^4*e^10)) + 2*a*b)/((
a^4 + 2*a^2*b^2 + b^4)*d^2*e^5))) - 3*((a^4 + a^2*b^2)*d*e^3*cos(2*d*x + 2*
c) + (a^4 + a^2*b^2)*d*e^3)*sqrt(((a^4 + 2*a^2*b^2 + b^4)*d^2*e^5*sqrt(-(a^
4 - 2*a^2*b^2 + b^4)/((a^8 + 4*a^6*b^2 + 6*a^4*b^4 + 4*a^2*b^6 + b^8)*d^4*e
^10)) + 2*a*b)/((a^4 + 2*a^2*b^2 + b^4)*d^2*e^5))*log(-(a^2 - b^2)*sqrt((e*
cos(2*d*x + 2*c) + e)/sin(2*d*x + 2*c)) - ((a^4*b + 2*a^2*b^3 + b^5)*d^3*e^
8*sqrt(-(a^4 - 2*a^2*b^2 + b^4)/((a^8 + 4*a^6*b^2 + 6*a^4*b^4 + 4*a^2*b^6 +
b^8)*d^4*e^10)) + (a^3 - a*b^2)*d*e^3)*sqrt(((a^4 + 2*a^2*b^2 + b^4)*d^2*e
^5*sqrt(-(a^4 - 2*a^2*b^2 + b^4)/((a^8 + 4*a^6*b^2 + 6*a^4*b^4 + 4*a^2*b^6
+ b^8)*d^4*e^10)) + 2*a*b)/((a^4 + 2*a^2*b^2 + b^4)*d^2*e^5))) - 3*((a^4 +
a^2*b^2)*d*e^3*cos(2*d*x + 2*c) + (a^4 + a^2*b^2)*d*e^3)*sqrt(-((a^4 + 2*a^
2*b^2 + b^4)*d^2*e^5*sqrt(-(a^4 - 2*a^2*b^2 + b^4)/((a^8 + 4*a^6*b^2 + 6*a^
4*b^4 + 4*a^2*b^6 + b^8)*d^4*e^10)) - 2*a*b)/((a^4 + 2*a^2*b^2 + b^4)*d^2*e
^5))*log(-(a^2 - b^2)*sqrt((e*cos(2*d*x + 2*c) + e)/sin(2*d*x + 2*c)) + ((a
^4*b + 2*a^2*b^3 + b^5)*d^3*e^8*sqrt(-(a^4 - 2*a^2*b^2 + b^4)/((a^8 + 4*a^6
*b^2 + 6*a^4*b^4 + 4*a^2*b^6 + b^8)*d^4*e^10)) - (a^3 - a*b^2)*d*e^3)*sqrt(
-((a^4 + 2*a^2*b^2 + b^4)*d^2*e^5*sqrt(-(a^4 - 2*a^2*b^2 + b^4)/((a^8 + 4*a
^6*b^2 + 6*a^4*b^4 + 4*a^2*b^6 + b^8)*d^4*e^10)) - 2*a*b)/((a^4 + 2*a^2*b^2
+ b^4)*d^2*e^5))) + 3*((a^4 + a^2*b^2)*d*e^3*cos(2*d*x + 2*c) + (a^4 + a^2
*b^2)*d*e^3)*sqrt(-((a^4 + 2*a^2*b^2 + b^4)*d^2*e^5*sqrt(-(a^4 - 2*a^2*b^2
+ b^4)/((a^8 + 4*a^6*b^2 + 6*a^4*b^4 + 4*a^2*b^6 + b^8)*d^4*e^10)) - 2*a*b)
/((a^4 + 2*a^2*b^2 + b^4)*d^2*e^5))*log(-(a^2 - b^2)*sqrt((e*cos(2*d*x + 2*
c) + e)/sin(2*d*x + 2*c)) - ((a^4*b + 2*a^2*b^3 + b^5)*d^3*e^8*sqrt(-(a^4 -
2*a^2*b^2 + b^4)/((a^8 + 4*a^6*b^2 + 6*a^4*b^4 + 4*a^2*b^6 + b^8)*d^4*e^10
)) - (a^3 - a*b^2)*d*e^3)*sqrt(-((a^4 + 2*a^2*b^2 + b^4)*d^2*e^5*sqrt(-(a^4
- 2*a^2*b^2 + b^4)/((a^8 + 4*a^6*b^2 + 6*a^4*b^4 + 4*a^2*b^6 + b^8)*d^4*e^
10)) - 2*a*b)/((a^4 + 2*a^2*b^2 + b^4)*d^2*e^5))) - 6*(b^3*e*cos(2*d*x + 2*
c) + b^3*e)*sqrt(-b/(a*e))*log(-(2*a*sqrt((e*cos(2*d*x + 2*c) + e)/sin(2*d*
x + 2*c))*sqrt(-b/(a*e))*sin(2*d*x + 2*c) - b*cos(2*d*x + 2*c) + a*sin(2*d*
x + 2*c) - b)/(b*cos(2*d*x + 2*c) + a*sin(2*d*x + 2*c) + b)) - 4*(a^3 + a*b
^2 - (a^3 + a*b^2)*cos(2*d*x + 2*c) - 3*(a^2*b + b^3)*sin(2*d*x + 2*c))*sqr
t((e*cos(2*d*x + 2*c) + e)/sin(2*d*x + 2*c)))/((a^4 + a^2*b^2)*d*e^3*cos(2*
d*x + 2*c) + (a^4 + a^2*b^2)*d*e^3), 1/6*(12*(b^3*e*cos(2*d*x + 2*c) + b^3*
e)*sqrt(b/(a*e))*arctan(a*sqrt((e*cos(2*d*x + 2*c) + e)/sin(2*d*x + 2*c))*s
qrt(b/(a*e))*sin(2*d*x + 2*c)/(b*cos(2*d*x + 2*c) + b)) - 3*((a^4 + a^2*b^2
)*d*e^3*cos(2*d*x + 2*c) + (a^4 + a^2*b^2)*d*e^3)*sqrt(((a^4 + 2*a^2*b^2 +
b^4)*d^2*e^5*sqrt(-(a^4 - 2*a^2*b^2 + b^4)/((a^8 + 4*a^6*b^2 + 6*a^4*b^4 +
4*a^2*b^6 + b^8)*d^4*e^10)) + 2*a*b)/((a^4 + 2*a^2*b^2 + b^4)*d^2*e^5))*log
(-(a^2 - b^2)*sqrt((e*cos(2*d*x + 2*c) + e)/sin(2*d*x + 2*c)) + ((a^4*b + 2
*a^2*b^3 + b^5)*d^3*e^8*sqrt(-(a^4 - 2*a^2*b^2 + b^4)/((a^8 + 4*a^6*b^2 + 6

```
*a^4*b^4 + 4*a^2*b^6 + b^8)*d^4*e^10)) + (a^3 - a*b^2)*d*e^3)*sqrt(((a^4 +
2*a^2*b^2 + b^4)*d^2*e^5*sqrt(-(a^4 - 2*a^2*b^2 + b^4)/((a^8 + 4*a^6*b^2 +
6*a^4*b^4 + 4*a^2*b^6 + b^8)*d^4*e^10)) + 2*a*b)/((a^4 + 2*a^2*b^2 + b^4)*d
^2*e^5))) + 3*((a^4 + a^2*b^2)*d*e^3*cos(2*d*x + 2*c) + (a^4 + a^2*b^2)*d*e
^3)*sqrt(((a^4 + 2*a^2*b^2 + b^4)*d^2*e^5*sqrt(-(a^4 - 2*a^2*b^2 + b^4)/((a
^8 + 4*a^6*b^2 + 6*a^4*b^4 + 4*a^2*b^6 + b^8)*d^4*e^10)) + 2*a*b)/((a^4 + 2
*a^2*b^2 + b^4)*d^2*e^5))*log(-(a^2 - b^2)*sqrt((e*cos(2*d*x + 2*c) + e)/si
n(2*d*x + 2*c)) - ((a^4*b + 2*a^2*b^3 + b^5)*d^3*e^8*sqrt(-(a^4 - 2*a^2*b^2
 + b^4)/((a^8 + 4*a^6*b^2 + 6*a^4*b^4 + 4*a^2*b^6 + b^8)*d^4*e^10)) + (a^3
- a*b^2)*d*e^3)*sqrt(((a^4 + 2*a^2*b^2 + b^4)*d^2*e^5*sqrt(-(a^4 - 2*a^2*b^
2 + b^4)/((a^8 + 4*a^6*b^2 + 6*a^4*b^4 + 4*a^2*b^6 + b^8)*d^4*e^10)) + 2*a*
b)/((a^4 + 2*a^2*b^2 + b^4)*d^2*e^5))) + 3*((a^4 + a^2*b^2)*d*e^3*cos(2*d*x
 + 2*c) + (a^4 + a^2*b^2)*d*e^3)*sqrt(-((a^4 + 2*a^2*b^2 + b^4)*d^2*e^5*sqr
t(-(a^4 - 2*a^2*b^2 + b^4)/((a^8 + 4*a^6*b^2 + 6*a^4*b^4 + 4*a^2*b^6 + b^8)
*d^4*e^10)) - 2*a*b)/((a^4 + 2*a^2*b^2 + b^4)*d^2*e^5))*log(-(a^2 - b^2)*sq
rt((e*cos(2*d*x + 2*c) + e)/sin(2*d*x + 2*c)) + ((a^4*b + 2*a^2*b^3 + b^5)*
d^3*e^8*sqrt(-(a^4 - 2*a^2*b^2 + b^4)/((a^8 + 4*a^6*b^2 + 6*a^4*b^4 + 4*a^2
*b^6 + b^8)*d^4*e^10)) - (a^3 - a*b^2)*d*e^3)*sqrt(-((a^4 + 2*a^2*b^2 + b^4
)*d^2*e^5*sqrt(-(a^4 - 2*a^2*b^2 + b^4)/((a^8 + 4*a^6*b^2 + 6*a^4*b^4 + 4*a
^2*b^6 + b^8)*d^4*e^10)) - 2*a*b)/((a^4 + 2*a^2*b^2 + b^4)*d^2*e^5))) - 3*(
(a^4 + a^2*b^2)*d*e^3*cos(2*d*x + 2*c) + (a^4 + a^2*b^2)*d*e^3)*sqrt(-((a^4
 + 2*a^2*b^2 + b^4)*d^2*e^5*sqrt(-(a^4 - 2*a^2*b^2 + b^4)/((a^8 + 4*a^6*b^2
 + 6*a^4*b^4 + 4*a^2*b^6 + b^8)*d^4*e^10)) - 2*a*b)/((a^4 + 2*a^2*b^2 + b^4
)*d^2*e^5))*log(-(a^2 - b^2)*sqrt((e*cos(2*d*x + 2*c) + e)/sin(2*d*x + 2*c)
) - ((a^4*b + 2*a^2*b^3 + b^5)*d^3*e^8*sqrt(-(a^4 - 2*a^2*b^2 + b^4)/((a^8
+ 4*a^6*b^2 + 6*a^4*b^4 + 4*a^2*b^6 + b^8)*d^4*e^10)) - (a^3 - a*b^2)*d*e^3
)*sqrt(-((a^4 + 2*a^2*b^2 + b^4)*d^2*e^5*sqrt(-(a^4 - 2*a^2*b^2 + b^4)/((a^
8 + 4*a^6*b^2 + 6*a^4*b^4 + 4*a^2*b^6 + b^8)*d^4*e^10)) - 2*a*b)/((a^4 + 2*
a^2*b^2 + b^4)*d^2*e^5))) + 4*(a^3 + a*b^2 - (a^3 + a*b^2)*cos(2*d*x + 2*c)
 - 3*(a^2*b + b^3)*sin(2*d*x + 2*c))*sqrt((e*cos(2*d*x + 2*c) + e)/sin(2*d*
x + 2*c)))/((a^4 + a^2*b^2)*d*e^3*cos(2*d*x + 2*c) + (a^4 + a^2*b^2)*d*e^3)
]
```

Sympy [F]

$$\int \frac{1}{(e\cot(c+dx))^{5/2}(a+b\cot(c+dx))}\,dx = \int \frac{1}{(e\cot(c+dx))^{\frac{5}{2}}\,(a+b\cot(c+dx))}\,dx$$

```
[In] integrate(1/(e*cot(d*x+c))**(5/2)/(a+b*cot(d*x+c)),x)
[Out] Integral(1/((e*cot(c + d*x))**(5/2)*(a + b*cot(c + d*x))), x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{1}{(e\cot(c+dx))^{5/2}(a+b\cot(c+dx))}\,dx = \text{Exception raised: ValueError}$$

```
[In] integrate(1/(e*cot(d*x+c))^(5/2)/(a+b*cot(d*x+c)),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(e>0)', see `assume?` for more detai
ls)Is e
```

Giac [F]

$$\int \frac{1}{(e\cot(c+dx))^{5/2}(a+b\cot(c+dx))}\,dx = \int \frac{1}{(b\cot(dx+c)+a)\,(e\cot(dx+c))^{\frac{5}{2}}}\,dx$$

```
[In] integrate(1/(e*cot(d*x+c))^(5/2)/(a+b*cot(d*x+c)),x, algorithm="giac")
```

```
[Out] integrate(1/((b*cot(d*x + c) + a)*(e*cot(d*x + c))^(5/2)), x)
```

Mupad [B] (verification not implemented)

Time = 14.96 (sec) , antiderivative size = 6042, normalized size of antiderivative = 17.21

$$\int \frac{1}{(e\cot(c+dx))^{5/2}(a+b\cot(c+dx))}\,dx = \text{Too large to display}$$

```
[In] int(1/((e*cot(c + d*x))^(5/2)*(a + b*cot(c + d*x))),x)
```

```
[Out] (2/(3*a*e) - (2*b*cot(c + d*x))/(a^2*e))/(d*(e*cot(c + d*x))^(3/2)) - atan(
((((e*cot(c + d*x))^(1/2)*(64*a^14*b^9*d^5*e^18 + 32*a^18*b^5*d^5*e^18))/2
+ ((1/(b^2*d^2*e^5*1i - a^2*d^2*e^5*1i + 2*a*b*d^2*e^5))^(1/2)*(((((1/(b^2*
d^2*e^5*1i - a^2*d^2*e^5*1i + 2*a*b*d^2*e^5))^(1/2)*(((e*cot(c + d*x))^(1/2
)*(1/(b^2*d^2*e^5*1i - a^2*d^2*e^5*1i + 2*a*b*d^2*e^5))^(1/2)*(512*a^18*b^9
*d^9*e^28 + 512*a^20*b^7*d^9*e^28 - 512*a^22*b^5*d^9*e^28 - 512*a^24*b^3*d^
9*e^28))/4 - 256*a^16*b^10*d^8*e^26 - 256*a^18*b^8*d^8*e^26 + 192*a^20*b^6*
d^8*e^26 + 128*a^22*b^4*d^8*e^26 - 64*a^24*b^2*d^8*e^26))/2 - ((e*cot(c + d
*x))^(1/2)*(512*a^15*b^10*d^7*e^23 + 448*a^19*b^6*d^7*e^23 - 128*a^21*b^4*d
^7*e^23 - 64*a^23*b^2*d^7*e^23))/2)*(1/(b^2*d^2*e^5*1i - a^2*d^2*e^5*1i + 2
*a*b*d^2*e^5))^(1/2))/2 + 192*a^15*b^9*d^6*e^21 - 16*a^19*b^5*d^6*e^21 - 16
*a^21*b^3*d^6*e^21))/2)*(1/(b^2*d^2*e^5*1i - a^2*d^2*e^5*1i + 2*a*b*d^2*e^5
```

))^(1/2)*1i + (((e*cot(c + d*x))^(1/2)*(64*a^14*b^9*d^5*e^18 + 32*a^18*b^5*d^5*e^18))/2 + ((1/(b^2*d^2*e^5*1i - a^2*d^2*e^5*1i + 2*a*b*d^2*e^5))^(1/2)*(((((1/(b^2*d^2*e^5*1i - a^2*d^2*e^5*1i + 2*a*b*d^2*e^5))^(1/2)*(((e*cot(c + d*x))^(1/2)*(1/(b^2*d^2*e^5*1i - a^2*d^2*e^5*1i + 2*a*b*d^2*e^5))^(1/2)*(512*a^18*b^9*d^9*e^28 + 512*a^20*b^7*d^9*e^28 - 512*a^22*b^5*d^9*e^28 - 512*a^24*b^3*d^9*e^28))/4 + 256*a^16*b^10*d^8*e^26 + 256*a^18*b^8*d^8*e^26 - 192*a^20*b^6*d^8*e^26 - 128*a^22*b^4*d^8*e^26 + 64*a^24*b^2*d^8*e^26))/2 - ((e*cot(c + d*x))^(1/2)*(512*a^15*b^10*d^7*e^23 + 448*a^19*b^6*d^7*e^23 - 128*a^21*b^4*d^7*e^23 - 64*a^23*b^2*d^7*e^23))/2)*(1/(b^2*d^2*e^5*1i - a^2*d^2*e^5*1i + 2*a*b*d^2*e^5))^(1/2))/2 - 192*a^15*b^9*d^6*e^21 + 16*a^19*b^5*d^6*e^21 + 16*a^21*b^3*d^6*e^21))/2)*(1/(b^2*d^2*e^5*1i - a^2*d^2*e^5*1i + 2*a*b*d^2*e^5))^(1/2)*1i)/((((e*cot(c + d*x))^(1/2)*(64*a^14*b^9*d^5*e^18 + 32*a^18*b^5*d^5*e^18))/2 + ((1/(b^2*d^2*e^5*1i - a^2*d^2*e^5*1i + 2*a*b*d^2*e^5))^(1/2)*(((((1/(b^2*d^2*e^5*1i - a^2*d^2*e^5*1i + 2*a*b*d^2*e^5))^(1/2)*(((e*cot(c + d*x))^(1/2)*(1/(b^2*d^2*e^5*1i - a^2*d^2*e^5*1i + 2*a*b*d^2*e^5))^(1/2)*(512*a^18*b^9*d^9*e^28 + 512*a^20*b^7*d^9*e^28 - 512*a^22*b^5*d^9*e^28 - 512*a^24*b^3*d^9*e^28))/4 + 256*a^16*b^10*d^8*e^26 + 256*a^18*b^8*d^8*e^26 - 192*a^20*b^6*d^8*e^26 - 128*a^22*b^4*d^8*e^26 + 64*a^24*b^2*d^8*e^26))/2 - ((e*cot(c + d*x))^(1/2)*(512*a^15*b^10*d^7*e^23 + 448*a^19*b^6*d^7*e^23 - 128*a^21*b^4*d^7*e^23 - 64*a^23*b^2*d^7*e^23))/2)*(1/(b^2*d^2*e^5*1i - a^2*d^2*e^5*1i + 2*a*b*d^2*e^5))^(1/2))/2 - 192*a^15*b^9*d^6*e^21 + 16*a^19*b^5*d^6*e^21 + 16*a^21*b^3*d^6*e^21))/2)*(1/(b^2*d^2*e^5*1i - a^2*d^2*e^5*1i + 2*a*b*d^2*e^5))^(1/2) - (((e*cot(c + d*x))^(1/2)*(64*a^14*b^9*d^5*e^18 + 32*a^18*b^5*d^5*e^18))/2 + ((1/(b^2*d^2*e^5*1i - a^2*d^2*e^5*1i + 2*a*b*d^2*e^5))^(1/2)*(((((1/(b^2*d^2*e^5*1i - a^2*d^2*e^5*1i + 2*a*b*d^2*e^5))^(1/2)*(((e*cot(c + d*x))^(1/2)*(1/(b^2*d^2*e^5*1i - a^2*d^2*e^5*1i + 2*a*b*d^2*e^5))^(1/2)*(512*a^18*b^9*d^9*e^28 + 512*a^20*b^7*d^9*e^28 - 512*a^22*b^5*d^9*e^28 - 512*a^24*b^3*d^9*e^28))/4 - 256*a^16*b^10*d^8*e^26 - 256*a^18*b^8*d^8*e^26 + 192*a^20*b^6*d^8*e^26 + 128*a^22*b^4*d^8*e^26 - 64*a^24*b^2*d^8*e^26))/2 - ((e*cot(c + d*x))^(1/2)*(512*a^15*b^10*d^7*e^23 + 448*a^19*b^6*d^7*e^23 - 128*a^21*b^4*d^7*e^23 - 64*a^23*b^2*d^7*e^23))/2)*(1/(b^2*d^2*e^5*1i - a^2*d^2*e^5*1i + 2*a*b*d^2*e^5))^(1/2))/2 + 192*a^15*b^9*d^6*e^21 - 16*a^19*b^5*d^6*e^21 - 16*a^21*b^3*d^6*e^21))/2)*(1/(b^2*d^2*e^5*1i - a^2*d^2*e^5*1i + 2*a*b*d^2*e^5))^(1/2) + 64*a^14*b^8*d^4*e^16))*(1/(b^2*d^2*e^5*1i - a^2*d^2*e^5*1i + 2*a*b*d^2*e^5))^(1/2)*1i - atan(((1i/(4*(b^2*d^2*e^5 - a^2*d^2*e^5 + a*b*d^2*e^5*2i)))^(1/2)*((e*cot(c + d*x))^(1/2)*(64*a^14*b^9*d^5*e^18 + 32*a^18*b^5*d^5*e^18) + (1i/(4*(b^2*d^2*e^5 - a^2*d^2*e^5 + a*b*d^2*e^5*2i)))^(1/2)*((1i/(4*(b^2*d^2*e^5 - a^2*d^2*e^5 + a*b*d^2*e^5*2i)))^(1/2)*((1i/(4*(b^2*d^2*e^5 - a^2*d^2*e^5 + a*b*d^2*e^5*2i)))^(1/2)*((1i/(4*(b^2*d^2*e^5 - a^2*d^2*e^5 + a*b*d^2*e^5*2i)))^(1/2)*(e*cot(c + d*x))^(1/2)*(512*a^18*b^9*d^9*e^28 + 512*a^20*b^7*d^9*e^28 - 512*a^22*b^5*d^9*e^28 - 512*a^24*b^3*d^9*e^28) - 512*a^16*b^10*d^8*e^26 - 512*a^18*b^8*d^8*e^26 + 384*a^20*b^6*d^8*e^26 + 256*a^22*b^4*d^8*e^26 - 128*a^24*b^2*d^8*e^26) - (e*cot(c + d*x))^(1/2)*(512*a^15*b^10*d^7*e^23 + 448*a^19*b^6*d^7*e^23 - 128*a^21*b^4*d^7*e^23 - 64*a^23*b^2*d^7*e^23)) + 384*a^15*b^9*d^6*e^

21 - 32*a^19*b^5*d^6*e^21 - 32*a^21*b^3*d^6*e^21))*1i + (1i/(4*(b^2*d^2*e^5
 - a^2*d^2*e^5 + a*b*d^2*e^5*2i)))^(1/2)*((e*cot(c + d*x))^(1/2)*(64*a^14*b
^9*d^5*e^18 + 32*a^18*b^5*d^5*e^18) + (1i/(4*(b^2*d^2*e^5 - a^2*d^2*e^5 + a
*b*d^2*e^5*2i)))^(1/2)*((1i/(4*(b^2*d^2*e^5 - a^2*d^2*e^5 + a*b*d^2*e^5*2i)
))^(1/2)*((1i/(4*(b^2*d^2*e^5 - a^2*d^2*e^5 + a*b*d^2*e^5*2i)))^(1/2)*((1i/
(4*(b^2*d^2*e^5 - a^2*d^2*e^5 + a*b*d^2*e^5*2i)))^(1/2)*(e*cot(c + d*x))^(1
/2)*(512*a^18*b^9*d^9*e^28 + 512*a^20*b^7*d^9*e^28 - 512*a^22*b^5*d^9*e^28
 - 512*a^24*b^3*d^9*e^28) + 512*a^16*b^10*d^8*e^26 + 512*a^18*b^8*d^8*e^26 -
 384*a^20*b^6*d^8*e^26 - 256*a^22*b^4*d^8*e^26 + 128*a^24*b^2*d^8*e^26) - (
e*cot(c + d*x))^(1/2)*(512*a^15*b^10*d^7*e^23 + 448*a^19*b^6*d^7*e^23 - 128
*a^21*b^4*d^7*e^23 - 64*a^23*b^2*d^7*e^23)) - 384*a^15*b^9*d^6*e^21 + 32*a^
19*b^5*d^6*e^21 + 32*a^21*b^3*d^6*e^21))*1i)/((1i/(4*(b^2*d^2*e^5 - a^2*d^2
*e^5 + a*b*d^2*e^5*2i)))^(1/2)*((e*cot(c + d*x))^(1/2)*(64*a^14*b^9*d^5*e^1
8 + 32*a^18*b^5*d^5*e^18) + (1i/(4*(b^2*d^2*e^5 - a^2*d^2*e^5 + a*b*d^2*e^5
2i)))^(1/2)((1i/(4*(b^2*d^2*e^5 - a^2*d^2*e^5 + a*b*d^2*e^5*2i)))^(1/2)*(
(1i/(4*(b^2*d^2*e^5 - a^2*d^2*e^5 + a*b*d^2*e^5*2i)))^(1/2)*((1i/(4*(b^2*d^
2*e^5 - a^2*d^2*e^5 + a*b*d^2*e^5*2i)))^(1/2)*(e*cot(c + d*x))^(1/2)*(512*a
^18*b^9*d^9*e^28 + 512*a^20*b^7*d^9*e^28 - 512*a^22*b^5*d^9*e^28 - 512*a^24
*b^3*d^9*e^28) + 512*a^16*b^10*d^8*e^26 + 512*a^18*b^8*d^8*e^26 - 384*a^20*
b^6*d^8*e^26 - 256*a^22*b^4*d^8*e^26 + 128*a^24*b^2*d^8*e^26) - (e*cot(c +
d*x))^(1/2)*(512*a^15*b^10*d^7*e^23 + 448*a^19*b^6*d^7*e^23 - 128*a^21*b^4*
d^7*e^23 - 64*a^23*b^2*d^7*e^23)) - 384*a^15*b^9*d^6*e^21 + 32*a^19*b^5*d^6
*e^21 + 32*a^21*b^3*d^6*e^21)) - (1i/(4*(b^2*d^2*e^5 - a^2*d^2*e^5 + a*b*d^
2*e^5*2i)))^(1/2)*((e*cot(c + d*x))^(1/2)*(64*a^14*b^9*d^5*e^18 + 32*a^18*b
^5*d^5*e^18) + (1i/(4*(b^2*d^2*e^5 - a^2*d^2*e^5 + a*b*d^2*e^5*2i)))^(1/2)*
((1i/(4*(b^2*d^2*e^5 - a^2*d^2*e^5 + a*b*d^2*e^5*2i)))^(1/2)*((1i/(4*(b^2*d
^2*e^5 - a^2*d^2*e^5 + a*b*d^2*e^5*2i)))^(1/2)*((1i/(4*(b^2*d^2*e^5 - a^2*d
^2*e^5 + a*b*d^2*e^5*2i)))^(1/2)*(e*cot(c + d*x))^(1/2)*(512*a^18*b^9*d^9*e
^28 + 512*a^20*b^7*d^9*e^28 - 512*a^22*b^5*d^9*e^28 - 512*a^24*b^3*d^9*e^28
) - 512*a^16*b^10*d^8*e^26 - 512*a^18*b^8*d^8*e^26 + 384*a^20*b^6*d^8*e^26
 + 256*a^22*b^4*d^8*e^26 - 128*a^24*b^2*d^8*e^26) - (e*cot(c + d*x))^(1/2)*(
512*a^15*b^10*d^7*e^23 + 448*a^19*b^6*d^7*e^23 - 128*a^21*b^4*d^7*e^23 - 64
*a^23*b^2*d^7*e^23)) + 384*a^15*b^9*d^6*e^21 - 32*a^19*b^5*d^6*e^21 - 32*a^
21*b^3*d^6*e^21)) + 64*a^14*b^8*d^4*e^16))*(1i/(4*(b^2*d^2*e^5 - a^2*d^2*e^
5 + a*b*d^2*e^5*2i)))^(1/2)*2i - (atan(((((e*cot(c + d*x))^(1/2)*(64*a^14*b
^9*d^5*e^18 + 32*a^18*b^5*d^5*e^18) - ((-a^5*b^7*e^5)^(1/2)*(32*a^19*b^5*d^
6*e^21 - 384*a^15*b^9*d^6*e^21 + 32*a^21*b^3*d^6*e^21 + (((e*cot(c + d*x))^
(1/2)*(512*a^15*b^10*d^7*e^23 + 448*a^19*b^6*d^7*e^23 - 128*a^21*b^4*d^7*e^
23 - 64*a^23*b^2*d^7*e^23) + ((-a^5*b^7*e^5)^(1/2)*(512*a^16*b^10*d^8*e^26
 + 512*a^18*b^8*d^8*e^26 - 384*a^20*b^6*d^8*e^26 - 256*a^22*b^4*d^8*e^26 + 1
28*a^24*b^2*d^8*e^26 - ((e*cot(c + d*x))^(1/2)*(-a^5*b^7*e^5)^(1/2)*(512*a^
18*b^9*d^9*e^28 + 512*a^20*b^7*d^9*e^28 - 512*a^22*b^5*d^9*e^28 - 512*a^24*
b^3*d^9*e^28))/(a^5*d*e^5*(a^2 + b^2))))/(a^5*d*e^5*(a^2 + b^2)))*(-a^5*b^7
*e^5)^(1/2))/(a^5*d*e^5*(a^2 + b^2))))/(a^5*d*e^5*(a^2 + b^2)))*(-a^5*b^7*e
^5)^(1/2)*1i)/(a^5*d*e^5*(a^2 + b^2)) + (((e*cot(c + d*x))^(1/2)*(64*a^14*b

```
^9*d^5*e^18 + 32*a^18*b^5*d^5*e^18) - ((-a^5*b^7*e^5)^(1/2)*(384*a^15*b^9*d
^6*e^21 - 32*a^19*b^5*d^6*e^21 - 32*a^21*b^3*d^6*e^21 + (((e*cot(c + d*x))^
(1/2)*(512*a^15*b^10*d^7*e^23 + 448*a^19*b^6*d^7*e^23 - 128*a^21*b^4*d^7*e^
23 - 64*a^23*b^2*d^7*e^23) - ((-a^5*b^7*e^5)^(1/2)*(512*a^16*b^10*d^8*e^26
+ 512*a^18*b^8*d^8*e^26 - 384*a^20*b^6*d^8*e^26 - 256*a^22*b^4*d^8*e^26 + 1
28*a^24*b^2*d^8*e^26 + ((e*cot(c + d*x))^(1/2)*(-a^5*b^7*e^5)^(1/2)*(512*a^
18*b^9*d^9*e^28 + 512*a^20*b^7*d^9*e^28 - 512*a^22*b^5*d^9*e^28 - 512*a^24*
b^3*d^9*e^28))/(a^5*d*e^5*(a^2 + b^2))))/(a^5*d*e^5*(a^2 + b^2)))*(-a^5*b^7
*e^5)^(1/2))/(a^5*d*e^5*(a^2 + b^2))))/(a^5*d*e^5*(a^2 + b^2)))*(-a^5*b^7*e
^5)^(1/2)*1i)/(a^5*d*e^5*(a^2 + b^2)))/(64*a^14*b^8*d^4*e^16 - (((e*cot(c +
 d*x))^(1/2)*(64*a^14*b^9*d^5*e^18 + 32*a^18*b^5*d^5*e^18) - ((-a^5*b^7*e^5
)^(1/2)*(32*a^19*b^5*d^6*e^21 - 384*a^15*b^9*d^6*e^21 + 32*a^21*b^3*d^6*e^2
1 + (((e*cot(c + d*x))^(1/2)*(512*a^15*b^10*d^7*e^23 + 448*a^19*b^6*d^7*e^2
3 - 128*a^21*b^4*d^7*e^23 - 64*a^23*b^2*d^7*e^23) + ((-a^5*b^7*e^5)^(1/2)*(
512*a^16*b^10*d^8*e^26 + 512*a^18*b^8*d^8*e^26 - 384*a^20*b^6*d^8*e^26 - 25
6*a^22*b^4*d^8*e^26 + 128*a^24*b^2*d^8*e^26 - ((e*cot(c + d*x))^(1/2)*(-a^5
*b^7*e^5)^(1/2)*(512*a^18*b^9*d^9*e^28 + 512*a^20*b^7*d^9*e^28 - 512*a^22*b
^5*d^9*e^28 - 512*a^24*b^3*d^9*e^28))/(a^5*d*e^5*(a^2 + b^2))))/(a^5*d*e^5*
(a^2 + b^2)))*(-a^5*b^7*e^5)^(1/2))/(a^5*d*e^5*(a^2 + b^2))))/(a^5*d*e^5*(a
^2 + b^2)))*(-a^5*b^7*e^5)^(1/2))/(a^5*d*e^5*(a^2 + b^2)) + (((e*cot(c + d*
x))^(1/2)*(64*a^14*b^9*d^5*e^18 + 32*a^18*b^5*d^5*e^18) - ((-a^5*b^7*e^5)^(
1/2)*(384*a^15*b^9*d^6*e^21 - 32*a^19*b^5*d^6*e^21 - 32*a^21*b^3*d^6*e^21 +
 (((e*cot(c + d*x))^(1/2)*(512*a^15*b^10*d^7*e^23 + 448*a^19*b^6*d^7*e^23 -
 128*a^21*b^4*d^7*e^23 - 64*a^23*b^2*d^7*e^23) - ((-a^5*b^7*e^5)^(1/2)*(512
*a^16*b^10*d^8*e^26 + 512*a^18*b^8*d^8*e^26 - 384*a^20*b^6*d^8*e^26 - 256*a
^22*b^4*d^8*e^26 + 128*a^24*b^2*d^8*e^26 + ((e*cot(c + d*x))^(1/2)*(-a^5*b^
7*e^5)^(1/2)*(512*a^18*b^9*d^9*e^28 + 512*a^20*b^7*d^9*e^28 - 512*a^22*b^5*
d^9*e^28 - 512*a^24*b^3*d^9*e^28))/(a^5*d*e^5*(a^2 + b^2))))/(a^5*d*e^5*(a^
2 + b^2)))*(-a^5*b^7*e^5)^(1/2))/(a^5*d*e^5*(a^2 + b^2))))/(a^5*d*e^5*(a^2
+ b^2)))*(-a^5*b^7*e^5)^(1/2))/(a^5*d*e^5*(a^2 + b^2))))*(-a^5*b^7*e^5)^(1/
2)*2i)/(a^5*d*e^5*(a^2 + b^2))
```

3.75 $\int \frac{(e\cot(c+dx))^{7/2}}{(a+b\cot(c+dx))^2}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 437

$$\begin{aligned}\int \frac{(e\cot(c+dx))^{7/2}}{(a+b\cot(c+dx))^2}\,dx &= \frac{a^{5/2}(3a^2+7b^2)\,e^{7/2}\arctan\left(\frac{\sqrt{b}\sqrt{e\cot(c+dx)}}{\sqrt{a}\sqrt{e}}\right)}{b^{5/2}\,(a^2+b^2)^2\,d} \\ &+ \frac{(a^2-2ab-b^2)\,e^{7/2}\arctan\left(1-\frac{\sqrt{2}\sqrt{e\cot(c+dx)}}{\sqrt{e}}\right)}{\sqrt{2}\,(a^2+b^2)^2\,d} \\ &- \frac{(a^2-2ab-b^2)\,e^{7/2}\arctan\left(1+\frac{\sqrt{2}\sqrt{e\cot(c+dx)}}{\sqrt{e}}\right)}{\sqrt{2}\,(a^2+b^2)^2\,d} \\ &- \frac{(3a^2+2b^2)\,e^3\sqrt{e\cot(c+dx)}}{b^2\,(a^2+b^2)\,d} + \frac{a^2e^2(e\cot(c+dx))^{3/2}}{b\,(a^2+b^2)\,d(a+b\cot(c+dx))} \\ &+ \frac{(a^2+2ab-b^2)\,e^{7/2}\log\left(\sqrt{e}+\sqrt{e}\cot(c+dx)-\sqrt{2}\sqrt{e\cot(c+dx)}\right)}{2\sqrt{2}\,(a^2+b^2)^2\,d} \\ &- \frac{(a^2+2ab-b^2)\,e^{7/2}\log\left(\sqrt{e}+\sqrt{e}\cot(c+dx)+\sqrt{2}\sqrt{e\cot(c+dx)}\right)}{2\sqrt{2}\,(a^2+b^2)^2\,d}\end{aligned}$$

```
[Out] a^(5/2)*(3*a^2+7*b^2)*e^(7/2)*arctan(b^(1/2)*(e*cot(d*x+c))^(1/2)/a^(1/2)/e
^(1/2))/b^(5/2)/(a^2+b^2)^2/d+a^2*e^2*(e*cot(d*x+c))^(3/2)/b/(a^2+b^2)/d/(a
+b*cot(d*x+c))+1/2*(a^2-2*a*b-b^2)*e^(7/2)*arctan(1-2^(1/2)*(e*cot(d*x+c))^
(1/2)/e^(1/2))/(a^2+b^2)^2/d*2^(1/2)-1/2*(a^2-2*a*b-b^2)*e^(7/2)*arctan(1+2
^(1/2)*(e*cot(d*x+c))^(1/2)/e^(1/2))/(a^2+b^2)^2/d*2^(1/2)+1/4*(a^2+2*a*b-b
^2)*e^(7/2)*ln(e^(1/2)+cot(d*x+c)*e^(1/2)-2^(1/2)*(e*cot(d*x+c))^(1/2))/(a^
2+b^2)^2/d*2^(1/2)-1/4*(a^2+2*a*b-b^2)*e^(7/2)*ln(e^(1/2)+cot(d*x+c)*e^(1/2
)+2^(1/2)*(e*cot(d*x+c))^(1/2))/(a^2+b^2)^2/d*2^(1/2)-(3*a^2+2*b^2)*e^3*(e*
cot(d*x+c))^(1/2)/b^2/(a^2+b^2)/d
```

Rubi [A] (verified)

Time = 1.22 (sec) , antiderivative size = 437, normalized size of antiderivative = 1.00, number of steps used = 16, number of rules used = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.520$, Rules used = {3646, 3728, 3734, 3615, 1182, 1176, 631, 210, 1179, 642, 3715, 65, 211}

$$\int \frac{(e\cot(c+dx))^{7/2}}{(a+b\cot(c+dx))^2}\,dx = \frac{e^{7/2}(a^2-2ab-b^2)\arctan\left(1-\frac{\sqrt{2}\sqrt{e\cot(c+dx)}}{\sqrt{e}}\right)}{\sqrt{2}d\,(a^2+b^2)^2} - \frac{e^{7/2}(a^2-2ab-b^2)\arctan\left(\frac{\sqrt{2}\sqrt{e\cot(c+dx)}}{\sqrt{e}}+1\right)}{\sqrt{2}d\,(a^2+b^2)^2} + \frac{e^{7/2}(a^2+2ab-b^2)\log\left(\sqrt{e}\cot(c+dx)-\sqrt{2}\sqrt{e\cot(c+dx)}+\sqrt{e}\right)}{2\sqrt{2}d\,(a^2+b^2)^2} - \frac{e^{7/2}(a^2+2ab-b^2)\log\left(\sqrt{e}\cot(c+dx)+\sqrt{2}\sqrt{e\cot(c+dx)}+\sqrt{e}\right)}{2\sqrt{2}d\,(a^2+b^2)^2} - \frac{e^3(3a^2+2b^2)\sqrt{e\cot(c+dx)}}{b^2d\,(a^2+b^2)} + \frac{a^2e^2(e\cot(c+dx))^{3/2}}{bd\,(a^2+b^2)\,(a+b\cot(c+dx))} + \frac{a^{5/2}e^{7/2}(3a^2+7b^2)\arctan\left(\frac{\sqrt{b}\sqrt{e\cot(c+dx)}}{\sqrt{a}\sqrt{e}}\right)}{b^{5/2}d\,(a^2+b^2)^2}$$

```
[In] Int[(e*Cot[c + d*x])^(7/2)/(a + b*Cot[c + d*x])^2,x]
```

```
[Out] (a^(5/2)*(3*a^2 + 7*b^2)*e^(7/2)*ArcTan[(Sqrt[b]*Sqrt[e*Cot[c + d*x]])/(Sqr
t[a]*Sqrt[e])])/(b^(5/2)*(a^2 + b^2)^2*d) + ((a^2 - 2*a*b - b^2)*e^(7/2)*Ar
cTan[1 - (Sqrt[2]*Sqrt[e*Cot[c + d*x]])/Sqrt[e]])/(Sqrt[2]*(a^2 + b^2)^2*d)
 - ((a^2 - 2*a*b - b^2)*e^(7/2)*ArcTan[1 + (Sqrt[2]*Sqrt[e*Cot[c + d*x]])/S
qrt[e]])/(Sqrt[2]*(a^2 + b^2)^2*d) - ((3*a^2 + 2*b^2)*e^3*Sqrt[e*Cot[c + d*
x]])/(b^2*(a^2 + b^2)*d) + (a^2*e^2*(e*Cot[c + d*x])^(3/2))/(b*(a^2 + b^2)*
d*(a + b*Cot[c + d*x])) + ((a^2 + 2*a*b - b^2)*e^(7/2)*Log[Sqrt[e] + Sqrt[e
]*Cot[c + d*x] - Sqrt[2]*Sqrt[e*Cot[c + d*x]]])/(2*Sqrt[2]*(a^2 + b^2)^2*d)
 - ((a^2 + 2*a*b - b^2)*e^(7/2)*Log[Sqrt[e] + Sqrt[e]*Cot[c + d*x] + Sqrt[2
]*Sqrt[e*Cot[c + d*x]]])/(2*Sqrt[2]*(a^2 + b^2)^2*d)
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1179

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 1182

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
a*c, 2]}, Dist[(d*q + a*e)/(2*a*c), Int[(q + c*x^2)/(a + c*x^4), x], x] + D
ist[(d*q - a*e)/(2*a*c), Int[(q - c*x^2)/(a + c*x^4), x], x]] /; FreeQ[{a,
c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0] && NeQ[c*d^2 - a*e^2, 0] && NegQ[(-a)
*c]
```

Rule 3615

```
Int[((c_) + (d_.)*tan[(e_.) + (f_.)*(x_)])/Sqrt[(b_.)*tan[(e_.) + (f_.)*(x_
)]], x_Symbol] :> Dist[2/f, Subst[Int[(b*c + d*x^2)/(b^2 + x^4), x], x, Sqr
t[b*Tan[e + f*x]]], x] /; FreeQ[{b, c, d, e, f}, x] && NeQ[c^2 - d^2, 0] &&
 NeQ[c^2 + d^2, 0]
```

Rule 3646

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*c - a*d)^2*(a + b*Tan[e + f*x])^(m
 - 2)*((c + d*Tan[e + f*x])^(n + 1)/(d*f*(n + 1)*(c^2 + d^2))), x] - Dist[1
/(d*(n + 1)*(c^2 + d^2)), Int[(a + b*Tan[e + f*x])^(m - 3)*(c + d*Tan[e + f
*x])^(n + 1)*Simp[a^2*d*(b*d*(m - 2) - a*c*(n + 1)) + b*(b*c - 2*a*d)*(b*c*
(m - 2) + a*d*(n + 1)) - d*(n + 1)*(3*a^2*b*c - b^3*c - a^3*d + 3*a*b^2*d)*
Tan[e + f*x] - b*(a*d*(2*b*c - a*d)*(m + n - 1) - b^2*(c^2*(m - 2) - d^2*(n
 + 1)))*Tan[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b
*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] && GtQ[m, 2] && LtQ[
n, -1] && IntegerQ[2*m]
```

Rule 3715

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*tan[(e_.)
+ (f_.)*(x_)])^(n_.)*((A_) + (C_.)*tan[(e_.) + (f_.)*(x_)]^2), x_Symbol] :>
 Dist[A/f, Subst[Int[(a + b*x)^m*(c + d*x)^n, x], x, Tan[e + f*x]], x] /; F
reeQ[{a, b, c, d, e, f, A, C, m, n}, x] && EqQ[A, C]
```

Rule 3728

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*tan[(e_.)
+ (f_.)*(x_)])^(n_)*((A_.) + (B_.)*tan[(e_.) + (f_.)*(x_)] + (C_.)*tan[(e_.
) + (f_.)*(x_)]^2), x_Symbol] :> Simp[C*(a + b*Tan[e + f*x])^m*((c + d*Tan[
e + f*x])^(n + 1)/(d*f*(m + n + 1))), x] + Dist[1/(d*(m + n + 1)), Int[(a +
 b*Tan[e + f*x])^(m - 1)*(c + d*Tan[e + f*x])^n*Simp[a*A*d*(m + n + 1) - C*
(b*c*m + a*d*(n + 1)) + d*(A*b + a*B - b*C)*(m + n + 1)*Tan[e + f*x] - (C*m
*(b*c - a*d) - b*B*d*(m + n + 1))*Tan[e + f*x]^2, x], x], x] /; FreeQ[{a, b
, c, d, e, f, A, B, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] &&
NeQ[c^2 + d^2, 0] && GtQ[m, 0] &&  !(IGtQ[n, 0] && ( !IntegerQ[m] || (EqQ[c
, 0] && NeQ[a, 0])))
```

Rule 3734

```
Int[(((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_)*((A_.) + (B_.)*tan[(e_.)
+ (f_.)*(x_)] + (C_.)*tan[(e_.) + (f_.)*(x_)]^2))/((a_.) + (b_.)*tan[(e_.)
+ (f_.)*(x_)]), x_Symbol] :> Dist[1/(a^2 + b^2), Int[(c + d*Tan[e + f*x])^n
*Simp[b*B + a*(A - C) + (a*B - b*(A - C))*Tan[e + f*x], x], x], x] + Dist[(
A*b^2 - a*b*B + a^2*C)/(a^2 + b^2), Int[(c + d*Tan[e + f*x])^n*((1 + Tan[e
+ f*x]^2)/(a + b*Tan[e + f*x])), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C
```

```
, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] &&
 !GtQ[n, 0] &&  !LeQ[n, -1]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{a^2e^2(e\cot(c+dx))^{3/2}}{b(a^2+b^2)d(a+b\cot(c+dx))} - \frac{\int \frac{\sqrt{e\cot(c+dx)}\left(-\frac{3}{2}a^2e^3+abe^3\cot(c+dx)-\frac{1}{2}(3a^2+2b^2)e^3\cot^2(c+dx)\right)}{a+b\cot(c+dx)}\,dx}{b(a^2+b^2)} \\
&= -\frac{(3a^2+2b^2)e^3\sqrt{e\cot(c+dx)}}{b^2(a^2+b^2)d} + \frac{a^2e^2(e\cot(c+dx))^{3/2}}{b(a^2+b^2)d(a+b\cot(c+dx))} + \frac{2\int \frac{-\frac{1}{4}a(3a^2+2b^2)e^4-\frac{1}{2}b^3e^4\cot(c+dx)-\frac{1}{4}a(3a^2+4b^2)e^4\cot^2(c+dx)}{\sqrt{e\cot(c+dx)}(a+b\cot(c+dx))}\,dx}{b^2(a^2+b^2)} \\
&= -\frac{(3a^2+2b^2)e^3\sqrt{e\cot(c+dx)}}{b^2(a^2+b^2)d} + \frac{a^2e^2(e\cot(c+dx))^{3/2}}{b(a^2+b^2)d(a+b\cot(c+dx))} + \frac{2\int \frac{\frac{1}{2}b^2(a^2-b^2)e^4-ab^3e^4\cot(c+dx)}{\sqrt{e\cot(c+dx)}}\,dx}{b^2(a^2+b^2)^2} - \frac{(a^3(3a^2+7b^2)e^4)\int \frac{1+\cot^2(c+dx)}{\sqrt{e\cot(c+dx)}(a+b\cot(c+dx))}\,dx}{2b^2(a^2+b^2)^2} \\
&= -\frac{(3a^2+2b^2)e^3\sqrt{e\cot(c+dx)}}{b^2(a^2+b^2)d} + \frac{a^2e^2(e\cot(c+dx))^{3/2}}{b(a^2+b^2)d(a+b\cot(c+dx))} + \frac{4\text{Subst}\left(\int \frac{-\frac{1}{2}b^2(a^2-b^2)e^5+ab^3e^4x^2}{e^2+x^4}\,dx, x, \sqrt{e\cot(c+dx)}\right)}{b^2(a^2+b^2)^2d} \\
&\quad - \frac{(a^3(3a^2+7b^2)e^4)\text{Subst}\left(\int \frac{1}{\sqrt{-ex}(a-bx)}\,dx, x, -\cot(c+dx)\right)}{2b^2(a^2+b^2)^2d} \\
&= -\frac{(3a^2+2b^2)e^3\sqrt{e\cot(c+dx)}}{b^2(a^2+b^2)d} + \frac{a^2e^2(e\cot(c+dx))^{3/2}}{b(a^2+b^2)d(a+b\cot(c+dx))} + \frac{(a^3(3a^2+7b^2)e^3)\text{Subst}\left(\int \frac{1}{a+\frac{bx^2}{e}}\,dx, x, \sqrt{e\cot(c+dx)}\right)}{b^2(a^2+b^2)^2d} \\
&\quad - \frac{((a^2-2ab-b^2)e^4)\text{Subst}\left(\int \frac{e+x^2}{e^2+x^4}\,dx, x, \sqrt{e\cot(c+dx)}\right)}{(a^2+b^2)^2d} - \frac{((a^2+2ab-b^2)e^4)\text{Subst}\left(\int \frac{e-x^2}{e^2+x^4}\,dx, x, \sqrt{e\cot(c+dx)}\right)}{(a^2+b^2)^2d}
\end{aligned}$$

$$=\frac{a^{5/2}(3a^2+7b^2)\,e^{7/2}\arctan\left(\frac{\sqrt{b}\sqrt{e\cot(c+dx)}}{\sqrt{a}\sqrt{e}}\right)}{b^{5/2}\left(a^2+b^2\right)^2 d} - \frac{(3a^2+2b^2)\,e^3\sqrt{e\cot(c+dx)}}{b^2\left(a^2+b^2\right) d} + \frac{a^2e^2(e\cot(c+dx))^{3/2}}{b\left(a^2+b^2\right) d(a+b\cot(c+dx))} + \frac{\left((a^2+2ab-b^2)\,e^{7/2}\right)\text{Subst}\left(\int \frac{\sqrt{2}\sqrt{e}+2x}{-e-\sqrt{2}\sqrt{e}x-x^2}\,dx, x, \sqrt{e\cot(c+dx)}\right)}{2\sqrt{2}\left(a^2+b^2\right)^2 d} + \frac{\left((a^2+2ab-b^2)\,e^{7/2}\right)\text{Subst}\left(\int \frac{\sqrt{2}\sqrt{e}-2x}{-e+\sqrt{2}\sqrt{e}x-x^2}\,dx, x, \sqrt{e\cot(c+dx)}\right)}{2\sqrt{2}\left(a^2+b^2\right)^2 d} - \frac{\left((a^2-2ab-b^2)\,e^4\right)\text{Subst}\left(\int \frac{1}{e-\sqrt{2}\sqrt{e}x+x^2}\,dx, x, \sqrt{e\cot(c+dx)}\right)}{2\left(a^2+b^2\right)^2 d} - \frac{\left((a^2-2ab-b^2)\,e^4\right)\text{Subst}\left(\int \frac{1}{e+\sqrt{2}\sqrt{e}x+x^2}\,dx, x, \sqrt{e\cot(c+dx)}\right)}{2\left(a^2+b^2\right)^2 d}$$

$$=\frac{a^{5/2}(3a^2+7b^2)\,e^{7/2}\arctan\left(\frac{\sqrt{b}\sqrt{e\cot(c+dx)}}{\sqrt{a}\sqrt{e}}\right)}{b^{5/2}\left(a^2+b^2\right)^2 d} - \frac{(3a^2+2b^2)\,e^3\sqrt{e\cot(c+dx)}}{b^2\left(a^2+b^2\right) d} + \frac{a^2e^2(e\cot(c+dx))^{3/2}}{b\left(a^2+b^2\right) d(a+b\cot(c+dx))} + \frac{(a^2+2ab-b^2)\,e^{7/2}\log\left(\sqrt{e}+\sqrt{e}\cot(c+dx)-\sqrt{2}\sqrt{e\cot(c+dx)}\right)}{2\sqrt{2}\left(a^2+b^2\right)^2 d} - \frac{(a^2+2ab-b^2)\,e^{7/2}\log\left(\sqrt{e}+\sqrt{e}\cot(c+dx)+\sqrt{2}\sqrt{e\cot(c+dx)}\right)}{2\sqrt{2}\left(a^2+b^2\right)^2 d} - \frac{\left((a^2-2ab-b^2)\,e^{7/2}\right)\text{Subst}\left(\int \frac{1}{-1-x^2}\,dx, x, 1-\frac{\sqrt{2}\sqrt{e\cot(c+dx)}}{\sqrt{e}}\right)}{\sqrt{2}\left(a^2+b^2\right)^2 d} + \frac{\left((a^2-2ab-b^2)\,e^{7/2}\right)\text{Subst}\left(\int \frac{1}{-1-x^2}\,dx, x, 1+\frac{\sqrt{2}\sqrt{e\cot(c+dx)}}{\sqrt{e}}\right)}{\sqrt{2}\left(a^2+b^2\right)^2 d}$$

$$
\begin{aligned}
&= \frac{a^{5/2}(3a^2+7b^2)\,e^{7/2}\arctan\left(\frac{\sqrt{b}\sqrt{e\cot(c+dx)}}{\sqrt{a}\sqrt{e}}\right)}{b^{5/2}\,(a^2+b^2)^2\,d} \\
&+ \frac{(a^2-2ab-b^2)\,e^{7/2}\arctan\left(1-\frac{\sqrt{2}\sqrt{e\cot(c+dx)}}{\sqrt{e}}\right)}{\sqrt{2}\,(a^2+b^2)^2\,d} \\
&- \frac{(a^2-2ab-b^2)\,e^{7/2}\arctan\left(1+\frac{\sqrt{2}\sqrt{e\cot(c+dx)}}{\sqrt{e}}\right)}{\sqrt{2}\,(a^2+b^2)^2\,d} \\
&- \frac{(3a^2+2b^2)\,e^3\sqrt{e\cot(c+dx)}}{b^2\,(a^2+b^2)\,d} + \frac{a^2e^2(e\cot(c+dx))^{3/2}}{b\,(a^2+b^2)\,d(a+b\cot(c+dx))} \\
&+ \frac{(a^2+2ab-b^2)\,e^{7/2}\log\left(\sqrt{e}+\sqrt{e}\cot(c+dx)-\sqrt{2}\sqrt{e\cot(c+dx)}\right)}{2\sqrt{2}\,(a^2+b^2)^2\,d} \\
&- \frac{(a^2+2ab-b^2)\,e^{7/2}\log\left(\sqrt{e}+\sqrt{e}\cot(c+dx)+\sqrt{2}\sqrt{e\cot(c+dx)}\right)}{2\sqrt{2}\,(a^2+b^2)^2\,d}
\end{aligned}
$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 6.19 (sec) , antiderivative size = 461, normalized size of antiderivative = 1.05

$$
\int \frac{(e\cot(c+dx))^{7/2}}{(a+b\cot(c+dx))^2}\,dx =
$$

$$
-\frac{(e\cot(c+dx))^{7/2}\left(\frac{4a^{9/2}\arctan\left(\frac{\sqrt{b}\sqrt{\cot(c+dx)}}{\sqrt{a}}\right)}{b^{5/2}(a^2+b^2)^2} - \frac{4a^4\sqrt{\cot(c+dx)}}{b^2(a^2+b^2)^2} + \frac{4a^3\cot^{\frac{3}{2}}(c+dx)}{3b(a^2+b^2)^2} - \frac{4a^2\cot^{\frac{5}{2}}(c+dx)}{5(a^2+b^2)^2} + \frac{4ab\cot^{\frac{7}{2}}(c+dx)}{7(a^2+b^2)^2} + \cdots\right.}{}
$$

```
[In] Integrate[(e*Cot[c + d*x])^(7/2)/(a + b*Cot[c + d*x])^2,x]
```

```
[Out] -(((e*Cot[c + d*x])^(7/2)*((4*a^(9/2)*ArcTan[(Sqrt[b]*Sqrt[Cot[c + d*x]])/S
qrt[a]])/(b^(5/2)*(a^2 + b^2)^2) - (4*a^4*Sqrt[Cot[c + d*x]])/(b^2*(a^2 + b
^2)^2) + (4*a^3*Cot[c + d*x]^(3/2))/(3*b*(a^2 + b^2)^2) - (4*a^2*Cot[c + d*
x]^(5/2))/(5*(a^2 + b^2)^2) + (4*a*b*Cot[c + d*x]^(7/2))/(7*(a^2 + b^2)^2)
+ (4*a*b*(7*Cot[c + d*x]^(3/2) - 3*Cot[c + d*x]^(7/2) - 7*Cot[c + d*x]^(3/2
)*Hypergeometric2F1[3/4, 1, 7/4, -Cot[c + d*x]^2]))/(21*(a^2 + b^2)^2) + (2
*b^2*Cot[c + d*x]^(9/2)*Hypergeometric2F1[2, 9/2, 11/2, -((b*Cot[c + d*x])/
a)])/(9*a^2*(a^2 + b^2)) - ((a - b)*(a + b)*(10*Sqrt[2]*ArcTan[1 - Sqrt[2]*
Sqrt[Cot[c + d*x]]] - 10*Sqrt[2]*ArcTan[1 + Sqrt[2]*Sqrt[Cot[c + d*x]]] + 4
0*Sqrt[Cot[c + d*x]] - 8*Cot[c + d*x]^(5/2) + 5*Sqrt[2]*Log[1 - Sqrt[2]*Sqr
t[Cot[c + d*x]] + Cot[c + d*x]] - 5*Sqrt[2]*Log[1 + Sqrt[2]*Sqrt[Cot[c + d*
x]] + Cot[c + d*x]]))/(20*(a^2 + b^2)^2)))/(d*Cot[c + d*x]^(7/2)))
```

Maple [A] (verified)

Time = 0.33 (sec) , antiderivative size = 409, normalized size of antiderivative = 0.94

method	result
derivativedivides	$-\frac{2e^3\left(\frac{\sqrt{e\cot(dx+c)}}{b^2}-\frac{a^3e\left(\frac{\left(-\frac{a^2}{2}-\frac{b^2}{2}\right)\sqrt{e\cot(dx+c)}}{e\cot(dx+c)b+ae}+\frac{\left(3a^2+7b^2\right)\arctan\left(\frac{\sqrt{e\cot(dx+c)}\,b}{\sqrt{aeb}}\right)}{2\sqrt{aeb}}\right)}{b^2\left(a^2+b^2\right)^2}+\frac{e\left(\left(a^2e-b^2e\right)\left(e^2\right)^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\frac{e}{e}\right.\right.\right.}{}\right.}{}$
default	$-\frac{2e^3\left(\frac{\sqrt{e\cot(dx+c)}}{b^2}-\frac{a^3e\left(\frac{\left(-\frac{a^2}{2}-\frac{b^2}{2}\right)\sqrt{e\cot(dx+c)}}{e\cot(dx+c)b+ae}+\frac{\left(3a^2+7b^2\right)\arctan\left(\frac{\sqrt{e\cot(dx+c)}\,b}{\sqrt{aeb}}\right)}{2\sqrt{aeb}}\right)}{b^2\left(a^2+b^2\right)^2}+\frac{e\left(\left(a^2e-b^2e\right)\left(e^2\right)^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\frac{e}{e}\right.\right.\right.}{}\right.}{}$

```
[In] int((e*cot(d*x+c))^(7/2)/(a+b*cot(d*x+c))^2,x,method=_RETURNVERBOSE)
```

```
[Out] -2/d*e^3*((e*cot(d*x+c))^(1/2)/b^2-a^3*e/b^2/(a^2+b^2)^2*((-1/2*a^2-1/2*b^2
)*(e*cot(d*x+c))^(1/2)/(e*cot(d*x+c)*b+a*e)+1/2*(3*a^2+7*b^2)/(a*e*b)^(1/2)
*arctan((e*cot(d*x+c))^(1/2)*b/(a*e*b)^(1/2)))+e/(a^2+b^2)^2*(1/8*(a^2*e-b^
2*e)*(e^2)^(1/4)/e^2*2^(1/2)*(ln((e*cot(d*x+c)+(e^2)^(1/4)*(e*cot(d*x+c))^(
1/2)*2^(1/2)+(e^2)^(1/2))/(e*cot(d*x+c)-(e^2)^(1/4)*(e*cot(d*x+c))^(1/2)*2^
(1/2)+(e^2)^(1/2)))+2*arctan(2^(1/2)/(e^2)^(1/4)*(e*cot(d*x+c))^(1/2)+1)-2*
arctan(-2^(1/2)/(e^2)^(1/4)*(e*cot(d*x+c))^(1/2)+1))-1/4*a*b/(e^2)^(1/4)*2^
(1/2)*(ln((e*cot(d*x+c)-(e^2)^(1/4)*(e*cot(d*x+c))^(1/2)*2^(1/2)+(e^2)^(1/2
))/(e*cot(d*x+c)+(e^2)^(1/4)*(e*cot(d*x+c))^(1/2)*2^(1/2)+(e^2)^(1/2)))+2*a
rctan(2^(1/2)/(e^2)^(1/4)*(e*cot(d*x+c))^(1/2)+1)-2*arctan(-2^(1/2)/(e^2)^(
1/4)*(e*cot(d*x+c))^(1/2)+1))))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 3170 vs. $2(372) = 744$.

Time = 0.71 (sec) , antiderivative size = 6403, normalized size of antiderivative = 14.65

$$\int \frac{(e\cot(c+dx))^{7/2}}{(a+b\cot(c+dx))^2}\,dx = \text{Too large to display}$$

```
[In] integrate((e*cot(d*x+c))^(7/2)/(a+b*cot(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] Too large to include
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(e\cot(c+dx))^{7/2}}{(a+b\cot(c+dx))^2}\,dx = \text{Timed out}$$

```
[In] integrate((e*cot(d*x+c))**(7/2)/(a+b*cot(d*x+c))**2,x)
```

```
[Out] Timed out
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{(e\cot(c+dx))^{7/2}}{(a+b\cot(c+dx))^2}\,dx = \text{Exception raised: ValueError}$$

```
[In] integrate((e*cot(d*x+c))^(7/2)/(a+b*cot(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(e>0)', see `assume?` for more detai
ls)Is e
```

Giac [F]

$$\int \frac{(e\cot(c+dx))^{7/2}}{(a+b\cot(c+dx))^2}\,dx = \int \frac{(e\cot(dx+c))^{\frac{7}{2}}}{(b\cot(dx+c)+a)^2}\,dx$$

```
[In] integrate((e*cot(d*x+c))^(7/2)/(a+b*cot(d*x+c))^2,x, algorithm="giac")
[Out] integrate((e*cot(d*x + c))^(7/2)/(b*cot(d*x + c) + a)^2, x)
```

Mupad [B] (verification not implemented)

Time = 16.41 (sec) , antiderivative size = 13244, normalized size of antiderivative = 30.31

$$\int \frac{(e\cot(c+dx))^{7/2}}{(a+b\cot(c+dx))^2}\,dx = \text{Too large to display}$$

```
[In] int((e*cot(c + d*x))^(7/2)/(a + b*cot(c + d*x))^2,x)
[Out] (atan(((((16*(e*cot(c + d*x))^(1/2)*(9*a^12*e^24 + 2*b^12*e^24 + 4*a^2*b^10
*e^24 + 2*a^4*b^8*e^24 - 49*a^6*b^6*e^24 + 7*a^8*b^4*e^24 + 33*a^10*b^2*e^2
4))/(b^11*d^4 + 4*a^2*b^9*d^4 + 6*a^4*b^7*d^4 + 4*a^6*b^5*d^4 + a^8*b^3*d^4
) + (((16*(30*a^6*b^8*d^2*e^21 - 224*a^4*b^10*d^2*e^21 - 18*a^14*d^2*e^21 +
 600*a^8*b^6*d^2*e^21 + 388*a^10*b^4*d^2*e^21 + 24*a^12*b^2*d^2*e^21))/(b^1
1*d^5 + 4*a^2*b^9*d^5 + 6*a^4*b^7*d^5 + 4*a^6*b^5*d^5 + a^8*b^3*d^5) - (((1
6*(e*cot(c + d*x))^(1/2)*(72*a^15*b*d^2*e^17 - 60*a*b^15*d^2*e^17 - 52*a^3*
b^13*d^2*e^17 + 72*a^5*b^11*d^2*e^17 + 448*a^7*b^9*d^2*e^17 + 1108*a^9*b^7*
d^2*e^17 + 1132*a^11*b^5*d^2*e^17 + 480*a^13*b^3*d^2*e^17))/(b^11*d^4 + 4*a
^2*b^9*d^4 + 6*a^4*b^7*d^4 + 4*a^6*b^5*d^4 + a^8*b^3*d^4) + (((16*(8*a*b^17
*d^4*e^14 + 96*a^3*b^15*d^4*e^14 + 360*a^5*b^13*d^4*e^14 + 640*a^7*b^11*d^4
*e^14 + 600*a^9*b^9*d^4*e^14 + 288*a^11*b^7*d^4*e^14 + 56*a^13*b^5*d^4*e^14
))/(b^11*d^5 + 4*a^2*b^9*d^5 + 6*a^4*b^7*d^5 + 4*a^6*b^5*d^5 + a^8*b^3*d^5)
 - (8*(e*cot(c + d*x))^(1/2)*(3*a^2 + 7*b^2)*(-a^5*b^5*e^7)^(1/2)*(32*b^20*
d^4*e^10 + 160*a^2*b^18*d^4*e^10 + 288*a^4*b^16*d^4*e^10 + 160*a^6*b^14*d^4
*e^10 - 160*a^8*b^12*d^4*e^10 - 288*a^10*b^10*d^4*e^10 - 160*a^12*b^8*d^4*e
^10 - 32*a^14*b^6*d^4*e^10))/((b^9*d + 2*a^2*b^7*d + a^4*b^5*d)*(b^11*d^4 +
 4*a^2*b^9*d^4 + 6*a^4*b^7*d^4 + 4*a^6*b^5*d^4 + a^8*b^3*d^4)))*(3*a^2 + 7*
b^2)*(-a^5*b^5*e^7)^(1/2))/(2*(b^9*d + 2*a^2*b^7*d + a^4*b^5*d)))*(3*a^2 +
7*b^2)*(-a^5*b^5*e^7)^(1/2))/(2*(b^9*d + 2*a^2*b^7*d + a^4*b^5*d)))*(3*a^2
+ 7*b^2)*(-a^5*b^5*e^7)^(1/2))/(2*(b^9*d + 2*a^2*b^7*d + a^4*b^5*d)))*(3*a^
2 + 7*b^2)*(-a^5*b^5*e^7)^(1/2)*1i)/(2*(b^9*d + 2*a^2*b^7*d + a^4*b^5*d)) +
 (((16*(e*cot(c + d*x))^(1/2)*(9*a^12*e^24 + 2*b^12*e^24 + 4*a^2*b^10*e^24
+ 2*a^4*b^8*e^24 - 49*a^6*b^6*e^24 + 7*a^8*b^4*e^24 + 33*a^10*b^2*e^24))/(b
^11*d^4 + 4*a^2*b^9*d^4 + 6*a^4*b^7*d^4 + 4*a^6*b^5*d^4 + a^8*b^3*d^4) - ((
(16*(30*a^6*b^8*d^2*e^21 - 224*a^4*b^10*d^2*e^21 - 18*a^14*d^2*e^21 + 600*a
```

^8*b^6*d^2*e^21 + 388*a^10*b^4*d^2*e^21 + 24*a^12*b^2*d^2*e^21))/(b^11*d^5 + 4*a^2*b^9*d^5 + 6*a^4*b^7*d^5 + 4*a^6*b^5*d^5 + a^8*b^3*d^5) + (((16*(e*cot(c + d*x))^(1/2)*(72*a^15*b*d^2*e^17 - 60*a*b^15*d^2*e^17 - 52*a^3*b^13*d^2*e^17 + 72*a^5*b^11*d^2*e^17 + 448*a^7*b^9*d^2*e^17 + 1108*a^9*b^7*d^2*e^17 + 1132*a^11*b^5*d^2*e^17 + 480*a^13*b^3*d^2*e^17))/(b^11*d^4 + 4*a^2*b^9*d^4 + 6*a^4*b^7*d^4 + 4*a^6*b^5*d^4 + a^8*b^3*d^4) - (((16*(8*a*b^17*d^4*e^14 + 96*a^3*b^15*d^4*e^14 + 360*a^5*b^13*d^4*e^14 + 640*a^7*b^11*d^4*e^14 + 600*a^9*b^9*d^4*e^14 + 288*a^11*b^7*d^4*e^14 + 56*a^13*b^5*d^4*e^14))/(b^11*d^5 + 4*a^2*b^9*d^5 + 6*a^4*b^7*d^5 + 4*a^6*b^5*d^5 + a^8*b^3*d^5) + (8*(e*cot(c + d*x))^(1/2)*(3*a^2 + 7*b^2)*(-a^5*b^5*e^7)^(1/2)*(32*b^20*d^4*e^10 + 160*a^2*b^18*d^4*e^10 + 288*a^4*b^16*d^4*e^10 + 160*a^6*b^14*d^4*e^10 - 160*a^8*b^12*d^4*e^10 - 288*a^10*b^10*d^4*e^10 - 160*a^12*b^8*d^4*e^10 - 32*a^14*b^6*d^4*e^10))/((b^9*d + 2*a^2*b^7*d + a^4*b^5*d)*(b^11*d^4 + 4*a^2*b^9*d^4 + 6*a^4*b^7*d^4 + 4*a^6*b^5*d^4 + a^8*b^3*d^4)))*(3*a^2 + 7*b^2)*(-a^5*b^5*e^7)^(1/2))/(2*(b^9*d + 2*a^2*b^7*d + a^4*b^5*d)))*(3*a^2 + 7*b^2)*(-a^5*b^5*e^7)^(1/2))/(2*(b^9*d + 2*a^2*b^7*d + a^4*b^5*d)))*(3*a^2 + 7*b^2)*(-a^5*b^5*e^7)^(1/2))/(2*(b^9*d + 2*a^2*b^7*d + a^4*b^5*d)))*(3*a^2 + 7*b^2)*(-a^5*b^5*e^7)^(1/2)*1i)/(2*(b^9*d + 2*a^2*b^7*d + a^4*b^5*d)))/((32*(7*a^3*b^7*e^28 + 3*a^5*b^5*e^28))/(b^11*d^5 + 4*a^2*b^9*d^5 + 6*a^4*b^7*d^5 + 4*a^6*b^5*d^5 + a^8*b^3*d^5) - (((16*(e*cot(c + d*x))^(1/2)*(9*a^12*e^24 + 2*b^12*e^24 + 4*a^2*b^10*e^24 + 2*a^4*b^8*e^24 - 49*a^6*b^6*e^24 + 7*a^8*b^4*e^24 + 33*a^10*b^2*e^24))/(b^11*d^4 + 4*a^2*b^9*d^4 + 6*a^4*b^7*d^4 + 4*a^6*b^5*d^4 + a^8*b^3*d^4) + (((16*(30*a^6*b^8*d^2*e^21 - 224*a^4*b^10*d^2*e^21 - 18*a^14*d^2*e^21 + 600*a^8*b^6*d^2*e^21 + 388*a^10*b^4*d^2*e^21 + 24*a^12*b^2*d^2*e^21))/(b^11*d^5 + 4*a^2*b^9*d^5 + 6*a^4*b^7*d^5 + 4*a^6*b^5*d^5 + a^8*b^3*d^5) - (((16*(e*cot(c + d*x))^(1/2)*(72*a^15*b*d^2*e^17 - 60*a*b^15*d^2*e^17 - 52*a^3*b^13*d^2*e^17 + 72*a^5*b^11*d^2*e^17 + 448*a^7*b^9*d^2*e^17 + 1108*a^9*b^7*d^2*e^17 + 1132*a^11*b^5*d^2*e^17 + 480*a^13*b^3*d^2*e^17))/(b^11*d^4 + 4*a^2*b^9*d^4 + 6*a^4*b^7*d^4 + 4*a^6*b^5*d^4 + a^8*b^3*d^4) + (((16*(8*a*b^17*d^4*e^14 + 96*a^3*b^15*d^4*e^14 + 360*a^5*b^13*d^4*e^14 + 640*a^7*b^11*d^4*e^14 + 600*a^9*b^9*d^4*e^14 + 288*a^11*b^7*d^4*e^14 + 56*a^13*b^5*d^4*e^14))/(b^11*d^5 + 4*a^2*b^9*d^5 + 6*a^4*b^7*d^5 + 4*a^6*b^5*d^5 + a^8*b^3*d^5) - (8*(e*cot(c + d*x))^(1/2)*(3*a^2 + 7*b^2)*(-a^5*b^5*e^7)^(1/2)*(32*b^20*d^4*e^10 + 160*a^2*b^18*d^4*e^10 + 288*a^4*b^16*d^4*e^10 + 160*a^6*b^14*d^4*e^10 - 160*a^8*b^12*d^4*e^10 - 288*a^10*b^10*d^4*e^10 - 160*a^12*b^8*d^4*e^10 - 32*a^14*b^6*d^4*e^10))/((b^9*d + 2*a^2*b^7*d + a^4*b^5*d)*(b^11*d^4 + 4*a^2*b^9*d^4 + 6*a^4*b^7*d^4 + 4*a^6*b^5*d^4 + a^8*b^3*d^4)))*(3*a^2 + 7*b^2)*(-a^5*b^5*e^7)^(1/2))/(2*(b^9*d + 2*a^2*b^7*d + a^4*b^5*d)))*(3*a^2 + 7*b^2)*(-a^5*b^5*e^7)^(1/2))/(2*(b^9*d + 2*a^2*b^7*d + a^4*b^5*d)))*(3*a^2 + 7*b^2)*(-a^5*b^5*e^7)^(1/2))/(2*(b^9*d + 2*a^2*b^7*d + a^4*b^5*d)))*(3*a^2 + 7*b^2)*(-a^5*b^5*e^7)^(1/2))/(2*(b^9*d + 2*a^2*b^7*d + a^4*b^5*d)) + (((16*(e*cot(c + d*x))^(1/2)*(9*a^12*e^24 + 2*b^12*e^24 + 4*a^2*b^10*e^24 + 2*a^4*b^8*e^24 - 49*a^6*b^6*e^24 + 7*a^8*b^4*e^24 + 33*a^10*b^2*e^24))/(b^11*d^4 + 4*a^2*b^9*d^4 + 6*a^4*b^7*d^4 + 4*a^6*b^5*d^4 + a^8*b^3*d^4) - (((16*(30*a^6*b^8*d^2*e^21 - 224*a^4*b^10*d^2*e^21 -

18*a^14*d^2*e^21 + 600*a^8*b^6*d^2*e^21 + 388*a^10*b^4*d^2*e^21 + 24*a^12*b^2*d^2*e^21))/(b^11*d^5 + 4*a^2*b^9*d^5 + 6*a^4*b^7*d^5 + 4*a^6*b^5*d^5 + a^8*b^3*d^5) + (((16*(e*cot(c + d*x))^(1/2)*(72*a^15*b*d^2*e^17 - 60*a*b^15*d^2*e^17 - 52*a^3*b^13*d^2*e^17 + 72*a^5*b^11*d^2*e^17 + 448*a^7*b^9*d^2*e^17 + 1108*a^9*b^7*d^2*e^17 + 1132*a^11*b^5*d^2*e^17 + 480*a^13*b^3*d^2*e^17))/(b^11*d^4 + 4*a^2*b^9*d^4 + 6*a^4*b^7*d^4 + 4*a^6*b^5*d^4 + a^8*b^3*d^4) - (((16*(8*a*b^17*d^4*e^14 + 96*a^3*b^15*d^4*e^14 + 360*a^5*b^13*d^4*e^14 + 640*a^7*b^11*d^4*e^14 + 600*a^9*b^9*d^4*e^14 + 288*a^11*b^7*d^4*e^14 + 56*a^13*b^5*d^4*e^14))/(b^11*d^5 + 4*a^2*b^9*d^5 + 6*a^4*b^7*d^5 + 4*a^6*b^5*d^5 + a^8*b^3*d^5) + (8*(e*cot(c + d*x))^(1/2)*(3*a^2 + 7*b^2)*(-a^5*b^5*e^7)^(1/2)*(32*b^20*d^4*e^10 + 160*a^2*b^18*d^4*e^10 + 288*a^4*b^16*d^4*e^10 + 160*a^6*b^14*d^4*e^10 - 160*a^8*b^12*d^4*e^10 - 288*a^10*b^10*d^4*e^10 - 160*a^12*b^8*d^4*e^10 - 32*a^14*b^6*d^4*e^10))/((b^9*d + 2*a^2*b^7*d + a^4*b^5*d)*(b^11*d^4 + 4*a^2*b^9*d^4 + 6*a^4*b^7*d^4 + 4*a^6*b^5*d^4 + a^8*b^3*d^4)))*(3*a^2 + 7*b^2)*(-a^5*b^5*e^7)^(1/2))/(2*(b^9*d + 2*a^2*b^7*d + a^4*b^5*d)))*(3*a^2 + 7*b^2)*(-a^5*b^5*e^7)^(1/2))/(2*(b^9*d + 2*a^2*b^7*d + a^4*b^5*d)))*(3*a^2 + 7*b^2)*(-a^5*b^5*e^7)^(1/2))/(2*(b^9*d + 2*a^2*b^7*d + a^4*b^5*d)))*(3*a^2 + 7*b^2)*(-a^5*b^5*e^7)^(1/2))/(2*(b^9*d + 2*a^2*b^7*d + a^4*b^5*d))))*(3*a^2 + 7*b^2)*(-a^5*b^5*e^7)^(1/2)*1i)/(b^9*d + 2*a^2*b^7*d + a^4*b^5*d) - atan((((((16*(8*a*b^17*d^4*e^14 + 96*a^3*b^15*d^4*e^14 + 360*a^5*b^13*d^4*e^14 + 640*a^7*b^11*d^4*e^14 + 600*a^9*b^9*d^4*e^14 + 288*a^11*b^7*d^4*e^14 + 56*a^13*b^5*d^4*e^14))/(b^11*d^5 + 4*a^2*b^9*d^5 + 6*a^4*b^7*d^5 + 4*a^6*b^5*d^5 + a^8*b^3*d^5) - (16*(e*cot(c + d*x))^(1/2)*(-e^7/(4*(a^4*d^2*1i + b^4*d^2*1i + 4*a*b^3*d^2 - 4*a^3*b*d^2 - a^2*b^2*d^2*6i)))^(1/2)*(32*b^20*d^4*e^10 + 160*a^2*b^18*d^4*e^10 + 288*a^4*b^16*d^4*e^10 + 160*a^6*b^14*d^4*e^10 - 160*a^8*b^12*d^4*e^10 - 288*a^10*b^10*d^4*e^10 - 160*a^12*b^8*d^4*e^10 - 32*a^14*b^6*d^4*e^10))/(b^11*d^4 + 4*a^2*b^9*d^4 + 6*a^4*b^7*d^4 + 4*a^6*b^5*d^4 + a^8*b^3*d^4))*(-e^7/(4*(a^4*d^2*1i + b^4*d^2*1i + 4*a*b^3*d^2 - 4*a^3*b*d^2 - a^2*b^2*d^2*6i)))^(1/2) + (16*(e*cot(c + d*x))^(1/2)*(72*a^15*b*d^2*e^17 - 60*a*b^15*d^2*e^17 - 52*a^3*b^13*d^2*e^17 + 72*a^5*b^11*d^2*e^17 + 448*a^7*b^9*d^2*e^17 + 1108*a^9*b^7*d^2*e^17 + 1132*a^11*b^5*d^2*e^17 + 480*a^13*b^3*d^2*e^17))/(b^11*d^4 + 4*a^2*b^9*d^4 + 6*a^4*b^7*d^4 + 4*a^6*b^5*d^4 + a^8*b^3*d^4))*(-e^7/(4*(a^4*d^2*1i + b^4*d^2*1i + 4*a*b^3*d^2 - 4*a^3*b*d^2 - a^2*b^2*d^2*6i)))^(1/2) - (16*(30*a^6*b^8*d^2*e^21 - 224*a^4*b^10*d^2*e^21 - 18*a^14*d^2*e^21 + 600*a^8*b^6*d^2*e^21 + 388*a^10*b^4*d^2*e^21 + 24*a^12*b^2*d^2*e^21))/(b^11*d^5 + 4*a^2*b^9*d^5 + 6*a^4*b^7*d^5 + 4*a^6*b^5*d^5 + a^8*b^3*d^5))*(-e^7/(4*(a^4*d^2*1i + b^4*d^2*1i + 4*a*b^3*d^2 - 4*a^3*b*d^2 - a^2*b^2*d^2*6i)))^(1/2) - (16*(e*cot(c + d*x))^(1/2)*(9*a^12*e^24 + 2*b^12*e^24 + 4*a^2*b^10*e^24 + 2*a^4*b^8*e^24 - 49*a^6*b^6*e^24 + 7*a^8*b^4*e^24 + 33*a^10*b^2*e^24))/(b^11*d^4 + 4*a^2*b^9*d^4 + 6*a^4*b^7*d^4 + 4*a^6*b^5*d^4 + a^8*b^3*d^4))*(-e^7/(4*(a^4*d^2*1i + b^4*d^2*1i + 4*a*b^3*d^2 - 4*a^3*b*d^2 - a^2*b^2*d^2*6i)))^(1/2)*1i - (((((16*(8*a*b^17*d^4*e^14 + 96*a^3*b^15*d^4*e^14 + 360*a^5*b^13*d^4*e^14 + 640*a^7*b^11*d^4*e^14 + 600*a^9*b^9*d^4*e^14 + 288*a^11*b^7*d^4*e^14 + 56*a^13*b^5*d^4*e^14))/(b^11*d^5 + 4*a^2*b^9*d^5 + 6*a^4*b^7*d^5 + 4*a^6*b^

5*d^5 + a^8*b^3*d^5) + (16*(e*cot(c + d*x))^(1/2)*(-e^7/(4*(a^4*d^2*1i + b^4*d^2*1i + 4*a*b^3*d^2 - 4*a^3*b*d^2 - a^2*b^2*d^2*6i)))^(1/2)*(32*b^20*d^4*e^10 + 160*a^2*b^18*d^4*e^10 + 288*a^4*b^16*d^4*e^10 + 160*a^6*b^14*d^4*e^10 - 160*a^8*b^12*d^4*e^10 - 288*a^10*b^10*d^4*e^10 - 160*a^12*b^8*d^4*e^10 - 32*a^14*b^6*d^4*e^10))/(b^11*d^4 + 4*a^2*b^9*d^4 + 6*a^4*b^7*d^4 + 4*a^6*b^5*d^4 + a^8*b^3*d^4))*(-e^7/(4*(a^4*d^2*1i + b^4*d^2*1i + 4*a*b^3*d^2 - 4*a^3*b*d^2 - a^2*b^2*d^2*6i)))^(1/2) - (16*(e*cot(c + d*x))^(1/2)*(72*a^15*b*d^2*e^17 - 60*a*b^15*d^2*e^17 - 52*a^3*b^13*d^2*e^17 + 72*a^5*b^11*d^2*e^17 + 448*a^7*b^9*d^2*e^17 + 1108*a^9*b^7*d^2*e^17 + 1132*a^11*b^5*d^2*e^17 + 480*a^13*b^3*d^2*e^17))/(b^11*d^4 + 4*a^2*b^9*d^4 + 6*a^4*b^7*d^4 + 4*a^6*b^5*d^4 + a^8*b^3*d^4))*(-e^7/(4*(a^4*d^2*1i + b^4*d^2*1i + 4*a*b^3*d^2 - 4*a^3*b*d^2 - a^2*b^2*d^2*6i)))^(1/2) - (16*(30*a^6*b^8*d^2*e^21 - 224*a^4*b^10*d^2*e^21 - 18*a^14*d^2*e^21 + 600*a^8*b^6*d^2*e^21 + 388*a^10*b^4*d^2*e^21 + 24*a^12*b^2*d^2*e^21))/(b^11*d^5 + 4*a^2*b^9*d^5 + 6*a^4*b^7*d^5 + 4*a^6*b^5*d^5 + a^8*b^3*d^5))*(-e^7/(4*(a^4*d^2*1i + b^4*d^2*1i + 4*a*b^3*d^2 - 4*a^3*b*d^2 - a^2*b^2*d^2*6i)))^(1/2) + (16*(e*cot(c + d*x))^(1/2)*(9*a^12*e^24 + 2*b^12*e^24 + 4*a^2*b^10*e^24 + 2*a^4*b^8*e^24 - 49*a^6*b^6*e^24 + 7*a^8*b^4*e^24 + 33*a^10*b^2*e^24))/(b^11*d^4 + 4*a^2*b^9*d^4 + 6*a^4*b^7*d^4 + 4*a^6*b^5*d^4 + a^8*b^3*d^4))*(-e^7/(4*(a^4*d^2*1i + b^4*d^2*1i + 4*a*b^3*d^2 - 4*a^3*b*d^2 - a^2*b^2*d^2*6i)))^(1/2)*1i)/((32*(7*a^3*b^7*e^28 + 3*a^5*b^5*e^28))/(b^11*d^5 + 4*a^2*b^9*d^5 + 6*a^4*b^7*d^5 + 4*a^6*b^5*d^5 + a^8*b^3*d^5) + (((((16*(8*a*b^17*d^4*e^14 + 96*a^3*b^15*d^4*e^14 + 360*a^5*b^13*d^4*e^14 + 640*a^7*b^11*d^4*e^14 + 600*a^9*b^9*d^4*e^14 + 288*a^11*b^7*d^4*e^14 + 56*a^13*b^5*d^4*e^14))/(b^11*d^5 + 4*a^2*b^9*d^5 + 6*a^4*b^7*d^5 + 4*a^6*b^5*d^5 + a^8*b^3*d^5) - (16*(e*cot(c + d*x))^(1/2)*(-e^7/(4*(a^4*d^2*1i + b^4*d^2*1i + 4*a*b^3*d^2 - 4*a^3*b*d^2 - a^2*b^2*d^2*6i)))^(1/2)*(32*b^20*d^4*e^10 + 160*a^2*b^18*d^4*e^10 + 288*a^4*b^16*d^4*e^10 + 160*a^6*b^14*d^4*e^10 - 160*a^8*b^12*d^4*e^10 - 288*a^10*b^10*d^4*e^10 - 160*a^12*b^8*d^4*e^10 - 32*a^14*b^6*d^4*e^10))/(b^11*d^4 + 4*a^2*b^9*d^4 + 6*a^4*b^7*d^4 + 4*a^6*b^5*d^4 + a^8*b^3*d^4))*(-e^7/(4*(a^4*d^2*1i + b^4*d^2*1i + 4*a*b^3*d^2 - 4*a^3*b*d^2 - a^2*b^2*d^2*6i)))^(1/2) + (16*(e*cot(c + d*x))^(1/2)*(72*a^15*b*d^2*e^17 - 60*a*b^15*d^2*e^17 - 52*a^3*b^13*d^2*e^17 + 72*a^5*b^11*d^2*e^17 + 448*a^7*b^9*d^2*e^17 + 1108*a^9*b^7*d^2*e^17 + 1132*a^11*b^5*d^2*e^17 + 480*a^13*b^3*d^2*e^17))/(b^11*d^4 + 4*a^2*b^9*d^4 + 6*a^4*b^7*d^4 + 4*a^6*b^5*d^4 + a^8*b^3*d^4))*(-e^7/(4*(a^4*d^2*1i + b^4*d^2*1i + 4*a*b^3*d^2 - 4*a^3*b*d^2 - a^2*b^2*d^2*6i)))^(1/2) - (16*(30*a^6*b^8*d^2*e^21 - 224*a^4*b^10*d^2*e^21 - 18*a^14*d^2*e^21 + 600*a^8*b^6*d^2*e^21 + 388*a^10*b^4*d^2*e^21 + 24*a^12*b^2*d^2*e^21))/(b^11*d^5 + 4*a^2*b^9*d^5 + 6*a^4*b^7*d^5 + 4*a^6*b^5*d^5 + a^8*b^3*d^5))*(-e^7/(4*(a^4*d^2*1i + b^4*d^2*1i + 4*a*b^3*d^2 - 4*a^3*b*d^2 - a^2*b^2*d^2*6i)))^(1/2) - (16*(e*cot(c + d*x))^(1/2)*(9*a^12*e^24 + 2*b^12*e^24 + 4*a^2*b^10*e^24 + 2*a^4*b^8*e^24 - 49*a^6*b^6*e^24 + 7*a^8*b^4*e^24 + 33*a^10*b^2*e^24))/(b^11*d^4 + 4*a^2*b^9*d^4 + 6*a^4*b^7*d^4 + 4*a^6*b^5*d^4 + a^8*b^3*d^4))*(-e^7/(4*(a^4*d^2*1i + b^4*d^2*1i + 4*a*b^3*d^2 - 4*a^3*b*d^2 - a^2*b^2*d^2*6i)))^(1/2) + (((((16*(8*a*b^17*d^4*e^14 + 96*a^3*b^15*d^4*e^14 + 360*a^5*b^13*d^4*e^14 + 64

0*a^7*b^11*d^4*e^14 + 600*a^9*b^9*d^4*e^14 + 288*a^11*b^7*d^4*e^14 + 56*a^1
3*b^5*d^4*e^14))/(b^11*d^5 + 4*a^2*b^9*d^5 + 6*a^4*b^7*d^5 + 4*a^6*b^5*d^5
+ a^8*b^3*d^5) + (16*(e*cot(c + d*x))^(1/2)*(-e^7/(4*(a^4*d^2*1i + b^4*d^2*
1i + 4*a*b^3*d^2 - 4*a^3*b*d^2 - a^2*b^2*d^2*6i)))^(1/2)*(32*b^20*d^4*e^10
+ 160*a^2*b^18*d^4*e^10 + 288*a^4*b^16*d^4*e^10 + 160*a^6*b^14*d^4*e^10 - 1
60*a^8*b^12*d^4*e^10 - 288*a^10*b^10*d^4*e^10 - 160*a^12*b^8*d^4*e^10 - 32*
a^14*b^6*d^4*e^10))/(b^11*d^4 + 4*a^2*b^9*d^4 + 6*a^4*b^7*d^4 + 4*a^6*b^5*d
^4 + a^8*b^3*d^4))*(-e^7/(4*(a^4*d^2*1i + b^4*d^2*1i + 4*a*b^3*d^2 - 4*a^3*
b*d^2 - a^2*b^2*d^2*6i)))^(1/2) - (16*(e*cot(c + d*x))^(1/2)*(72*a^15*b*d^2
*e^17 - 60*a*b^15*d^2*e^17 - 52*a^3*b^13*d^2*e^17 + 72*a^5*b^11*d^2*e^17 +
448*a^7*b^9*d^2*e^17 + 1108*a^9*b^7*d^2*e^17 + 1132*a^11*b^5*d^2*e^17 + 480
*a^13*b^3*d^2*e^17))/(b^11*d^4 + 4*a^2*b^9*d^4 + 6*a^4*b^7*d^4 + 4*a^6*b^5*
d^4 + a^8*b^3*d^4))*(-e^7/(4*(a^4*d^2*1i + b^4*d^2*1i + 4*a*b^3*d^2 - 4*a^3
*b*d^2 - a^2*b^2*d^2*6i)))^(1/2) - (16*(30*a^6*b^8*d^2*e^21 - 224*a^4*b^10*
d^2*e^21 - 18*a^14*d^2*e^21 + 600*a^8*b^6*d^2*e^21 + 388*a^10*b^4*d^2*e^21
+ 24*a^12*b^2*d^2*e^21))/(b^11*d^5 + 4*a^2*b^9*d^5 + 6*a^4*b^7*d^5 + 4*a^6*
b^5*d^5 + a^8*b^3*d^5))*(-e^7/(4*(a^4*d^2*1i + b^4*d^2*1i + 4*a*b^3*d^2 - 4
*a^3*b*d^2 - a^2*b^2*d^2*6i)))^(1/2) + (16*(e*cot(c + d*x))^(1/2)*(9*a^12*e
^24 + 2*b^12*e^24 + 4*a^2*b^10*e^24 + 2*a^4*b^8*e^24 - 49*a^6*b^6*e^24 + 7*
a^8*b^4*e^24 + 33*a^10*b^2*e^24))/(b^11*d^4 + 4*a^2*b^9*d^4 + 6*a^4*b^7*d^4
 + 4*a^6*b^5*d^4 + a^8*b^3*d^4))*(-e^7/(4*(a^4*d^2*1i + b^4*d^2*1i + 4*a*b^
3*d^2 - 4*a^3*b*d^2 - a^2*b^2*d^2*6i)))^(1/2)))*(-e^7/(4*(a^4*d^2*1i + b^4*
d^2*1i + 4*a*b^3*d^2 - 4*a^3*b*d^2 - a^2*b^2*d^2*6i)))^(1/2)*2i - (2*e^3*(e
*cot(c + d*x))^(1/2))/(b^2*d) - atan(((((((16*(8*a*b^17*d^4*e^14 + 96*a^3*b
^15*d^4*e^14 + 360*a^5*b^13*d^4*e^14 + 640*a^7*b^11*d^4*e^14 + 600*a^9*b^9*
d^4*e^14 + 288*a^11*b^7*d^4*e^14 + 56*a^13*b^5*d^4*e^14))/(b^11*d^5 + 4*a^2
*b^9*d^5 + 6*a^4*b^7*d^5 + 4*a^6*b^5*d^5 + a^8*b^3*d^5) - (16*(e*cot(c + d*
x))^(1/2)*(-(e^7*1i)/(4*(a^4*d^2 + b^4*d^2 + a*b^3*d^2*4i - a^3*b*d^2*4i -
6*a^2*b^2*d^2)))^(1/2)*(32*b^20*d^4*e^10 + 160*a^2*b^18*d^4*e^10 + 288*a^4*
b^16*d^4*e^10 + 160*a^6*b^14*d^4*e^10 - 160*a^8*b^12*d^4*e^10 - 288*a^10*b^
10*d^4*e^10 - 160*a^12*b^8*d^4*e^10 - 32*a^14*b^6*d^4*e^10))/(b^11*d^4 + 4*
a^2*b^9*d^4 + 6*a^4*b^7*d^4 + 4*a^6*b^5*d^4 + a^8*b^3*d^4))*(-(e^7*1i)/(4*(
a^4*d^2 + b^4*d^2 + a*b^3*d^2*4i - a^3*b*d^2*4i - 6*a^2*b^2*d^2)))^(1/2) +
(16*(e*cot(c + d*x))^(1/2)*(72*a^15*b*d^2*e^17 - 60*a*b^15*d^2*e^17 - 52*a^
3*b^13*d^2*e^17 + 72*a^5*b^11*d^2*e^17 + 448*a^7*b^9*d^2*e^17 + 1108*a^9*b^
7*d^2*e^17 + 1132*a^11*b^5*d^2*e^17 + 480*a^13*b^3*d^2*e^17))/(b^11*d^4 + 4
*a^2*b^9*d^4 + 6*a^4*b^7*d^4 + 4*a^6*b^5*d^4 + a^8*b^3*d^4))*(-(e^7*1i)/(4*
(a^4*d^2 + b^4*d^2 + a*b^3*d^2*4i - a^3*b*d^2*4i - 6*a^2*b^2*d^2)))^(1/2) -
 (16*(30*a^6*b^8*d^2*e^21 - 224*a^4*b^10*d^2*e^21 - 18*a^14*d^2*e^21 + 600*
a^8*b^6*d^2*e^21 + 388*a^10*b^4*d^2*e^21 + 24*a^12*b^2*d^2*e^21))/(b^11*d^5
 + 4*a^2*b^9*d^5 + 6*a^4*b^7*d^5 + 4*a^6*b^5*d^5 + a^8*b^3*d^5))*(-(e^7*1i)
/(4*(a^4*d^2 + b^4*d^2 + a*b^3*d^2*4i - a^3*b*d^2*4i - 6*a^2*b^2*d^2)))^(1/
2) - (16*(e*cot(c + d*x))^(1/2)*(9*a^12*e^24 + 2*b^12*e^24 + 4*a^2*b^10*e^2
4 + 2*a^4*b^8*e^24 - 49*a^6*b^6*e^24 + 7*a^8*b^4*e^24 + 33*a^10*b^2*e^24))/
(b^11*d^4 + 4*a^2*b^9*d^4 + 6*a^4*b^7*d^4 + 4*a^6*b^5*d^4 + a^8*b^3*d^4))*(

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-(e^7*1i)/(4*(a^4*d^2 + b^4*d^2 + a*b^3*d^2*4i - a^3*b*d^2*4i - 6*a^2*b^2*d
^2)))^(1/2)*1i - (((((16*(8*a*b^17*d^4*e^14 + 96*a^3*b^15*d^4*e^14 + 360*a^
5*b^13*d^4*e^14 + 640*a^7*b^11*d^4*e^14 + 600*a^9*b^9*d^4*e^14 + 288*a^11*b
^7*d^4*e^14 + 56*a^13*b^5*d^4*e^14))/(b^11*d^5 + 4*a^2*b^9*d^5 + 6*a^4*b^7*
d^5 + 4*a^6*b^5*d^5 + a^8*b^3*d^5) + (16*(e*cot(c + d*x))^(1/2)*(-(e^7*1i)/
(4*(a^4*d^2 + b^4*d^2 + a*b^3*d^2*4i - a^3*b*d^2*4i - 6*a^2*b^2*d^2)))^(1/2
)*(32*b^20*d^4*e^10 + 160*a^2*b^18*d^4*e^10 + 288*a^4*b^16*d^4*e^10 + 160*a
^6*b^14*d^4*e^10 - 160*a^8*b^12*d^4*e^10 - 288*a^10*b^10*d^4*e^10 - 160*a^1
2*b^8*d^4*e^10 - 32*a^14*b^6*d^4*e^10))/(b^11*d^4 + 4*a^2*b^9*d^4 + 6*a^4*b
^7*d^4 + 4*a^6*b^5*d^4 + a^8*b^3*d^4))*(-(e^7*1i)/(4*(a^4*d^2 + b^4*d^2 + a
*b^3*d^2*4i - a^3*b*d^2*4i - 6*a^2*b^2*d^2)))^(1/2) - (16*(e*cot(c + d*x))^
(1/2)*(72*a^15*b*d^2*e^17 - 60*a*b^15*d^2*e^17 - 52*a^3*b^13*d^2*e^17 + 72*
a^5*b^11*d^2*e^17 + 448*a^7*b^9*d^2*e^17 + 1108*a^9*b^7*d^2*e^17 + 1132*a^1
1*b^5*d^2*e^17 + 480*a^13*b^3*d^2*e^17))/(b^11*d^4 + 4*a^2*b^9*d^4 + 6*a^4*
b^7*d^4 + 4*a^6*b^5*d^4 + a^8*b^3*d^4))*(-(e^7*1i)/(4*(a^4*d^2 + b^4*d^2 +
a*b^3*d^2*4i - a^3*b*d^2*4i - 6*a^2*b^2*d^2)))^(1/2) - (16*(30*a^6*b^8*d^2*
e^21 - 224*a^4*b^10*d^2*e^21 - 18*a^14*d^2*e^21 + 600*a^8*b^6*d^2*e^21 + 38
8*a^10*b^4*d^2*e^21 + 24*a^12*b^2*d^2*e^21))/(b^11*d^5 + 4*a^2*b^9*d^5 + 6*
a^4*b^7*d^5 + 4*a^6*b^5*d^5 + a^8*b^3*d^5))*(-(e^7*1i)/(4*(a^4*d^2 + b^4*d^
2 + a*b^3*d^2*4i - a^3*b*d^2*4i - 6*a^2*b^2*d^2)))^(1/2) + (16*(e*cot(c + d
*x))^(1/2)*(9*a^12*e^24 + 2*b^12*e^24 + 4*a^2*b^10*e^24 + 2*a^4*b^8*e^24 -
49*a^6*b^6*e^24 + 7*a^8*b^4*e^24 + 33*a^10*b^2*e^24))/(b^11*d^4 + 4*a^2*b^9
*d^4 + 6*a^4*b^7*d^4 + 4*a^6*b^5*d^4 + a^8*b^3*d^4))*(-(e^7*1i)/(4*(a^4*d^2
 + b^4*d^2 + a*b^3*d^2*4i - a^3*b*d^2*4i - 6*a^2*b^2*d^2)))^(1/2)*1i)/((32*
(7*a^3*b^7*e^28 + 3*a^5*b^5*e^28))/(b^11*d^5 + 4*a^2*b^9*d^5 + 6*a^4*b^7*d^
5 + 4*a^6*b^5*d^5 + a^8*b^3*d^5) + (((((16*(8*a*b^17*d^4*e^14 + 96*a^3*b^15
*d^4*e^14 + 360*a^5*b^13*d^4*e^14 + 640*a^7*b^11*d^4*e^14 + 600*a^9*b^9*d^4
*e^14 + 288*a^11*b^7*d^4*e^14 + 56*a^13*b^5*d^4*e^14))/(b^11*d^5 + 4*a^2*b^
9*d^5 + 6*a^4*b^7*d^5 + 4*a^6*b^5*d^5 + a^8*b^3*d^5) - (16*(e*cot(c + d*x))
^(1/2)*(-(e^7*1i)/(4*(a^4*d^2 + b^4*d^2 + a*b^3*d^2*4i - a^3*b*d^2*4i - 6*a
^2*b^2*d^2)))^(1/2)*(32*b^20*d^4*e^10 + 160*a^2*b^18*d^4*e^10 + 288*a^4*b^1
6*d^4*e^10 + 160*a^6*b^14*d^4*e^10 - 160*a^8*b^12*d^4*e^10 - 288*a^10*b^10*
d^4*e^10 - 160*a^12*b^8*d^4*e^10 - 32*a^14*b^6*d^4*e^10))/(b^11*d^4 + 4*a^2
*b^9*d^4 + 6*a^4*b^7*d^4 + 4*a^6*b^5*d^4 + a^8*b^3*d^4))*(-(e^7*1i)/(4*(a^4
*d^2 + b^4*d^2 + a*b^3*d^2*4i - a^3*b*d^2*4i - 6*a^2*b^2*d^2)))^(1/2) + (16
*(e*cot(c + d*x))^(1/2)*(72*a^15*b*d^2*e^17 - 60*a*b^15*d^2*e^17 - 52*a^3*b
^13*d^2*e^17 + 72*a^5*b^11*d^2*e^17 + 448*a^7*b^9*d^2*e^17 + 1108*a^9*b^7*d
^2*e^17 + 1132*a^11*b^5*d^2*e^17 + 480*a^13*b^3*d^2*e^17))/(b^11*d^4 + 4*a^
2*b^9*d^4 + 6*a^4*b^7*d^4 + 4*a^6*b^5*d^4 + a^8*b^3*d^4))*(-(e^7*1i)/(4*(a^
4*d^2 + b^4*d^2 + a*b^3*d^2*4i - a^3*b*d^2*4i - 6*a^2*b^2*d^2)))^(1/2) - (1
6*(30*a^6*b^8*d^2*e^21 - 224*a^4*b^10*d^2*e^21 - 18*a^14*d^2*e^21 + 600*a^8
*b^6*d^2*e^21 + 388*a^10*b^4*d^2*e^21 + 24*a^12*b^2*d^2*e^21))/(b^11*d^5 +
4*a^2*b^9*d^5 + 6*a^4*b^7*d^5 + 4*a^6*b^5*d^5 + a^8*b^3*d^5))*(-(e^7*1i)/(4
*(a^4*d^2 + b^4*d^2 + a*b^3*d^2*4i - a^3*b*d^2*4i - 6*a^2*b^2*d^2)))^(1/2)
- (16*(e*cot(c + d*x))^(1/2)*(9*a^12*e^24 + 2*b^12*e^24 + 4*a^2*b^10*e^24 +
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2*a^4*b^8*e^24 - 49*a^6*b^6*e^24 + 7*a^8*b^4*e^24 + 33*a^10*b^2*e^24))/(b^11*d^4 + 4*a^2*b^9*d^4 + 6*a^4*b^7*d^4 + 4*a^6*b^5*d^4 + a^8*b^3*d^4))*(-(e^7*1i)/(4*(a^4*d^2 + b^4*d^2 + a*b^3*d^2*4i - a^3*b*d^2*4i - 6*a^2*b^2*d^2)))^(1/2) + (((((16*(8*a*b^17*d^4*e^14 + 96*a^3*b^15*d^4*e^14 + 360*a^5*b^13*d^4*e^14 + 640*a^7*b^11*d^4*e^14 + 600*a^9*b^9*d^4*e^14 + 288*a^11*b^7*d^4*e^14 + 56*a^13*b^5*d^4*e^14))/(b^11*d^5 + 4*a^2*b^9*d^5 + 6*a^4*b^7*d^5 + 4*a^6*b^5*d^5 + a^8*b^3*d^5) + (16*(e*cot(c + d*x))^(1/2)*(-(e^7*1i)/(4*(a^4*d^2 + b^4*d^2 + a*b^3*d^2*4i - a^3*b*d^2*4i - 6*a^2*b^2*d^2)))^(1/2)*(32*b^20*d^4*e^10 + 160*a^2*b^18*d^4*e^10 + 288*a^4*b^16*d^4*e^10 + 160*a^6*b^14*d^4*e^10 - 160*a^8*b^12*d^4*e^10 - 288*a^10*b^10*d^4*e^10 - 160*a^12*b^8*d^4*e^10 - 32*a^14*b^6*d^4*e^10))/(b^11*d^4 + 4*a^2*b^9*d^4 + 6*a^4*b^7*d^4 + 4*a^6*b^5*d^4 + a^8*b^3*d^4))*(-(e^7*1i)/(4*(a^4*d^2 + b^4*d^2 + a*b^3*d^2*4i - a^3*b*d^2*4i - 6*a^2*b^2*d^2)))^(1/2) - (16*(e*cot(c + d*x))^(1/2)*(72*a^15*b*d^2*e^17 - 60*a*b^15*d^2*e^17 - 52*a^3*b^13*d^2*e^17 + 72*a^5*b^11*d^2*e^17 + 448*a^7*b^9*d^2*e^17 + 1108*a^9*b^7*d^2*e^17 + 1132*a^11*b^5*d^2*e^17 + 480*a^13*b^3*d^2*e^17))/(b^11*d^4 + 4*a^2*b^9*d^4 + 6*a^4*b^7*d^4 + 4*a^6*b^5*d^4 + a^8*b^3*d^4))*(-(e^7*1i)/(4*(a^4*d^2 + b^4*d^2 + a*b^3*d^2*4i - a^3*b*d^2*4i - 6*a^2*b^2*d^2)))^(1/2) - (16*(30*a^6*b^8*d^2*e^21 - 224*a^4*b^10*d^2*e^21 - 18*a^14*d^2*e^21 + 600*a^8*b^6*d^2*e^21 + 388*a^10*b^4*d^2*e^21 + 24*a^12*b^2*d^2*e^21))/(b^11*d^5 + 4*a^2*b^9*d^5 + 6*a^4*b^7*d^5 + 4*a^6*b^5*d^5 + a^8*b^3*d^5))*(-(e^7*1i)/(4*(a^4*d^2 + b^4*d^2 + a*b^3*d^2*4i - a^3*b*d^2*4i - 6*a^2*b^2*d^2)))^(1/2) + (16*(e*cot(c + d*x))^(1/2)*(9*a^12*e^24 + 2*b^12*e^24 + 4*a^2*b^10*e^24 + 2*a^4*b^8*e^24 - 49*a^6*b^6*e^24 + 7*a^8*b^4*e^24 + 33*a^10*b^2*e^24))/(b^11*d^4 + 4*a^2*b^9*d^4 + 6*a^4*b^7*d^4 + 4*a^6*b^5*d^4 + a^8*b^3*d^4))*(-(e^7*1i)/(4*(a^4*d^2 + b^4*d^2 + a*b^3*d^2*4i - a^3*b*d^2*4i - 6*a^2*b^2*d^2)))^(1/2)))*(-(e^7*1i)/(4*(a^4*d^2 + b^4*d^2 + a*b^3*d^2*4i - a^3*b*d^2*4i - 6*a^2*b^2*d^2)))^(1/2)*2i - (a^3*e^4*(e*cot(c + d*x))^(1/2))/((a^2 + b^2)*(a*b^2*d*e + b^3*d*e*cot(c + d*x)))

3.76 $\int \frac{(e\cot(c+dx))^{5/2}}{(a+b\cot(c+dx))^2}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 393

$$\begin{aligned}\int \frac{(e\cot(c+dx))^{5/2}}{(a+b\cot(c+dx))^2}\,dx &= -\frac{a^{3/2}(a^2+5b^2)\,e^{5/2}\arctan\left(\frac{\sqrt{b}\sqrt{e\cot(c+dx)}}{\sqrt{a}\sqrt{e}}\right)}{b^{3/2}\,(a^2+b^2)^2\,d} \\ &- \frac{(a^2+2ab-b^2)\,e^{5/2}\arctan\left(1-\frac{\sqrt{2}\sqrt{e\cot(c+dx)}}{\sqrt{e}}\right)}{\sqrt{2}\,(a^2+b^2)^2\,d} \\ &+ \frac{(a^2+2ab-b^2)\,e^{5/2}\arctan\left(1+\frac{\sqrt{2}\sqrt{e\cot(c+dx)}}{\sqrt{e}}\right)}{\sqrt{2}\,(a^2+b^2)^2\,d} + \frac{a^2e^2\sqrt{e\cot(c+dx)}}{b\,(a^2+b^2)\,d(a+b\cot(c+dx))} \\ &+ \frac{(a^2-2ab-b^2)\,e^{5/2}\log\left(\sqrt{e}+\sqrt{e}\cot(c+dx)-\sqrt{2}\sqrt{e\cot(c+dx)}\right)}{2\sqrt{2}\,(a^2+b^2)^2\,d} \\ &- \frac{(a^2-2ab-b^2)\,e^{5/2}\log\left(\sqrt{e}+\sqrt{e}\cot(c+dx)+\sqrt{2}\sqrt{e\cot(c+dx)}\right)}{2\sqrt{2}\,(a^2+b^2)^2\,d}\end{aligned}$$

```
[Out] -a^(3/2)*(a^2+5*b^2)*e^(5/2)*arctan(b^(1/2)*(e*cot(d*x+c))^(1/2)/a^(1/2)/e^
(1/2))/b^(3/2)/(a^2+b^2)^2/d-1/2*(a^2+2*a*b-b^2)*e^(5/2)*arctan(1-2^(1/2)*(
e*cot(d*x+c))^(1/2)/e^(1/2))/(a^2+b^2)^2/d*2^(1/2)+1/2*(a^2+2*a*b-b^2)*e^(5
/2)*arctan(1+2^(1/2)*(e*cot(d*x+c))^(1/2)/e^(1/2))/(a^2+b^2)^2/d*2^(1/2)+1/
4*(a^2-2*a*b-b^2)*e^(5/2)*ln(e^(1/2)+cot(d*x+c)*e^(1/2)-2^(1/2)*(e*cot(d*x+
c))^(1/2))/(a^2+b^2)^2/d*2^(1/2)-1/4*(a^2-2*a*b-b^2)*e^(5/2)*ln(e^(1/2)+cot
(d*x+c)*e^(1/2)+2^(1/2)*(e*cot(d*x+c))^(1/2))/(a^2+b^2)^2/d*2^(1/2)+a^2*e^2
*(e*cot(d*x+c))^(1/2)/b/(a^2+b^2)/d/(a+b*cot(d*x+c))
```

Rubi [A] (verified)

Time = 1.09 (sec) , antiderivative size = 393, normalized size of antiderivative = 1.00, number of steps used = 15, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.480, Rules used = {3646, 3734, 3615, 1182, 1176, 631, 210, 1179, 642, 3715, 65, 211}

$$\int \frac{(e\cot(c+dx))^{5/2}}{(a+b\cot(c+dx))^2}\,dx = -\frac{e^{5/2}(a^2+2ab-b^2)\arctan\left(1-\frac{\sqrt{2}\sqrt{e\cot(c+dx)}}{\sqrt{e}}\right)}{\sqrt{2}d\,(a^2+b^2)^2}$$
$$+\frac{e^{5/2}(a^2+2ab-b^2)\arctan\left(\frac{\sqrt{2}\sqrt{e\cot(c+dx)}}{\sqrt{e}}+1\right)}{\sqrt{2}d\,(a^2+b^2)^2}$$
$$+\frac{e^{5/2}(a^2-2ab-b^2)\log\left(\sqrt{e}\cot(c+dx)-\sqrt{2}\sqrt{e\cot(c+dx)}+\sqrt{e}\right)}{2\sqrt{2}d\,(a^2+b^2)^2}$$
$$-\frac{e^{5/2}(a^2-2ab-b^2)\log\left(\sqrt{e}\cot(c+dx)+\sqrt{2}\sqrt{e\cot(c+dx)}+\sqrt{e}\right)}{2\sqrt{2}d\,(a^2+b^2)^2}$$
$$+\frac{a^2e^2\sqrt{e\cot(c+dx)}}{bd\,(a^2+b^2)\,(a+b\cot(c+dx))}-\frac{a^{3/2}e^{5/2}(a^2+5b^2)\arctan\left(\frac{\sqrt{b}\sqrt{e\cot(c+dx)}}{\sqrt{a}\sqrt{e}}\right)}{b^{3/2}d\,(a^2+b^2)^2}$$

```
[In] Int[(e*Cot[c + d*x])^(5/2)/(a + b*Cot[c + d*x])^2,x]
```

```
[Out] -((a^(3/2)*(a^2 + 5*b^2)*e^(5/2)*ArcTan[(Sqrt[b]*Sqrt[e*Cot[c + d*x]])/(Sqr
t[a]*Sqrt[e])])/(b^(3/2)*(a^2 + b^2)^2*d)) - ((a^2 + 2*a*b - b^2)*e^(5/2)*A
rcTan[1 - (Sqrt[2]*Sqrt[e*Cot[c + d*x]])/Sqrt[e]])/(Sqrt[2]*(a^2 + b^2)^2*d
) + ((a^2 + 2*a*b - b^2)*e^(5/2)*ArcTan[1 + (Sqrt[2]*Sqrt[e*Cot[c + d*x]])/
Sqrt[e]])/(Sqrt[2]*(a^2 + b^2)^2*d) + (a^2*e^2*Sqrt[e*Cot[c + d*x]])/(b*(a^
2 + b^2)*d*(a + b*Cot[c + d*x])) + ((a^2 - 2*a*b - b^2)*e^(5/2)*Log[Sqrt[e]
 + Sqrt[e]*Cot[c + d*x] - Sqrt[2]*Sqrt[e*Cot[c + d*x]]])/(2*Sqrt[2]*(a^2 +
b^2)^2*d) - ((a^2 - 2*a*b - b^2)*e^(5/2)*Log[Sqrt[e] + Sqrt[e]*Cot[c + d*x]
 + Sqrt[2]*Sqrt[e*Cot[c + d*x]]])/(2*Sqrt[2]*(a^2 + b^2)^2*d)
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1179

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 1182

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
a*c, 2]}, Dist[(d*q + a*e)/(2*a*c), Int[(q + c*x^2)/(a + c*x^4), x], x] + D
ist[(d*q - a*e)/(2*a*c), Int[(q - c*x^2)/(a + c*x^4), x], x]] /; FreeQ[{a,
c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0] && NeQ[c*d^2 - a*e^2, 0] && NegQ[(-a)
*c]
```

Rule 3615

```
Int[((c_) + (d_.)*tan[(e_.) + (f_.)*(x_)])/Sqrt[(b_.)*tan[(e_.) + (f_.)*(x_
)]], x_Symbol] :> Dist[2/f, Subst[Int[(b*c + d*x^2)/(b^2 + x^4), x], x, Sqr
t[b*Tan[e + f*x]]], x] /; FreeQ[{b, c, d, e, f}, x] && NeQ[c^2 - d^2, 0] &&
 NeQ[c^2 + d^2, 0]
```

Rule 3646

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*c - a*d)^2*(a + b*Tan[e + f*x])^(m
 - 2)*((c + d*Tan[e + f*x])^(n + 1)/(d*f*(n + 1)*(c^2 + d^2))), x] - Dist[1
/(d*(n + 1)*(c^2 + d^2)), Int[(a + b*Tan[e + f*x])^(m - 3)*(c + d*Tan[e + f
*x])^(n + 1)*Simp[a^2*d*(b*d*(m - 2) - a*c*(n + 1)) + b*(b*c - 2*a*d)*(b*c*
(m - 2) + a*d*(n + 1)) - d*(n + 1)*(3*a^2*b*c - b^3*c - a^3*d + 3*a*b^2*d)*
Tan[e + f*x] - b*(a*d*(2*b*c - a*d)*(m + n - 1) - b^2*(c^2*(m - 2) - d^2*(n
 + 1)))*Tan[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b
*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] && GtQ[m, 2] && LtQ[
n, -1] && IntegerQ[2*m]
```

Rule 3715

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*tan[(e_.)
+ (f_.)*(x_)])^(n_.)*((A_) + (C_.)*tan[(e_.) + (f_.)*(x_)]^2), x_Symbol] :>
 Dist[A/f, Subst[Int[(a + b*x)^m*(c + d*x)^n, x], x, Tan[e + f*x]], x] /; F
reeQ[{a, b, c, d, e, f, A, C, m, n}, x] && EqQ[A, C]
```

Rule 3734

```
Int[(((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_)*((A_.) + (B_.)*tan[(e_.)
+ (f_.)*(x_)] + (C_.)*tan[(e_.) + (f_.)*(x_)]^2))/((a_.) + (b_.)*tan[(e_.)
+ (f_.)*(x_)]), x_Symbol] :> Dist[1/(a^2 + b^2), Int[(c + d*Tan[e + f*x])^n
*Simp[b*B + a*(A - C) + (a*B - b*(A - C))*Tan[e + f*x], x], x], x] + Dist[(
A*b^2 - a*b*B + a^2*C)/(a^2 + b^2), Int[(c + d*Tan[e + f*x])^n*((1 + Tan[e
+ f*x]^2)/(a + b*Tan[e + f*x])), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C
, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] &&
 !GtQ[n, 0] &&  !LeQ[n, -1]
```

Rubi steps

$$\text{integral} = \frac{a^2e^2\sqrt{e\cot(c+dx)}}{b\,(a^2+b^2)\,d(a+b\cot(c+dx))} - \frac{\int \frac{-\frac{1}{2}a^2e^3+abe^3\cot(c+dx)-\frac{1}{2}(a^2+2b^2)e^3\cot^2(c+dx)}{\sqrt{e\cot(c+dx)}(a+b\cot(c+dx))}\,dx}{b\,(a^2+b^2)}$$

$$= \frac{a^2e^2\sqrt{e\cot(c+dx)}}{b\,(a^2+b^2)\,d(a+b\cot(c+dx))} - \frac{\int \frac{2ab^2e^3+b(a^2-b^2)e^3\cot(c+dx)}{\sqrt{e\cot(c+dx)}}\,dx}{b\,(a^2+b^2)^2} + \frac{(a^2(a^2+5b^2)\,e^3)\int \frac{1+\cot^2(c+dx)}{\sqrt{e\cot(c+dx)}(a+b\cot(c+dx))}\,dx}{2b\,(a^2+b^2)^2}$$

$$= \frac{a^2e^2\sqrt{e\cot(c+dx)}}{b(a^2+b^2)d(a+b\cot(c+dx))} - \frac{2\text{Subst}\left(\int \frac{-2ab^2e^4-b(a^2-b^2)e^3x^2}{e^2+x^4}\,dx, x, \sqrt{e\cot(c+dx)}\right)}{b(a^2+b^2)^2 d} + \frac{(a^2(a^2+5b^2)e^3)\text{Subst}\left(\int \frac{1}{\sqrt{-ex}(a-bx)}\,dx, x, -\cot(c+dx)\right)}{2b(a^2+b^2)^2 d}$$

$$= \frac{a^2e^2\sqrt{e\cot(c+dx)}}{b(a^2+b^2)d(a+b\cot(c+dx))} - \frac{(a^2(a^2+5b^2)e^2)\text{Subst}\left(\int \frac{1}{a+\frac{bx^2}{e}}\,dx, x, \sqrt{e\cot(c+dx)}\right)}{b(a^2+b^2)^2 d} - \frac{((a^2-2ab-b^2)e^3)\text{Subst}\left(\int \frac{e-x^2}{e^2+x^4}\,dx, x, \sqrt{e\cot(c+dx)}\right)}{(a^2+b^2)^2 d} + \frac{((a^2+2ab-b^2)e^3)\text{Subst}\left(\int \frac{e+x^2}{e^2+x^4}\,dx, x, \sqrt{e\cot(c+dx)}\right)}{(a^2+b^2)^2 d}$$

$$= -\frac{a^{3/2}(a^2+5b^2)e^{5/2}\arctan\left(\frac{\sqrt{b}\sqrt{e\cot(c+dx)}}{\sqrt{a}\sqrt{e}}\right)}{b^{3/2}(a^2+b^2)^2 d} + \frac{a^2e^2\sqrt{e\cot(c+dx)}}{b(a^2+b^2)d(a+b\cot(c+dx))} + \frac{((a^2-2ab-b^2)e^{5/2})\text{Subst}\left(\int \frac{\sqrt{2}\sqrt{e}+2x}{-e-\sqrt{2}\sqrt{e}x-x^2}\,dx, x, \sqrt{e\cot(c+dx)}\right)}{2\sqrt{2}(a^2+b^2)^2 d} + \frac{((a^2-2ab-b^2)e^{5/2})\text{Subst}\left(\int \frac{\sqrt{2}\sqrt{e}-2x}{-e+\sqrt{2}\sqrt{e}x-x^2}\,dx, x, \sqrt{e\cot(c+dx)}\right)}{2\sqrt{2}(a^2+b^2)^2 d} + \frac{((a^2+2ab-b^2)e^3)\text{Subst}\left(\int \frac{1}{e-\sqrt{2}\sqrt{e}x+x^2}\,dx, x, \sqrt{e\cot(c+dx)}\right)}{2(a^2+b^2)^2 d} + \frac{((a^2+2ab-b^2)e^3)\text{Subst}\left(\int \frac{1}{e+\sqrt{2}\sqrt{e}x+x^2}\,dx, x, \sqrt{e\cot(c+dx)}\right)}{2(a^2+b^2)^2 d}$$

$$= -\frac{a^{3/2}(a^2+5b^2)\,e^{5/2}\arctan\left(\frac{\sqrt{b}\sqrt{e\cot(c+dx)}}{\sqrt{a}\sqrt{e}}\right)}{b^{3/2}\left(a^2+b^2\right)^2 d} + \frac{a^2e^2\sqrt{e\cot(c+dx)}}{b\left(a^2+b^2\right)d(a+b\cot(c+dx))}$$
$$+ \frac{(a^2-2ab-b^2)\,e^{5/2}\log\left(\sqrt{e}+\sqrt{e}\cot(c+dx)-\sqrt{2}\sqrt{e\cot(c+dx)}\right)}{2\sqrt{2}\left(a^2+b^2\right)^2 d}$$
$$- \frac{(a^2-2ab-b^2)\,e^{5/2}\log\left(\sqrt{e}+\sqrt{e}\cot(c+dx)+\sqrt{2}\sqrt{e\cot(c+dx)}\right)}{2\sqrt{2}\left(a^2+b^2\right)^2 d}$$
$$+ \frac{\left((a^2+2ab-b^2)\,e^{5/2}\right)\text{Subst}\left(\int\frac{1}{-1-x^2}\,dx, x, 1-\frac{\sqrt{2}\sqrt{e\cot(c+dx)}}{\sqrt{e}}\right)}{\sqrt{2}\left(a^2+b^2\right)^2 d}$$
$$- \frac{\left((a^2+2ab-b^2)\,e^{5/2}\right)\text{Subst}\left(\int\frac{1}{-1-x^2}\,dx, x, 1+\frac{\sqrt{2}\sqrt{e\cot(c+dx)}}{\sqrt{e}}\right)}{\sqrt{2}\left(a^2+b^2\right)^2 d}$$

$$= -\frac{a^{3/2}(a^2+5b^2)\,e^{5/2}\arctan\left(\frac{\sqrt{b}\sqrt{e\cot(c+dx)}}{\sqrt{a}\sqrt{e}}\right)}{b^{3/2}\left(a^2+b^2\right)^2 d}$$
$$- \frac{(a^2+2ab-b^2)\,e^{5/2}\arctan\left(1-\frac{\sqrt{2}\sqrt{e\cot(c+dx)}}{\sqrt{e}}\right)}{\sqrt{2}\left(a^2+b^2\right)^2 d}$$
$$+ \frac{(a^2+2ab-b^2)\,e^{5/2}\arctan\left(1+\frac{\sqrt{2}\sqrt{e\cot(c+dx)}}{\sqrt{e}}\right)}{\sqrt{2}\left(a^2+b^2\right)^2 d} + \frac{a^2e^2\sqrt{e\cot(c+dx)}}{b\left(a^2+b^2\right)d(a+b\cot(c+dx))}$$
$$+ \frac{(a^2-2ab-b^2)\,e^{5/2}\log\left(\sqrt{e}+\sqrt{e}\cot(c+dx)-\sqrt{2}\sqrt{e\cot(c+dx)}\right)}{2\sqrt{2}\left(a^2+b^2\right)^2 d}$$
$$- \frac{(a^2-2ab-b^2)\,e^{5/2}\log\left(\sqrt{e}+\sqrt{e}\cot(c+dx)+\sqrt{2}\sqrt{e\cot(c+dx)}\right)}{2\sqrt{2}\left(a^2+b^2\right)^2 d}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 3.05 (sec) , antiderivative size = 390, normalized size of antiderivative = 0.99

$$\int \frac{(e\cot(c+dx))^{5/2}}{(a+b\cot(c+dx))^2}\,dx =$$
$$-\frac{(e\cot(c+dx))^{5/2}\left(-28a^2b^{3/2}(a^2-b^2)\cot^{\frac{3}{2}}(c+dx)\,\text{Hypergeometric2F1}\left(\frac{3}{4},1,\frac{7}{4},-\cot^2(c+dx)\right)+12b^{7/2}(a\right.}{}$$

```
[In] Integrate[(e*Cot[c + d*x])^(5/2)/(a + b*Cot[c + d*x])^2,x]
```

```
[Out] -1/42*((e*Cot[c + d*x])^(5/2)*(-28*a^2*b^(3/2)*(a^2 - b^2)*Cot[c + d*x]^(3/
2)*Hypergeometric2F1[3/4, 1, 7/4, -Cot[c + d*x]^2] + 12*b^(7/2)*(a^2 + b^2)
```

```
*Cot[c + d*x]^(7/2)*Hypergeometric2F1[2, 7/2, 9/2, -((b*Cot[c + d*x])/a)] -
 7*a^2*(-6*Sqrt[2]*a*b^(5/2)*ArcTan[1 - Sqrt[2]*Sqrt[Cot[c + d*x]]] + 6*Sqr
t[2]*a*b^(5/2)*ArcTan[1 + Sqrt[2]*Sqrt[Cot[c + d*x]]] + 24*a^(7/2)*ArcTan[(
Sqrt[b]*Sqrt[Cot[c + d*x]])/Sqrt[a]] - 24*a^3*Sqrt[b]*Sqrt[Cot[c + d*x]] -
24*a*b^(5/2)*Sqrt[Cot[c + d*x]] + 4*a^2*b^(3/2)*Cot[c + d*x]^(3/2) + 4*b^(7
/2)*Cot[c + d*x]^(3/2) - 3*Sqrt[2]*a*b^(5/2)*Log[1 - Sqrt[2]*Sqrt[Cot[c + d
*x]] + Cot[c + d*x]] + 3*Sqrt[2]*a*b^(5/2)*Log[1 + Sqrt[2]*Sqrt[Cot[c + d*x
]] + Cot[c + d*x]])))/(a^2*b^(3/2)*(a^2 + b^2)^2*d*Cot[c + d*x]^(5/2))
```

Maple [A] (verified)

Time = 0.30 (sec) , antiderivative size = 387, normalized size of antiderivative = 0.98

method	result
derivativedivides	$-\frac{2e^3\left(\frac{a^2\left(-\frac{(a^2+b^2)\sqrt{e\cot(dx+c)}}{2b(e\cot(dx+c)b+ae)}+\frac{(a^2+5b^2)\arctan\left(\frac{\sqrt{e\cot(dx+c)}\,b}{\sqrt{aeb}}\right)}{2b\sqrt{aeb}}\right)}{(a^2+b^2)^2}+\frac{-\frac{ab(e^2)^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\frac{e\cot(dx+c)+(e^2)^{\frac{1}{4}}\sqrt{e\cot(dx}}{e\cot(dx+c)-(e^2)^{\frac{1}{4}}\sqrt{e\cot(dx}}\right.\right.}{}}{}\right.}{} \cdots$
default	$-\frac{2e^3\left(\frac{a^2\left(-\frac{(a^2+b^2)\sqrt{e\cot(dx+c)}}{2b(e\cot(dx+c)b+ae)}+\frac{(a^2+5b^2)\arctan\left(\frac{\sqrt{e\cot(dx+c)}\,b}{\sqrt{aeb}}\right)}{2b\sqrt{aeb}}\right)}{(a^2+b^2)^2}+\frac{-\frac{ab(e^2)^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\frac{e\cot(dx+c)+(e^2)^{\frac{1}{4}}\sqrt{e\cot(dx}}{e\cot(dx+c)-(e^2)^{\frac{1}{4}}\sqrt{e\cot(dx}}\right.\right.}{}}{}\right.}{} \cdots$

```
[In] int((e*cot(d*x+c))^(5/2)/(a+b*cot(d*x+c))^2,x,method=_RETURNVERBOSE)
```

```
[Out] -2/d*e^3*(a^2/(a^2+b^2)^2*(-1/2*(a^2+b^2)/b*(e*cot(d*x+c))^(1/2)/(e*cot(d*x
+c)*b+a*e)+1/2*(a^2+5*b^2)/b/(a*e*b)^(1/2)*arctan((e*cot(d*x+c))^(1/2)*b/(a
*e*b)^(1/2)))+1/(a^2+b^2)^2*(-1/4*a/e*b*(e^2)^(1/4)*2^(1/2)*(ln((e*cot(d*x+
c)+(e^2)^(1/4)*(e*cot(d*x+c))^(1/2)*2^(1/2)+(e^2)^(1/2))/(e*cot(d*x+c)-(e^2
)^(1/4)*(e*cot(d*x+c))^(1/2)*2^(1/2)+(e^2)^(1/2)))+2*arctan(2^(1/2)/(e^2)^(
1/4)*(e*cot(d*x+c))^(1/2)+1)-2*arctan(-2^(1/2)/(e^2)^(1/4)*(e*cot(d*x+c))^(
1/2)+1))+1/8*(-a^2+b^2)/(e^2)^(1/4)*2^(1/2)*(ln((e*cot(d*x+c)-(e^2)^(1/4)*(
e*cot(d*x+c))^(1/2)*2^(1/2)+(e^2)^(1/2))/(e*cot(d*x+c)+(e^2)^(1/4)*(e*cot(d
*x+c))^(1/2)*2^(1/2)+(e^2)^(1/2)))+2*arctan(2^(1/2)/(e^2)^(1/4)*(e*cot(d*x+
c))^(1/2)+1)-2*arctan(-2^(1/2)/(e^2)^(1/4)*(e*cot(d*x+c))^(1/2)+1))))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 3098 vs. 2(330) = 660.

Time = 0.59 (sec) , antiderivative size = 6258, normalized size of antiderivative = 15.92

$$\int \frac{(e\cot(c+dx))^{5/2}}{(a+b\cot(c+dx))^2}\,dx = \text{Too large to display}$$

```
[In] integrate((e*cot(d*x+c))^(5/2)/(a+b*cot(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] Too large to include
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(e\cot(c+dx))^{5/2}}{(a+b\cot(c+dx))^2}\,dx = \text{Timed out}$$

```
[In] integrate((e*cot(d*x+c))**(5/2)/(a+b*cot(d*x+c))**2,x)
```

```
[Out] Timed out
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{(e\cot(c+dx))^{5/2}}{(a+b\cot(c+dx))^2}\,dx = \text{Exception raised: ValueError}$$

```
[In] integrate((e*cot(d*x+c))^(5/2)/(a+b*cot(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(e>0)', see `assume?` for more detai
ls)Is e
```

Giac [F]

$$\int \frac{(e\cot(c+dx))^{5/2}}{(a+b\cot(c+dx))^2}\,dx = \int \frac{(e\cot(dx+c))^{\frac{5}{2}}}{(b\cot(dx+c)+a)^2}\,dx$$

```
[In] integrate((e*cot(d*x+c))^(5/2)/(a+b*cot(d*x+c))^2,x, algorithm="giac")
[Out] integrate((e*cot(d*x + c))^(5/2)/(b*cot(d*x + c) + a)^2, x)
```

Mupad [B] (verification not implemented)

Time = 15.91 (sec) , antiderivative size = 12617, normalized size of antiderivative = 32.10

$$\int \frac{(e\cot(c+dx))^{5/2}}{(a+b\cot(c+dx))^2}\,dx = \text{Too large to display}$$

```
[In] int((e*cot(c + d*x))^(5/2)/(a + b*cot(c + d*x))^2,x)
[Out] atan((((((8*(96*a^2*b^14*d^4*e^13 + 480*a^4*b^12*d^4*e^13 + 960*a^6*b^10*d
^4*e^13 + 960*a^8*b^8*d^4*e^13 + 480*a^10*b^6*d^4*e^13 + 96*a^12*b^4*d^4*e^
13))/(b^9*d^5 + a^8*b*d^5 + 4*a^2*b^7*d^5 + 6*a^4*b^5*d^5 + 4*a^6*b^3*d^5)
- (16*(e*cot(c + d*x))^(1/2)*((e^5*1i)/(4*(a^4*d^2 + b^4*d^2 + a*b^3*d^2*4i
 - a^3*b*d^2*4i - 6*a^2*b^2*d^2)))^(1/2)*(32*b^18*d^4*e^10 + 160*a^2*b^16*d
^4*e^10 + 288*a^4*b^14*d^4*e^10 + 160*a^6*b^12*d^4*e^10 - 160*a^8*b^10*d^4*
e^10 - 288*a^10*b^8*d^4*e^10 - 160*a^12*b^6*d^4*e^10 - 32*a^14*b^4*d^4*e^10
))/(b^9*d^4 + a^8*b*d^4 + 4*a^2*b^7*d^4 + 6*a^4*b^5*d^4 + 4*a^6*b^3*d^4))*(
(e^5*1i)/(4*(a^4*d^2 + b^4*d^2 + a*b^3*d^2*4i - a^3*b*d^2*4i - 6*a^2*b^2*d^
2)))^(1/2) + (16*(e*cot(c + d*x))^(1/2)*(60*a*b^13*d^2*e^15 + 8*a^13*b*d^2*
e^15 + 52*a^3*b^11*d^2*e^15 + 128*a^5*b^9*d^2*e^15 + 424*a^7*b^7*d^2*e^15 +
 380*a^9*b^5*d^2*e^15 + 100*a^11*b^3*d^2*e^15))/(b^9*d^4 + a^8*b*d^4 + 4*a^
2*b^7*d^4 + 6*a^4*b^5*d^4 + 4*a^6*b^3*d^4))*((e^5*1i)/(4*(a^4*d^2 + b^4*d^2
 + a*b^3*d^2*4i - a^3*b*d^2*4i - 6*a^2*b^2*d^2)))^(1/2) + (8*(4*a*b^11*d^2*
e^18 + 16*a^11*b*d^2*e^18 - 304*a^3*b^9*d^2*e^18 - 120*a^5*b^7*d^2*e^18 + 3
20*a^7*b^5*d^2*e^18 + 148*a^9*b^3*d^2*e^18))/(b^9*d^5 + a^8*b*d^5 + 4*a^2*b
^7*d^5 + 6*a^4*b^5*d^5 + 4*a^6*b^3*d^5))*((e^5*1i)/(4*(a^4*d^2 + b^4*d^2 +
a*b^3*d^2*4i - a^3*b*d^2*4i - 6*a^2*b^2*d^2)))^(1/2) + (16*(e*cot(c + d*x))
^(1/2)*(a^10*e^20 - 2*b^10*e^20 - 4*a^2*b^8*e^20 - 27*a^4*b^6*e^20 + 15*a^6
*b^4*e^20 + 9*a^8*b^2*e^20))/(b^9*d^4 + a^8*b*d^4 + 4*a^2*b^7*d^4 + 6*a^4*b
^5*d^4 + 4*a^6*b^3*d^4))*((e^5*1i)/(4*(a^4*d^2 + b^4*d^2 + a*b^3*d^2*4i - a
^3*b*d^2*4i - 6*a^2*b^2*d^2)))^(1/2)*1i - (((((8*(96*a^2*b^14*d^4*e^13 + 48
0*a^4*b^12*d^4*e^13 + 960*a^6*b^10*d^4*e^13 + 960*a^8*b^8*d^4*e^13 + 480*a^
10*b^6*d^4*e^13 + 96*a^12*b^4*d^4*e^13))/(b^9*d^5 + a^8*b*d^5 + 4*a^2*b^7*d
^5 + 6*a^4*b^5*d^5 + 4*a^6*b^3*d^5) + (16*(e*cot(c + d*x))^(1/2)*((e^5*1i)/
(4*(a^4*d^2 + b^4*d^2 + a*b^3*d^2*4i - a^3*b*d^2*4i - 6*a^2*b^2*d^2)))^(1/2
```

)*(32*b^18*d^4*e^10 + 160*a^2*b^16*d^4*e^10 + 288*a^4*b^14*d^4*e^10 + 160*a
^6*b^12*d^4*e^10 - 160*a^8*b^10*d^4*e^10 - 288*a^10*b^8*d^4*e^10 - 160*a^12
*b^6*d^4*e^10 - 32*a^14*b^4*d^4*e^10))/(b^9*d^4 + a^8*b*d^4 + 4*a^2*b^7*d^4
 + 6*a^4*b^5*d^4 + 4*a^6*b^3*d^4))*((e^5*1i)/(4*(a^4*d^2 + b^4*d^2 + a*b^3*
d^2*4i - a^3*b*d^2*4i - 6*a^2*b^2*d^2)))^(1/2) - (16*(e*cot(c + d*x))^(1/2)
*(60*a*b^13*d^2*e^15 + 8*a^13*b*d^2*e^15 + 52*a^3*b^11*d^2*e^15 + 128*a^5*b
^9*d^2*e^15 + 424*a^7*b^7*d^2*e^15 + 380*a^9*b^5*d^2*e^15 + 100*a^11*b^3*d^
2*e^15))/(b^9*d^4 + a^8*b*d^4 + 4*a^2*b^7*d^4 + 6*a^4*b^5*d^4 + 4*a^6*b^3*d
^4))*((e^5*1i)/(4*(a^4*d^2 + b^4*d^2 + a*b^3*d^2*4i - a^3*b*d^2*4i - 6*a^2*
b^2*d^2)))^(1/2) + (8*(4*a*b^11*d^2*e^18 + 16*a^11*b*d^2*e^18 - 304*a^3*b^9
*d^2*e^18 - 120*a^5*b^7*d^2*e^18 + 320*a^7*b^5*d^2*e^18 + 148*a^9*b^3*d^2*e
^18))/(b^9*d^5 + a^8*b*d^5 + 4*a^2*b^7*d^5 + 6*a^4*b^5*d^5 + 4*a^6*b^3*d^5)
)*((e^5*1i)/(4*(a^4*d^2 + b^4*d^2 + a*b^3*d^2*4i - a^3*b*d^2*4i - 6*a^2*b^2
d^2)))^(1/2) - (16(e*cot(c + d*x))^(1/2)*(a^10*e^20 - 2*b^10*e^20 - 4*a^2
*b^8*e^20 - 27*a^4*b^6*e^20 + 15*a^6*b^4*e^20 + 9*a^8*b^2*e^20))/(b^9*d^4 +
 a^8*b*d^4 + 4*a^2*b^7*d^4 + 6*a^4*b^5*d^4 + 4*a^6*b^3*d^4))*((e^5*1i)/(4*(
a^4*d^2 + b^4*d^2 + a*b^3*d^2*4i - a^3*b*d^2*4i - 6*a^2*b^2*d^2)))^(1/2)*1i
)/((16*(a^8*e^23 + 10*a^2*b^6*e^23 + 27*a^4*b^4*e^23 + 10*a^6*b^2*e^23))/(b
^9*d^5 + a^8*b*d^5 + 4*a^2*b^7*d^5 + 6*a^4*b^5*d^5 + 4*a^6*b^3*d^5) + (((((
8*(96*a^2*b^14*d^4*e^13 + 480*a^4*b^12*d^4*e^13 + 960*a^6*b^10*d^4*e^13 + 9
60*a^8*b^8*d^4*e^13 + 480*a^10*b^6*d^4*e^13 + 96*a^12*b^4*d^4*e^13))/(b^9*d
^5 + a^8*b*d^5 + 4*a^2*b^7*d^5 + 6*a^4*b^5*d^5 + 4*a^6*b^3*d^5) - (16*(e*co
t(c + d*x))^(1/2)*((e^5*1i)/(4*(a^4*d^2 + b^4*d^2 + a*b^3*d^2*4i - a^3*b*d^
2*4i - 6*a^2*b^2*d^2)))^(1/2)*(32*b^18*d^4*e^10 + 160*a^2*b^16*d^4*e^10 + 2
88*a^4*b^14*d^4*e^10 + 160*a^6*b^12*d^4*e^10 - 160*a^8*b^10*d^4*e^10 - 288*
a^10*b^8*d^4*e^10 - 160*a^12*b^6*d^4*e^10 - 32*a^14*b^4*d^4*e^10))/(b^9*d^4
 + a^8*b*d^4 + 4*a^2*b^7*d^4 + 6*a^4*b^5*d^4 + 4*a^6*b^3*d^4))*((e^5*1i)/(4
*(a^4*d^2 + b^4*d^2 + a*b^3*d^2*4i - a^3*b*d^2*4i - 6*a^2*b^2*d^2)))^(1/2)
+ (16*(e*cot(c + d*x))^(1/2)*(60*a*b^13*d^2*e^15 + 8*a^13*b*d^2*e^15 + 52*a
^3*b^11*d^2*e^15 + 128*a^5*b^9*d^2*e^15 + 424*a^7*b^7*d^2*e^15 + 380*a^9*b^
5*d^2*e^15 + 100*a^11*b^3*d^2*e^15))/(b^9*d^4 + a^8*b*d^4 + 4*a^2*b^7*d^4 +
 6*a^4*b^5*d^4 + 4*a^6*b^3*d^4))*((e^5*1i)/(4*(a^4*d^2 + b^4*d^2 + a*b^3*d^
2*4i - a^3*b*d^2*4i - 6*a^2*b^2*d^2)))^(1/2) + (8*(4*a*b^11*d^2*e^18 + 16*a
^11*b*d^2*e^18 - 304*a^3*b^9*d^2*e^18 - 120*a^5*b^7*d^2*e^18 + 320*a^7*b^5*
d^2*e^18 + 148*a^9*b^3*d^2*e^18))/(b^9*d^5 + a^8*b*d^5 + 4*a^2*b^7*d^5 + 6*
a^4*b^5*d^5 + 4*a^6*b^3*d^5))*((e^5*1i)/(4*(a^4*d^2 + b^4*d^2 + a*b^3*d^2*4
i - a^3*b*d^2*4i - 6*a^2*b^2*d^2)))^(1/2) + (16*(e*cot(c + d*x))^(1/2)*(a^1
0*e^20 - 2*b^10*e^20 - 4*a^2*b^8*e^20 - 27*a^4*b^6*e^20 + 15*a^6*b^4*e^20 +
 9*a^8*b^2*e^20))/(b^9*d^4 + a^8*b*d^4 + 4*a^2*b^7*d^4 + 6*a^4*b^5*d^4 + 4*
a^6*b^3*d^4))*((e^5*1i)/(4*(a^4*d^2 + b^4*d^2 + a*b^3*d^2*4i - a^3*b*d^2*4i
 - 6*a^2*b^2*d^2)))^(1/2) + (((((8*(96*a^2*b^14*d^4*e^13 + 480*a^4*b^12*d^4
*e^13 + 960*a^6*b^10*d^4*e^13 + 960*a^8*b^8*d^4*e^13 + 480*a^10*b^6*d^4*e^1
3 + 96*a^12*b^4*d^4*e^13))/(b^9*d^5 + a^8*b*d^5 + 4*a^2*b^7*d^5 + 6*a^4*b^5
*d^5 + 4*a^6*b^3*d^5) + (16*(e*cot(c + d*x))^(1/2)*((e^5*1i)/(4*(a^4*d^2 +
b^4*d^2 + a*b^3*d^2*4i - a^3*b*d^2*4i - 6*a^2*b^2*d^2)))^(1/2)*(32*b^18*d^4

*e^10 + 160*a^2*b^16*d^4*e^10 + 288*a^4*b^14*d^4*e^10 + 160*a^6*b^12*d^4*e^
10 - 160*a^8*b^10*d^4*e^10 - 288*a^10*b^8*d^4*e^10 - 160*a^12*b^6*d^4*e^10
- 32*a^14*b^4*d^4*e^10))/(b^9*d^4 + a^8*b*d^4 + 4*a^2*b^7*d^4 + 6*a^4*b^5*d
^4 + 4*a^6*b^3*d^4))*((e^5*1i)/(4*(a^4*d^2 + b^4*d^2 + a*b^3*d^2*4i - a^3*b
*d^2*4i - 6*a^2*b^2*d^2)))^(1/2) - (16*(e*cot(c + d*x))^(1/2)*(60*a*b^13*d^
2*e^15 + 8*a^13*b*d^2*e^15 + 52*a^3*b^11*d^2*e^15 + 128*a^5*b^9*d^2*e^15 +
424*a^7*b^7*d^2*e^15 + 380*a^9*b^5*d^2*e^15 + 100*a^11*b^3*d^2*e^15))/(b^9*
d^4 + a^8*b*d^4 + 4*a^2*b^7*d^4 + 6*a^4*b^5*d^4 + 4*a^6*b^3*d^4))*((e^5*1i)
/(4*(a^4*d^2 + b^4*d^2 + a*b^3*d^2*4i - a^3*b*d^2*4i - 6*a^2*b^2*d^2)))^(1/
2) + (8*(4*a*b^11*d^2*e^18 + 16*a^11*b*d^2*e^18 - 304*a^3*b^9*d^2*e^18 - 12
0*a^5*b^7*d^2*e^18 + 320*a^7*b^5*d^2*e^18 + 148*a^9*b^3*d^2*e^18))/(b^9*d^5
 + a^8*b*d^5 + 4*a^2*b^7*d^5 + 6*a^4*b^5*d^5 + 4*a^6*b^3*d^5))*((e^5*1i)/(4
*(a^4*d^2 + b^4*d^2 + a*b^3*d^2*4i - a^3*b*d^2*4i - 6*a^2*b^2*d^2)))^(1/2)
- (16*(e*cot(c + d*x))^(1/2)*(a^10*e^20 - 2*b^10*e^20 - 4*a^2*b^8*e^20 - 27
*a^4*b^6*e^20 + 15*a^6*b^4*e^20 + 9*a^8*b^2*e^20))/(b^9*d^4 + a^8*b*d^4 + 4
*a^2*b^7*d^4 + 6*a^4*b^5*d^4 + 4*a^6*b^3*d^4))*((e^5*1i)/(4*(a^4*d^2 + b^4*
d^2 + a*b^3*d^2*4i - a^3*b*d^2*4i - 6*a^2*b^2*d^2)))^(1/2)))*((e^5*1i)/(4*(
a^4*d^2 + b^4*d^2 + a*b^3*d^2*4i - a^3*b*d^2*4i - 6*a^2*b^2*d^2)))^(1/2)*2i
 + atan((((((8*(96*a^2*b^14*d^4*e^13 + 480*a^4*b^12*d^4*e^13 + 960*a^6*b^1
0*d^4*e^13 + 960*a^8*b^8*d^4*e^13 + 480*a^10*b^6*d^4*e^13 + 96*a^12*b^4*d^4
*e^13))/(b^9*d^5 + a^8*b*d^5 + 4*a^2*b^7*d^5 + 6*a^4*b^5*d^5 + 4*a^6*b^3*d^
5) - (16*(e*cot(c + d*x))^(1/2)*(e^5/(4*(a^4*d^2*1i + b^4*d^2*1i + 4*a*b^3*
d^2 - 4*a^3*b*d^2 - a^2*b^2*d^2*6i)))^(1/2)*(32*b^18*d^4*e^10 + 160*a^2*b^1
6*d^4*e^10 + 288*a^4*b^14*d^4*e^10 + 160*a^6*b^12*d^4*e^10 - 160*a^8*b^10*d
^4*e^10 - 288*a^10*b^8*d^4*e^10 - 160*a^12*b^6*d^4*e^10 - 32*a^14*b^4*d^4*e
^10))/(b^9*d^4 + a^8*b*d^4 + 4*a^2*b^7*d^4 + 6*a^4*b^5*d^4 + 4*a^6*b^3*d^4)
)*(e^5/(4*(a^4*d^2*1i + b^4*d^2*1i + 4*a*b^3*d^2 - 4*a^3*b*d^2 - a^2*b^2*d^
2*6i)))^(1/2) + (16*(e*cot(c + d*x))^(1/2)*(60*a*b^13*d^2*e^15 + 8*a^13*b*d
^2*e^15 + 52*a^3*b^11*d^2*e^15 + 128*a^5*b^9*d^2*e^15 + 424*a^7*b^7*d^2*e^1
5 + 380*a^9*b^5*d^2*e^15 + 100*a^11*b^3*d^2*e^15))/(b^9*d^4 + a^8*b*d^4 + 4
*a^2*b^7*d^4 + 6*a^4*b^5*d^4 + 4*a^6*b^3*d^4))*(e^5/(4*(a^4*d^2*1i + b^4*d^
2*1i + 4*a*b^3*d^2 - 4*a^3*b*d^2 - a^2*b^2*d^2*6i)))^(1/2) + (8*(4*a*b^11*d
^2*e^18 + 16*a^11*b*d^2*e^18 - 304*a^3*b^9*d^2*e^18 - 120*a^5*b^7*d^2*e^18
+ 320*a^7*b^5*d^2*e^18 + 148*a^9*b^3*d^2*e^18))/(b^9*d^5 + a^8*b*d^5 + 4*a^
2*b^7*d^5 + 6*a^4*b^5*d^5 + 4*a^6*b^3*d^5))*(e^5/(4*(a^4*d^2*1i + b^4*d^2*1
i + 4*a*b^3*d^2 - 4*a^3*b*d^2 - a^2*b^2*d^2*6i)))^(1/2) + (16*(e*cot(c + d*
x))^(1/2)*(a^10*e^20 - 2*b^10*e^20 - 4*a^2*b^8*e^20 - 27*a^4*b^6*e^20 + 15*
a^6*b^4*e^20 + 9*a^8*b^2*e^20))/(b^9*d^4 + a^8*b*d^4 + 4*a^2*b^7*d^4 + 6*a^
4*b^5*d^4 + 4*a^6*b^3*d^4))*(e^5/(4*(a^4*d^2*1i + b^4*d^2*1i + 4*a*b^3*d^2
- 4*a^3*b*d^2 - a^2*b^2*d^2*6i)))^(1/2)*1i - (((((8*(96*a^2*b^14*d^4*e^13 +
 480*a^4*b^12*d^4*e^13 + 960*a^6*b^10*d^4*e^13 + 960*a^8*b^8*d^4*e^13 + 480
*a^10*b^6*d^4*e^13 + 96*a^12*b^4*d^4*e^13))/(b^9*d^5 + a^8*b*d^5 + 4*a^2*b^
7*d^5 + 6*a^4*b^5*d^5 + 4*a^6*b^3*d^5) + (16*(e*cot(c + d*x))^(1/2)*(e^5/(4
*(a^4*d^2*1i + b^4*d^2*1i + 4*a*b^3*d^2 - 4*a^3*b*d^2 - a^2*b^2*d^2*6i)))^(
1/2)*(32*b^18*d^4*e^10 + 160*a^2*b^16*d^4*e^10 + 288*a^4*b^14*d^4*e^10 + 16

0*a^6*b^12*d^4*e^10 - 160*a^8*b^10*d^4*e^10 - 288*a^10*b^8*d^4*e^10 - 160*a
^12*b^6*d^4*e^10 - 32*a^14*b^4*d^4*e^10))/(b^9*d^4 + a^8*b*d^4 + 4*a^2*b^7*
d^4 + 6*a^4*b^5*d^4 + 4*a^6*b^3*d^4))*(e^5/(4*(a^4*d^2*1i + b^4*d^2*1i + 4*
a*b^3*d^2 - 4*a^3*b*d^2 - a^2*b^2*d^2*6i)))^(1/2) - (16*(e*cot(c + d*x))^(1
/2)*(60*a*b^13*d^2*e^15 + 8*a^13*b*d^2*e^15 + 52*a^3*b^11*d^2*e^15 + 128*a^
5*b^9*d^2*e^15 + 424*a^7*b^7*d^2*e^15 + 380*a^9*b^5*d^2*e^15 + 100*a^11*b^3
*d^2*e^15))/(b^9*d^4 + a^8*b*d^4 + 4*a^2*b^7*d^4 + 6*a^4*b^5*d^4 + 4*a^6*b^
3*d^4))*(e^5/(4*(a^4*d^2*1i + b^4*d^2*1i + 4*a*b^3*d^2 - 4*a^3*b*d^2 - a^2*
b^2*d^2*6i)))^(1/2) + (8*(4*a*b^11*d^2*e^18 + 16*a^11*b*d^2*e^18 - 304*a^3*
b^9*d^2*e^18 - 120*a^5*b^7*d^2*e^18 + 320*a^7*b^5*d^2*e^18 + 148*a^9*b^3*d^
2*e^18))/(b^9*d^5 + a^8*b*d^5 + 4*a^2*b^7*d^5 + 6*a^4*b^5*d^5 + 4*a^6*b^3*d
^5))*(e^5/(4*(a^4*d^2*1i + b^4*d^2*1i + 4*a*b^3*d^2 - 4*a^3*b*d^2 - a^2*b^2
*d^2*6i)))^(1/2) - (16*(e*cot(c + d*x))^(1/2)*(a^10*e^20 - 2*b^10*e^20 - 4*
a^2*b^8*e^20 - 27*a^4*b^6*e^20 + 15*a^6*b^4*e^20 + 9*a^8*b^2*e^20))/(b^9*d^
4 + a^8*b*d^4 + 4*a^2*b^7*d^4 + 6*a^4*b^5*d^4 + 4*a^6*b^3*d^4))*(e^5/(4*(a^
4*d^2*1i + b^4*d^2*1i + 4*a*b^3*d^2 - 4*a^3*b*d^2 - a^2*b^2*d^2*6i)))^(1/2)
1i)/((16(a^8*e^23 + 10*a^2*b^6*e^23 + 27*a^4*b^4*e^23 + 10*a^6*b^2*e^23))
/(b^9*d^5 + a^8*b*d^5 + 4*a^2*b^7*d^5 + 6*a^4*b^5*d^5 + 4*a^6*b^3*d^5) + ((
(((8*(96*a^2*b^14*d^4*e^13 + 480*a^4*b^12*d^4*e^13 + 960*a^6*b^10*d^4*e^13
+ 960*a^8*b^8*d^4*e^13 + 480*a^10*b^6*d^4*e^13 + 96*a^12*b^4*d^4*e^13))/(b^
9*d^5 + a^8*b*d^5 + 4*a^2*b^7*d^5 + 6*a^4*b^5*d^5 + 4*a^6*b^3*d^5) - (16*(e
*cot(c + d*x))^(1/2)*(e^5/(4*(a^4*d^2*1i + b^4*d^2*1i + 4*a*b^3*d^2 - 4*a^3
*b*d^2 - a^2*b^2*d^2*6i)))^(1/2)*(32*b^18*d^4*e^10 + 160*a^2*b^16*d^4*e^10
+ 288*a^4*b^14*d^4*e^10 + 160*a^6*b^12*d^4*e^10 - 160*a^8*b^10*d^4*e^10 - 2
88*a^10*b^8*d^4*e^10 - 160*a^12*b^6*d^4*e^10 - 32*a^14*b^4*d^4*e^10))/(b^9*
d^4 + a^8*b*d^4 + 4*a^2*b^7*d^4 + 6*a^4*b^5*d^4 + 4*a^6*b^3*d^4))*(e^5/(4*(
a^4*d^2*1i + b^4*d^2*1i + 4*a*b^3*d^2 - 4*a^3*b*d^2 - a^2*b^2*d^2*6i)))^(1/
2) + (16*(e*cot(c + d*x))^(1/2)*(60*a*b^13*d^2*e^15 + 8*a^13*b*d^2*e^15 + 5
2*a^3*b^11*d^2*e^15 + 128*a^5*b^9*d^2*e^15 + 424*a^7*b^7*d^2*e^15 + 380*a^9
*b^5*d^2*e^15 + 100*a^11*b^3*d^2*e^15))/(b^9*d^4 + a^8*b*d^4 + 4*a^2*b^7*d^
4 + 6*a^4*b^5*d^4 + 4*a^6*b^3*d^4))*(e^5/(4*(a^4*d^2*1i + b^4*d^2*1i + 4*a*
b^3*d^2 - 4*a^3*b*d^2 - a^2*b^2*d^2*6i)))^(1/2) + (8*(4*a*b^11*d^2*e^18 + 1
6*a^11*b*d^2*e^18 - 304*a^3*b^9*d^2*e^18 - 120*a^5*b^7*d^2*e^18 + 320*a^7*b
^5*d^2*e^18 + 148*a^9*b^3*d^2*e^18))/(b^9*d^5 + a^8*b*d^5 + 4*a^2*b^7*d^5 +
 6*a^4*b^5*d^5 + 4*a^6*b^3*d^5))*(e^5/(4*(a^4*d^2*1i + b^4*d^2*1i + 4*a*b^3
*d^2 - 4*a^3*b*d^2 - a^2*b^2*d^2*6i)))^(1/2) + (16*(e*cot(c + d*x))^(1/2)*(
a^10*e^20 - 2*b^10*e^20 - 4*a^2*b^8*e^20 - 27*a^4*b^6*e^20 + 15*a^6*b^4*e^2
0 + 9*a^8*b^2*e^20))/(b^9*d^4 + a^8*b*d^4 + 4*a^2*b^7*d^4 + 6*a^4*b^5*d^4 +
 4*a^6*b^3*d^4))*(e^5/(4*(a^4*d^2*1i + b^4*d^2*1i + 4*a*b^3*d^2 - 4*a^3*b*d
^2 - a^2*b^2*d^2*6i)))^(1/2) + (((((8*(96*a^2*b^14*d^4*e^13 + 480*a^4*b^12*
d^4*e^13 + 960*a^6*b^10*d^4*e^13 + 960*a^8*b^8*d^4*e^13 + 480*a^10*b^6*d^4*
e^13 + 96*a^12*b^4*d^4*e^13))/(b^9*d^5 + a^8*b*d^5 + 4*a^2*b^7*d^5 + 6*a^4*
b^5*d^5 + 4*a^6*b^3*d^5) + (16*(e*cot(c + d*x))^(1/2)*(e^5/(4*(a^4*d^2*1i +
 b^4*d^2*1i + 4*a*b^3*d^2 - 4*a^3*b*d^2 - a^2*b^2*d^2*6i)))^(1/2)*(32*b^18*
d^4*e^10 + 160*a^2*b^16*d^4*e^10 + 288*a^4*b^14*d^4*e^10 + 160*a^6*b^12*d^4

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*e^10 - 160*a^8*b^10*d^4*e^10 - 288*a^10*b^8*d^4*e^10 - 160*a^12*b^6*d^4*e^
10 - 32*a^14*b^4*d^4*e^10))/(b^9*d^4 + a^8*b*d^4 + 4*a^2*b^7*d^4 + 6*a^4*b^
5*d^4 + 4*a^6*b^3*d^4))*(e^5/(4*(a^4*d^2*1i + b^4*d^2*1i + 4*a*b^3*d^2 - 4*
a^3*b*d^2 - a^2*b^2*d^2*6i)))^(1/2) - (16*(e*cot(c + d*x))^(1/2)*(60*a*b^13
*d^2*e^15 + 8*a^13*b*d^2*e^15 + 52*a^3*b^11*d^2*e^15 + 128*a^5*b^9*d^2*e^15
 + 424*a^7*b^7*d^2*e^15 + 380*a^9*b^5*d^2*e^15 + 100*a^11*b^3*d^2*e^15))/(b
^9*d^4 + a^8*b*d^4 + 4*a^2*b^7*d^4 + 6*a^4*b^5*d^4 + 4*a^6*b^3*d^4))*(e^5/(
4*(a^4*d^2*1i + b^4*d^2*1i + 4*a*b^3*d^2 - 4*a^3*b*d^2 - a^2*b^2*d^2*6i)))^
(1/2) + (8*(4*a*b^11*d^2*e^18 + 16*a^11*b*d^2*e^18 - 304*a^3*b^9*d^2*e^18 -
 120*a^5*b^7*d^2*e^18 + 320*a^7*b^5*d^2*e^18 + 148*a^9*b^3*d^2*e^18))/(b^9*
d^5 + a^8*b*d^5 + 4*a^2*b^7*d^5 + 6*a^4*b^5*d^5 + 4*a^6*b^3*d^5))*(e^5/(4*(
a^4*d^2*1i + b^4*d^2*1i + 4*a*b^3*d^2 - 4*a^3*b*d^2 - a^2*b^2*d^2*6i)))^(1/
2) - (16*(e*cot(c + d*x))^(1/2)*(a^10*e^20 - 2*b^10*e^20 - 4*a^2*b^8*e^20 -
 27*a^4*b^6*e^20 + 15*a^6*b^4*e^20 + 9*a^8*b^2*e^20))/(b^9*d^4 + a^8*b*d^4
+ 4*a^2*b^7*d^4 + 6*a^4*b^5*d^4 + 4*a^6*b^3*d^4))*(e^5/(4*(a^4*d^2*1i + b^4
*d^2*1i + 4*a*b^3*d^2 - 4*a^3*b*d^2 - a^2*b^2*d^2*6i)))^(1/2)))*(e^5/(4*(a^
4*d^2*1i + b^4*d^2*1i + 4*a*b^3*d^2 - 4*a^3*b*d^2 - a^2*b^2*d^2*6i)))^(1/2)
*2i + (atan((((a^2 + 5*b^2)*((16*(e*cot(c + d*x))^(1/2)*(a^10*e^20 - 2*b^10
*e^20 - 4*a^2*b^8*e^20 - 27*a^4*b^6*e^20 + 15*a^6*b^4*e^20 + 9*a^8*b^2*e^20
))/(b^9*d^4 + a^8*b*d^4 + 4*a^2*b^7*d^4 + 6*a^4*b^5*d^4 + 4*a^6*b^3*d^4) +
((a^2 + 5*b^2)*((8*(4*a*b^11*d^2*e^18 + 16*a^11*b*d^2*e^18 - 304*a^3*b^9*d^
2*e^18 - 120*a^5*b^7*d^2*e^18 + 320*a^7*b^5*d^2*e^18 + 148*a^9*b^3*d^2*e^18
))/(b^9*d^5 + a^8*b*d^5 + 4*a^2*b^7*d^5 + 6*a^4*b^5*d^5 + 4*a^6*b^3*d^5) +
(((16*(e*cot(c + d*x))^(1/2)*(60*a*b^13*d^2*e^15 + 8*a^13*b*d^2*e^15 + 52*a
^3*b^11*d^2*e^15 + 128*a^5*b^9*d^2*e^15 + 424*a^7*b^7*d^2*e^15 + 380*a^9*b^
5*d^2*e^15 + 100*a^11*b^3*d^2*e^15))/(b^9*d^4 + a^8*b*d^4 + 4*a^2*b^7*d^4 +
 6*a^4*b^5*d^4 + 4*a^6*b^3*d^4) + (((8*(96*a^2*b^14*d^4*e^13 + 480*a^4*b^12
*d^4*e^13 + 960*a^6*b^10*d^4*e^13 + 960*a^8*b^8*d^4*e^13 + 480*a^10*b^6*d^4
*e^13 + 96*a^12*b^4*d^4*e^13))/(b^9*d^5 + a^8*b*d^5 + 4*a^2*b^7*d^5 + 6*a^4
*b^5*d^5 + 4*a^6*b^3*d^5) - (8*(e*cot(c + d*x))^(1/2)*(a^2 + 5*b^2)*(-a^3*b
^3*e^5)^(1/2)*(32*b^18*d^4*e^10 + 160*a^2*b^16*d^4*e^10 + 288*a^4*b^14*d^4*
e^10 + 160*a^6*b^12*d^4*e^10 - 160*a^8*b^10*d^4*e^10 - 288*a^10*b^8*d^4*e^1
0 - 160*a^12*b^6*d^4*e^10 - 32*a^14*b^4*d^4*e^10))/((b^7*d + 2*a^2*b^5*d +
a^4*b^3*d)*(b^9*d^4 + a^8*b*d^4 + 4*a^2*b^7*d^4 + 6*a^4*b^5*d^4 + 4*a^6*b^3
*d^4)))*(a^2 + 5*b^2)*(-a^3*b^3*e^5)^(1/2))/(2*(b^7*d + 2*a^2*b^5*d + a^4*b
^3*d)))*(a^2 + 5*b^2)*(-a^3*b^3*e^5)^(1/2))/(2*(b^7*d + 2*a^2*b^5*d + a^4*b
^3*d)))*(-a^3*b^3*e^5)^(1/2))/(2*(b^7*d + 2*a^2*b^5*d + a^4*b^3*d)))*(-a^3*
b^3*e^5)^(1/2)*1i)/(2*(b^7*d + 2*a^2*b^5*d + a^4*b^3*d)) + ((a^2 + 5*b^2)*(
(16*(e*cot(c + d*x))^(1/2)*(a^10*e^20 - 2*b^10*e^20 - 4*a^2*b^8*e^20 - 27*a
^4*b^6*e^20 + 15*a^6*b^4*e^20 + 9*a^8*b^2*e^20))/(b^9*d^4 + a^8*b*d^4 + 4*a
^2*b^7*d^4 + 6*a^4*b^5*d^4 + 4*a^6*b^3*d^4) - ((a^2 + 5*b^2)*((8*(4*a*b^11*
d^2*e^18 + 16*a^11*b*d^2*e^18 - 304*a^3*b^9*d^2*e^18 - 120*a^5*b^7*d^2*e^18
 + 320*a^7*b^5*d^2*e^18 + 148*a^9*b^3*d^2*e^18))/(b^9*d^5 + a^8*b*d^5 + 4*a
^2*b^7*d^5 + 6*a^4*b^5*d^5 + 4*a^6*b^3*d^5) - (((16*(e*cot(c + d*x))^(1/2)*
(60*a*b^13*d^2*e^15 + 8*a^13*b*d^2*e^15 + 52*a^3*b^11*d^2*e^15 + 128*a^5*b^
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9*d^2*e^15 + 424*a^7*b^7*d^2*e^15 + 380*a^9*b^5*d^2*e^15 + 100*a^11*b^3*d^2
*e^15))/(b^9*d^4 + a^8*b*d^4 + 4*a^2*b^7*d^4 + 6*a^4*b^5*d^4 + 4*a^6*b^3*d^
4) - (((8*(96*a^2*b^14*d^4*e^13 + 480*a^4*b^12*d^4*e^13 + 960*a^6*b^10*d^4*
e^13 + 960*a^8*b^8*d^4*e^13 + 480*a^10*b^6*d^4*e^13 + 96*a^12*b^4*d^4*e^13)
)/(b^9*d^5 + a^8*b*d^5 + 4*a^2*b^7*d^5 + 6*a^4*b^5*d^5 + 4*a^6*b^3*d^5) + (
8*(e*cot(c + d*x))^(1/2)*(a^2 + 5*b^2)*(-a^3*b^3*e^5)^(1/2)*(32*b^18*d^4*e^
10 + 160*a^2*b^16*d^4*e^10 + 288*a^4*b^14*d^4*e^10 + 160*a^6*b^12*d^4*e^10
- 160*a^8*b^10*d^4*e^10 - 288*a^10*b^8*d^4*e^10 - 160*a^12*b^6*d^4*e^10 - 3
2*a^14*b^4*d^4*e^10))/((b^7*d + 2*a^2*b^5*d + a^4*b^3*d)*(b^9*d^4 + a^8*b*d
^4 + 4*a^2*b^7*d^4 + 6*a^4*b^5*d^4 + 4*a^6*b^3*d^4)))*(a^2 + 5*b^2)*(-a^3*b
^3*e^5)^(1/2))/(2*(b^7*d + 2*a^2*b^5*d + a^4*b^3*d)))*(a^2 + 5*b^2)*(-a^3*b
^3*e^5)^(1/2))/(2*(b^7*d + 2*a^2*b^5*d + a^4*b^3*d)))*(-a^3*b^3*e^5)^(1/2))
/(2*(b^7*d + 2*a^2*b^5*d + a^4*b^3*d)))*(-a^3*b^3*e^5)^(1/2)*1i)/(2*(b^7*d
+ 2*a^2*b^5*d + a^4*b^3*d)))/((16*(a^8*e^23 + 10*a^2*b^6*e^23 + 27*a^4*b^4*
e^23 + 10*a^6*b^2*e^23))/(b^9*d^5 + a^8*b*d^5 + 4*a^2*b^7*d^5 + 6*a^4*b^5*d
^5 + 4*a^6*b^3*d^5) + ((a^2 + 5*b^2)*((16*(e*cot(c + d*x))^(1/2)*(a^10*e^20
- 2*b^10*e^20 - 4*a^2*b^8*e^20 - 27*a^4*b^6*e^20 + 15*a^6*b^4*e^20 + 9*a^8
*b^2*e^20))/(b^9*d^4 + a^8*b*d^4 + 4*a^2*b^7*d^4 + 6*a^4*b^5*d^4 + 4*a^6*b^
3*d^4) + ((a^2 + 5*b^2)*((8*(4*a*b^11*d^2*e^18 + 16*a^11*b*d^2*e^18 - 304*a
^3*b^9*d^2*e^18 - 120*a^5*b^7*d^2*e^18 + 320*a^7*b^5*d^2*e^18 + 148*a^9*b^3
*d^2*e^18))/(b^9*d^5 + a^8*b*d^5 + 4*a^2*b^7*d^5 + 6*a^4*b^5*d^5 + 4*a^6*b^
3*d^5) + (((16*(e*cot(c + d*x))^(1/2)*(60*a*b^13*d^2*e^15 + 8*a^13*b*d^2*e^
15 + 52*a^3*b^11*d^2*e^15 + 128*a^5*b^9*d^2*e^15 + 424*a^7*b^7*d^2*e^15 + 3
80*a^9*b^5*d^2*e^15 + 100*a^11*b^3*d^2*e^15))/(b^9*d^4 + a^8*b*d^4 + 4*a^2*
b^7*d^4 + 6*a^4*b^5*d^4 + 4*a^6*b^3*d^4) + (((8*(96*a^2*b^14*d^4*e^13 + 480
*a^4*b^12*d^4*e^13 + 960*a^6*b^10*d^4*e^13 + 960*a^8*b^8*d^4*e^13 + 480*a^1
0*b^6*d^4*e^13 + 96*a^12*b^4*d^4*e^13))/(b^9*d^5 + a^8*b*d^5 + 4*a^2*b^7*d^
5 + 6*a^4*b^5*d^5 + 4*a^6*b^3*d^5) - (8*(e*cot(c + d*x))^(1/2)*(a^2 + 5*b^2
)*(-a^3*b^3*e^5)^(1/2)*(32*b^18*d^4*e^10 + 160*a^2*b^16*d^4*e^10 + 288*a^4*
b^14*d^4*e^10 + 160*a^6*b^12*d^4*e^10 - 160*a^8*b^10*d^4*e^10 - 288*a^10*b^
8*d^4*e^10 - 160*a^12*b^6*d^4*e^10 - 32*a^14*b^4*d^4*e^10))/((b^7*d + 2*a^2
*b^5*d + a^4*b^3*d)*(b^9*d^4 + a^8*b*d^4 + 4*a^2*b^7*d^4 + 6*a^4*b^5*d^4 +
4*a^6*b^3*d^4)))*(a^2 + 5*b^2)*(-a^3*b^3*e^5)^(1/2))/(2*(b^7*d + 2*a^2*b^5*
d + a^4*b^3*d)))*(a^2 + 5*b^2)*(-a^3*b^3*e^5)^(1/2))/(2*(b^7*d + 2*a^2*b^5*
d + a^4*b^3*d)))*(-a^3*b^3*e^5)^(1/2))/(2*(b^7*d + 2*a^2*b^5*d + a^4*b^3*d)
))*(-a^3*b^3*e^5)^(1/2))/(2*(b^7*d + 2*a^2*b^5*d + a^4*b^3*d)) - ((a^2 + 5*
b^2)*((16*(e*cot(c + d*x))^(1/2)*(a^10*e^20 - 2*b^10*e^20 - 4*a^2*b^8*e^20
- 27*a^4*b^6*e^20 + 15*a^6*b^4*e^20 + 9*a^8*b^2*e^20))/(b^9*d^4 + a^8*b*d^4
+ 4*a^2*b^7*d^4 + 6*a^4*b^5*d^4 + 4*a^6*b^3*d^4) - ((a^2 + 5*b^2)*((8*(4*a
*b^11*d^2*e^18 + 16*a^11*b*d^2*e^18 - 304*a^3*b^9*d^2*e^18 - 120*a^5*b^7*d^
2*e^18 + 320*a^7*b^5*d^2*e^18 + 148*a^9*b^3*d^2*e^18))/(b^9*d^5 + a^8*b*d^5
+ 4*a^2*b^7*d^5 + 6*a^4*b^5*d^5 + 4*a^6*b^3*d^5) - (((16*(e*cot(c + d*x))^
(1/2)*(60*a*b^13*d^2*e^15 + 8*a^13*b*d^2*e^15 + 52*a^3*b^11*d^2*e^15 + 128*
a^5*b^9*d^2*e^15 + 424*a^7*b^7*d^2*e^15 + 380*a^9*b^5*d^2*e^15 + 100*a^11*b
^3*d^2*e^15))/(b^9*d^4 + a^8*b*d^4 + 4*a^2*b^7*d^4 + 6*a^4*b^5*d^4 + 4*a^6*

```
b^3*d^4) - (((8*(96*a^2*b^14*d^4*e^13 + 480*a^4*b^12*d^4*e^13 + 960*a^6*b^1
0*d^4*e^13 + 960*a^8*b^8*d^4*e^13 + 480*a^10*b^6*d^4*e^13 + 96*a^12*b^4*d^4
*e^13))/(b^9*d^5 + a^8*b*d^5 + 4*a^2*b^7*d^5 + 6*a^4*b^5*d^5 + 4*a^6*b^3*d^
5) + (8*(e*cot(c + d*x))^(1/2)*(a^2 + 5*b^2)*(-a^3*b^3*e^5)^(1/2)*(32*b^18*
d^4*e^10 + 160*a^2*b^16*d^4*e^10 + 288*a^4*b^14*d^4*e^10 + 160*a^6*b^12*d^4
*e^10 - 160*a^8*b^10*d^4*e^10 - 288*a^10*b^8*d^4*e^10 - 160*a^12*b^6*d^4*e^
10 - 32*a^14*b^4*d^4*e^10))/((b^7*d + 2*a^2*b^5*d + a^4*b^3*d)*(b^9*d^4 + a
^8*b*d^4 + 4*a^2*b^7*d^4 + 6*a^4*b^5*d^4 + 4*a^6*b^3*d^4)))*(a^2 + 5*b^2)*(
-a^3*b^3*e^5)^(1/2))/(2*(b^7*d + 2*a^2*b^5*d + a^4*b^3*d)))*(a^2 + 5*b^2)*(
-a^3*b^3*e^5)^(1/2))/(2*(b^7*d + 2*a^2*b^5*d + a^4*b^3*d)))*(-a^3*b^3*e^5)^
(1/2))/(2*(b^7*d + 2*a^2*b^5*d + a^4*b^3*d)))*(-a^3*b^3*e^5)^(1/2))/(2*(b^7
*d + 2*a^2*b^5*d + a^4*b^3*d))))*(a^2 + 5*b^2)*(-a^3*b^3*e^5)^(1/2)*1i)/(b^
7*d + 2*a^2*b^5*d + a^4*b^3*d) + (a^2*e^3*(e*cot(c + d*x))^(1/2))/(b*(a*d*e
 + b*d*e*cot(c + d*x))*(a^2 + b^2))
```

3.77 $\int \frac{(e\cot(c+dx))^{3/2}}{(a+b\cot(c+dx))^2}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 387

$$\int \frac{(e\cot(c+dx))^{3/2}}{(a+b\cot(c+dx))^2}\,dx = -\frac{\sqrt{a}(a^2-3b^2)\,e^{3/2}\arctan\left(\frac{\sqrt{b}\sqrt{e\cot(c+dx)}}{\sqrt{a}\sqrt{e}}\right)}{\sqrt{b}\,(a^2+b^2)^2\,d} - \frac{(a^2-2ab-b^2)\,e^{3/2}\arctan\left(1-\frac{\sqrt{2}\sqrt{e\cot(c+dx)}}{\sqrt{e}}\right)}{\sqrt{2}\,(a^2+b^2)^2\,d} + \frac{(a^2-2ab-b^2)\,e^{3/2}\arctan\left(1+\frac{\sqrt{2}\sqrt{e\cot(c+dx)}}{\sqrt{e}}\right)}{\sqrt{2}\,(a^2+b^2)^2\,d} - \frac{ae\sqrt{e\cot(c+dx)}}{(a^2+b^2)\,d(a+b\cot(c+dx))} - \frac{(a^2+2ab-b^2)\,e^{3/2}\log\left(\sqrt{e}+\sqrt{e}\cot(c+dx)-\sqrt{2}\sqrt{e\cot(c+dx)}\right)}{2\sqrt{2}\,(a^2+b^2)^2\,d} + \frac{(a^2+2ab-b^2)\,e^{3/2}\log\left(\sqrt{e}+\sqrt{e}\cot(c+dx)+\sqrt{2}\sqrt{e\cot(c+dx)}\right)}{2\sqrt{2}\,(a^2+b^2)^2\,d}$$

```
[Out] -1/2*(a^2-2*a*b-b^2)*e^(3/2)*arctan(1-2^(1/2)*(e*cot(d*x+c))^(1/2)/e^(1/2))
/(a^2+b^2)^2/d*2^(1/2)+1/2*(a^2-2*a*b-b^2)*e^(3/2)*arctan(1+2^(1/2)*(e*cot(
d*x+c))^(1/2)/e^(1/2))/(a^2+b^2)^2/d*2^(1/2)-1/4*(a^2+2*a*b-b^2)*e^(3/2)*ln
(e^(1/2)+cot(d*x+c)*e^(1/2)-2^(1/2)*(e*cot(d*x+c))^(1/2))/(a^2+b^2)^2/d*2^(
1/2)+1/4*(a^2+2*a*b-b^2)*e^(3/2)*ln(e^(1/2)+cot(d*x+c)*e^(1/2)+2^(1/2)*(e*c
ot(d*x+c))^(1/2))/(a^2+b^2)^2/d*2^(1/2)-(a^2-3*b^2)*e^(3/2)*arctan(b^(1/2)*
(e*cot(d*x+c))^(1/2)/a^(1/2)/e^(1/2))*a^(1/2)/(a^2+b^2)^2/d/b^(1/2)-a*e*(e*
cot(d*x+c))^(1/2)/(a^2+b^2)/d/(a+b*cot(d*x+c))
```

Rubi [A] (verified)

Time = 1.07 (sec) , antiderivative size = 387, normalized size of antiderivative = 1.00, number of steps used = 15, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.480, Rules used = {3648, 3734, 3615, 1182, 1176, 631, 210, 1179, 642, 3715, 65, 211}

$$\int \frac{(e\cot(c+dx))^{3/2}}{(a+b\cot(c+dx))^2}\,dx = -\frac{\sqrt{a}e^{3/2}(a^2-3b^2)\arctan\left(\frac{\sqrt{b}\sqrt{e\cot(c+dx)}}{\sqrt{a}\sqrt{e}}\right)}{\sqrt{b}d\,(a^2+b^2)^2} - \frac{e^{3/2}(a^2-2ab-b^2)\arctan\left(1-\frac{\sqrt{2}\sqrt{e\cot(c+dx)}}{\sqrt{e}}\right)}{\sqrt{2}d\,(a^2+b^2)^2} + \frac{e^{3/2}(a^2-2ab-b^2)\arctan\left(\frac{\sqrt{2}\sqrt{e\cot(c+dx)}}{\sqrt{e}}+1\right)}{\sqrt{2}d\,(a^2+b^2)^2} - \frac{e^{3/2}(a^2+2ab-b^2)\log\left(\sqrt{e}\cot(c+dx)-\sqrt{2}\sqrt{e\cot(c+dx)}+\sqrt{e}\right)}{2\sqrt{2}d\,(a^2+b^2)^2} + \frac{e^{3/2}(a^2+2ab-b^2)\log\left(\sqrt{e}\cot(c+dx)+\sqrt{2}\sqrt{e\cot(c+dx)}+\sqrt{e}\right)}{2\sqrt{2}d\,(a^2+b^2)^2} - \frac{ae\sqrt{e\cot(c+dx)}}{d\,(a^2+b^2)\,(a+b\cot(c+dx))}$$

```
[In] Int[(e*Cot[c + d*x])^(3/2)/(a + b*Cot[c + d*x])^2,x]
```

```
[Out] -((Sqrt[a]*(a^2 - 3*b^2)*e^(3/2)*ArcTan[(Sqrt[b]*Sqrt[e*Cot[c + d*x]])/(Sqr
t[a]*Sqrt[e])])/(Sqrt[b]*(a^2 + b^2)^2*d)) - ((a^2 - 2*a*b - b^2)*e^(3/2)*A
rcTan[1 - (Sqrt[2]*Sqrt[e*Cot[c + d*x]])/Sqrt[e]])/(Sqrt[2]*(a^2 + b^2)^2*d
) + ((a^2 - 2*a*b - b^2)*e^(3/2)*ArcTan[1 + (Sqrt[2]*Sqrt[e*Cot[c + d*x]])/
Sqrt[e]])/(Sqrt[2]*(a^2 + b^2)^2*d) - (a*e*Sqrt[e*Cot[c + d*x]])/((a^2 + b^
2)*d*(a + b*Cot[c + d*x])) - ((a^2 + 2*a*b - b^2)*e^(3/2)*Log[Sqrt[e] + Sqr
t[e]*Cot[c + d*x] - Sqrt[2]*Sqrt[e*Cot[c + d*x]]])/(2*Sqrt[2]*(a^2 + b^2)^2
*d) + ((a^2 + 2*a*b - b^2)*e^(3/2)*Log[Sqrt[e] + Sqrt[e]*Cot[c + d*x] + Sqr
t[2]*Sqrt[e*Cot[c + d*x]]])/(2*Sqrt[2]*(a^2 + b^2)^2*d)
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
```

```
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1179

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 1182

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
a*c, 2]}, Dist[(d*q + a*e)/(2*a*c), Int[(q + c*x^2)/(a + c*x^4), x], x] + D
ist[(d*q - a*e)/(2*a*c), Int[(q - c*x^2)/(a + c*x^4), x], x]] /; FreeQ[{a,
c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0] && NeQ[c*d^2 - a*e^2, 0] && NegQ[(-a)
*c]
```

Rule 3615

```
Int[((c_) + (d_.)*tan[(e_.) + (f_.)*(x_)])/Sqrt[(b_.)*tan[(e_.) + (f_.)*(x_
)]], x_Symbol] :> Dist[2/f, Subst[Int[(b*c + d*x^2)/(b^2 + x^4), x], x, Sqr
```

```
t[b*Tan[e + f*x]]], x] /; FreeQ[{b, c, d, e, f}, x] && NeQ[c^2 - d^2, 0] &&
 NeQ[c^2 + d^2, 0]
```

Rule 3648

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*c - a*d)*(a + b*Tan[e + f*x])^(m +
 1)*((c + d*Tan[e + f*x])^(n - 1)/(f*(m + 1)*(a^2 + b^2))), x] + Dist[1/((m
 + 1)*(a^2 + b^2)), Int[(a + b*Tan[e + f*x])^(m + 1)*(c + d*Tan[e + f*x])^(
n - 2)*Simp[a*c^2*(m + 1) + a*d^2*(n - 1) + b*c*d*(m - n + 2) - (b*c^2 - 2*
a*c*d - b*d^2)*(m + 1)*Tan[e + f*x] - d*(b*c - a*d)*(m + n)*Tan[e + f*x]^2,
 x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2
 + b^2, 0] && NeQ[c^2 + d^2, 0] && LtQ[m, -1] && LtQ[1, n, 2] && IntegerQ[2
*m]
```

Rule 3715

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*tan[(e_.)
+ (f_.)*(x_)])^(n_.)*((A_) + (C_.)*tan[(e_.) + (f_.)*(x_)]^2), x_Symbol] :>
 Dist[A/f, Subst[Int[(a + b*x)^m*(c + d*x)^n, x], x, Tan[e + f*x]], x] /; F
reeQ[{a, b, c, d, e, f, A, C, m, n}, x] && EqQ[A, C]
```

Rule 3734

```
Int[(((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_)*((A_.) + (B_.)*tan[(e_.)
+ (f_.)*(x_)] + (C_.)*tan[(e_.) + (f_.)*(x_)]^2))/((a_.) + (b_.)*tan[(e_.)
+ (f_.)*(x_)]), x_Symbol] :> Dist[1/(a^2 + b^2), Int[(c + d*Tan[e + f*x])^n
*Simp[b*B + a*(A - C) + (a*B - b*(A - C))*Tan[e + f*x], x], x], x] + Dist[(
A*b^2 - a*b*B + a^2*C)/(a^2 + b^2), Int[(c + d*Tan[e + f*x])^n*((1 + Tan[e
+ f*x]^2)/(a + b*Tan[e + f*x])), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C
, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] &&
 !GtQ[n, 0] &&  !LeQ[n, -1]
```

Rubi steps

$$\begin{aligned}\text{integral} &= -\frac{ae\sqrt{e\cot(c+dx)}}{(a^2+b^2)\,d(a+b\cot(c+dx))} - \frac{\int \frac{\frac{ae^2}{2}-be^2\cot(c+dx)-\frac{1}{2}ae^2\cot^2(c+dx)}{\sqrt{e\cot(c+dx)}(a+b\cot(c+dx))}\,dx}{a^2+b^2}\\ &= -\frac{ae\sqrt{e\cot(c+dx)}}{(a^2+b^2)\,d(a+b\cot(c+dx))} - \frac{\int \frac{(a^2-b^2)e^2-2abe^2\cot(c+dx)}{\sqrt{e\cot(c+dx)}}\,dx}{(a^2+b^2)^2}\\ &\quad + \frac{(a(a^2-3b^2)\,e^2)\int \frac{1+\cot^2(c+dx)}{\sqrt{e\cot(c+dx)}(a+b\cot(c+dx))}\,dx}{2\,(a^2+b^2)^2}\end{aligned}$$

$$
\begin{aligned}
&= -\frac{ae\sqrt{e\cot(c+dx)}}{(a^2+b^2)\,d(a+b\cot(c+dx))} \\
&\quad -\frac{2\text{Subst}\left(\int \frac{-((a^2-b^2)e^3)+2abe^2x^2}{e^2+x^4}\,dx, x, \sqrt{e\cot(c+dx)}\right)}{(a^2+b^2)^2\,d} \\
&\quad +\frac{(a(a^2-3b^2)\,e^2)\,\text{Subst}\left(\int \frac{1}{\sqrt{-ex}(a-bx)}\,dx, x, -\cot(c+dx)\right)}{2\,(a^2+b^2)^2\,d} \\
&= -\frac{ae\sqrt{e\cot(c+dx)}}{(a^2+b^2)\,d(a+b\cot(c+dx))} \\
&\quad -\frac{(a(a^2-3b^2)\,e)\,\text{Subst}\left(\int \frac{1}{a+\frac{bx^2}{e}}\,dx, x, \sqrt{e\cot(c+dx)}\right)}{(a^2+b^2)^2\,d} \\
&\quad +\frac{((a^2-2ab-b^2)\,e^2)\,\text{Subst}\left(\int \frac{e+x^2}{e^2+x^4}\,dx, x, \sqrt{e\cot(c+dx)}\right)}{(a^2+b^2)^2\,d} \\
&\quad +\frac{((a^2+2ab-b^2)\,e^2)\,\text{Subst}\left(\int \frac{e-x^2}{e^2+x^4}\,dx, x, \sqrt{e\cot(c+dx)}\right)}{(a^2+b^2)^2\,d} \\
&= -\frac{\sqrt{a}(a^2-3b^2)\,e^{3/2}\arctan\left(\frac{\sqrt{b}\sqrt{e\cot(c+dx)}}{\sqrt{a}\sqrt{e}}\right)}{\sqrt{b}\,(a^2+b^2)^2\,d} - \frac{ae\sqrt{e\cot(c+dx)}}{(a^2+b^2)\,d(a+b\cot(c+dx))} \\
&\quad -\frac{((a^2+2ab-b^2)\,e^{3/2})\,\text{Subst}\left(\int \frac{\sqrt{2}\sqrt{e}+2x}{-e-\sqrt{2}\sqrt{e}x-x^2}\,dx, x, \sqrt{e\cot(c+dx)}\right)}{2\sqrt{2}\,(a^2+b^2)^2\,d} \\
&\quad -\frac{((a^2+2ab-b^2)\,e^{3/2})\,\text{Subst}\left(\int \frac{\sqrt{2}\sqrt{e}-2x}{-e+\sqrt{2}\sqrt{e}x-x^2}\,dx, x, \sqrt{e\cot(c+dx)}\right)}{2\sqrt{2}\,(a^2+b^2)^2\,d} \\
&\quad +\frac{((a^2-2ab-b^2)\,e^2)\,\text{Subst}\left(\int \frac{1}{e-\sqrt{2}\sqrt{e}x+x^2}\,dx, x, \sqrt{e\cot(c+dx)}\right)}{2\,(a^2+b^2)^2\,d} \\
&\quad +\frac{((a^2-2ab-b^2)\,e^2)\,\text{Subst}\left(\int \frac{1}{e+\sqrt{2}\sqrt{e}x+x^2}\,dx, x, \sqrt{e\cot(c+dx)}\right)}{2\,(a^2+b^2)^2\,d}
\end{aligned}
$$

$$= -\frac{\sqrt{a}(a^2-3b^2)\,e^{3/2}\arctan\left(\frac{\sqrt{b}\sqrt{e\cot(c+dx)}}{\sqrt{a}\sqrt{e}}\right)}{\sqrt{b}\,(a^2+b^2)^2\,d} - \frac{ae\sqrt{e\cot(c+dx)}}{(a^2+b^2)\,d(a+b\cot(c+dx))}$$
$$-\frac{(a^2+2ab-b^2)\,e^{3/2}\log\left(\sqrt{e}+\sqrt{e}\cot(c+dx)-\sqrt{2}\sqrt{e\cot(c+dx)}\right)}{2\sqrt{2}\,(a^2+b^2)^2\,d}$$
$$+\frac{(a^2+2ab-b^2)\,e^{3/2}\log\left(\sqrt{e}+\sqrt{e}\cot(c+dx)+\sqrt{2}\sqrt{e\cot(c+dx)}\right)}{2\sqrt{2}\,(a^2+b^2)^2\,d}$$
$$+\frac{\left((a^2-2ab-b^2)\,e^{3/2}\right)\mathrm{Subst}\left(\int\frac{1}{-1-x^2}\,dx,x,1-\frac{\sqrt{2}\sqrt{e\cot(c+dx)}}{\sqrt{e}}\right)}{\sqrt{2}\,(a^2+b^2)^2\,d}$$
$$-\frac{\left((a^2-2ab-b^2)\,e^{3/2}\right)\mathrm{Subst}\left(\int\frac{1}{-1-x^2}\,dx,x,1+\frac{\sqrt{2}\sqrt{e\cot(c+dx)}}{\sqrt{e}}\right)}{\sqrt{2}\,(a^2+b^2)^2\,d}$$

$$= -\frac{\sqrt{a}(a^2-3b^2)\,e^{3/2}\arctan\left(\frac{\sqrt{b}\sqrt{e\cot(c+dx)}}{\sqrt{a}\sqrt{e}}\right)}{\sqrt{b}\,(a^2+b^2)^2\,d}$$
$$-\frac{(a^2-2ab-b^2)\,e^{3/2}\arctan\left(1-\frac{\sqrt{2}\sqrt{e\cot(c+dx)}}{\sqrt{e}}\right)}{\sqrt{2}\,(a^2+b^2)^2\,d}$$
$$+\frac{(a^2-2ab-b^2)\,e^{3/2}\arctan\left(1+\frac{\sqrt{2}\sqrt{e\cot(c+dx)}}{\sqrt{e}}\right)}{\sqrt{2}\,(a^2+b^2)^2\,d} - \frac{ae\sqrt{e\cot(c+dx)}}{(a^2+b^2)\,d(a+b\cot(c+dx))}$$
$$-\frac{(a^2+2ab-b^2)\,e^{3/2}\log\left(\sqrt{e}+\sqrt{e}\cot(c+dx)-\sqrt{2}\sqrt{e\cot(c+dx)}\right)}{2\sqrt{2}\,(a^2+b^2)^2\,d}$$
$$+\frac{(a^2+2ab-b^2)\,e^{3/2}\log\left(\sqrt{e}+\sqrt{e}\cot(c+dx)+\sqrt{2}\sqrt{e\cot(c+dx)}\right)}{2\sqrt{2}\,(a^2+b^2)^2\,d}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 3.30 (sec) , antiderivative size = 321, normalized size of antiderivative = 0.83

$$\int\frac{(e\cot(c+dx))^{3/2}}{(a+b\cot(c+dx))^2}\,dx =$$
$$-\frac{(e\cot(c+dx))^{3/2}\left(\frac{240a^{5/2}\arctan\left(\frac{\sqrt{b}\sqrt{\cot(c+dx)}}{\sqrt{a}}\right)}{\sqrt{b}}-240a^2\sqrt{\cot(c+dx)}+80ab\cot^{\frac{3}{2}}(c+dx)+80ab\cot^{\frac{3}{2}}(c+d\right.}{}$$

```
[In] Integrate[(e*Cot[c + d*x])^(3/2)/(a + b*Cot[c + d*x])^2,x]
```

```
[Out] -1/60*((e*Cot[c + d*x])^(3/2)*((240*a^(5/2)*ArcTan[(Sqrt[b]*Sqrt[Cot[c + d*
x]])/Sqrt[a]])/Sqrt[b] - 240*a^2*Sqrt[Cot[c + d*x]] + 80*a*b*Cot[c + d*x]^(
3/2) + 80*a*b*Cot[c + d*x]^(3/2)*(-1 + Hypergeometric2F1[3/4, 1, 7/4, -Cot[
c + d*x]^2]) + (24*b^2*(a^2 + b^2)*Cot[c + d*x]^(5/2)*Hypergeometric2F1[2,
5/2, 7/2, -((b*Cot[c + d*x])/a)])/a^2 + 15*(a - b)*(a + b)*(2*Sqrt[2]*ArcTa
n[1 - Sqrt[2]*Sqrt[Cot[c + d*x]]] - 2*Sqrt[2]*ArcTan[1 + Sqrt[2]*Sqrt[Cot[c
 + d*x]]] + 8*Sqrt[Cot[c + d*x]] + Sqrt[2]*Log[1 - Sqrt[2]*Sqrt[Cot[c + d*x
]] + Cot[c + d*x]] - Sqrt[2]*Log[1 + Sqrt[2]*Sqrt[Cot[c + d*x]] + Cot[c + d
*x]])))/((a^2 + b^2)^2*d*Cot[c + d*x]^(3/2))
```

Maple [A] (verified)

Time = 0.04 (sec) , antiderivative size = 391, normalized size of antiderivative = 1.01

method	result
derivativedivides	$-\frac{2e^3\left(\frac{a\left(\frac{\left(\frac{a^2}{2}+\frac{b^2}{2}\right)\sqrt{e\cot(dx+c)}}{e\cot(dx+c)b+ae}+\frac{(a^2-3b^2)\arctan\left(\frac{\sqrt{e\cot(dx+c)}\,b}{\sqrt{aeb}}\right)}{2\sqrt{aeb}}\right)}{(a^2+b^2)^2 e}+\frac{(-a^2e+b^2e)(e^2)^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\frac{e\cot(dx+c)+(e^2)^{\frac{1}{4}}\sqrt{e}\cdots}{e\cot(dx+c)-(e^2)^{\frac{1}{4}}\sqrt{e}\cdots}\right)\cdots\right)}{\cdots}\cdots\right)}{\cdots}$
default	$-\frac{2e^3\left(\frac{a\left(\frac{\left(\frac{a^2}{2}+\frac{b^2}{2}\right)\sqrt{e\cot(dx+c)}}{e\cot(dx+c)b+ae}+\frac{(a^2-3b^2)\arctan\left(\frac{\sqrt{e\cot(dx+c)}\,b}{\sqrt{aeb}}\right)}{2\sqrt{aeb}}\right)}{(a^2+b^2)^2 e}+\frac{(-a^2e+b^2e)(e^2)^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\frac{e\cot(dx+c)+(e^2)^{\frac{1}{4}}\sqrt{e}\cdots}{e\cot(dx+c)-(e^2)^{\frac{1}{4}}\sqrt{e}\cdots}\right)\cdots\right)}{\cdots}\cdots\right)}{\cdots}$

```
[In] int((e*cot(d*x+c))^(3/2)/(a+b*cot(d*x+c))^2,x,method=_RETURNVERBOSE)
```

```
[Out] -2/d*e^3*(a/(a^2+b^2)^2/e*((1/2*a^2+1/2*b^2)*(e*cot(d*x+c))^(1/2)/(e*cot(d*
x+c)*b+a*e)+1/2*(a^2-3*b^2)/(a*e*b)^(1/2)*arctan((e*cot(d*x+c))^(1/2)*b/(a*
e*b)^(1/2)))+1/e/(a^2+b^2)^2*(1/8*(-a^2*e+b^2*e)*(e^2)^(1/4)/e^2*2^(1/2)*(l
n((e*cot(d*x+c)+(e^2)^(1/4)*(e*cot(d*x+c))^(1/2)*2^(1/2)+(e^2)^(1/2))/(e*co
t(d*x+c)-(e^2)^(1/4)*(e*cot(d*x+c))^(1/2)*2^(1/2)+(e^2)^(1/2)))+2*arctan(2^
(1/2)/(e^2)^(1/4)*(e*cot(d*x+c))^(1/2)+1)-2*arctan(-2^(1/2)/(e^2)^(1/4)*(e*
cot(d*x+c))^(1/2)+1))+1/4*a*b/(e^2)^(1/4)*2^(1/2)*(ln((e*cot(d*x+c)-(e^2)^(
1/4)*(e*cot(d*x+c))^(1/2)*2^(1/2)+(e^2)^(1/2))/(e*cot(d*x+c)+(e^2)^(1/4)*(e
*cot(d*x+c))^(1/2)*2^(1/2)+(e^2)^(1/2)))+2*arctan(2^(1/2)/(e^2)^(1/4)*(e*co
t(d*x+c))^(1/2)+1)-2*arctan(-2^(1/2)/(e^2)^(1/4)*(e*cot(d*x+c))^(1/2)+1)))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 3044 vs. $2(324) = 648$.

Time = 0.47 (sec) , antiderivative size = 6150, normalized size of antiderivative = 15.89

$$\int \frac{(e\cot(c+dx))^{3/2}}{(a+b\cot(c+dx))^2}\,dx = \text{Too large to display}$$

```
[In] integrate((e*cot(d*x+c))^(3/2)/(a+b*cot(d*x+c))^2,x, algorithm="fricas")
[Out] Too large to include
```

Sympy [F]

$$\int \frac{(e\cot(c+dx))^{3/2}}{(a+b\cot(c+dx))^2}\,dx = \int \frac{(e\cot{(c+dx)})^{\frac{3}{2}}}{(a+b\cot{(c+dx)})^2}\,dx$$

```
[In] integrate((e*cot(d*x+c))**(3/2)/(a+b*cot(d*x+c))**2,x)
[Out] Integral((e*cot(c + d*x))**(3/2)/(a + b*cot(c + d*x))**2, x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{(e\cot(c+dx))^{3/2}}{(a+b\cot(c+dx))^2}\,dx = \text{Exception raised: ValueError}$$

```
[In] integrate((e*cot(d*x+c))^(3/2)/(a+b*cot(d*x+c))^2,x, algorithm="maxima")
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(e>0)', see `assume?` for more detai
ls)Is e
```

Giac [F]

$$\int \frac{(e\cot(c+dx))^{3/2}}{(a+b\cot(c+dx))^2}\,dx = \int \frac{(e\cot(dx+c))^{\frac{3}{2}}}{(b\cot(dx+c)+a)^2}\,dx$$

```
[In] integrate((e*cot(d*x+c))^(3/2)/(a+b*cot(d*x+c))^2,x, algorithm="giac")
[Out] integrate((e*cot(d*x + c))^(3/2)/(b*cot(d*x + c) + a)^2, x)
```

Mupad [B] (verification not implemented)

Time = 16.25 (sec) , antiderivative size = 11953, normalized size of antiderivative = 30.89

$$\int \frac{(e\cot(c+dx))^{3/2}}{(a+b\cot(c+dx))^2}\,dx = \text{Too large to display}$$

```
[In] int((e*cot(c + d*x))^(3/2)/(a + b*cot(c + d*x))^2,x)
[Out] (atan((((a^2 - 3*b^2)*((16*(e*cot(c + d*x))^(1/2)*(2*b^9*e^16 + a^8*b*e^16
- 5*a^2*b^7*e^16 + 17*a^4*b^5*e^16 - 7*a^6*b^3*e^16))/(a^8*d^4 + b^8*d^4 +
4*a^2*b^6*d^4 + 6*a^4*b^4*d^4 + 4*a^6*b^2*d^4) + ((a^2 - 3*b^2)*((16*(2*a^1
0*b*d^2*e^15 - 78*a^2*b^9*d^2*e^15 + 8*a^4*b^7*d^2*e^15 + 60*a^6*b^5*d^2*e^
15 - 24*a^8*b^3*d^2*e^15))/(a^8*d^5 + b^8*d^5 + 4*a^2*b^6*d^5 + 6*a^4*b^4*d
^5 + 4*a^6*b^2*d^5) - ((a^2 - 3*b^2)*((16*(e*cot(c + d*x))^(1/2)*(20*a^3*b^
10*d^2*e^13 - 60*a*b^12*d^2*e^13 + 168*a^5*b^8*d^2*e^13 + 40*a^7*b^6*d^2*e^
13 - 44*a^9*b^4*d^2*e^13 + 4*a^11*b^2*d^2*e^13))/(a^8*d^4 + b^8*d^4 + 4*a^2
*b^6*d^4 + 6*a^4*b^4*d^4 + 4*a^6*b^2*d^4) + ((a^2 - 3*b^2)*((16*(40*a*b^14*
d^4*e^12 + 192*a^3*b^12*d^4*e^12 + 360*a^5*b^10*d^4*e^12 + 320*a^7*b^8*d^4*
e^12 + 120*a^9*b^6*d^4*e^12 - 8*a^13*b^2*d^4*e^12))/(a^8*d^5 + b^8*d^5 + 4*
a^2*b^6*d^5 + 6*a^4*b^4*d^5 + 4*a^6*b^2*d^5) - (8*(e*cot(c + d*x))^(1/2)*(a
^2 - 3*b^2)*(-a*b*e^3)^(1/2)*(32*b^17*d^4*e^10 + 160*a^2*b^15*d^4*e^10 + 28
8*a^4*b^13*d^4*e^10 + 160*a^6*b^11*d^4*e^10 - 160*a^8*b^9*d^4*e^10 - 288*a^
10*b^7*d^4*e^10 - 160*a^12*b^5*d^4*e^10 - 32*a^14*b^3*d^4*e^10))/((b^5*d +
2*a^2*b^3*d + a^4*b*d)*(a^8*d^4 + b^8*d^4 + 4*a^2*b^6*d^4 + 6*a^4*b^4*d^4 +
 4*a^6*b^2*d^4)))*(-a*b*e^3)^(1/2))/(2*(b^5*d + 2*a^2*b^3*d + a^4*b*d)))*(-
a*b*e^3)^(1/2))/(2*(b^5*d + 2*a^2*b^3*d + a^4*b*d)))*(-a*b*e^3)^(1/2))/(2*(
b^5*d + 2*a^2*b^3*d + a^4*b*d)))*(-a*b*e^3)^(1/2)*1i)/(2*(b^5*d + 2*a^2*b^3
*d + a^4*b*d)) + ((a^2 - 3*b^2)*((16*(e*cot(c + d*x))^(1/2)*(2*b^9*e^16 + a
^8*b*e^16 - 5*a^2*b^7*e^16 + 17*a^4*b^5*e^16 - 7*a^6*b^3*e^16))/(a^8*d^4 +
b^8*d^4 + 4*a^2*b^6*d^4 + 6*a^4*b^4*d^4 + 4*a^6*b^2*d^4) - ((a^2 - 3*b^2)*(
(16*(2*a^10*b*d^2*e^15 - 78*a^2*b^9*d^2*e^15 + 8*a^4*b^7*d^2*e^15 + 60*a^6*
b^5*d^2*e^15 - 24*a^8*b^3*d^2*e^15))/(a^8*d^5 + b^8*d^5 + 4*a^2*b^6*d^5 + 6
*a^4*b^4*d^5 + 4*a^6*b^2*d^5) + ((a^2 - 3*b^2)*((16*(e*cot(c + d*x))^(1/2)*
(20*a^3*b^10*d^2*e^13 - 60*a*b^12*d^2*e^13 + 168*a^5*b^8*d^2*e^13 + 40*a^7*
```

```
b^6*d^2*e^13 - 44*a^9*b^4*d^2*e^13 + 4*a^11*b^2*d^2*e^13))/(a^8*d^4 + b^8*d
^4 + 4*a^2*b^6*d^4 + 6*a^4*b^4*d^4 + 4*a^6*b^2*d^4) - ((a^2 - 3*b^2)*((16*(
40*a*b^14*d^4*e^12 + 192*a^3*b^12*d^4*e^12 + 360*a^5*b^10*d^4*e^12 + 320*a^
7*b^8*d^4*e^12 + 120*a^9*b^6*d^4*e^12 - 8*a^13*b^2*d^4*e^12))/(a^8*d^5 + b^
8*d^5 + 4*a^2*b^6*d^5 + 6*a^4*b^4*d^5 + 4*a^6*b^2*d^5) + (8*(e*cot(c + d*x)
)^(1/2)*(a^2 - 3*b^2)*(-a*b*e^3)^(1/2)*(32*b^17*d^4*e^10 + 160*a^2*b^15*d^4
*e^10 + 288*a^4*b^13*d^4*e^10 + 160*a^6*b^11*d^4*e^10 - 160*a^8*b^9*d^4*e^1
0 - 288*a^10*b^7*d^4*e^10 - 160*a^12*b^5*d^4*e^10 - 32*a^14*b^3*d^4*e^10))/
((b^5*d + 2*a^2*b^3*d + a^4*b*d)*(a^8*d^4 + b^8*d^4 + 4*a^2*b^6*d^4 + 6*a^4
*b^4*d^4 + 4*a^6*b^2*d^4)))*(-a*b*e^3)^(1/2))/(2*(b^5*d + 2*a^2*b^3*d + a^4
*b*d)))*(-a*b*e^3)^(1/2))/(2*(b^5*d + 2*a^2*b^3*d + a^4*b*d)))*(-a*b*e^3)^(
1/2))/(2*(b^5*d + 2*a^2*b^3*d + a^4*b*d)))*(-a*b*e^3)^(1/2)*1i)/(2*(b^5*d +
 2*a^2*b^3*d + a^4*b*d)))/((32*(3*a*b^6*e^18 - a^3*b^4*e^18))/(a^8*d^5 + b^
8*d^5 + 4*a^2*b^6*d^5 + 6*a^4*b^4*d^5 + 4*a^6*b^2*d^5) - ((a^2 - 3*b^2)*((1
6*(e*cot(c + d*x))^(1/2)*(2*b^9*e^16 + a^8*b*e^16 - 5*a^2*b^7*e^16 + 17*a^4
*b^5*e^16 - 7*a^6*b^3*e^16))/(a^8*d^4 + b^8*d^4 + 4*a^2*b^6*d^4 + 6*a^4*b^4
*d^4 + 4*a^6*b^2*d^4) + ((a^2 - 3*b^2)*((16*(2*a^10*b*d^2*e^15 - 78*a^2*b^9
*d^2*e^15 + 8*a^4*b^7*d^2*e^15 + 60*a^6*b^5*d^2*e^15 - 24*a^8*b^3*d^2*e^15)
)/(a^8*d^5 + b^8*d^5 + 4*a^2*b^6*d^5 + 6*a^4*b^4*d^5 + 4*a^6*b^2*d^5) - ((a
^2 - 3*b^2)*((16*(e*cot(c + d*x))^(1/2)*(20*a^3*b^10*d^2*e^13 - 60*a*b^12*d
^2*e^13 + 168*a^5*b^8*d^2*e^13 + 40*a^7*b^6*d^2*e^13 - 44*a^9*b^4*d^2*e^13
+ 4*a^11*b^2*d^2*e^13))/(a^8*d^4 + b^8*d^4 + 4*a^2*b^6*d^4 + 6*a^4*b^4*d^4
+ 4*a^6*b^2*d^4) + ((a^2 - 3*b^2)*((16*(40*a*b^14*d^4*e^12 + 192*a^3*b^12*d
^4*e^12 + 360*a^5*b^10*d^4*e^12 + 320*a^7*b^8*d^4*e^12 + 120*a^9*b^6*d^4*e^
12 - 8*a^13*b^2*d^4*e^12))/(a^8*d^5 + b^8*d^5 + 4*a^2*b^6*d^5 + 6*a^4*b^4*d
^5 + 4*a^6*b^2*d^5) - (8*(e*cot(c + d*x))^(1/2)*(a^2 - 3*b^2)*(-a*b*e^3)^(1
/2)*(32*b^17*d^4*e^10 + 160*a^2*b^15*d^4*e^10 + 288*a^4*b^13*d^4*e^10 + 160
*a^6*b^11*d^4*e^10 - 160*a^8*b^9*d^4*e^10 - 288*a^10*b^7*d^4*e^10 - 160*a^1
2*b^5*d^4*e^10 - 32*a^14*b^3*d^4*e^10))/((b^5*d + 2*a^2*b^3*d + a^4*b*d)*(a
^8*d^4 + b^8*d^4 + 4*a^2*b^6*d^4 + 6*a^4*b^4*d^4 + 4*a^6*b^2*d^4)))*(-a*b*e
^3)^(1/2))/(2*(b^5*d + 2*a^2*b^3*d + a^4*b*d)))*(-a*b*e^3)^(1/2))/(2*(b^5*d
 + 2*a^2*b^3*d + a^4*b*d)))*(-a*b*e^3)^(1/2))/(2*(b^5*d + 2*a^2*b^3*d + a^4
*b*d)))*(-a*b*e^3)^(1/2))/(2*(b^5*d + 2*a^2*b^3*d + a^4*b*d)) + ((a^2 - 3*b
^2)*((16*(e*cot(c + d*x))^(1/2)*(2*b^9*e^16 + a^8*b*e^16 - 5*a^2*b^7*e^16 +
 17*a^4*b^5*e^16 - 7*a^6*b^3*e^16))/(a^8*d^4 + b^8*d^4 + 4*a^2*b^6*d^4 + 6*
a^4*b^4*d^4 + 4*a^6*b^2*d^4) - ((a^2 - 3*b^2)*((16*(2*a^10*b*d^2*e^15 - 78*
a^2*b^9*d^2*e^15 + 8*a^4*b^7*d^2*e^15 + 60*a^6*b^5*d^2*e^15 - 24*a^8*b^3*d^
2*e^15))/(a^8*d^5 + b^8*d^5 + 4*a^2*b^6*d^5 + 6*a^4*b^4*d^5 + 4*a^6*b^2*d^5
) + ((a^2 - 3*b^2)*((16*(e*cot(c + d*x))^(1/2)*(20*a^3*b^10*d^2*e^13 - 60*a
*b^12*d^2*e^13 + 168*a^5*b^8*d^2*e^13 + 40*a^7*b^6*d^2*e^13 - 44*a^9*b^4*d^
2*e^13 + 4*a^11*b^2*d^2*e^13))/(a^8*d^4 + b^8*d^4 + 4*a^2*b^6*d^4 + 6*a^4*b
^4*d^4 + 4*a^6*b^2*d^4) - ((a^2 - 3*b^2)*((16*(40*a*b^14*d^4*e^12 + 192*a^3
*b^12*d^4*e^12 + 360*a^5*b^10*d^4*e^12 + 320*a^7*b^8*d^4*e^12 + 120*a^9*b^6
*d^4*e^12 - 8*a^13*b^2*d^4*e^12))/(a^8*d^5 + b^8*d^5 + 4*a^2*b^6*d^5 + 6*a^
4*b^4*d^5 + 4*a^6*b^2*d^5) + (8*(e*cot(c + d*x))^(1/2)*(a^2 - 3*b^2)*(-a*b*
```

e^3)^(1/2)*(32*b^17*d^4*e^10 + 160*a^2*b^15*d^4*e^10 + 288*a^4*b^13*d^4*e^1
0 + 160*a^6*b^11*d^4*e^10 - 160*a^8*b^9*d^4*e^10 - 288*a^10*b^7*d^4*e^10 -
160*a^12*b^5*d^4*e^10 - 32*a^14*b^3*d^4*e^10))/((b^5*d + 2*a^2*b^3*d + a^4*
b*d)*(a^8*d^4 + b^8*d^4 + 4*a^2*b^6*d^4 + 6*a^4*b^4*d^4 + 4*a^6*b^2*d^4))*
(-a*b*e^3)^(1/2))/(2*(b^5*d + 2*a^2*b^3*d + a^4*b*d)))*(-a*b*e^3)^(1/2))/(2
*(b^5*d + 2*a^2*b^3*d + a^4*b*d)))*(-a*b*e^3)^(1/2))/(2*(b^5*d + 2*a^2*b^3*
d + a^4*b*d)))*(-a*b*e^3)^(1/2))/(2*(b^5*d + 2*a^2*b^3*d + a^4*b*d))))*(a^2
 - 3*b^2)*(-a*b*e^3)^(1/2)*1i)/(b^5*d + 2*a^2*b^3*d + a^4*b*d) - atan((((((
(16*(40*a*b^14*d^4*e^12 + 192*a^3*b^12*d^4*e^12 + 360*a^5*b^10*d^4*e^12 + 3
20*a^7*b^8*d^4*e^12 + 120*a^9*b^6*d^4*e^12 - 8*a^13*b^2*d^4*e^12))/(a^8*d^5
 + b^8*d^5 + 4*a^2*b^6*d^5 + 6*a^4*b^4*d^5 + 4*a^6*b^2*d^5) - (16*(e*cot(c
+ d*x))^(1/2)*(-e^3/(4*(a^4*d^2*1i + b^4*d^2*1i + 4*a*b^3*d^2 - 4*a^3*b*d^2
 - a^2*b^2*d^2*6i)))^(1/2)*(32*b^17*d^4*e^10 + 160*a^2*b^15*d^4*e^10 + 288*
a^4*b^13*d^4*e^10 + 160*a^6*b^11*d^4*e^10 - 160*a^8*b^9*d^4*e^10 - 288*a^10
*b^7*d^4*e^10 - 160*a^12*b^5*d^4*e^10 - 32*a^14*b^3*d^4*e^10))/(a^8*d^4 + b
^8*d^4 + 4*a^2*b^6*d^4 + 6*a^4*b^4*d^4 + 4*a^6*b^2*d^4))*(-e^3/(4*(a^4*d^2*
1i + b^4*d^2*1i + 4*a*b^3*d^2 - 4*a^3*b*d^2 - a^2*b^2*d^2*6i)))^(1/2) + (16
*(e*cot(c + d*x))^(1/2)*(20*a^3*b^10*d^2*e^13 - 60*a*b^12*d^2*e^13 + 168*a^
5*b^8*d^2*e^13 + 40*a^7*b^6*d^2*e^13 - 44*a^9*b^4*d^2*e^13 + 4*a^11*b^2*d^2
*e^13))/(a^8*d^4 + b^8*d^4 + 4*a^2*b^6*d^4 + 6*a^4*b^4*d^4 + 4*a^6*b^2*d^4)
)*(-e^3/(4*(a^4*d^2*1i + b^4*d^2*1i + 4*a*b^3*d^2 - 4*a^3*b*d^2 - a^2*b^2*d
^2*6i)))^(1/2) - (16*(2*a^10*b*d^2*e^15 - 78*a^2*b^9*d^2*e^15 + 8*a^4*b^7*d
^2*e^15 + 60*a^6*b^5*d^2*e^15 - 24*a^8*b^3*d^2*e^15))/(a^8*d^5 + b^8*d^5 +
4*a^2*b^6*d^5 + 6*a^4*b^4*d^5 + 4*a^6*b^2*d^5))*(-e^3/(4*(a^4*d^2*1i + b^4*
d^2*1i + 4*a*b^3*d^2 - 4*a^3*b*d^2 - a^2*b^2*d^2*6i)))^(1/2) - (16*(e*cot(c
 + d*x))^(1/2)*(2*b^9*e^16 + a^8*b*e^16 - 5*a^2*b^7*e^16 + 17*a^4*b^5*e^16
- 7*a^6*b^3*e^16))/(a^8*d^4 + b^8*d^4 + 4*a^2*b^6*d^4 + 6*a^4*b^4*d^4 + 4*a
^6*b^2*d^4))*(-e^3/(4*(a^4*d^2*1i + b^4*d^2*1i + 4*a*b^3*d^2 - 4*a^3*b*d^2
- a^2*b^2*d^2*6i)))^(1/2)*1i - (((((16*(40*a*b^14*d^4*e^12 + 192*a^3*b^12*d
^4*e^12 + 360*a^5*b^10*d^4*e^12 + 320*a^7*b^8*d^4*e^12 + 120*a^9*b^6*d^4*e^
12 - 8*a^13*b^2*d^4*e^12))/(a^8*d^5 + b^8*d^5 + 4*a^2*b^6*d^5 + 6*a^4*b^4*d
^5 + 4*a^6*b^2*d^5) + (16*(e*cot(c + d*x))^(1/2)*(-e^3/(4*(a^4*d^2*1i + b^4
*d^2*1i + 4*a*b^3*d^2 - 4*a^3*b*d^2 - a^2*b^2*d^2*6i)))^(1/2)*(32*b^17*d^4*
e^10 + 160*a^2*b^15*d^4*e^10 + 288*a^4*b^13*d^4*e^10 + 160*a^6*b^11*d^4*e^1
0 - 160*a^8*b^9*d^4*e^10 - 288*a^10*b^7*d^4*e^10 - 160*a^12*b^5*d^4*e^10 -
32*a^14*b^3*d^4*e^10))/(a^8*d^4 + b^8*d^4 + 4*a^2*b^6*d^4 + 6*a^4*b^4*d^4 +
 4*a^6*b^2*d^4))*(-e^3/(4*(a^4*d^2*1i + b^4*d^2*1i + 4*a*b^3*d^2 - 4*a^3*b*
d^2 - a^2*b^2*d^2*6i)))^(1/2) - (16*(e*cot(c + d*x))^(1/2)*(20*a^3*b^10*d^2
*e^13 - 60*a*b^12*d^2*e^13 + 168*a^5*b^8*d^2*e^13 + 40*a^7*b^6*d^2*e^13 - 4
4*a^9*b^4*d^2*e^13 + 4*a^11*b^2*d^2*e^13))/(a^8*d^4 + b^8*d^4 + 4*a^2*b^6*d
^4 + 6*a^4*b^4*d^4 + 4*a^6*b^2*d^4))*(-e^3/(4*(a^4*d^2*1i + b^4*d^2*1i + 4*
a*b^3*d^2 - 4*a^3*b*d^2 - a^2*b^2*d^2*6i)))^(1/2) - (16*(2*a^10*b*d^2*e^15
- 78*a^2*b^9*d^2*e^15 + 8*a^4*b^7*d^2*e^15 + 60*a^6*b^5*d^2*e^15 - 24*a^8*b
^3*d^2*e^15))/(a^8*d^5 + b^8*d^5 + 4*a^2*b^6*d^5 + 6*a^4*b^4*d^5 + 4*a^6*b^
2*d^5))*(-e^3/(4*(a^4*d^2*1i + b^4*d^2*1i + 4*a*b^3*d^2 - 4*a^3*b*d^2 - a^2

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*b^2*d^2*6i)))^(1/2) + (16*(e*cot(c + d*x))^(1/2)*(2*b^9*e^16 + a^8*b*e^16
- 5*a^2*b^7*e^16 + 17*a^4*b^5*e^16 - 7*a^6*b^3*e^16))/(a^8*d^4 + b^8*d^4 +
4*a^2*b^6*d^4 + 6*a^4*b^4*d^4 + 4*a^6*b^2*d^4))*(-e^3/(4*(a^4*d^2*1i + b^4*
d^2*1i + 4*a*b^3*d^2 - 4*a^3*b*d^2 - a^2*b^2*d^2*6i)))^(1/2)*1i)/((32*(3*a*
b^6*e^18 - a^3*b^4*e^18))/(a^8*d^5 + b^8*d^5 + 4*a^2*b^6*d^5 + 6*a^4*b^4*d^
5 + 4*a^6*b^2*d^5) + (((((16*(40*a*b^14*d^4*e^12 + 192*a^3*b^12*d^4*e^12 +
360*a^5*b^10*d^4*e^12 + 320*a^7*b^8*d^4*e^12 + 120*a^9*b^6*d^4*e^12 - 8*a^1
3*b^2*d^4*e^12))/(a^8*d^5 + b^8*d^5 + 4*a^2*b^6*d^5 + 6*a^4*b^4*d^5 + 4*a^6
*b^2*d^5) - (16*(e*cot(c + d*x))^(1/2)*(-e^3/(4*(a^4*d^2*1i + b^4*d^2*1i +
4*a*b^3*d^2 - 4*a^3*b*d^2 - a^2*b^2*d^2*6i)))^(1/2)*(32*b^17*d^4*e^10 + 160
*a^2*b^15*d^4*e^10 + 288*a^4*b^13*d^4*e^10 + 160*a^6*b^11*d^4*e^10 - 160*a^
8*b^9*d^4*e^10 - 288*a^10*b^7*d^4*e^10 - 160*a^12*b^5*d^4*e^10 - 32*a^14*b^
3*d^4*e^10))/(a^8*d^4 + b^8*d^4 + 4*a^2*b^6*d^4 + 6*a^4*b^4*d^4 + 4*a^6*b^2
*d^4))*(-e^3/(4*(a^4*d^2*1i + b^4*d^2*1i + 4*a*b^3*d^2 - 4*a^3*b*d^2 - a^2*
b^2*d^2*6i)))^(1/2) + (16*(e*cot(c + d*x))^(1/2)*(20*a^3*b^10*d^2*e^13 - 60
*a*b^12*d^2*e^13 + 168*a^5*b^8*d^2*e^13 + 40*a^7*b^6*d^2*e^13 - 44*a^9*b^4*
d^2*e^13 + 4*a^11*b^2*d^2*e^13))/(a^8*d^4 + b^8*d^4 + 4*a^2*b^6*d^4 + 6*a^4
*b^4*d^4 + 4*a^6*b^2*d^4))*(-e^3/(4*(a^4*d^2*1i + b^4*d^2*1i + 4*a*b^3*d^2
- 4*a^3*b*d^2 - a^2*b^2*d^2*6i)))^(1/2) - (16*(2*a^10*b*d^2*e^15 - 78*a^2*b
^9*d^2*e^15 + 8*a^4*b^7*d^2*e^15 + 60*a^6*b^5*d^2*e^15 - 24*a^8*b^3*d^2*e^1
5))/(a^8*d^5 + b^8*d^5 + 4*a^2*b^6*d^5 + 6*a^4*b^4*d^5 + 4*a^6*b^2*d^5))*(-
e^3/(4*(a^4*d^2*1i + b^4*d^2*1i + 4*a*b^3*d^2 - 4*a^3*b*d^2 - a^2*b^2*d^2*6
i)))^(1/2) - (16*(e*cot(c + d*x))^(1/2)*(2*b^9*e^16 + a^8*b*e^16 - 5*a^2*b^
7*e^16 + 17*a^4*b^5*e^16 - 7*a^6*b^3*e^16))/(a^8*d^4 + b^8*d^4 + 4*a^2*b^6*
d^4 + 6*a^4*b^4*d^4 + 4*a^6*b^2*d^4))*(-e^3/(4*(a^4*d^2*1i + b^4*d^2*1i + 4
*a*b^3*d^2 - 4*a^3*b*d^2 - a^2*b^2*d^2*6i)))^(1/2) + (((((16*(40*a*b^14*d^4
*e^12 + 192*a^3*b^12*d^4*e^12 + 360*a^5*b^10*d^4*e^12 + 320*a^7*b^8*d^4*e^1
2 + 120*a^9*b^6*d^4*e^12 - 8*a^13*b^2*d^4*e^12))/(a^8*d^5 + b^8*d^5 + 4*a^2
*b^6*d^5 + 6*a^4*b^4*d^5 + 4*a^6*b^2*d^5) + (16*(e*cot(c + d*x))^(1/2)*(-e^
3/(4*(a^4*d^2*1i + b^4*d^2*1i + 4*a*b^3*d^2 - 4*a^3*b*d^2 - a^2*b^2*d^2*6i)
))^(1/2)*(32*b^17*d^4*e^10 + 160*a^2*b^15*d^4*e^10 + 288*a^4*b^13*d^4*e^10
+ 160*a^6*b^11*d^4*e^10 - 160*a^8*b^9*d^4*e^10 - 288*a^10*b^7*d^4*e^10 - 16
0*a^12*b^5*d^4*e^10 - 32*a^14*b^3*d^4*e^10))/(a^8*d^4 + b^8*d^4 + 4*a^2*b^6
*d^4 + 6*a^4*b^4*d^4 + 4*a^6*b^2*d^4))*(-e^3/(4*(a^4*d^2*1i + b^4*d^2*1i +
4*a*b^3*d^2 - 4*a^3*b*d^2 - a^2*b^2*d^2*6i)))^(1/2) - (16*(e*cot(c + d*x))^
(1/2)*(20*a^3*b^10*d^2*e^13 - 60*a*b^12*d^2*e^13 + 168*a^5*b^8*d^2*e^13 + 4
0*a^7*b^6*d^2*e^13 - 44*a^9*b^4*d^2*e^13 + 4*a^11*b^2*d^2*e^13))/(a^8*d^4 +
 b^8*d^4 + 4*a^2*b^6*d^4 + 6*a^4*b^4*d^4 + 4*a^6*b^2*d^4))*(-e^3/(4*(a^4*d^
2*1i + b^4*d^2*1i + 4*a*b^3*d^2 - 4*a^3*b*d^2 - a^2*b^2*d^2*6i)))^(1/2) - (
16*(2*a^10*b*d^2*e^15 - 78*a^2*b^9*d^2*e^15 + 8*a^4*b^7*d^2*e^15 + 60*a^6*b
^5*d^2*e^15 - 24*a^8*b^3*d^2*e^15))/(a^8*d^5 + b^8*d^5 + 4*a^2*b^6*d^5 + 6*
a^4*b^4*d^5 + 4*a^6*b^2*d^5))*(-e^3/(4*(a^4*d^2*1i + b^4*d^2*1i + 4*a*b^3*d
^2 - 4*a^3*b*d^2 - a^2*b^2*d^2*6i)))^(1/2) + (16*(e*cot(c + d*x))^(1/2)*(2*
b^9*e^16 + a^8*b*e^16 - 5*a^2*b^7*e^16 + 17*a^4*b^5*e^16 - 7*a^6*b^3*e^16))
/(a^8*d^4 + b^8*d^4 + 4*a^2*b^6*d^4 + 6*a^4*b^4*d^4 + 4*a^6*b^2*d^4))*(-e^3
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/(4*(a^4*d^2*1i + b^4*d^2*1i + 4*a*b^3*d^2 - 4*a^3*b*d^2 - a^2*b^2*d^2*6i))
)^(1/2)))*(-e^3/(4*(a^4*d^2*1i + b^4*d^2*1i + 4*a*b^3*d^2 - 4*a^3*b*d^2 - a
^2*b^2*d^2*6i)))^(1/2)*2i - atan((((((((16*(40*a*b^14*d^4*e^12 + 192*a^3*b^1
2*d^4*e^12 + 360*a^5*b^10*d^4*e^12 + 320*a^7*b^8*d^4*e^12 + 120*a^9*b^6*d^4
*e^12 - 8*a^13*b^2*d^4*e^12))/(a^8*d^5 + b^8*d^5 + 4*a^2*b^6*d^5 + 6*a^4*b^
4*d^5 + 4*a^6*b^2*d^5) - (16*(e*cot(c + d*x))^(1/2)*(-(e^3*1i)/(4*(a^4*d^2
+ b^4*d^2 + a*b^3*d^2*4i - a^3*b*d^2*4i - 6*a^2*b^2*d^2)))^(1/2)*(32*b^17*d
^4*e^10 + 160*a^2*b^15*d^4*e^10 + 288*a^4*b^13*d^4*e^10 + 160*a^6*b^11*d^4*
e^10 - 160*a^8*b^9*d^4*e^10 - 288*a^10*b^7*d^4*e^10 - 160*a^12*b^5*d^4*e^10
 - 32*a^14*b^3*d^4*e^10))/(a^8*d^4 + b^8*d^4 + 4*a^2*b^6*d^4 + 6*a^4*b^4*d^
4 + 4*a^6*b^2*d^4))*(-(e^3*1i)/(4*(a^4*d^2 + b^4*d^2 + a*b^3*d^2*4i - a^3*b
*d^2*4i - 6*a^2*b^2*d^2)))^(1/2) + (16*(e*cot(c + d*x))^(1/2)*(20*a^3*b^10*
d^2*e^13 - 60*a*b^12*d^2*e^13 + 168*a^5*b^8*d^2*e^13 + 40*a^7*b^6*d^2*e^13
- 44*a^9*b^4*d^2*e^13 + 4*a^11*b^2*d^2*e^13))/(a^8*d^4 + b^8*d^4 + 4*a^2*b^
6*d^4 + 6*a^4*b^4*d^4 + 4*a^6*b^2*d^4))*(-(e^3*1i)/(4*(a^4*d^2 + b^4*d^2 +
a*b^3*d^2*4i - a^3*b*d^2*4i - 6*a^2*b^2*d^2)))^(1/2) - (16*(2*a^10*b*d^2*e^
15 - 78*a^2*b^9*d^2*e^15 + 8*a^4*b^7*d^2*e^15 + 60*a^6*b^5*d^2*e^15 - 24*a^
8*b^3*d^2*e^15))/(a^8*d^5 + b^8*d^5 + 4*a^2*b^6*d^5 + 6*a^4*b^4*d^5 + 4*a^6
*b^2*d^5))*(-(e^3*1i)/(4*(a^4*d^2 + b^4*d^2 + a*b^3*d^2*4i - a^3*b*d^2*4i -
 6*a^2*b^2*d^2)))^(1/2) - (16*(e*cot(c + d*x))^(1/2)*(2*b^9*e^16 + a^8*b*e^
16 - 5*a^2*b^7*e^16 + 17*a^4*b^5*e^16 - 7*a^6*b^3*e^16))/(a^8*d^4 + b^8*d^4
 + 4*a^2*b^6*d^4 + 6*a^4*b^4*d^4 + 4*a^6*b^2*d^4))*(-(e^3*1i)/(4*(a^4*d^2 +
 b^4*d^2 + a*b^3*d^2*4i - a^3*b*d^2*4i - 6*a^2*b^2*d^2)))^(1/2)*1i - (((((1
6*(40*a*b^14*d^4*e^12 + 192*a^3*b^12*d^4*e^12 + 360*a^5*b^10*d^4*e^12 + 320
*a^7*b^8*d^4*e^12 + 120*a^9*b^6*d^4*e^12 - 8*a^13*b^2*d^4*e^12))/(a^8*d^5 +
 b^8*d^5 + 4*a^2*b^6*d^5 + 6*a^4*b^4*d^5 + 4*a^6*b^2*d^5) + (16*(e*cot(c +
d*x))^(1/2)*(-(e^3*1i)/(4*(a^4*d^2 + b^4*d^2 + a*b^3*d^2*4i - a^3*b*d^2*4i
- 6*a^2*b^2*d^2)))^(1/2)*(32*b^17*d^4*e^10 + 160*a^2*b^15*d^4*e^10 + 288*a^
4*b^13*d^4*e^10 + 160*a^6*b^11*d^4*e^10 - 160*a^8*b^9*d^4*e^10 - 288*a^10*b
^7*d^4*e^10 - 160*a^12*b^5*d^4*e^10 - 32*a^14*b^3*d^4*e^10))/(a^8*d^4 + b^8
*d^4 + 4*a^2*b^6*d^4 + 6*a^4*b^4*d^4 + 4*a^6*b^2*d^4))*(-(e^3*1i)/(4*(a^4*d
^2 + b^4*d^2 + a*b^3*d^2*4i - a^3*b*d^2*4i - 6*a^2*b^2*d^2)))^(1/2) - (16*(
e*cot(c + d*x))^(1/2)*(20*a^3*b^10*d^2*e^13 - 60*a*b^12*d^2*e^13 + 168*a^5*
b^8*d^2*e^13 + 40*a^7*b^6*d^2*e^13 - 44*a^9*b^4*d^2*e^13 + 4*a^11*b^2*d^2*e
^13))/(a^8*d^4 + b^8*d^4 + 4*a^2*b^6*d^4 + 6*a^4*b^4*d^4 + 4*a^6*b^2*d^4))*
(-(e^3*1i)/(4*(a^4*d^2 + b^4*d^2 + a*b^3*d^2*4i - a^3*b*d^2*4i - 6*a^2*b^2*
d^2)))^(1/2) - (16*(2*a^10*b*d^2*e^15 - 78*a^2*b^9*d^2*e^15 + 8*a^4*b^7*d^2
*e^15 + 60*a^6*b^5*d^2*e^15 - 24*a^8*b^3*d^2*e^15))/(a^8*d^5 + b^8*d^5 + 4*
a^2*b^6*d^5 + 6*a^4*b^4*d^5 + 4*a^6*b^2*d^5))*(-(e^3*1i)/(4*(a^4*d^2 + b^4*
d^2 + a*b^3*d^2*4i - a^3*b*d^2*4i - 6*a^2*b^2*d^2)))^(1/2) + (16*(e*cot(c +
 d*x))^(1/2)*(2*b^9*e^16 + a^8*b*e^16 - 5*a^2*b^7*e^16 + 17*a^4*b^5*e^16 -
7*a^6*b^3*e^16))/(a^8*d^4 + b^8*d^4 + 4*a^2*b^6*d^4 + 6*a^4*b^4*d^4 + 4*a^6
*b^2*d^4))*(-(e^3*1i)/(4*(a^4*d^2 + b^4*d^2 + a*b^3*d^2*4i - a^3*b*d^2*4i -
 6*a^2*b^2*d^2)))^(1/2)*1i)/((32*(3*a*b^6*e^18 - a^3*b^4*e^18))/(a^8*d^5 +
b^8*d^5 + 4*a^2*b^6*d^5 + 6*a^4*b^4*d^5 + 4*a^6*b^2*d^5) + (((((16*(40*a*b^

14*d^4*e^12 + 192*a^3*b^12*d^4*e^12 + 360*a^5*b^10*d^4*e^12 + 320*a^7*b^8*d
^4*e^12 + 120*a^9*b^6*d^4*e^12 - 8*a^13*b^2*d^4*e^12))/(a^8*d^5 + b^8*d^5 +
 4*a^2*b^6*d^5 + 6*a^4*b^4*d^5 + 4*a^6*b^2*d^5) - (16*(e*cot(c + d*x))^(1/2
)*(-(e^3*1i)/(4*(a^4*d^2 + b^4*d^2 + a*b^3*d^2*4i - a^3*b*d^2*4i - 6*a^2*b^
2*d^2)))^(1/2)*(32*b^17*d^4*e^10 + 160*a^2*b^15*d^4*e^10 + 288*a^4*b^13*d^4
*e^10 + 160*a^6*b^11*d^4*e^10 - 160*a^8*b^9*d^4*e^10 - 288*a^10*b^7*d^4*e^1
0 - 160*a^12*b^5*d^4*e^10 - 32*a^14*b^3*d^4*e^10))/(a^8*d^4 + b^8*d^4 + 4*a
^2*b^6*d^4 + 6*a^4*b^4*d^4 + 4*a^6*b^2*d^4))*(-(e^3*1i)/(4*(a^4*d^2 + b^4*d
^2 + a*b^3*d^2*4i - a^3*b*d^2*4i - 6*a^2*b^2*d^2)))^(1/2) + (16*(e*cot(c +
d*x))^(1/2)*(20*a^3*b^10*d^2*e^13 - 60*a*b^12*d^2*e^13 + 168*a^5*b^8*d^2*e^
13 + 40*a^7*b^6*d^2*e^13 - 44*a^9*b^4*d^2*e^13 + 4*a^11*b^2*d^2*e^13))/(a^8
*d^4 + b^8*d^4 + 4*a^2*b^6*d^4 + 6*a^4*b^4*d^4 + 4*a^6*b^2*d^4))*(-(e^3*1i)
/(4*(a^4*d^2 + b^4*d^2 + a*b^3*d^2*4i - a^3*b*d^2*4i - 6*a^2*b^2*d^2)))^(1/
2) - (16*(2*a^10*b*d^2*e^15 - 78*a^2*b^9*d^2*e^15 + 8*a^4*b^7*d^2*e^15 + 60
*a^6*b^5*d^2*e^15 - 24*a^8*b^3*d^2*e^15))/(a^8*d^5 + b^8*d^5 + 4*a^2*b^6*d^
5 + 6*a^4*b^4*d^5 + 4*a^6*b^2*d^5))*(-(e^3*1i)/(4*(a^4*d^2 + b^4*d^2 + a*b^
3*d^2*4i - a^3*b*d^2*4i - 6*a^2*b^2*d^2)))^(1/2) - (16*(e*cot(c + d*x))^(1/
2)*(2*b^9*e^16 + a^8*b*e^16 - 5*a^2*b^7*e^16 + 17*a^4*b^5*e^16 - 7*a^6*b^3*
e^16))/(a^8*d^4 + b^8*d^4 + 4*a^2*b^6*d^4 + 6*a^4*b^4*d^4 + 4*a^6*b^2*d^4))
*(-(e^3*1i)/(4*(a^4*d^2 + b^4*d^2 + a*b^3*d^2*4i - a^3*b*d^2*4i - 6*a^2*b^2
d^2)))^(1/2) + (((((16(40*a*b^14*d^4*e^12 + 192*a^3*b^12*d^4*e^12 + 360*a
^5*b^10*d^4*e^12 + 320*a^7*b^8*d^4*e^12 + 120*a^9*b^6*d^4*e^12 - 8*a^13*b^2
*d^4*e^12))/(a^8*d^5 + b^8*d^5 + 4*a^2*b^6*d^5 + 6*a^4*b^4*d^5 + 4*a^6*b^2*
d^5) + (16*(e*cot(c + d*x))^(1/2)*(-(e^3*1i)/(4*(a^4*d^2 + b^4*d^2 + a*b^3*
d^2*4i - a^3*b*d^2*4i - 6*a^2*b^2*d^2)))^(1/2)*(32*b^17*d^4*e^10 + 160*a^2*
b^15*d^4*e^10 + 288*a^4*b^13*d^4*e^10 + 160*a^6*b^11*d^4*e^10 - 160*a^8*b^9
*d^4*e^10 - 288*a^10*b^7*d^4*e^10 - 160*a^12*b^5*d^4*e^10 - 32*a^14*b^3*d^4
*e^10))/(a^8*d^4 + b^8*d^4 + 4*a^2*b^6*d^4 + 6*a^4*b^4*d^4 + 4*a^6*b^2*d^4)
)*(-(e^3*1i)/(4*(a^4*d^2 + b^4*d^2 + a*b^3*d^2*4i - a^3*b*d^2*4i - 6*a^2*b^
2*d^2)))^(1/2) - (16*(e*cot(c + d*x))^(1/2)*(20*a^3*b^10*d^2*e^13 - 60*a*b^
12*d^2*e^13 + 168*a^5*b^8*d^2*e^13 + 40*a^7*b^6*d^2*e^13 - 44*a^9*b^4*d^2*e
^13 + 4*a^11*b^2*d^2*e^13))/(a^8*d^4 + b^8*d^4 + 4*a^2*b^6*d^4 + 6*a^4*b^4*
d^4 + 4*a^6*b^2*d^4))*(-(e^3*1i)/(4*(a^4*d^2 + b^4*d^2 + a*b^3*d^2*4i - a^3
*b*d^2*4i - 6*a^2*b^2*d^2)))^(1/2) - (16*(2*a^10*b*d^2*e^15 - 78*a^2*b^9*d^
2*e^15 + 8*a^4*b^7*d^2*e^15 + 60*a^6*b^5*d^2*e^15 - 24*a^8*b^3*d^2*e^15))/(
a^8*d^5 + b^8*d^5 + 4*a^2*b^6*d^5 + 6*a^4*b^4*d^5 + 4*a^6*b^2*d^5))*(-(e^3*
1i)/(4*(a^4*d^2 + b^4*d^2 + a*b^3*d^2*4i - a^3*b*d^2*4i - 6*a^2*b^2*d^2)))^
(1/2) + (16*(e*cot(c + d*x))^(1/2)*(2*b^9*e^16 + a^8*b*e^16 - 5*a^2*b^7*e^1
6 + 17*a^4*b^5*e^16 - 7*a^6*b^3*e^16))/(a^8*d^4 + b^8*d^4 + 4*a^2*b^6*d^4 +
 6*a^4*b^4*d^4 + 4*a^6*b^2*d^4))*(-(e^3*1i)/(4*(a^4*d^2 + b^4*d^2 + a*b^3*d
^2*4i - a^3*b*d^2*4i - 6*a^2*b^2*d^2)))^(1/2)))*(-(e^3*1i)/(4*(a^4*d^2 + b^
4*d^2 + a*b^3*d^2*4i - a^3*b*d^2*4i - 6*a^2*b^2*d^2)))^(1/2)*2i - (a*e^2*(e
*cot(c + d*x))^(1/2))/((a*d*e + b*d*e*cot(c + d*x))*(a^2 + b^2))

3.78 $\int \frac{\sqrt{e\cot(c+dx)}}{(a+b\cot(c+dx))^2}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 386

$$\begin{aligned}
\int \frac{\sqrt{e\cot(c+dx)}}{(a+b\cot(c+dx))^2}\,dx
&= \frac{\sqrt{b}(3a^2-b^2)\sqrt{e}\arctan\left(\frac{\sqrt{b}\sqrt{e\cot(c+dx)}}{\sqrt{a}\sqrt{e}}\right)}{\sqrt{a}\,(a^2+b^2)^2\,d} \\
&\quad+\frac{(a^2+2ab-b^2)\sqrt{e}\arctan\left(1-\frac{\sqrt{2}\sqrt{e\cot(c+dx)}}{\sqrt{e}}\right)}{\sqrt{2}\,(a^2+b^2)^2\,d} \\
&\quad-\frac{(a^2+2ab-b^2)\sqrt{e}\arctan\left(1+\frac{\sqrt{2}\sqrt{e\cot(c+dx)}}{\sqrt{e}}\right)}{\sqrt{2}\,(a^2+b^2)^2\,d}+\frac{b\sqrt{e\cot(c+dx)}}{(a^2+b^2)\,d(a+b\cot(c+dx))} \\
&\quad-\frac{(a^2-2ab-b^2)\sqrt{e}\log\left(\sqrt{e}+\sqrt{e}\cot(c+dx)-\sqrt{2}\sqrt{e\cot(c+dx)}\right)}{2\sqrt{2}\,(a^2+b^2)^2\,d} \\
&\quad+\frac{(a^2-2ab-b^2)\sqrt{e}\log\left(\sqrt{e}+\sqrt{e}\cot(c+dx)+\sqrt{2}\sqrt{e\cot(c+dx)}\right)}{2\sqrt{2}\,(a^2+b^2)^2\,d}
\end{aligned}$$

```
[Out] 1/2*(a^2+2*a*b-b^2)*arctan(1-2^(1/2)*(e*cot(d*x+c))^(1/2)/e^(1/2))*e^(1/2)/
(a^2+b^2)^2/d*2^(1/2)-1/2*(a^2+2*a*b-b^2)*arctan(1+2^(1/2)*(e*cot(d*x+c))^(
1/2)/e^(1/2))*e^(1/2)/(a^2+b^2)^2/d*2^(1/2)-1/4*(a^2-2*a*b-b^2)*ln(e^(1/2)+
cot(d*x+c)*e^(1/2)-2^(1/2)*(e*cot(d*x+c))^(1/2))*e^(1/2)/(a^2+b^2)^2/d*2^(1
/2)+1/4*(a^2-2*a*b-b^2)*ln(e^(1/2)+cot(d*x+c)*e^(1/2)+2^(1/2)*(e*cot(d*x+c)
)^(1/2))*e^(1/2)/(a^2+b^2)^2/d*2^(1/2)+(3*a^2-b^2)*arctan(b^(1/2)*(e*cot(d*
x+c))^(1/2)/a^(1/2)/e^(1/2))*b^(1/2)*e^(1/2)/(a^2+b^2)^2/d/a^(1/2)+b*(e*cot
(d*x+c))^(1/2)/(a^2+b^2)/d/(a+b*cot(d*x+c))
```

Rubi [A] (verified)

Time = 0.78 (sec) , antiderivative size = 386, normalized size of antiderivative = 1.00, number of steps used = 15, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.480, Rules used = {3649, 3734, 3615, 1182, 1176, 631, 210, 1179, 642, 3715, 65, 211}

$$\begin{aligned}
&\int \frac{\sqrt{e\cot(c+dx)}}{(a+b\cot(c+dx))^2}\,dx \\
&=\frac{\sqrt{b}\sqrt{e}(3a^2-b^2)\arctan\left(\frac{\sqrt{b}\sqrt{e\cot(c+dx)}}{\sqrt{a}\sqrt{e}}\right)}{\sqrt{a}d\,(a^2+b^2)^2}+\frac{\sqrt{e}(a^2+2ab-b^2)\arctan\left(1-\frac{\sqrt{2}\sqrt{e\cot(c+dx)}}{\sqrt{e}}\right)}{\sqrt{2}d\,(a^2+b^2)^2} \\
&\quad-\frac{\sqrt{e}(a^2+2ab-b^2)\arctan\left(\frac{\sqrt{2}\sqrt{e\cot(c+dx)}}{\sqrt{e}}+1\right)}{\sqrt{2}d\,(a^2+b^2)^2}+\frac{b\sqrt{e\cot(c+dx)}}{d\,(a^2+b^2)\,(a+b\cot(c+dx))} \\
&\quad-\frac{\sqrt{e}(a^2-2ab-b^2)\log\left(\sqrt{e}\cot(c+dx)-\sqrt{2}\sqrt{e\cot(c+dx)}+\sqrt{e}\right)}{2\sqrt{2}d\,(a^2+b^2)^2} \\
&\quad+\frac{\sqrt{e}(a^2-2ab-b^2)\log\left(\sqrt{e}\cot(c+dx)+\sqrt{2}\sqrt{e\cot(c+dx)}+\sqrt{e}\right)}{2\sqrt{2}d\,(a^2+b^2)^2}
\end{aligned}$$

```
[In] Int[Sqrt[e*Cot[c + d*x]]/(a + b*Cot[c + d*x])^2,x]
```

```
[Out] (Sqrt[b]*(3*a^2 - b^2)*Sqrt[e]*ArcTan[(Sqrt[b]*Sqrt[e*Cot[c + d*x]])/(Sqrt[
a]*Sqrt[e])])/(Sqrt[a]*(a^2 + b^2)^2*d) + ((a^2 + 2*a*b - b^2)*Sqrt[e]*ArcT
an[1 - (Sqrt[2]*Sqrt[e*Cot[c + d*x]])/Sqrt[e]])/(Sqrt[2]*(a^2 + b^2)^2*d) -
 ((a^2 + 2*a*b - b^2)*Sqrt[e]*ArcTan[1 + (Sqrt[2]*Sqrt[e*Cot[c + d*x]])/Sqr
t[e]])/(Sqrt[2]*(a^2 + b^2)^2*d) + (b*Sqrt[e*Cot[c + d*x]])/((a^2 + b^2)*d*
(a + b*Cot[c + d*x])) - ((a^2 - 2*a*b - b^2)*Sqrt[e]*Log[Sqrt[e] + Sqrt[e]*
Cot[c + d*x] - Sqrt[2]*Sqrt[e*Cot[c + d*x]]])/(2*Sqrt[2]*(a^2 + b^2)^2*d) +
 ((a^2 - 2*a*b - b^2)*Sqrt[e]*Log[Sqrt[e] + Sqrt[e]*Cot[c + d*x] + Sqrt[2]*
Sqrt[e*Cot[c + d*x]]])/(2*Sqrt[2]*(a^2 + b^2)^2*d)
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1179

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 1182

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
a*c, 2]}, Dist[(d*q + a*e)/(2*a*c), Int[(q + c*x^2)/(a + c*x^4), x], x] + D
ist[(d*q - a*e)/(2*a*c), Int[(q - c*x^2)/(a + c*x^4), x], x]] /; FreeQ[{a,
c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0] && NeQ[c*d^2 - a*e^2, 0] && NegQ[(-a)
*c]
```

Rule 3615

```
Int[((c_) + (d_.)*tan[(e_.) + (f_.)*(x_)])/Sqrt[(b_.)*tan[(e_.) + (f_.)*(x_
)]], x_Symbol] :> Dist[2/f, Subst[Int[(b*c + d*x^2)/(b^2 + x^4), x], x, Sqr
t[b*Tan[e + f*x]]], x] /; FreeQ[{b, c, d, e, f}, x] && NeQ[c^2 - d^2, 0] &&
 NeQ[c^2 + d^2, 0]
```

Rule 3649

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)])^(n_), x_Symbol] :> Simp[b*(a + b*Tan[e + f*x])^(m + 1)*((c +
d*Tan[e + f*x])^n/(f*(m + 1)*(a^2 + b^2))), x] + Dist[1/((m + 1)*(a^2 + b^2
)), Int[(a + b*Tan[e + f*x])^(m + 1)*(c + d*Tan[e + f*x])^(n - 1)*Simp[a*c*
(m + 1) - b*d*n - (b*c - a*d)*(m + 1)*Tan[e + f*x] - b*d*(m + n + 1)*Tan[e
+ f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] &
& NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] && LtQ[m, -1] && GtQ[n, 0] && Inte
gerQ[2*m]
```

Rule 3715

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*tan[(e_.)
+ (f_.)*(x_)])^(n_.)*((A_) + (C_.)*tan[(e_.) + (f_.)*(x_)]^2), x_Symbol] :>
 Dist[A/f, Subst[Int[(a + b*x)^m*(c + d*x)^n, x], x, Tan[e + f*x]], x] /; F
reeQ[{a, b, c, d, e, f, A, C, m, n}, x] && EqQ[A, C]
```

Rule 3734

```
Int[(((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_)*((A_.) + (B_.)*tan[(e_.)
+ (f_.)*(x_)] + (C_.)*tan[(e_.) + (f_.)*(x_)]^2))/((a_.) + (b_.)*tan[(e_.)
+ (f_.)*(x_)]), x_Symbol] :> Dist[1/(a^2 + b^2), Int[(c + d*Tan[e + f*x])^n
*Simp[b*B + a*(A - C) + (a*B - b*(A - C))*Tan[e + f*x], x], x], x] + Dist[(
A*b^2 - a*b*B + a^2*C)/(a^2 + b^2), Int[(c + d*Tan[e + f*x])^n*((1 + Tan[e
+ f*x]^2)/(a + b*Tan[e + f*x])), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C
, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] &&
 !GtQ[n, 0] &&  !LeQ[n, -1]
```

Rubi steps

$$\text{integral} = \frac{b\sqrt{e\cot(c+dx)}}{(a^2+b^2)\,d(a+b\cot(c+dx))} - \frac{\int \frac{-\frac{be}{2}-ae\cot(c+dx)+\frac{1}{2}be\cot^2(c+dx)}{\sqrt{e\cot(c+dx)}(a+b\cot(c+dx))}\,dx}{a^2+b^2}$$

$$= \frac{b\sqrt{e\cot(c+dx)}}{(a^2+b^2)\,d(a+b\cot(c+dx))} - \frac{\int \frac{-2abe-(a^2-b^2)e\cot(c+dx)}{\sqrt{e\cot(c+dx)}}\,dx}{(a^2+b^2)^2} - \frac{(b(3a^2-b^2)\,e)\int \frac{1+\cot^2(c+dx)}{\sqrt{e\cot(c+dx)}(a+b\cot(c+dx))}\,dx}{2\,(a^2+b^2)^2}$$

$$= \frac{b\sqrt{e\cot(c+dx)}}{(a^2+b^2)\,d(a+b\cot(c+dx))} - \frac{2\mathrm{Subst}\left(\int \frac{2abe^2+(a^2-b^2)ex^2}{e^2+x^4}\,dx, x, \sqrt{e\cot(c+dx)}\right)}{(a^2+b^2)^2\,d} - \frac{(b(3a^2-b^2)\,e)\,\mathrm{Subst}\left(\int \frac{1}{\sqrt{-ex}(a-bx)}\,dx, x, -\cot(c+dx)\right)}{2\,(a^2+b^2)^2\,d}$$

$$=\frac{b\sqrt{e\cot(c+dx)}}{(a^2+b^2)\,d(a+b\cot(c+dx))}$$
$$+\frac{(b(3a^2-b^2))\,\text{Subst}\left(\int\frac{1}{a+\frac{bx^2}{e}}\,dx,x,\sqrt{e\cot(c+dx)}\right)}{(a^2+b^2)^2\,d}$$
$$+\frac{((a^2-2ab-b^2)\,e)\,\text{Subst}\left(\int\frac{e-x^2}{e^2+x^4}\,dx,x,\sqrt{e\cot(c+dx)}\right)}{(a^2+b^2)^2\,d}$$
$$-\frac{((a^2+2ab-b^2)\,e)\,\text{Subst}\left(\int\frac{e+x^2}{e^2+x^4}\,dx,x,\sqrt{e\cot(c+dx)}\right)}{(a^2+b^2)^2\,d}$$

$$=\frac{\sqrt{b}(3a^2-b^2)\,\sqrt{e}\arctan\left(\frac{\sqrt{b}\sqrt{e\cot(c+dx)}}{\sqrt{a}\sqrt{e}}\right)}{\sqrt{a}\,(a^2+b^2)^2\,d}+\frac{b\sqrt{e\cot(c+dx)}}{(a^2+b^2)\,d(a+b\cot(c+dx))}$$
$$-\frac{((a^2-2ab-b^2)\,\sqrt{e})\,\text{Subst}\left(\int\frac{\sqrt{2}\sqrt{e}+2x}{-e-\sqrt{2}\sqrt{e}x-x^2}\,dx,x,\sqrt{e\cot(c+dx)}\right)}{2\sqrt{2}\,(a^2+b^2)^2\,d}$$
$$-\frac{((a^2-2ab-b^2)\,\sqrt{e})\,\text{Subst}\left(\int\frac{\sqrt{2}\sqrt{e}-2x}{-e+\sqrt{2}\sqrt{e}x-x^2}\,dx,x,\sqrt{e\cot(c+dx)}\right)}{2\sqrt{2}\,(a^2+b^2)^2\,d}$$
$$-\frac{((a^2+2ab-b^2)\,e)\,\text{Subst}\left(\int\frac{1}{e-\sqrt{2}\sqrt{e}x+x^2}\,dx,x,\sqrt{e\cot(c+dx)}\right)}{2\,(a^2+b^2)^2\,d}$$
$$-\frac{((a^2+2ab-b^2)\,e)\,\text{Subst}\left(\int\frac{1}{e+\sqrt{2}\sqrt{e}x+x^2}\,dx,x,\sqrt{e\cot(c+dx)}\right)}{2\,(a^2+b^2)^2\,d}$$

$$=\frac{\sqrt{b}(3a^2-b^2)\,\sqrt{e}\arctan\left(\frac{\sqrt{b}\sqrt{e\cot(c+dx)}}{\sqrt{a}\sqrt{e}}\right)}{\sqrt{a}\,(a^2+b^2)^2\,d}+\frac{b\sqrt{e\cot(c+dx)}}{(a^2+b^2)\,d(a+b\cot(c+dx))}$$
$$-\frac{(a^2-2ab-b^2)\,\sqrt{e}\log\left(\sqrt{e}+\sqrt{e}\cot(c+dx)-\sqrt{2}\sqrt{e\cot(c+dx)}\right)}{2\sqrt{2}\,(a^2+b^2)^2\,d}$$
$$+\frac{(a^2-2ab-b^2)\,\sqrt{e}\log\left(\sqrt{e}+\sqrt{e}\cot(c+dx)+\sqrt{2}\sqrt{e\cot(c+dx)}\right)}{2\sqrt{2}\,(a^2+b^2)^2\,d}$$
$$-\frac{((a^2+2ab-b^2)\,\sqrt{e})\,\text{Subst}\left(\int\frac{1}{-1-x^2}\,dx,x,1-\frac{\sqrt{2}\sqrt{e\cot(c+dx)}}{\sqrt{e}}\right)}{\sqrt{2}\,(a^2+b^2)^2\,d}$$
$$+\frac{((a^2+2ab-b^2)\,\sqrt{e})\,\text{Subst}\left(\int\frac{1}{-1-x^2}\,dx,x,1+\frac{\sqrt{2}\sqrt{e\cot(c+dx)}}{\sqrt{e}}\right)}{\sqrt{2}\,(a^2+b^2)^2\,d}$$

$$
= \frac{\sqrt{b}(3a^2-b^2)\sqrt{e}\arctan\left(\frac{\sqrt{b}\sqrt{e\cot(c+dx)}}{\sqrt{a}\sqrt{e}}\right)}{\sqrt{a}\,(a^2+b^2)^2\,d}
+ \frac{(a^2+2ab-b^2)\sqrt{e}\arctan\left(1-\frac{\sqrt{2}\sqrt{e\cot(c+dx)}}{\sqrt{e}}\right)}{\sqrt{2}\,(a^2+b^2)^2\,d}
- \frac{(a^2+2ab-b^2)\sqrt{e}\arctan\left(1+\frac{\sqrt{2}\sqrt{e\cot(c+dx)}}{\sqrt{e}}\right)}{\sqrt{2}\,(a^2+b^2)^2\,d}
+ \frac{b\sqrt{e\cot(c+dx)}}{(a^2+b^2)\,d(a+b\cot(c+dx))}
- \frac{(a^2-2ab-b^2)\sqrt{e}\log\left(\sqrt{e}+\sqrt{e}\cot(c+dx)-\sqrt{2}\sqrt{e\cot(c+dx)}\right)}{2\sqrt{2}\,(a^2+b^2)^2\,d}
+ \frac{(a^2-2ab-b^2)\sqrt{e}\log\left(\sqrt{e}+\sqrt{e}\cot(c+dx)+\sqrt{2}\sqrt{e\cot(c+dx)}\right)}{2\sqrt{2}\,(a^2+b^2)^2\,d}
$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 6.11 (sec) , antiderivative size = 399, normalized size of antiderivative = 1.03

$$
\int \frac{\sqrt{e\cot(c+dx)}}{(a+b\cot(c+dx))^2}\,dx =
- \frac{\sqrt{e\cot(c+dx)}\left(-\frac{4a^{3/2}\sqrt{b}\arctan\left(\frac{\sqrt{b}\sqrt{\cot(c+dx)}}{\sqrt{a}}\right)}{(a^2+b^2)^2} + \frac{4ab\sqrt{\cot(c+dx)}}{(a^2+b^2)^2} - \frac{\sqrt{b}\left(-a\arctan\left(\frac{\sqrt{b}\sqrt{\cot(c+dx)}}{\sqrt{a}}\right)+\sqrt{a}\sqrt{b}\sqrt{\cot(c+dx)}-b\right.}{\sqrt{a}(a^2+b^2)(a+b\cot(c+}\right.}{}
$$

```
[In] Integrate[Sqrt[e*Cot[c + d*x]]/(a + b*Cot[c + d*x])^2,x]
```

```
[Out] -((Sqrt[e*Cot[c + d*x]]*((-4*a^(3/2)*Sqrt[b]*ArcTan[(Sqrt[b]*Sqrt[Cot[c + d
*x]])/Sqrt[a]])/(a^2 + b^2)^2 + (4*a*b*Sqrt[Cot[c + d*x]])/(a^2 + b^2)^2 -
(Sqrt[b]*(-(a*ArcTan[(Sqrt[b]*Sqrt[Cot[c + d*x]])/Sqrt[a]]) + Sqrt[a]*Sqrt[
b]*Sqrt[Cot[c + d*x]] - b*ArcTan[(Sqrt[b]*Sqrt[Cot[c + d*x]])/Sqrt[a]]*Cot[
c + d*x]))/(Sqrt[a]*(a^2 + b^2)*(a + b*Cot[c + d*x])) + (2*(a - b)*(a + b)*
Cot[c + d*x]^(3/2)*Hypergeometric2F1[3/4, 1, 7/4, -Cot[c + d*x]^2])/(3*(a^2
 + b^2)^2) - (a*b*(2*Sqrt[2]*ArcTan[1 - Sqrt[2]*Sqrt[Cot[c + d*x]]] - 2*Sqr
t[2]*ArcTan[1 + Sqrt[2]*Sqrt[Cot[c + d*x]]] + 8*Sqrt[Cot[c + d*x]] + Sqrt[2
]*Log[1 - Sqrt[2]*Sqrt[Cot[c + d*x]] + Cot[c + d*x]] - Sqrt[2]*Log[1 + Sqrt
[2]*Sqrt[Cot[c + d*x]] + Cot[c + d*x]]))/(2*(a^2 + b^2)^2)))/(d*Sqrt[Cot[c
+ d*x]]))
```

Maple [A] (verified)

Time = 0.04 (sec) , antiderivative size = 391, normalized size of antiderivative = 1.01

method	result
derivativedivides	$-\frac{2e^3\left(-\frac{b\left(\frac{\left(\frac{a^2}{2}+\frac{b^2}{2}\right)\sqrt{e\cot(dx+c)}}{e\cot(dx+c)b+ae}+\frac{(3a^2-b^2)\arctan\left(\frac{\sqrt{e\cot(dx+c)}\,b}{\sqrt{aeb}}\right)}{2\sqrt{aeb}}\right)}{e^2(a^2+b^2)^2}+\frac{ab(e^2)^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\frac{e\cot(dx+c)+(e^2)^{\frac{1}{4}}\sqrt{e\cot(dx+c}}{e\cot(dx+c)-(e^2)^{\frac{1}{4}}\sqrt{e\cot(dx+c}}\right.\right.}{}\right.}{}$
default	$-\frac{2e^3\left(-\frac{b\left(\frac{\left(\frac{a^2}{2}+\frac{b^2}{2}\right)\sqrt{e\cot(dx+c)}}{e\cot(dx+c)b+ae}+\frac{(3a^2-b^2)\arctan\left(\frac{\sqrt{e\cot(dx+c)}\,b}{\sqrt{aeb}}\right)}{2\sqrt{aeb}}\right)}{e^2(a^2+b^2)^2}+\frac{ab(e^2)^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\frac{e\cot(dx+c)+(e^2)^{\frac{1}{4}}\sqrt{e\cot(dx+c}}{e\cot(dx+c)-(e^2)^{\frac{1}{4}}\sqrt{e\cot(dx+c}}\right.\right.}{}\right.}{}$

```
[In] int((e*cot(d*x+c))^(1/2)/(a+b*cot(d*x+c))^2,x,method=_RETURNVERBOSE)
```

```
[Out] -2/d*e^3*(-b/e^2/(a^2+b^2)^2*((1/2*a^2+1/2*b^2)*(e*cot(d*x+c))^(1/2)/(e*cot
(d*x+c)*b+a*e)+1/2*(3*a^2-b^2)/(a*e*b)^(1/2)*arctan((e*cot(d*x+c))^(1/2)*b/
(a*e*b)^(1/2)))+1/e^2/(a^2+b^2)^2*(1/4*a/e*b*(e^2)^(1/4)*2^(1/2)*(ln((e*cot
(d*x+c)+(e^2)^(1/4)*(e*cot(d*x+c))^(1/2)*2^(1/2)+(e^2)^(1/2))/(e*cot(d*x+c)
-(e^2)^(1/4)*(e*cot(d*x+c))^(1/2)*2^(1/2)+(e^2)^(1/2)))+2*arctan(2^(1/2)/(e
^2)^(1/4)*(e*cot(d*x+c))^(1/2)+1)-2*arctan(-2^(1/2)/(e^2)^(1/4)*(e*cot(d*x+
c))^(1/2)+1))+1/8*(a^2-b^2)/(e^2)^(1/4)*2^(1/2)*(ln((e*cot(d*x+c)-(e^2)^(1/
4)*(e*cot(d*x+c))^(1/2)*2^(1/2)+(e^2)^(1/2))/(e*cot(d*x+c)+(e^2)^(1/4)*(e*c
ot(d*x+c))^(1/2)*2^(1/2)+(e^2)^(1/2)))+2*arctan(2^(1/2)/(e^2)^(1/4)*(e*cot(
d*x+c))^(1/2)+1)-2*arctan(-2^(1/2)/(e^2)^(1/4)*(e*cot(d*x+c))^(1/2)+1))))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 3031 vs. 2(323) = 646.

Time = 0.45 (sec) , antiderivative size = 6104, normalized size of antiderivative = 15.81

$$\int \frac{\sqrt{e\cot(c+dx)}}{(a+b\cot(c+dx))^2}\,dx = \text{Too large to display}$$

```
[In] integrate((e*cot(d*x+c))^(1/2)/(a+b*cot(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] Too large to include
```

Sympy [F]

$$\int \frac{\sqrt{e\cot(c+dx)}}{(a+b\cot(c+dx))^2}\,dx = \int \frac{\sqrt{e\cot(c+dx)}}{(a+b\cot(c+dx))^2}\,dx$$

```
[In] integrate((e*cot(d*x+c))**(1/2)/(a+b*cot(d*x+c))**2,x)
[Out] Integral(sqrt(e*cot(c + d*x))/(a + b*cot(c + d*x))**2, x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{\sqrt{e\cot(c+dx)}}{(a+b\cot(c+dx))^2}\,dx = \text{Exception raised: ValueError}$$

```
[In] integrate((e*cot(d*x+c))^(1/2)/(a+b*cot(d*x+c))^2,x, algorithm="maxima")
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(e>0)', see `assume?` for more detai
ls)Is e
```

Giac [F]

$$\int \frac{\sqrt{e\cot(c+dx)}}{(a+b\cot(c+dx))^2}\,dx = \int \frac{\sqrt{e\cot(dx+c)}}{(b\cot(dx+c)+a)^2}\,dx$$

```
[In] integrate((e*cot(d*x+c))^(1/2)/(a+b*cot(d*x+c))^2,x, algorithm="giac")
[Out] integrate(sqrt(e*cot(d*x + c))/(b*cot(d*x + c) + a)^2, x)
```

Mupad [B] (verification not implemented)

Time = 15.69 (sec) , antiderivative size = 11731, normalized size of antiderivative = 30.39

$$\int \frac{\sqrt{e\cot(c+dx)}}{(a+b\cot(c+dx))^2}\,dx = \text{Too large to display}$$

```
[In] int((e*cot(c + d*x))^(1/2)/(a + b*cot(c + d*x))^2,x)
[Out] (b*e*(e*cot(c + d*x))^(1/2))/((a*d*e + b*d*e*cot(c + d*x))*(a^2 + b^2)) - a
tan((((((8*(320*a^6*b^9*d^4*e^11 - 96*a^2*b^13*d^4*e^11 - 32*b^15*d^4*e^11
```

+ 480*a^8*b^7*d^4*e^11 + 288*a^10*b^5*d^4*e^11 + 64*a^12*b^3*d^4*e^11))/(a
^8*d^5 + b^8*d^5 + 4*a^2*b^6*d^5 + 6*a^4*b^4*d^5 + 4*a^6*b^2*d^5) - (16*(e*
cot(c + d*x))^(1/2)*(e/(4*(a^4*d^2*1i + b^4*d^2*1i + 4*a*b^3*d^2 - 4*a^3*b*
d^2 - a^2*b^2*d^2*6i)))^(1/2)*(32*b^17*d^4*e^10 + 160*a^2*b^15*d^4*e^10 + 2
88*a^4*b^13*d^4*e^10 + 160*a^6*b^11*d^4*e^10 - 160*a^8*b^9*d^4*e^10 - 288*a
^10*b^7*d^4*e^10 - 160*a^12*b^5*d^4*e^10 - 32*a^14*b^3*d^4*e^10))/(a^8*d^4
+ b^8*d^4 + 4*a^2*b^6*d^4 + 6*a^4*b^4*d^4 + 4*a^6*b^2*d^4))*(e/(4*(a^4*d^2*
1i + b^4*d^2*1i + 4*a*b^3*d^2 - 4*a^3*b*d^2 - a^2*b^2*d^2*6i)))^(1/2) + (16
*(e*cot(c + d*x))^(1/2)*(68*a*b^12*d^2*e^11 + 20*a^3*b^10*d^2*e^11 - 88*a^5
*b^8*d^2*e^11 + 40*a^7*b^6*d^2*e^11 + 84*a^9*b^4*d^2*e^11 + 4*a^11*b^2*d^2*
e^11))/(a^8*d^4 + b^8*d^4 + 4*a^2*b^6*d^4 + 6*a^4*b^4*d^4 + 4*a^6*b^2*d^4))
(e/(4(a^4*d^2*1i + b^4*d^2*1i + 4*a*b^3*d^2 - 4*a^3*b*d^2 - a^2*b^2*d^2*6
i)))^(1/2) + (8*(52*a*b^10*d^2*e^12 - 128*a^3*b^8*d^2*e^12 - 24*a^5*b^6*d^2
*e^12 + 160*a^7*b^4*d^2*e^12 + 4*a^9*b^2*d^2*e^12))/(a^8*d^5 + b^8*d^5 + 4*
a^2*b^6*d^5 + 6*a^4*b^4*d^5 + 4*a^6*b^2*d^5))*(e/(4*(a^4*d^2*1i + b^4*d^2*1
i + 4*a*b^3*d^2 - 4*a^3*b*d^2 - a^2*b^2*d^2*6i)))^(1/2) - (16*(e*cot(c + d*
x))^(1/2)*(3*b^9*e^12 - 3*a^2*b^7*e^12 + 17*a^4*b^5*e^12 - 9*a^6*b^3*e^12))
/(a^8*d^4 + b^8*d^4 + 4*a^2*b^6*d^4 + 6*a^4*b^4*d^4 + 4*a^6*b^2*d^4))*(e/(4
*(a^4*d^2*1i + b^4*d^2*1i + 4*a*b^3*d^2 - 4*a^3*b*d^2 - a^2*b^2*d^2*6i)))^(
1/2)*1i - (((((8*(320*a^6*b^9*d^4*e^11 - 96*a^2*b^13*d^4*e^11 - 32*b^15*d^4
*e^11 + 480*a^8*b^7*d^4*e^11 + 288*a^10*b^5*d^4*e^11 + 64*a^12*b^3*d^4*e^11
))/(a^8*d^5 + b^8*d^5 + 4*a^2*b^6*d^5 + 6*a^4*b^4*d^5 + 4*a^6*b^2*d^5) + (1
6*(e*cot(c + d*x))^(1/2)*(e/(4*(a^4*d^2*1i + b^4*d^2*1i + 4*a*b^3*d^2 - 4*a
^3*b*d^2 - a^2*b^2*d^2*6i)))^(1/2)*(32*b^17*d^4*e^10 + 160*a^2*b^15*d^4*e^1
0 + 288*a^4*b^13*d^4*e^10 + 160*a^6*b^11*d^4*e^10 - 160*a^8*b^9*d^4*e^10 -
288*a^10*b^7*d^4*e^10 - 160*a^12*b^5*d^4*e^10 - 32*a^14*b^3*d^4*e^10))/(a^8
*d^4 + b^8*d^4 + 4*a^2*b^6*d^4 + 6*a^4*b^4*d^4 + 4*a^6*b^2*d^4))*(e/(4*(a^4
*d^2*1i + b^4*d^2*1i + 4*a*b^3*d^2 - 4*a^3*b*d^2 - a^2*b^2*d^2*6i)))^(1/2)
- (16*(e*cot(c + d*x))^(1/2)*(68*a*b^12*d^2*e^11 + 20*a^3*b^10*d^2*e^11 - 8
8*a^5*b^8*d^2*e^11 + 40*a^7*b^6*d^2*e^11 + 84*a^9*b^4*d^2*e^11 + 4*a^11*b^2
*d^2*e^11))/(a^8*d^4 + b^8*d^4 + 4*a^2*b^6*d^4 + 6*a^4*b^4*d^4 + 4*a^6*b^2*
d^4))*(e/(4*(a^4*d^2*1i + b^4*d^2*1i + 4*a*b^3*d^2 - 4*a^3*b*d^2 - a^2*b^2*
d^2*6i)))^(1/2) + (8*(52*a*b^10*d^2*e^12 - 128*a^3*b^8*d^2*e^12 - 24*a^5*b^
6*d^2*e^12 + 160*a^7*b^4*d^2*e^12 + 4*a^9*b^2*d^2*e^12))/(a^8*d^5 + b^8*d^5
+ 4*a^2*b^6*d^5 + 6*a^4*b^4*d^5 + 4*a^6*b^2*d^5))*(e/(4*(a^4*d^2*1i + b^4*
d^2*1i + 4*a*b^3*d^2 - 4*a^3*b*d^2 - a^2*b^2*d^2*6i)))^(1/2) + (16*(e*cot(c
+ d*x))^(1/2)*(3*b^9*e^12 - 3*a^2*b^7*e^12 + 17*a^4*b^5*e^12 - 9*a^6*b^3*e
^12))/(a^8*d^4 + b^8*d^4 + 4*a^2*b^6*d^4 + 6*a^4*b^4*d^4 + 4*a^6*b^2*d^4))*
(e/(4*(a^4*d^2*1i + b^4*d^2*1i + 4*a*b^3*d^2 - 4*a^3*b*d^2 - a^2*b^2*d^2*6i
)))^(1/2)*1i)/((((((8*(320*a^6*b^9*d^4*e^11 - 96*a^2*b^13*d^4*e^11 - 32*b^1
5*d^4*e^11 + 480*a^8*b^7*d^4*e^11 + 288*a^10*b^5*d^4*e^11 + 64*a^12*b^3*d^4
*e^11))/(a^8*d^5 + b^8*d^5 + 4*a^2*b^6*d^5 + 6*a^4*b^4*d^5 + 4*a^6*b^2*d^5)
- (16*(e*cot(c + d*x))^(1/2)*(e/(4*(a^4*d^2*1i + b^4*d^2*1i + 4*a*b^3*d^2
- 4*a^3*b*d^2 - a^2*b^2*d^2*6i)))^(1/2)*(32*b^17*d^4*e^10 + 160*a^2*b^15*d^
4*e^10 + 288*a^4*b^13*d^4*e^10 + 160*a^6*b^11*d^4*e^10 - 160*a^8*b^9*d^4*e^

```
10 - 288*a^10*b^7*d^4*e^10 - 160*a^12*b^5*d^4*e^10 - 32*a^14*b^3*d^4*e^10))
/(a^8*d^4 + b^8*d^4 + 4*a^2*b^6*d^4 + 6*a^4*b^4*d^4 + 4*a^6*b^2*d^4))*(e/(4
*(a^4*d^2*1i + b^4*d^2*1i + 4*a*b^3*d^2 - 4*a^3*b*d^2 - a^2*b^2*d^2*6i)))^(
1/2) + (16*(e*cot(c + d*x))^(1/2)*(68*a*b^12*d^2*e^11 + 20*a^3*b^10*d^2*e^1
1 - 88*a^5*b^8*d^2*e^11 + 40*a^7*b^6*d^2*e^11 + 84*a^9*b^4*d^2*e^11 + 4*a^1
1*b^2*d^2*e^11))/(a^8*d^4 + b^8*d^4 + 4*a^2*b^6*d^4 + 6*a^4*b^4*d^4 + 4*a^6
*b^2*d^4))*(e/(4*(a^4*d^2*1i + b^4*d^2*1i + 4*a*b^3*d^2 - 4*a^3*b*d^2 - a^2
*b^2*d^2*6i)))^(1/2) + (8*(52*a*b^10*d^2*e^12 - 128*a^3*b^8*d^2*e^12 - 24*a
^5*b^6*d^2*e^12 + 160*a^7*b^4*d^2*e^12 + 4*a^9*b^2*d^2*e^12))/(a^8*d^5 + b^
8*d^5 + 4*a^2*b^6*d^5 + 6*a^4*b^4*d^5 + 4*a^6*b^2*d^5))*(e/(4*(a^4*d^2*1i +
 b^4*d^2*1i + 4*a*b^3*d^2 - 4*a^3*b*d^2 - a^2*b^2*d^2*6i)))^(1/2) - (16*(e*
cot(c + d*x))^(1/2)*(3*b^9*e^12 - 3*a^2*b^7*e^12 + 17*a^4*b^5*e^12 - 9*a^6*
b^3*e^12))/(a^8*d^4 + b^8*d^4 + 4*a^2*b^6*d^4 + 6*a^4*b^4*d^4 + 4*a^6*b^2*d
^4))*(e/(4*(a^4*d^2*1i + b^4*d^2*1i + 4*a*b^3*d^2 - 4*a^3*b*d^2 - a^2*b^2*d
^2*6i)))^(1/2) + (((((8*(320*a^6*b^9*d^4*e^11 - 96*a^2*b^13*d^4*e^11 - 32*b
^15*d^4*e^11 + 480*a^8*b^7*d^4*e^11 + 288*a^10*b^5*d^4*e^11 + 64*a^12*b^3*d
^4*e^11))/(a^8*d^5 + b^8*d^5 + 4*a^2*b^6*d^5 + 6*a^4*b^4*d^5 + 4*a^6*b^2*d^
5) + (16*(e*cot(c + d*x))^(1/2)*(e/(4*(a^4*d^2*1i + b^4*d^2*1i + 4*a*b^3*d^
2 - 4*a^3*b*d^2 - a^2*b^2*d^2*6i)))^(1/2)*(32*b^17*d^4*e^10 + 160*a^2*b^15*
d^4*e^10 + 288*a^4*b^13*d^4*e^10 + 160*a^6*b^11*d^4*e^10 - 160*a^8*b^9*d^4*
e^10 - 288*a^10*b^7*d^4*e^10 - 160*a^12*b^5*d^4*e^10 - 32*a^14*b^3*d^4*e^10
))/(a^8*d^4 + b^8*d^4 + 4*a^2*b^6*d^4 + 6*a^4*b^4*d^4 + 4*a^6*b^2*d^4))*(e/
(4*(a^4*d^2*1i + b^4*d^2*1i + 4*a*b^3*d^2 - 4*a^3*b*d^2 - a^2*b^2*d^2*6i)))
^(1/2) - (16*(e*cot(c + d*x))^(1/2)*(68*a*b^12*d^2*e^11 + 20*a^3*b^10*d^2*e
^11 - 88*a^5*b^8*d^2*e^11 + 40*a^7*b^6*d^2*e^11 + 84*a^9*b^4*d^2*e^11 + 4*a
^11*b^2*d^2*e^11))/(a^8*d^4 + b^8*d^4 + 4*a^2*b^6*d^4 + 6*a^4*b^4*d^4 + 4*a
^6*b^2*d^4))*(e/(4*(a^4*d^2*1i + b^4*d^2*1i + 4*a*b^3*d^2 - 4*a^3*b*d^2 - a
^2*b^2*d^2*6i)))^(1/2) + (8*(52*a*b^10*d^2*e^12 - 128*a^3*b^8*d^2*e^12 - 24
*a^5*b^6*d^2*e^12 + 160*a^7*b^4*d^2*e^12 + 4*a^9*b^2*d^2*e^12))/(a^8*d^5 +
b^8*d^5 + 4*a^2*b^6*d^5 + 6*a^4*b^4*d^5 + 4*a^6*b^2*d^5))*(e/(4*(a^4*d^2*1i
 + b^4*d^2*1i + 4*a*b^3*d^2 - 4*a^3*b*d^2 - a^2*b^2*d^2*6i)))^(1/2) + (16*(
e*cot(c + d*x))^(1/2)*(3*b^9*e^12 - 3*a^2*b^7*e^12 + 17*a^4*b^5*e^12 - 9*a^
6*b^3*e^12))/(a^8*d^4 + b^8*d^4 + 4*a^2*b^6*d^4 + 6*a^4*b^4*d^4 + 4*a^6*b^2
*d^4))*(e/(4*(a^4*d^2*1i + b^4*d^2*1i + 4*a*b^3*d^2 - 4*a^3*b*d^2 - a^2*b^2
*d^2*6i)))^(1/2) - (16*(b^7*e^13 - 9*a^4*b^3*e^13))/(a^8*d^5 + b^8*d^5 + 4*
a^2*b^6*d^5 + 6*a^4*b^4*d^5 + 4*a^6*b^2*d^5)))*(e/(4*(a^4*d^2*1i + b^4*d^2*
1i + 4*a*b^3*d^2 - 4*a^3*b*d^2 - a^2*b^2*d^2*6i)))^(1/2)*2i - (atan((((3*a^
2 - b^2)*((16*(e*cot(c + d*x))^(1/2)*(3*b^9*e^12 - 3*a^2*b^7*e^12 + 17*a^4*
b^5*e^12 - 9*a^6*b^3*e^12))/(a^8*d^4 + b^8*d^4 + 4*a^2*b^6*d^4 + 6*a^4*b^4*
d^4 + 4*a^6*b^2*d^4) - (((8*(52*a*b^10*d^2*e^12 - 128*a^3*b^8*d^2*e^12 - 24
*a^5*b^6*d^2*e^12 + 160*a^7*b^4*d^2*e^12 + 4*a^9*b^2*d^2*e^12))/(a^8*d^5 +
b^8*d^5 + 4*a^2*b^6*d^5 + 6*a^4*b^4*d^5 + 4*a^6*b^2*d^5) + (((16*(e*cot(c +
 d*x))^(1/2)*(68*a*b^12*d^2*e^11 + 20*a^3*b^10*d^2*e^11 - 88*a^5*b^8*d^2*e^
11 + 40*a^7*b^6*d^2*e^11 + 84*a^9*b^4*d^2*e^11 + 4*a^11*b^2*d^2*e^11))/(a^8
*d^4 + b^8*d^4 + 4*a^2*b^6*d^4 + 6*a^4*b^4*d^4 + 4*a^6*b^2*d^4) + (((8*(320
```

*a^6*b^9*d^4*e^11 - 96*a^2*b^13*d^4*e^11 - 32*b^15*d^4*e^11 + 480*a^8*b^7*d^4*e^11 + 288*a^10*b^5*d^4*e^11 + 64*a^12*b^3*d^4*e^11))/(a^8*d^5 + b^8*d^5 + 4*a^2*b^6*d^5 + 6*a^4*b^4*d^5 + 4*a^6*b^2*d^5) - (8*(e*cot(c + d*x))^(1/2)*(3*a^2 - b^2)*(-a*b*e)^(1/2)*(32*b^17*d^4*e^10 + 160*a^2*b^15*d^4*e^10 + 288*a^4*b^13*d^4*e^10 + 160*a^6*b^11*d^4*e^10 - 160*a^8*b^9*d^4*e^10 - 288*a^10*b^7*d^4*e^10 - 160*a^12*b^5*d^4*e^10 - 32*a^14*b^3*d^4*e^10))/((a^5*d + 2*a^3*b^2*d + a*b^4*d)*(a^8*d^4 + b^8*d^4 + 4*a^2*b^6*d^4 + 6*a^4*b^4*d^4 + 4*a^6*b^2*d^4)))*(3*a^2 - b^2)*(-a*b*e)^(1/2))/(2*(a^5*d + 2*a^3*b^2*d + a*b^4*d)))*(3*a^2 - b^2)*(-a*b*e)^(1/2))/(2*(a^5*d + 2*a^3*b^2*d + a*b^4*d)))*(3*a^2 - b^2)*(-a*b*e)^(1/2))/(2*(a^5*d + 2*a^3*b^2*d + a*b^4*d)))*(-a*b*e)^(1/2)*1i)/(2*(a^5*d + 2*a^3*b^2*d + a*b^4*d)) + ((3*a^2 - b^2)*((16*(e*cot(c + d*x))^(1/2)*(3*b^9*e^12 - 3*a^2*b^7*e^12 + 17*a^4*b^5*e^12 - 9*a^6*b^3*e^12))/(a^8*d^4 + b^8*d^4 + 4*a^2*b^6*d^4 + 6*a^4*b^4*d^4 + 4*a^6*b^2*d^4) + (((8*(52*a*b^10*d^2*e^12 - 128*a^3*b^8*d^2*e^12 - 24*a^5*b^6*d^2*e^12 + 160*a^7*b^4*d^2*e^12 + 4*a^9*b^2*d^2*e^12))/(a^8*d^5 + b^8*d^5 + 4*a^2*b^6*d^5 + 6*a^4*b^4*d^5 + 4*a^6*b^2*d^5) - (((16*(e*cot(c + d*x))^(1/2)*(68*a*b^12*d^2*e^11 + 20*a^3*b^10*d^2*e^11 - 88*a^5*b^8*d^2*e^11 + 40*a^7*b^6*d^2*e^11 + 84*a^9*b^4*d^2*e^11 + 4*a^11*b^2*d^2*e^11))/(a^8*d^4 + b^8*d^4 + 4*a^2*b^6*d^4 + 6*a^4*b^4*d^4 + 4*a^6*b^2*d^4) - (((8*(320*a^6*b^9*d^4*e^11 - 96*a^2*b^13*d^4*e^11 - 32*b^15*d^4*e^11 + 480*a^8*b^7*d^4*e^11 + 288*a^10*b^5*d^4*e^11 + 64*a^12*b^3*d^4*e^11))/(a^8*d^5 + b^8*d^5 + 4*a^2*b^6*d^5 + 6*a^4*b^4*d^5 + 4*a^6*b^2*d^5) + (8*(e*cot(c + d*x))^(1/2)*(3*a^2 - b^2)*(-a*b*e)^(1/2)*(32*b^17*d^4*e^10 + 160*a^2*b^15*d^4*e^10 + 288*a^4*b^13*d^4*e^10 + 160*a^6*b^11*d^4*e^10 - 160*a^8*b^9*d^4*e^10 - 288*a^10*b^7*d^4*e^10 - 160*a^12*b^5*d^4*e^10 - 32*a^14*b^3*d^4*e^10))/((a^5*d + 2*a^3*b^2*d + a*b^4*d)*(a^8*d^4 + b^8*d^4 + 4*a^2*b^6*d^4 + 6*a^4*b^4*d^4 + 4*a^6*b^2*d^4)))*(3*a^2 - b^2)*(-a*b*e)^(1/2))/(2*(a^5*d + 2*a^3*b^2*d + a*b^4*d)))*(3*a^2 - b^2)*(-a*b*e)^(1/2))/(2*(a^5*d + 2*a^3*b^2*d + a*b^4*d)))*(3*a^2 - b^2)*(-a*b*e)^(1/2))/(2*(a^5*d + 2*a^3*b^2*d + a*b^4*d)))*(-a*b*e)^(1/2)*1i)/(2*(a^5*d + 2*a^3*b^2*d + a*b^4*d)))/((16*(b^7*e^13 - 9*a^4*b^3*e^13))/(a^8*d^5 + b^8*d^5 + 4*a^2*b^6*d^5 + 6*a^4*b^4*d^5 + 4*a^6*b^2*d^5) + ((3*a^2 - b^2)*((16*(e*cot(c + d*x))^(1/2)*(3*b^9*e^12 - 3*a^2*b^7*e^12 + 17*a^4*b^5*e^12 - 9*a^6*b^3*e^12))/(a^8*d^4 + b^8*d^4 + 4*a^2*b^6*d^4 + 6*a^4*b^4*d^4 + 4*a^6*b^2*d^4) - (((8*(52*a*b^10*d^2*e^12 - 128*a^3*b^8*d^2*e^12 - 24*a^5*b^6*d^2*e^12 + 160*a^7*b^4*d^2*e^12 + 4*a^9*b^2*d^2*e^12))/(a^8*d^5 + b^8*d^5 + 4*a^2*b^6*d^5 + 6*a^4*b^4*d^5 + 4*a^6*b^2*d^5) + (((16*(e*cot(c + d*x))^(1/2)*(68*a*b^12*d^2*e^11 + 20*a^3*b^10*d^2*e^11 - 88*a^5*b^8*d^2*e^11 + 40*a^7*b^6*d^2*e^11 + 84*a^9*b^4*d^2*e^11 + 4*a^11*b^2*d^2*e^11))/(a^8*d^4 + b^8*d^4 + 4*a^2*b^6*d^4 + 6*a^4*b^4*d^4 + 4*a^6*b^2*d^4) + (((8*(320*a^6*b^9*d^4*e^11 - 96*a^2*b^13*d^4*e^11 - 32*b^15*d^4*e^11 + 480*a^8*b^7*d^4*e^11 + 288*a^10*b^5*d^4*e^11 + 64*a^12*b^3*d^4*e^11))/(a^8*d^5 + b^8*d^5 + 4*a^2*b^6*d^5 + 6*a^4*b^4*d^5 + 4*a^6*b^2*d^5) - (8*(e*cot(c + d*x))^(1/2)*(3*a^2 - b^2)*(-a*b*e)^(1/2)*(32*b^17*d^4*e^10 + 160*a^2*b^15*d^4*e^10 + 288*a^4*b^13*d^4*e^10 + 160*a^6*b^11*d^4*e^10 - 160*a^8*b^9*d^4*e^10 - 288*a^10*b^7*d^4*e^10 - 160*a^12*b^5*d^4*e^10 - 32*a^14*b^3*d^4*e^10))/((a^5*d +

2*a^3*b^2*d + a*b^4*d)*(a^8*d^4 + b^8*d^4 + 4*a^2*b^6*d^4 + 6*a^4*b^4*d^4 + 4*a^6*b^2*d^4)))*(3*a^2 - b^2)*(-a*b*e)^(1/2))/(2*(a^5*d + 2*a^3*b^2*d + a*b^4*d)))*(3*a^2 - b^2)*(-a*b*e)^(1/2))/(2*(a^5*d + 2*a^3*b^2*d + a*b^4*d)))*(3*a^2 - b^2)*(-a*b*e)^(1/2))/(2*(a^5*d + 2*a^3*b^2*d + a*b^4*d)))*(-a*b *e)^(1/2))/(2*(a^5*d + 2*a^3*b^2*d + a*b^4*d)) - ((3*a^2 - b^2)*((16*(e*cot (c + d*x))^(1/2)*(3*b^9*e^12 - 3*a^2*b^7*e^12 + 17*a^4*b^5*e^12 - 9*a^6*b^3 *e^12))/(a^8*d^4 + b^8*d^4 + 4*a^2*b^6*d^4 + 6*a^4*b^4*d^4 + 4*a^6*b^2*d^4) + (((8*(52*a*b^10*d^2*e^12 - 128*a^3*b^8*d^2*e^12 - 24*a^5*b^6*d^2*e^12 + 160*a^7*b^4*d^2*e^12 + 4*a^9*b^2*d^2*e^12))/(a^8*d^5 + b^8*d^5 + 4*a^2*b^6* d^5 + 6*a^4*b^4*d^5 + 4*a^6*b^2*d^5) - (((16*(e*cot(c + d*x))^(1/2)*(68*a*b ^12*d^2*e^11 + 20*a^3*b^10*d^2*e^11 - 88*a^5*b^8*d^2*e^11 + 40*a^7*b^6*d^2* e^11 + 84*a^9*b^4*d^2*e^11 + 4*a^11*b^2*d^2*e^11))/(a^8*d^4 + b^8*d^4 + 4*a ^2*b^6*d^4 + 6*a^4*b^4*d^4 + 4*a^6*b^2*d^4) - (((8*(320*a^6*b^9*d^4*e^11 - 96*a^2*b^13*d^4*e^11 - 32*b^15*d^4*e^11 + 480*a^8*b^7*d^4*e^11 + 288*a^10*b ^5*d^4*e^11 + 64*a^12*b^3*d^4*e^11))/(a^8*d^5 + b^8*d^5 + 4*a^2*b^6*d^5 + 6 *a^4*b^4*d^5 + 4*a^6*b^2*d^5) + (8*(e*cot(c + d*x))^(1/2)*(3*a^2 - b^2)*(-a *b*e)^(1/2)*(32*b^17*d^4*e^10 + 160*a^2*b^15*d^4*e^10 + 288*a^4*b^13*d^4*e^ 10 + 160*a^6*b^11*d^4*e^10 - 160*a^8*b^9*d^4*e^10 - 288*a^10*b^7*d^4*e^10 - 160*a^12*b^5*d^4*e^10 - 32*a^14*b^3*d^4*e^10))/((a^5*d + 2*a^3*b^2*d + a*b ^4*d)*(a^8*d^4 + b^8*d^4 + 4*a^2*b^6*d^4 + 6*a^4*b^4*d^4 + 4*a^6*b^2*d^4))) *(3*a^2 - b^2)*(-a*b*e)^(1/2))/(2*(a^5*d + 2*a^3*b^2*d + a*b^4*d)))*(3*a^2 - b^2)*(-a*b*e)^(1/2))/(2*(a^5*d + 2*a^3*b^2*d + a*b^4*d)))*(3*a^2 - b^2)*(-a*b*e)^(1/2))/(2*(a^5*d + 2*a^3*b^2*d + a*b^4*d)))*(-a*b*e)^(1/2))/(2*(a^5 *d + 2*a^3*b^2*d + a*b^4*d))))*(3*a^2 - b^2)*(-a*b*e)^(1/2)*1i)/(a^5*d + 2* a^3*b^2*d + a*b^4*d) - atan((((((8*(320*a^6*b^9*d^4*e^11 - 96*a^2*b^13*d^4 *e^11 - 32*b^15*d^4*e^11 + 480*a^8*b^7*d^4*e^11 + 288*a^10*b^5*d^4*e^11 + 6 4*a^12*b^3*d^4*e^11))/(a^8*d^5 + b^8*d^5 + 4*a^2*b^6*d^5 + 6*a^4*b^4*d^5 + 4*a^6*b^2*d^5) - (16*(e*cot(c + d*x))^(1/2)*((e*1i)/(4*(a^4*d^2 + b^4*d^2 + a*b^3*d^2*4i - a^3*b*d^2*4i - 6*a^2*b^2*d^2)))^(1/2)*(32*b^17*d^4*e^10 + 1 60*a^2*b^15*d^4*e^10 + 288*a^4*b^13*d^4*e^10 + 160*a^6*b^11*d^4*e^10 - 160* a^8*b^9*d^4*e^10 - 288*a^10*b^7*d^4*e^10 - 160*a^12*b^5*d^4*e^10 - 32*a^14* b^3*d^4*e^10))/(a^8*d^4 + b^8*d^4 + 4*a^2*b^6*d^4 + 6*a^4*b^4*d^4 + 4*a^6*b ^2*d^4))*((e*1i)/(4*(a^4*d^2 + b^4*d^2 + a*b^3*d^2*4i - a^3*b*d^2*4i - 6*a^ 2*b^2*d^2)))^(1/2) + (16*(e*cot(c + d*x))^(1/2)*(68*a*b^12*d^2*e^11 + 20*a^ 3*b^10*d^2*e^11 - 88*a^5*b^8*d^2*e^11 + 40*a^7*b^6*d^2*e^11 + 84*a^9*b^4*d^ 2*e^11 + 4*a^11*b^2*d^2*e^11))/(a^8*d^4 + b^8*d^4 + 4*a^2*b^6*d^4 + 6*a^4*b ^4*d^4 + 4*a^6*b^2*d^4))*((e*1i)/(4*(a^4*d^2 + b^4*d^2 + a*b^3*d^2*4i - a^3 *b*d^2*4i - 6*a^2*b^2*d^2)))^(1/2) + (8*(52*a*b^10*d^2*e^12 - 128*a^3*b^8*d ^2*e^12 - 24*a^5*b^6*d^2*e^12 + 160*a^7*b^4*d^2*e^12 + 4*a^9*b^2*d^2*e^12)) /(a^8*d^5 + b^8*d^5 + 4*a^2*b^6*d^5 + 6*a^4*b^4*d^5 + 4*a^6*b^2*d^5))*((e*1 i)/(4*(a^4*d^2 + b^4*d^2 + a*b^3*d^2*4i - a^3*b*d^2*4i - 6*a^2*b^2*d^2)))^(1/2) - (16*(e*cot(c + d*x))^(1/2)*(3*b^9*e^12 - 3*a^2*b^7*e^12 + 17*a^4*b^5 *e^12 - 9*a^6*b^3*e^12))/(a^8*d^4 + b^8*d^4 + 4*a^2*b^6*d^4 + 6*a^4*b^4*d^4 + 4*a^6*b^2*d^4))*((e*1i)/(4*(a^4*d^2 + b^4*d^2 + a*b^3*d^2*4i - a^3*b*d^2 *4i - 6*a^2*b^2*d^2)))^(1/2)*1i - (((((8*(320*a^6*b^9*d^4*e^11 - 96*a^2*b^1

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3*d^4*e^11 - 32*b^15*d^4*e^11 + 480*a^8*b^7*d^4*e^11 + 288*a^10*b^5*d^4*e^1
1 + 64*a^12*b^3*d^4*e^11))/(a^8*d^5 + b^8*d^5 + 4*a^2*b^6*d^5 + 6*a^4*b^4*d
^5 + 4*a^6*b^2*d^5) + (16*(e*cot(c + d*x))^(1/2)*((e*1i)/(4*(a^4*d^2 + b^4*
d^2 + a*b^3*d^2*4i - a^3*b*d^2*4i - 6*a^2*b^2*d^2)))^(1/2)*(32*b^17*d^4*e^1
0 + 160*a^2*b^15*d^4*e^10 + 288*a^4*b^13*d^4*e^10 + 160*a^6*b^11*d^4*e^10 -
 160*a^8*b^9*d^4*e^10 - 288*a^10*b^7*d^4*e^10 - 160*a^12*b^5*d^4*e^10 - 32*
a^14*b^3*d^4*e^10))/(a^8*d^4 + b^8*d^4 + 4*a^2*b^6*d^4 + 6*a^4*b^4*d^4 + 4*
a^6*b^2*d^4))*((e*1i)/(4*(a^4*d^2 + b^4*d^2 + a*b^3*d^2*4i - a^3*b*d^2*4i -
 6*a^2*b^2*d^2)))^(1/2) - (16*(e*cot(c + d*x))^(1/2)*(68*a*b^12*d^2*e^11 +
20*a^3*b^10*d^2*e^11 - 88*a^5*b^8*d^2*e^11 + 40*a^7*b^6*d^2*e^11 + 84*a^9*b
^4*d^2*e^11 + 4*a^11*b^2*d^2*e^11))/(a^8*d^4 + b^8*d^4 + 4*a^2*b^6*d^4 + 6*
a^4*b^4*d^4 + 4*a^6*b^2*d^4))*((e*1i)/(4*(a^4*d^2 + b^4*d^2 + a*b^3*d^2*4i
- a^3*b*d^2*4i - 6*a^2*b^2*d^2)))^(1/2) + (8*(52*a*b^10*d^2*e^12 - 128*a^3*
b^8*d^2*e^12 - 24*a^5*b^6*d^2*e^12 + 160*a^7*b^4*d^2*e^12 + 4*a^9*b^2*d^2*e
^12))/(a^8*d^5 + b^8*d^5 + 4*a^2*b^6*d^5 + 6*a^4*b^4*d^5 + 4*a^6*b^2*d^5))*
((e*1i)/(4*(a^4*d^2 + b^4*d^2 + a*b^3*d^2*4i - a^3*b*d^2*4i - 6*a^2*b^2*d^2
)))^(1/2) + (16*(e*cot(c + d*x))^(1/2)*(3*b^9*e^12 - 3*a^2*b^7*e^12 + 17*a^
4*b^5*e^12 - 9*a^6*b^3*e^12))/(a^8*d^4 + b^8*d^4 + 4*a^2*b^6*d^4 + 6*a^4*b^
4*d^4 + 4*a^6*b^2*d^4))*((e*1i)/(4*(a^4*d^2 + b^4*d^2 + a*b^3*d^2*4i - a^3*
b*d^2*4i - 6*a^2*b^2*d^2)))^(1/2)*1i)/((((((8*(320*a^6*b^9*d^4*e^11 - 96*a^
2*b^13*d^4*e^11 - 32*b^15*d^4*e^11 + 480*a^8*b^7*d^4*e^11 + 288*a^10*b^5*d^
4*e^11 + 64*a^12*b^3*d^4*e^11))/(a^8*d^5 + b^8*d^5 + 4*a^2*b^6*d^5 + 6*a^4*
b^4*d^5 + 4*a^6*b^2*d^5) - (16*(e*cot(c + d*x))^(1/2)*((e*1i)/(4*(a^4*d^2 +
 b^4*d^2 + a*b^3*d^2*4i - a^3*b*d^2*4i - 6*a^2*b^2*d^2)))^(1/2)*(32*b^17*d^
4*e^10 + 160*a^2*b^15*d^4*e^10 + 288*a^4*b^13*d^4*e^10 + 160*a^6*b^11*d^4*e
^10 - 160*a^8*b^9*d^4*e^10 - 288*a^10*b^7*d^4*e^10 - 160*a^12*b^5*d^4*e^10
- 32*a^14*b^3*d^4*e^10))/(a^8*d^4 + b^8*d^4 + 4*a^2*b^6*d^4 + 6*a^4*b^4*d^4
 + 4*a^6*b^2*d^4))*((e*1i)/(4*(a^4*d^2 + b^4*d^2 + a*b^3*d^2*4i - a^3*b*d^2
*4i - 6*a^2*b^2*d^2)))^(1/2) + (16*(e*cot(c + d*x))^(1/2)*(68*a*b^12*d^2*e^
11 + 20*a^3*b^10*d^2*e^11 - 88*a^5*b^8*d^2*e^11 + 40*a^7*b^6*d^2*e^11 + 84*
a^9*b^4*d^2*e^11 + 4*a^11*b^2*d^2*e^11))/(a^8*d^4 + b^8*d^4 + 4*a^2*b^6*d^4
 + 6*a^4*b^4*d^4 + 4*a^6*b^2*d^4))*((e*1i)/(4*(a^4*d^2 + b^4*d^2 + a*b^3*d^
2*4i - a^3*b*d^2*4i - 6*a^2*b^2*d^2)))^(1/2) + (8*(52*a*b^10*d^2*e^12 - 128
*a^3*b^8*d^2*e^12 - 24*a^5*b^6*d^2*e^12 + 160*a^7*b^4*d^2*e^12 + 4*a^9*b^2*
d^2*e^12))/(a^8*d^5 + b^8*d^5 + 4*a^2*b^6*d^5 + 6*a^4*b^4*d^5 + 4*a^6*b^2*d
^5))*((e*1i)/(4*(a^4*d^2 + b^4*d^2 + a*b^3*d^2*4i - a^3*b*d^2*4i - 6*a^2*b^
2*d^2)))^(1/2) - (16*(e*cot(c + d*x))^(1/2)*(3*b^9*e^12 - 3*a^2*b^7*e^12 +
17*a^4*b^5*e^12 - 9*a^6*b^3*e^12))/(a^8*d^4 + b^8*d^4 + 4*a^2*b^6*d^4 + 6*a
^4*b^4*d^4 + 4*a^6*b^2*d^4))*((e*1i)/(4*(a^4*d^2 + b^4*d^2 + a*b^3*d^2*4i -
 a^3*b*d^2*4i - 6*a^2*b^2*d^2)))^(1/2) + (((((8*(320*a^6*b^9*d^4*e^11 - 96*
a^2*b^13*d^4*e^11 - 32*b^15*d^4*e^11 + 480*a^8*b^7*d^4*e^11 + 288*a^10*b^5*
d^4*e^11 + 64*a^12*b^3*d^4*e^11))/(a^8*d^5 + b^8*d^5 + 4*a^2*b^6*d^5 + 6*a^
4*b^4*d^5 + 4*a^6*b^2*d^5) + (16*(e*cot(c + d*x))^(1/2)*((e*1i)/(4*(a^4*d^2
 + b^4*d^2 + a*b^3*d^2*4i - a^3*b*d^2*4i - 6*a^2*b^2*d^2)))^(1/2)*(32*b^17*
d^4*e^10 + 160*a^2*b^15*d^4*e^10 + 288*a^4*b^13*d^4*e^10 + 160*a^6*b^11*d^4
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```
*e^10 - 160*a^8*b^9*d^4*e^10 - 288*a^10*b^7*d^4*e^10 - 160*a^12*b^5*d^4*e^1
0 - 32*a^14*b^3*d^4*e^10))/(a^8*d^4 + b^8*d^4 + 4*a^2*b^6*d^4 + 6*a^4*b^4*d
^4 + 4*a^6*b^2*d^4))*((e*1i)/(4*(a^4*d^2 + b^4*d^2 + a*b^3*d^2*4i - a^3*b*d
^2*4i - 6*a^2*b^2*d^2)))^(1/2) - (16*(e*cot(c + d*x))^(1/2)*(68*a*b^12*d^2*
e^11 + 20*a^3*b^10*d^2*e^11 - 88*a^5*b^8*d^2*e^11 + 40*a^7*b^6*d^2*e^11 + 8
4*a^9*b^4*d^2*e^11 + 4*a^11*b^2*d^2*e^11))/(a^8*d^4 + b^8*d^4 + 4*a^2*b^6*d
^4 + 6*a^4*b^4*d^4 + 4*a^6*b^2*d^4))*((e*1i)/(4*(a^4*d^2 + b^4*d^2 + a*b^3*
d^2*4i - a^3*b*d^2*4i - 6*a^2*b^2*d^2)))^(1/2) + (8*(52*a*b^10*d^2*e^12 - 1
28*a^3*b^8*d^2*e^12 - 24*a^5*b^6*d^2*e^12 + 160*a^7*b^4*d^2*e^12 + 4*a^9*b^
2*d^2*e^12))/(a^8*d^5 + b^8*d^5 + 4*a^2*b^6*d^5 + 6*a^4*b^4*d^5 + 4*a^6*b^2
*d^5))*((e*1i)/(4*(a^4*d^2 + b^4*d^2 + a*b^3*d^2*4i - a^3*b*d^2*4i - 6*a^2*
b^2*d^2)))^(1/2) + (16*(e*cot(c + d*x))^(1/2)*(3*b^9*e^12 - 3*a^2*b^7*e^12
+ 17*a^4*b^5*e^12 - 9*a^6*b^3*e^12))/(a^8*d^4 + b^8*d^4 + 4*a^2*b^6*d^4 + 6
*a^4*b^4*d^4 + 4*a^6*b^2*d^4))*((e*1i)/(4*(a^4*d^2 + b^4*d^2 + a*b^3*d^2*4i
 - a^3*b*d^2*4i - 6*a^2*b^2*d^2)))^(1/2) - (16*(b^7*e^13 - 9*a^4*b^3*e^13))
/(a^8*d^5 + b^8*d^5 + 4*a^2*b^6*d^5 + 6*a^4*b^4*d^5 + 4*a^6*b^2*d^5)))*((e*
1i)/(4*(a^4*d^2 + b^4*d^2 + a*b^3*d^2*4i - a^3*b*d^2*4i - 6*a^2*b^2*d^2)))^
(1/2)*2i
```

3.79 $\int \frac{1}{\sqrt{e\cot(c+dx)}(a+b\cot(c+dx))^2}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 394

$$\begin{aligned}
&\int \frac{1}{\sqrt{e\cot(c+dx)}(a+b\cot(c+dx))^2}\,dx\\
&= -\frac{b^{3/2}(5a^2+b^2)\arctan\left(\frac{\sqrt{b}\sqrt{e\cot(c+dx)}}{\sqrt{a}\sqrt{e}}\right)}{a^{3/2}\left(a^2+b^2\right)^2 d\sqrt{e}}+\frac{(a^2-2ab-b^2)\arctan\left(1-\frac{\sqrt{2}\sqrt{e\cot(c+dx)}}{\sqrt{e}}\right)}{\sqrt{2}\left(a^2+b^2\right)^2 d\sqrt{e}}\\
&\quad-\frac{(a^2-2ab-b^2)\arctan\left(1+\frac{\sqrt{2}\sqrt{e\cot(c+dx)}}{\sqrt{e}}\right)}{\sqrt{2}\left(a^2+b^2\right)^2 d\sqrt{e}}-\frac{b^2\sqrt{e\cot(c+dx)}}{a\left(a^2+b^2\right)de(a+b\cot(c+dx))}\\
&\quad+\frac{(a^2+2ab-b^2)\log\left(\sqrt{e}+\sqrt{e}\cot(c+dx)-\sqrt{2}\sqrt{e\cot(c+dx)}\right)}{2\sqrt{2}\left(a^2+b^2\right)^2 d\sqrt{e}}\\
&\quad-\frac{(a^2+2ab-b^2)\log\left(\sqrt{e}+\sqrt{e}\cot(c+dx)+\sqrt{2}\sqrt{e\cot(c+dx)}\right)}{2\sqrt{2}\left(a^2+b^2\right)^2 d\sqrt{e}}
\end{aligned}$$

```
[Out] -b^(3/2)*(5*a^2+b^2)*arctan(b^(1/2)*(e*cot(d*x+c))^(1/2)/a^(1/2)/e^(1/2))/a
^(3/2)/(a^2+b^2)^2/d/e^(1/2)+1/2*(a^2-2*a*b-b^2)*arctan(1-2^(1/2)*(e*cot(d*
x+c))^(1/2)/e^(1/2))/(a^2+b^2)^2/d*2^(1/2)/e^(1/2)-1/2*(a^2-2*a*b-b^2)*arct
an(1+2^(1/2)*(e*cot(d*x+c))^(1/2)/e^(1/2))/(a^2+b^2)^2/d*2^(1/2)/e^(1/2)+1/
4*(a^2+2*a*b-b^2)*ln(e^(1/2)+cot(d*x+c)*e^(1/2)-2^(1/2)*(e*cot(d*x+c))^(1/2
))/(a^2+b^2)^2/d*2^(1/2)/e^(1/2)-1/4*(a^2+2*a*b-b^2)*ln(e^(1/2)+cot(d*x+c)*
e^(1/2)+2^(1/2)*(e*cot(d*x+c))^(1/2))/(a^2+b^2)^2/d*2^(1/2)/e^(1/2)-b^2*(e*
cot(d*x+c))^(1/2)/a/(a^2+b^2)/d/e/(a+b*cot(d*x+c))
```

Rubi [A] (verified)

Time = 0.87 (sec) , antiderivative size = 394, normalized size of antiderivative = 1.00, number of steps used = 15, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.480, Rules used = {3650, 3734, 3615, 1182, 1176, 631, 210, 1179, 642, 3715, 65, 211}

$$\int \frac{1}{\sqrt{e\cot(c+dx)}(a+b\cot(c+dx))^2}\,dx$$
$$=\frac{(a^2-2ab-b^2)\arctan\left(1-\frac{\sqrt{2}\sqrt{e\cot(c+dx)}}{\sqrt{e}}\right)}{\sqrt{2}d\sqrt{e}\,(a^2+b^2)^2}-\frac{(a^2-2ab-b^2)\arctan\left(\frac{\sqrt{2}\sqrt{e\cot(c+dx)}}{\sqrt{e}}+1\right)}{\sqrt{2}d\sqrt{e}\,(a^2+b^2)^2}-\frac{b^2\sqrt{e\cot(c+dx)}}{ade\,(a^2+b^2)\,(a+b\cot(c+dx))}+\frac{(a^2+2ab-b^2)\log\left(\sqrt{e}\cot(c+dx)-\sqrt{2}\sqrt{e\cot(c+dx)}+\sqrt{e}\right)}{2\sqrt{2}d\sqrt{e}\,(a^2+b^2)^2}-\frac{(a^2+2ab-b^2)\log\left(\sqrt{e}\cot(c+dx)+\sqrt{2}\sqrt{e\cot(c+dx)}+\sqrt{e}\right)}{2\sqrt{2}d\sqrt{e}\,(a^2+b^2)^2}-\frac{b^{3/2}(5a^2+b^2)\arctan\left(\frac{\sqrt{b}\sqrt{e\cot(c+dx)}}{\sqrt{a}\sqrt{e}}\right)}{a^{3/2}d\sqrt{e}\,(a^2+b^2)^2}$$

```
[In] Int[1/(Sqrt[e*Cot[c + d*x]]*(a + b*Cot[c + d*x])^2),x]
```

```
[Out] -((b^(3/2)*(5*a^2 + b^2)*ArcTan[(Sqrt[b]*Sqrt[e*Cot[c + d*x]])/(Sqrt[a]*Sqr
t[e])])/(a^(3/2)*(a^2 + b^2)^2*d*Sqrt[e])) + ((a^2 - 2*a*b - b^2)*ArcTan[1
- (Sqrt[2]*Sqrt[e*Cot[c + d*x]])/Sqrt[e]])/(Sqrt[2]*(a^2 + b^2)^2*d*Sqrt[e]
) - ((a^2 - 2*a*b - b^2)*ArcTan[1 + (Sqrt[2]*Sqrt[e*Cot[c + d*x]])/Sqrt[e]]
)/(Sqrt[2]*(a^2 + b^2)^2*d*Sqrt[e]) - (b^2*Sqrt[e*Cot[c + d*x]])/(a*(a^2 +
b^2)*d*e*(a + b*Cot[c + d*x])) + ((a^2 + 2*a*b - b^2)*Log[Sqrt[e] + Sqrt[e]
*Cot[c + d*x] - Sqrt[2]*Sqrt[e*Cot[c + d*x]]])/(2*Sqrt[2]*(a^2 + b^2)^2*d*S
qrt[e]) - ((a^2 + 2*a*b - b^2)*Log[Sqrt[e] + Sqrt[e]*Cot[c + d*x] + Sqrt[2]
*Sqrt[e*Cot[c + d*x]]])/(2*Sqrt[2]*(a^2 + b^2)^2*d*Sqrt[e])
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
```

```
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1179

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 1182

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
a*c, 2]}, Dist[(d*q + a*e)/(2*a*c), Int[(q + c*x^2)/(a + c*x^4), x], x] + D
ist[(d*q - a*e)/(2*a*c), Int[(q - c*x^2)/(a + c*x^4), x], x]] /; FreeQ[{a,
c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0] && NeQ[c*d^2 - a*e^2, 0] && NegQ[(-a)
*c]
```

Rule 3615

```
Int[((c_) + (d_.)*tan[(e_.) + (f_.)*(x_)])/Sqrt[(b_.)*tan[(e_.) + (f_.)*(x_
)]], x_Symbol] :> Dist[2/f, Subst[Int[(b*c + d*x^2)/(b^2 + x^4), x], x, Sqr
```

```
t[b*Tan[e + f*x]]], x] /; FreeQ[{b, c, d, e, f}, x] && NeQ[c^2 - d^2, 0] &&
 NeQ[c^2 + d^2, 0]
```

Rule 3650

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)])^(n_), x_Symbol] :> Simp[b^2*(a + b*Tan[e + f*x])^(m + 1)*((c
+ d*Tan[e + f*x])^(n + 1)/(f*(m + 1)*(a^2 + b^2)*(b*c - a*d))), x] + Dist[1
/((m + 1)*(a^2 + b^2)*(b*c - a*d)), Int[(a + b*Tan[e + f*x])^(m + 1)*(c + d
*Tan[e + f*x])^n*Simp[a*(b*c - a*d)*(m + 1) - b^2*d*(m + n + 2) - b*(b*c -
a*d)*(m + 1)*Tan[e + f*x] - b^2*d*(m + n + 2)*Tan[e + f*x]^2, x], x], x] /;
 FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0]
&& NeQ[c^2 + d^2, 0] && IntegerQ[2*m] && LtQ[m, -1] && (LtQ[n, 0] || Intege
rQ[m]) &&  !(ILtQ[n, -1] && ( !IntegerQ[m] || (EqQ[c, 0] && NeQ[a, 0])))
```

Rule 3715

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*tan[(e_.)
+ (f_.)*(x_)])^(n_.)*((A_) + (C_.)*tan[(e_.) + (f_.)*(x_)]^2), x_Symbol] :>
 Dist[A/f, Subst[Int[(a + b*x)^m*(c + d*x)^n, x], x, Tan[e + f*x]], x] /; F
reeQ[{a, b, c, d, e, f, A, C, m, n}, x] && EqQ[A, C]
```

Rule 3734

```
Int[(((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_)*((A_.) + (B_.)*tan[(e_.)
+ (f_.)*(x_)] + (C_.)*tan[(e_.) + (f_.)*(x_)]^2))/((a_.) + (b_.)*tan[(e_.)
+ (f_.)*(x_)]), x_Symbol] :> Dist[1/(a^2 + b^2), Int[(c + d*Tan[e + f*x])^n
*Simp[b*B + a*(A - C) + (a*B - b*(A - C))*Tan[e + f*x], x], x], x] + Dist[(
A*b^2 - a*b*B + a^2*C)/(a^2 + b^2), Int[(c + d*Tan[e + f*x])^n*((1 + Tan[e
+ f*x]^2)/(a + b*Tan[e + f*x])), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C
, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] &&
 !GtQ[n, 0] &&  !LeQ[n, -1]
```

Rubi steps

$$\begin{aligned}\text{integral} &= -\frac{b^2\sqrt{e\cot(c+dx)}}{a\left(a^2+b^2\right)de(a+b\cot(c+dx))}-\frac{\int\frac{-\frac{1}{2}\left(2a^2+b^2\right)e+abe\cot(c+dx)-\frac{1}{2}b^2e\cot^2(c+dx)}{\sqrt{e\cot(c+dx)}(a+b\cot(c+dx))}\,dx}{a\left(a^2+b^2\right)e}\\ &= -\frac{b^2\sqrt{e\cot(c+dx)}}{a\left(a^2+b^2\right)de(a+b\cot(c+dx))}\\ &\quad+\frac{\left(b^2\left(5a^2+b^2\right)\right)\int\frac{1+\cot^2(c+dx)}{\sqrt{e\cot(c+dx)}(a+b\cot(c+dx))}\,dx}{2a\left(a^2+b^2\right)^2}-\frac{\int\frac{-a\left(a^2-b^2\right)e+2a^2be\cot(c+dx)}{\sqrt{e\cot(c+dx)}}\,dx}{a\left(a^2+b^2\right)^2e}\end{aligned}$$

$$= -\frac{b^2\sqrt{e\cot(c+dx)}}{a(a^2+b^2)de(a+b\cot(c+dx))} + \frac{(b^2(5a^2+b^2))\,\text{Subst}\left(\int\frac{1}{\sqrt{-ex}(a-bx)}\,dx, x, -\cot(c+dx)\right)}{2a(a^2+b^2)^2 d} - \frac{2\text{Subst}\left(\int\frac{a(a^2-b^2)e^2-2a^2bex^2}{e^2+x^4}\,dx, x, \sqrt{e\cot(c+dx)}\right)}{a(a^2+b^2)^2 de}$$

$$= -\frac{b^2\sqrt{e\cot(c+dx)}}{a(a^2+b^2)de(a+b\cot(c+dx))} - \frac{(a^2-2ab-b^2)\,\text{Subst}\left(\int\frac{e+x^2}{e^2+x^4}\,dx, x, \sqrt{e\cot(c+dx)}\right)}{(a^2+b^2)^2 d} - \frac{(a^2+2ab-b^2)\,\text{Subst}\left(\int\frac{e-x^2}{e^2+x^4}\,dx, x, \sqrt{e\cot(c+dx)}\right)}{(a^2+b^2)^2 d} - \frac{(b^2(5a^2+b^2))\,\text{Subst}\left(\int\frac{1}{a+\frac{bx^2}{e}}\,dx, x, \sqrt{e\cot(c+dx)}\right)}{a(a^2+b^2)^2 de}$$

$$= -\frac{b^{3/2}(5a^2+b^2)\arctan\left(\frac{\sqrt{b}\sqrt{e\cot(c+dx)}}{\sqrt{a}\sqrt{e}}\right)}{a^{3/2}(a^2+b^2)^2 d\sqrt{e}} - \frac{b^2\sqrt{e\cot(c+dx)}}{a(a^2+b^2)de(a+b\cot(c+dx))} - \frac{(a^2-2ab-b^2)\,\text{Subst}\left(\int\frac{1}{e-\sqrt{2}\sqrt{e}x+x^2}\,dx, x, \sqrt{e\cot(c+dx)}\right)}{2(a^2+b^2)^2 d} - \frac{(a^2-2ab-b^2)\,\text{Subst}\left(\int\frac{1}{e+\sqrt{2}\sqrt{e}x+x^2}\,dx, x, \sqrt{e\cot(c+dx)}\right)}{2(a^2+b^2)^2 d} + \frac{(a^2+2ab-b^2)\,\text{Subst}\left(\int\frac{\sqrt{2}\sqrt{e}+2x}{-e-\sqrt{2}\sqrt{e}x-x^2}\,dx, x, \sqrt{e\cot(c+dx)}\right)}{2\sqrt{2}(a^2+b^2)^2 d\sqrt{e}} + \frac{(a^2+2ab-b^2)\,\text{Subst}\left(\int\frac{\sqrt{2}\sqrt{e}-2x}{-e+\sqrt{2}\sqrt{e}x-x^2}\,dx, x, \sqrt{e\cot(c+dx)}\right)}{2\sqrt{2}(a^2+b^2)^2 d\sqrt{e}}$$

$$= -\frac{b^{3/2}(5a^2+b^2)\arctan\left(\frac{\sqrt{b}\sqrt{e\cot(c+dx)}}{\sqrt{a}\sqrt{e}}\right)}{a^{3/2}(a^2+b^2)^2 d\sqrt{e}} - \frac{b^2\sqrt{e\cot(c+dx)}}{a(a^2+b^2)de(a+b\cot(c+dx))}$$
$$+ \frac{(a^2+2ab-b^2)\log\left(\sqrt{e}+\sqrt{e}\cot(c+dx)-\sqrt{2}\sqrt{e\cot(c+dx)}\right)}{2\sqrt{2}(a^2+b^2)^2 d\sqrt{e}}$$
$$- \frac{(a^2+2ab-b^2)\log\left(\sqrt{e}+\sqrt{e}\cot(c+dx)+\sqrt{2}\sqrt{e\cot(c+dx)}\right)}{2\sqrt{2}(a^2+b^2)^2 d\sqrt{e}}$$
$$- \frac{(a^2-2ab-b^2)\,\text{Subst}\left(\int \frac{1}{-1-x^2}\,dx, x, 1-\frac{\sqrt{2}\sqrt{e\cot(c+dx)}}{\sqrt{e}}\right)}{\sqrt{2}(a^2+b^2)^2 d\sqrt{e}}$$
$$+ \frac{(a^2-2ab-b^2)\,\text{Subst}\left(\int \frac{1}{-1-x^2}\,dx, x, 1+\frac{\sqrt{2}\sqrt{e\cot(c+dx)}}{\sqrt{e}}\right)}{\sqrt{2}(a^2+b^2)^2 d\sqrt{e}}$$

$$= -\frac{b^{3/2}(5a^2+b^2)\arctan\left(\frac{\sqrt{b}\sqrt{e\cot(c+dx)}}{\sqrt{a}\sqrt{e}}\right)}{a^{3/2}(a^2+b^2)^2 d\sqrt{e}}$$
$$+ \frac{(a^2-2ab-b^2)\arctan\left(1-\frac{\sqrt{2}\sqrt{e\cot(c+dx)}}{\sqrt{e}}\right)}{\sqrt{2}(a^2+b^2)^2 d\sqrt{e}}$$
$$- \frac{(a^2-2ab-b^2)\arctan\left(1+\frac{\sqrt{2}\sqrt{e\cot(c+dx)}}{\sqrt{e}}\right)}{\sqrt{2}(a^2+b^2)^2 d\sqrt{e}} - \frac{b^2\sqrt{e\cot(c+dx)}}{a(a^2+b^2)de(a+b\cot(c+dx))}$$
$$+ \frac{(a^2+2ab-b^2)\log\left(\sqrt{e}+\sqrt{e}\cot(c+dx)-\sqrt{2}\sqrt{e\cot(c+dx)}\right)}{2\sqrt{2}(a^2+b^2)^2 d\sqrt{e}}$$
$$- \frac{(a^2+2ab-b^2)\log\left(\sqrt{e}+\sqrt{e}\cot(c+dx)+\sqrt{2}\sqrt{e\cot(c+dx)}\right)}{2\sqrt{2}(a^2+b^2)^2 d\sqrt{e}}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 2.79 (sec) , antiderivative size = 295, normalized size of antiderivative = 0.75

$$\int \frac{1}{\sqrt{e\cot(c+dx)}(a+b\cot(c+dx))^2}\,dx =$$
$$-\frac{\sqrt{\cot(c+dx)}\left(48\sqrt{a}b^{3/2}\arctan\left(\frac{\sqrt{b}\sqrt{\cot(c+dx)}}{\sqrt{a}}\right)+\frac{12b^{3/2}(a^2+b^2)\arctan\left(\frac{\sqrt{b}\sqrt{\cot(c+dx)}}{\sqrt{a}}\right)}{a^{3/2}}+\frac{12b^2(a^2+b^2)\sqrt{\cot(c+dx)}}{a(a+b\cot(c+dx))}\right.}{}$$

```
[In] Integrate[1/(Sqrt[e*Cot[c + d*x]]*(a + b*Cot[c + d*x])^2),x]
```

```
[Out] -1/12*(Sqrt[Cot[c + d*x]]*(48*Sqrt[a]*b^(3/2)*ArcTan[(Sqrt[b]*Sqrt[Cot[c +
d*x]])/Sqrt[a]] + (12*b^(3/2)*(a^2 + b^2)*ArcTan[(Sqrt[b]*Sqrt[Cot[c + d*x]
])/Sqrt[a]])/a^(3/2) + (12*b^2*(a^2 + b^2)*Sqrt[Cot[c + d*x]])/(a*(a + b*Co
t[c + d*x])) - 16*a*b*Cot[c + d*x]^(3/2)*Hypergeometric2F1[3/4, 1, 7/4, -Co
t[c + d*x]^2] - 3*Sqrt[2]*(a - b)*(a + b)*(2*ArcTan[1 - Sqrt[2]*Sqrt[Cot[c
+ d*x]]] - 2*ArcTan[1 + Sqrt[2]*Sqrt[Cot[c + d*x]]] + Log[1 - Sqrt[2]*Sqrt[
Cot[c + d*x]] + Cot[c + d*x]] - Log[1 + Sqrt[2]*Sqrt[Cot[c + d*x]] + Cot[c
+ d*x]])))/((a^2 + b^2)^2*d*Sqrt[e*Cot[c + d*x]])
```

Maple [A] (verified)

Time = 0.04 (sec) , antiderivative size = 396, normalized size of antiderivative = 1.01

method	result
derivativedivides	$-\frac{2e^3\left(\frac{b^2\left(\frac{(a^2+b^2)\sqrt{e\cot(dx+c)}}{2a(e\cot(dx+c)b+ae)}+\frac{(5a^2+b^2)\arctan\left(\frac{\sqrt{e\cot(dx+c)}\,b}{\sqrt{aeb}}\right)}{2a\sqrt{aeb}}\right)}{e^3(a^2+b^2)^2}+\frac{(a^2e-b^2e)(e^2)^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\frac{e\cot(dx+c)+(e^2)^{\frac{1}{4}}\sqrt{e\cot}}{e\cot(dx+c)-(e^2)^{\frac{1}{4}}\sqrt{e\cot}}\right.\right.}{}\right.}{}$
default	$-\frac{2e^3\left(\frac{b^2\left(\frac{(a^2+b^2)\sqrt{e\cot(dx+c)}}{2a(e\cot(dx+c)b+ae)}+\frac{(5a^2+b^2)\arctan\left(\frac{\sqrt{e\cot(dx+c)}\,b}{\sqrt{aeb}}\right)}{2a\sqrt{aeb}}\right)}{e^3(a^2+b^2)^2}+\frac{(a^2e-b^2e)(e^2)^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\frac{e\cot(dx+c)+(e^2)^{\frac{1}{4}}\sqrt{e\cot}}{e\cot(dx+c)-(e^2)^{\frac{1}{4}}\sqrt{e\cot}}\right.\right.}{}\right.}{}$

```
[In] int(1/(e*cot(d*x+c))^(1/2)/(a+b*cot(d*x+c))^2,x,method=_RETURNVERBOSE)
```

```
[Out] -2/d*e^3*(b^2/e^3/(a^2+b^2)^2*(1/2*(a^2+b^2)/a*(e*cot(d*x+c))^(1/2)/(e*cot(
d*x+c)*b+a*e)+1/2*(5*a^2+b^2)/a/(a*e*b)^(1/2)*arctan((e*cot(d*x+c))^(1/2)*b
/(a*e*b)^(1/2)))+1/e^3/(a^2+b^2)^2*(1/8*(a^2*e-b^2*e)*(e^2)^(1/4)/e^2*2^(1/
2)*(ln((e*cot(d*x+c)+(e^2)^(1/4)*(e*cot(d*x+c))^(1/2)*2^(1/2)+(e^2)^(1/2))/
(e*cot(d*x+c)-(e^2)^(1/4)*(e*cot(d*x+c))^(1/2)*2^(1/2)+(e^2)^(1/2)))+2*arct
an(2^(1/2)/(e^2)^(1/4)*(e*cot(d*x+c))^(1/2)+1)-2*arctan(-2^(1/2)/(e^2)^(1/4
)*(e*cot(d*x+c))^(1/2)+1))-1/4*a*b/(e^2)^(1/4)*2^(1/2)*(ln((e*cot(d*x+c)-(e
^2)^(1/4)*(e*cot(d*x+c))^(1/2)*2^(1/2)+(e^2)^(1/2))/(e*cot(d*x+c)+(e^2)^(1/
4)*(e*cot(d*x+c))^(1/2)*2^(1/2)+(e^2)^(1/2)))+2*arctan(2^(1/2)/(e^2)^(1/4)*
(e*cot(d*x+c))^(1/2)+1)-2*arctan(-2^(1/2)/(e^2)^(1/4)*(e*cot(d*x+c))^(1/2)+
1))))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 3102 vs. 2(331) = 662.

Time = 0.61 (sec) , antiderivative size = 6248, normalized size of antiderivative = 15.86

$$\int \frac{1}{\sqrt{e\cot(c+dx)}(a+b\cot(c+dx))^2}\,dx = \text{Too large to display}$$

```
[In] integrate(1/(e*cot(d*x+c))^(1/2)/(a+b*cot(d*x+c))^2,x, algorithm="fricas")
[Out] Too large to include
```

Sympy [F]

$$\int \frac{1}{\sqrt{e\cot(c+dx)}(a+b\cot(c+dx))^2}\,dx = \int \frac{1}{\sqrt{e\cot{(c+dx)}}\,(a+b\cot{(c+dx)})^2}\,dx$$

```
[In] integrate(1/(e*cot(d*x+c))**(1/2)/(a+b*cot(d*x+c))**2,x)
[Out] Integral(1/(sqrt(e*cot(c + d*x))*(a + b*cot(c + d*x))**2), x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{1}{\sqrt{e\cot(c+dx)}(a+b\cot(c+dx))^2}\,dx = \text{Exception raised: ValueError}$$

```
[In] integrate(1/(e*cot(d*x+c))^(1/2)/(a+b*cot(d*x+c))^2,x, algorithm="maxima")
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(e>0)', see `assume?` for more detai
ls)Is e
```

Giac [F]

$$\int \frac{1}{\sqrt{e\cot(c+dx)}(a+b\cot(c+dx))^2}\,dx = \int \frac{1}{(b\cot{(dx+c)}+a)^2\sqrt{e\cot{(dx+c)}}}\,dx$$

```
[In] integrate(1/(e*cot(d*x+c))^(1/2)/(a+b*cot(d*x+c))^2,x, algorithm="giac")
[Out] integrate(1/((b*cot(d*x + c) + a)^2*sqrt(e*cot(d*x + c))), x)
```

Mupad [B] (verification not implemented)

Time = 20.82 (sec) , antiderivative size = 9400, normalized size of antiderivative = 23.86

$$\int \frac{1}{\sqrt{e\cot(c+dx)}(a+b\cot(c+dx))^2}\,dx = \text{Too large to display}$$

```
[In] int(1/((e*cot(c + d*x))^(1/2)*(a + b*cot(c + d*x))^2),x)
```

```
[Out] (log(- ((((((((((128*b^2*e^10*(2*b^6 - a^6 + 9*a^2*b^4 + 6*a^4*b^2))/(a*d) -
 256*b^3*e^10*(e*cot(c + d*x))^(1/2)*(a^2 - b^2)*(a^2 + b^2)^2*(1i/(d^2*e*(
a*1i - b)^4))^(1/2))*(1i/(d^2*e*(a*1i - b)^4))^(1/2))/2 - (64*b^2*e^9*(e*co
t(c + d*x))^(1/2)*(2*b^8 - a^8 + 5*a^2*b^6 + 67*a^4*b^4 - a^6*b^2))/(a*d^2*
(a^2 + b^2)^2))*(1i/(d^2*e*(a*1i - b)^4))^(1/2))/2 - (32*b^5*e^9*(25*a^6 +
b^6 - 13*a^2*b^4 - 85*a^4*b^2))/(a^2*d^3*(a^2 + b^2)^3))*(1i/(d^2*e*(a*1i -
 b)^4))^(1/2))/2 - (16*b^5*e^8*(e*cot(c + d*x))^(1/2)*(b^6 - 27*a^6 + 7*a^2
*b^4 + 11*a^4*b^2))/(a^2*d^4*(a^2 + b^2)^4))*(1i/(d^2*e*(a*1i - b)^4))^(1/2
))/2 - (16*b^6*e^8*(5*a^2 + b^2))/(a*d^5*(a^2 + b^2)^4))*(-1/(a^4*d^2*e*1i
+ b^4*d^2*e*1i - a^2*b^2*d^2*e*6i + 4*a*b^3*d^2*e - 4*a^3*b*d^2*e))^(1/2))/
2 - log(- ((((((((((128*b^2*e^10*(2*b^6 - a^6 + 9*a^2*b^4 + 6*a^4*b^2))/(a*d
) + 256*b^3*e^10*(e*cot(c + d*x))^(1/2)*(a^2 - b^2)*(a^2 + b^2)^2*(1i/(d^2*
e*(a*1i - b)^4))^(1/2))*(1i/(d^2*e*(a*1i - b)^4))^(1/2))/2 + (64*b^2*e^9*(e
*cot(c + d*x))^(1/2)*(2*b^8 - a^8 + 5*a^2*b^6 + 67*a^4*b^4 - a^6*b^2))/(a*d
^2*(a^2 + b^2)^2))*(1i/(d^2*e*(a*1i - b)^4))^(1/2))/2 - (32*b^5*e^9*(25*a^6
 + b^6 - 13*a^2*b^4 - 85*a^4*b^2))/(a^2*d^3*(a^2 + b^2)^3))*(1i/(d^2*e*(a*1
i - b)^4))^(1/2))/2 + (16*b^5*e^8*(e*cot(c + d*x))^(1/2)*(b^6 - 27*a^6 + 7*
a^2*b^4 + 11*a^4*b^2))/(a^2*d^4*(a^2 + b^2)^4))*(1i/(d^2*e*(a*1i - b)^4))^(
1/2))/2 - (16*b^6*e^8*(5*a^2 + b^2))/(a*d^5*(a^2 + b^2)^4))*(-1/(4*(a^4*d^2
*e*1i + b^4*d^2*e*1i - a^2*b^2*d^2*e*6i + 4*a*b^3*d^2*e - 4*a^3*b*d^2*e)))^
(1/2) + atan(((-1i/(4*(a^4*d^2*e + b^4*d^2*e - 6*a^2*b^2*d^2*e + a*b^3*d^2*
e*4i - a^3*b*d^2*e*4i)))^(1/2)*(((16*(24*a^2*b^11*d^2*e^9 - 2*b^13*d^2*e^9
+ 196*a^4*b^9*d^2*e^9 + 120*a^6*b^7*d^2*e^9 - 50*a^8*b^5*d^2*e^9))/(a^10*d^
5 + a^2*b^8*d^5 + 4*a^4*b^6*d^5 + 6*a^6*b^4*d^5 + 4*a^8*b^2*d^5) + (-1i/(4*
(a^4*d^2*e + b^4*d^2*e - 6*a^2*b^2*d^2*e + a*b^3*d^2*e*4i - a^3*b*d^2*e*4i)
))^(1/2)*((-1i/(4*(a^4*d^2*e + b^4*d^2*e - 6*a^2*b^2*d^2*e + a*b^3*d^2*e*4i
 - a^3*b*d^2*e*4i)))^(1/2)*((16*(16*a*b^16*d^4*e^10 + 136*a^3*b^14*d^4*e^10
 + 432*a^5*b^12*d^4*e^10 + 680*a^7*b^10*d^4*e^10 + 560*a^9*b^8*d^4*e^10 + 2
16*a^11*b^6*d^4*e^10 + 16*a^13*b^4*d^4*e^10 - 8*a^15*b^2*d^4*e^10))/(a^10*d
^5 + a^2*b^8*d^5 + 4*a^4*b^6*d^5 + 6*a^6*b^4*d^5 + 4*a^8*b^2*d^5) - (16*(-1
i/(4*(a^4*d^2*e + b^4*d^2*e - 6*a^2*b^2*d^2*e + a*b^3*d^2*e*4i - a^3*b*d^2*
e*4i)))^(1/2)*(e*cot(c + d*x))^(1/2)*(32*a^2*b^17*d^4*e^10 + 160*a^4*b^15*d
^4*e^10 + 288*a^6*b^13*d^4*e^10 + 160*a^8*b^11*d^4*e^10 - 160*a^10*b^9*d^4*
e^10 - 288*a^12*b^7*d^4*e^10 - 160*a^14*b^5*d^4*e^10 - 32*a^16*b^3*d^4*e^10
))/(a^10*d^4 + a^2*b^8*d^4 + 4*a^4*b^6*d^4 + 6*a^6*b^4*d^4 + 4*a^8*b^2*d^4)
) + (16*(e*cot(c + d*x))^(1/2)*(8*a*b^14*d^2*e^9 + 36*a^3*b^12*d^2*e^9 + 31
```

6*a^5*b^10*d^2*e^9 + 552*a^7*b^8*d^2*e^9 + 256*a^9*b^6*d^2*e^9 - 12*a^11*b^4*d^2*e^9 - 4*a^13*b^2*d^2*e^9))/(a^10*d^4 + a^2*b^8*d^4 + 4*a^4*b^6*d^4 + 6*a^6*b^4*d^4 + 4*a^8*b^2*d^4)))*(-1i/(4*(a^4*d^2*e + b^4*d^2*e - 6*a^2*b^2*d^2*e + a*b^3*d^2*e*4i - a^3*b*d^2*e*4i)))^(1/2) + (16*(e*cot(c + d*x))^(1/2)*(b^11*e^8 + 7*a^2*b^9*e^8 + 11*a^4*b^7*e^8 - 27*a^6*b^5*e^8))/(a^10*d^4 + a^2*b^8*d^4 + 4*a^4*b^6*d^4 + 6*a^6*b^4*d^4 + 4*a^8*b^2*d^4))*1i - (-1i/(4*(a^4*d^2*e + b^4*d^2*e - 6*a^2*b^2*d^2*e + a*b^3*d^2*e*4i - a^3*b*d^2*e*4i)))^(1/2)*(((16*(24*a^2*b^11*d^2*e^9 - 2*b^13*d^2*e^9 + 196*a^4*b^9*d^2*e^9 + 120*a^6*b^7*d^2*e^9 - 50*a^8*b^5*d^2*e^9))/(a^10*d^5 + a^2*b^8*d^5 + 4*a^4*b^6*d^5 + 6*a^6*b^4*d^5 + 4*a^8*b^2*d^5) + (-1i/(4*(a^4*d^2*e + b^4*d^2*e - 6*a^2*b^2*d^2*e + a*b^3*d^2*e*4i - a^3*b*d^2*e*4i)))^(1/2)*((-1i/(4*(a^4*d^2*e + b^4*d^2*e - 6*a^2*b^2*d^2*e + a*b^3*d^2*e*4i - a^3*b*d^2*e*4i)))^(1/2)*((16*(16*a*b^16*d^4*e^10 + 136*a^3*b^14*d^4*e^10 + 432*a^5*b^12*d^4*e^10 + 680*a^7*b^10*d^4*e^10 + 560*a^9*b^8*d^4*e^10 + 216*a^11*b^6*d^4*e^10 + 16*a^13*b^4*d^4*e^10 - 8*a^15*b^2*d^4*e^10))/(a^10*d^5 + a^2*b^8*d^5 + 4*a^4*b^6*d^5 + 6*a^6*b^4*d^5 + 4*a^8*b^2*d^5) + (16*(-1i/(4*(a^4*d^2*e + b^4*d^2*e - 6*a^2*b^2*d^2*e + a*b^3*d^2*e*4i - a^3*b*d^2*e*4i)))^(1/2)*(e*cot(c + d*x))^(1/2)*(32*a^2*b^17*d^4*e^10 + 160*a^4*b^15*d^4*e^10 + 288*a^6*b^13*d^4*e^10 + 160*a^8*b^11*d^4*e^10 - 160*a^10*b^9*d^4*e^10 - 288*a^12*b^7*d^4*e^10 - 160*a^14*b^5*d^4*e^10 - 32*a^16*b^3*d^4*e^10))/(a^10*d^4 + a^2*b^8*d^4 + 4*a^4*b^6*d^4 + 6*a^6*b^4*d^4 + 4*a^8*b^2*d^4)) - (16*(e*cot(c + d*x))^(1/2)*(8*a*b^14*d^2*e^9 + 36*a^3*b^12*d^2*e^9 + 316*a^5*b^10*d^2*e^9 + 552*a^7*b^8*d^2*e^9 + 256*a^9*b^6*d^2*e^9 - 12*a^11*b^4*d^2*e^9 - 4*a^13*b^2*d^2*e^9))/(a^10*d^4 + a^2*b^8*d^4 + 4*a^4*b^6*d^4 + 6*a^6*b^4*d^4 + 4*a^8*b^2*d^4)))*(-1i/(4*(a^4*d^2*e + b^4*d^2*e - 6*a^2*b^2*d^2*e + a*b^3*d^2*e*4i - a^3*b*d^2*e*4i)))^(1/2) - (16*(e*cot(c + d*x))^(1/2)*(b^11*e^8 + 7*a^2*b^9*e^8 + 11*a^4*b^7*e^8 - 27*a^6*b^5*e^8))/(a^10*d^4 + a^2*b^8*d^4 + 4*a^4*b^6*d^4 + 6*a^6*b^4*d^4 + 4*a^8*b^2*d^4))*1i)/((-1i/(4*(a^4*d^2*e + b^4*d^2*e - 6*a^2*b^2*d^2*e + a*b^3*d^2*e*4i - a^3*b*d^2*e*4i)))^(1/2)*(((16*(24*a^2*b^11*d^2*e^9 - 2*b^13*d^2*e^9 + 196*a^4*b^9*d^2*e^9 + 120*a^6*b^7*d^2*e^9 - 50*a^8*b^5*d^2*e^9))/(a^10*d^5 + a^2*b^8*d^5 + 4*a^4*b^6*d^5 + 6*a^6*b^4*d^5 + 4*a^8*b^2*d^5) + (-1i/(4*(a^4*d^2*e + b^4*d^2*e - 6*a^2*b^2*d^2*e + a*b^3*d^2*e*4i - a^3*b*d^2*e*4i)))^(1/2)*((-1i/(4*(a^4*d^2*e + b^4*d^2*e - 6*a^2*b^2*d^2*e + a*b^3*d^2*e*4i - a^3*b*d^2*e*4i)))^(1/2)*((16*(16*a*b^16*d^4*e^10 + 136*a^3*b^14*d^4*e^10 + 432*a^5*b^12*d^4*e^10 + 680*a^7*b^10*d^4*e^10 + 560*a^9*b^8*d^4*e^10 + 216*a^11*b^6*d^4*e^10 + 16*a^13*b^4*d^4*e^10 - 8*a^15*b^2*d^4*e^10))/(a^10*d^5 + a^2*b^8*d^5 + 4*a^4*b^6*d^5 + 6*a^6*b^4*d^5 + 4*a^8*b^2*d^5) - (16*(-1i/(4*(a^4*d^2*e + b^4*d^2*e - 6*a^2*b^2*d^2*e + a*b^3*d^2*e*4i - a^3*b*d^2*e*4i)))^(1/2)*(e*cot(c + d*x))^(1/2)*(32*a^2*b^17*d^4*e^10 + 160*a^4*b^15*d^4*e^10 + 288*a^6*b^13*d^4*e^10 + 160*a^8*b^11*d^4*e^10 - 160*a^10*b^9*d^4*e^10 - 288*a^12*b^7*d^4*e^10 - 160*a^14*b^5*d^4*e^10 - 32*a^16*b^3*d^4*e^10))/(a^10*d^4 + a^2*b^8*d^4 + 4*a^4*b^6*d^4 + 6*a^6*b^4*d^4 + 4*a^8*b^2*d^4)) + (16*(e*cot(c + d*x))^(1/2)*(8*a*b^14*d^2*e^9 + 36*a^3*b^12*d^2*e^9 + 316*a^5*b^10*d^2*e^9 + 552*a^7*b^8*d^2*e^9 + 256*a^9*b^6*d^2*e^9 - 12*a^11*b^4*d^2*e^9 - 4*a^13*b^2*d^2*e^9))/(a^10

*d^4 + a^2*b^8*d^4 + 4*a^4*b^6*d^4 + 6*a^6*b^4*d^4 + 4*a^8*b^2*d^4)))*(-1i/
(4*(a^4*d^2*e + b^4*d^2*e - 6*a^2*b^2*d^2*e + a*b^3*d^2*e*4i - a^3*b*d^2*e*
4i)))^(1/2) + (16*(e*cot(c + d*x))^(1/2)*(b^11*e^8 + 7*a^2*b^9*e^8 + 11*a^4
*b^7*e^8 - 27*a^6*b^5*e^8))/(a^10*d^4 + a^2*b^8*d^4 + 4*a^4*b^6*d^4 + 6*a^6
*b^4*d^4 + 4*a^8*b^2*d^4)) + (-1i/(4*(a^4*d^2*e + b^4*d^2*e - 6*a^2*b^2*d^2
*e + a*b^3*d^2*e*4i - a^3*b*d^2*e*4i)))^(1/2)*(((16*(24*a^2*b^11*d^2*e^9 -
2*b^13*d^2*e^9 + 196*a^4*b^9*d^2*e^9 + 120*a^6*b^7*d^2*e^9 - 50*a^8*b^5*d^2
*e^9))/(a^10*d^5 + a^2*b^8*d^5 + 4*a^4*b^6*d^5 + 6*a^6*b^4*d^5 + 4*a^8*b^2*
d^5) + (-1i/(4*(a^4*d^2*e + b^4*d^2*e - 6*a^2*b^2*d^2*e + a*b^3*d^2*e*4i -
a^3*b*d^2*e*4i)))^(1/2)*((-1i/(4*(a^4*d^2*e + b^4*d^2*e - 6*a^2*b^2*d^2*e +
a*b^3*d^2*e*4i - a^3*b*d^2*e*4i)))^(1/2)*((16*(16*a*b^16*d^4*e^10 + 136*a^
3*b^14*d^4*e^10 + 432*a^5*b^12*d^4*e^10 + 680*a^7*b^10*d^4*e^10 + 560*a^9*b
^8*d^4*e^10 + 216*a^11*b^6*d^4*e^10 + 16*a^13*b^4*d^4*e^10 - 8*a^15*b^2*d^4
*e^10))/(a^10*d^5 + a^2*b^8*d^5 + 4*a^4*b^6*d^5 + 6*a^6*b^4*d^5 + 4*a^8*b^2
d^5) + (16(-1i/(4*(a^4*d^2*e + b^4*d^2*e - 6*a^2*b^2*d^2*e + a*b^3*d^2*e*
4i - a^3*b*d^2*e*4i)))^(1/2)*(e*cot(c + d*x))^(1/2)*(32*a^2*b^17*d^4*e^10 +
160*a^4*b^15*d^4*e^10 + 288*a^6*b^13*d^4*e^10 + 160*a^8*b^11*d^4*e^10 - 16
0*a^10*b^9*d^4*e^10 - 288*a^12*b^7*d^4*e^10 - 160*a^14*b^5*d^4*e^10 - 32*a^
16*b^3*d^4*e^10))/(a^10*d^4 + a^2*b^8*d^4 + 4*a^4*b^6*d^4 + 6*a^6*b^4*d^4 +
4*a^8*b^2*d^4)) - (16*(e*cot(c + d*x))^(1/2)*(8*a*b^14*d^2*e^9 + 36*a^3*b^
12*d^2*e^9 + 316*a^5*b^10*d^2*e^9 + 552*a^7*b^8*d^2*e^9 + 256*a^9*b^6*d^2*e
^9 - 12*a^11*b^4*d^2*e^9 - 4*a^13*b^2*d^2*e^9))/(a^10*d^4 + a^2*b^8*d^4 + 4
*a^4*b^6*d^4 + 6*a^6*b^4*d^4 + 4*a^8*b^2*d^4)))*(-1i/(4*(a^4*d^2*e + b^4*d^
2*e - 6*a^2*b^2*d^2*e + a*b^3*d^2*e*4i - a^3*b*d^2*e*4i)))^(1/2) - (16*(e*c
ot(c + d*x))^(1/2)*(b^11*e^8 + 7*a^2*b^9*e^8 + 11*a^4*b^7*e^8 - 27*a^6*b^5*
e^8))/(a^10*d^4 + a^2*b^8*d^4 + 4*a^4*b^6*d^4 + 6*a^6*b^4*d^4 + 4*a^8*b^2*d
^4)) + (32*(a*b^8*e^8 + 5*a^3*b^6*e^8))/(a^10*d^5 + a^2*b^8*d^5 + 4*a^4*b^6
*d^5 + 6*a^6*b^4*d^5 + 4*a^8*b^2*d^5)))*(-1i/(4*(a^4*d^2*e + b^4*d^2*e - 6*
a^2*b^2*d^2*e + a*b^3*d^2*e*4i - a^3*b*d^2*e*4i)))^(1/2)*2i + (atan((((5*a^
2 + b^2)*((16*(e*cot(c + d*x))^(1/2)*(b^11*e^8 + 7*a^2*b^9*e^8 + 11*a^4*b^7
*e^8 - 27*a^6*b^5*e^8))/(a^10*d^4 + a^2*b^8*d^4 + 4*a^4*b^6*d^4 + 6*a^6*b^4
*d^4 + 4*a^8*b^2*d^4) + ((5*a^2 + b^2)*((16*(24*a^2*b^11*d^2*e^9 - 2*b^13*d
^2*e^9 + 196*a^4*b^9*d^2*e^9 + 120*a^6*b^7*d^2*e^9 - 50*a^8*b^5*d^2*e^9))/(
a^10*d^5 + a^2*b^8*d^5 + 4*a^4*b^6*d^5 + 6*a^6*b^4*d^5 + 4*a^8*b^2*d^5) + (
(5*a^2 + b^2)*((16*(e*cot(c + d*x))^(1/2)*(8*a*b^14*d^2*e^9 + 36*a^3*b^12*d
^2*e^9 + 316*a^5*b^10*d^2*e^9 + 552*a^7*b^8*d^2*e^9 + 256*a^9*b^6*d^2*e^9 -
12*a^11*b^4*d^2*e^9 - 4*a^13*b^2*d^2*e^9))/(a^10*d^4 + a^2*b^8*d^4 + 4*a^4
*b^6*d^4 + 6*a^6*b^4*d^4 + 4*a^8*b^2*d^4) + ((5*a^2 + b^2)*((16*(16*a*b^16*
d^4*e^10 + 136*a^3*b^14*d^4*e^10 + 432*a^5*b^12*d^4*e^10 + 680*a^7*b^10*d^4
*e^10 + 560*a^9*b^8*d^4*e^10 + 216*a^11*b^6*d^4*e^10 + 16*a^13*b^4*d^4*e^10
- 8*a^15*b^2*d^4*e^10))/(a^10*d^5 + a^2*b^8*d^5 + 4*a^4*b^6*d^5 + 6*a^6*b^
4*d^5 + 4*a^8*b^2*d^5) - (8*(e*cot(c + d*x))^(1/2)*(5*a^2 + b^2)*(-a^3*b^3*
e)^(1/2)*(32*a^2*b^17*d^4*e^10 + 160*a^4*b^15*d^4*e^10 + 288*a^6*b^13*d^4*e
^10 + 160*a^8*b^11*d^4*e^10 - 160*a^10*b^9*d^4*e^10 - 288*a^12*b^7*d^4*e^10
- 160*a^14*b^5*d^4*e^10 - 32*a^16*b^3*d^4*e^10))/((a^7*d*e + a^3*b^4*d*e +

2*a^5*b^2*d*e)*(a^10*d^4 + a^2*b^8*d^4 + 4*a^4*b^6*d^4 + 6*a^6*b^4*d^4 + 4*a^8*b^2*d^4)))*(-a^3*b^3*e)^(1/2))/(2*(a^7*d*e + a^3*b^4*d*e + 2*a^5*b^2*d*e)))*(-a^3*b^3*e)^(1/2))/(2*(a^7*d*e + a^3*b^4*d*e + 2*a^5*b^2*d*e)))*(-a^3*b^3*e)^(1/2))/(2*(a^7*d*e + a^3*b^4*d*e + 2*a^5*b^2*d*e)))*(-a^3*b^3*e)^(1/2)*1i)/(2*(a^7*d*e + a^3*b^4*d*e + 2*a^5*b^2*d*e)) + ((5*a^2 + b^2)*((16*(e*cot(c + d*x))^(1/2)*(b^11*e^8 + 7*a^2*b^9*e^8 + 11*a^4*b^7*e^8 - 27*a^6*b^5*e^8))/(a^10*d^4 + a^2*b^8*d^4 + 4*a^4*b^6*d^4 + 6*a^6*b^4*d^4 + 4*a^8*b^2*d^4) - ((5*a^2 + b^2)*((16*(24*a^2*b^11*d^2*e^9 - 2*b^13*d^2*e^9 + 196*a^4*b^9*d^2*e^9 + 120*a^6*b^7*d^2*e^9 - 50*a^8*b^5*d^2*e^9))/(a^10*d^5 + a^2*b^8*d^5 + 4*a^4*b^6*d^5 + 6*a^6*b^4*d^5 + 4*a^8*b^2*d^5) - ((5*a^2 + b^2)*((16*(e*cot(c + d*x))^(1/2)*(8*a*b^14*d^2*e^9 + 36*a^3*b^12*d^2*e^9 + 316*a^5*b^10*d^2*e^9 + 552*a^7*b^8*d^2*e^9 + 256*a^9*b^6*d^2*e^9 - 12*a^11*b^4*d^2*e^9 - 4*a^13*b^2*d^2*e^9))/(a^10*d^4 + a^2*b^8*d^4 + 4*a^4*b^6*d^4 + 6*a^6*b^4*d^4 + 4*a^8*b^2*d^4) - ((5*a^2 + b^2)*((16*(16*a*b^16*d^4*e^10 + 136*a^3*b^14*d^4*e^10 + 432*a^5*b^12*d^4*e^10 + 680*a^7*b^10*d^4*e^10 + 560*a^9*b^8*d^4*e^10 + 216*a^11*b^6*d^4*e^10 + 16*a^13*b^4*d^4*e^10 - 8*a^15*b^2*d^4*e^10))/(a^10*d^5 + a^2*b^8*d^5 + 4*a^4*b^6*d^5 + 6*a^6*b^4*d^5 + 4*a^8*b^2*d^5) + (8*(e*cot(c + d*x))^(1/2)*(5*a^2 + b^2)*(-a^3*b^3*e)^(1/2)*(32*a^2*b^17*d^4*e^10 + 160*a^4*b^15*d^4*e^10 + 288*a^6*b^13*d^4*e^10 + 160*a^8*b^11*d^4*e^10 - 160*a^10*b^9*d^4*e^10 - 288*a^12*b^7*d^4*e^10 - 160*a^14*b^5*d^4*e^10 - 32*a^16*b^3*d^4*e^10))/((a^7*d*e + a^3*b^4*d*e + 2*a^5*b^2*d*e)*(a^10*d^4 + a^2*b^8*d^4 + 4*a^4*b^6*d^4 + 6*a^6*b^4*d^4 + 4*a^8*b^2*d^4)))*(-a^3*b^3*e)^(1/2))/(2*(a^7*d*e + a^3*b^4*d*e + 2*a^5*b^2*d*e)))*(-a^3*b^3*e)^(1/2))/(2*(a^7*d*e + a^3*b^4*d*e + 2*a^5*b^2*d*e)))*(-a^3*b^3*e)^(1/2))/(2*(a^7*d*e + a^3*b^4*d*e + 2*a^5*b^2*d*e)))*(-a^3*b^3*e)^(1/2)*1i)/(2*(a^7*d*e + a^3*b^4*d*e + 2*a^5*b^2*d*e)))/((32*(a*b^8*e^8 + 5*a^3*b^6*e^8))/(a^10*d^5 + a^2*b^8*d^5 + 4*a^4*b^6*d^5 + 6*a^6*b^4*d^5 + 4*a^8*b^2*d^5) + ((5*a^2 + b^2)*((16*(e*cot(c + d*x))^(1/2)*(b^11*e^8 + 7*a^2*b^9*e^8 + 11*a^4*b^7*e^8 - 27*a^6*b^5*e^8))/(a^10*d^4 + a^2*b^8*d^4 + 4*a^4*b^6*d^4 + 6*a^6*b^4*d^4 + 4*a^8*b^2*d^4) + ((5*a^2 + b^2)*((16*(24*a^2*b^11*d^2*e^9 - 2*b^13*d^2*e^9 + 196*a^4*b^9*d^2*e^9 + 120*a^6*b^7*d^2*e^9 - 50*a^8*b^5*d^2*e^9))/(a^10*d^5 + a^2*b^8*d^5 + 4*a^4*b^6*d^5 + 6*a^6*b^4*d^5 + 4*a^8*b^2*d^5) + ((5*a^2 + b^2)*((16*(e*cot(c + d*x))^(1/2)*(8*a*b^14*d^2*e^9 + 36*a^3*b^12*d^2*e^9 + 316*a^5*b^10*d^2*e^9 + 552*a^7*b^8*d^2*e^9 + 256*a^9*b^6*d^2*e^9 - 12*a^11*b^4*d^2*e^9 - 4*a^13*b^2*d^2*e^9))/(a^10*d^4 + a^2*b^8*d^4 + 4*a^4*b^6*d^4 + 6*a^6*b^4*d^4 + 4*a^8*b^2*d^4) + ((5*a^2 + b^2)*((16*(16*a*b^16*d^4*e^10 + 136*a^3*b^14*d^4*e^10 + 432*a^5*b^12*d^4*e^10 + 680*a^7*b^10*d^4*e^10 + 560*a^9*b^8*d^4*e^10 + 216*a^11*b^6*d^4*e^10 + 16*a^13*b^4*d^4*e^10 - 8*a^15*b^2*d^4*e^10))/(a^10*d^5 + a^2*b^8*d^5 + 4*a^4*b^6*d^5 + 6*a^6*b^4*d^5 + 4*a^8*b^2*d^5) - (8*(e*cot(c + d*x))^(1/2)*(5*a^2 + b^2)*(-a^3*b^3*e)^(1/2)*(32*a^2*b^17*d^4*e^10 + 160*a^4*b^15*d^4*e^10 + 288*a^6*b^13*d^4*e^10 + 160*a^8*b^11*d^4*e^10 - 160*a^10*b^9*d^4*e^10 - 288*a^12*b^7*d^4*e^10 - 160*a^14*b^5*d^4*e^10 - 32*a^16*b^3*d^4*e^10))/((a^7*d*e + a^3*b^4*d*e + 2*a^5*b^2*d*e)*(a^10*d^4 + a^2*b^8*d^4 + 4*a^4*b^6*d^4 + 6*a^6*b^4*d^4 + 4*a^8*b^2*d^4)))*(-a^3*b^3*e)^(1/2))/(2*(a^7*d*e + a^3*b^4*d*e + 2*a^5*

```
b^2*d*e)))*(-a^3*b^3*e)^(1/2))/(2*(a^7*d*e + a^3*b^4*d*e + 2*a^5*b^2*d*e)))
*(-a^3*b^3*e)^(1/2))/(2*(a^7*d*e + a^3*b^4*d*e + 2*a^5*b^2*d*e)))*(-a^3*b^3
*e)^(1/2))/(2*(a^7*d*e + a^3*b^4*d*e + 2*a^5*b^2*d*e)) - ((5*a^2 + b^2)*((1
6*(e*cot(c + d*x))^(1/2)*(b^11*e^8 + 7*a^2*b^9*e^8 + 11*a^4*b^7*e^8 - 27*a^
6*b^5*e^8))/(a^10*d^4 + a^2*b^8*d^4 + 4*a^4*b^6*d^4 + 6*a^6*b^4*d^4 + 4*a^8
*b^2*d^4) - ((5*a^2 + b^2)*((16*(24*a^2*b^11*d^2*e^9 - 2*b^13*d^2*e^9 + 196
*a^4*b^9*d^2*e^9 + 120*a^6*b^7*d^2*e^9 - 50*a^8*b^5*d^2*e^9))/(a^10*d^5 + a
^2*b^8*d^5 + 4*a^4*b^6*d^5 + 6*a^6*b^4*d^5 + 4*a^8*b^2*d^5) - ((5*a^2 + b^2
)*((16*(e*cot(c + d*x))^(1/2)*(8*a*b^14*d^2*e^9 + 36*a^3*b^12*d^2*e^9 + 316
*a^5*b^10*d^2*e^9 + 552*a^7*b^8*d^2*e^9 + 256*a^9*b^6*d^2*e^9 - 12*a^11*b^4
*d^2*e^9 - 4*a^13*b^2*d^2*e^9))/(a^10*d^4 + a^2*b^8*d^4 + 4*a^4*b^6*d^4 + 6
*a^6*b^4*d^4 + 4*a^8*b^2*d^4) - ((5*a^2 + b^2)*((16*(16*a*b^16*d^4*e^10 + 1
36*a^3*b^14*d^4*e^10 + 432*a^5*b^12*d^4*e^10 + 680*a^7*b^10*d^4*e^10 + 560*
a^9*b^8*d^4*e^10 + 216*a^11*b^6*d^4*e^10 + 16*a^13*b^4*d^4*e^10 - 8*a^15*b^
2*d^4*e^10))/(a^10*d^5 + a^2*b^8*d^5 + 4*a^4*b^6*d^5 + 6*a^6*b^4*d^5 + 4*a^
8*b^2*d^5) + (8*(e*cot(c + d*x))^(1/2)*(5*a^2 + b^2)*(-a^3*b^3*e)^(1/2)*(32
*a^2*b^17*d^4*e^10 + 160*a^4*b^15*d^4*e^10 + 288*a^6*b^13*d^4*e^10 + 160*a^
8*b^11*d^4*e^10 - 160*a^10*b^9*d^4*e^10 - 288*a^12*b^7*d^4*e^10 - 160*a^14*
b^5*d^4*e^10 - 32*a^16*b^3*d^4*e^10))/((a^7*d*e + a^3*b^4*d*e + 2*a^5*b^2*d
*e)*(a^10*d^4 + a^2*b^8*d^4 + 4*a^4*b^6*d^4 + 6*a^6*b^4*d^4 + 4*a^8*b^2*d^4
)))*(-a^3*b^3*e)^(1/2))/(2*(a^7*d*e + a^3*b^4*d*e + 2*a^5*b^2*d*e)))*(-a^3*
b^3*e)^(1/2))/(2*(a^7*d*e + a^3*b^4*d*e + 2*a^5*b^2*d*e)))*(-a^3*b^3*e)^(1/
2))/(2*(a^7*d*e + a^3*b^4*d*e + 2*a^5*b^2*d*e)))*(-a^3*b^3*e)^(1/2))/(2*(a^
7*d*e + a^3*b^4*d*e + 2*a^5*b^2*d*e))))*(5*a^2 + b^2)*(-a^3*b^3*e)^(1/2)*1i
)/(a^7*d*e + a^3*b^4*d*e + 2*a^5*b^2*d*e) - (b^2*(e*cot(c + d*x))^(1/2))/(a
*(a*d*e + b*d*e*cot(c + d*x))*(a^2 + b^2))
```

3.80 $\int \frac{1}{(e\cot(c+dx))^{3/2}(a+b\cot(c+dx))^2}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 437

$$\int \frac{1}{(e\cot(c+dx))^{3/2}(a+b\cot(c+dx))^2}\,dx = \frac{b^{5/2}(7a^2+3b^2)\arctan\left(\frac{\sqrt{b}\sqrt{e\cot(c+dx)}}{\sqrt{a}\sqrt{e}}\right)}{a^{5/2}(a^2+b^2)^2 de^{3/2}} - \frac{(a^2+2ab-b^2)\arctan\left(1-\frac{\sqrt{2}\sqrt{e\cot(c+dx)}}{\sqrt{e}}\right)}{\sqrt{2}(a^2+b^2)^2 de^{3/2}} + \frac{(a^2+2ab-b^2)\arctan\left(1+\frac{\sqrt{2}\sqrt{e\cot(c+dx)}}{\sqrt{e}}\right)}{\sqrt{2}(a^2+b^2)^2 de^{3/2}} + \frac{2a^2+3b^2}{a^2(a^2+b^2)de\sqrt{e\cot(c+dx)}} - \frac{b^2}{a(a^2+b^2)de\sqrt{e\cot(c+dx)}(a+b\cot(c+dx))} + \frac{(a^2-2ab-b^2)\log\left(\sqrt{e}+\sqrt{e}\cot(c+dx)-\sqrt{2}\sqrt{e\cot(c+dx)}\right)}{2\sqrt{2}(a^2+b^2)^2 de^{3/2}} - \frac{(a^2-2ab-b^2)\log\left(\sqrt{e}+\sqrt{e}\cot(c+dx)+\sqrt{2}\sqrt{e\cot(c+dx)}\right)}{2\sqrt{2}(a^2+b^2)^2 de^{3/2}}$$

```
[Out] b^(5/2)*(7*a^2+3*b^2)*arctan(b^(1/2)*(e*cot(d*x+c))^(1/2)/a^(1/2)/e^(1/2))/
a^(5/2)/(a^2+b^2)^2/d/e^(3/2)-1/2*(a^2+2*a*b-b^2)*arctan(1-2^(1/2)*(e*cot(d
*x+c))^(1/2)/e^(1/2))/(a^2+b^2)^2/d/e^(3/2)*2^(1/2)+1/2*(a^2+2*a*b-b^2)*arc
tan(1+2^(1/2)*(e*cot(d*x+c))^(1/2)/e^(1/2))/(a^2+b^2)^2/d/e^(3/2)*2^(1/2)+1
/4*(a^2-2*a*b-b^2)*ln(e^(1/2)+cot(d*x+c)*e^(1/2)-2^(1/2)*(e*cot(d*x+c))^(1/
2))/(a^2+b^2)^2/d/e^(3/2)*2^(1/2)-1/4*(a^2-2*a*b-b^2)*ln(e^(1/2)+cot(d*x+c)
*e^(1/2)+2^(1/2)*(e*cot(d*x+c))^(1/2))/(a^2+b^2)^2/d/e^(3/2)*2^(1/2)+(2*a^2
+3*b^2)/a^2/(a^2+b^2)/d/e/(e*cot(d*x+c))^(1/2)-b^2/a/(a^2+b^2)/d/e/(a+b*cot
(d*x+c))/(e*cot(d*x+c))^(1/2)
```

Rubi [A] (verified)

Time = 1.29 (sec) , antiderivative size = 437, normalized size of antiderivative = 1.00, number of steps used = 16, number of rules used = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.520, Rules used = {3650, 3730, 3734, 3615, 1182, 1176, 631, 210, 1179, 642, 3715, 65, 211}

$$\begin{aligned}&\int \frac{1}{(e\cot(c+dx))^{3/2}(a+b\cot(c+dx))^2}\,dx=\\&-\frac{(a^2+2ab-b^2)\arctan\left(1-\frac{\sqrt{2}\sqrt{e\cot(c+dx)}}{\sqrt{e}}\right)}{\sqrt{2}de^{3/2}\left(a^2+b^2\right)^2}\\&+\frac{(a^2+2ab-b^2)\arctan\left(\frac{\sqrt{2}\sqrt{e\cot(c+dx)}}{\sqrt{e}}+1\right)}{\sqrt{2}de^{3/2}\left(a^2+b^2\right)^2}\\&+\frac{(a^2-2ab-b^2)\log\left(\sqrt{e}\cot(c+dx)-\sqrt{2}\sqrt{e\cot(c+dx)}+\sqrt{e}\right)}{2\sqrt{2}de^{3/2}\left(a^2+b^2\right)^2}\\&-\frac{(a^2-2ab-b^2)\log\left(\sqrt{e}\cot(c+dx)+\sqrt{2}\sqrt{e\cot(c+dx)}+\sqrt{e}\right)}{2\sqrt{2}de^{3/2}\left(a^2+b^2\right)^2}\\&-\frac{b^2}{ade\left(a^2+b^2\right)\sqrt{e\cot(c+dx)}(a+b\cot(c+dx))}\\&+\frac{2a^2+3b^2}{a^2de\left(a^2+b^2\right)\sqrt{e\cot(c+dx)}}+\frac{b^{5/2}(7a^2+3b^2)\arctan\left(\frac{\sqrt{b}\sqrt{e\cot(c+dx)}}{\sqrt{a}\sqrt{e}}\right)}{a^{5/2}de^{3/2}\left(a^2+b^2\right)^2}\end{aligned}$$

```
[In] Int[1/((e*Cot[c + d*x])^(3/2)*(a + b*Cot[c + d*x])^2),x]
```

```
[Out] (b^(5/2)*(7*a^2 + 3*b^2)*ArcTan[(Sqrt[b]*Sqrt[e*Cot[c + d*x]])/(Sqrt[a]*Sqr
t[e])])/(a^(5/2)*(a^2 + b^2)^2*d*e^(3/2)) - ((a^2 + 2*a*b - b^2)*ArcTan[1 -
 (Sqrt[2]*Sqrt[e*Cot[c + d*x]])/Sqrt[e]])/(Sqrt[2]*(a^2 + b^2)^2*d*e^(3/2))
 + ((a^2 + 2*a*b - b^2)*ArcTan[1 + (Sqrt[2]*Sqrt[e*Cot[c + d*x]])/Sqrt[e]])
/(Sqrt[2]*(a^2 + b^2)^2*d*e^(3/2)) + (2*a^2 + 3*b^2)/(a^2*(a^2 + b^2)*d*e*S
qrt[e*Cot[c + d*x]]) - b^2/(a*(a^2 + b^2)*d*e*Sqrt[e*Cot[c + d*x]]*(a + b*C
ot[c + d*x])) + ((a^2 - 2*a*b - b^2)*Log[Sqrt[e] + Sqrt[e]*Cot[c + d*x] - S
qrt[2]*Sqrt[e*Cot[c + d*x]]])/(2*Sqrt[2]*(a^2 + b^2)^2*d*e^(3/2)) - ((a^2 -
 2*a*b - b^2)*Log[Sqrt[e] + Sqrt[e]*Cot[c + d*x] + Sqrt[2]*Sqrt[e*Cot[c + d
*x]]])/(2*Sqrt[2]*(a^2 + b^2)^2*d*e^(3/2))
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1179

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 1182

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
a*c, 2]}, Dist[(d*q + a*e)/(2*a*c), Int[(q + c*x^2)/(a + c*x^4), x], x] + D
ist[(d*q - a*e)/(2*a*c), Int[(q - c*x^2)/(a + c*x^4), x], x]] /; FreeQ[{a,
c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0] && NeQ[c*d^2 - a*e^2, 0] && NegQ[(-a)
*c]
```

Rule 3615

```
Int[((c_) + (d_.)*tan[(e_.) + (f_.)*(x_)])/Sqrt[(b_.)*tan[(e_.) + (f_.)*(x_
)]], x_Symbol] :> Dist[2/f, Subst[Int[(b*c + d*x^2)/(b^2 + x^4), x], x, Sqr
t[b*Tan[e + f*x]]], x] /; FreeQ[{b, c, d, e, f}, x] && NeQ[c^2 - d^2, 0] &&
 NeQ[c^2 + d^2, 0]
```

Rule 3650

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)])^(n_), x_Symbol] :> Simp[b^2*(a + b*Tan[e + f*x])^(m + 1)*((c
+ d*Tan[e + f*x])^(n + 1)/(f*(m + 1)*(a^2 + b^2)*(b*c - a*d))), x] + Dist[1
/((m + 1)*(a^2 + b^2)*(b*c - a*d)), Int[(a + b*Tan[e + f*x])^(m + 1)*(c + d
*Tan[e + f*x])^n*Simp[a*(b*c - a*d)*(m + 1) - b^2*d*(m + n + 2) - b*(b*c -
a*d)*(m + 1)*Tan[e + f*x] - b^2*d*(m + n + 2)*Tan[e + f*x]^2, x], x], x] /;
 FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0]
&& NeQ[c^2 + d^2, 0] && IntegerQ[2*m] && LtQ[m, -1] && (LtQ[n, 0] || Intege
rQ[m]) &&  !(ILtQ[n, -1] && ( !IntegerQ[m] || (EqQ[c, 0] && NeQ[a, 0])))
```

Rule 3715

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*tan[(e_.)
+ (f_.)*(x_)])^(n_.)*((A_) + (C_.)*tan[(e_.) + (f_.)*(x_)]^2), x_Symbol] :>
 Dist[A/f, Subst[Int[(a + b*x)^m*(c + d*x)^n, x], x, Tan[e + f*x]], x] /; F
reeQ[{a, b, c, d, e, f, A, C, m, n}, x] && EqQ[A, C]
```

Rule 3730

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*tan[(e_.) + (f_.)*(x_)] + (C_.)*tan[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> Simp[(A*b^2 - a*(b*B - a*C))*(a + b*Tan[e +
 f*x])^(m + 1)*((c + d*Tan[e + f*x])^(n + 1)/(f*(m + 1)*(b*c - a*d)*(a^2 +
b^2))), x] + Dist[1/((m + 1)*(b*c - a*d)*(a^2 + b^2)), Int[(a + b*Tan[e + f
*x])^(m + 1)*(c + d*Tan[e + f*x])^n*Simp[A*(a*(b*c - a*d)*(m + 1) - b^2*d*(
m + n + 2)) + (b*B - a*C)*(b*c*(m + 1) + a*d*(n + 1)) - (m + 1)*(b*c - a*d)
*(A*b - a*B - b*C)*Tan[e + f*x] - d*(A*b^2 - a*(b*B - a*C))*(m + n + 2)*Tan
[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, n}, x] && NeQ[
b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] && LtQ[m, -1] &&  !
(ILtQ[n, -1] && ( !IntegerQ[m] || (EqQ[c, 0] && NeQ[a, 0])))
```

Rule 3734

```
Int[(((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_)*((A_.) + (B_.)*tan[(e_.)
+ (f_.)*(x_)] + (C_.)*tan[(e_.) + (f_.)*(x_)]^2))/((a_.) + (b_.)*tan[(e_.)
+ (f_.)*(x_)]), x_Symbol] :> Dist[1/(a^2 + b^2), Int[(c + d*Tan[e + f*x])^n
*Simp[b*B + a*(A - C) + (a*B - b*(A - C))*Tan[e + f*x], x], x], x] + Dist[(
```

```
A*b^2 - a*b*B + a^2*C)/(a^2 + b^2), Int[(c + d*Tan[e + f*x])^n*((1 + Tan[e
+ f*x]^2)/(a + b*Tan[e + f*x])), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C
, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] &&
 !GtQ[n, 0] &&  !LeQ[n, -1]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\frac{b^2}{a(a^2+b^2)de\sqrt{e\cot(c+dx)}(a+b\cot(c+dx))} - \frac{\int \frac{-\frac{1}{2}(2a^2+3b^2)e+abe\cot(c+dx)-\frac{3}{2}b^2e\cot^2(c+dx)}{(e\cot(c+dx))^{3/2}(a+b\cot(c+dx))}\,dx}{a(a^2+b^2)e}\\
&= \frac{2a^2+3b^2}{a^2(a^2+b^2)de\sqrt{e\cot(c+dx)}} - \frac{b^2}{a(a^2+b^2)de\sqrt{e\cot(c+dx)}(a+b\cot(c+dx))}\\
&\quad - \frac{2\int \frac{\frac{1}{4}b(4a^2+3b^2)e^3+\frac{1}{2}a^3e^3\cot(c+dx)+\frac{1}{4}b(2a^2+3b^2)e^3\cot^2(c+dx)}{\sqrt{e\cot(c+dx)}(a+b\cot(c+dx))}\,dx}{a^2(a^2+b^2)e^4}\\
&= \frac{2a^2+3b^2}{a^2(a^2+b^2)de\sqrt{e\cot(c+dx)}} - \frac{b^2}{a(a^2+b^2)de\sqrt{e\cot(c+dx)}(a+b\cot(c+dx))}\\
&\quad - \frac{2\int \frac{a^3be^3+\frac{1}{2}a^2(a^2-b^2)e^3\cot(c+dx)}{\sqrt{e\cot(c+dx)}}\,dx}{a^2(a^2+b^2)^2e^4} - \frac{(b^3(7a^2+3b^2))\int \frac{1+\cot^2(c+dx)}{\sqrt{e\cot(c+dx)}(a+b\cot(c+dx))}\,dx}{2a^2(a^2+b^2)^2e}\\
&= \frac{2a^2+3b^2}{a^2(a^2+b^2)de\sqrt{e\cot(c+dx)}} - \frac{b^2}{a(a^2+b^2)de\sqrt{e\cot(c+dx)}(a+b\cot(c+dx))}\\
&\quad - \frac{4\text{Subst}\left(\int \frac{-a^3be^4-\frac{1}{2}a^2(a^2-b^2)e^3x^2}{e^2+x^4}\,dx, x, \sqrt{e\cot(c+dx)}\right)}{a^2(a^2+b^2)^2de^4}\\
&\quad - \frac{(b^3(7a^2+3b^2))\,\text{Subst}\left(\int \frac{1}{\sqrt{-ex}(a-bx)}\,dx, x, -\cot(c+dx)\right)}{2a^2(a^2+b^2)^2de}\\
&= \frac{2a^2+3b^2}{a^2(a^2+b^2)de\sqrt{e\cot(c+dx)}} - \frac{b^2}{a(a^2+b^2)de\sqrt{e\cot(c+dx)}(a+b\cot(c+dx))}\\
&\quad + \frac{(b^3(7a^2+3b^2))\,\text{Subst}\left(\int \frac{1}{a+\frac{bx^2}{e}}\,dx, x, \sqrt{e\cot(c+dx)}\right)}{a^2(a^2+b^2)^2de^2}\\
&\quad - \frac{(a^2-2ab-b^2)\,\text{Subst}\left(\int \frac{e-x^2}{e^2+x^4}\,dx, x, \sqrt{e\cot(c+dx)}\right)}{(a^2+b^2)^2de}\\
&\quad + \frac{(a^2+2ab-b^2)\,\text{Subst}\left(\int \frac{e+x^2}{e^2+x^4}\,dx, x, \sqrt{e\cot(c+dx)}\right)}{(a^2+b^2)^2de}
\end{aligned}$$

$$= \frac{b^{5/2}(7a^2+3b^2)\arctan\left(\frac{\sqrt{b}\sqrt{e\cot(c+dx)}}{\sqrt{a}\sqrt{e}}\right)}{a^{5/2}\left(a^2+b^2\right)^2 de^{3/2}} + \frac{2a^2+3b^2}{a^2\left(a^2+b^2\right) de\sqrt{e\cot(c+dx)}} - \frac{b^2}{a\left(a^2+b^2\right) de\sqrt{e\cot(c+dx)}(a+b\cot(c+dx))} + \frac{(a^2-2ab-b^2)\,\text{Subst}\left(\int \frac{\sqrt{2}\sqrt{e}+2x}{-e-\sqrt{2}\sqrt{e}x-x^2}\,dx, x, \sqrt{e\cot(c+dx)}\right)}{2\sqrt{2}\left(a^2+b^2\right)^2 de^{3/2}} + \frac{(a^2-2ab-b^2)\,\text{Subst}\left(\int \frac{\sqrt{2}\sqrt{e}-2x}{-e+\sqrt{2}\sqrt{e}x-x^2}\,dx, x, \sqrt{e\cot(c+dx)}\right)}{2\sqrt{2}\left(a^2+b^2\right)^2 de^{3/2}} + \frac{(a^2+2ab-b^2)\,\text{Subst}\left(\int \frac{1}{e-\sqrt{2}\sqrt{e}x+x^2}\,dx, x, \sqrt{e\cot(c+dx)}\right)}{2\left(a^2+b^2\right)^2 de} + \frac{(a^2+2ab-b^2)\,\text{Subst}\left(\int \frac{1}{e+\sqrt{2}\sqrt{e}x+x^2}\,dx, x, \sqrt{e\cot(c+dx)}\right)}{2\left(a^2+b^2\right)^2 de}$$

$$= \frac{b^{5/2}(7a^2+3b^2)\arctan\left(\frac{\sqrt{b}\sqrt{e\cot(c+dx)}}{\sqrt{a}\sqrt{e}}\right)}{a^{5/2}\left(a^2+b^2\right)^2 de^{3/2}} + \frac{2a^2+3b^2}{a^2\left(a^2+b^2\right) de\sqrt{e\cot(c+dx)}} - \frac{b^2}{a\left(a^2+b^2\right) de\sqrt{e\cot(c+dx)}(a+b\cot(c+dx))} + \frac{(a^2-2ab-b^2)\log\left(\sqrt{e}+\sqrt{e}\cot(c+dx)-\sqrt{2}\sqrt{e\cot(c+dx)}\right)}{2\sqrt{2}\left(a^2+b^2\right)^2 de^{3/2}} - \frac{(a^2-2ab-b^2)\log\left(\sqrt{e}+\sqrt{e}\cot(c+dx)+\sqrt{2}\sqrt{e\cot(c+dx)}\right)}{2\sqrt{2}\left(a^2+b^2\right)^2 de^{3/2}} + \frac{(a^2+2ab-b^2)\,\text{Subst}\left(\int \frac{1}{-1-x^2}\,dx, x, 1-\frac{\sqrt{2}\sqrt{e\cot(c+dx)}}{\sqrt{e}}\right)}{\sqrt{2}\left(a^2+b^2\right)^2 de^{3/2}} - \frac{(a^2+2ab-b^2)\,\text{Subst}\left(\int \frac{1}{-1-x^2}\,dx, x, 1+\frac{\sqrt{2}\sqrt{e\cot(c+dx)}}{\sqrt{e}}\right)}{\sqrt{2}\left(a^2+b^2\right)^2 de^{3/2}}$$

$$
\begin{aligned}
&= \frac{b^{5/2}(7a^2+3b^2)\arctan\left(\frac{\sqrt{b}\sqrt{e\cot(c+dx)}}{\sqrt{a}\sqrt{e}}\right)}{a^{5/2}\,(a^2+b^2)^2\,de^{3/2}} \\
&\quad - \frac{(a^2+2ab-b^2)\arctan\left(1-\frac{\sqrt{2}\sqrt{e\cot(c+dx)}}{\sqrt{e}}\right)}{\sqrt{2}\,(a^2+b^2)^2\,de^{3/2}} \\
&\quad + \frac{(a^2+2ab-b^2)\arctan\left(1+\frac{\sqrt{2}\sqrt{e\cot(c+dx)}}{\sqrt{e}}\right)}{\sqrt{2}\,(a^2+b^2)^2\,de^{3/2}} + \frac{2a^2+3b^2}{a^2\,(a^2+b^2)\,de\sqrt{e\cot(c+dx)}} \\
&\quad - \frac{b^2}{a\,(a^2+b^2)\,de\sqrt{e\cot(c+dx)}(a+b\cot(c+dx))} \\
&\quad + \frac{(a^2-2ab-b^2)\log\left(\sqrt{e}+\sqrt{e}\cot(c+dx)-\sqrt{2}\sqrt{e\cot(c+dx)}\right)}{2\sqrt{2}\,(a^2+b^2)^2\,de^{3/2}} \\
&\quad - \frac{(a^2-2ab-b^2)\log\left(\sqrt{e}+\sqrt{e}\cot(c+dx)+\sqrt{2}\sqrt{e\cot(c+dx)}\right)}{2\sqrt{2}\,(a^2+b^2)^2\,de^{3/2}}
\end{aligned}
$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 0.61 (sec) , antiderivative size = 244, normalized size of antiderivative = 0.56

$$
\int \frac{1}{(e\cot(c+dx))^{3/2}(a+b\cot(c+dx))^2}\,dx = \frac{8a^2b^2\,\text{Hypergeometric2F1}\left(-\frac{1}{2},1,\frac{1}{2},-\frac{b\cot(c+dx)}{a}\right)+4b^2(a^2+}{}
$$

```
[In] Integrate[1/((e*Cot[c + d*x])^(3/2)*(a + b*Cot[c + d*x])^2),x]
```

```
[Out] (8*a^2*b^2*Hypergeometric2F1[-1/2, 1, 1/2, -((b*Cot[c + d*x])/a)] + 4*b^2*(
a^2 + b^2)*Hypergeometric2F1[-1/2, 2, 1/2, -((b*Cot[c + d*x])/a)] + a^2*(4*
(a^2 - b^2)*Hypergeometric2F1[-1/4, 1, 3/4, -Cot[c + d*x]^2] + Sqrt[2]*a*b*
Sqrt[Cot[c + d*x]]*(-2*ArcTan[1 - Sqrt[2]*Sqrt[Cot[c + d*x]]] + 2*ArcTan[1
+ Sqrt[2]*Sqrt[Cot[c + d*x]]] - Log[1 - Sqrt[2]*Sqrt[Cot[c + d*x]] + Cot[c
+ d*x]] + Log[1 + Sqrt[2]*Sqrt[Cot[c + d*x]] + Cot[c + d*x]])))/(2*a^2*(a^2
 + b^2)^2*d*e*Sqrt[e*Cot[c + d*x]])
```

Maple [A] (verified)

Time = 0.05 (sec) , antiderivative size = 414, normalized size of antiderivative = 0.95

method	result
derivativedivides	$-\frac{2e^3\left(-\frac{b^3\left(\frac{\left(\frac{a^2}{2}+\frac{b^2}{2}\right)\sqrt{e\cot(dx+c)}}{e\cot(dx+c)b+ae}+\frac{(7a^2+3b^2)\arctan\left(\frac{\sqrt{e\cot(dx+c)}\,b}{\sqrt{aeb}}\right)}{2\sqrt{aeb}}\right)}{a^2e^4(a^2+b^2)^2}+\frac{-\frac{ab(e^2)^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\frac{e\cot(dx+c)+(e^2)^{\frac{1}{4}}\sqrt{e\cot(d\cdots}}{e\cot(dx+c)-(e^2)^{\frac{1}{4}}\sqrt{e\cot(d\cdots}}\right.\right.}{\cdots}}{\cdots}\right.}{\cdots}$
default	$-\frac{2e^3\left(-\frac{b^3\left(\frac{\left(\frac{a^2}{2}+\frac{b^2}{2}\right)\sqrt{e\cot(dx+c)}}{e\cot(dx+c)b+ae}+\frac{(7a^2+3b^2)\arctan\left(\frac{\sqrt{e\cot(dx+c)}\,b}{\sqrt{aeb}}\right)}{2\sqrt{aeb}}\right)}{a^2e^4(a^2+b^2)^2}+\frac{-\frac{ab(e^2)^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\frac{e\cot(dx+c)+(e^2)^{\frac{1}{4}}\sqrt{e\cot(d\cdots}}{e\cot(dx+c)-(e^2)^{\frac{1}{4}}\sqrt{e\cot(d\cdots}}\right.\right.}{\cdots}}{\cdots}\right.}{\cdots}$

```
[In] int(1/(e*cot(d*x+c))^(3/2)/(a+b*cot(d*x+c))^2,x,method=_RETURNVERBOSE)
[Out] -2/d*e^3*(-b^3/a^2/e^4/(a^2+b^2)^2*((1/2*a^2+1/2*b^2)*(e*cot(d*x+c))^(1/2)/
(e*cot(d*x+c)*b+a*e)+1/2*(7*a^2+3*b^2)/(a*e*b)^(1/2)*arctan((e*cot(d*x+c))^
(1/2)*b/(a*e*b)^(1/2)))+1/(a^2+b^2)^2/e^4*(-1/4*a/e*b*(e^2)^(1/4)*2^(1/2)*(
ln((e*cot(d*x+c)+(e^2)^(1/4)*(e*cot(d*x+c))^(1/2)*2^(1/2)+(e^2)^(1/2))/(e*c
ot(d*x+c)-(e^2)^(1/4)*(e*cot(d*x+c))^(1/2)*2^(1/2)+(e^2)^(1/2)))+2*arctan(2
^(1/2)/(e^2)^(1/4)*(e*cot(d*x+c))^(1/2)+1)-2*arctan(-2^(1/2)/(e^2)^(1/4)*(e
*cot(d*x+c))^(1/2)+1))+1/8*(-a^2+b^2)/(e^2)^(1/4)*2^(1/2)*(ln((e*cot(d*x+c)
-(e^2)^(1/4)*(e*cot(d*x+c))^(1/2)*2^(1/2)+(e^2)^(1/2))/(e*cot(d*x+c)+(e^2)^
(1/4)*(e*cot(d*x+c))^(1/2)*2^(1/2)+(e^2)^(1/2)))+2*arctan(2^(1/2)/(e^2)^(1/
4)*(e*cot(d*x+c))^(1/2)+1)-2*arctan(-2^(1/2)/(e^2)^(1/4)*(e*cot(d*x+c))^(1/
2)+1)))-1/a^2/e^4/(e*cot(d*x+c))^(1/2))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 3239 vs. $2(372) = 744$.

Time = 0.72 (sec) , antiderivative size = 6519, normalized size of antiderivative = 14.92

$$\int \frac{1}{(e\cot(c+dx))^{3/2}(a+b\cot(c+dx))^2}\,dx = \text{Too large to display}$$

```
[In] integrate(1/(e*cot(d*x+c))^(3/2)/(a+b*cot(d*x+c))^2,x, algorithm="fricas")
[Out] Too large to include
```

Sympy [F]

$$\int \frac{1}{(e\cot(c+dx))^{3/2}(a+b\cot(c+dx))^2}\,dx = \int \frac{1}{(e\cot{(c+dx)})^{\frac{3}{2}}\,(a+b\cot{(c+dx)})^2}\,dx$$

```
[In] integrate(1/(e*cot(d*x+c))**(3/2)/(a+b*cot(d*x+c))**2,x)
```

```
[Out] Integral(1/((e*cot(c + d*x))**(3/2)*(a + b*cot(c + d*x))**2), x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{1}{(e\cot(c+dx))^{3/2}(a+b\cot(c+dx))^2}\,dx = \text{Exception raised: ValueError}$$

```
[In] integrate(1/(e*cot(d*x+c))^(3/2)/(a+b*cot(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(e>0)', see `assume?` for more detai
ls)Is e
```

Giac [F(-1)]

Timed out.

$$\int \frac{1}{(e\cot(c+dx))^{3/2}(a+b\cot(c+dx))^2}\,dx = \text{Timed out}$$

```
[In] integrate(1/(e*cot(d*x+c))^(3/2)/(a+b*cot(d*x+c))^2,x, algorithm="giac")
```

```
[Out] Timed out
```

Mupad [B] (verification not implemented)

Time = 16.75 (sec) , antiderivative size = 15251, normalized size of antiderivative = 34.90

$$\int \frac{1}{(e\cot(c+dx))^{3/2}(a+b\cot(c+dx))^2}\,dx = \text{Too large to display}$$

```
[In] int(1/((e*cot(c + d*x))^(3/2)*(a + b*cot(c + d*x))^2),x)
```

```
[Out] (2/a + (b*cot(c + d*x)*(2*a^2 + 3*b^2))/(a^2*(a^2 + b^2)))/(b*d*(e*cot(c +
d*x))^(3/2) + a*d*e*(e*cot(c + d*x))^(1/2)) - atan((((e*cot(c + d*x))^(1/2)
```

```
*(144*a^14*b^23*d^5*e^13 + 1248*a^16*b^21*d^5*e^13 + 4224*a^18*b^19*d^5*e^1
3 + 6720*a^20*b^17*d^5*e^13 + 3872*a^22*b^15*d^5*e^13 - 2816*a^24*b^13*d^5*
e^13 - 5632*a^26*b^11*d^5*e^13 - 3136*a^28*b^9*d^5*e^13 - 560*a^30*b^7*d^5*
e^13 + 32*a^32*b^5*d^5*e^13) + (1i/(4*(a^4*d^2*e^3 + b^4*d^2*e^3 + a*b^3*d^
2*e^3*4i - a^3*b*d^2*e^3*4i - 6*a^2*b^2*d^2*e^3)))^(1/2)*(26496*a^25*b^14*d
^6*e^15 - 1152*a^15*b^24*d^6*e^15 - 8448*a^17*b^22*d^6*e^15 - 23776*a^19*b^
20*d^6*e^15 - 29664*a^21*b^18*d^6*e^15 - 6528*a^23*b^16*d^6*e^15 - ((e*cot(
c + d*x))^(1/2)*(1152*a^15*b^26*d^7*e^16 + 13440*a^17*b^24*d^7*e^16 + 69056
*a^19*b^22*d^7*e^16 + 202752*a^21*b^20*d^7*e^16 + 372800*a^23*b^18*d^7*e^16
 + 443136*a^25*b^16*d^7*e^16 + 337792*a^27*b^14*d^7*e^16 + 156160*a^29*b^12
*d^7*e^16 + 37632*a^31*b^10*d^7*e^16 + 3200*a^33*b^8*d^7*e^16 + 704*a^35*b^
6*d^7*e^16 + 512*a^37*b^4*d^7*e^16 + 64*a^39*b^2*d^7*e^16) + (1i/(4*(a^4*d^
2*e^3 + b^4*d^2*e^3 + a*b^3*d^2*e^3*4i - a^3*b*d^2*e^3*4i - 6*a^2*b^2*d^2*e
^3)))^(1/2)*(768*a^16*b^27*d^8*e^18 - (e*cot(c + d*x))^(1/2)*(1i/(4*(a^4*d^
2*e^3 + b^4*d^2*e^3 + a*b^3*d^2*e^3*4i - a^3*b*d^2*e^3*4i - 6*a^2*b^2*d^2*e
^3)))^(1/2)*(512*a^18*b^27*d^9*e^19 + 5120*a^20*b^25*d^9*e^19 + 22528*a^22*
b^23*d^9*e^19 + 56320*a^24*b^21*d^9*e^19 + 84480*a^26*b^19*d^9*e^19 + 67584
*a^28*b^17*d^9*e^19 - 67584*a^32*b^13*d^9*e^19 - 84480*a^34*b^11*d^9*e^19 -
 56320*a^36*b^9*d^9*e^19 - 22528*a^38*b^7*d^9*e^19 - 5120*a^40*b^5*d^9*e^19
 - 512*a^42*b^3*d^9*e^19) + 8704*a^18*b^25*d^8*e^18 + 44288*a^20*b^23*d^8*e
^18 + 133120*a^22*b^21*d^8*e^18 + 261120*a^24*b^19*d^8*e^18 + 347136*a^26*b
^17*d^8*e^18 + 311808*a^28*b^15*d^8*e^18 + 178176*a^30*b^13*d^8*e^18 + 4992
0*a^32*b^11*d^8*e^18 - 7680*a^34*b^9*d^8*e^18 - 12032*a^36*b^7*d^8*e^18 - 4
096*a^38*b^5*d^8*e^18 - 512*a^40*b^3*d^8*e^18))*(1i/(4*(a^4*d^2*e^3 + b^4*d
^2*e^3 + a*b^3*d^2*e^3*4i - a^3*b*d^2*e^3*4i - 6*a^2*b^2*d^2*e^3)))^(1/2) +
 33984*a^27*b^12*d^6*e^15 + 18624*a^29*b^10*d^6*e^15 + 5376*a^31*b^8*d^6*e^
15 + 1152*a^33*b^6*d^6*e^15 + 288*a^35*b^4*d^6*e^15 + 32*a^37*b^2*d^6*e^15)
)*(1i/(4*(a^4*d^2*e^3 + b^4*d^2*e^3 + a*b^3*d^2*e^3*4i - a^3*b*d^2*e^3*4i -
 6*a^2*b^2*d^2*e^3)))^(1/2)*1i + ((e*cot(c + d*x))^(1/2)*(144*a^14*b^23*d^5
*e^13 + 1248*a^16*b^21*d^5*e^13 + 4224*a^18*b^19*d^5*e^13 + 6720*a^20*b^17*
d^5*e^13 + 3872*a^22*b^15*d^5*e^13 - 2816*a^24*b^13*d^5*e^13 - 5632*a^26*b^
11*d^5*e^13 - 3136*a^28*b^9*d^5*e^13 - 560*a^30*b^7*d^5*e^13 + 32*a^32*b^5*
d^5*e^13) - (1i/(4*(a^4*d^2*e^3 + b^4*d^2*e^3 + a*b^3*d^2*e^3*4i - a^3*b*d^
2*e^3*4i - 6*a^2*b^2*d^2*e^3)))^(1/2)*(((e*cot(c + d*x))^(1/2)*(1152*a^15*b
^26*d^7*e^16 + 13440*a^17*b^24*d^7*e^16 + 69056*a^19*b^22*d^7*e^16 + 202752
*a^21*b^20*d^7*e^16 + 372800*a^23*b^18*d^7*e^16 + 443136*a^25*b^16*d^7*e^16
 + 337792*a^27*b^14*d^7*e^16 + 156160*a^29*b^12*d^7*e^16 + 37632*a^31*b^10*
d^7*e^16 + 3200*a^33*b^8*d^7*e^16 + 704*a^35*b^6*d^7*e^16 + 512*a^37*b^4*d^
7*e^16 + 64*a^39*b^2*d^7*e^16) - (1i/(4*(a^4*d^2*e^3 + b^4*d^2*e^3 + a*b^3*
d^2*e^3*4i - a^3*b*d^2*e^3*4i - 6*a^2*b^2*d^2*e^3)))^(1/2)*((e*cot(c + d*x)
)^(1/2)*(1i/(4*(a^4*d^2*e^3 + b^4*d^2*e^3 + a*b^3*d^2*e^3*4i - a^3*b*d^2*e^
3*4i - 6*a^2*b^2*d^2*e^3)))^(1/2)*(512*a^18*b^27*d^9*e^19 + 5120*a^20*b^25*
d^9*e^19 + 22528*a^22*b^23*d^9*e^19 + 56320*a^24*b^21*d^9*e^19 + 84480*a^26
*b^19*d^9*e^19 + 67584*a^28*b^17*d^9*e^19 - 67584*a^32*b^13*d^9*e^19 - 8448
0*a^34*b^11*d^9*e^19 - 56320*a^36*b^9*d^9*e^19 - 22528*a^38*b^7*d^9*e^19 -
```

5120*a^40*b^5*d^9*e^19 - 512*a^42*b^3*d^9*e^19) + 768*a^16*b^27*d^8*e^18 + 8704*a^18*b^25*d^8*e^18 + 44288*a^20*b^23*d^8*e^18 + 133120*a^22*b^21*d^8*e^18 + 261120*a^24*b^19*d^8*e^18 + 347136*a^26*b^17*d^8*e^18 + 311808*a^28*b^15*d^8*e^18 + 178176*a^30*b^13*d^8*e^18 + 49920*a^32*b^11*d^8*e^18 - 7680*a^34*b^9*d^8*e^18 - 12032*a^36*b^7*d^8*e^18 - 4096*a^38*b^5*d^8*e^18 - 512*a^40*b^3*d^8*e^18))*(1i/(4*(a^4*d^2*e^3 + b^4*d^2*e^3 + a*b^3*d^2*e^3*4i - a^3*b*d^2*e^3*4i - 6*a^2*b^2*d^2*e^3)))^(1/2) - 1152*a^15*b^24*d^6*e^15 - 8448*a^17*b^22*d^6*e^15 - 23776*a^19*b^20*d^6*e^15 - 29664*a^21*b^18*d^6*e^15 - 6528*a^23*b^16*d^6*e^15 + 26496*a^25*b^14*d^6*e^15 + 33984*a^27*b^12*d^6*e^15 + 18624*a^29*b^10*d^6*e^15 + 5376*a^31*b^8*d^6*e^15 + 1152*a^33*b^6*d^6*e^15 + 288*a^35*b^4*d^6*e^15 + 32*a^37*b^2*d^6*e^15))*(1i/(4*(a^4*d^2*e^3 + b^4*d^2*e^3 + a*b^3*d^2*e^3*4i - a^3*b*d^2*e^3*4i - 6*a^2*b^2*d^2*e^3)))^(1/2)*1i)/(((e*cot(c + d*x))^(1/2)*(144*a^14*b^23*d^5*e^13 + 1248*a^16*b^21*d^5*e^13 + 4224*a^18*b^19*d^5*e^13 + 6720*a^20*b^17*d^5*e^13 + 3872*a^22*b^15*d^5*e^13 - 2816*a^24*b^13*d^5*e^13 - 5632*a^26*b^11*d^5*e^13 - 3136*a^28*b^9*d^5*e^13 - 560*a^30*b^7*d^5*e^13 + 32*a^32*b^5*d^5*e^13) + (1i/(4*(a^4*d^2*e^3 + b^4*d^2*e^3 + a*b^3*d^2*e^3*4i - a^3*b*d^2*e^3*4i - 6*a^2*b^2*d^2*e^3)))^(1/2)*(26496*a^25*b^14*d^6*e^15 - 1152*a^15*b^24*d^6*e^15 - 8448*a^17*b^22*d^6*e^15 - 23776*a^19*b^20*d^6*e^15 - 29664*a^21*b^18*d^6*e^15 - 6528*a^23*b^16*d^6*e^15 - ((e*cot(c + d*x))^(1/2)*(1152*a^15*b^26*d^7*e^16 + 13440*a^17*b^24*d^7*e^16 + 69056*a^19*b^22*d^7*e^16 + 202752*a^21*b^20*d^7*e^16 + 372800*a^23*b^18*d^7*e^16 + 443136*a^25*b^16*d^7*e^16 + 337792*a^27*b^14*d^7*e^16 + 156160*a^29*b^12*d^7*e^16 + 37632*a^31*b^10*d^7*e^16 + 3200*a^33*b^8*d^7*e^16 + 704*a^35*b^6*d^7*e^16 + 512*a^37*b^4*d^7*e^16 + 64*a^39*b^2*d^7*e^16) + (1i/(4*(a^4*d^2*e^3 + b^4*d^2*e^3 + a*b^3*d^2*e^3*4i - a^3*b*d^2*e^3*4i - 6*a^2*b^2*d^2*e^3)))^(1/2)*(768*a^16*b^27*d^8*e^18 - (e*cot(c + d*x))^(1/2)*(1i/(4*(a^4*d^2*e^3 + b^4*d^2*e^3 + a*b^3*d^2*e^3*4i - a^3*b*d^2*e^3*4i - 6*a^2*b^2*d^2*e^3)))^(1/2)*(512*a^18*b^27*d^9*e^19 + 5120*a^20*b^25*d^9*e^19 + 22528*a^22*b^23*d^9*e^19 + 56320*a^24*b^21*d^9*e^19 + 84480*a^26*b^19*d^9*e^19 + 67584*a^28*b^17*d^9*e^19 - 67584*a^32*b^13*d^9*e^19 - 84480*a^34*b^11*d^9*e^19 - 56320*a^36*b^9*d^9*e^19 - 22528*a^38*b^7*d^9*e^19 - 5120*a^40*b^5*d^9*e^19 - 512*a^42*b^3*d^9*e^19) + 8704*a^18*b^25*d^8*e^18 + 44288*a^20*b^23*d^8*e^18 + 133120*a^22*b^21*d^8*e^18 + 261120*a^24*b^19*d^8*e^18 + 347136*a^26*b^17*d^8*e^18 + 311808*a^28*b^15*d^8*e^18 + 178176*a^30*b^13*d^8*e^18 + 49920*a^32*b^11*d^8*e^18 - 7680*a^34*b^9*d^8*e^18 - 12032*a^36*b^7*d^8*e^18 - 4096*a^38*b^5*d^8*e^18 - 512*a^40*b^3*d^8*e^18))*(1i/(4*(a^4*d^2*e^3 + b^4*d^2*e^3 + a*b^3*d^2*e^3*4i - a^3*b*d^2*e^3*4i - 6*a^2*b^2*d^2*e^3)))^(1/2) + 33984*a^27*b^12*d^6*e^15 + 18624*a^29*b^10*d^6*e^15 + 5376*a^31*b^8*d^6*e^15 + 1152*a^33*b^6*d^6*e^15 + 288*a^35*b^4*d^6*e^15 + 32*a^37*b^2*d^6*e^15))*(1i/(4*(a^4*d^2*e^3 + b^4*d^2*e^3 + a*b^3*d^2*e^3*4i - a^3*b*d^2*e^3*4i - 6*a^2*b^2*d^2*e^3)))^(1/2) - ((e*cot(c + d*x))^(1/2)*(144*a^14*b^23*d^5*e^13 + 1248*a^16*b^21*d^5*e^13 + 4224*a^18*b^19*d^5*e^13 + 6720*a^20*b^17*d^5*e^13 + 3872*a^22*b^15*d^5*e^13 - 2816*a^24*b^13*d^5*e^13 - 5632*a^26*b^11*d^5*e^13 - 3136*a^28*b^9*d^5*e^13 - 560*a^30*b^7*d^5*e^13 + 32*a^32*b^5*d^5*e^13) - (1i/(4*(a^4*d^2*e^3 + b^4*d^2

```
*e^3 + a*b^3*d^2*e^3*4i - a^3*b*d^2*e^3*4i - 6*a^2*b^2*d^2*e^3)))^(1/2)*(((
e*cot(c + d*x))^(1/2)*(1152*a^15*b^26*d^7*e^16 + 13440*a^17*b^24*d^7*e^16 +
 69056*a^19*b^22*d^7*e^16 + 202752*a^21*b^20*d^7*e^16 + 372800*a^23*b^18*d^
7*e^16 + 443136*a^25*b^16*d^7*e^16 + 337792*a^27*b^14*d^7*e^16 + 156160*a^2
9*b^12*d^7*e^16 + 37632*a^31*b^10*d^7*e^16 + 3200*a^33*b^8*d^7*e^16 + 704*a
^35*b^6*d^7*e^16 + 512*a^37*b^4*d^7*e^16 + 64*a^39*b^2*d^7*e^16) - (1i/(4*(
a^4*d^2*e^3 + b^4*d^2*e^3 + a*b^3*d^2*e^3*4i - a^3*b*d^2*e^3*4i - 6*a^2*b^2
*d^2*e^3)))^(1/2)*((e*cot(c + d*x))^(1/2)*(1i/(4*(a^4*d^2*e^3 + b^4*d^2*e^3
 + a*b^3*d^2*e^3*4i - a^3*b*d^2*e^3*4i - 6*a^2*b^2*d^2*e^3)))^(1/2)*(512*a^
18*b^27*d^9*e^19 + 5120*a^20*b^25*d^9*e^19 + 22528*a^22*b^23*d^9*e^19 + 563
20*a^24*b^21*d^9*e^19 + 84480*a^26*b^19*d^9*e^19 + 67584*a^28*b^17*d^9*e^19
 - 67584*a^32*b^13*d^9*e^19 - 84480*a^34*b^11*d^9*e^19 - 56320*a^36*b^9*d^9
*e^19 - 22528*a^38*b^7*d^9*e^19 - 5120*a^40*b^5*d^9*e^19 - 512*a^42*b^3*d^9
*e^19) + 768*a^16*b^27*d^8*e^18 + 8704*a^18*b^25*d^8*e^18 + 44288*a^20*b^23
*d^8*e^18 + 133120*a^22*b^21*d^8*e^18 + 261120*a^24*b^19*d^8*e^18 + 347136*
a^26*b^17*d^8*e^18 + 311808*a^28*b^15*d^8*e^18 + 178176*a^30*b^13*d^8*e^18
+ 49920*a^32*b^11*d^8*e^18 - 7680*a^34*b^9*d^8*e^18 - 12032*a^36*b^7*d^8*e^
18 - 4096*a^38*b^5*d^8*e^18 - 512*a^40*b^3*d^8*e^18))*(1i/(4*(a^4*d^2*e^3 +
 b^4*d^2*e^3 + a*b^3*d^2*e^3*4i - a^3*b*d^2*e^3*4i - 6*a^2*b^2*d^2*e^3)))^(
1/2) - 1152*a^15*b^24*d^6*e^15 - 8448*a^17*b^22*d^6*e^15 - 23776*a^19*b^20*
d^6*e^15 - 29664*a^21*b^18*d^6*e^15 - 6528*a^23*b^16*d^6*e^15 + 26496*a^25*
b^14*d^6*e^15 + 33984*a^27*b^12*d^6*e^15 + 18624*a^29*b^10*d^6*e^15 + 5376*
a^31*b^8*d^6*e^15 + 1152*a^33*b^6*d^6*e^15 + 288*a^35*b^4*d^6*e^15 + 32*a^3
7*b^2*d^6*e^15))*(1i/(4*(a^4*d^2*e^3 + b^4*d^2*e^3 + a*b^3*d^2*e^3*4i - a^3
*b*d^2*e^3*4i - 6*a^2*b^2*d^2*e^3)))^(1/2) + 144*a^14*b^21*d^4*e^12 + 1296*
a^16*b^19*d^4*e^12 + 4880*a^18*b^17*d^4*e^12 + 10000*a^20*b^15*d^4*e^12 + 1
2080*a^22*b^13*d^4*e^12 + 8624*a^24*b^11*d^4*e^12 + 3376*a^26*b^9*d^4*e^12
+ 560*a^28*b^7*d^4*e^12))*(1i/(4*(a^4*d^2*e^3 + b^4*d^2*e^3 + a*b^3*d^2*e^3
*4i - a^3*b*d^2*e^3*4i - 6*a^2*b^2*d^2*e^3)))^(1/2)*2i - atan(((1/(a^4*d^2*
e^3*1i + b^4*d^2*e^3*1i + 4*a*b^3*d^2*e^3 - 4*a^3*b*d^2*e^3 - a^2*b^2*d^2*e
^3*6i))^(1/2)*(((e*cot(c + d*x))^(1/2)*(144*a^14*b^23*d^5*e^13 + 1248*a^16*
b^21*d^5*e^13 + 4224*a^18*b^19*d^5*e^13 + 6720*a^20*b^17*d^5*e^13 + 3872*a^
22*b^15*d^5*e^13 - 2816*a^24*b^13*d^5*e^13 - 5632*a^26*b^11*d^5*e^13 - 3136
*a^28*b^9*d^5*e^13 - 560*a^30*b^7*d^5*e^13 + 32*a^32*b^5*d^5*e^13))/2 + ((1
/(a^4*d^2*e^3*1i + b^4*d^2*e^3*1i + 4*a*b^3*d^2*e^3 - 4*a^3*b*d^2*e^3 - a^2
*b^2*d^2*e^3*6i))^(1/2)*(13248*a^25*b^14*d^6*e^15 - 576*a^15*b^24*d^6*e^15
- 4224*a^17*b^22*d^6*e^15 - 11888*a^19*b^20*d^6*e^15 - 14832*a^21*b^18*d^6*
e^15 - 3264*a^23*b^16*d^6*e^15 - ((((1/(a^4*d^2*e^3*1i + b^4*d^2*e^3*1i + 4
*a*b^3*d^2*e^3 - 4*a^3*b*d^2*e^3 - a^2*b^2*d^2*e^3*6i))^(1/2)*(384*a^16*b^2
7*d^8*e^18 - ((e*cot(c + d*x))^(1/2)*(1/(a^4*d^2*e^3*1i + b^4*d^2*e^3*1i +
4*a*b^3*d^2*e^3 - 4*a^3*b*d^2*e^3 - a^2*b^2*d^2*e^3*6i))^(1/2)*(512*a^18*b^
27*d^9*e^19 + 5120*a^20*b^25*d^9*e^19 + 22528*a^22*b^23*d^9*e^19 + 56320*a^
24*b^21*d^9*e^19 + 84480*a^26*b^19*d^9*e^19 + 67584*a^28*b^17*d^9*e^19 - 67
584*a^32*b^13*d^9*e^19 - 84480*a^34*b^11*d^9*e^19 - 56320*a^36*b^9*d^9*e^19
 - 22528*a^38*b^7*d^9*e^19 - 5120*a^40*b^5*d^9*e^19 - 512*a^42*b^3*d^9*e^19
```

))/4 + 4352*a^18*b^25*d^8*e^18 + 22144*a^20*b^23*d^8*e^18 + 66560*a^22*b^21
*d^8*e^18 + 130560*a^24*b^19*d^8*e^18 + 173568*a^26*b^17*d^8*e^18 + 155904*
a^28*b^15*d^8*e^18 + 89088*a^30*b^13*d^8*e^18 + 24960*a^32*b^11*d^8*e^18 -
3840*a^34*b^9*d^8*e^18 - 6016*a^36*b^7*d^8*e^18 - 2048*a^38*b^5*d^8*e^18 -
256*a^40*b^3*d^8*e^18))/2 + ((e*cot(c + d*x))^(1/2)*(1152*a^15*b^26*d^7*e^1
6 + 13440*a^17*b^24*d^7*e^16 + 69056*a^19*b^22*d^7*e^16 + 202752*a^21*b^20*
d^7*e^16 + 372800*a^23*b^18*d^7*e^16 + 443136*a^25*b^16*d^7*e^16 + 337792*a
^27*b^14*d^7*e^16 + 156160*a^29*b^12*d^7*e^16 + 37632*a^31*b^10*d^7*e^16 +
3200*a^33*b^8*d^7*e^16 + 704*a^35*b^6*d^7*e^16 + 512*a^37*b^4*d^7*e^16 + 64
*a^39*b^2*d^7*e^16))/2)*(1/(a^4*d^2*e^3*1i + b^4*d^2*e^3*1i + 4*a*b^3*d^2*e
^3 - 4*a^3*b*d^2*e^3 - a^2*b^2*d^2*e^3*6i))^(1/2))/2 + 16992*a^27*b^12*d^6*
e^15 + 9312*a^29*b^10*d^6*e^15 + 2688*a^31*b^8*d^6*e^15 + 576*a^33*b^6*d^6*
e^15 + 144*a^35*b^4*d^6*e^15 + 16*a^37*b^2*d^6*e^15))/2)*1i + (1/(a^4*d^2*e
^3*1i + b^4*d^2*e^3*1i + 4*a*b^3*d^2*e^3 - 4*a^3*b*d^2*e^3 - a^2*b^2*d^2*e^
3*6i))^(1/2)*(((e*cot(c + d*x))^(1/2)*(144*a^14*b^23*d^5*e^13 + 1248*a^16*b
^21*d^5*e^13 + 4224*a^18*b^19*d^5*e^13 + 6720*a^20*b^17*d^5*e^13 + 3872*a^2
2*b^15*d^5*e^13 - 2816*a^24*b^13*d^5*e^13 - 5632*a^26*b^11*d^5*e^13 - 3136*
a^28*b^9*d^5*e^13 - 560*a^30*b^7*d^5*e^13 + 32*a^32*b^5*d^5*e^13))/2 - ((1/
(a^4*d^2*e^3*1i + b^4*d^2*e^3*1i + 4*a*b^3*d^2*e^3 - 4*a^3*b*d^2*e^3 - a^2*
b^2*d^2*e^3*6i))^(1/2)*(13248*a^25*b^14*d^6*e^15 - 576*a^15*b^24*d^6*e^15 -
4224*a^17*b^22*d^6*e^15 - 11888*a^19*b^20*d^6*e^15 - 14832*a^21*b^18*d^6*e
^15 - 3264*a^23*b^16*d^6*e^15 - ((((1/(a^4*d^2*e^3*1i + b^4*d^2*e^3*1i + 4*
a*b^3*d^2*e^3 - 4*a^3*b*d^2*e^3 - a^2*b^2*d^2*e^3*6i))^(1/2)*(((e*cot(c + d
x))^(1/2)(1/(a^4*d^2*e^3*1i + b^4*d^2*e^3*1i + 4*a*b^3*d^2*e^3 - 4*a^3*b*
d^2*e^3 - a^2*b^2*d^2*e^3*6i))^(1/2)*(512*a^18*b^27*d^9*e^19 + 5120*a^20*b^
25*d^9*e^19 + 22528*a^22*b^23*d^9*e^19 + 56320*a^24*b^21*d^9*e^19 + 84480*a
^26*b^19*d^9*e^19 + 67584*a^28*b^17*d^9*e^19 - 67584*a^32*b^13*d^9*e^19 - 8
4480*a^34*b^11*d^9*e^19 - 56320*a^36*b^9*d^9*e^19 - 22528*a^38*b^7*d^9*e^19
- 5120*a^40*b^5*d^9*e^19 - 512*a^42*b^3*d^9*e^19))/4 + 384*a^16*b^27*d^8*e
^18 + 4352*a^18*b^25*d^8*e^18 + 22144*a^20*b^23*d^8*e^18 + 66560*a^22*b^21*
d^8*e^18 + 130560*a^24*b^19*d^8*e^18 + 173568*a^26*b^17*d^8*e^18 + 155904*a
^28*b^15*d^8*e^18 + 89088*a^30*b^13*d^8*e^18 + 24960*a^32*b^11*d^8*e^18 - 3
840*a^34*b^9*d^8*e^18 - 6016*a^36*b^7*d^8*e^18 - 2048*a^38*b^5*d^8*e^18 - 2
56*a^40*b^3*d^8*e^18))/2 - ((e*cot(c + d*x))^(1/2)*(1152*a^15*b^26*d^7*e^16
+ 13440*a^17*b^24*d^7*e^16 + 69056*a^19*b^22*d^7*e^16 + 202752*a^21*b^20*d
^7*e^16 + 372800*a^23*b^18*d^7*e^16 + 443136*a^25*b^16*d^7*e^16 + 337792*a^
27*b^14*d^7*e^16 + 156160*a^29*b^12*d^7*e^16 + 37632*a^31*b^10*d^7*e^16 + 3
200*a^33*b^8*d^7*e^16 + 704*a^35*b^6*d^7*e^16 + 512*a^37*b^4*d^7*e^16 + 64*
a^39*b^2*d^7*e^16))/2)*(1/(a^4*d^2*e^3*1i + b^4*d^2*e^3*1i + 4*a*b^3*d^2*e^
3 - 4*a^3*b*d^2*e^3 - a^2*b^2*d^2*e^3*6i))^(1/2))/2 + 16992*a^27*b^12*d^6*e
^15 + 9312*a^29*b^10*d^6*e^15 + 2688*a^31*b^8*d^6*e^15 + 576*a^33*b^6*d^6*e
^15 + 144*a^35*b^4*d^6*e^15 + 16*a^37*b^2*d^6*e^15))/2)*1i)/((1/(a^4*d^2*e^
3*1i + b^4*d^2*e^3*1i + 4*a*b^3*d^2*e^3 - 4*a^3*b*d^2*e^3 - a^2*b^2*d^2*e^3
6i))^(1/2)(((e*cot(c + d*x))^(1/2)*(144*a^14*b^23*d^5*e^13 + 1248*a^16*b^
21*d^5*e^13 + 4224*a^18*b^19*d^5*e^13 + 6720*a^20*b^17*d^5*e^13 + 3872*a^22

*b^15*d^5*e^13 - 2816*a^24*b^13*d^5*e^13 - 5632*a^26*b^11*d^5*e^13 - 3136*a
^28*b^9*d^5*e^13 - 560*a^30*b^7*d^5*e^13 + 32*a^32*b^5*d^5*e^13))/2 + ((1/(
a^4*d^2*e^3*1i + b^4*d^2*e^3*1i + 4*a*b^3*d^2*e^3 - 4*a^3*b*d^2*e^3 - a^2*b
^2*d^2*e^3*6i))^(1/2)*(13248*a^25*b^14*d^6*e^15 - 576*a^15*b^24*d^6*e^15 -
4224*a^17*b^22*d^6*e^15 - 11888*a^19*b^20*d^6*e^15 - 14832*a^21*b^18*d^6*e^
15 - 3264*a^23*b^16*d^6*e^15 - ((((1/(a^4*d^2*e^3*1i + b^4*d^2*e^3*1i + 4*a
*b^3*d^2*e^3 - 4*a^3*b*d^2*e^3 - a^2*b^2*d^2*e^3*6i))^(1/2)*(384*a^16*b^27*
d^8*e^18 - ((e*cot(c + d*x))^(1/2)*(1/(a^4*d^2*e^3*1i + b^4*d^2*e^3*1i + 4*
a*b^3*d^2*e^3 - 4*a^3*b*d^2*e^3 - a^2*b^2*d^2*e^3*6i))^(1/2)*(512*a^18*b^27
*d^9*e^19 + 5120*a^20*b^25*d^9*e^19 + 22528*a^22*b^23*d^9*e^19 + 56320*a^24
*b^21*d^9*e^19 + 84480*a^26*b^19*d^9*e^19 + 67584*a^28*b^17*d^9*e^19 - 6758
4*a^32*b^13*d^9*e^19 - 84480*a^34*b^11*d^9*e^19 - 56320*a^36*b^9*d^9*e^19 -
22528*a^38*b^7*d^9*e^19 - 5120*a^40*b^5*d^9*e^19 - 512*a^42*b^3*d^9*e^19))
/4 + 4352*a^18*b^25*d^8*e^18 + 22144*a^20*b^23*d^8*e^18 + 66560*a^22*b^21*d
^8*e^18 + 130560*a^24*b^19*d^8*e^18 + 173568*a^26*b^17*d^8*e^18 + 155904*a^
28*b^15*d^8*e^18 + 89088*a^30*b^13*d^8*e^18 + 24960*a^32*b^11*d^8*e^18 - 38
40*a^34*b^9*d^8*e^18 - 6016*a^36*b^7*d^8*e^18 - 2048*a^38*b^5*d^8*e^18 - 25
6*a^40*b^3*d^8*e^18))/2 + ((e*cot(c + d*x))^(1/2)*(1152*a^15*b^26*d^7*e^16
+ 13440*a^17*b^24*d^7*e^16 + 69056*a^19*b^22*d^7*e^16 + 202752*a^21*b^20*d^
7*e^16 + 372800*a^23*b^18*d^7*e^16 + 443136*a^25*b^16*d^7*e^16 + 337792*a^2
7*b^14*d^7*e^16 + 156160*a^29*b^12*d^7*e^16 + 37632*a^31*b^10*d^7*e^16 + 32
00*a^33*b^8*d^7*e^16 + 704*a^35*b^6*d^7*e^16 + 512*a^37*b^4*d^7*e^16 + 64*a
^39*b^2*d^7*e^16))/2)*(1/(a^4*d^2*e^3*1i + b^4*d^2*e^3*1i + 4*a*b^3*d^2*e^3
 - 4*a^3*b*d^2*e^3 - a^2*b^2*d^2*e^3*6i))^(1/2))/2 + 16992*a^27*b^12*d^6*e^
15 + 9312*a^29*b^10*d^6*e^15 + 2688*a^31*b^8*d^6*e^15 + 576*a^33*b^6*d^6*e^
15 + 144*a^35*b^4*d^6*e^15 + 16*a^37*b^2*d^6*e^15))/2) - (1/(a^4*d^2*e^3*1i
 + b^4*d^2*e^3*1i + 4*a*b^3*d^2*e^3 - 4*a^3*b*d^2*e^3 - a^2*b^2*d^2*e^3*6i)
)^(1/2)*(((e*cot(c + d*x))^(1/2)*(144*a^14*b^23*d^5*e^13 + 1248*a^16*b^21*d
^5*e^13 + 4224*a^18*b^19*d^5*e^13 + 6720*a^20*b^17*d^5*e^13 + 3872*a^22*b^1
5*d^5*e^13 - 2816*a^24*b^13*d^5*e^13 - 5632*a^26*b^11*d^5*e^13 - 3136*a^28*
b^9*d^5*e^13 - 560*a^30*b^7*d^5*e^13 + 32*a^32*b^5*d^5*e^13))/2 - ((1/(a^4*
d^2*e^3*1i + b^4*d^2*e^3*1i + 4*a*b^3*d^2*e^3 - 4*a^3*b*d^2*e^3 - a^2*b^2*d
^2*e^3*6i))^(1/2)*(13248*a^25*b^14*d^6*e^15 - 576*a^15*b^24*d^6*e^15 - 4224
*a^17*b^22*d^6*e^15 - 11888*a^19*b^20*d^6*e^15 - 14832*a^21*b^18*d^6*e^15 -
3264*a^23*b^16*d^6*e^15 - ((((1/(a^4*d^2*e^3*1i + b^4*d^2*e^3*1i + 4*a*b^3
*d^2*e^3 - 4*a^3*b*d^2*e^3 - a^2*b^2*d^2*e^3*6i))^(1/2)*(((e*cot(c + d*x))^
(1/2)*(1/(a^4*d^2*e^3*1i + b^4*d^2*e^3*1i + 4*a*b^3*d^2*e^3 - 4*a^3*b*d^2*e
^3 - a^2*b^2*d^2*e^3*6i))^(1/2)*(512*a^18*b^27*d^9*e^19 + 5120*a^20*b^25*d^
9*e^19 + 22528*a^22*b^23*d^9*e^19 + 56320*a^24*b^21*d^9*e^19 + 84480*a^26*b
^19*d^9*e^19 + 67584*a^28*b^17*d^9*e^19 - 67584*a^32*b^13*d^9*e^19 - 84480*
a^34*b^11*d^9*e^19 - 56320*a^36*b^9*d^9*e^19 - 22528*a^38*b^7*d^9*e^19 - 51
20*a^40*b^5*d^9*e^19 - 512*a^42*b^3*d^9*e^19))/4 + 384*a^16*b^27*d^8*e^18 +
4352*a^18*b^25*d^8*e^18 + 22144*a^20*b^23*d^8*e^18 + 66560*a^22*b^21*d^8*e
^18 + 130560*a^24*b^19*d^8*e^18 + 173568*a^26*b^17*d^8*e^18 + 155904*a^28*b
^15*d^8*e^18 + 89088*a^30*b^13*d^8*e^18 + 24960*a^32*b^11*d^8*e^18 - 3840*a

^34*b^9*d^8*e^18 - 6016*a^36*b^7*d^8*e^18 - 2048*a^38*b^5*d^8*e^18 - 256*a^40*b^3*d^8*e^18))/2 - ((e*cot(c + d*x))^(1/2)*(1152*a^15*b^26*d^7*e^16 + 13440*a^17*b^24*d^7*e^16 + 69056*a^19*b^22*d^7*e^16 + 202752*a^21*b^20*d^7*e^16 + 372800*a^23*b^18*d^7*e^16 + 443136*a^25*b^16*d^7*e^16 + 337792*a^27*b^14*d^7*e^16 + 156160*a^29*b^12*d^7*e^16 + 37632*a^31*b^10*d^7*e^16 + 3200*a^33*b^8*d^7*e^16 + 704*a^35*b^6*d^7*e^16 + 512*a^37*b^4*d^7*e^16 + 64*a^39*b^2*d^7*e^16))/2)*(1/(a^4*d^2*e^3*1i + b^4*d^2*e^3*1i + 4*a*b^3*d^2*e^3 - 4*a^3*b*d^2*e^3 - a^2*b^2*d^2*e^3*6i))^(1/2))/2 + 16992*a^27*b^12*d^6*e^15 + 9312*a^29*b^10*d^6*e^15 + 2688*a^31*b^8*d^6*e^15 + 576*a^33*b^6*d^6*e^15 + 144*a^35*b^4*d^6*e^15 + 16*a^37*b^2*d^6*e^15))/2) + 144*a^14*b^21*d^4*e^12 + 1296*a^16*b^19*d^4*e^12 + 4880*a^18*b^17*d^4*e^12 + 10000*a^20*b^15*d^4*e^12 + 12080*a^22*b^13*d^4*e^12 + 8624*a^24*b^11*d^4*e^12 + 3376*a^26*b^9*d^4*e^12 + 560*a^28*b^7*d^4*e^12))*(1/(a^4*d^2*e^3*1i + b^4*d^2*e^3*1i + 4*a*b^3*d^2*e^3 - 4*a^3*b*d^2*e^3 - a^2*b^2*d^2*e^3*6i))^(1/2)*1i - (atan(((((e*cot(c + d*x))^(1/2)*(144*a^14*b^23*d^5*e^13 + 1248*a^16*b^21*d^5*e^13 + 4224*a^18*b^19*d^5*e^13 + 6720*a^20*b^17*d^5*e^13 + 3872*a^22*b^15*d^5*e^13 - 2816*a^24*b^13*d^5*e^13 - 5632*a^26*b^11*d^5*e^13 - 3136*a^28*b^9*d^5*e^13 - 560*a^30*b^7*d^5*e^13 + 32*a^32*b^5*d^5*e^13) + ((7*a^2 + 3*b^2)*(-a^5*b^5*e^3)^(1/2)*(26496*a^25*b^14*d^6*e^15 - 8448*a^17*b^22*d^6*e^15 - 23776*a^19*b^20*d^6*e^15 - 29664*a^21*b^18*d^6*e^15 - 6528*a^23*b^16*d^6*e^15 - 1152*a^15*b^24*d^6*e^15 + 33984*a^27*b^12*d^6*e^15 + 18624*a^29*b^10*d^6*e^15 + 5376*a^31*b^8*d^6*e^15 + 1152*a^33*b^6*d^6*e^15 + 288*a^35*b^4*d^6*e^15 + 32*a^37*b^2*d^6*e^15 - ((7*a^2 + 3*b^2)*((e*cot(c + d*x))^(1/2)*(1152*a^15*b^26*d^7*e^16 + 13440*a^17*b^24*d^7*e^16 + 69056*a^19*b^22*d^7*e^16 + 202752*a^21*b^20*d^7*e^16 + 372800*a^23*b^18*d^7*e^16 + 443136*a^25*b^16*d^7*e^16 + 337792*a^27*b^14*d^7*e^16 + 156160*a^29*b^12*d^7*e^16 + 37632*a^31*b^10*d^7*e^16 + 3200*a^33*b^8*d^7*e^16 + 704*a^35*b^6*d^7*e^16 + 512*a^37*b^4*d^7*e^16 + 64*a^39*b^2*d^7*e^16) + ((7*a^2 + 3*b^2)*(-a^5*b^5*e^3)^(1/2)*(768*a^16*b^27*d^8*e^18 + 8704*a^18*b^25*d^8*e^18 + 44288*a^20*b^23*d^8*e^18 + 133120*a^22*b^21*d^8*e^18 + 261120*a^24*b^19*d^8*e^18 + 347136*a^26*b^17*d^8*e^18 + 311808*a^28*b^15*d^8*e^18 + 178176*a^30*b^13*d^8*e^18 + 49920*a^32*b^11*d^8*e^18 - 7680*a^34*b^9*d^8*e^18 - 12032*a^36*b^7*d^8*e^18 - 4096*a^38*b^5*d^8*e^18 - 512*a^40*b^3*d^8*e^18 - ((e*cot(c + d*x))^(1/2)*(7*a^2 + 3*b^2)*(-a^5*b^5*e^3)^(1/2)*(512*a^18*b^27*d^9*e^19 + 5120*a^20*b^25*d^9*e^19 + 22528*a^22*b^23*d^9*e^19 + 56320*a^24*b^21*d^9*e^19 + 84480*a^26*b^19*d^9*e^19 + 67584*a^28*b^17*d^9*e^19 - 67584*a^32*b^13*d^9*e^19 - 84480*a^34*b^11*d^9*e^19 - 56320*a^36*b^9*d^9*e^19 - 22528*a^38*b^7*d^9*e^19 - 5120*a^40*b^5*d^9*e^19 - 512*a^42*b^3*d^9*e^19))/(2*(a^9*d*e^3 + a^5*b^4*d*e^3 + 2*a^7*b^2*d*e^3))))/(2*(a^9*d*e^3 + a^5*b^4*d*e^3 + 2*a^7*b^2*d*e^3)))*(-a^5*b^5*e^3)^(1/2))/(2*(a^9*d*e^3 + a^5*b^4*d*e^3 + 2*a^7*b^2*d*e^3))))/(2*(a^9*d*e^3 + a^5*b^4*d*e^3 + 2*a^7*b^2*d*e^3)))*(7*a^2 + 3*b^2)*(-a^5*b^5*e^3)^(1/2)*1i)/(2*(a^9*d*e^3 + a^5*b^4*d*e^3 + 2*a^7*b^2*d*e^3)) + (((e*cot(c + d*x))^(1/2)*(144*a^14*b^23*d^5*e^13 + 1248*a^16*b^21*d^5*e^13 + 4224*a^18*b^19*d^5*e^13 + 6720*a^20*b^17*d^5*e^13 + 3872*a^22*b^15*d^5*e^13 - 2816*a^24*b^13*d^5*e^13 - 5632*a^26*b^11*d^5*e^13 - 3136*a^28*b^9*d^5*e^13 -

560*a^30*b^7*d^5*e^13 + 32*a^32*b^5*d^5*e^13) - ((7*a^2 + 3*b^2)*(-a^5*b^5*
e^3)^(1/2)*(26496*a^25*b^14*d^6*e^15 - 8448*a^17*b^22*d^6*e^15 - 23776*a^19
*b^20*d^6*e^15 - 29664*a^21*b^18*d^6*e^15 - 6528*a^23*b^16*d^6*e^15 - 1152*
a^15*b^24*d^6*e^15 + 33984*a^27*b^12*d^6*e^15 + 18624*a^29*b^10*d^6*e^15 +
5376*a^31*b^8*d^6*e^15 + 1152*a^33*b^6*d^6*e^15 + 288*a^35*b^4*d^6*e^15 + 3
2*a^37*b^2*d^6*e^15 + ((7*a^2 + 3*b^2)*((e*cot(c + d*x))^(1/2)*(1152*a^15*b
^26*d^7*e^16 + 13440*a^17*b^24*d^7*e^16 + 69056*a^19*b^22*d^7*e^16 + 202752
*a^21*b^20*d^7*e^16 + 372800*a^23*b^18*d^7*e^16 + 443136*a^25*b^16*d^7*e^16
 + 337792*a^27*b^14*d^7*e^16 + 156160*a^29*b^12*d^7*e^16 + 37632*a^31*b^10*
d^7*e^16 + 3200*a^33*b^8*d^7*e^16 + 704*a^35*b^6*d^7*e^16 + 512*a^37*b^4*d^
7*e^16 + 64*a^39*b^2*d^7*e^16) - ((7*a^2 + 3*b^2)*(-a^5*b^5*e^3)^(1/2)*(768
*a^16*b^27*d^8*e^18 + 8704*a^18*b^25*d^8*e^18 + 44288*a^20*b^23*d^8*e^18 +
133120*a^22*b^21*d^8*e^18 + 261120*a^24*b^19*d^8*e^18 + 347136*a^26*b^17*d^
8*e^18 + 311808*a^28*b^15*d^8*e^18 + 178176*a^30*b^13*d^8*e^18 + 49920*a^32
*b^11*d^8*e^18 - 7680*a^34*b^9*d^8*e^18 - 12032*a^36*b^7*d^8*e^18 - 4096*a^
38*b^5*d^8*e^18 - 512*a^40*b^3*d^8*e^18 + ((e*cot(c + d*x))^(1/2)*(7*a^2 +
3*b^2)*(-a^5*b^5*e^3)^(1/2)*(512*a^18*b^27*d^9*e^19 + 5120*a^20*b^25*d^9*e^
19 + 22528*a^22*b^23*d^9*e^19 + 56320*a^24*b^21*d^9*e^19 + 84480*a^26*b^19*
d^9*e^19 + 67584*a^28*b^17*d^9*e^19 - 67584*a^32*b^13*d^9*e^19 - 84480*a^34
*b^11*d^9*e^19 - 56320*a^36*b^9*d^9*e^19 - 22528*a^38*b^7*d^9*e^19 - 5120*a
^40*b^5*d^9*e^19 - 512*a^42*b^3*d^9*e^19))/(2*(a^9*d*e^3 + a^5*b^4*d*e^3 +
2*a^7*b^2*d*e^3))))/(2*(a^9*d*e^3 + a^5*b^4*d*e^3 + 2*a^7*b^2*d*e^3)))*(-a^
5*b^5*e^3)^(1/2))/(2*(a^9*d*e^3 + a^5*b^4*d*e^3 + 2*a^7*b^2*d*e^3))))/(2*(a
^9*d*e^3 + a^5*b^4*d*e^3 + 2*a^7*b^2*d*e^3)))*(7*a^2 + 3*b^2)*(-a^5*b^5*e^3
)^(1/2)*1i)/(2*(a^9*d*e^3 + a^5*b^4*d*e^3 + 2*a^7*b^2*d*e^3)))/((((e*cot(c
+ d*x))^(1/2)*(144*a^14*b^23*d^5*e^13 + 1248*a^16*b^21*d^5*e^13 + 4224*a^18
*b^19*d^5*e^13 + 6720*a^20*b^17*d^5*e^13 + 3872*a^22*b^15*d^5*e^13 - 2816*a
^24*b^13*d^5*e^13 - 5632*a^26*b^11*d^5*e^13 - 3136*a^28*b^9*d^5*e^13 - 560*
a^30*b^7*d^5*e^13 + 32*a^32*b^5*d^5*e^13) + ((7*a^2 + 3*b^2)*(-a^5*b^5*e^3)
^(1/2)*(26496*a^25*b^14*d^6*e^15 - 8448*a^17*b^22*d^6*e^15 - 23776*a^19*b^2
0*d^6*e^15 - 29664*a^21*b^18*d^6*e^15 - 6528*a^23*b^16*d^6*e^15 - 1152*a^15
*b^24*d^6*e^15 + 33984*a^27*b^12*d^6*e^15 + 18624*a^29*b^10*d^6*e^15 + 5376
*a^31*b^8*d^6*e^15 + 1152*a^33*b^6*d^6*e^15 + 288*a^35*b^4*d^6*e^15 + 32*a^
37*b^2*d^6*e^15 - ((7*a^2 + 3*b^2)*((e*cot(c + d*x))^(1/2)*(1152*a^15*b^26*
d^7*e^16 + 13440*a^17*b^24*d^7*e^16 + 69056*a^19*b^22*d^7*e^16 + 202752*a^2
1*b^20*d^7*e^16 + 372800*a^23*b^18*d^7*e^16 + 443136*a^25*b^16*d^7*e^16 + 3
37792*a^27*b^14*d^7*e^16 + 156160*a^29*b^12*d^7*e^16 + 37632*a^31*b^10*d^7*
e^16 + 3200*a^33*b^8*d^7*e^16 + 704*a^35*b^6*d^7*e^16 + 512*a^37*b^4*d^7*e^
16 + 64*a^39*b^2*d^7*e^16) + ((7*a^2 + 3*b^2)*(-a^5*b^5*e^3)^(1/2)*(768*a^1
6*b^27*d^8*e^18 + 8704*a^18*b^25*d^8*e^18 + 44288*a^20*b^23*d^8*e^18 + 1331
20*a^22*b^21*d^8*e^18 + 261120*a^24*b^19*d^8*e^18 + 347136*a^26*b^17*d^8*e^
18 + 311808*a^28*b^15*d^8*e^18 + 178176*a^30*b^13*d^8*e^18 + 49920*a^32*b^1
1*d^8*e^18 - 7680*a^34*b^9*d^8*e^18 - 12032*a^36*b^7*d^8*e^18 - 4096*a^38*b
^5*d^8*e^18 - 512*a^40*b^3*d^8*e^18 - ((e*cot(c + d*x))^(1/2)*(7*a^2 + 3*b^
2)*(-a^5*b^5*e^3)^(1/2)*(512*a^18*b^27*d^9*e^19 + 5120*a^20*b^25*d^9*e^19 +

22528*a^22*b^23*d^9*e^19 + 56320*a^24*b^21*d^9*e^19 + 84480*a^26*b^19*d^9*e^19 + 67584*a^28*b^17*d^9*e^19 - 67584*a^32*b^13*d^9*e^19 - 84480*a^34*b^11*d^9*e^19 - 56320*a^36*b^9*d^9*e^19 - 22528*a^38*b^7*d^9*e^19 - 5120*a^40*b^5*d^9*e^19 - 512*a^42*b^3*d^9*e^19))/(2*(a^9*d*e^3 + a^5*b^4*d*e^3 + 2*a^7*b^2*d*e^3))))/(2*(a^9*d*e^3 + a^5*b^4*d*e^3 + 2*a^7*b^2*d*e^3)))*(-a^5*b^5*e^3)^(1/2))/(2*(a^9*d*e^3 + a^5*b^4*d*e^3 + 2*a^7*b^2*d*e^3))))/(2*(a^9*d*e^3 + a^5*b^4*d*e^3 + 2*a^7*b^2*d*e^3)))*(7*a^2 + 3*b^2)*(-a^5*b^5*e^3)^(1/2))/(2*(a^9*d*e^3 + a^5*b^4*d*e^3 + 2*a^7*b^2*d*e^3)) - (((e*cot(c + d*x))^(1/2)*(144*a^14*b^23*d^5*e^13 + 1248*a^16*b^21*d^5*e^13 + 4224*a^18*b^19*d^5*e^13 + 6720*a^20*b^17*d^5*e^13 + 3872*a^22*b^15*d^5*e^13 - 2816*a^24*b^13*d^5*e^13 - 5632*a^26*b^11*d^5*e^13 - 3136*a^28*b^9*d^5*e^13 - 560*a^30*b^7*d^5*e^13 + 32*a^32*b^5*d^5*e^13) - ((7*a^2 + 3*b^2)*(-a^5*b^5*e^3)^(1/2)*(26496*a^25*b^14*d^6*e^15 - 8448*a^17*b^22*d^6*e^15 - 23776*a^19*b^20*d^6*e^15 - 29664*a^21*b^18*d^6*e^15 - 6528*a^23*b^16*d^6*e^15 - 1152*a^15*b^24*d^6*e^15 + 33984*a^27*b^12*d^6*e^15 + 18624*a^29*b^10*d^6*e^15 + 5376*a^31*b^8*d^6*e^15 + 1152*a^33*b^6*d^6*e^15 + 288*a^35*b^4*d^6*e^15 + 32*a^37*b^2*d^6*e^15 + ((7*a^2 + 3*b^2)*((e*cot(c + d*x))^(1/2)*(1152*a^15*b^26*d^7*e^16 + 13440*a^17*b^24*d^7*e^16 + 69056*a^19*b^22*d^7*e^16 + 202752*a^21*b^20*d^7*e^16 + 372800*a^23*b^18*d^7*e^16 + 443136*a^25*b^16*d^7*e^16 + 337792*a^27*b^14*d^7*e^16 + 156160*a^29*b^12*d^7*e^16 + 37632*a^31*b^10*d^7*e^16 + 3200*a^33*b^8*d^7*e^16 + 704*a^35*b^6*d^7*e^16 + 512*a^37*b^4*d^7*e^16 + 64*a^39*b^2*d^7*e^16) - ((7*a^2 + 3*b^2)*(-a^5*b^5*e^3)^(1/2)*(768*a^16*b^27*d^8*e^18 + 8704*a^18*b^25*d^8*e^18 + 44288*a^20*b^23*d^8*e^18 + 133120*a^22*b^21*d^8*e^18 + 261120*a^24*b^19*d^8*e^18 + 347136*a^26*b^17*d^8*e^18 + 311808*a^28*b^15*d^8*e^18 + 178176*a^30*b^13*d^8*e^18 + 49920*a^32*b^11*d^8*e^18 - 7680*a^34*b^9*d^8*e^18 - 12032*a^36*b^7*d^8*e^18 - 4096*a^38*b^5*d^8*e^18 - 512*a^40*b^3*d^8*e^18 + ((e*cot(c + d*x))^(1/2)*(7*a^2 + 3*b^2)*(-a^5*b^5*e^3)^(1/2)*(512*a^18*b^27*d^9*e^19 + 5120*a^20*b^25*d^9*e^19 + 22528*a^22*b^23*d^9*e^19 + 56320*a^24*b^21*d^9*e^19 + 84480*a^26*b^19*d^9*e^19 + 67584*a^28*b^17*d^9*e^19 - 67584*a^32*b^13*d^9*e^19 - 84480*a^34*b^11*d^9*e^19 - 56320*a^36*b^9*d^9*e^19 - 22528*a^38*b^7*d^9*e^19 - 5120*a^40*b^5*d^9*e^19 - 512*a^42*b^3*d^9*e^19))/(2*(a^9*d*e^3 + a^5*b^4*d*e^3 + 2*a^7*b^2*d*e^3))))/(2*(a^9*d*e^3 + a^5*b^4*d*e^3 + 2*a^7*b^2*d*e^3)))*(-a^5*b^5*e^3)^(1/2))/(2*(a^9*d*e^3 + a^5*b^4*d*e^3 + 2*a^7*b^2*d*e^3))))/(2*(a^9*d*e^3 + a^5*b^4*d*e^3 + 2*a^7*b^2*d*e^3)))*(7*a^2 + 3*b^2)*(-a^5*b^5*e^3)^(1/2))/(2*(a^9*d*e^3 + a^5*b^4*d*e^3 + 2*a^7*b^2*d*e^3)) + 144*a^14*b^21*d^4*e^12 + 1296*a^16*b^19*d^4*e^12 + 4880*a^18*b^17*d^4*e^12 + 10000*a^20*b^15*d^4*e^12 + 12080*a^22*b^13*d^4*e^12 + 8624*a^24*b^11*d^4*e^12 + 3376*a^26*b^9*d^4*e^12 + 560*a^28*b^7*d^4*e^12))*(7*a^2 + 3*b^2)*(-a^5*b^5*e^3)^(1/2)*1i)/(a^9*d*e^3 + a^5*b^4*d*e^3 + 2*a^7*b^2*d*e^3)

3.81 $\int \frac{(e\cot(c+dx))^{9/2}}{(a+b\cot(c+dx))^3}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 529

$$\int \frac{(e\cot(c+dx))^{9/2}}{(a+b\cot(c+dx))^3}\,dx = \frac{a^{5/2}(15a^4+46a^2b^2+63b^4)\,e^{9/2}\arctan\left(\frac{\sqrt{b}\sqrt{e\cot(c+dx)}}{\sqrt{a}\sqrt{e}}\right)}{4b^{7/2}\left(a^2+b^2\right)^3 d}$$
$$+\frac{(a-b)\left(a^2+4ab+b^2\right)e^{9/2}\arctan\left(1-\frac{\sqrt{2}\sqrt{e\cot(c+dx)}}{\sqrt{e}}\right)}{\sqrt{2}\left(a^2+b^2\right)^3 d}$$
$$-\frac{(a-b)\left(a^2+4ab+b^2\right)e^{9/2}\arctan\left(1+\frac{\sqrt{2}\sqrt{e\cot(c+dx)}}{\sqrt{e}}\right)}{\sqrt{2}\left(a^2+b^2\right)^3 d}$$
$$-\frac{(15a^4+31a^2b^2+8b^4)\,e^4\sqrt{e\cot(c+dx)}}{4b^3\left(a^2+b^2\right)^2 d}$$
$$+\frac{a^2e^2(e\cot(c+dx))^{5/2}}{2b\left(a^2+b^2\right)d(a+b\cot(c+dx))^2}+\frac{a^2(5a^2+13b^2)\,e^3(e\cot(c+dx))^{3/2}}{4b^2\left(a^2+b^2\right)^2 d(a+b\cot(c+dx))}$$
$$-\frac{(a+b)\left(a^2-4ab+b^2\right)e^{9/2}\log\left(\sqrt{e}+\sqrt{e}\cot(c+dx)-\sqrt{2}\sqrt{e\cot(c+dx)}\right)}{2\sqrt{2}\left(a^2+b^2\right)^3 d}$$
$$+\frac{(a+b)\left(a^2-4ab+b^2\right)e^{9/2}\log\left(\sqrt{e}+\sqrt{e}\cot(c+dx)+\sqrt{2}\sqrt{e\cot(c+dx)}\right)}{2\sqrt{2}\left(a^2+b^2\right)^3 d}$$

```
[Out] 1/4*a^(5/2)*(15*a^4+46*a^2*b^2+63*b^4)*e^(9/2)*arctan(b^(1/2)*(e*cot(d*x+c)
)^(1/2)/a^(1/2)/e^(1/2))/b^(7/2)/(a^2+b^2)^3/d+1/2*a^2*e^2*(e*cot(d*x+c))^(
5/2)/b/(a^2+b^2)/d/(a+b*cot(d*x+c))^2+1/4*a^2*(5*a^2+13*b^2)*e^3*(e*cot(d*x
+c))^(3/2)/b^2/(a^2+b^2)^2/d/(a+b*cot(d*x+c))+1/2*(a-b)*(a^2+4*a*b+b^2)*e^(
9/2)*arctan(1-2^(1/2)*(e*cot(d*x+c))^(1/2)/e^(1/2))/(a^2+b^2)^3/d*2^(1/2)-1
/2*(a-b)*(a^2+4*a*b+b^2)*e^(9/2)*arctan(1+2^(1/2)*(e*cot(d*x+c))^(1/2)/e^(1
/2))/(a^2+b^2)^3/d*2^(1/2)-1/4*(a+b)*(a^2-4*a*b+b^2)*e^(9/2)*ln(e^(1/2)+cot
```

```
(d*x+c)*e^(1/2)-2^(1/2)*(e*cot(d*x+c))^(1/2))/(a^2+b^2)^3/d*2^(1/2)+1/4*(a+
b)*(a^2-4*a*b+b^2)*e^(9/2)*ln(e^(1/2)+cot(d*x+c)*e^(1/2)+2^(1/2)*(e*cot(d*x
+c))^(1/2))/(a^2+b^2)^3/d*2^(1/2)-1/4*(15*a^4+31*a^2*b^2+8*b^4)*e^4*(e*cot(
d*x+c))^(1/2)/b^3/(a^2+b^2)^2/d
```

Rubi [A] (verified)

Time = 1.83 (sec) , antiderivative size = 529, normalized size of antiderivative = 1.00, number of steps used = 17, number of rules used = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.560, Rules used = {3646, 3726, 3728, 3734, 3615, 1182, 1176, 631, 210, 1179, 642, 3715, 65, 211}

$$\begin{aligned}
\int \frac{(e\cot(c+dx))^{9/2}}{(a+b\cot(c+dx))^3}\,dx = {} & \frac{e^{9/2}(a-b)\left(a^2+4ab+b^2\right)\arctan\left(1-\frac{\sqrt{2}\sqrt{e\cot(c+dx)}}{\sqrt{e}}\right)}{\sqrt{2}d\left(a^2+b^2\right)^3} \\
& - \frac{e^{9/2}(a-b)\left(a^2+4ab+b^2\right)\arctan\left(\frac{\sqrt{2}\sqrt{e\cot(c+dx)}}{\sqrt{e}}+1\right)}{\sqrt{2}d\left(a^2+b^2\right)^3} \\
& - \frac{e^{9/2}(a+b)\left(a^2-4ab+b^2\right)\log\left(\sqrt{e}\cot(c+dx)-\sqrt{2}\sqrt{e\cot(c+dx)}+\sqrt{e}\right)}{2\sqrt{2}d\left(a^2+b^2\right)^3} \\
& + \frac{e^{9/2}(a+b)\left(a^2-4ab+b^2\right)\log\left(\sqrt{e}\cot(c+dx)+\sqrt{2}\sqrt{e\cot(c+dx)}+\sqrt{e}\right)}{2\sqrt{2}d\left(a^2+b^2\right)^3} \\
& + \frac{a^2e^3\left(5a^2+13b^2\right)(e\cot(c+dx))^{3/2}}{4b^2d\left(a^2+b^2\right)^2(a+b\cot(c+dx))} + \frac{a^2e^2(e\cot(c+dx))^{5/2}}{2bd\left(a^2+b^2\right)(a+b\cot(c+dx))^2} \\
& - \frac{e^4\left(15a^4+31a^2b^2+8b^4\right)\sqrt{e\cot(c+dx)}}{4b^3d\left(a^2+b^2\right)^2} \\
& + \frac{a^{5/2}e^{9/2}\left(15a^4+46a^2b^2+63b^4\right)\arctan\left(\frac{\sqrt{b}\sqrt{e\cot(c+dx)}}{\sqrt{a}\sqrt{e}}\right)}{4b^{7/2}d\left(a^2+b^2\right)^3}
\end{aligned}$$

```
[In] Int[(e*Cot[c + d*x])^(9/2)/(a + b*Cot[c + d*x])^3,x]
```

```
[Out] (a^(5/2)*(15*a^4 + 46*a^2*b^2 + 63*b^4)*e^(9/2)*ArcTan[(Sqrt[b]*Sqrt[e*Cot[
c + d*x]])/(Sqrt[a]*Sqrt[e])])/(4*b^(7/2)*(a^2 + b^2)^3*d) + ((a - b)*(a^2
+ 4*a*b + b^2)*e^(9/2)*ArcTan[1 - (Sqrt[2]*Sqrt[e*Cot[c + d*x]])/Sqrt[e]])/
(Sqrt[2]*(a^2 + b^2)^3*d) - ((a - b)*(a^2 + 4*a*b + b^2)*e^(9/2)*ArcTan[1 +
 (Sqrt[2]*Sqrt[e*Cot[c + d*x]])/Sqrt[e]])/(Sqrt[2]*(a^2 + b^2)^3*d) - ((15*
a^4 + 31*a^2*b^2 + 8*b^4)*e^4*Sqrt[e*Cot[c + d*x]])/(4*b^3*(a^2 + b^2)^2*d)
 + (a^2*e^2*(e*Cot[c + d*x])^(5/2))/(2*b*(a^2 + b^2)*d*(a + b*Cot[c + d*x])
^2) + (a^2*(5*a^2 + 13*b^2)*e^3*(e*Cot[c + d*x])^(3/2))/(4*b^2*(a^2 + b^2)^
2*d*(a + b*Cot[c + d*x])) - ((a + b)*(a^2 - 4*a*b + b^2)*e^(9/2)*Log[Sqrt[e
] + Sqrt[e]*Cot[c + d*x] - Sqrt[2]*Sqrt[e*Cot[c + d*x]]])/(2*Sqrt[2]*(a^2 +
 b^2)^3*d) + ((a + b)*(a^2 - 4*a*b + b^2)*e^(9/2)*Log[Sqrt[e] + Sqrt[e]*Cot
[c + d*x] + Sqrt[2]*Sqrt[e*Cot[c + d*x]]])/(2*Sqrt[2]*(a^2 + b^2)^3*d)
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1179

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 1182

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
a*c, 2]}, Dist[(d*q + a*e)/(2*a*c), Int[(q + c*x^2)/(a + c*x^4), x], x] + D
ist[(d*q - a*e)/(2*a*c), Int[(q - c*x^2)/(a + c*x^4), x], x]] /; FreeQ[{a,
c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0] && NeQ[c*d^2 - a*e^2, 0] && NegQ[(-a)
*c]
```

Rule 3615

```
Int[((c_) + (d_.)*tan[(e_.) + (f_.)*(x_)])/Sqrt[(b_.)*tan[(e_.) + (f_.)*(x_
)]], x_Symbol] :> Dist[2/f, Subst[Int[(b*c + d*x^2)/(b^2 + x^4), x], x, Sqr
t[b*Tan[e + f*x]]], x] /; FreeQ[{b, c, d, e, f}, x] && NeQ[c^2 - d^2, 0] &&
 NeQ[c^2 + d^2, 0]
```

Rule 3646

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*c - a*d)^2*(a + b*Tan[e + f*x])^(m
 - 2)*((c + d*Tan[e + f*x])^(n + 1)/(d*f*(n + 1)*(c^2 + d^2))), x] - Dist[1
/(d*(n + 1)*(c^2 + d^2)), Int[(a + b*Tan[e + f*x])^(m - 3)*(c + d*Tan[e + f
*x])^(n + 1)*Simp[a^2*d*(b*d*(m - 2) - a*c*(n + 1)) + b*(b*c - 2*a*d)*(b*c*
(m - 2) + a*d*(n + 1)) - d*(n + 1)*(3*a^2*b*c - b^3*c - a^3*d + 3*a*b^2*d)*
Tan[e + f*x] - b*(a*d*(2*b*c - a*d)*(m + n - 1) - b^2*(c^2*(m - 2) - d^2*(n
 + 1)))*Tan[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b
*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] && GtQ[m, 2] && LtQ[
n, -1] && IntegerQ[2*m]
```

Rule 3715

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*tan[(e_.)
+ (f_.)*(x_)])^(n_.)*((A_) + (C_.)*tan[(e_.) + (f_.)*(x_)]^2), x_Symbol] :>
 Dist[A/f, Subst[Int[(a + b*x)^m*(c + d*x)^n, x], x, Tan[e + f*x]], x] /; F
reeQ[{a, b, c, d, e, f, A, C, m, n}, x] && EqQ[A, C]
```

Rule 3726

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*tan[(e_.) + (f_.)*(x_)] + (C_.)*tan[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> Simp[(A*d^2 + c*(c*C - B*d))*(a + b*Tan[e +
 f*x])^m*((c + d*Tan[e + f*x])^(n + 1)/(d*f*(n + 1)*(c^2 + d^2))), x] - Dis
t[1/(d*(n + 1)*(c^2 + d^2)), Int[(a + b*Tan[e + f*x])^(m - 1)*(c + d*Tan[e
+ f*x])^(n + 1)*Simp[A*d*(b*d*m - a*c*(n + 1)) + (c*C - B*d)*(b*c*m + a*d*(
n + 1)) - d*(n + 1)*((A - C)*(b*c - a*d) + B*(a*c + b*d))*Tan[e + f*x] - b*
(d*(B*c - A*d)*(m + n + 1) - C*(c^2*m - d^2*(n + 1)))*Tan[e + f*x]^2, x], x
], x] /; FreeQ[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] && NeQ[
```

```
a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] && GtQ[m, 0] && LtQ[n, -1]
```

Rule 3728

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*tan[(e_.)
+ (f_.)*(x_)])^(n_)*((A_.) + (B_.)*tan[(e_.) + (f_.)*(x_)] + (C_.)*tan[(e_.
) + (f_.)*(x_)]^2), x_Symbol] :> Simp[C*(a + b*Tan[e + f*x])^m*((c + d*Tan[
e + f*x])^(n + 1)/(d*f*(m + n + 1))), x] + Dist[1/(d*(m + n + 1)), Int[(a +
 b*Tan[e + f*x])^(m - 1)*(c + d*Tan[e + f*x])^n*Simp[a*A*d*(m + n + 1) - C*
(b*c*m + a*d*(n + 1)) + d*(A*b + a*B - b*C)*(m + n + 1)*Tan[e + f*x] - (C*m
*(b*c - a*d) - b*B*d*(m + n + 1))*Tan[e + f*x]^2, x], x], x] /; FreeQ[{a, b
, c, d, e, f, A, B, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] &&
NeQ[c^2 + d^2, 0] && GtQ[m, 0] &&  !(IGtQ[n, 0] && ( !IntegerQ[m] || (EqQ[c
, 0] && NeQ[a, 0])))
```

Rule 3734

```
Int[(((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_)*((A_.) + (B_.)*tan[(e_.)
+ (f_.)*(x_)] + (C_.)*tan[(e_.) + (f_.)*(x_)]^2))/((a_.) + (b_.)*tan[(e_.)
+ (f_.)*(x_)]), x_Symbol] :> Dist[1/(a^2 + b^2), Int[(c + d*Tan[e + f*x])^n
*Simp[b*B + a*(A - C) + (a*B - b*(A - C))*Tan[e + f*x], x], x], x] + Dist[(
A*b^2 - a*b*B + a^2*C)/(a^2 + b^2), Int[(c + d*Tan[e + f*x])^n*((1 + Tan[e
+ f*x]^2)/(a + b*Tan[e + f*x])), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C
, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] &&
 !GtQ[n, 0] &&  !LeQ[n, -1]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{a^2e^2(e\cot(c+dx))^{5/2}}{2b\left(a^2+b^2\right)d(a+b\cot(c+dx))^2} - \frac{\int \frac{(e\cot(c+dx))^{3/2}\left(-\frac{5}{2}a^2e^3+2abe^3\cot(c+dx)-\frac{1}{2}\left(5a^2+4b^2\right)e^3\cot^2(c+dx)\right)}{(a+b\cot(c+dx))^2}\,dx}{2b\left(a^2+b^2\right)} \\
&= \frac{a^2e^2(e\cot(c+dx))^{5/2}}{2b\left(a^2+b^2\right)d(a+b\cot(c+dx))^2} + \frac{a^2\left(5a^2+13b^2\right)e^3(e\cot(c+dx))^{3/2}}{4b^2\left(a^2+b^2\right)^2d(a+b\cot(c+dx))} \\
&\quad + \frac{\int \frac{\sqrt{e\cot(c+dx)}\left(\frac{3}{4}a^2\left(5a^2+13b^2\right)e^4-4ab^3e^4\cot(c+dx)+\frac{1}{4}\left(15a^4+31a^2b^2+8b^4\right)e^4\cot^2(c+dx)\right)}{a+b\cot(c+dx)}\,dx}{2b^2\left(a^2+b^2\right)^2}
\end{aligned}$$

$$
\begin{aligned}
&= -\frac{\left(15a^4+31a^2b^2+8b^4\right)e^4\sqrt{e\cot(c+dx)}}{4b^3\left(a^2+b^2\right)^2 d} \\
&\quad + \frac{a^2e^2(e\cot(c+dx))^{5/2}}{2b\left(a^2+b^2\right)d(a+b\cot(c+dx))^2} + \frac{a^2\left(5a^2+13b^2\right)e^3(e\cot(c+dx))^{3/2}}{4b^2\left(a^2+b^2\right)^2 d(a+b\cot(c+dx))} \\
&\quad - \frac{\int \frac{\frac{1}{8}a\left(15a^4+31a^2b^2+8b^4\right)e^5-b^3\left(a^2-b^2\right)e^5\cot(c+dx)+\frac{1}{8}a\left(15a^4+31a^2b^2+24b^4\right)e^5\cot^2(c+dx)}{\sqrt{e\cot(c+dx)}(a+b\cot(c+dx))}\,dx}{b^3\left(a^2+b^2\right)^2}
\end{aligned}
$$

$$
\begin{aligned}
&= -\frac{\left(15a^4+31a^2b^2+8b^4\right)e^4\sqrt{e\cot(c+dx)}}{4b^3\left(a^2+b^2\right)^2 d} + \frac{a^2e^2(e\cot(c+dx))^{5/2}}{2b\left(a^2+b^2\right)d(a+b\cot(c+dx))^2} \\
&\quad + \frac{a^2\left(5a^2+13b^2\right)e^3(e\cot(c+dx))^{3/2}}{4b^2\left(a^2+b^2\right)^2 d(a+b\cot(c+dx))} - \frac{\int \frac{-b^4\left(3a^2-b^2\right)e^5-ab^3\left(a^2-3b^2\right)e^5\cot(c+dx)}{\sqrt{e\cot(c+dx)}}\,dx}{b^3\left(a^2+b^2\right)^3} \\
&\quad - \frac{\left(a^3\left(15a^4+46a^2b^2+63b^4\right)e^5\right)\int \frac{1+\cot^2(c+dx)}{\sqrt{e\cot(c+dx)}(a+b\cot(c+dx))}\,dx}{8b^3\left(a^2+b^2\right)^3}
\end{aligned}
$$

$$
\begin{aligned}
&= -\frac{\left(15a^4+31a^2b^2+8b^4\right)e^4\sqrt{e\cot(c+dx)}}{4b^3\left(a^2+b^2\right)^2 d} \\
&\quad + \frac{a^2e^2(e\cot(c+dx))^{5/2}}{2b\left(a^2+b^2\right)d(a+b\cot(c+dx))^2} + \frac{a^2\left(5a^2+13b^2\right)e^3(e\cot(c+dx))^{3/2}}{4b^2\left(a^2+b^2\right)^2 d(a+b\cot(c+dx))} \\
&\quad - \frac{2\mathrm{Subst}\left(\int \frac{b^4\left(3a^2-b^2\right)e^6+ab^3\left(a^2-3b^2\right)e^5x^2}{e^2+x^4}\,dx, x, \sqrt{e\cot(c+dx)}\right)}{b^3\left(a^2+b^2\right)^3 d} \\
&\quad - \frac{\left(a^3\left(15a^4+46a^2b^2+63b^4\right)e^5\right)\mathrm{Subst}\left(\int \frac{1}{\sqrt{-ex}(a-bx)}\,dx, x, -\cot(c+dx)\right)}{8b^3\left(a^2+b^2\right)^3 d}
\end{aligned}
$$

$$
\begin{aligned}
&= -\frac{\left(15a^4+31a^2b^2+8b^4\right)e^4\sqrt{e\cot(c+dx)}}{4b^3\left(a^2+b^2\right)^2 d} \\
&\quad + \frac{a^2e^2(e\cot(c+dx))^{5/2}}{2b\left(a^2+b^2\right)d(a+b\cot(c+dx))^2} + \frac{a^2\left(5a^2+13b^2\right)e^3(e\cot(c+dx))^{3/2}}{4b^2\left(a^2+b^2\right)^2 d(a+b\cot(c+dx))} \\
&\quad + \frac{\left(a^3\left(15a^4+46a^2b^2+63b^4\right)e^4\right)\mathrm{Subst}\left(\int \frac{1}{a+\frac{bx^2}{e}}\,dx, x, \sqrt{e\cot(c+dx)}\right)}{4b^3\left(a^2+b^2\right)^3 d} \\
&\quad + \frac{\left((a+b)\left(a^2-4ab+b^2\right)e^5\right)\mathrm{Subst}\left(\int \frac{e-x^2}{e^2+x^4}\,dx, x, \sqrt{e\cot(c+dx)}\right)}{\left(a^2+b^2\right)^3 d} \\
&\quad - \frac{\left((a-b)\left(a^2+4ab+b^2\right)e^5\right)\mathrm{Subst}\left(\int \frac{e+x^2}{e^2+x^4}\,dx, x, \sqrt{e\cot(c+dx)}\right)}{\left(a^2+b^2\right)^3 d}
\end{aligned}
$$

$$= \frac{a^{5/2}(15a^4+46a^2b^2+63b^4)\,e^{9/2}\arctan\left(\frac{\sqrt{b}\sqrt{e\cot(c+dx)}}{\sqrt{a}\sqrt{e}}\right)}{4b^{7/2}\left(a^2+b^2\right)^3 d} - \frac{(15a^4+31a^2b^2+8b^4)\,e^4\sqrt{e\cot(c+dx)}}{4b^3\left(a^2+b^2\right)^2 d} + \frac{a^2e^2(e\cot(c+dx))^{5/2}}{2b\left(a^2+b^2\right)d(a+b\cot(c+dx))^2} + \frac{a^2(5a^2+13b^2)\,e^3(e\cot(c+dx))^{3/2}}{4b^2\left(a^2+b^2\right)^2 d(a+b\cot(c+dx))} - \frac{\left((a+b)\left(a^2-4ab+b^2\right)e^{9/2}\right)\mathrm{Subst}\left(\int\frac{\sqrt{2}\sqrt{e}+2x}{-e-\sqrt{2}\sqrt{e}x-x^2}\,dx,x,\sqrt{e\cot(c+dx)}\right)}{2\sqrt{2}\left(a^2+b^2\right)^3 d} - \frac{\left((a+b)\left(a^2-4ab+b^2\right)e^{9/2}\right)\mathrm{Subst}\left(\int\frac{\sqrt{2}\sqrt{e}-2x}{-e+\sqrt{2}\sqrt{e}x-x^2}\,dx,x,\sqrt{e\cot(c+dx)}\right)}{2\sqrt{2}\left(a^2+b^2\right)^3 d} - \frac{\left((a-b)\left(a^2+4ab+b^2\right)e^5\right)\mathrm{Subst}\left(\int\frac{1}{e-\sqrt{2}\sqrt{e}x+x^2}\,dx,x,\sqrt{e\cot(c+dx)}\right)}{2\left(a^2+b^2\right)^3 d} - \frac{\left((a-b)\left(a^2+4ab+b^2\right)e^5\right)\mathrm{Subst}\left(\int\frac{1}{e+\sqrt{2}\sqrt{e}x+x^2}\,dx,x,\sqrt{e\cot(c+dx)}\right)}{2\left(a^2+b^2\right)^3 d}$$

$$= \frac{a^{5/2}(15a^4+46a^2b^2+63b^4)\,e^{9/2}\arctan\left(\frac{\sqrt{b}\sqrt{e\cot(c+dx)}}{\sqrt{a}\sqrt{e}}\right)}{4b^{7/2}\left(a^2+b^2\right)^3 d} - \frac{(15a^4+31a^2b^2+8b^4)\,e^4\sqrt{e\cot(c+dx)}}{4b^3\left(a^2+b^2\right)^2 d} + \frac{a^2e^2(e\cot(c+dx))^{5/2}}{2b\left(a^2+b^2\right)d(a+b\cot(c+dx))^2} + \frac{a^2(5a^2+13b^2)\,e^3(e\cot(c+dx))^{3/2}}{4b^2\left(a^2+b^2\right)^2 d(a+b\cot(c+dx))} - \frac{(a+b)\left(a^2-4ab+b^2\right)e^{9/2}\log\left(\sqrt{e}+\sqrt{e}\cot(c+dx)-\sqrt{2}\sqrt{e\cot(c+dx)}\right)}{2\sqrt{2}\left(a^2+b^2\right)^3 d} + \frac{(a+b)\left(a^2-4ab+b^2\right)e^{9/2}\log\left(\sqrt{e}+\sqrt{e}\cot(c+dx)+\sqrt{2}\sqrt{e\cot(c+dx)}\right)}{2\sqrt{2}\left(a^2+b^2\right)^3 d} - \frac{\left((a-b)\left(a^2+4ab+b^2\right)e^{9/2}\right)\mathrm{Subst}\left(\int\frac{1}{-1-x^2}\,dx,x,1-\frac{\sqrt{2}\sqrt{e\cot(c+dx)}}{\sqrt{e}}\right)}{\sqrt{2}\left(a^2+b^2\right)^3 d} + \frac{\left((a-b)\left(a^2+4ab+b^2\right)e^{9/2}\right)\mathrm{Subst}\left(\int\frac{1}{-1-x^2}\,dx,x,1+\frac{\sqrt{2}\sqrt{e\cot(c+dx)}}{\sqrt{e}}\right)}{\sqrt{2}\left(a^2+b^2\right)^3 d}$$

$$= \frac{a^{5/2}(15a^4+46a^2b^2+63b^4)\,e^{9/2}\arctan\left(\frac{\sqrt{b}\sqrt{e\cot(c+dx)}}{\sqrt{a}\sqrt{e}}\right)}{4b^{7/2}\left(a^2+b^2\right)^3 d}$$
$$+ \frac{(a-b)\left(a^2+4ab+b^2\right)e^{9/2}\arctan\left(1-\frac{\sqrt{2}\sqrt{e\cot(c+dx)}}{\sqrt{e}}\right)}{\sqrt{2}\left(a^2+b^2\right)^3 d}$$
$$- \frac{(a-b)\left(a^2+4ab+b^2\right)e^{9/2}\arctan\left(1+\frac{\sqrt{2}\sqrt{e\cot(c+dx)}}{\sqrt{e}}\right)}{\sqrt{2}\left(a^2+b^2\right)^3 d}$$
$$- \frac{(15a^4+31a^2b^2+8b^4)\,e^4\sqrt{e\cot(c+dx)}}{4b^3\left(a^2+b^2\right)^2 d}$$
$$+ \frac{a^2e^2(e\cot(c+dx))^{5/2}}{2b\left(a^2+b^2\right)d(a+b\cot(c+dx))^2} + \frac{a^2(5a^2+13b^2)\,e^3(e\cot(c+dx))^{3/2}}{4b^2\left(a^2+b^2\right)^2 d(a+b\cot(c+dx))}$$
$$- \frac{(a+b)\left(a^2-4ab+b^2\right)e^{9/2}\log\left(\sqrt{e}+\sqrt{e}\cot(c+dx)-\sqrt{2}\sqrt{e\cot(c+dx)}\right)}{2\sqrt{2}\left(a^2+b^2\right)^3 d}$$
$$+ \frac{(a+b)\left(a^2-4ab+b^2\right)e^{9/2}\log\left(\sqrt{e}+\sqrt{e}\cot(c+dx)+\sqrt{2}\sqrt{e\cot(c+dx)}\right)}{2\sqrt{2}\left(a^2+b^2\right)^3 d}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 6.32 (sec) , antiderivative size = 626, normalized size of antiderivative = 1.18

$$\int \frac{(e\cot(c+dx))^{9/2}}{(a+b\cot(c+dx))^3}\,dx =$$

$$-\frac{(e\cot(c+dx))^{9/2}\left(-\frac{2a^{9/2}(3a^2-b^2)\arctan\left(\frac{\sqrt{b}\sqrt{\cot(c+dx)}}{\sqrt{a}}\right)}{b^{7/2}(a^2+b^2)^3}+\frac{2a^4(3a^2-b^2)\sqrt{\cot(c+dx)}}{b^3(a^2+b^2)^3}-\frac{2a^3(3a^2-b^2)\cot^{\frac{3}{2}}(c+dx)}{3b^2(a^2+b^2)^3}+\frac{2a^2(3a^2-}{5b}\right.}{}$$

```
[In] Integrate[(e*Cot[c + d*x])^(9/2)/(a + b*Cot[c + d*x])^3,x]
```

```
[Out] -(((e*Cot[c + d*x])^(9/2)*((-2*a^(9/2)*(3*a^2 - b^2)*ArcTan[(Sqrt[b]*Sqrt[C
ot[c + d*x]])/Sqrt[a]])/(b^(7/2)*(a^2 + b^2)^3) + (2*a^4*(3*a^2 - b^2)*Sqrt
[Cot[c + d*x]])/(b^3*(a^2 + b^2)^3) - (2*a^3*(3*a^2 - b^2)*Cot[c + d*x]^(3/
2))/(3*b^2*(a^2 + b^2)^3) + (2*a^2*(3*a^2 - b^2)*Cot[c + d*x]^(5/2))/(5*b*(
a^2 + b^2)^3) - (2*a*(3*a^2 - b^2)*Cot[c + d*x]^(7/2))/(7*(a^2 + b^2)^3) +
(2*b*(3*a^2 - b^2)*Cot[c + d*x]^(9/2))/(9*(a^2 + b^2)^3) - (2*a*(a^2 - 3*b^
2)*(7*Cot[c + d*x]^(3/2) - 3*Cot[c + d*x]^(7/2) - 7*Cot[c + d*x]^(3/2)*Hype
rgeometric2F1[3/4, 1, 7/4, -Cot[c + d*x]^2]))/(21*(a^2 + b^2)^3) + (4*b^2*C
ot[c + d*x]^(11/2)*Hypergeometric2F1[2, 11/2, 13/2, -((b*Cot[c + d*x])/a)])
/(11*a*(a^2 + b^2)^2) + (2*b^2*Cot[c + d*x]^(11/2)*Hypergeometric2F1[3, 11/
```

```
2, 13/2, -((b*Cot[c + d*x])/a)])/(11*a^3*(a^2 + b^2)) - (b*(3*a^2 - b^2)*(9
0*Sqrt[2]*ArcTan[1 - Sqrt[2]*Sqrt[Cot[c + d*x]]] - 90*Sqrt[2]*ArcTan[1 + Sq
rt[2]*Sqrt[Cot[c + d*x]]] + 360*Sqrt[Cot[c + d*x]] - 72*Cot[c + d*x]^(5/2)
+ 40*Cot[c + d*x]^(9/2) + 45*Sqrt[2]*Log[1 - Sqrt[2]*Sqrt[Cot[c + d*x]] + C
ot[c + d*x]] - 45*Sqrt[2]*Log[1 + Sqrt[2]*Sqrt[Cot[c + d*x]] + Cot[c + d*x]
]))/(180*(a^2 + b^2)^3)))/(d*Cot[c + d*x]^(9/2)))
```

Maple [A] (verified)

Time = 1.20 (sec) , antiderivative size = 471, normalized size of antiderivative = 0.89

method	result
derivativedivides	$-\frac{2e^4\left(\frac{\sqrt{e\cot(dx+c)}}{b^3}-\frac{a^3 e\left(\frac{\left(-\frac{9}{8}a^4b-\frac{13}{4}a^2b^3-\frac{17}{8}b^5\right)(e\cot(dx+c))^{\frac{3}{2}}-\frac{ae\left(7a^4+22a^2b^2+15b^4\right)\sqrt{e\cot(dx+c)}}{8}}{(e\cot(dx+c)b+ae)^2}+\frac{\left(15a^4+46a^2b^2+63 \cdots\right.}{\cdots}\right)}{b^3\left(a^2+b^2\right)^3}\cdots\right)}{\cdots}$
default	$-\frac{2e^4\left(\frac{\sqrt{e\cot(dx+c)}}{b^3}-\frac{a^3 e\left(\frac{\left(-\frac{9}{8}a^4b-\frac{13}{4}a^2b^3-\frac{17}{8}b^5\right)(e\cot(dx+c))^{\frac{3}{2}}-\frac{ae\left(7a^4+22a^2b^2+15b^4\right)\sqrt{e\cot(dx+c)}}{8}}{(e\cot(dx+c)b+ae)^2}+\frac{\left(15a^4+46a^2b^2+63 \cdots\right.}{\cdots}\right)}{b^3\left(a^2+b^2\right)^3}\cdots\right)}{\cdots}$

```
[In] int((e*cot(d*x+c))^(9/2)/(a+b*cot(d*x+c))^3,x,method=_RETURNVERBOSE)
```

```
[Out] -2/d*e^4*((e*cot(d*x+c))^(1/2)/b^3-a^3*e/b^3/(a^2+b^2)^3*(((-9/8*a^4*b-13/4
*a^2*b^3-17/8*b^5)*(e*cot(d*x+c))^(3/2)-1/8*a*e*(7*a^4+22*a^2*b^2+15*b^4)*(
e*cot(d*x+c))^(1/2))/(e*cot(d*x+c)*b+a*e)^2+1/8*(15*a^4+46*a^2*b^2+63*b^4)/
(a*e*b)^(1/2)*arctan((e*cot(d*x+c))^(1/2)*b/(a*e*b)^(1/2)))+e/(a^2+b^2)^3*(
1/8*(3*a^2*b*e-b^3*e)*(e^2)^(1/4)/e^2*2^(1/2)*(ln((e*cot(d*x+c)+(e^2)^(1/4)
*(e*cot(d*x+c))^(1/2)*2^(1/2)+(e^2)^(1/2))/(e*cot(d*x+c)-(e^2)^(1/4)*(e*cot
(d*x+c))^(1/2)*2^(1/2)+(e^2)^(1/2)))+2*arctan(2^(1/2)/(e^2)^(1/4)*(e*cot(d*
x+c))^(1/2)+1)-2*arctan(-2^(1/2)/(e^2)^(1/4)*(e*cot(d*x+c))^(1/2)+1))+1/8*(
a^3-3*a*b^2)/(e^2)^(1/4)*2^(1/2)*(ln((e*cot(d*x+c)-(e^2)^(1/4)*(e*cot(d*x+c
))^(1/2)*2^(1/2)+(e^2)^(1/2))/(e*cot(d*x+c)+(e^2)^(1/4)*(e*cot(d*x+c))^(1/2
)*2^(1/2)+(e^2)^(1/2)))+2*arctan(2^(1/2)/(e^2)^(1/4)*(e*cot(d*x+c))^(1/2)+1
)-2*arctan(-2^(1/2)/(e^2)^(1/4)*(e*cot(d*x+c))^(1/2)+1))))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 4519 vs. $2(454) = 908$.

Time = 2.08 (sec) , antiderivative size = 9101, normalized size of antiderivative = 17.20

$$\int \frac{(e\cot(c+dx))^{9/2}}{(a+b\cot(c+dx))^3}\,dx = \text{Too large to display}$$

```
[In] integrate((e*cot(d*x+c))^(9/2)/(a+b*cot(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] Too large to include
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(e\cot(c+dx))^{9/2}}{(a+b\cot(c+dx))^3}\,dx = \text{Timed out}$$

```
[In] integrate((e*cot(d*x+c))**(9/2)/(a+b*cot(d*x+c))**3,x)
```

```
[Out] Timed out
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{(e\cot(c+dx))^{9/2}}{(a+b\cot(c+dx))^3}\,dx = \text{Exception raised: ValueError}$$

```
[In] integrate((e*cot(d*x+c))^(9/2)/(a+b*cot(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(e>0)', see `assume?` for more detai
ls)Is e
```

Giac [F]

$$\int \frac{(e\cot(c+dx))^{9/2}}{(a+b\cot(c+dx))^3}\,dx = \int \frac{(e\cot(dx+c))^{\frac{9}{2}}}{(b\cot(dx+c)+a)^3}\,dx$$

[In] integrate((e*cot(d*x+c))^(9/2)/(a+b*cot(d*x+c))^3,x, algorithm="giac")

[Out] integrate((e*cot(d*x + c))^(9/2)/(b*cot(d*x + c) + a)^3, x)

Mupad [B] (verification not implemented)

Time = 23.64 (sec) , antiderivative size = 20651, normalized size of antiderivative = 39.04

$$\int \frac{(e\cot(c+dx))^{9/2}}{(a+b\cot(c+dx))^3}\,dx = \text{Too large to display}$$

[In] int((e*cot(c + d*x))^(9/2)/(a + b*cot(c + d*x))^3,x)

[Out] atan((((((((128*a*b^26*d^4*e^15 + 3648*a^3*b^24*d^4*e^15 + 25536*a^5*b^22*d^4*e^15 + 88320*a^7*b^20*d^4*e^15 + 182784*a^9*b^18*d^4*e^15 + 244608*a^11*b^16*d^4*e^15 + 217728*a^13*b^14*d^4*e^15 + 128256*a^15*b^12*d^4*e^15 + 48000*a^17*b^10*d^4*e^15 + 10304*a^19*b^8*d^4*e^15 + 960*a^21*b^6*d^4*e^15)/(b^21*d^5 + 8*a^2*b^19*d^5 + 28*a^4*b^17*d^5 + 56*a^6*b^15*d^5 + 70*a^8*b^13*d^5 + 56*a^10*b^11*d^5 + 28*a^12*b^9*d^5 + 8*a^14*b^7*d^5 + a^16*b^5*d^5) + ((e*cot(c + d*x))^(1/2)*(-(e^9*1i)/(4*(b^6*d^2 - a^6*d^2 + a*b^5*d^2*6i + a^5*b*d^2*6i - 15*a^2*b^4*d^2 - a^3*b^3*d^2*20i + 15*a^4*b^2*d^2)))^(1/2)*(512*b^30*d^4*e^10 + 4608*a^2*b^28*d^4*e^10 + 17920*a^4*b^26*d^4*e^10 + 38400*a^6*b^24*d^4*e^10 + 46080*a^8*b^22*d^4*e^10 + 21504*a^10*b^20*d^4*e^10 - 21504*a^12*b^18*d^4*e^10 - 46080*a^14*b^16*d^4*e^10 - 38400*a^16*b^14*d^4*e^10 - 17920*a^18*b^12*d^4*e^10 - 4608*a^20*b^10*d^4*e^10 - 512*a^22*b^8*d^4*e^10))/(b^21*d^4 + 8*a^2*b^19*d^4 + 28*a^4*b^17*d^4 + 56*a^6*b^15*d^4 + 70*a^8*b^13*d^4 + 56*a^10*b^11*d^4 + 28*a^12*b^9*d^4 + 8*a^14*b^7*d^4 + a^16*b^5*d^4))*(-(e^9*1i)/(4*(b^6*d^2 - a^6*d^2 + a*b^5*d^2*6i + a^5*b*d^2*6i - 15*a^2*b^4*d^2 - a^3*b^3*d^2*20i + 15*a^4*b^2*d^2)))^(1/2) - ((e*cot(c + d*x))^(1/2)*(1800*a^23*b*d^2*e^19 - 1472*a*b^23*d^2*e^19 - 1024*a^3*b^21*d^2*e^19 + 8448*a^5*b^19*d^2*e^19 + 46088*a^7*b^17*d^2*e^19 + 177344*a^9*b^15*d^2*e^19 + 402912*a^11*b^13*d^2*e^19 + 541632*a^13*b^11*d^2*e^19 + 455472*a^15*b^9*d^2*e^19 + 248064*a^17*b^7*d^2*e^19 + 87008*a^19*b^5*d^2*e^19 + 18240*a^21*b^3*d^2*e^19))/(b^21*d^4 + 8*a^2*b^19*d^4 + 28*a^4*b^17*d^4 + 56*a^6*b^15*d^4 + 70*a^8*b^13*d^4 + 56*a^10*b^11*d^4 + 28*a^12*b^9*d^4 + 8*a^14*b^7*d^4 + a^16*b^5*d^4))*(-(e^9*1i)/(4*(b^6*d^2 - a^6*d^2 + a*b^5*d^2*6i + a^5*b*d^2*6i - 15*a^2*b^4*d^2 - a^3*b^3*d^2*20i + 15*a^4*b^2*d^2)))^(1/2) + (2250*a^20*b*d^2*e^24 + 32*a^2*b^19*d^2*e^24 + 12288*a^4*b^17*d^2*e^24 - 10974*a^6*b^15*d^2*e^24 - 105162*a^8*b^13*d^2*e^24 - 150758*a^10*b^11*d^2*e^24

 - 85314*a^12*b^9*d^2*e^24 - 3578*a^14*b^7*d^2*e^24 + 22210*a^16*b^5*d^2*e^
24 + 11550*a^18*b^3*d^2*e^24)/(b^21*d^5 + 8*a^2*b^19*d^5 + 28*a^4*b^17*d^5
+ 56*a^6*b^15*d^5 + 70*a^8*b^13*d^5 + 56*a^10*b^11*d^5 + 28*a^12*b^9*d^5 +
8*a^14*b^7*d^5 + a^16*b^5*d^5))*(-(e^9*1i)/(4*(b^6*d^2 - a^6*d^2 + a*b^5*d^
2*6i + a^5*b*d^2*6i - 15*a^2*b^4*d^2 - a^3*b^3*d^2*20i + 15*a^4*b^2*d^2)))^
(1/2) + ((e*cot(c + d*x))^(1/2)*(32*b^18*e^28 - 225*a^18*e^28 + 128*a^2*b^1
6*e^28 + 192*a^4*b^14*e^28 - 3841*a^6*b^12*e^28 + 18050*a^8*b^10*e^28 + 268
01*a^10*b^8*e^28 + 16860*a^12*b^6*e^28 + 4049*a^14*b^4*e^28 - 30*a^16*b^2*e
^28))/(b^21*d^4 + 8*a^2*b^19*d^4 + 28*a^4*b^17*d^4 + 56*a^6*b^15*d^4 + 70*a
^8*b^13*d^4 + 56*a^10*b^11*d^4 + 28*a^12*b^9*d^4 + 8*a^14*b^7*d^4 + a^16*b^
5*d^4))*(-(e^9*1i)/(4*(b^6*d^2 - a^6*d^2 + a*b^5*d^2*6i + a^5*b*d^2*6i - 15
*a^2*b^4*d^2 - a^3*b^3*d^2*20i + 15*a^4*b^2*d^2)))^(1/2)*1i - (((((128*a*b^
26*d^4*e^15 + 3648*a^3*b^24*d^4*e^15 + 25536*a^5*b^22*d^4*e^15 + 88320*a^7*
b^20*d^4*e^15 + 182784*a^9*b^18*d^4*e^15 + 244608*a^11*b^16*d^4*e^15 + 2177
28*a^13*b^14*d^4*e^15 + 128256*a^15*b^12*d^4*e^15 + 48000*a^17*b^10*d^4*e^1
5 + 10304*a^19*b^8*d^4*e^15 + 960*a^21*b^6*d^4*e^15)/(b^21*d^5 + 8*a^2*b^19
*d^5 + 28*a^4*b^17*d^5 + 56*a^6*b^15*d^5 + 70*a^8*b^13*d^5 + 56*a^10*b^11*d
^5 + 28*a^12*b^9*d^5 + 8*a^14*b^7*d^5 + a^16*b^5*d^5) - ((e*cot(c + d*x))^(
1/2)*(-(e^9*1i)/(4*(b^6*d^2 - a^6*d^2 + a*b^5*d^2*6i + a^5*b*d^2*6i - 15*a^
2*b^4*d^2 - a^3*b^3*d^2*20i + 15*a^4*b^2*d^2)))^(1/2)*(512*b^30*d^4*e^10 +
4608*a^2*b^28*d^4*e^10 + 17920*a^4*b^26*d^4*e^10 + 38400*a^6*b^24*d^4*e^10
+ 46080*a^8*b^22*d^4*e^10 + 21504*a^10*b^20*d^4*e^10 - 21504*a^12*b^18*d^4*
e^10 - 46080*a^14*b^16*d^4*e^10 - 38400*a^16*b^14*d^4*e^10 - 17920*a^18*b^1
2*d^4*e^10 - 4608*a^20*b^10*d^4*e^10 - 512*a^22*b^8*d^4*e^10))/(b^21*d^4 +
8*a^2*b^19*d^4 + 28*a^4*b^17*d^4 + 56*a^6*b^15*d^4 + 70*a^8*b^13*d^4 + 56*a
^10*b^11*d^4 + 28*a^12*b^9*d^4 + 8*a^14*b^7*d^4 + a^16*b^5*d^4))*(-(e^9*1i)
/(4*(b^6*d^2 - a^6*d^2 + a*b^5*d^2*6i + a^5*b*d^2*6i - 15*a^2*b^4*d^2 - a^3
*b^3*d^2*20i + 15*a^4*b^2*d^2)))^(1/2) + ((e*cot(c + d*x))^(1/2)*(1800*a^23
*b*d^2*e^19 - 1472*a*b^23*d^2*e^19 - 1024*a^3*b^21*d^2*e^19 + 8448*a^5*b^19
*d^2*e^19 + 46088*a^7*b^17*d^2*e^19 + 177344*a^9*b^15*d^2*e^19 + 402912*a^1
1*b^13*d^2*e^19 + 541632*a^13*b^11*d^2*e^19 + 455472*a^15*b^9*d^2*e^19 + 24
8064*a^17*b^7*d^2*e^19 + 87008*a^19*b^5*d^2*e^19 + 18240*a^21*b^3*d^2*e^19)
)/(b^21*d^4 + 8*a^2*b^19*d^4 + 28*a^4*b^17*d^4 + 56*a^6*b^15*d^4 + 70*a^8*b
^13*d^4 + 56*a^10*b^11*d^4 + 28*a^12*b^9*d^4 + 8*a^14*b^7*d^4 + a^16*b^5*d^
4))*(-(e^9*1i)/(4*(b^6*d^2 - a^6*d^2 + a*b^5*d^2*6i + a^5*b*d^2*6i - 15*a^2
*b^4*d^2 - a^3*b^3*d^2*20i + 15*a^4*b^2*d^2)))^(1/2) + (2250*a^20*b*d^2*e^2
4 + 32*a^2*b^19*d^2*e^24 + 12288*a^4*b^17*d^2*e^24 - 10974*a^6*b^15*d^2*e^2
4 - 105162*a^8*b^13*d^2*e^24 - 150758*a^10*b^11*d^2*e^24 - 85314*a^12*b^9*d
^2*e^24 - 3578*a^14*b^7*d^2*e^24 + 22210*a^16*b^5*d^2*e^24 + 11550*a^18*b^3
*d^2*e^24)/(b^21*d^5 + 8*a^2*b^19*d^5 + 28*a^4*b^17*d^5 + 56*a^6*b^15*d^5 +
 70*a^8*b^13*d^5 + 56*a^10*b^11*d^5 + 28*a^12*b^9*d^5 + 8*a^14*b^7*d^5 + a^
16*b^5*d^5))*(-(e^9*1i)/(4*(b^6*d^2 - a^6*d^2 + a*b^5*d^2*6i + a^5*b*d^2*6i
 - 15*a^2*b^4*d^2 - a^3*b^3*d^2*20i + 15*a^4*b^2*d^2)))^(1/2) - ((e*cot(c +
 d*x))^(1/2)*(32*b^18*e^28 - 225*a^18*e^28 + 128*a^2*b^16*e^28 + 192*a^4*b^
14*e^28 - 3841*a^6*b^12*e^28 + 18050*a^8*b^10*e^28 + 26801*a^10*b^8*e^28 +

16860*a^12*b^6*e^28 + 4049*a^14*b^4*e^28 - 30*a^16*b^2*e^28))/(b^21*d^4 + 8
*a^2*b^19*d^4 + 28*a^4*b^17*d^4 + 56*a^6*b^15*d^4 + 70*a^8*b^13*d^4 + 56*a^
10*b^11*d^4 + 28*a^12*b^9*d^4 + 8*a^14*b^7*d^4 + a^16*b^5*d^4))*(-(e^9*1i)/
(4*(b^6*d^2 - a^6*d^2 + a*b^5*d^2*6i + a^5*b*d^2*6i - 15*a^2*b^4*d^2 - a^3*
b^3*d^2*20i + 15*a^4*b^2*d^2)))^(1/2)*1i)/((225*a^15*e^33 + 504*a^3*b^12*e^
33 + 872*a^5*b^10*e^33 + 4457*a^7*b^8*e^33 + 5916*a^9*b^6*e^33 + 4006*a^11*
b^4*e^33 + 1380*a^13*b^2*e^33)/(b^21*d^5 + 8*a^2*b^19*d^5 + 28*a^4*b^17*d^5
 + 56*a^6*b^15*d^5 + 70*a^8*b^13*d^5 + 56*a^10*b^11*d^5 + 28*a^12*b^9*d^5 +
 8*a^14*b^7*d^5 + a^16*b^5*d^5) + (((((128*a*b^26*d^4*e^15 + 3648*a^3*b^24*
d^4*e^15 + 25536*a^5*b^22*d^4*e^15 + 88320*a^7*b^20*d^4*e^15 + 182784*a^9*b
^18*d^4*e^15 + 244608*a^11*b^16*d^4*e^15 + 217728*a^13*b^14*d^4*e^15 + 1282
56*a^15*b^12*d^4*e^15 + 48000*a^17*b^10*d^4*e^15 + 10304*a^19*b^8*d^4*e^15
+ 960*a^21*b^6*d^4*e^15)/(b^21*d^5 + 8*a^2*b^19*d^5 + 28*a^4*b^17*d^5 + 56*
a^6*b^15*d^5 + 70*a^8*b^13*d^5 + 56*a^10*b^11*d^5 + 28*a^12*b^9*d^5 + 8*a^1
4*b^7*d^5 + a^16*b^5*d^5) + ((e*cot(c + d*x))^(1/2)*(-(e^9*1i)/(4*(b^6*d^2
- a^6*d^2 + a*b^5*d^2*6i + a^5*b*d^2*6i - 15*a^2*b^4*d^2 - a^3*b^3*d^2*20i
+ 15*a^4*b^2*d^2)))^(1/2)*(512*b^30*d^4*e^10 + 4608*a^2*b^28*d^4*e^10 + 179
20*a^4*b^26*d^4*e^10 + 38400*a^6*b^24*d^4*e^10 + 46080*a^8*b^22*d^4*e^10 +
21504*a^10*b^20*d^4*e^10 - 21504*a^12*b^18*d^4*e^10 - 46080*a^14*b^16*d^4*e
^10 - 38400*a^16*b^14*d^4*e^10 - 17920*a^18*b^12*d^4*e^10 - 4608*a^20*b^10*
d^4*e^10 - 512*a^22*b^8*d^4*e^10))/(b^21*d^4 + 8*a^2*b^19*d^4 + 28*a^4*b^17
*d^4 + 56*a^6*b^15*d^4 + 70*a^8*b^13*d^4 + 56*a^10*b^11*d^4 + 28*a^12*b^9*d
^4 + 8*a^14*b^7*d^4 + a^16*b^5*d^4))*(-(e^9*1i)/(4*(b^6*d^2 - a^6*d^2 + a*b
^5*d^2*6i + a^5*b*d^2*6i - 15*a^2*b^4*d^2 - a^3*b^3*d^2*20i + 15*a^4*b^2*d^
2)))^(1/2) - ((e*cot(c + d*x))^(1/2)*(1800*a^23*b*d^2*e^19 - 1472*a*b^23*d^
2*e^19 - 1024*a^3*b^21*d^2*e^19 + 8448*a^5*b^19*d^2*e^19 + 46088*a^7*b^17*d
^2*e^19 + 177344*a^9*b^15*d^2*e^19 + 402912*a^11*b^13*d^2*e^19 + 541632*a^1
3*b^11*d^2*e^19 + 455472*a^15*b^9*d^2*e^19 + 248064*a^17*b^7*d^2*e^19 + 870
08*a^19*b^5*d^2*e^19 + 18240*a^21*b^3*d^2*e^19))/(b^21*d^4 + 8*a^2*b^19*d^4
 + 28*a^4*b^17*d^4 + 56*a^6*b^15*d^4 + 70*a^8*b^13*d^4 + 56*a^10*b^11*d^4 +
 28*a^12*b^9*d^4 + 8*a^14*b^7*d^4 + a^16*b^5*d^4))*(-(e^9*1i)/(4*(b^6*d^2 -
 a^6*d^2 + a*b^5*d^2*6i + a^5*b*d^2*6i - 15*a^2*b^4*d^2 - a^3*b^3*d^2*20i +
 15*a^4*b^2*d^2)))^(1/2) + (2250*a^20*b*d^2*e^24 + 32*a^2*b^19*d^2*e^24 + 1
2288*a^4*b^17*d^2*e^24 - 10974*a^6*b^15*d^2*e^24 - 105162*a^8*b^13*d^2*e^24
 - 150758*a^10*b^11*d^2*e^24 - 85314*a^12*b^9*d^2*e^24 - 3578*a^14*b^7*d^2*
e^24 + 22210*a^16*b^5*d^2*e^24 + 11550*a^18*b^3*d^2*e^24)/(b^21*d^5 + 8*a^2
*b^19*d^5 + 28*a^4*b^17*d^5 + 56*a^6*b^15*d^5 + 70*a^8*b^13*d^5 + 56*a^10*b
^11*d^5 + 28*a^12*b^9*d^5 + 8*a^14*b^7*d^5 + a^16*b^5*d^5))*(-(e^9*1i)/(4*(
b^6*d^2 - a^6*d^2 + a*b^5*d^2*6i + a^5*b*d^2*6i - 15*a^2*b^4*d^2 - a^3*b^3*
d^2*20i + 15*a^4*b^2*d^2)))^(1/2) + ((e*cot(c + d*x))^(1/2)*(32*b^18*e^28 -
 225*a^18*e^28 + 128*a^2*b^16*e^28 + 192*a^4*b^14*e^28 - 3841*a^6*b^12*e^28
 + 18050*a^8*b^10*e^28 + 26801*a^10*b^8*e^28 + 16860*a^12*b^6*e^28 + 4049*a
^14*b^4*e^28 - 30*a^16*b^2*e^28))/(b^21*d^4 + 8*a^2*b^19*d^4 + 28*a^4*b^17*
d^4 + 56*a^6*b^15*d^4 + 70*a^8*b^13*d^4 + 56*a^10*b^11*d^4 + 28*a^12*b^9*d^
4 + 8*a^14*b^7*d^4 + a^16*b^5*d^4))*(-(e^9*1i)/(4*(b^6*d^2 - a^6*d^2 + a*b^

5*d^2*6i + a^5*b*d^2*6i - 15*a^2*b^4*d^2 - a^3*b^3*d^2*20i + 15*a^4*b^2*d^2
)))^(1/2) + (((((128*a*b^26*d^4*e^15 + 3648*a^3*b^24*d^4*e^15 + 25536*a^5*b
^22*d^4*e^15 + 88320*a^7*b^20*d^4*e^15 + 182784*a^9*b^18*d^4*e^15 + 244608*
a^11*b^16*d^4*e^15 + 217728*a^13*b^14*d^4*e^15 + 128256*a^15*b^12*d^4*e^15
+ 48000*a^17*b^10*d^4*e^15 + 10304*a^19*b^8*d^4*e^15 + 960*a^21*b^6*d^4*e^1
5)/(b^21*d^5 + 8*a^2*b^19*d^5 + 28*a^4*b^17*d^5 + 56*a^6*b^15*d^5 + 70*a^8*
b^13*d^5 + 56*a^10*b^11*d^5 + 28*a^12*b^9*d^5 + 8*a^14*b^7*d^5 + a^16*b^5*d
^5) - ((e*cot(c + d*x))^(1/2)*(-(e^9*1i)/(4*(b^6*d^2 - a^6*d^2 + a*b^5*d^2*
6i + a^5*b*d^2*6i - 15*a^2*b^4*d^2 - a^3*b^3*d^2*20i + 15*a^4*b^2*d^2)))^(1
/2)*(512*b^30*d^4*e^10 + 4608*a^2*b^28*d^4*e^10 + 17920*a^4*b^26*d^4*e^10 +
38400*a^6*b^24*d^4*e^10 + 46080*a^8*b^22*d^4*e^10 + 21504*a^10*b^20*d^4*e^
10 - 21504*a^12*b^18*d^4*e^10 - 46080*a^14*b^16*d^4*e^10 - 38400*a^16*b^14*
d^4*e^10 - 17920*a^18*b^12*d^4*e^10 - 4608*a^20*b^10*d^4*e^10 - 512*a^22*b^
8*d^4*e^10))/(b^21*d^4 + 8*a^2*b^19*d^4 + 28*a^4*b^17*d^4 + 56*a^6*b^15*d^4
+ 70*a^8*b^13*d^4 + 56*a^10*b^11*d^4 + 28*a^12*b^9*d^4 + 8*a^14*b^7*d^4 +
a^16*b^5*d^4))*(-(e^9*1i)/(4*(b^6*d^2 - a^6*d^2 + a*b^5*d^2*6i + a^5*b*d^2*
6i - 15*a^2*b^4*d^2 - a^3*b^3*d^2*20i + 15*a^4*b^2*d^2)))^(1/2) + ((e*cot(c
+ d*x))^(1/2)*(1800*a^23*b*d^2*e^19 - 1472*a*b^23*d^2*e^19 - 1024*a^3*b^21
*d^2*e^19 + 8448*a^5*b^19*d^2*e^19 + 46088*a^7*b^17*d^2*e^19 + 177344*a^9*b
^15*d^2*e^19 + 402912*a^11*b^13*d^2*e^19 + 541632*a^13*b^11*d^2*e^19 + 4554
72*a^15*b^9*d^2*e^19 + 248064*a^17*b^7*d^2*e^19 + 87008*a^19*b^5*d^2*e^19 +
18240*a^21*b^3*d^2*e^19))/(b^21*d^4 + 8*a^2*b^19*d^4 + 28*a^4*b^17*d^4 + 5
6*a^6*b^15*d^4 + 70*a^8*b^13*d^4 + 56*a^10*b^11*d^4 + 28*a^12*b^9*d^4 + 8*a
^14*b^7*d^4 + a^16*b^5*d^4))*(-(e^9*1i)/(4*(b^6*d^2 - a^6*d^2 + a*b^5*d^2*6
i + a^5*b*d^2*6i - 15*a^2*b^4*d^2 - a^3*b^3*d^2*20i + 15*a^4*b^2*d^2)))^(1/
2) + (2250*a^20*b*d^2*e^24 + 32*a^2*b^19*d^2*e^24 + 12288*a^4*b^17*d^2*e^24
- 10974*a^6*b^15*d^2*e^24 - 105162*a^8*b^13*d^2*e^24 - 150758*a^10*b^11*d^
2*e^24 - 85314*a^12*b^9*d^2*e^24 - 3578*a^14*b^7*d^2*e^24 + 22210*a^16*b^5*
d^2*e^24 + 11550*a^18*b^3*d^2*e^24)/(b^21*d^5 + 8*a^2*b^19*d^5 + 28*a^4*b^1
7*d^5 + 56*a^6*b^15*d^5 + 70*a^8*b^13*d^5 + 56*a^10*b^11*d^5 + 28*a^12*b^9*
d^5 + 8*a^14*b^7*d^5 + a^16*b^5*d^5))*(-(e^9*1i)/(4*(b^6*d^2 - a^6*d^2 + a*
b^5*d^2*6i + a^5*b*d^2*6i - 15*a^2*b^4*d^2 - a^3*b^3*d^2*20i + 15*a^4*b^2*d
^2)))^(1/2) - ((e*cot(c + d*x))^(1/2)*(32*b^18*e^28 - 225*a^18*e^28 + 128*a
^2*b^16*e^28 + 192*a^4*b^14*e^28 - 3841*a^6*b^12*e^28 + 18050*a^8*b^10*e^28
+ 26801*a^10*b^8*e^28 + 16860*a^12*b^6*e^28 + 4049*a^14*b^4*e^28 - 30*a^16
*b^2*e^28))/(b^21*d^4 + 8*a^2*b^19*d^4 + 28*a^4*b^17*d^4 + 56*a^6*b^15*d^4
+ 70*a^8*b^13*d^4 + 56*a^10*b^11*d^4 + 28*a^12*b^9*d^4 + 8*a^14*b^7*d^4 + a
^16*b^5*d^4))*(-(e^9*1i)/(4*(b^6*d^2 - a^6*d^2 + a*b^5*d^2*6i + a^5*b*d^2*6
i - 15*a^2*b^4*d^2 - a^3*b^3*d^2*20i + 15*a^4*b^2*d^2)))^(1/2)))*(-(e^9*1i)
/(4*(b^6*d^2 - a^6*d^2 + a*b^5*d^2*6i + a^5*b*d^2*6i - 15*a^2*b^4*d^2 - a^3
*b^3*d^2*20i + 15*a^4*b^2*d^2)))^(1/2)*2i - (((e*cot(c + d*x))^(1/2)*(7*a^6
*e^6 + 15*a^4*b^2*e^6))/(4*(a^4 + b^4 + 2*a^2*b^2)) + (b*(e*cot(c + d*x))^(
3/2)*(9*a^5*e^5 + 17*a^3*b^2*e^5))/(4*(a^4 + b^4 + 2*a^2*b^2)))/(a^2*b^3*d*
e^2 + b^5*d*e^2*cot(c + d*x)^2 + 2*a*b^4*d*e^2*cot(c + d*x)) + atan((((((1
28*a*b^26*d^4*e^15 + 3648*a^3*b^24*d^4*e^15 + 25536*a^5*b^22*d^4*e^15 + 883

20*a^7*b^20*d^4*e^15 + 182784*a^9*b^18*d^4*e^15 + 244608*a^11*b^16*d^4*e^15 + 217728*a^13*b^14*d^4*e^15 + 128256*a^15*b^12*d^4*e^15 + 48000*a^17*b^10*d^4*e^15 + 10304*a^19*b^8*d^4*e^15 + 960*a^21*b^6*d^4*e^15)/(b^21*d^5 + 8*a^2*b^19*d^5 + 28*a^4*b^17*d^5 + 56*a^6*b^15*d^5 + 70*a^8*b^13*d^5 + 56*a^10*b^11*d^5 + 28*a^12*b^9*d^5 + 8*a^14*b^7*d^5 + a^16*b^5*d^5) + ((e*cot(c + d*x))^(1/2)*(-e^9/(4*(b^6*d^2*1i - a^6*d^2*1i + 6*a*b^5*d^2 + 6*a^5*b*d^2 - a^2*b^4*d^2*15i - 20*a^3*b^3*d^2 + a^4*b^2*d^2*15i)))^(1/2)*(512*b^30*d^4*e^10 + 4608*a^2*b^28*d^4*e^10 + 17920*a^4*b^26*d^4*e^10 + 38400*a^6*b^24*d^4*e^10 + 46080*a^8*b^22*d^4*e^10 + 21504*a^10*b^20*d^4*e^10 - 21504*a^12*b^18*d^4*e^10 - 46080*a^14*b^16*d^4*e^10 - 38400*a^16*b^14*d^4*e^10 - 17920*a^18*b^12*d^4*e^10 - 4608*a^20*b^10*d^4*e^10 - 512*a^22*b^8*d^4*e^10))/(b^21*d^4 + 8*a^2*b^19*d^4 + 28*a^4*b^17*d^4 + 56*a^6*b^15*d^4 + 70*a^8*b^13*d^4 + 56*a^10*b^11*d^4 + 28*a^12*b^9*d^4 + 8*a^14*b^7*d^4 + a^16*b^5*d^4))*(-e^9/(4*(b^6*d^2*1i - a^6*d^2*1i + 6*a*b^5*d^2 + 6*a^5*b*d^2 - a^2*b^4*d^2*15i - 20*a^3*b^3*d^2 + a^4*b^2*d^2*15i)))^(1/2) - ((e*cot(c + d*x))^(1/2)*(1800*a^23*b*d^2*e^19 - 1472*a*b^23*d^2*e^19 - 1024*a^3*b^21*d^2*e^19 + 8448*a^5*b^19*d^2*e^19 + 46088*a^7*b^17*d^2*e^19 + 177344*a^9*b^15*d^2*e^19 + 402912*a^11*b^13*d^2*e^19 + 541632*a^13*b^11*d^2*e^19 + 455472*a^15*b^9*d^2*e^19 + 248064*a^17*b^7*d^2*e^19 + 87008*a^19*b^5*d^2*e^19 + 18240*a^21*b^3*d^2*e^19))/(b^21*d^4 + 8*a^2*b^19*d^4 + 28*a^4*b^17*d^4 + 56*a^6*b^15*d^4 + 70*a^8*b^13*d^4 + 56*a^10*b^11*d^4 + 28*a^12*b^9*d^4 + 8*a^14*b^7*d^4 + a^16*b^5*d^4))*(-e^9/(4*(b^6*d^2*1i - a^6*d^2*1i + 6*a*b^5*d^2 + 6*a^5*b*d^2 - a^2*b^4*d^2*15i - 20*a^3*b^3*d^2 + a^4*b^2*d^2*15i)))^(1/2) + (2250*a^20*b*d^2*e^24 + 32*a^2*b^19*d^2*e^24 + 12288*a^4*b^17*d^2*e^24 - 10974*a^6*b^15*d^2*e^24 - 105162*a^8*b^13*d^2*e^24 - 150758*a^10*b^11*d^2*e^24 - 85314*a^12*b^9*d^2*e^24 - 3578*a^14*b^7*d^2*e^24 + 22210*a^16*b^5*d^2*e^24 + 11550*a^18*b^3*d^2*e^24)/(b^21*d^5 + 8*a^2*b^19*d^5 + 28*a^4*b^17*d^5 + 56*a^6*b^15*d^5 + 70*a^8*b^13*d^5 + 56*a^10*b^11*d^5 + 28*a^12*b^9*d^5 + 8*a^14*b^7*d^5 + a^16*b^5*d^5))*(-e^9/(4*(b^6*d^2*1i - a^6*d^2*1i + 6*a*b^5*d^2 + 6*a^5*b*d^2 - a^2*b^4*d^2*15i - 20*a^3*b^3*d^2 + a^4*b^2*d^2*15i)))^(1/2) + ((e*cot(c + d*x))^(1/2)*(32*b^18*e^28 - 225*a^18*e^28 + 128*a^2*b^16*e^28 + 192*a^4*b^14*e^28 - 3841*a^6*b^12*e^28 + 18050*a^8*b^10*e^28 + 26801*a^10*b^8*e^28 + 16860*a^12*b^6*e^28 + 4049*a^14*b^4*e^28 - 30*a^16*b^2*e^28))/(b^21*d^4 + 8*a^2*b^19*d^4 + 28*a^4*b^17*d^4 + 56*a^6*b^15*d^4 + 70*a^8*b^13*d^4 + 56*a^10*b^11*d^4 + 28*a^12*b^9*d^4 + 8*a^14*b^7*d^4 + a^16*b^5*d^4))*(-e^9/(4*(b^6*d^2*1i - a^6*d^2*1i + 6*a*b^5*d^2 + 6*a^5*b*d^2 - a^2*b^4*d^2*15i - 20*a^3*b^3*d^2 + a^4*b^2*d^2*15i)))^(1/2)*1i - (((((128*a*b^26*d^4*e^15 + 3648*a^3*b^24*d^4*e^15 + 25536*a^5*b^22*d^4*e^15 + 88320*a^7*b^20*d^4*e^15 + 182784*a^9*b^18*d^4*e^15 + 244608*a^11*b^16*d^4*e^15 + 217728*a^13*b^14*d^4*e^15 + 128256*a^15*b^12*d^4*e^15 + 48000*a^17*b^10*d^4*e^15 + 10304*a^19*b^8*d^4*e^15 + 960*a^21*b^6*d^4*e^15)/(b^21*d^5 + 8*a^2*b^19*d^5 + 28*a^4*b^17*d^5 + 56*a^6*b^15*d^5 + 70*a^8*b^13*d^5 + 56*a^10*b^11*d^5 + 28*a^12*b^9*d^5 + 8*a^14*b^7*d^5 + a^16*b^5*d^5) - ((e*cot(c + d*x))^(1/2)*(-e^9/(4*(b^6*d^2*1i - a^6*d^2*1i + 6*a*b^5*d^2 + 6*a^5*b*d^2 - a^2*b^4*d^2*15i - 20*a^3*b^3*d^2 + a^4*b^2*d^2*15i)))^(1/2)*(512*b^30*d^4*e^10 + 4608*a^2*b^2

8*d^4*e^10 + 17920*a^4*b^26*d^4*e^10 + 38400*a^6*b^24*d^4*e^10 + 46080*a^8*
b^22*d^4*e^10 + 21504*a^10*b^20*d^4*e^10 - 21504*a^12*b^18*d^4*e^10 - 46080
*a^14*b^16*d^4*e^10 - 38400*a^16*b^14*d^4*e^10 - 17920*a^18*b^12*d^4*e^10 -
 4608*a^20*b^10*d^4*e^10 - 512*a^22*b^8*d^4*e^10))/(b^21*d^4 + 8*a^2*b^19*d
^4 + 28*a^4*b^17*d^4 + 56*a^6*b^15*d^4 + 70*a^8*b^13*d^4 + 56*a^10*b^11*d^4
 + 28*a^12*b^9*d^4 + 8*a^14*b^7*d^4 + a^16*b^5*d^4))*(-e^9/(4*(b^6*d^2*1i -
 a^6*d^2*1i + 6*a*b^5*d^2 + 6*a^5*b*d^2 - a^2*b^4*d^2*15i - 20*a^3*b^3*d^2
+ a^4*b^2*d^2*15i)))^(1/2) + ((e*cot(c + d*x))^(1/2)*(1800*a^23*b*d^2*e^19
- 1472*a*b^23*d^2*e^19 - 1024*a^3*b^21*d^2*e^19 + 8448*a^5*b^19*d^2*e^19 +
46088*a^7*b^17*d^2*e^19 + 177344*a^9*b^15*d^2*e^19 + 402912*a^11*b^13*d^2*e
^19 + 541632*a^13*b^11*d^2*e^19 + 455472*a^15*b^9*d^2*e^19 + 248064*a^17*b^
7*d^2*e^19 + 87008*a^19*b^5*d^2*e^19 + 18240*a^21*b^3*d^2*e^19))/(b^21*d^4
+ 8*a^2*b^19*d^4 + 28*a^4*b^17*d^4 + 56*a^6*b^15*d^4 + 70*a^8*b^13*d^4 + 56
*a^10*b^11*d^4 + 28*a^12*b^9*d^4 + 8*a^14*b^7*d^4 + a^16*b^5*d^4))*(-e^9/(4
*(b^6*d^2*1i - a^6*d^2*1i + 6*a*b^5*d^2 + 6*a^5*b*d^2 - a^2*b^4*d^2*15i - 2
0*a^3*b^3*d^2 + a^4*b^2*d^2*15i)))^(1/2) + (2250*a^20*b*d^2*e^24 + 32*a^2*b
^19*d^2*e^24 + 12288*a^4*b^17*d^2*e^24 - 10974*a^6*b^15*d^2*e^24 - 105162*a
^8*b^13*d^2*e^24 - 150758*a^10*b^11*d^2*e^24 - 85314*a^12*b^9*d^2*e^24 - 35
78*a^14*b^7*d^2*e^24 + 22210*a^16*b^5*d^2*e^24 + 11550*a^18*b^3*d^2*e^24)/(
b^21*d^5 + 8*a^2*b^19*d^5 + 28*a^4*b^17*d^5 + 56*a^6*b^15*d^5 + 70*a^8*b^13
*d^5 + 56*a^10*b^11*d^5 + 28*a^12*b^9*d^5 + 8*a^14*b^7*d^5 + a^16*b^5*d^5))
(-e^9/(4(b^6*d^2*1i - a^6*d^2*1i + 6*a*b^5*d^2 + 6*a^5*b*d^2 - a^2*b^4*d^
2*15i - 20*a^3*b^3*d^2 + a^4*b^2*d^2*15i)))^(1/2) - ((e*cot(c + d*x))^(1/2)
*(32*b^18*e^28 - 225*a^18*e^28 + 128*a^2*b^16*e^28 + 192*a^4*b^14*e^28 - 38
41*a^6*b^12*e^28 + 18050*a^8*b^10*e^28 + 26801*a^10*b^8*e^28 + 16860*a^12*b
^6*e^28 + 4049*a^14*b^4*e^28 - 30*a^16*b^2*e^28))/(b^21*d^4 + 8*a^2*b^19*d^
4 + 28*a^4*b^17*d^4 + 56*a^6*b^15*d^4 + 70*a^8*b^13*d^4 + 56*a^10*b^11*d^4
+ 28*a^12*b^9*d^4 + 8*a^14*b^7*d^4 + a^16*b^5*d^4))*(-e^9/(4*(b^6*d^2*1i -
a^6*d^2*1i + 6*a*b^5*d^2 + 6*a^5*b*d^2 - a^2*b^4*d^2*15i - 20*a^3*b^3*d^2 +
 a^4*b^2*d^2*15i)))^(1/2)*1i)/((225*a^15*e^33 + 504*a^3*b^12*e^33 + 872*a^5
*b^10*e^33 + 4457*a^7*b^8*e^33 + 5916*a^9*b^6*e^33 + 4006*a^11*b^4*e^33 + 1
380*a^13*b^2*e^33)/(b^21*d^5 + 8*a^2*b^19*d^5 + 28*a^4*b^17*d^5 + 56*a^6*b^
15*d^5 + 70*a^8*b^13*d^5 + 56*a^10*b^11*d^5 + 28*a^12*b^9*d^5 + 8*a^14*b^7*
d^5 + a^16*b^5*d^5) + (((((128*a*b^26*d^4*e^15 + 3648*a^3*b^24*d^4*e^15 + 2
5536*a^5*b^22*d^4*e^15 + 88320*a^7*b^20*d^4*e^15 + 182784*a^9*b^18*d^4*e^15
 + 244608*a^11*b^16*d^4*e^15 + 217728*a^13*b^14*d^4*e^15 + 128256*a^15*b^12
*d^4*e^15 + 48000*a^17*b^10*d^4*e^15 + 10304*a^19*b^8*d^4*e^15 + 960*a^21*b
^6*d^4*e^15)/(b^21*d^5 + 8*a^2*b^19*d^5 + 28*a^4*b^17*d^5 + 56*a^6*b^15*d^5
 + 70*a^8*b^13*d^5 + 56*a^10*b^11*d^5 + 28*a^12*b^9*d^5 + 8*a^14*b^7*d^5 +
a^16*b^5*d^5) + ((e*cot(c + d*x))^(1/2)*(-e^9/(4*(b^6*d^2*1i - a^6*d^2*1i +
 6*a*b^5*d^2 + 6*a^5*b*d^2 - a^2*b^4*d^2*15i - 20*a^3*b^3*d^2 + a^4*b^2*d^2
15i)))^(1/2)(512*b^30*d^4*e^10 + 4608*a^2*b^28*d^4*e^10 + 17920*a^4*b^26*
d^4*e^10 + 38400*a^6*b^24*d^4*e^10 + 46080*a^8*b^22*d^4*e^10 + 21504*a^10*b
^20*d^4*e^10 - 21504*a^12*b^18*d^4*e^10 - 46080*a^14*b^16*d^4*e^10 - 38400*
a^16*b^14*d^4*e^10 - 17920*a^18*b^12*d^4*e^10 - 4608*a^20*b^10*d^4*e^10 - 5

12*a^22*b^8*d^4*e^10))/(b^21*d^4 + 8*a^2*b^19*d^4 + 28*a^4*b^17*d^4 + 56*a^6*b^15*d^4 + 70*a^8*b^13*d^4 + 56*a^10*b^11*d^4 + 28*a^12*b^9*d^4 + 8*a^14*b^7*d^4 + a^16*b^5*d^4))*(-e^9/(4*(b^6*d^2*1i - a^6*d^2*1i + 6*a*b^5*d^2 + 6*a^5*b*d^2 - a^2*b^4*d^2*15i - 20*a^3*b^3*d^2 + a^4*b^2*d^2*15i)))^(1/2) - ((e*cot(c + d*x))^(1/2)*(1800*a^23*b*d^2*e^19 - 1472*a*b^23*d^2*e^19 - 1024*a^3*b^21*d^2*e^19 + 8448*a^5*b^19*d^2*e^19 + 46088*a^7*b^17*d^2*e^19 + 177344*a^9*b^15*d^2*e^19 + 402912*a^11*b^13*d^2*e^19 + 541632*a^13*b^11*d^2*e^19 + 455472*a^15*b^9*d^2*e^19 + 248064*a^17*b^7*d^2*e^19 + 87008*a^19*b^5*d^2*e^19 + 18240*a^21*b^3*d^2*e^19))/(b^21*d^4 + 8*a^2*b^19*d^4 + 28*a^4*b^17*d^4 + 56*a^6*b^15*d^4 + 70*a^8*b^13*d^4 + 56*a^10*b^11*d^4 + 28*a^12*b^9*d^4 + 8*a^14*b^7*d^4 + a^16*b^5*d^4))*(-e^9/(4*(b^6*d^2*1i - a^6*d^2*1i + 6*a*b^5*d^2 + 6*a^5*b*d^2 - a^2*b^4*d^2*15i - 20*a^3*b^3*d^2 + a^4*b^2*d^2*15i)))^(1/2) + (2250*a^20*b*d^2*e^24 + 32*a^2*b^19*d^2*e^24 + 12288*a^4*b^17*d^2*e^24 - 10974*a^6*b^15*d^2*e^24 - 105162*a^8*b^13*d^2*e^24 - 150758*a^10*b^11*d^2*e^24 - 85314*a^12*b^9*d^2*e^24 - 3578*a^14*b^7*d^2*e^24 + 22210*a^16*b^5*d^2*e^24 + 11550*a^18*b^3*d^2*e^24)/(b^21*d^5 + 8*a^2*b^19*d^5 + 28*a^4*b^17*d^5 + 56*a^6*b^15*d^5 + 70*a^8*b^13*d^5 + 56*a^10*b^11*d^5 + 28*a^12*b^9*d^5 + 8*a^14*b^7*d^5 + a^16*b^5*d^5))*(-e^9/(4*(b^6*d^2*1i - a^6*d^2*1i + 6*a*b^5*d^2 + 6*a^5*b*d^2 - a^2*b^4*d^2*15i - 20*a^3*b^3*d^2 + a^4*b^2*d^2*15i)))^(1/2) + ((e*cot(c + d*x))^(1/2)*(32*b^18*e^28 - 225*a^18*e^28 + 128*a^2*b^16*e^28 + 192*a^4*b^14*e^28 - 3841*a^6*b^12*e^28 + 18050*a^8*b^10*e^28 + 26801*a^10*b^8*e^28 + 16860*a^12*b^6*e^28 + 4049*a^14*b^4*e^28 - 30*a^16*b^2*e^28))/(b^21*d^4 + 8*a^2*b^19*d^4 + 28*a^4*b^17*d^4 + 56*a^6*b^15*d^4 + 70*a^8*b^13*d^4 + 56*a^10*b^11*d^4 + 28*a^12*b^9*d^4 + 8*a^14*b^7*d^4 + a^16*b^5*d^4))*(-e^9/(4*(b^6*d^2*1i - a^6*d^2*1i + 6*a*b^5*d^2 + 6*a^5*b*d^2 - a^2*b^4*d^2*15i - 20*a^3*b^3*d^2 + a^4*b^2*d^2*15i)))^(1/2) + (((((128*a*b^26*d^4*e^15 + 3648*a^3*b^24*d^4*e^15 + 25536*a^5*b^22*d^4*e^15 + 88320*a^7*b^20*d^4*e^15 + 182784*a^9*b^18*d^4*e^15 + 244608*a^11*b^16*d^4*e^15 + 217728*a^13*b^14*d^4*e^15 + 128256*a^15*b^12*d^4*e^15 + 48000*a^17*b^10*d^4*e^15 + 10304*a^19*b^8*d^4*e^15 + 960*a^21*b^6*d^4*e^15)/(b^21*d^5 + 8*a^2*b^19*d^5 + 28*a^4*b^17*d^5 + 56*a^6*b^15*d^5 + 70*a^8*b^13*d^5 + 56*a^10*b^11*d^5 + 28*a^12*b^9*d^5 + 8*a^14*b^7*d^5 + a^16*b^5*d^5) - ((e*cot(c + d*x))^(1/2)*(-e^9/(4*(b^6*d^2*1i - a^6*d^2*1i + 6*a*b^5*d^2 + 6*a^5*b*d^2 - a^2*b^4*d^2*15i - 20*a^3*b^3*d^2 + a^4*b^2*d^2*15i)))^(1/2)*(512*b^30*d^4*e^10 + 4608*a^2*b^28*d^4*e^10 + 17920*a^4*b^26*d^4*e^10 + 38400*a^6*b^24*d^4*e^10 + 46080*a^8*b^22*d^4*e^10 + 21504*a^10*b^20*d^4*e^10 - 21504*a^12*b^18*d^4*e^10 - 46080*a^14*b^16*d^4*e^10 - 38400*a^16*b^14*d^4*e^10 - 17920*a^18*b^12*d^4*e^10 - 4608*a^20*b^10*d^4*e^10 - 512*a^22*b^8*d^4*e^10))/(b^21*d^4 + 8*a^2*b^19*d^4 + 28*a^4*b^17*d^4 + 56*a^6*b^15*d^4 + 70*a^8*b^13*d^4 + 56*a^10*b^11*d^4 + 28*a^12*b^9*d^4 + 8*a^14*b^7*d^4 + a^16*b^5*d^4))*(-e^9/(4*(b^6*d^2*1i - a^6*d^2*1i + 6*a*b^5*d^2 + 6*a^5*b*d^2 - a^2*b^4*d^2*15i - 20*a^3*b^3*d^2 + a^4*b^2*d^2*15i)))^(1/2) + ((e*cot(c + d*x))^(1/2)*(1800*a^23*b*d^2*e^19 - 1472*a*b^23*d^2*e^19 - 1024*a^3*b^21*d^2*e^19 + 8448*a^5*b^19*d^2*e^19 + 46088*a^7*b^17*d^2*e^19 + 177344*a^9*b^15*d^2*e^19 + 402912*a^11*b^13*d^2*e^19 + 541632*a^13*b^11*d^2*e^19 + 455472*a^15*b^9*

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d^2*e^19 + 248064*a^17*b^7*d^2*e^19 + 87008*a^19*b^5*d^2*e^19 + 18240*a^21*
b^3*d^2*e^19))/(b^21*d^4 + 8*a^2*b^19*d^4 + 28*a^4*b^17*d^4 + 56*a^6*b^15*d
^4 + 70*a^8*b^13*d^4 + 56*a^10*b^11*d^4 + 28*a^12*b^9*d^4 + 8*a^14*b^7*d^4
+ a^16*b^5*d^4))*(-e^9/(4*(b^6*d^2*1i - a^6*d^2*1i + 6*a*b^5*d^2 + 6*a^5*b*
d^2 - a^2*b^4*d^2*15i - 20*a^3*b^3*d^2 + a^4*b^2*d^2*15i)))^(1/2) + (2250*a
^20*b*d^2*e^24 + 32*a^2*b^19*d^2*e^24 + 12288*a^4*b^17*d^2*e^24 - 10974*a^6
*b^15*d^2*e^24 - 105162*a^8*b^13*d^2*e^24 - 150758*a^10*b^11*d^2*e^24 - 853
14*a^12*b^9*d^2*e^24 - 3578*a^14*b^7*d^2*e^24 + 22210*a^16*b^5*d^2*e^24 + 1
1550*a^18*b^3*d^2*e^24)/(b^21*d^5 + 8*a^2*b^19*d^5 + 28*a^4*b^17*d^5 + 56*a
^6*b^15*d^5 + 70*a^8*b^13*d^5 + 56*a^10*b^11*d^5 + 28*a^12*b^9*d^5 + 8*a^14
*b^7*d^5 + a^16*b^5*d^5))*(-e^9/(4*(b^6*d^2*1i - a^6*d^2*1i + 6*a*b^5*d^2 +
 6*a^5*b*d^2 - a^2*b^4*d^2*15i - 20*a^3*b^3*d^2 + a^4*b^2*d^2*15i)))^(1/2)
- ((e*cot(c + d*x))^(1/2)*(32*b^18*e^28 - 225*a^18*e^28 + 128*a^2*b^16*e^28
 + 192*a^4*b^14*e^28 - 3841*a^6*b^12*e^28 + 18050*a^8*b^10*e^28 + 26801*a^1
0*b^8*e^28 + 16860*a^12*b^6*e^28 + 4049*a^14*b^4*e^28 - 30*a^16*b^2*e^28))/
(b^21*d^4 + 8*a^2*b^19*d^4 + 28*a^4*b^17*d^4 + 56*a^6*b^15*d^4 + 70*a^8*b^1
3*d^4 + 56*a^10*b^11*d^4 + 28*a^12*b^9*d^4 + 8*a^14*b^7*d^4 + a^16*b^5*d^4)
)*(-e^9/(4*(b^6*d^2*1i - a^6*d^2*1i + 6*a*b^5*d^2 + 6*a^5*b*d^2 - a^2*b^4*d
^2*15i - 20*a^3*b^3*d^2 + a^4*b^2*d^2*15i)))^(1/2)))*(-e^9/(4*(b^6*d^2*1i -
 a^6*d^2*1i + 6*a*b^5*d^2 + 6*a^5*b*d^2 - a^2*b^4*d^2*15i - 20*a^3*b^3*d^2
+ a^4*b^2*d^2*15i)))^(1/2)*2i - (2*e^4*(e*cot(c + d*x))^(1/2))/(b^3*d) + (a
tan((((((e*cot(c + d*x))^(1/2)*(32*b^18*e^28 - 225*a^18*e^28 + 128*a^2*b^16
*e^28 + 192*a^4*b^14*e^28 - 3841*a^6*b^12*e^28 + 18050*a^8*b^10*e^28 + 2680
1*a^10*b^8*e^28 + 16860*a^12*b^6*e^28 + 4049*a^14*b^4*e^28 - 30*a^16*b^2*e^
28))/(b^21*d^4 + 8*a^2*b^19*d^4 + 28*a^4*b^17*d^4 + 56*a^6*b^15*d^4 + 70*a^
8*b^13*d^4 + 56*a^10*b^11*d^4 + 28*a^12*b^9*d^4 + 8*a^14*b^7*d^4 + a^16*b^5
*d^4) - (((2250*a^20*b*d^2*e^24 + 32*a^2*b^19*d^2*e^24 + 12288*a^4*b^17*d^2
*e^24 - 10974*a^6*b^15*d^2*e^24 - 105162*a^8*b^13*d^2*e^24 - 150758*a^10*b^
11*d^2*e^24 - 85314*a^12*b^9*d^2*e^24 - 3578*a^14*b^7*d^2*e^24 + 22210*a^16
*b^5*d^2*e^24 + 11550*a^18*b^3*d^2*e^24)/(b^21*d^5 + 8*a^2*b^19*d^5 + 28*a^
4*b^17*d^5 + 56*a^6*b^15*d^5 + 70*a^8*b^13*d^5 + 56*a^10*b^11*d^5 + 28*a^12
*b^9*d^5 + 8*a^14*b^7*d^5 + a^16*b^5*d^5) + ((((e*cot(c + d*x))^(1/2)*(1800
*a^23*b*d^2*e^19 - 1472*a*b^23*d^2*e^19 - 1024*a^3*b^21*d^2*e^19 + 8448*a^5
*b^19*d^2*e^19 + 46088*a^7*b^17*d^2*e^19 + 177344*a^9*b^15*d^2*e^19 + 40291
2*a^11*b^13*d^2*e^19 + 541632*a^13*b^11*d^2*e^19 + 455472*a^15*b^9*d^2*e^19
 + 248064*a^17*b^7*d^2*e^19 + 87008*a^19*b^5*d^2*e^19 + 18240*a^21*b^3*d^2*
e^19))/(b^21*d^4 + 8*a^2*b^19*d^4 + 28*a^4*b^17*d^4 + 56*a^6*b^15*d^4 + 70*
a^8*b^13*d^4 + 56*a^10*b^11*d^4 + 28*a^12*b^9*d^4 + 8*a^14*b^7*d^4 + a^16*b
^5*d^4) + (((128*a*b^26*d^4*e^15 + 3648*a^3*b^24*d^4*e^15 + 25536*a^5*b^22*
d^4*e^15 + 88320*a^7*b^20*d^4*e^15 + 182784*a^9*b^18*d^4*e^15 + 244608*a^11
*b^16*d^4*e^15 + 217728*a^13*b^14*d^4*e^15 + 128256*a^15*b^12*d^4*e^15 + 48
000*a^17*b^10*d^4*e^15 + 10304*a^19*b^8*d^4*e^15 + 960*a^21*b^6*d^4*e^15)/(
b^21*d^5 + 8*a^2*b^19*d^5 + 28*a^4*b^17*d^5 + 56*a^6*b^15*d^5 + 70*a^8*b^13
*d^5 + 56*a^10*b^11*d^5 + 28*a^12*b^9*d^5 + 8*a^14*b^7*d^5 + a^16*b^5*d^5)
- ((e*cot(c + d*x))^(1/2)*(15*a^4 + 63*b^4 + 46*a^2*b^2)*(-a^5*b^7*e^9)^(1/
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2)*(512*b^30*d^4*e^10 + 4608*a^2*b^28*d^4*e^10 + 17920*a^4*b^26*d^4*e^10 +
38400*a^6*b^24*d^4*e^10 + 46080*a^8*b^22*d^4*e^10 + 21504*a^10*b^20*d^4*e^1
0 - 21504*a^12*b^18*d^4*e^10 - 46080*a^14*b^16*d^4*e^10 - 38400*a^16*b^14*d
^4*e^10 - 17920*a^18*b^12*d^4*e^10 - 4608*a^20*b^10*d^4*e^10 - 512*a^22*b^8
*d^4*e^10))/(8*(b^13*d + 3*a^2*b^11*d + 3*a^4*b^9*d + a^6*b^7*d)*(b^21*d^4
+ 8*a^2*b^19*d^4 + 28*a^4*b^17*d^4 + 56*a^6*b^15*d^4 + 70*a^8*b^13*d^4 + 56
*a^10*b^11*d^4 + 28*a^12*b^9*d^4 + 8*a^14*b^7*d^4 + a^16*b^5*d^4)))*(15*a^4
 + 63*b^4 + 46*a^2*b^2)*(-a^5*b^7*e^9)^(1/2))/(8*(b^13*d + 3*a^2*b^11*d + 3
*a^4*b^9*d + a^6*b^7*d)))*(15*a^4 + 63*b^4 + 46*a^2*b^2)*(-a^5*b^7*e^9)^(1/
2))/(8*(b^13*d + 3*a^2*b^11*d + 3*a^4*b^9*d + a^6*b^7*d)))*(15*a^4 + 63*b^4
 + 46*a^2*b^2)*(-a^5*b^7*e^9)^(1/2))/(8*(b^13*d + 3*a^2*b^11*d + 3*a^4*b^9*
d + a^6*b^7*d)))*(15*a^4 + 63*b^4 + 46*a^2*b^2)*(-a^5*b^7*e^9)^(1/2)*1i)/(8
*(b^13*d + 3*a^2*b^11*d + 3*a^4*b^9*d + a^6*b^7*d)) + ((((e*cot(c + d*x))^(
1/2)*(32*b^18*e^28 - 225*a^18*e^28 + 128*a^2*b^16*e^28 + 192*a^4*b^14*e^28
- 3841*a^6*b^12*e^28 + 18050*a^8*b^10*e^28 + 26801*a^10*b^8*e^28 + 16860*a^
12*b^6*e^28 + 4049*a^14*b^4*e^28 - 30*a^16*b^2*e^28))/(b^21*d^4 + 8*a^2*b^1
9*d^4 + 28*a^4*b^17*d^4 + 56*a^6*b^15*d^4 + 70*a^8*b^13*d^4 + 56*a^10*b^11*
d^4 + 28*a^12*b^9*d^4 + 8*a^14*b^7*d^4 + a^16*b^5*d^4) + (((2250*a^20*b*d^2
*e^24 + 32*a^2*b^19*d^2*e^24 + 12288*a^4*b^17*d^2*e^24 - 10974*a^6*b^15*d^2
*e^24 - 105162*a^8*b^13*d^2*e^24 - 150758*a^10*b^11*d^2*e^24 - 85314*a^12*b
^9*d^2*e^24 - 3578*a^14*b^7*d^2*e^24 + 22210*a^16*b^5*d^2*e^24 + 11550*a^18
*b^3*d^2*e^24)/(b^21*d^5 + 8*a^2*b^19*d^5 + 28*a^4*b^17*d^5 + 56*a^6*b^15*d
^5 + 70*a^8*b^13*d^5 + 56*a^10*b^11*d^5 + 28*a^12*b^9*d^5 + 8*a^14*b^7*d^5
+ a^16*b^5*d^5) - ((((e*cot(c + d*x))^(1/2)*(1800*a^23*b*d^2*e^19 - 1472*a*
b^23*d^2*e^19 - 1024*a^3*b^21*d^2*e^19 + 8448*a^5*b^19*d^2*e^19 + 46088*a^7
*b^17*d^2*e^19 + 177344*a^9*b^15*d^2*e^19 + 402912*a^11*b^13*d^2*e^19 + 541
632*a^13*b^11*d^2*e^19 + 455472*a^15*b^9*d^2*e^19 + 248064*a^17*b^7*d^2*e^1
9 + 87008*a^19*b^5*d^2*e^19 + 18240*a^21*b^3*d^2*e^19))/(b^21*d^4 + 8*a^2*b
^19*d^4 + 28*a^4*b^17*d^4 + 56*a^6*b^15*d^4 + 70*a^8*b^13*d^4 + 56*a^10*b^1
1*d^4 + 28*a^12*b^9*d^4 + 8*a^14*b^7*d^4 + a^16*b^5*d^4) - (((128*a*b^26*d^
4*e^15 + 3648*a^3*b^24*d^4*e^15 + 25536*a^5*b^22*d^4*e^15 + 88320*a^7*b^20*
d^4*e^15 + 182784*a^9*b^18*d^4*e^15 + 244608*a^11*b^16*d^4*e^15 + 217728*a^
13*b^14*d^4*e^15 + 128256*a^15*b^12*d^4*e^15 + 48000*a^17*b^10*d^4*e^15 + 1
0304*a^19*b^8*d^4*e^15 + 960*a^21*b^6*d^4*e^15)/(b^21*d^5 + 8*a^2*b^19*d^5
+ 28*a^4*b^17*d^5 + 56*a^6*b^15*d^5 + 70*a^8*b^13*d^5 + 56*a^10*b^11*d^5 +
28*a^12*b^9*d^5 + 8*a^14*b^7*d^5 + a^16*b^5*d^5) + ((e*cot(c + d*x))^(1/2)*
(15*a^4 + 63*b^4 + 46*a^2*b^2)*(-a^5*b^7*e^9)^(1/2)*(512*b^30*d^4*e^10 + 46
08*a^2*b^28*d^4*e^10 + 17920*a^4*b^26*d^4*e^10 + 38400*a^6*b^24*d^4*e^10 +
46080*a^8*b^22*d^4*e^10 + 21504*a^10*b^20*d^4*e^10 - 21504*a^12*b^18*d^4*e^
10 - 46080*a^14*b^16*d^4*e^10 - 38400*a^16*b^14*d^4*e^10 - 17920*a^18*b^12*
d^4*e^10 - 4608*a^20*b^10*d^4*e^10 - 512*a^22*b^8*d^4*e^10))/(8*(b^13*d + 3
*a^2*b^11*d + 3*a^4*b^9*d + a^6*b^7*d)*(b^21*d^4 + 8*a^2*b^19*d^4 + 28*a^4*
b^17*d^4 + 56*a^6*b^15*d^4 + 70*a^8*b^13*d^4 + 56*a^10*b^11*d^4 + 28*a^12*b
^9*d^4 + 8*a^14*b^7*d^4 + a^16*b^5*d^4)))*(15*a^4 + 63*b^4 + 46*a^2*b^2)*(-
a^5*b^7*e^9)^(1/2))/(8*(b^13*d + 3*a^2*b^11*d + 3*a^4*b^9*d + a^6*b^7*d)))*

(15*a^4 + 63*b^4 + 46*a^2*b^2)*(-a^5*b^7*e^9)^(1/2))/(8*(b^13*d + 3*a^2*b^11*d + 3*a^4*b^9*d + a^6*b^7*d)))*(15*a^4 + 63*b^4 + 46*a^2*b^2)*(-a^5*b^7*e^9)^(1/2))/(8*(b^13*d + 3*a^2*b^11*d + 3*a^4*b^9*d + a^6*b^7*d)))*(15*a^4 + 63*b^4 + 46*a^2*b^2)*(-a^5*b^7*e^9)^(1/2)*1i)/(8*(b^13*d + 3*a^2*b^11*d + 3*a^4*b^9*d + a^6*b^7*d)))/((225*a^15*e^33 + 504*a^3*b^12*e^33 + 872*a^5*b^10*e^33 + 4457*a^7*b^8*e^33 + 5916*a^9*b^6*e^33 + 4006*a^11*b^4*e^33 + 1380*a^13*b^2*e^33)/(b^21*d^5 + 8*a^2*b^19*d^5 + 28*a^4*b^17*d^5 + 56*a^6*b^15*d^5 + 70*a^8*b^13*d^5 + 56*a^10*b^11*d^5 + 28*a^12*b^9*d^5 + 8*a^14*b^7*d^5 + a^16*b^5*d^5) - ((((e*cot(c + d*x))^(1/2)*(32*b^18*e^28 - 225*a^18*e^28 + 128*a^2*b^16*e^28 + 192*a^4*b^14*e^28 - 3841*a^6*b^12*e^28 + 18050*a^8*b^10*e^28 + 26801*a^10*b^8*e^28 + 16860*a^12*b^6*e^28 + 4049*a^14*b^4*e^28 - 30*a^16*b^2*e^28))/(b^21*d^4 + 8*a^2*b^19*d^4 + 28*a^4*b^17*d^4 + 56*a^6*b^15*d^4 + 70*a^8*b^13*d^4 + 56*a^10*b^11*d^4 + 28*a^12*b^9*d^4 + 8*a^14*b^7*d^4 + a^16*b^5*d^4) - (((2250*a^20*b*d^2*e^24 + 32*a^2*b^19*d^2*e^24 + 12288*a^4*b^17*d^2*e^24 - 10974*a^6*b^15*d^2*e^24 - 105162*a^8*b^13*d^2*e^24 - 150758*a^10*b^11*d^2*e^24 - 85314*a^12*b^9*d^2*e^24 - 3578*a^14*b^7*d^2*e^24 + 22210*a^16*b^5*d^2*e^24 + 11550*a^18*b^3*d^2*e^24)/(b^21*d^5 + 8*a^2*b^19*d^5 + 28*a^4*b^17*d^5 + 56*a^6*b^15*d^5 + 70*a^8*b^13*d^5 + 56*a^10*b^11*d^5 + 28*a^12*b^9*d^5 + 8*a^14*b^7*d^5 + a^16*b^5*d^5) + ((((e*cot(c + d*x))^(1/2)*(1800*a^23*b*d^2*e^19 - 1472*a*b^23*d^2*e^19 - 1024*a^3*b^21*d^2*e^19 + 8448*a^5*b^19*d^2*e^19 + 46088*a^7*b^17*d^2*e^19 + 177344*a^9*b^15*d^2*e^19 + 402912*a^11*b^13*d^2*e^19 + 541632*a^13*b^11*d^2*e^19 + 455472*a^15*b^9*d^2*e^19 + 248064*a^17*b^7*d^2*e^19 + 87008*a^19*b^5*d^2*e^19 + 18240*a^21*b^3*d^2*e^19))/(b^21*d^4 + 8*a^2*b^19*d^4 + 28*a^4*b^17*d^4 + 56*a^6*b^15*d^4 + 70*a^8*b^13*d^4 + 56*a^10*b^11*d^4 + 28*a^12*b^9*d^4 + 8*a^14*b^7*d^4 + a^16*b^5*d^4) + (((128*a*b^26*d^4*e^15 + 3648*a^3*b^24*d^4*e^15 + 25536*a^5*b^22*d^4*e^15 + 88320*a^7*b^20*d^4*e^15 + 182784*a^9*b^18*d^4*e^15 + 244608*a^11*b^16*d^4*e^15 + 217728*a^13*b^14*d^4*e^15 + 128256*a^15*b^12*d^4*e^15 + 48000*a^17*b^10*d^4*e^15 + 10304*a^19*b^8*d^4*e^15 + 960*a^21*b^6*d^4*e^15)/(b^21*d^5 + 8*a^2*b^19*d^5 + 28*a^4*b^17*d^5 + 56*a^6*b^15*d^5 + 70*a^8*b^13*d^5 + 56*a^10*b^11*d^5 + 28*a^12*b^9*d^5 + 8*a^14*b^7*d^5 + a^16*b^5*d^5) - ((e*cot(c + d*x))^(1/2)*(15*a^4 + 63*b^4 + 46*a^2*b^2)*(-a^5*b^7*e^9)^(1/2)*(512*b^30*d^4*e^10 + 4608*a^2*b^28*d^4*e^10 + 17920*a^4*b^26*d^4*e^10 + 38400*a^6*b^24*d^4*e^10 + 46080*a^8*b^22*d^4*e^10 + 21504*a^10*b^20*d^4*e^10 - 21504*a^12*b^18*d^4*e^10 - 46080*a^14*b^16*d^4*e^10 - 38400*a^16*b^14*d^4*e^10 - 17920*a^18*b^12*d^4*e^10 - 4608*a^20*b^10*d^4*e^10 - 512*a^22*b^8*d^4*e^10))/(8*(b^13*d + 3*a^2*b^11*d + 3*a^4*b^9*d + a^6*b^7*d)*(b^21*d^4 + 8*a^2*b^19*d^4 + 28*a^4*b^17*d^4 + 56*a^6*b^15*d^4 + 70*a^8*b^13*d^4 + 56*a^10*b^11*d^4 + 28*a^12*b^9*d^4 + 8*a^14*b^7*d^4 + a^16*b^5*d^4)))*(15*a^4 + 63*b^4 + 46*a^2*b^2)*(-a^5*b^7*e^9)^(1/2))/(8*(b^13*d + 3*a^2*b^11*d + 3*a^4*b^9*d + a^6*b^7*d)))*(15*a^4 + 63*b^4 + 46*a^2*b^2)*(-a^5*b^7*e^9)^(1/2))/(8*(b^13*d + 3*a^2*b^11*d + 3*a^4*b^9*d + a^6*b^7*d)))*(15*a^4 + 63*b^4 + 46*a^2*b^2)*(-a^5*b^7*e^9)^(1/2))/(8*(b^13*d + 3*a^2*b^11*d + 3*a^4*b^9*d + a^6*b^7*d)))*(15*a^4 + 63*b^4 + 46*a^2*b^2)*(-a^5*b^7*e^9)^(1/2))/(8*(b^13*d + 3*a^2*b^11*d + 3*a^4*b^9*d + a^6*b^7*d)) + ((((e*cot(

c + d*x))^(1/2)*(32*b^18*e^28 - 225*a^18*e^28 + 128*a^2*b^16*e^28 + 192*a^4 *b^14*e^28 - 3841*a^6*b^12*e^28 + 18050*a^8*b^10*e^28 + 26801*a^10*b^8*e^28 + 16860*a^12*b^6*e^28 + 4049*a^14*b^4*e^28 - 30*a^16*b^2*e^28))/(b^21*d^4 + 8*a^2*b^19*d^4 + 28*a^4*b^17*d^4 + 56*a^6*b^15*d^4 + 70*a^8*b^13*d^4 + 56 *a^10*b^11*d^4 + 28*a^12*b^9*d^4 + 8*a^14*b^7*d^4 + a^16*b^5*d^4) + (((2250 *a^20*b*d^2*e^24 + 32*a^2*b^19*d^2*e^24 + 12288*a^4*b^17*d^2*e^24 - 10974*a ^6*b^15*d^2*e^24 - 105162*a^8*b^13*d^2*e^24 - 150758*a^10*b^11*d^2*e^24 - 8 5314*a^12*b^9*d^2*e^24 - 3578*a^14*b^7*d^2*e^24 + 22210*a^16*b^5*d^2*e^24 + 11550*a^18*b^3*d^2*e^24)/(b^21*d^5 + 8*a^2*b^19*d^5 + 28*a^4*b^17*d^5 + 56 *a^6*b^15*d^5 + 70*a^8*b^13*d^5 + 56*a^10*b^11*d^5 + 28*a^12*b^9*d^5 + 8*a^ 14*b^7*d^5 + a^16*b^5*d^5) - ((((e*cot(c + d*x))^(1/2)*(1800*a^23*b*d^2*e^1 9 - 1472*a*b^23*d^2*e^19 - 1024*a^3*b^21*d^2*e^19 + 8448*a^5*b^19*d^2*e^19 + 46088*a^7*b^17*d^2*e^19 + 177344*a^9*b^15*d^2*e^19 + 402912*a^11*b^13*d^2 *e^19 + 541632*a^13*b^11*d^2*e^19 + 455472*a^15*b^9*d^2*e^19 + 248064*a^17* b^7*d^2*e^19 + 87008*a^19*b^5*d^2*e^19 + 18240*a^21*b^3*d^2*e^19))/(b^21*d^ 4 + 8*a^2*b^19*d^4 + 28*a^4*b^17*d^4 + 56*a^6*b^15*d^4 + 70*a^8*b^13*d^4 + 56*a^10*b^11*d^4 + 28*a^12*b^9*d^4 + 8*a^14*b^7*d^4 + a^16*b^5*d^4) - (((12 8*a*b^26*d^4*e^15 + 3648*a^3*b^24*d^4*e^15 + 25536*a^5*b^22*d^4*e^15 + 8832 0*a^7*b^20*d^4*e^15 + 182784*a^9*b^18*d^4*e^15 + 244608*a^11*b^16*d^4*e^15 + 217728*a^13*b^14*d^4*e^15 + 128256*a^15*b^12*d^4*e^15 + 48000*a^17*b^10*d ^4*e^15 + 10304*a^19*b^8*d^4*e^15 + 960*a^21*b^6*d^4*e^15)/(b^21*d^5 + 8*a^ 2*b^19*d^5 + 28*a^4*b^17*d^5 + 56*a^6*b^15*d^5 + 70*a^8*b^13*d^5 + 56*a^10* b^11*d^5 + 28*a^12*b^9*d^5 + 8*a^14*b^7*d^5 + a^16*b^5*d^5) + ((e*cot(c + d *x))^(1/2)*(15*a^4 + 63*b^4 + 46*a^2*b^2)*(-a^5*b^7*e^9)^(1/2)*(512*b^30*d^ 4*e^10 + 4608*a^2*b^28*d^4*e^10 + 17920*a^4*b^26*d^4*e^10 + 38400*a^6*b^24* d^4*e^10 + 46080*a^8*b^22*d^4*e^10 + 21504*a^10*b^20*d^4*e^10 - 21504*a^12* b^18*d^4*e^10 - 46080*a^14*b^16*d^4*e^10 - 38400*a^16*b^14*d^4*e^10 - 17920 *a^18*b^12*d^4*e^10 - 4608*a^20*b^10*d^4*e^10 - 512*a^22*b^8*d^4*e^10))/(8* (b^13*d + 3*a^2*b^11*d + 3*a^4*b^9*d + a^6*b^7*d)*(b^21*d^4 + 8*a^2*b^19*d^ 4 + 28*a^4*b^17*d^4 + 56*a^6*b^15*d^4 + 70*a^8*b^13*d^4 + 56*a^10*b^11*d^4 + 28*a^12*b^9*d^4 + 8*a^14*b^7*d^4 + a^16*b^5*d^4)))*(15*a^4 + 63*b^4 + 46* a^2*b^2)*(-a^5*b^7*e^9)^(1/2))/(8*(b^13*d + 3*a^2*b^11*d + 3*a^4*b^9*d + a^ 6*b^7*d)))*(15*a^4 + 63*b^4 + 46*a^2*b^2)*(-a^5*b^7*e^9)^(1/2))/(8*(b^13*d + 3*a^2*b^11*d + 3*a^4*b^9*d + a^6*b^7*d)))*(15*a^4 + 63*b^4 + 46*a^2*b^2)* (-a^5*b^7*e^9)^(1/2))/(8*(b^13*d + 3*a^2*b^11*d + 3*a^4*b^9*d + a^6*b^7*d)))*(15*a^4 + 63*b^4 + 46*a^2*b^2)*(-a^5*b^7*e^9)^(1/2))/(8*(b^13*d + 3*a^2*b ^11*d + 3*a^4*b^9*d + a^6*b^7*d))))*(15*a^4 + 63*b^4 + 46*a^2*b^2)*(-a^5*b^ 7*e^9)^(1/2)*1i)/(4*(b^13*d + 3*a^2*b^11*d + 3*a^4*b^9*d + a^6*b^7*d))

3.82 $\int \frac{(e\cot(c+dx))^{7/2}}{(a+b\cot(c+dx))^3}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 476

$$\begin{aligned}
\int \frac{(e\cot(c+dx))^{7/2}}{(a+b\cot(c+dx))^3}\,dx &= -\frac{a^{3/2}(3a^4+6a^2b^2+35b^4)\,e^{7/2}\arctan\left(\frac{\sqrt{b}\sqrt{e\cot(c+dx)}}{\sqrt{a}\sqrt{e}}\right)}{4b^{5/2}\left(a^2+b^2\right)^3 d} \\
&+ \frac{(a+b)\left(a^2-4ab+b^2\right)e^{7/2}\arctan\left(1-\frac{\sqrt{2}\sqrt{e\cot(c+dx)}}{\sqrt{e}}\right)}{\sqrt{2}\left(a^2+b^2\right)^3 d} \\
&- \frac{(a+b)\left(a^2-4ab+b^2\right)e^{7/2}\arctan\left(1+\frac{\sqrt{2}\sqrt{e\cot(c+dx)}}{\sqrt{e}}\right)}{\sqrt{2}\left(a^2+b^2\right)^3 d} \\
&+ \frac{a^2e^2(e\cot(c+dx))^{3/2}}{2b\left(a^2+b^2\right)d(a+b\cot(c+dx))^2} + \frac{a^2(3a^2+11b^2)\,e^3\sqrt{e\cot(c+dx)}}{4b^2\left(a^2+b^2\right)^2 d(a+b\cot(c+dx))} \\
&+ \frac{(a-b)\left(a^2+4ab+b^2\right)e^{7/2}\log\left(\sqrt{e}+\sqrt{e}\cot(c+dx)-\sqrt{2}\sqrt{e\cot(c+dx)}\right)}{2\sqrt{2}\left(a^2+b^2\right)^3 d} \\
&- \frac{(a-b)\left(a^2+4ab+b^2\right)e^{7/2}\log\left(\sqrt{e}+\sqrt{e}\cot(c+dx)+\sqrt{2}\sqrt{e\cot(c+dx)}\right)}{2\sqrt{2}\left(a^2+b^2\right)^3 d}
\end{aligned}$$

```
[Out] -1/4*a^(3/2)*(3*a^4+6*a^2*b^2+35*b^4)*e^(7/2)*arctan(b^(1/2)*(e*cot(d*x+c))
^(1/2)/a^(1/2)/e^(1/2))/b^(5/2)/(a^2+b^2)^3/d+1/2*a^2*e^2*(e*cot(d*x+c))^(3
/2)/b/(a^2+b^2)/d/(a+b*cot(d*x+c))^2+1/2*(a+b)*(a^2-4*a*b+b^2)*e^(7/2)*arct
an(1-2^(1/2)*(e*cot(d*x+c))^(1/2)/e^(1/2))/(a^2+b^2)^3/d*2^(1/2)-1/2*(a+b)*
(a^2-4*a*b+b^2)*e^(7/2)*arctan(1+2^(1/2)*(e*cot(d*x+c))^(1/2)/e^(1/2))/(a^2
+b^2)^3/d*2^(1/2)+1/4*(a-b)*(a^2+4*a*b+b^2)*e^(7/2)*ln(e^(1/2)+cot(d*x+c)*e
^(1/2)-2^(1/2)*(e*cot(d*x+c))^(1/2))/(a^2+b^2)^3/d*2^(1/2)-1/4*(a-b)*(a^2+4
*a*b+b^2)*e^(7/2)*ln(e^(1/2)+cot(d*x+c)*e^(1/2)+2^(1/2)*(e*cot(d*x+c))^(1/2
))/(a^2+b^2)^3/d*2^(1/2)+1/4*a^2*(3*a^2+11*b^2)*e^3*(e*cot(d*x+c))^(1/2)/b^
2/(a^2+b^2)^2/d/(a+b*cot(d*x+c))
```

Rubi [A] (verified)

Time = 1.36 (sec) , antiderivative size = 476, normalized size of antiderivative = 1.00, number of steps used = 16, number of rules used = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.520, Rules used = {3646, 3726, 3734, 3615, 1182, 1176, 631, 210, 1179, 642, 3715, 65, 211}

$$\int \frac{(e\cot(c+dx))^{7/2}}{(a+b\cot(c+dx))^3}\,dx=\frac{e^{7/2}(a+b)\left(a^2-4ab+b^2\right)\arctan\left(1-\frac{\sqrt{2}\sqrt{e\cot(c+dx)}}{\sqrt{e}}\right)}{\sqrt{2}d\left(a^2+b^2\right)^3}-\frac{e^{7/2}(a+b)\left(a^2-4ab+b^2\right)\arctan\left(\frac{\sqrt{2}\sqrt{e\cot(c+dx)}}{\sqrt{e}}+1\right)}{\sqrt{2}d\left(a^2+b^2\right)^3}+\frac{e^{7/2}(a-b)\left(a^2+4ab+b^2\right)\log\left(\sqrt{e}\cot(c+dx)-\sqrt{2}\sqrt{e\cot(c+dx)}+\sqrt{e}\right)}{2\sqrt{2}d\left(a^2+b^2\right)^3}-\frac{e^{7/2}(a-b)\left(a^2+4ab+b^2\right)\log\left(\sqrt{e}\cot(c+dx)+\sqrt{2}\sqrt{e\cot(c+dx)}+\sqrt{e}\right)}{2\sqrt{2}d\left(a^2+b^2\right)^3}+\frac{a^2e^3\left(3a^2+11b^2\right)\sqrt{e\cot(c+dx)}}{4b^2d\left(a^2+b^2\right)^2(a+b\cot(c+dx))}+\frac{a^2e^2(e\cot(c+dx))^{3/2}}{2bd\left(a^2+b^2\right)(a+b\cot(c+dx))^2}-\frac{a^{3/2}e^{7/2}\left(3a^4+6a^2b^2+35b^4\right)\arctan\left(\frac{\sqrt{b}\sqrt{e\cot(c+dx)}}{\sqrt{a}\sqrt{e}}\right)}{4b^{5/2}d\left(a^2+b^2\right)^3}$$

```
[In] Int[(e*Cot[c + d*x])^(7/2)/(a + b*Cot[c + d*x])^3,x]
```

```
[Out] -1/4*(a^(3/2)*(3*a^4 + 6*a^2*b^2 + 35*b^4)*e^(7/2)*ArcTan[(Sqrt[b]*Sqrt[e*C
ot[c + d*x]])/(Sqrt[a]*Sqrt[e])])/(b^(5/2)*(a^2 + b^2)^3*d) + ((a + b)*(a^2
 - 4*a*b + b^2)*e^(7/2)*ArcTan[1 - (Sqrt[2]*Sqrt[e*Cot[c + d*x]])/Sqrt[e]])
/(Sqrt[2]*(a^2 + b^2)^3*d) - ((a + b)*(a^2 - 4*a*b + b^2)*e^(7/2)*ArcTan[1
+ (Sqrt[2]*Sqrt[e*Cot[c + d*x]])/Sqrt[e]])/(Sqrt[2]*(a^2 + b^2)^3*d) + (a^2
*e^2*(e*Cot[c + d*x])^(3/2))/(2*b*(a^2 + b^2)*d*(a + b*Cot[c + d*x])^2) + (
a^2*(3*a^2 + 11*b^2)*e^3*Sqrt[e*Cot[c + d*x]])/(4*b^2*(a^2 + b^2)^2*d*(a +
b*Cot[c + d*x])) + ((a - b)*(a^2 + 4*a*b + b^2)*e^(7/2)*Log[Sqrt[e] + Sqrt[
e]*Cot[c + d*x] - Sqrt[2]*Sqrt[e*Cot[c + d*x]]])/(2*Sqrt[2]*(a^2 + b^2)^3*d
) - ((a - b)*(a^2 + 4*a*b + b^2)*e^(7/2)*Log[Sqrt[e] + Sqrt[e]*Cot[c + d*x]
 + Sqrt[2]*Sqrt[e*Cot[c + d*x]]])/(2*Sqrt[2]*(a^2 + b^2)^3*d)
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1179

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 1182

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
a*c, 2]}, Dist[(d*q + a*e)/(2*a*c), Int[(q + c*x^2)/(a + c*x^4), x], x] + D
ist[(d*q - a*e)/(2*a*c), Int[(q - c*x^2)/(a + c*x^4), x], x]] /; FreeQ[{a,
c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0] && NeQ[c*d^2 - a*e^2, 0] && NegQ[(-a)
*c]
```

Rule 3615

```
Int[((c_) + (d_.)*tan[(e_.) + (f_.)*(x_)])/Sqrt[(b_.)*tan[(e_.) + (f_.)*(x_
)]], x_Symbol] :> Dist[2/f, Subst[Int[(b*c + d*x^2)/(b^2 + x^4), x], x, Sqr
t[b*Tan[e + f*x]]], x] /; FreeQ[{b, c, d, e, f}, x] && NeQ[c^2 - d^2, 0] &&
 NeQ[c^2 + d^2, 0]
```

Rule 3646

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*c - a*d)^2*(a + b*Tan[e + f*x])^(m
 - 2)*((c + d*Tan[e + f*x])^(n + 1)/(d*f*(n + 1)*(c^2 + d^2))), x] - Dist[1
/(d*(n + 1)*(c^2 + d^2)), Int[(a + b*Tan[e + f*x])^(m - 3)*(c + d*Tan[e + f
*x])^(n + 1)*Simp[a^2*d*(b*d*(m - 2) - a*c*(n + 1)) + b*(b*c - 2*a*d)*(b*c*
(m - 2) + a*d*(n + 1)) - d*(n + 1)*(3*a^2*b*c - b^3*c - a^3*d + 3*a*b^2*d)*
Tan[e + f*x] - b*(a*d*(2*b*c - a*d)*(m + n - 1) - b^2*(c^2*(m - 2) - d^2*(n
 + 1)))*Tan[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b
*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] && GtQ[m, 2] && LtQ[
n, -1] && IntegerQ[2*m]
```

Rule 3715

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*tan[(e_.)
+ (f_.)*(x_)])^(n_.)*((A_) + (C_.)*tan[(e_.) + (f_.)*(x_)]^2), x_Symbol] :>
 Dist[A/f, Subst[Int[(a + b*x)^m*(c + d*x)^n, x], x, Tan[e + f*x]], x] /; F
reeQ[{a, b, c, d, e, f, A, C, m, n}, x] && EqQ[A, C]
```

Rule 3726

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*tan[(e_.) + (f_.)*(x_)] + (C_.)*tan[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> Simp[(A*d^2 + c*(c*C - B*d))*(a + b*Tan[e +
 f*x])^m*((c + d*Tan[e + f*x])^(n + 1)/(d*f*(n + 1)*(c^2 + d^2))), x] - Dis
t[1/(d*(n + 1)*(c^2 + d^2)), Int[(a + b*Tan[e + f*x])^(m - 1)*(c + d*Tan[e
+ f*x])^(n + 1)*Simp[A*d*(b*d*m - a*c*(n + 1)) + (c*C - B*d)*(b*c*m + a*d*(
n + 1)) - d*(n + 1)*((A - C)*(b*c - a*d) + B*(a*c + b*d))*Tan[e + f*x] - b*
(d*(B*c - A*d)*(m + n + 1) - C*(c^2*m - d^2*(n + 1)))*Tan[e + f*x]^2, x], x
], x] /; FreeQ[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] && NeQ[
a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] && GtQ[m, 0] && LtQ[n, -1]
```

Rule 3734

```
Int[(((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_)*((A_.) + (B_.)*tan[(e_.)
+ (f_.)*(x_)] + (C_.)*tan[(e_.) + (f_.)*(x_)]^2))/((a_.) + (b_.)*tan[(e_.)
+ (f_.)*(x_)]), x_Symbol] :> Dist[1/(a^2 + b^2), Int[(c + d*Tan[e + f*x])^n
*Simp[b*B + a*(A - C) + (a*B - b*(A - C))*Tan[e + f*x], x], x], x] + Dist[(
A*b^2 - a*b*B + a^2*C)/(a^2 + b^2), Int[(c + d*Tan[e + f*x])^n*((1 + Tan[e
+ f*x]^2)/(a + b*Tan[e + f*x])), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C
```

```
, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] &&
 !GtQ[n, 0] &&  !LeQ[n, -1]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{a^2e^2(e\cot(c+dx))^{3/2}}{2b\left(a^2+b^2\right)d(a+b\cot(c+dx))^2} - \frac{\int \frac{\sqrt{e\cot(c+dx)}\left(-\frac{3}{2}a^2e^3+2abe^3\cot(c+dx)-\frac{1}{2}\left(3a^2+4b^2\right)e^3\cot^2(c+dx)\right)}{(a+b\cot(c+dx))^2}\,dx}{2b\left(a^2+b^2\right)} \\
&= \frac{a^2e^2(e\cot(c+dx))^{3/2}}{2b\left(a^2+b^2\right)d(a+b\cot(c+dx))^2} + \frac{a^2\left(3a^2+11b^2\right)e^3\sqrt{e\cot(c+dx)}}{4b^2\left(a^2+b^2\right)^2d(a+b\cot(c+dx))} + \frac{\int \frac{\frac{1}{4}a^2\left(3a^2+11b^2\right)e^4-4ab^3e^4\cot(c+dx)+\frac{1}{4}\left(3a^4+3a^2b^2+8b^4\right)e^4\cot^2(c+dx)}{\sqrt{e\cot(c+dx)}(a+b\cot(c+dx))}\,dx}{2b^2\left(a^2+b^2\right)^2} \\
&= \frac{a^2e^2(e\cot(c+dx))^{3/2}}{2b\left(a^2+b^2\right)d(a+b\cot(c+dx))^2} + \frac{a^2\left(3a^2+11b^2\right)e^3\sqrt{e\cot(c+dx)}}{4b^2\left(a^2+b^2\right)^2d(a+b\cot(c+dx))} + \frac{\int \frac{2ab^2\left(a^2-3b^2\right)e^4-2b^3\left(3a^2-b^2\right)e^4\cot(c+dx)}{\sqrt{e\cot(c+dx)}}\,dx}{2b^2\left(a^2+b^2\right)^3} + \frac{\left(a^2\left(3a^4+6a^2b^2+35b^4\right)e^4\right)\int \frac{1+\cot^2(c+dx)}{\sqrt{e\cot(c+dx)}(a+b\cot(c+dx))}\,dx}{8b^2\left(a^2+b^2\right)^3} \\
&= \frac{a^2e^2(e\cot(c+dx))^{3/2}}{2b\left(a^2+b^2\right)d(a+b\cot(c+dx))^2} + \frac{a^2\left(3a^2+11b^2\right)e^3\sqrt{e\cot(c+dx)}}{4b^2\left(a^2+b^2\right)^2d(a+b\cot(c+dx))} + \frac{\text{Subst}\left(\int \frac{-2ab^2\left(a^2-3b^2\right)e^5+2b^3\left(3a^2-b^2\right)e^4x^2}{e^2+x^4}\,dx,x,\sqrt{e\cot(c+dx)}\right)}{b^2\left(a^2+b^2\right)^3d} + \frac{\left(a^2\left(3a^4+6a^2b^2+35b^4\right)e^4\right)\text{Subst}\left(\int \frac{1}{\sqrt{-ex}(a-bx)}\,dx,x,-\cot(c+dx)\right)}{8b^2\left(a^2+b^2\right)^3d} \\
&= \frac{a^2e^2(e\cot(c+dx))^{3/2}}{2b\left(a^2+b^2\right)d(a+b\cot(c+dx))^2} + \frac{a^2\left(3a^2+11b^2\right)e^3\sqrt{e\cot(c+dx)}}{4b^2\left(a^2+b^2\right)^2d(a+b\cot(c+dx))} - \frac{\left(a^2\left(3a^4+6a^2b^2+35b^4\right)e^3\right)\text{Subst}\left(\int \frac{1}{a+\frac{bx^2}{e}}\,dx,x,\sqrt{e\cot(c+dx)}\right)}{4b^2\left(a^2+b^2\right)^3d} \\
&\quad - \frac{\left((a+b)\left(a^2-4ab+b^2\right)e^4\right)\text{Subst}\left(\int \frac{e+x^2}{e^2+x^4}\,dx,x,\sqrt{e\cot(c+dx)}\right)}{\left(a^2+b^2\right)^3d} - \frac{\left((a-b)\left(a^2+4ab+b^2\right)e^4\right)\text{Subst}\left(\int \frac{e-x^2}{e^2+x^4}\,dx,x,\sqrt{e\cot(c+dx)}\right)}{\left(a^2+b^2\right)^3d}
\end{aligned}$$

$$= -\frac{a^{3/2}(3a^4+6a^2b^2+35b^4)\,e^{7/2}\arctan\left(\frac{\sqrt{b}\sqrt{e\cot(c+dx)}}{\sqrt{a}\sqrt{e}}\right)}{4b^{5/2}\,(a^2+b^2)^3\,d}$$
$$+\frac{a^2e^2(e\cot(c+dx))^{3/2}}{2b\,(a^2+b^2)\,d(a+b\cot(c+dx))^2}+\frac{a^2(3a^2+11b^2)\,e^3\sqrt{e\cot(c+dx)}}{4b^2\,(a^2+b^2)^2\,d(a+b\cot(c+dx))}$$
$$+\frac{((a-b)\,(a^2+4ab+b^2)\,e^{7/2})\,\mathrm{Subst}\left(\int\frac{\sqrt{2}\sqrt{e}+2x}{-e-\sqrt{2}\sqrt{e}x-x^2}\,dx,x,\sqrt{e\cot(c+dx)}\right)}{2\sqrt{2}\,(a^2+b^2)^3\,d}$$
$$+\frac{((a-b)\,(a^2+4ab+b^2)\,e^{7/2})\,\mathrm{Subst}\left(\int\frac{\sqrt{2}\sqrt{e}-2x}{-e+\sqrt{2}\sqrt{e}x-x^2}\,dx,x,\sqrt{e\cot(c+dx)}\right)}{2\sqrt{2}\,(a^2+b^2)^3\,d}$$
$$-\frac{((a+b)\,(a^2-4ab+b^2)\,e^4)\,\mathrm{Subst}\left(\int\frac{1}{e-\sqrt{2}\sqrt{e}x+x^2}\,dx,x,\sqrt{e\cot(c+dx)}\right)}{2\,(a^2+b^2)^3\,d}$$
$$-\frac{((a+b)\,(a^2-4ab+b^2)\,e^4)\,\mathrm{Subst}\left(\int\frac{1}{e+\sqrt{2}\sqrt{e}x+x^2}\,dx,x,\sqrt{e\cot(c+dx)}\right)}{2\,(a^2+b^2)^3\,d}$$

$$= -\frac{a^{3/2}(3a^4+6a^2b^2+35b^4)\,e^{7/2}\arctan\left(\frac{\sqrt{b}\sqrt{e\cot(c+dx)}}{\sqrt{a}\sqrt{e}}\right)}{4b^{5/2}\,(a^2+b^2)^3\,d}$$
$$+\frac{a^2e^2(e\cot(c+dx))^{3/2}}{2b\,(a^2+b^2)\,d(a+b\cot(c+dx))^2}+\frac{a^2(3a^2+11b^2)\,e^3\sqrt{e\cot(c+dx)}}{4b^2\,(a^2+b^2)^2\,d(a+b\cot(c+dx))}$$
$$+\frac{(a-b)\,(a^2+4ab+b^2)\,e^{7/2}\log\left(\sqrt{e}+\sqrt{e}\cot(c+dx)-\sqrt{2}\sqrt{e\cot(c+dx)}\right)}{2\sqrt{2}\,(a^2+b^2)^3\,d}$$
$$-\frac{(a-b)\,(a^2+4ab+b^2)\,e^{7/2}\log\left(\sqrt{e}+\sqrt{e}\cot(c+dx)+\sqrt{2}\sqrt{e\cot(c+dx)}\right)}{2\sqrt{2}\,(a^2+b^2)^3\,d}$$
$$-\frac{((a+b)\,(a^2-4ab+b^2)\,e^{7/2})\,\mathrm{Subst}\left(\int\frac{1}{-1-x^2}\,dx,x,1-\frac{\sqrt{2}\sqrt{e\cot(c+dx)}}{\sqrt{e}}\right)}{\sqrt{2}\,(a^2+b^2)^3\,d}$$
$$+\frac{((a+b)\,(a^2-4ab+b^2)\,e^{7/2})\,\mathrm{Subst}\left(\int\frac{1}{-1-x^2}\,dx,x,1+\frac{\sqrt{2}\sqrt{e\cot(c+dx)}}{\sqrt{e}}\right)}{\sqrt{2}\,(a^2+b^2)^3\,d}$$

$$= -\frac{a^{3/2}(3a^4+6a^2b^2+35b^4)\,e^{7/2}\arctan\left(\frac{\sqrt{b}\sqrt{e\cot(c+dx)}}{\sqrt{a}\sqrt{e}}\right)}{4b^{5/2}\left(a^2+b^2\right)^3 d} + \frac{(a+b)\left(a^2-4ab+b^2\right)e^{7/2}\arctan\left(1-\frac{\sqrt{2}\sqrt{e\cot(c+dx)}}{\sqrt{e}}\right)}{\sqrt{2}\left(a^2+b^2\right)^3 d} - \frac{(a+b)\left(a^2-4ab+b^2\right)e^{7/2}\arctan\left(1+\frac{\sqrt{2}\sqrt{e\cot(c+dx)}}{\sqrt{e}}\right)}{\sqrt{2}\left(a^2+b^2\right)^3 d} + \frac{a^2e^2(e\cot(c+dx))^{3/2}}{2b\left(a^2+b^2\right)d(a+b\cot(c+dx))^2} + \frac{a^2(3a^2+11b^2)\,e^3\sqrt{e\cot(c+dx)}}{4b^2\left(a^2+b^2\right)^2 d(a+b\cot(c+dx))} + \frac{(a-b)\left(a^2+4ab+b^2\right)e^{7/2}\log\left(\sqrt{e}+\sqrt{e}\cot(c+dx)-\sqrt{2}\sqrt{e\cot(c+dx)}\right)}{2\sqrt{2}\left(a^2+b^2\right)^3 d} - \frac{(a-b)\left(a^2+4ab+b^2\right)e^{7/2}\log\left(\sqrt{e}+\sqrt{e}\cot(c+dx)+\sqrt{2}\sqrt{e\cot(c+dx)}\right)}{2\sqrt{2}\left(a^2+b^2\right)^3 d}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 6.25 (sec) , antiderivative size = 574, normalized size of antiderivative = 1.21

$$\int \frac{(e\cot(c+dx))^{7/2}}{(a+b\cot(c+dx))^3}\,dx =$$

$$-\frac{(e\cot(c+dx))^{7/2}\left(\frac{2a^{7/2}(3a^2-b^2)\arctan\left(\frac{\sqrt{b}\sqrt{\cot(c+dx)}}{\sqrt{a}}\right)}{b^{5/2}(a^2+b^2)^3} - \frac{2a^3(3a^2-b^2)\sqrt{\cot(c+dx)}}{b^2(a^2+b^2)^3} + \frac{2a^2(3a^2-b^2)\cot^{\frac{3}{2}}(c+dx)}{3b(a^2+b^2)^3} - \frac{2a(3a^2-b^2)}{5(a^2}\right.}{}$$

```
[In] Integrate[(e*Cot[c + d*x])^(7/2)/(a + b*Cot[c + d*x])^3,x]
```

```
[Out] -(((e*Cot[c + d*x])^(7/2)*((2*a^(7/2)*(3*a^2 - b^2)*ArcTan[(Sqrt[b]*Sqrt[Co
t[c + d*x]])/Sqrt[a]])/(b^(5/2)*(a^2 + b^2)^3) - (2*a^3*(3*a^2 - b^2)*Sqrt[
Cot[c + d*x]])/(b^2*(a^2 + b^2)^3) + (2*a^2*(3*a^2 - b^2)*Cot[c + d*x]^(3/2
))/(3*b*(a^2 + b^2)^3) - (2*a*(3*a^2 - b^2)*Cot[c + d*x]^(5/2))/(5*(a^2 + b
^2)^3) + (2*b*(3*a^2 - b^2)*Cot[c + d*x]^(7/2))/(7*(a^2 + b^2)^3) + (2*b*(3
*a^2 - b^2)*(7*Cot[c + d*x]^(3/2) - 3*Cot[c + d*x]^(7/2) - 7*Cot[c + d*x]^(
3/2)*Hypergeometric2F1[3/4, 1, 7/4, -Cot[c + d*x]^2]))/(21*(a^2 + b^2)^3) +
 (4*b^2*Cot[c + d*x]^(9/2)*Hypergeometric2F1[2, 9/2, 11/2, -((b*Cot[c + d*x
])/a)])/(9*a*(a^2 + b^2)^2) + (2*b^2*Cot[c + d*x]^(9/2)*Hypergeometric2F1[3
, 9/2, 11/2, -((b*Cot[c + d*x])/a)])/(9*a^3*(a^2 + b^2)) - (a*(a^2 - 3*b^2)
*(10*Sqrt[2]*ArcTan[1 - Sqrt[2]*Sqrt[Cot[c + d*x]]] - 10*Sqrt[2]*ArcTan[1 +
 Sqrt[2]*Sqrt[Cot[c + d*x]]] + 40*Sqrt[Cot[c + d*x]] - 8*Cot[c + d*x]^(5/2)
 + 5*Sqrt[2]*Log[1 - Sqrt[2]*Sqrt[Cot[c + d*x]] + Cot[c + d*x]] - 5*Sqrt[2]
```

```
*Log[1 + Sqrt[2]*Sqrt[Cot[c + d*x]] + Cot[c + d*x]]))/(20*(a^2 + b^2)^3)))/
(d*Cot[c + d*x]^(7/2)))
```

Maple [A] (verified)

Time = 1.16 (sec) , antiderivative size = 460, normalized size of antiderivative = 0.97

method	result
derivativedivides	$-\frac{2e^4\left(-\frac{a^2\left(\frac{\frac{(5a^4+18a^2b^2+13b^4)(e\cot(dx+c))^{\frac{3}{2}}}{8b}+\frac{ae(3a^4+14a^2b^2+11b^4)\sqrt{e\cot(dx+c)}}{8b^2}}{(e\cot(dx+c)b+ae)^2}-\frac{(3a^4+6a^2b^2+35b^4)\arctan\left(\frac{\sqrt{e\cot(dx\cdots}}{\sqrt{aeb}}\right)}{8b^2\sqrt{aeb}}\cdots\right)}{(a^2+b^2)^3}\cdots\right)}{\cdots}$
default	$-\frac{2e^4\left(-\frac{a^2\left(\frac{\frac{(5a^4+18a^2b^2+13b^4)(e\cot(dx+c))^{\frac{3}{2}}}{8b}+\frac{ae(3a^4+14a^2b^2+11b^4)\sqrt{e\cot(dx+c)}}{8b^2}}{(e\cot(dx+c)b+ae)^2}-\frac{(3a^4+6a^2b^2+35b^4)\arctan\left(\frac{\sqrt{e\cot(dx\cdots}}{\sqrt{aeb}}\right)}{8b^2\sqrt{aeb}}\cdots\right)}{(a^2+b^2)^3}\cdots\right)}{\cdots}$

```
[In] int((e*cot(d*x+c))^(7/2)/(a+b*cot(d*x+c))^3,x,method=_RETURNVERBOSE)
```

```
[Out] -2/d*e^4*(-a^2/(a^2+b^2)^3*((1/8*(5*a^4+18*a^2*b^2+13*b^4)/b*(e*cot(d*x+c))
^(3/2)+1/8*a*e*(3*a^4+14*a^2*b^2+11*b^4)/b^2*(e*cot(d*x+c))^(1/2))/(e*cot(d
*x+c)*b+a*e)^2-1/8*(3*a^4+6*a^2*b^2+35*b^4)/b^2/(a*e*b)^(1/2)*arctan((e*cot
(d*x+c))^(1/2)*b/(a*e*b)^(1/2)))+1/(a^2+b^2)^3*(1/8*(a^3*e-3*a*b^2*e)*(e^2)
^(1/4)/e^2*2^(1/2)*(ln((e*cot(d*x+c)+(e^2)^(1/4)*(e*cot(d*x+c))^(1/2)*2^(1/
2)+(e^2)^(1/2))/(e*cot(d*x+c)-(e^2)^(1/4)*(e*cot(d*x+c))^(1/2)*2^(1/2)+(e^2
)^(1/2)))+2*arctan(2^(1/2)/(e^2)^(1/4)*(e*cot(d*x+c))^(1/2)+1)-2*arctan(-2^
(1/2)/(e^2)^(1/4)*(e*cot(d*x+c))^(1/2)+1))+1/8*(-3*a^2*b+b^3)/(e^2)^(1/4)*2
^(1/2)*(ln((e*cot(d*x+c)-(e^2)^(1/4)*(e*cot(d*x+c))^(1/2)*2^(1/2)+(e^2)^(1/
2))/(e*cot(d*x+c)+(e^2)^(1/4)*(e*cot(d*x+c))^(1/2)*2^(1/2)+(e^2)^(1/2)))+2*
arctan(2^(1/2)/(e^2)^(1/4)*(e*cot(d*x+c))^(1/2)+1)-2*arctan(-2^(1/2)/(e^2)^
(1/4)*(e*cot(d*x+c))^(1/2)+1))))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 4483 vs. $2(405) = 810$.

Time = 63.17 (sec) , antiderivative size = 9029, normalized size of antiderivative = 18.97

$$\int \frac{(e\cot(c+dx))^{7/2}}{(a+b\cot(c+dx))^3}\,dx = \text{Too large to display}$$

```
[In] integrate((e*cot(d*x+c))^(7/2)/(a+b*cot(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] Too large to include
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(e\cot(c+dx))^{7/2}}{(a+b\cot(c+dx))^3}\,dx = \text{Timed out}$$

```
[In] integrate((e*cot(d*x+c))**(7/2)/(a+b*cot(d*x+c))**3,x)
```

```
[Out] Timed out
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{(e\cot(c+dx))^{7/2}}{(a+b\cot(c+dx))^3}\,dx = \text{Exception raised: ValueError}$$

```
[In] integrate((e*cot(d*x+c))^(7/2)/(a+b*cot(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(e>0)', see `assume?` for more detai
ls)Is e
```

Giac [F]

$$\int \frac{(e\cot(c+dx))^{7/2}}{(a+b\cot(c+dx))^3}\,dx = \int \frac{(e\cot(dx+c))^{\frac{7}{2}}}{(b\cot(dx+c)+a)^3}\,dx$$

[In] integrate((e*cot(d*x+c))^(7/2)/(a+b*cot(d*x+c))^3,x, algorithm="giac")

[Out] integrate((e*cot(d*x + c))^(7/2)/(b*cot(d*x + c) + a)^3, x)

Mupad [B] (verification not implemented)

Time = 20.49 (sec) , antiderivative size = 20089, normalized size of antiderivative = 42.20

$$\int \frac{(e\cot(c+dx))^{7/2}}{(a+b\cot(c+dx))^3}\,dx = \text{Too large to display}$$

[In] int((e*cot(c + d*x))^(7/2)/(a + b*cot(c + d*x))^3,x)

[Out] (((e*cot(c + d*x))^(1/2)*(3*a^5*e^5 + 11*a^3*b^2*e^5))/(4*b^2*(a^4 + b^4 + 2*a^2*b^2)) + (e^4*(e*cot(c + d*x))^(3/2)*(5*a^4 + 13*a^2*b^2))/(4*b*(a^4 + b^4 + 2*a^2*b^2)))/(a^2*d*e^2 + b^2*d*e^2*cot(c + d*x)^2 + 2*a*b*d*e^2*cot(c + d*x)) - atan(((((32*a*b^18*d^2*e^21 - 18*a^19*d^2*e^21 - 6528*a^3*b^16*d^2*e^21 + 2758*a^5*b^14*d^2*e^21 + 26482*a^7*b^12*d^2*e^21 + 21582*a^9*b^10*d^2*e^21 + 7594*a^11*b^8*d^2*e^21 + 3314*a^13*b^6*d^2*e^21 + 246*a^15*b^4*d^2*e^21 + 90*a^17*b^2*d^2*e^21)/(b^19*d^5 + 8*a^2*b^17*d^5 + 28*a^4*b^15*d^5 + 56*a^6*b^13*d^5 + 70*a^8*b^11*d^5 + 56*a^10*b^9*d^5 + 28*a^12*b^7*d^5 + 8*a^14*b^5*d^5 + a^16*b^3*d^5) + (((1600*a^2*b^23*d^4*e^14 + 12864*a^4*b^21*d^4*e^14 + 45312*a^6*b^19*d^4*e^14 + 91392*a^8*b^17*d^4*e^14 + 115584*a^10*b^15*d^4*e^14 + 94080*a^12*b^13*d^4*e^14 + 48384*a^14*b^11*d^4*e^14 + 14592*a^16*b^9*d^4*e^14 + 2112*a^18*b^7*d^4*e^14 + 64*a^20*b^5*d^4*e^14)/(b^19*d^5 + 8*a^2*b^17*d^5 + 28*a^4*b^15*d^5 + 56*a^6*b^13*d^5 + 70*a^8*b^11*d^5 + 56*a^10*b^9*d^5 + 28*a^12*b^7*d^5 + 8*a^14*b^5*d^5 + a^16*b^3*d^5) + ((e*cot(c + d*x))^(1/2)*((e^7*1i)/(4*(b^6*d^2 - a^6*d^2 + a*b^5*d^2*6i + a^5*b*d^2*6i - 15*a^2*b^4*d^2 - a^3*b^3*d^2*20i + 15*a^4*b^2*d^2)))^(1/2)*(512*b^28*d^4*e^10 + 4608*a^2*b^26*d^4*e^10 + 17920*a^4*b^24*d^4*e^10 + 38400*a^6*b^22*d^4*e^10 + 46080*a^8*b^20*d^4*e^10 + 21504*a^10*b^18*d^4*e^10 - 21504*a^12*b^16*d^4*e^10 - 46080*a^14*b^14*d^4*e^10 - 38400*a^16*b^12*d^4*e^10 - 17920*a^18*b^10*d^4*e^10 - 4608*a^20*b^8*d^4*e^10 - 512*a^22*b^6*d^4*e^10))/(b^19*d^4 + 8*a^2*b^17*d^4 + 28*a^4*b^15*d^4 + 56*a^6*b^13*d^4 + 70*a^8*b^11*d^4 + 56*a^10*b^9*d^4 + 28*a^12*b^7*d^4 + 8*a^14*b^5*d^4 + a^16*b^3*d^4))*((e^7*1i)/(4*(b^6*d^2 - a^6*d^2 + a*b^5*d^2*6i + a^5*b*d^2*6i - 15*a^2*b^4*d^2 - a^3*b^3*d^2*20i + 15*a^4*b^2*d^2)))^(1/2) - ((e*cot(c + d*x))^(1/2)*(1472*a*b^21*d^2*e^17 + 72*a^21*b*d^2*e^17 + 1024*a^3*b^19*d^2*e^17 + 1352*a^5*b^17*d^2*e^17 + 28224*a^7*b^15*d^2*e^17 + 70240*a^9*b^13*d^2*e^17

+ 72640*a^11*b^11*d^2*e^17 + 39088*a^13*b^9*d^2*e^17 + 13248*a^15*b^7*d^2*e
^17 + 3488*a^17*b^5*d^2*e^17 + 576*a^19*b^3*d^2*e^17))/(b^19*d^4 + 8*a^2*b^
17*d^4 + 28*a^4*b^15*d^4 + 56*a^6*b^13*d^4 + 70*a^8*b^11*d^4 + 56*a^10*b^9*
d^4 + 28*a^12*b^7*d^4 + 8*a^14*b^5*d^4 + a^16*b^3*d^4))*((e^7*1i)/(4*(b^6*d
^2 - a^6*d^2 + a*b^5*d^2*6i + a^5*b*d^2*6i - 15*a^2*b^4*d^2 - a^3*b^3*d^2*2
0i + 15*a^4*b^2*d^2)))^(1/2))*((e^7*1i)/(4*(b^6*d^2 - a^6*d^2 + a*b^5*d^2*6
i + a^5*b*d^2*6i - 15*a^2*b^4*d^2 - a^3*b^3*d^2*20i + 15*a^4*b^2*d^2)))^(1/
2) + ((e*cot(c + d*x))^(1/2)*(9*a^16*e^24 + 32*b^16*e^24 + 128*a^2*b^14*e^2
4 + 1417*a^4*b^12*e^24 - 6802*a^6*b^10*e^24 - 1017*a^8*b^8*e^24 - 1020*a^10
*b^6*e^24 + 39*a^12*b^4*e^24 - 18*a^14*b^2*e^24))/(b^19*d^4 + 8*a^2*b^17*d^
4 + 28*a^4*b^15*d^4 + 56*a^6*b^13*d^4 + 70*a^8*b^11*d^4 + 56*a^10*b^9*d^4 +
28*a^12*b^7*d^4 + 8*a^14*b^5*d^4 + a^16*b^3*d^4))*((e^7*1i)/(4*(b^6*d^2 -
a^6*d^2 + a*b^5*d^2*6i + a^5*b*d^2*6i - 15*a^2*b^4*d^2 - a^3*b^3*d^2*20i +
15*a^4*b^2*d^2)))^(1/2)*1i - (((32*a*b^18*d^2*e^21 - 18*a^19*d^2*e^21 - 652
8*a^3*b^16*d^2*e^21 + 2758*a^5*b^14*d^2*e^21 + 26482*a^7*b^12*d^2*e^21 + 21
582*a^9*b^10*d^2*e^21 + 7594*a^11*b^8*d^2*e^21 + 3314*a^13*b^6*d^2*e^21 + 2
46*a^15*b^4*d^2*e^21 + 90*a^17*b^2*d^2*e^21)/(b^19*d^5 + 8*a^2*b^17*d^5 + 2
8*a^4*b^15*d^5 + 56*a^6*b^13*d^5 + 70*a^8*b^11*d^5 + 56*a^10*b^9*d^5 + 28*a
^12*b^7*d^5 + 8*a^14*b^5*d^5 + a^16*b^3*d^5) + (((1600*a^2*b^23*d^4*e^14 +
12864*a^4*b^21*d^4*e^14 + 45312*a^6*b^19*d^4*e^14 + 91392*a^8*b^17*d^4*e^14
+ 115584*a^10*b^15*d^4*e^14 + 94080*a^12*b^13*d^4*e^14 + 48384*a^14*b^11*d
^4*e^14 + 14592*a^16*b^9*d^4*e^14 + 2112*a^18*b^7*d^4*e^14 + 64*a^20*b^5*d^
4*e^14)/(b^19*d^5 + 8*a^2*b^17*d^5 + 28*a^4*b^15*d^5 + 56*a^6*b^13*d^5 + 70
*a^8*b^11*d^5 + 56*a^10*b^9*d^5 + 28*a^12*b^7*d^5 + 8*a^14*b^5*d^5 + a^16*b
^3*d^5) - ((e*cot(c + d*x))^(1/2)*((e^7*1i)/(4*(b^6*d^2 - a^6*d^2 + a*b^5*d
^2*6i + a^5*b*d^2*6i - 15*a^2*b^4*d^2 - a^3*b^3*d^2*20i + 15*a^4*b^2*d^2)))
^(1/2)*(512*b^28*d^4*e^10 + 4608*a^2*b^26*d^4*e^10 + 17920*a^4*b^24*d^4*e^1
0 + 38400*a^6*b^22*d^4*e^10 + 46080*a^8*b^20*d^4*e^10 + 21504*a^10*b^18*d^4
*e^10 - 21504*a^12*b^16*d^4*e^10 - 46080*a^14*b^14*d^4*e^10 - 38400*a^16*b^
12*d^4*e^10 - 17920*a^18*b^10*d^4*e^10 - 4608*a^20*b^8*d^4*e^10 - 512*a^22*
b^6*d^4*e^10))/(b^19*d^4 + 8*a^2*b^17*d^4 + 28*a^4*b^15*d^4 + 56*a^6*b^13*d
^4 + 70*a^8*b^11*d^4 + 56*a^10*b^9*d^4 + 28*a^12*b^7*d^4 + 8*a^14*b^5*d^4 +
a^16*b^3*d^4))*((e^7*1i)/(4*(b^6*d^2 - a^6*d^2 + a*b^5*d^2*6i + a^5*b*d^2*
6i - 15*a^2*b^4*d^2 - a^3*b^3*d^2*20i + 15*a^4*b^2*d^2)))^(1/2) + ((e*cot(c
+ d*x))^(1/2)*(1472*a*b^21*d^2*e^17 + 72*a^21*b*d^2*e^17 + 1024*a^3*b^19*d
^2*e^17 + 1352*a^5*b^17*d^2*e^17 + 28224*a^7*b^15*d^2*e^17 + 70240*a^9*b^13
*d^2*e^17 + 72640*a^11*b^11*d^2*e^17 + 39088*a^13*b^9*d^2*e^17 + 13248*a^15
*b^7*d^2*e^17 + 3488*a^17*b^5*d^2*e^17 + 576*a^19*b^3*d^2*e^17))/(b^19*d^4
+ 8*a^2*b^17*d^4 + 28*a^4*b^15*d^4 + 56*a^6*b^13*d^4 + 70*a^8*b^11*d^4 + 56
*a^10*b^9*d^4 + 28*a^12*b^7*d^4 + 8*a^14*b^5*d^4 + a^16*b^3*d^4))*((e^7*1i)
/(4*(b^6*d^2 - a^6*d^2 + a*b^5*d^2*6i + a^5*b*d^2*6i - 15*a^2*b^4*d^2 - a^3
*b^3*d^2*20i + 15*a^4*b^2*d^2)))^(1/2))*((e^7*1i)/(4*(b^6*d^2 - a^6*d^2 + a
*b^5*d^2*6i + a^5*b*d^2*6i - 15*a^2*b^4*d^2 - a^3*b^3*d^2*20i + 15*a^4*b^2*
d^2)))^(1/2) - ((e*cot(c + d*x))^(1/2)*(9*a^16*e^24 + 32*b^16*e^24 + 128*a^
2*b^14*e^24 + 1417*a^4*b^12*e^24 - 6802*a^6*b^10*e^24 - 1017*a^8*b^8*e^24 -

1020*a^10*b^6*e^24 + 39*a^12*b^4*e^24 - 18*a^14*b^2*e^24))/(b^19*d^4 + 8*a
^2*b^17*d^4 + 28*a^4*b^15*d^4 + 56*a^6*b^13*d^4 + 70*a^8*b^11*d^4 + 56*a^10
*b^9*d^4 + 28*a^12*b^7*d^4 + 8*a^14*b^5*d^4 + a^16*b^3*d^4))*((e^7*1i)/(4*(
b^6*d^2 - a^6*d^2 + a*b^5*d^2*6i + a^5*b*d^2*6i - 15*a^2*b^4*d^2 - a^3*b^3*
d^2*20i + 15*a^4*b^2*d^2)))^(1/2)*1i)/((9*a^12*b*e^28 + 280*a^2*b^11*e^28 +
1553*a^4*b^9*e^28 + 492*a^6*b^7*e^28 + 270*a^8*b^5*e^28 + 36*a^10*b^3*e^28
)/(b^19*d^5 + 8*a^2*b^17*d^5 + 28*a^4*b^15*d^5 + 56*a^6*b^13*d^5 + 70*a^8*b
^11*d^5 + 56*a^10*b^9*d^5 + 28*a^12*b^7*d^5 + 8*a^14*b^5*d^5 + a^16*b^3*d^5
) + (((32*a*b^18*d^2*e^21 - 18*a^19*d^2*e^21 - 6528*a^3*b^16*d^2*e^21 + 275
8*a^5*b^14*d^2*e^21 + 26482*a^7*b^12*d^2*e^21 + 21582*a^9*b^10*d^2*e^21 + 7
594*a^11*b^8*d^2*e^21 + 3314*a^13*b^6*d^2*e^21 + 246*a^15*b^4*d^2*e^21 + 90
*a^17*b^2*d^2*e^21)/(b^19*d^5 + 8*a^2*b^17*d^5 + 28*a^4*b^15*d^5 + 56*a^6*b
^13*d^5 + 70*a^8*b^11*d^5 + 56*a^10*b^9*d^5 + 28*a^12*b^7*d^5 + 8*a^14*b^5*
d^5 + a^16*b^3*d^5) + (((1600*a^2*b^23*d^4*e^14 + 12864*a^4*b^21*d^4*e^14 +
45312*a^6*b^19*d^4*e^14 + 91392*a^8*b^17*d^4*e^14 + 115584*a^10*b^15*d^4*e
^14 + 94080*a^12*b^13*d^4*e^14 + 48384*a^14*b^11*d^4*e^14 + 14592*a^16*b^9*
d^4*e^14 + 2112*a^18*b^7*d^4*e^14 + 64*a^20*b^5*d^4*e^14)/(b^19*d^5 + 8*a^2
*b^17*d^5 + 28*a^4*b^15*d^5 + 56*a^6*b^13*d^5 + 70*a^8*b^11*d^5 + 56*a^10*b
^9*d^5 + 28*a^12*b^7*d^5 + 8*a^14*b^5*d^5 + a^16*b^3*d^5) + ((e*cot(c + d*x
))^(1/2)*((e^7*1i)/(4*(b^6*d^2 - a^6*d^2 + a*b^5*d^2*6i + a^5*b*d^2*6i - 15
*a^2*b^4*d^2 - a^3*b^3*d^2*20i + 15*a^4*b^2*d^2)))^(1/2)*(512*b^28*d^4*e^10
+ 4608*a^2*b^26*d^4*e^10 + 17920*a^4*b^24*d^4*e^10 + 38400*a^6*b^22*d^4*e^
10 + 46080*a^8*b^20*d^4*e^10 + 21504*a^10*b^18*d^4*e^10 - 21504*a^12*b^16*d
^4*e^10 - 46080*a^14*b^14*d^4*e^10 - 38400*a^16*b^12*d^4*e^10 - 17920*a^18*
b^10*d^4*e^10 - 4608*a^20*b^8*d^4*e^10 - 512*a^22*b^6*d^4*e^10))/(b^19*d^4
+ 8*a^2*b^17*d^4 + 28*a^4*b^15*d^4 + 56*a^6*b^13*d^4 + 70*a^8*b^11*d^4 + 56
*a^10*b^9*d^4 + 28*a^12*b^7*d^4 + 8*a^14*b^5*d^4 + a^16*b^3*d^4))*((e^7*1i)
/(4*(b^6*d^2 - a^6*d^2 + a*b^5*d^2*6i + a^5*b*d^2*6i - 15*a^2*b^4*d^2 - a^3
*b^3*d^2*20i + 15*a^4*b^2*d^2)))^(1/2) - ((e*cot(c + d*x))^(1/2)*(1472*a*b^
21*d^2*e^17 + 72*a^21*b*d^2*e^17 + 1024*a^3*b^19*d^2*e^17 + 1352*a^5*b^17*d
^2*e^17 + 28224*a^7*b^15*d^2*e^17 + 70240*a^9*b^13*d^2*e^17 + 72640*a^11*b^
11*d^2*e^17 + 39088*a^13*b^9*d^2*e^17 + 13248*a^15*b^7*d^2*e^17 + 3488*a^17
*b^5*d^2*e^17 + 576*a^19*b^3*d^2*e^17))/(b^19*d^4 + 8*a^2*b^17*d^4 + 28*a^4
*b^15*d^4 + 56*a^6*b^13*d^4 + 70*a^8*b^11*d^4 + 56*a^10*b^9*d^4 + 28*a^12*b
^7*d^4 + 8*a^14*b^5*d^4 + a^16*b^3*d^4))*((e^7*1i)/(4*(b^6*d^2 - a^6*d^2 +
a*b^5*d^2*6i + a^5*b*d^2*6i - 15*a^2*b^4*d^2 - a^3*b^3*d^2*20i + 15*a^4*b^2
d^2)))^(1/2))((e^7*1i)/(4*(b^6*d^2 - a^6*d^2 + a*b^5*d^2*6i + a^5*b*d^2*6
i - 15*a^2*b^4*d^2 - a^3*b^3*d^2*20i + 15*a^4*b^2*d^2)))^(1/2) + ((e*cot(c
+ d*x))^(1/2)*(9*a^16*e^24 + 32*b^16*e^24 + 128*a^2*b^14*e^24 + 1417*a^4*b^
12*e^24 - 6802*a^6*b^10*e^24 - 1017*a^8*b^8*e^24 - 1020*a^10*b^6*e^24 + 39*
a^12*b^4*e^24 - 18*a^14*b^2*e^24))/(b^19*d^4 + 8*a^2*b^17*d^4 + 28*a^4*b^15
*d^4 + 56*a^6*b^13*d^4 + 70*a^8*b^11*d^4 + 56*a^10*b^9*d^4 + 28*a^12*b^7*d^
4 + 8*a^14*b^5*d^4 + a^16*b^3*d^4))*((e^7*1i)/(4*(b^6*d^2 - a^6*d^2 + a*b^5
*d^2*6i + a^5*b*d^2*6i - 15*a^2*b^4*d^2 - a^3*b^3*d^2*20i + 15*a^4*b^2*d^2)
))^(1/2) + (((32*a*b^18*d^2*e^21 - 18*a^19*d^2*e^21 - 6528*a^3*b^16*d^2*e^2

1 + 2758*a^5*b^14*d^2*e^21 + 26482*a^7*b^12*d^2*e^21 + 21582*a^9*b^10*d^2*e
^21 + 7594*a^11*b^8*d^2*e^21 + 3314*a^13*b^6*d^2*e^21 + 246*a^15*b^4*d^2*e^
21 + 90*a^17*b^2*d^2*e^21)/(b^19*d^5 + 8*a^2*b^17*d^5 + 28*a^4*b^15*d^5 + 5
6*a^6*b^13*d^5 + 70*a^8*b^11*d^5 + 56*a^10*b^9*d^5 + 28*a^12*b^7*d^5 + 8*a^
14*b^5*d^5 + a^16*b^3*d^5) + (((1600*a^2*b^23*d^4*e^14 + 12864*a^4*b^21*d^4
*e^14 + 45312*a^6*b^19*d^4*e^14 + 91392*a^8*b^17*d^4*e^14 + 115584*a^10*b^1
5*d^4*e^14 + 94080*a^12*b^13*d^4*e^14 + 48384*a^14*b^11*d^4*e^14 + 14592*a^
16*b^9*d^4*e^14 + 2112*a^18*b^7*d^4*e^14 + 64*a^20*b^5*d^4*e^14)/(b^19*d^5
+ 8*a^2*b^17*d^5 + 28*a^4*b^15*d^5 + 56*a^6*b^13*d^5 + 70*a^8*b^11*d^5 + 56
*a^10*b^9*d^5 + 28*a^12*b^7*d^5 + 8*a^14*b^5*d^5 + a^16*b^3*d^5) - ((e*cot(
c + d*x))^(1/2)*((e^7*1i)/(4*(b^6*d^2 - a^6*d^2 + a*b^5*d^2*6i + a^5*b*d^2*
6i - 15*a^2*b^4*d^2 - a^3*b^3*d^2*20i + 15*a^4*b^2*d^2)))^(1/2)*(512*b^28*d
^4*e^10 + 4608*a^2*b^26*d^4*e^10 + 17920*a^4*b^24*d^4*e^10 + 38400*a^6*b^22
*d^4*e^10 + 46080*a^8*b^20*d^4*e^10 + 21504*a^10*b^18*d^4*e^10 - 21504*a^12
*b^16*d^4*e^10 - 46080*a^14*b^14*d^4*e^10 - 38400*a^16*b^12*d^4*e^10 - 1792
0*a^18*b^10*d^4*e^10 - 4608*a^20*b^8*d^4*e^10 - 512*a^22*b^6*d^4*e^10))/(b^
19*d^4 + 8*a^2*b^17*d^4 + 28*a^4*b^15*d^4 + 56*a^6*b^13*d^4 + 70*a^8*b^11*d
^4 + 56*a^10*b^9*d^4 + 28*a^12*b^7*d^4 + 8*a^14*b^5*d^4 + a^16*b^3*d^4))*((
e^7*1i)/(4*(b^6*d^2 - a^6*d^2 + a*b^5*d^2*6i + a^5*b*d^2*6i - 15*a^2*b^4*d^
2 - a^3*b^3*d^2*20i + 15*a^4*b^2*d^2)))^(1/2) + ((e*cot(c + d*x))^(1/2)*(14
72*a*b^21*d^2*e^17 + 72*a^21*b*d^2*e^17 + 1024*a^3*b^19*d^2*e^17 + 1352*a^5
*b^17*d^2*e^17 + 28224*a^7*b^15*d^2*e^17 + 70240*a^9*b^13*d^2*e^17 + 72640*
a^11*b^11*d^2*e^17 + 39088*a^13*b^9*d^2*e^17 + 13248*a^15*b^7*d^2*e^17 + 34
88*a^17*b^5*d^2*e^17 + 576*a^19*b^3*d^2*e^17))/(b^19*d^4 + 8*a^2*b^17*d^4 +
28*a^4*b^15*d^4 + 56*a^6*b^13*d^4 + 70*a^8*b^11*d^4 + 56*a^10*b^9*d^4 + 28
*a^12*b^7*d^4 + 8*a^14*b^5*d^4 + a^16*b^3*d^4))*((e^7*1i)/(4*(b^6*d^2 - a^6
*d^2 + a*b^5*d^2*6i + a^5*b*d^2*6i - 15*a^2*b^4*d^2 - a^3*b^3*d^2*20i + 15*
a^4*b^2*d^2)))^(1/2))*((e^7*1i)/(4*(b^6*d^2 - a^6*d^2 + a*b^5*d^2*6i + a^5*
b*d^2*6i - 15*a^2*b^4*d^2 - a^3*b^3*d^2*20i + 15*a^4*b^2*d^2)))^(1/2) - ((e
*cot(c + d*x))^(1/2)*(9*a^16*e^24 + 32*b^16*e^24 + 128*a^2*b^14*e^24 + 1417
*a^4*b^12*e^24 - 6802*a^6*b^10*e^24 - 1017*a^8*b^8*e^24 - 1020*a^10*b^6*e^2
4 + 39*a^12*b^4*e^24 - 18*a^14*b^2*e^24))/(b^19*d^4 + 8*a^2*b^17*d^4 + 28*a
^4*b^15*d^4 + 56*a^6*b^13*d^4 + 70*a^8*b^11*d^4 + 56*a^10*b^9*d^4 + 28*a^12
*b^7*d^4 + 8*a^14*b^5*d^4 + a^16*b^3*d^4))*((e^7*1i)/(4*(b^6*d^2 - a^6*d^2
+ a*b^5*d^2*6i + a^5*b*d^2*6i - 15*a^2*b^4*d^2 - a^3*b^3*d^2*20i + 15*a^4*b
^2*d^2)))^(1/2)))*((e^7*1i)/(4*(b^6*d^2 - a^6*d^2 + a*b^5*d^2*6i + a^5*b*d^
2*6i - 15*a^2*b^4*d^2 - a^3*b^3*d^2*20i + 15*a^4*b^2*d^2)))^(1/2)*2i - atan
(((((32*a*b^18*d^2*e^21 - 18*a^19*d^2*e^21 - 6528*a^3*b^16*d^2*e^21 + 2758*
a^5*b^14*d^2*e^21 + 26482*a^7*b^12*d^2*e^21 + 21582*a^9*b^10*d^2*e^21 + 759
4*a^11*b^8*d^2*e^21 + 3314*a^13*b^6*d^2*e^21 + 246*a^15*b^4*d^2*e^21 + 90*a
^17*b^2*d^2*e^21)/(b^19*d^5 + 8*a^2*b^17*d^5 + 28*a^4*b^15*d^5 + 56*a^6*b^1
3*d^5 + 70*a^8*b^11*d^5 + 56*a^10*b^9*d^5 + 28*a^12*b^7*d^5 + 8*a^14*b^5*d^
5 + a^16*b^3*d^5) + (((1600*a^2*b^23*d^4*e^14 + 12864*a^4*b^21*d^4*e^14 + 4
5312*a^6*b^19*d^4*e^14 + 91392*a^8*b^17*d^4*e^14 + 115584*a^10*b^15*d^4*e^1
4 + 94080*a^12*b^13*d^4*e^14 + 48384*a^14*b^11*d^4*e^14 + 14592*a^16*b^9*d^

4*e^14 + 2112*a^18*b^7*d^4*e^14 + 64*a^20*b^5*d^4*e^14)/(b^19*d^5 + 8*a^2*b^17*d^5 + 28*a^4*b^15*d^5 + 56*a^6*b^13*d^5 + 70*a^8*b^11*d^5 + 56*a^10*b^9*d^5 + 28*a^12*b^7*d^5 + 8*a^14*b^5*d^5 + a^16*b^3*d^5) + ((e*cot(c + d*x))^(1/2)*(e^7/(4*(b^6*d^2*1i - a^6*d^2*1i + 6*a*b^5*d^2 + 6*a^5*b*d^2 - a^2*b^4*d^2*15i - 20*a^3*b^3*d^2 + a^4*b^2*d^2*15i)))^(1/2)*(512*b^28*d^4*e^10 + 4608*a^2*b^26*d^4*e^10 + 17920*a^4*b^24*d^4*e^10 + 38400*a^6*b^22*d^4*e^10 + 46080*a^8*b^20*d^4*e^10 + 21504*a^10*b^18*d^4*e^10 - 21504*a^12*b^16*d^4*e^10 - 46080*a^14*b^14*d^4*e^10 - 38400*a^16*b^12*d^4*e^10 - 17920*a^18*b^10*d^4*e^10 - 4608*a^20*b^8*d^4*e^10 - 512*a^22*b^6*d^4*e^10))/(b^19*d^4 + 8*a^2*b^17*d^4 + 28*a^4*b^15*d^4 + 56*a^6*b^13*d^4 + 70*a^8*b^11*d^4 + 56*a^10*b^9*d^4 + 28*a^12*b^7*d^4 + 8*a^14*b^5*d^4 + a^16*b^3*d^4))*(e^7/(4*(b^6*d^2*1i - a^6*d^2*1i + 6*a*b^5*d^2 + 6*a^5*b*d^2 - a^2*b^4*d^2*15i - 20*a^3*b^3*d^2 + a^4*b^2*d^2*15i)))^(1/2) - ((e*cot(c + d*x))^(1/2)*(1472*a*b^21*d^2*e^17 + 72*a^21*b*d^2*e^17 + 1024*a^3*b^19*d^2*e^17 + 1352*a^5*b^17*d^2*e^17 + 28224*a^7*b^15*d^2*e^17 + 70240*a^9*b^13*d^2*e^17 + 72640*a^11*b^11*d^2*e^17 + 39088*a^13*b^9*d^2*e^17 + 13248*a^15*b^7*d^2*e^17 + 3488*a^17*b^5*d^2*e^17 + 576*a^19*b^3*d^2*e^17))/(b^19*d^4 + 8*a^2*b^17*d^4 + 28*a^4*b^15*d^4 + 56*a^6*b^13*d^4 + 70*a^8*b^11*d^4 + 56*a^10*b^9*d^4 + 28*a^12*b^7*d^4 + 8*a^14*b^5*d^4 + a^16*b^3*d^4))*(e^7/(4*(b^6*d^2*1i - a^6*d^2*1i + 6*a*b^5*d^2 + 6*a^5*b*d^2 - a^2*b^4*d^2*15i - 20*a^3*b^3*d^2 + a^4*b^2*d^2*15i)))^(1/2))*(e^7/(4*(b^6*d^2*1i - a^6*d^2*1i + 6*a*b^5*d^2 + 6*a^5*b*d^2 - a^2*b^4*d^2*15i - 20*a^3*b^3*d^2 + a^4*b^2*d^2*15i)))^(1/2) + ((e*cot(c + d*x))^(1/2)*(9*a^16*e^24 + 32*b^16*e^24 + 128*a^2*b^14*e^24 + 1417*a^4*b^12*e^24 - 6802*a^6*b^10*e^24 - 1017*a^8*b^8*e^24 - 1020*a^10*b^6*e^24 + 39*a^12*b^4*e^24 - 18*a^14*b^2*e^24))/(b^19*d^4 + 8*a^2*b^17*d^4 + 28*a^4*b^15*d^4 + 56*a^6*b^13*d^4 + 70*a^8*b^11*d^4 + 56*a^10*b^9*d^4 + 28*a^12*b^7*d^4 + 8*a^14*b^5*d^4 + a^16*b^3*d^4))*(e^7/(4*(b^6*d^2*1i - a^6*d^2*1i + 6*a*b^5*d^2 + 6*a^5*b*d^2 - a^2*b^4*d^2*15i - 20*a^3*b^3*d^2 + a^4*b^2*d^2*15i)))^(1/2)*1i - (((32*a*b^18*d^2*e^21 - 18*a^19*d^2*e^21 - 6528*a^3*b^16*d^2*e^21 + 2758*a^5*b^14*d^2*e^21 + 26482*a^7*b^12*d^2*e^21 + 21582*a^9*b^10*d^2*e^21 + 7594*a^11*b^8*d^2*e^21 + 3314*a^13*b^6*d^2*e^21 + 246*a^15*b^4*d^2*e^21 + 90*a^17*b^2*d^2*e^21)/(b^19*d^5 + 8*a^2*b^17*d^5 + 28*a^4*b^15*d^5 + 56*a^6*b^13*d^5 + 70*a^8*b^11*d^5 + 56*a^10*b^9*d^5 + 28*a^12*b^7*d^5 + 8*a^14*b^5*d^5 + a^16*b^3*d^5) + (((1600*a^2*b^23*d^4*e^14 + 12864*a^4*b^21*d^4*e^14 + 45312*a^6*b^19*d^4*e^14 + 91392*a^8*b^17*d^4*e^14 + 115584*a^10*b^15*d^4*e^14 + 94080*a^12*b^13*d^4*e^14 + 48384*a^14*b^11*d^4*e^14 + 14592*a^16*b^9*d^4*e^14 + 2112*a^18*b^7*d^4*e^14 + 64*a^20*b^5*d^4*e^14)/(b^19*d^5 + 8*a^2*b^17*d^5 + 28*a^4*b^15*d^5 + 56*a^6*b^13*d^5 + 70*a^8*b^11*d^5 + 56*a^10*b^9*d^5 + 28*a^12*b^7*d^5 + 8*a^14*b^5*d^5 + a^16*b^3*d^5) - ((e*cot(c + d*x))^(1/2)*(e^7/(4*(b^6*d^2*1i - a^6*d^2*1i + 6*a*b^5*d^2 + 6*a^5*b*d^2 - a^2*b^4*d^2*15i - 20*a^3*b^3*d^2 + a^4*b^2*d^2*15i)))^(1/2)*(512*b^28*d^4*e^10 + 4608*a^2*b^26*d^4*e^10 + 17920*a^4*b^24*d^4*e^10 + 38400*a^6*b^22*d^4*e^10 + 46080*a^8*b^20*d^4*e^10 + 21504*a^10*b^18*d^4*e^10 - 21504*a^12*b^16*d^4*e^10 - 46080*a^14*b^14*d^4*e^10 - 38400*a^16*b^12*d^4*e^10 - 17920*a^18*b^10*d^4*e^10 - 4608*a^20*b^8*d^4*e^10 - 512*a^22*b^6*d^4*e^10))/(b

^19*d^4 + 8*a^2*b^17*d^4 + 28*a^4*b^15*d^4 + 56*a^6*b^13*d^4 + 70*a^8*b^11*
d^4 + 56*a^10*b^9*d^4 + 28*a^12*b^7*d^4 + 8*a^14*b^5*d^4 + a^16*b^3*d^4))*(
e^7/(4*(b^6*d^2*1i - a^6*d^2*1i + 6*a*b^5*d^2 + 6*a^5*b*d^2 - a^2*b^4*d^2*1
5i - 20*a^3*b^3*d^2 + a^4*b^2*d^2*15i)))^(1/2) + ((e*cot(c + d*x))^(1/2)*(1
472*a*b^21*d^2*e^17 + 72*a^21*b*d^2*e^17 + 1024*a^3*b^19*d^2*e^17 + 1352*a^
5*b^17*d^2*e^17 + 28224*a^7*b^15*d^2*e^17 + 70240*a^9*b^13*d^2*e^17 + 72640
*a^11*b^11*d^2*e^17 + 39088*a^13*b^9*d^2*e^17 + 13248*a^15*b^7*d^2*e^17 + 3
488*a^17*b^5*d^2*e^17 + 576*a^19*b^3*d^2*e^17))/(b^19*d^4 + 8*a^2*b^17*d^4
+ 28*a^4*b^15*d^4 + 56*a^6*b^13*d^4 + 70*a^8*b^11*d^4 + 56*a^10*b^9*d^4 + 2
8*a^12*b^7*d^4 + 8*a^14*b^5*d^4 + a^16*b^3*d^4))*(e^7/(4*(b^6*d^2*1i - a^6*
d^2*1i + 6*a*b^5*d^2 + 6*a^5*b*d^2 - a^2*b^4*d^2*15i - 20*a^3*b^3*d^2 + a^4
*b^2*d^2*15i)))^(1/2))*(e^7/(4*(b^6*d^2*1i - a^6*d^2*1i + 6*a*b^5*d^2 + 6*a
^5*b*d^2 - a^2*b^4*d^2*15i - 20*a^3*b^3*d^2 + a^4*b^2*d^2*15i)))^(1/2) - ((
e*cot(c + d*x))^(1/2)*(9*a^16*e^24 + 32*b^16*e^24 + 128*a^2*b^14*e^24 + 141
7*a^4*b^12*e^24 - 6802*a^6*b^10*e^24 - 1017*a^8*b^8*e^24 - 1020*a^10*b^6*e^
24 + 39*a^12*b^4*e^24 - 18*a^14*b^2*e^24))/(b^19*d^4 + 8*a^2*b^17*d^4 + 28*
a^4*b^15*d^4 + 56*a^6*b^13*d^4 + 70*a^8*b^11*d^4 + 56*a^10*b^9*d^4 + 28*a^1
2*b^7*d^4 + 8*a^14*b^5*d^4 + a^16*b^3*d^4))*(e^7/(4*(b^6*d^2*1i - a^6*d^2*1
i + 6*a*b^5*d^2 + 6*a^5*b*d^2 - a^2*b^4*d^2*15i - 20*a^3*b^3*d^2 + a^4*b^2*
d^2*15i)))^(1/2)*1i)/((9*a^12*b*e^28 + 280*a^2*b^11*e^28 + 1553*a^4*b^9*e^2
8 + 492*a^6*b^7*e^28 + 270*a^8*b^5*e^28 + 36*a^10*b^3*e^28)/(b^19*d^5 + 8*a
^2*b^17*d^5 + 28*a^4*b^15*d^5 + 56*a^6*b^13*d^5 + 70*a^8*b^11*d^5 + 56*a^10
*b^9*d^5 + 28*a^12*b^7*d^5 + 8*a^14*b^5*d^5 + a^16*b^3*d^5) + (((32*a*b^18*
d^2*e^21 - 18*a^19*d^2*e^21 - 6528*a^3*b^16*d^2*e^21 + 2758*a^5*b^14*d^2*e^
21 + 26482*a^7*b^12*d^2*e^21 + 21582*a^9*b^10*d^2*e^21 + 7594*a^11*b^8*d^2*
e^21 + 3314*a^13*b^6*d^2*e^21 + 246*a^15*b^4*d^2*e^21 + 90*a^17*b^2*d^2*e^2
1)/(b^19*d^5 + 8*a^2*b^17*d^5 + 28*a^4*b^15*d^5 + 56*a^6*b^13*d^5 + 70*a^8*
b^11*d^5 + 56*a^10*b^9*d^5 + 28*a^12*b^7*d^5 + 8*a^14*b^5*d^5 + a^16*b^3*d^
5) + (((1600*a^2*b^23*d^4*e^14 + 12864*a^4*b^21*d^4*e^14 + 45312*a^6*b^19*d
^4*e^14 + 91392*a^8*b^17*d^4*e^14 + 115584*a^10*b^15*d^4*e^14 + 94080*a^12*
b^13*d^4*e^14 + 48384*a^14*b^11*d^4*e^14 + 14592*a^16*b^9*d^4*e^14 + 2112*a
^18*b^7*d^4*e^14 + 64*a^20*b^5*d^4*e^14)/(b^19*d^5 + 8*a^2*b^17*d^5 + 28*a^
4*b^15*d^5 + 56*a^6*b^13*d^5 + 70*a^8*b^11*d^5 + 56*a^10*b^9*d^5 + 28*a^12*
b^7*d^5 + 8*a^14*b^5*d^5 + a^16*b^3*d^5) + ((e*cot(c + d*x))^(1/2)*(e^7/(4*
(b^6*d^2*1i - a^6*d^2*1i + 6*a*b^5*d^2 + 6*a^5*b*d^2 - a^2*b^4*d^2*15i - 20
*a^3*b^3*d^2 + a^4*b^2*d^2*15i)))^(1/2)*(512*b^28*d^4*e^10 + 4608*a^2*b^26*
d^4*e^10 + 17920*a^4*b^24*d^4*e^10 + 38400*a^6*b^22*d^4*e^10 + 46080*a^8*b^
20*d^4*e^10 + 21504*a^10*b^18*d^4*e^10 - 21504*a^12*b^16*d^4*e^10 - 46080*a
^14*b^14*d^4*e^10 - 38400*a^16*b^12*d^4*e^10 - 17920*a^18*b^10*d^4*e^10 - 4
608*a^20*b^8*d^4*e^10 - 512*a^22*b^6*d^4*e^10))/(b^19*d^4 + 8*a^2*b^17*d^4
+ 28*a^4*b^15*d^4 + 56*a^6*b^13*d^4 + 70*a^8*b^11*d^4 + 56*a^10*b^9*d^4 + 2
8*a^12*b^7*d^4 + 8*a^14*b^5*d^4 + a^16*b^3*d^4))*(e^7/(4*(b^6*d^2*1i - a^6*
d^2*1i + 6*a*b^5*d^2 + 6*a^5*b*d^2 - a^2*b^4*d^2*15i - 20*a^3*b^3*d^2 + a^4
*b^2*d^2*15i)))^(1/2) - ((e*cot(c + d*x))^(1/2)*(1472*a*b^21*d^2*e^17 + 72*
a^21*b*d^2*e^17 + 1024*a^3*b^19*d^2*e^17 + 1352*a^5*b^17*d^2*e^17 + 28224*a

^7*b^15*d^2*e^17 + 70240*a^9*b^13*d^2*e^17 + 72640*a^11*b^11*d^2*e^17 + 390
88*a^13*b^9*d^2*e^17 + 13248*a^15*b^7*d^2*e^17 + 3488*a^17*b^5*d^2*e^17 + 5
76*a^19*b^3*d^2*e^17))/(b^19*d^4 + 8*a^2*b^17*d^4 + 28*a^4*b^15*d^4 + 56*a^
6*b^13*d^4 + 70*a^8*b^11*d^4 + 56*a^10*b^9*d^4 + 28*a^12*b^7*d^4 + 8*a^14*b
^5*d^4 + a^16*b^3*d^4))*(e^7/(4*(b^6*d^2*1i - a^6*d^2*1i + 6*a*b^5*d^2 + 6*
a^5*b*d^2 - a^2*b^4*d^2*15i - 20*a^3*b^3*d^2 + a^4*b^2*d^2*15i)))^(1/2))*(e
^7/(4*(b^6*d^2*1i - a^6*d^2*1i + 6*a*b^5*d^2 + 6*a^5*b*d^2 - a^2*b^4*d^2*15
i - 20*a^3*b^3*d^2 + a^4*b^2*d^2*15i)))^(1/2) + ((e*cot(c + d*x))^(1/2)*(9*
a^16*e^24 + 32*b^16*e^24 + 128*a^2*b^14*e^24 + 1417*a^4*b^12*e^24 - 6802*a^
6*b^10*e^24 - 1017*a^8*b^8*e^24 - 1020*a^10*b^6*e^24 + 39*a^12*b^4*e^24 - 1
8*a^14*b^2*e^24))/(b^19*d^4 + 8*a^2*b^17*d^4 + 28*a^4*b^15*d^4 + 56*a^6*b^1
3*d^4 + 70*a^8*b^11*d^4 + 56*a^10*b^9*d^4 + 28*a^12*b^7*d^4 + 8*a^14*b^5*d^
4 + a^16*b^3*d^4))*(e^7/(4*(b^6*d^2*1i - a^6*d^2*1i + 6*a*b^5*d^2 + 6*a^5*b
*d^2 - a^2*b^4*d^2*15i - 20*a^3*b^3*d^2 + a^4*b^2*d^2*15i)))^(1/2) + (((32*
a*b^18*d^2*e^21 - 18*a^19*d^2*e^21 - 6528*a^3*b^16*d^2*e^21 + 2758*a^5*b^14
*d^2*e^21 + 26482*a^7*b^12*d^2*e^21 + 21582*a^9*b^10*d^2*e^21 + 7594*a^11*b
^8*d^2*e^21 + 3314*a^13*b^6*d^2*e^21 + 246*a^15*b^4*d^2*e^21 + 90*a^17*b^2*
d^2*e^21)/(b^19*d^5 + 8*a^2*b^17*d^5 + 28*a^4*b^15*d^5 + 56*a^6*b^13*d^5 +
70*a^8*b^11*d^5 + 56*a^10*b^9*d^5 + 28*a^12*b^7*d^5 + 8*a^14*b^5*d^5 + a^16
*b^3*d^5) + (((1600*a^2*b^23*d^4*e^14 + 12864*a^4*b^21*d^4*e^14 + 45312*a^6
*b^19*d^4*e^14 + 91392*a^8*b^17*d^4*e^14 + 115584*a^10*b^15*d^4*e^14 + 9408
0*a^12*b^13*d^4*e^14 + 48384*a^14*b^11*d^4*e^14 + 14592*a^16*b^9*d^4*e^14 +
2112*a^18*b^7*d^4*e^14 + 64*a^20*b^5*d^4*e^14)/(b^19*d^5 + 8*a^2*b^17*d^5
+ 28*a^4*b^15*d^5 + 56*a^6*b^13*d^5 + 70*a^8*b^11*d^5 + 56*a^10*b^9*d^5 + 2
8*a^12*b^7*d^5 + 8*a^14*b^5*d^5 + a^16*b^3*d^5) - ((e*cot(c + d*x))^(1/2)*(
e^7/(4*(b^6*d^2*1i - a^6*d^2*1i + 6*a*b^5*d^2 + 6*a^5*b*d^2 - a^2*b^4*d^2*1
5i - 20*a^3*b^3*d^2 + a^4*b^2*d^2*15i)))^(1/2)*(512*b^28*d^4*e^10 + 4608*a^
2*b^26*d^4*e^10 + 17920*a^4*b^24*d^4*e^10 + 38400*a^6*b^22*d^4*e^10 + 46080
*a^8*b^20*d^4*e^10 + 21504*a^10*b^18*d^4*e^10 - 21504*a^12*b^16*d^4*e^10 -
46080*a^14*b^14*d^4*e^10 - 38400*a^16*b^12*d^4*e^10 - 17920*a^18*b^10*d^4*e
^10 - 4608*a^20*b^8*d^4*e^10 - 512*a^22*b^6*d^4*e^10))/(b^19*d^4 + 8*a^2*b^
17*d^4 + 28*a^4*b^15*d^4 + 56*a^6*b^13*d^4 + 70*a^8*b^11*d^4 + 56*a^10*b^9*
d^4 + 28*a^12*b^7*d^4 + 8*a^14*b^5*d^4 + a^16*b^3*d^4))*(e^7/(4*(b^6*d^2*1i
- a^6*d^2*1i + 6*a*b^5*d^2 + 6*a^5*b*d^2 - a^2*b^4*d^2*15i - 20*a^3*b^3*d^
2 + a^4*b^2*d^2*15i)))^(1/2) + ((e*cot(c + d*x))^(1/2)*(1472*a*b^21*d^2*e^1
7 + 72*a^21*b*d^2*e^17 + 1024*a^3*b^19*d^2*e^17 + 1352*a^5*b^17*d^2*e^17 +
28224*a^7*b^15*d^2*e^17 + 70240*a^9*b^13*d^2*e^17 + 72640*a^11*b^11*d^2*e^1
7 + 39088*a^13*b^9*d^2*e^17 + 13248*a^15*b^7*d^2*e^17 + 3488*a^17*b^5*d^2*e
^17 + 576*a^19*b^3*d^2*e^17))/(b^19*d^4 + 8*a^2*b^17*d^4 + 28*a^4*b^15*d^4
+ 56*a^6*b^13*d^4 + 70*a^8*b^11*d^4 + 56*a^10*b^9*d^4 + 28*a^12*b^7*d^4 + 8
*a^14*b^5*d^4 + a^16*b^3*d^4))*(e^7/(4*(b^6*d^2*1i - a^6*d^2*1i + 6*a*b^5*d
^2 + 6*a^5*b*d^2 - a^2*b^4*d^2*15i - 20*a^3*b^3*d^2 + a^4*b^2*d^2*15i)))^(1
/2))*(e^7/(4*(b^6*d^2*1i - a^6*d^2*1i + 6*a*b^5*d^2 + 6*a^5*b*d^2 - a^2*b^4
*d^2*15i - 20*a^3*b^3*d^2 + a^4*b^2*d^2*15i)))^(1/2) - ((e*cot(c + d*x))^(1
/2)*(9*a^16*e^24 + 32*b^16*e^24 + 128*a^2*b^14*e^24 + 1417*a^4*b^12*e^24 -

6802*a^6*b^10*e^24 - 1017*a^8*b^8*e^24 - 1020*a^10*b^6*e^24 + 39*a^12*b^4*e
^24 - 18*a^14*b^2*e^24))/(b^19*d^4 + 8*a^2*b^17*d^4 + 28*a^4*b^15*d^4 + 56*
a^6*b^13*d^4 + 70*a^8*b^11*d^4 + 56*a^10*b^9*d^4 + 28*a^12*b^7*d^4 + 8*a^14
*b^5*d^4 + a^16*b^3*d^4))*(e^7/(4*(b^6*d^2*1i - a^6*d^2*1i + 6*a*b^5*d^2 +
6*a^5*b*d^2 - a^2*b^4*d^2*15i - 20*a^3*b^3*d^2 + a^4*b^2*d^2*15i)))^(1/2)))
(e^7/(4(b^6*d^2*1i - a^6*d^2*1i + 6*a*b^5*d^2 + 6*a^5*b*d^2 - a^2*b^4*d^2
*15i - 20*a^3*b^3*d^2 + a^4*b^2*d^2*15i)))^(1/2)*2i - (atan((((((e*cot(c +
d*x))^(1/2)*(9*a^16*e^24 + 32*b^16*e^24 + 128*a^2*b^14*e^24 + 1417*a^4*b^12
*e^24 - 6802*a^6*b^10*e^24 - 1017*a^8*b^8*e^24 - 1020*a^10*b^6*e^24 + 39*a^
12*b^4*e^24 - 18*a^14*b^2*e^24))/(b^19*d^4 + 8*a^2*b^17*d^4 + 28*a^4*b^15*d
^4 + 56*a^6*b^13*d^4 + 70*a^8*b^11*d^4 + 56*a^10*b^9*d^4 + 28*a^12*b^7*d^4
+ 8*a^14*b^5*d^4 + a^16*b^3*d^4) - (((32*a*b^18*d^2*e^21 - 18*a^19*d^2*e^21
 - 6528*a^3*b^16*d^2*e^21 + 2758*a^5*b^14*d^2*e^21 + 26482*a^7*b^12*d^2*e^2
1 + 21582*a^9*b^10*d^2*e^21 + 7594*a^11*b^8*d^2*e^21 + 3314*a^13*b^6*d^2*e^
21 + 246*a^15*b^4*d^2*e^21 + 90*a^17*b^2*d^2*e^21)/(b^19*d^5 + 8*a^2*b^17*d
^5 + 28*a^4*b^15*d^5 + 56*a^6*b^13*d^5 + 70*a^8*b^11*d^5 + 56*a^10*b^9*d^5
+ 28*a^12*b^7*d^5 + 8*a^14*b^5*d^5 + a^16*b^3*d^5) + ((((e*cot(c + d*x))^(1
/2)*(1472*a*b^21*d^2*e^17 + 72*a^21*b*d^2*e^17 + 1024*a^3*b^19*d^2*e^17 + 1
352*a^5*b^17*d^2*e^17 + 28224*a^7*b^15*d^2*e^17 + 70240*a^9*b^13*d^2*e^17 +
 72640*a^11*b^11*d^2*e^17 + 39088*a^13*b^9*d^2*e^17 + 13248*a^15*b^7*d^2*e^
17 + 3488*a^17*b^5*d^2*e^17 + 576*a^19*b^3*d^2*e^17))/(b^19*d^4 + 8*a^2*b^1
7*d^4 + 28*a^4*b^15*d^4 + 56*a^6*b^13*d^4 + 70*a^8*b^11*d^4 + 56*a^10*b^9*d
^4 + 28*a^12*b^7*d^4 + 8*a^14*b^5*d^4 + a^16*b^3*d^4) + (((1600*a^2*b^23*d^
4*e^14 + 12864*a^4*b^21*d^4*e^14 + 45312*a^6*b^19*d^4*e^14 + 91392*a^8*b^17
*d^4*e^14 + 115584*a^10*b^15*d^4*e^14 + 94080*a^12*b^13*d^4*e^14 + 48384*a^
14*b^11*d^4*e^14 + 14592*a^16*b^9*d^4*e^14 + 2112*a^18*b^7*d^4*e^14 + 64*a^
20*b^5*d^4*e^14)/(b^19*d^5 + 8*a^2*b^17*d^5 + 28*a^4*b^15*d^5 + 56*a^6*b^13
*d^5 + 70*a^8*b^11*d^5 + 56*a^10*b^9*d^5 + 28*a^12*b^7*d^5 + 8*a^14*b^5*d^5
 + a^16*b^3*d^5) - ((e*cot(c + d*x))^(1/2)*(3*a^4 + 35*b^4 + 6*a^2*b^2)*(-a
^3*b^5*e^7)^(1/2)*(512*b^28*d^4*e^10 + 4608*a^2*b^26*d^4*e^10 + 17920*a^4*b
^24*d^4*e^10 + 38400*a^6*b^22*d^4*e^10 + 46080*a^8*b^20*d^4*e^10 + 21504*a^
10*b^18*d^4*e^10 - 21504*a^12*b^16*d^4*e^10 - 46080*a^14*b^14*d^4*e^10 - 38
400*a^16*b^12*d^4*e^10 - 17920*a^18*b^10*d^4*e^10 - 4608*a^20*b^8*d^4*e^10
- 512*a^22*b^6*d^4*e^10))/(8*(b^11*d + 3*a^2*b^9*d + 3*a^4*b^7*d + a^6*b^5*
d)*(b^19*d^4 + 8*a^2*b^17*d^4 + 28*a^4*b^15*d^4 + 56*a^6*b^13*d^4 + 70*a^8*
b^11*d^4 + 56*a^10*b^9*d^4 + 28*a^12*b^7*d^4 + 8*a^14*b^5*d^4 + a^16*b^3*d^
4)))*(3*a^4 + 35*b^4 + 6*a^2*b^2)*(-a^3*b^5*e^7)^(1/2))/(8*(b^11*d + 3*a^2*
b^9*d + 3*a^4*b^7*d + a^6*b^5*d)))*(3*a^4 + 35*b^4 + 6*a^2*b^2)*(-a^3*b^5*e
^7)^(1/2))/(8*(b^11*d + 3*a^2*b^9*d + 3*a^4*b^7*d + a^6*b^5*d)))*(3*a^4 + 3
5*b^4 + 6*a^2*b^2)*(-a^3*b^5*e^7)^(1/2))/(8*(b^11*d + 3*a^2*b^9*d + 3*a^4*b
^7*d + a^6*b^5*d)))*(3*a^4 + 35*b^4 + 6*a^2*b^2)*(-a^3*b^5*e^7)^(1/2)*1i)/(
8*(b^11*d + 3*a^2*b^9*d + 3*a^4*b^7*d + a^6*b^5*d)) + ((((e*cot(c + d*x))^(
1/2)*(9*a^16*e^24 + 32*b^16*e^24 + 128*a^2*b^14*e^24 + 1417*a^4*b^12*e^24 -
 6802*a^6*b^10*e^24 - 1017*a^8*b^8*e^24 - 1020*a^10*b^6*e^24 + 39*a^12*b^4*
e^24 - 18*a^14*b^2*e^24))/(b^19*d^4 + 8*a^2*b^17*d^4 + 28*a^4*b^15*d^4 + 56

*a^6*b^13*d^4 + 70*a^8*b^11*d^4 + 56*a^10*b^9*d^4 + 28*a^12*b^7*d^4 + 8*a^1
4*b^5*d^4 + a^16*b^3*d^4) + (((32*a*b^18*d^2*e^21 - 18*a^19*d^2*e^21 - 6528
*a^3*b^16*d^2*e^21 + 2758*a^5*b^14*d^2*e^21 + 26482*a^7*b^12*d^2*e^21 + 215
82*a^9*b^10*d^2*e^21 + 7594*a^11*b^8*d^2*e^21 + 3314*a^13*b^6*d^2*e^21 + 24
6*a^15*b^4*d^2*e^21 + 90*a^17*b^2*d^2*e^21)/(b^19*d^5 + 8*a^2*b^17*d^5 + 28
*a^4*b^15*d^5 + 56*a^6*b^13*d^5 + 70*a^8*b^11*d^5 + 56*a^10*b^9*d^5 + 28*a^
12*b^7*d^5 + 8*a^14*b^5*d^5 + a^16*b^3*d^5) - ((((e*cot(c + d*x))^(1/2)*(14
72*a*b^21*d^2*e^17 + 72*a^21*b*d^2*e^17 + 1024*a^3*b^19*d^2*e^17 + 1352*a^5
*b^17*d^2*e^17 + 28224*a^7*b^15*d^2*e^17 + 70240*a^9*b^13*d^2*e^17 + 72640*
a^11*b^11*d^2*e^17 + 39088*a^13*b^9*d^2*e^17 + 13248*a^15*b^7*d^2*e^17 + 34
88*a^17*b^5*d^2*e^17 + 576*a^19*b^3*d^2*e^17))/(b^19*d^4 + 8*a^2*b^17*d^4 +
28*a^4*b^15*d^4 + 56*a^6*b^13*d^4 + 70*a^8*b^11*d^4 + 56*a^10*b^9*d^4 + 28
*a^12*b^7*d^4 + 8*a^14*b^5*d^4 + a^16*b^3*d^4) - (((1600*a^2*b^23*d^4*e^14
+ 12864*a^4*b^21*d^4*e^14 + 45312*a^6*b^19*d^4*e^14 + 91392*a^8*b^17*d^4*e^
14 + 115584*a^10*b^15*d^4*e^14 + 94080*a^12*b^13*d^4*e^14 + 48384*a^14*b^11
*d^4*e^14 + 14592*a^16*b^9*d^4*e^14 + 2112*a^18*b^7*d^4*e^14 + 64*a^20*b^5*
d^4*e^14)/(b^19*d^5 + 8*a^2*b^17*d^5 + 28*a^4*b^15*d^5 + 56*a^6*b^13*d^5 +
70*a^8*b^11*d^5 + 56*a^10*b^9*d^5 + 28*a^12*b^7*d^5 + 8*a^14*b^5*d^5 + a^16
*b^3*d^5) + ((e*cot(c + d*x))^(1/2)*(3*a^4 + 35*b^4 + 6*a^2*b^2)*(-a^3*b^5*
e^7)^(1/2)*(512*b^28*d^4*e^10 + 4608*a^2*b^26*d^4*e^10 + 17920*a^4*b^24*d^4
*e^10 + 38400*a^6*b^22*d^4*e^10 + 46080*a^8*b^20*d^4*e^10 + 21504*a^10*b^18
*d^4*e^10 - 21504*a^12*b^16*d^4*e^10 - 46080*a^14*b^14*d^4*e^10 - 38400*a^1
6*b^12*d^4*e^10 - 17920*a^18*b^10*d^4*e^10 - 4608*a^20*b^8*d^4*e^10 - 512*a
^22*b^6*d^4*e^10))/(8*(b^11*d + 3*a^2*b^9*d + 3*a^4*b^7*d + a^6*b^5*d)*(b^1
9*d^4 + 8*a^2*b^17*d^4 + 28*a^4*b^15*d^4 + 56*a^6*b^13*d^4 + 70*a^8*b^11*d^
4 + 56*a^10*b^9*d^4 + 28*a^12*b^7*d^4 + 8*a^14*b^5*d^4 + a^16*b^3*d^4)))*(3
*a^4 + 35*b^4 + 6*a^2*b^2)*(-a^3*b^5*e^7)^(1/2))/(8*(b^11*d + 3*a^2*b^9*d +
3*a^4*b^7*d + a^6*b^5*d)))*(3*a^4 + 35*b^4 + 6*a^2*b^2)*(-a^3*b^5*e^7)^(1/
2))/(8*(b^11*d + 3*a^2*b^9*d + 3*a^4*b^7*d + a^6*b^5*d)))*(3*a^4 + 35*b^4 +
6*a^2*b^2)*(-a^3*b^5*e^7)^(1/2))/(8*(b^11*d + 3*a^2*b^9*d + 3*a^4*b^7*d +
a^6*b^5*d)))*(3*a^4 + 35*b^4 + 6*a^2*b^2)*(-a^3*b^5*e^7)^(1/2)*1i)/(8*(b^11
*d + 3*a^2*b^9*d + 3*a^4*b^7*d + a^6*b^5*d)))/((9*a^12*b*e^28 + 280*a^2*b^1
1*e^28 + 1553*a^4*b^9*e^28 + 492*a^6*b^7*e^28 + 270*a^8*b^5*e^28 + 36*a^10*
b^3*e^28)/(b^19*d^5 + 8*a^2*b^17*d^5 + 28*a^4*b^15*d^5 + 56*a^6*b^13*d^5 +
70*a^8*b^11*d^5 + 56*a^10*b^9*d^5 + 28*a^12*b^7*d^5 + 8*a^14*b^5*d^5 + a^16
*b^3*d^5) - ((((e*cot(c + d*x))^(1/2)*(9*a^16*e^24 + 32*b^16*e^24 + 128*a^2
*b^14*e^24 + 1417*a^4*b^12*e^24 - 6802*a^6*b^10*e^24 - 1017*a^8*b^8*e^24 -
1020*a^10*b^6*e^24 + 39*a^12*b^4*e^24 - 18*a^14*b^2*e^24))/(b^19*d^4 + 8*a^
2*b^17*d^4 + 28*a^4*b^15*d^4 + 56*a^6*b^13*d^4 + 70*a^8*b^11*d^4 + 56*a^10*
b^9*d^4 + 28*a^12*b^7*d^4 + 8*a^14*b^5*d^4 + a^16*b^3*d^4) - (((32*a*b^18*d
^2*e^21 - 18*a^19*d^2*e^21 - 6528*a^3*b^16*d^2*e^21 + 2758*a^5*b^14*d^2*e^2
1 + 26482*a^7*b^12*d^2*e^21 + 21582*a^9*b^10*d^2*e^21 + 7594*a^11*b^8*d^2*e
^21 + 3314*a^13*b^6*d^2*e^21 + 246*a^15*b^4*d^2*e^21 + 90*a^17*b^2*d^2*e^21
)/(b^19*d^5 + 8*a^2*b^17*d^5 + 28*a^4*b^15*d^5 + 56*a^6*b^13*d^5 + 70*a^8*b
^11*d^5 + 56*a^10*b^9*d^5 + 28*a^12*b^7*d^5 + 8*a^14*b^5*d^5 + a^16*b^3*d^5

) + ((((e*cot(c + d*x))^(1/2)*(1472*a*b^21*d^2*e^17 + 72*a^21*b*d^2*e^17 +
1024*a^3*b^19*d^2*e^17 + 1352*a^5*b^17*d^2*e^17 + 28224*a^7*b^15*d^2*e^17 +
70240*a^9*b^13*d^2*e^17 + 72640*a^11*b^11*d^2*e^17 + 39088*a^13*b^9*d^2*e^
17 + 13248*a^15*b^7*d^2*e^17 + 3488*a^17*b^5*d^2*e^17 + 576*a^19*b^3*d^2*e^
17))/(b^19*d^4 + 8*a^2*b^17*d^4 + 28*a^4*b^15*d^4 + 56*a^6*b^13*d^4 + 70*a^
8*b^11*d^4 + 56*a^10*b^9*d^4 + 28*a^12*b^7*d^4 + 8*a^14*b^5*d^4 + a^16*b^3*
d^4) + (((1600*a^2*b^23*d^4*e^14 + 12864*a^4*b^21*d^4*e^14 + 45312*a^6*b^19
*d^4*e^14 + 91392*a^8*b^17*d^4*e^14 + 115584*a^10*b^15*d^4*e^14 + 94080*a^1
2*b^13*d^4*e^14 + 48384*a^14*b^11*d^4*e^14 + 14592*a^16*b^9*d^4*e^14 + 2112
*a^18*b^7*d^4*e^14 + 64*a^20*b^5*d^4*e^14)/(b^19*d^5 + 8*a^2*b^17*d^5 + 28*
a^4*b^15*d^5 + 56*a^6*b^13*d^5 + 70*a^8*b^11*d^5 + 56*a^10*b^9*d^5 + 28*a^1
2*b^7*d^5 + 8*a^14*b^5*d^5 + a^16*b^3*d^5) - ((e*cot(c + d*x))^(1/2)*(3*a^4
+ 35*b^4 + 6*a^2*b^2)*(-a^3*b^5*e^7)^(1/2)*(512*b^28*d^4*e^10 + 4608*a^2*b
^26*d^4*e^10 + 17920*a^4*b^24*d^4*e^10 + 38400*a^6*b^22*d^4*e^10 + 46080*a^
8*b^20*d^4*e^10 + 21504*a^10*b^18*d^4*e^10 - 21504*a^12*b^16*d^4*e^10 - 460
80*a^14*b^14*d^4*e^10 - 38400*a^16*b^12*d^4*e^10 - 17920*a^18*b^10*d^4*e^10
- 4608*a^20*b^8*d^4*e^10 - 512*a^22*b^6*d^4*e^10))/(8*(b^11*d + 3*a^2*b^9*
d + 3*a^4*b^7*d + a^6*b^5*d)*(b^19*d^4 + 8*a^2*b^17*d^4 + 28*a^4*b^15*d^4 +
56*a^6*b^13*d^4 + 70*a^8*b^11*d^4 + 56*a^10*b^9*d^4 + 28*a^12*b^7*d^4 + 8*
a^14*b^5*d^4 + a^16*b^3*d^4)))*(3*a^4 + 35*b^4 + 6*a^2*b^2)*(-a^3*b^5*e^7)^
(1/2))/(8*(b^11*d + 3*a^2*b^9*d + 3*a^4*b^7*d + a^6*b^5*d)))*(3*a^4 + 35*b^
4 + 6*a^2*b^2)*(-a^3*b^5*e^7)^(1/2))/(8*(b^11*d + 3*a^2*b^9*d + 3*a^4*b^7*d
+ a^6*b^5*d)))*(3*a^4 + 35*b^4 + 6*a^2*b^2)*(-a^3*b^5*e^7)^(1/2))/(8*(b^11
*d + 3*a^2*b^9*d + 3*a^4*b^7*d + a^6*b^5*d)))*(3*a^4 + 35*b^4 + 6*a^2*b^2)*
(-a^3*b^5*e^7)^(1/2))/(8*(b^11*d + 3*a^2*b^9*d + 3*a^4*b^7*d + a^6*b^5*d))
+ ((((e*cot(c + d*x))^(1/2)*(9*a^16*e^24 + 32*b^16*e^24 + 128*a^2*b^14*e^24
+ 1417*a^4*b^12*e^24 - 6802*a^6*b^10*e^24 - 1017*a^8*b^8*e^24 - 1020*a^10*
b^6*e^24 + 39*a^12*b^4*e^24 - 18*a^14*b^2*e^24))/(b^19*d^4 + 8*a^2*b^17*d^4
+ 28*a^4*b^15*d^4 + 56*a^6*b^13*d^4 + 70*a^8*b^11*d^4 + 56*a^10*b^9*d^4 +
28*a^12*b^7*d^4 + 8*a^14*b^5*d^4 + a^16*b^3*d^4) + (((32*a*b^18*d^2*e^21 -
18*a^19*d^2*e^21 - 6528*a^3*b^16*d^2*e^21 + 2758*a^5*b^14*d^2*e^21 + 26482*
a^7*b^12*d^2*e^21 + 21582*a^9*b^10*d^2*e^21 + 7594*a^11*b^8*d^2*e^21 + 3314
*a^13*b^6*d^2*e^21 + 246*a^15*b^4*d^2*e^21 + 90*a^17*b^2*d^2*e^21)/(b^19*d^
5 + 8*a^2*b^17*d^5 + 28*a^4*b^15*d^5 + 56*a^6*b^13*d^5 + 70*a^8*b^11*d^5 +
56*a^10*b^9*d^5 + 28*a^12*b^7*d^5 + 8*a^14*b^5*d^5 + a^16*b^3*d^5) - ((((e*
cot(c + d*x))^(1/2)*(1472*a*b^21*d^2*e^17 + 72*a^21*b*d^2*e^17 + 1024*a^3*b
^19*d^2*e^17 + 1352*a^5*b^17*d^2*e^17 + 28224*a^7*b^15*d^2*e^17 + 70240*a^9
*b^13*d^2*e^17 + 72640*a^11*b^11*d^2*e^17 + 39088*a^13*b^9*d^2*e^17 + 13248
*a^15*b^7*d^2*e^17 + 3488*a^17*b^5*d^2*e^17 + 576*a^19*b^3*d^2*e^17))/(b^19
*d^4 + 8*a^2*b^17*d^4 + 28*a^4*b^15*d^4 + 56*a^6*b^13*d^4 + 70*a^8*b^11*d^4
+ 56*a^10*b^9*d^4 + 28*a^12*b^7*d^4 + 8*a^14*b^5*d^4 + a^16*b^3*d^4) - (((
1600*a^2*b^23*d^4*e^14 + 12864*a^4*b^21*d^4*e^14 + 45312*a^6*b^19*d^4*e^14
+ 91392*a^8*b^17*d^4*e^14 + 115584*a^10*b^15*d^4*e^14 + 94080*a^12*b^13*d^4
*e^14 + 48384*a^14*b^11*d^4*e^14 + 14592*a^16*b^9*d^4*e^14 + 2112*a^18*b^7*
d^4*e^14 + 64*a^20*b^5*d^4*e^14)/(b^19*d^5 + 8*a^2*b^17*d^5 + 28*a^4*b^15*d

```
^5 + 56*a^6*b^13*d^5 + 70*a^8*b^11*d^5 + 56*a^10*b^9*d^5 + 28*a^12*b^7*d^5
+ 8*a^14*b^5*d^5 + a^16*b^3*d^5) + ((e*cot(c + d*x))^(1/2)*(3*a^4 + 35*b^4
+ 6*a^2*b^2)*(-a^3*b^5*e^7)^(1/2)*(512*b^28*d^4*e^10 + 4608*a^2*b^26*d^4*e^
10 + 17920*a^4*b^24*d^4*e^10 + 38400*a^6*b^22*d^4*e^10 + 46080*a^8*b^20*d^4
*e^10 + 21504*a^10*b^18*d^4*e^10 - 21504*a^12*b^16*d^4*e^10 - 46080*a^14*b^
14*d^4*e^10 - 38400*a^16*b^12*d^4*e^10 - 17920*a^18*b^10*d^4*e^10 - 4608*a^
20*b^8*d^4*e^10 - 512*a^22*b^6*d^4*e^10))/(8*(b^11*d + 3*a^2*b^9*d + 3*a^4*
b^7*d + a^6*b^5*d)*(b^19*d^4 + 8*a^2*b^17*d^4 + 28*a^4*b^15*d^4 + 56*a^6*b^
13*d^4 + 70*a^8*b^11*d^4 + 56*a^10*b^9*d^4 + 28*a^12*b^7*d^4 + 8*a^14*b^5*d
^4 + a^16*b^3*d^4)))*(3*a^4 + 35*b^4 + 6*a^2*b^2)*(-a^3*b^5*e^7)^(1/2))/(8*
(b^11*d + 3*a^2*b^9*d + 3*a^4*b^7*d + a^6*b^5*d)))*(3*a^4 + 35*b^4 + 6*a^2*
b^2)*(-a^3*b^5*e^7)^(1/2))/(8*(b^11*d + 3*a^2*b^9*d + 3*a^4*b^7*d + a^6*b^5
*d)))*(3*a^4 + 35*b^4 + 6*a^2*b^2)*(-a^3*b^5*e^7)^(1/2))/(8*(b^11*d + 3*a^2
*b^9*d + 3*a^4*b^7*d + a^6*b^5*d)))*(3*a^4 + 35*b^4 + 6*a^2*b^2)*(-a^3*b^5*
e^7)^(1/2))/(8*(b^11*d + 3*a^2*b^9*d + 3*a^4*b^7*d + a^6*b^5*d))))*(3*a^4 +
 35*b^4 + 6*a^2*b^2)*(-a^3*b^5*e^7)^(1/2)*1i)/(4*(b^11*d + 3*a^2*b^9*d + 3*
a^4*b^7*d + a^6*b^5*d))
```

3.83 $\int \frac{(e\cot(c+dx))^{5/2}}{(a+b\cot(c+dx))^3}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 470

$$\int \frac{(e\cot(c+dx))^{5/2}}{(a+b\cot(c+dx))^3}\,dx = -\frac{\sqrt{a}(a^4+18a^2b^2-15b^4)\,e^{5/2}\arctan\left(\frac{\sqrt{b}\sqrt{e\cot(c+dx)}}{\sqrt{a}\sqrt{e}}\right)}{4b^{3/2}\left(a^2+b^2\right)^3 d}$$
$$-\frac{(a-b)\left(a^2+4ab+b^2\right)e^{5/2}\arctan\left(1-\frac{\sqrt{2}\sqrt{e\cot(c+dx)}}{\sqrt{e}}\right)}{\sqrt{2}\left(a^2+b^2\right)^3 d}$$
$$+\frac{(a-b)\left(a^2+4ab+b^2\right)e^{5/2}\arctan\left(1+\frac{\sqrt{2}\sqrt{e\cot(c+dx)}}{\sqrt{e}}\right)}{\sqrt{2}\left(a^2+b^2\right)^3 d}$$
$$+\frac{a^2e^2\sqrt{e\cot(c+dx)}}{2b\left(a^2+b^2\right)d(a+b\cot(c+dx))^2}-\frac{a(a^2+9b^2)\,e^2\sqrt{e\cot(c+dx)}}{4b\left(a^2+b^2\right)^2 d(a+b\cot(c+dx))}$$
$$+\frac{(a+b)\left(a^2-4ab+b^2\right)e^{5/2}\log\left(\sqrt{e}+\sqrt{e}\cot(c+dx)-\sqrt{2}\sqrt{e\cot(c+dx)}\right)}{2\sqrt{2}\left(a^2+b^2\right)^3 d}$$
$$-\frac{(a+b)\left(a^2-4ab+b^2\right)e^{5/2}\log\left(\sqrt{e}+\sqrt{e}\cot(c+dx)+\sqrt{2}\sqrt{e\cot(c+dx)}\right)}{2\sqrt{2}\left(a^2+b^2\right)^3 d}$$

```
[Out] -1/2*(a-b)*(a^2+4*a*b+b^2)*e^(5/2)*arctan(1-2^(1/2)*(e*cot(d*x+c))^(1/2)/e^
(1/2))/(a^2+b^2)^3/d*2^(1/2)+1/2*(a-b)*(a^2+4*a*b+b^2)*e^(5/2)*arctan(1+2^(
1/2)*(e*cot(d*x+c))^(1/2)/e^(1/2))/(a^2+b^2)^3/d*2^(1/2)+1/4*(a+b)*(a^2-4*a
*b+b^2)*e^(5/2)*ln(e^(1/2)+cot(d*x+c)*e^(1/2)-2^(1/2)*(e*cot(d*x+c))^(1/2))
/(a^2+b^2)^3/d*2^(1/2)-1/4*(a+b)*(a^2-4*a*b+b^2)*e^(5/2)*ln(e^(1/2)+cot(d*x
+c)*e^(1/2)+2^(1/2)*(e*cot(d*x+c))^(1/2))/(a^2+b^2)^3/d*2^(1/2)-1/4*(a^4+18
*a^2*b^2-15*b^4)*e^(5/2)*arctan(b^(1/2)*(e*cot(d*x+c))^(1/2)/a^(1/2)/e^(1/2
))*a^(1/2)/b^(3/2)/(a^2+b^2)^3/d+1/2*a^2*e^2*(e*cot(d*x+c))^(1/2)/b/(a^2+b^
2)/d/(a+b*cot(d*x+c))^2-1/4*a*(a^2+9*b^2)*e^2*(e*cot(d*x+c))^(1/2)/b/(a^2+b
^2)^2/d/(a+b*cot(d*x+c))
```

Rubi [A] (verified)

Time = 1.40 (sec) , antiderivative size = 470, normalized size of antiderivative = 1.00, number of steps used = 16, number of rules used = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.520, Rules used = {3646, 3730, 3734, 3615, 1182, 1176, 631, 210, 1179, 642, 3715, 65, 211}

$$\int \frac{(e\cot(c+dx))^{5/2}}{(a+b\cot(c+dx))^3}\,dx = -\frac{e^{5/2}(a-b)\left(a^2+4ab+b^2\right)\arctan\left(1-\frac{\sqrt{2}\sqrt{e\cot(c+dx)}}{\sqrt{e}}\right)}{\sqrt{2}d\left(a^2+b^2\right)^3} + \frac{e^{5/2}(a-b)\left(a^2+4ab+b^2\right)\arctan\left(\frac{\sqrt{2}\sqrt{e\cot(c+dx)}}{\sqrt{e}}+1\right)}{\sqrt{2}d\left(a^2+b^2\right)^3} + \frac{e^{5/2}(a+b)\left(a^2-4ab+b^2\right)\log\left(\sqrt{e}\cot(c+dx)-\sqrt{2}\sqrt{e\cot(c+dx)}+\sqrt{e}\right)}{2\sqrt{2}d\left(a^2+b^2\right)^3} - \frac{e^{5/2}(a+b)\left(a^2-4ab+b^2\right)\log\left(\sqrt{e}\cot(c+dx)+\sqrt{2}\sqrt{e\cot(c+dx)}+\sqrt{e}\right)}{2\sqrt{2}d\left(a^2+b^2\right)^3} - \frac{ae^2\left(a^2+9b^2\right)\sqrt{e\cot(c+dx)}}{4bd\left(a^2+b^2\right)^2\left(a+b\cot(c+dx)\right)} + \frac{a^2e^2\sqrt{e\cot(c+dx)}}{2bd\left(a^2+b^2\right)\left(a+b\cot(c+dx)\right)^2} - \frac{\sqrt{a}e^{5/2}\left(a^4+18a^2b^2-15b^4\right)\arctan\left(\frac{\sqrt{b}\sqrt{e\cot(c+dx)}}{\sqrt{a}\sqrt{e}}\right)}{4b^{3/2}d\left(a^2+b^2\right)^3}$$

```
[In] Int[(e*Cot[c + d*x])^(5/2)/(a + b*Cot[c + d*x])^3,x]
```

```
[Out] -1/4*(Sqrt[a]*(a^4 + 18*a^2*b^2 - 15*b^4)*e^(5/2)*ArcTan[(Sqrt[b]*Sqrt[e*Co
t[c + d*x]])/(Sqrt[a]*Sqrt[e])])/(b^(3/2)*(a^2 + b^2)^3*d) - ((a - b)*(a^2
+ 4*a*b + b^2)*e^(5/2)*ArcTan[1 - (Sqrt[2]*Sqrt[e*Cot[c + d*x]])/Sqrt[e]])/
(Sqrt[2]*(a^2 + b^2)^3*d) + ((a - b)*(a^2 + 4*a*b + b^2)*e^(5/2)*ArcTan[1 +
 (Sqrt[2]*Sqrt[e*Cot[c + d*x]])/Sqrt[e]])/(Sqrt[2]*(a^2 + b^2)^3*d) + (a^2*
e^2*Sqrt[e*Cot[c + d*x]])/(2*b*(a^2 + b^2)*d*(a + b*Cot[c + d*x])^2) - (a*(
a^2 + 9*b^2)*e^2*Sqrt[e*Cot[c + d*x]])/(4*b*(a^2 + b^2)^2*d*(a + b*Cot[c +
d*x])) + ((a + b)*(a^2 - 4*a*b + b^2)*e^(5/2)*Log[Sqrt[e] + Sqrt[e]*Cot[c +
 d*x] - Sqrt[2]*Sqrt[e*Cot[c + d*x]]])/(2*Sqrt[2]*(a^2 + b^2)^3*d) - ((a +
b)*(a^2 - 4*a*b + b^2)*e^(5/2)*Log[Sqrt[e] + Sqrt[e]*Cot[c + d*x] + Sqrt[2]
*Sqrt[e*Cot[c + d*x]]])/(2*Sqrt[2]*(a^2 + b^2)^3*d)
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
```

```
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1179

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 1182

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
a*c, 2]}, Dist[(d*q + a*e)/(2*a*c), Int[(q + c*x^2)/(a + c*x^4), x], x] + D
ist[(d*q - a*e)/(2*a*c), Int[(q - c*x^2)/(a + c*x^4), x], x]] /; FreeQ[{a,
c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0] && NeQ[c*d^2 - a*e^2, 0] && NegQ[(-a)
```

```
*c]
```

Rule 3615

```
Int[((c_) + (d_.)*tan[(e_.) + (f_.)*(x_)])/Sqrt[(b_.)*tan[(e_.) + (f_.)*(x_
)]], x_Symbol] :> Dist[2/f, Subst[Int[(b*c + d*x^2)/(b^2 + x^4), x], x, Sqr
t[b*Tan[e + f*x]]], x] /; FreeQ[{b, c, d, e, f}, x] && NeQ[c^2 - d^2, 0] &&
 NeQ[c^2 + d^2, 0]
```

Rule 3646

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*c - a*d)^2*(a + b*Tan[e + f*x])^(m
 - 2)*((c + d*Tan[e + f*x])^(n + 1)/(d*f*(n + 1)*(c^2 + d^2))), x] - Dist[1
/(d*(n + 1)*(c^2 + d^2)), Int[(a + b*Tan[e + f*x])^(m - 3)*(c + d*Tan[e + f
*x])^(n + 1)*Simp[a^2*d*(b*d*(m - 2) - a*c*(n + 1)) + b*(b*c - 2*a*d)*(b*c*
(m - 2) + a*d*(n + 1)) - d*(n + 1)*(3*a^2*b*c - b^3*c - a^3*d + 3*a*b^2*d)*
Tan[e + f*x] - b*(a*d*(2*b*c - a*d)*(m + n - 1) - b^2*(c^2*(m - 2) - d^2*(n
 + 1)))*Tan[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b
*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] && GtQ[m, 2] && LtQ[
n, -1] && IntegerQ[2*m]
```

Rule 3715

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*tan[(e_.)
+ (f_.)*(x_)])^(n_.)*((A_) + (C_.)*tan[(e_.) + (f_.)*(x_)]^2), x_Symbol] :>
 Dist[A/f, Subst[Int[(a + b*x)^m*(c + d*x)^n, x], x, Tan[e + f*x]], x] /; F
reeQ[{a, b, c, d, e, f, A, C, m, n}, x] && EqQ[A, C]
```

Rule 3730

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*tan[(e_.) + (f_.)*(x_)] + (C_.)*tan[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> Simp[(A*b^2 - a*(b*B - a*C))*(a + b*Tan[e +
 f*x])^(m + 1)*((c + d*Tan[e + f*x])^(n + 1)/(f*(m + 1)*(b*c - a*d)*(a^2 +
b^2))), x] + Dist[1/((m + 1)*(b*c - a*d)*(a^2 + b^2)), Int[(a + b*Tan[e + f
*x])^(m + 1)*(c + d*Tan[e + f*x])^n*Simp[A*(a*(b*c - a*d)*(m + 1) - b^2*d*(
m + n + 2)) + (b*B - a*C)*(b*c*(m + 1) + a*d*(n + 1)) - (m + 1)*(b*c - a*d)
*(A*b - a*B - b*C)*Tan[e + f*x] - d*(A*b^2 - a*(b*B - a*C))*(m + n + 2)*Tan
[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, n}, x] && NeQ[
b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] && LtQ[m, -1] &&  !
(ILtQ[n, -1] && ( !IntegerQ[m] || (EqQ[c, 0] && NeQ[a, 0])))
```

Rule 3734

```
Int[(((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_)*((A_.) + (B_.)*tan[(e_.)
+ (f_.)*(x_)] + (C_.)*tan[(e_.) + (f_.)*(x_)]^2))/((a_.) + (b_.)*tan[(e_.)
```

```
+ (f_.)*(x_)]), x_Symbol] :> Dist[1/(a^2 + b^2), Int[(c + d*Tan[e + f*x])^n
*Simp[b*B + a*(A - C) + (a*B - b*(A - C))*Tan[e + f*x], x], x], x] + Dist[(
A*b^2 - a*b*B + a^2*C)/(a^2 + b^2), Int[(c + d*Tan[e + f*x])^n*((1 + Tan[e
+ f*x]^2)/(a + b*Tan[e + f*x])), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C
, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] &&
 !GtQ[n, 0] &&  !LeQ[n, -1]
```

Rubi steps

$$\text{integral} = \frac{a^2e^2\sqrt{e\cot(c+dx)}}{2b(a^2+b^2)d(a+b\cot(c+dx))^2} - \frac{\int \frac{-\frac{1}{2}a^2e^3+2abe^3\cot(c+dx)-\frac{1}{2}(a^2+4b^2)e^3\cot^2(c+dx)}{\sqrt{e\cot(c+dx)}(a+b\cot(c+dx))^2}\,dx}{2b(a^2+b^2)}$$

$$= \frac{a^2e^2\sqrt{e\cot(c+dx)}}{2b(a^2+b^2)d(a+b\cot(c+dx))^2} - \frac{a(a^2+9b^2)e^2\sqrt{e\cot(c+dx)}}{4b(a^2+b^2)^2d(a+b\cot(c+dx))} + \frac{\int \frac{\frac{1}{4}a^2(a^2-7b^2)e^4-2ab(a^2-b^2)e^4\cot(c+dx)+\frac{1}{4}a^2(a^2+9b^2)e^4\cot^2(c+dx)}{\sqrt{e\cot(c+dx)}(a+b\cot(c+dx))}\,dx}{2ab(a^2+b^2)^2e}$$

$$= \frac{a^2e^2\sqrt{e\cot(c+dx)}}{2b(a^2+b^2)d(a+b\cot(c+dx))^2} - \frac{a(a^2+9b^2)e^2\sqrt{e\cot(c+dx)}}{4b(a^2+b^2)^2d(a+b\cot(c+dx))} + \frac{\int \frac{-2ab^2(3a^2-b^2)e^4-2a^2b(a^2-3b^2)e^4\cot(c+dx)}{\sqrt{e\cot(c+dx)}}\,dx}{2ab(a^2+b^2)^3e} + \frac{(a(a^4+18a^2b^2-15b^4)e^3)\int \frac{1+\cot^2(c+dx)}{\sqrt{e\cot(c+dx)}(a+b\cot(c+dx))}\,dx}{8b(a^2+b^2)^3}$$

$$= \frac{a^2e^2\sqrt{e\cot(c+dx)}}{2b(a^2+b^2)d(a+b\cot(c+dx))^2} - \frac{a(a^2+9b^2)e^2\sqrt{e\cot(c+dx)}}{4b(a^2+b^2)^2d(a+b\cot(c+dx))} + \frac{\text{Subst}\left(\int \frac{2ab^2(3a^2-b^2)e^5+2a^2b(a^2-3b^2)e^4x^2}{e^2+x^4}\,dx, x, \sqrt{e\cot(c+dx)}\right)}{ab(a^2+b^2)^3de} + \frac{(a(a^4+18a^2b^2-15b^4)e^3)\,\text{Subst}\left(\int \frac{1}{\sqrt{-ex}(a-bx)}\,dx, x, -\cot(c+dx)\right)}{8b(a^2+b^2)^3d}$$

$$= \frac{a^2e^2\sqrt{e\cot(c+dx)}}{2b(a^2+b^2)d(a+b\cot(c+dx))^2} - \frac{a(a^2+9b^2)e^2\sqrt{e\cot(c+dx)}}{4b(a^2+b^2)^2d(a+b\cot(c+dx))}$$
$$- \frac{(a(a^4+18a^2b^2-15b^4)e^2)\operatorname{Subst}\left(\int \frac{1}{a+\frac{bx^2}{e}}\,dx, x, \sqrt{e\cot(c+dx)}\right)}{4b(a^2+b^2)^3d}$$
$$- \frac{((a+b)(a^2-4ab+b^2)e^3)\operatorname{Subst}\left(\int \frac{e-x^2}{e^2+x^4}\,dx, x, \sqrt{e\cot(c+dx)}\right)}{(a^2+b^2)^3d}$$
$$+ \frac{((a-b)(a^2+4ab+b^2)e^3)\operatorname{Subst}\left(\int \frac{e+x^2}{e^2+x^4}\,dx, x, \sqrt{e\cot(c+dx)}\right)}{(a^2+b^2)^3d}$$

$$= -\frac{\sqrt{a}(a^4+18a^2b^2-15b^4)e^{5/2}\arctan\left(\frac{\sqrt{b}\sqrt{e\cot(c+dx)}}{\sqrt{a}\sqrt{e}}\right)}{4b^{3/2}(a^2+b^2)^3d}$$
$$+ \frac{a^2e^2\sqrt{e\cot(c+dx)}}{2b(a^2+b^2)d(a+b\cot(c+dx))^2} - \frac{a(a^2+9b^2)e^2\sqrt{e\cot(c+dx)}}{4b(a^2+b^2)^2d(a+b\cot(c+dx))}$$
$$+ \frac{((a+b)(a^2-4ab+b^2)e^{5/2})\operatorname{Subst}\left(\int \frac{\sqrt{2}\sqrt{e}+2x}{-e-\sqrt{2}\sqrt{e}x-x^2}\,dx, x, \sqrt{e\cot(c+dx)}\right)}{2\sqrt{2}(a^2+b^2)^3d}$$
$$+ \frac{((a+b)(a^2-4ab+b^2)e^{5/2})\operatorname{Subst}\left(\int \frac{\sqrt{2}\sqrt{e}-2x}{-e+\sqrt{2}\sqrt{e}x-x^2}\,dx, x, \sqrt{e\cot(c+dx)}\right)}{2\sqrt{2}(a^2+b^2)^3d}$$
$$+ \frac{((a-b)(a^2+4ab+b^2)e^3)\operatorname{Subst}\left(\int \frac{1}{e-\sqrt{2}\sqrt{e}x+x^2}\,dx, x, \sqrt{e\cot(c+dx)}\right)}{2(a^2+b^2)^3d}$$
$$+ \frac{((a-b)(a^2+4ab+b^2)e^3)\operatorname{Subst}\left(\int \frac{1}{e+\sqrt{2}\sqrt{e}x+x^2}\,dx, x, \sqrt{e\cot(c+dx)}\right)}{2(a^2+b^2)^3d}$$

$$= -\frac{\sqrt{a}(a^4+18a^2b^2-15b^4)\,e^{5/2}\arctan\left(\frac{\sqrt{b}\sqrt{e\cot(c+dx)}}{\sqrt{a}\sqrt{e}}\right)}{4b^{3/2}\left(a^2+b^2\right)^3 d} + \frac{a^2e^2\sqrt{e\cot(c+dx)}}{2b\left(a^2+b^2\right)d(a+b\cot(c+dx))^2} - \frac{a(a^2+9b^2)\,e^2\sqrt{e\cot(c+dx)}}{4b\left(a^2+b^2\right)^2 d(a+b\cot(c+dx))} + \frac{(a+b)\left(a^2-4ab+b^2\right)e^{5/2}\log\left(\sqrt{e}+\sqrt{e}\cot(c+dx)-\sqrt{2}\sqrt{e\cot(c+dx)}\right)}{2\sqrt{2}\left(a^2+b^2\right)^3 d} - \frac{(a+b)\left(a^2-4ab+b^2\right)e^{5/2}\log\left(\sqrt{e}+\sqrt{e}\cot(c+dx)+\sqrt{2}\sqrt{e\cot(c+dx)}\right)}{2\sqrt{2}\left(a^2+b^2\right)^3 d} + \frac{\left((a-b)\left(a^2+4ab+b^2\right)e^{5/2}\right)\text{Subst}\left(\int\frac{1}{-1-x^2}\,dx, x, 1-\frac{\sqrt{2}\sqrt{e\cot(c+dx)}}{\sqrt{e}}\right)}{\sqrt{2}\left(a^2+b^2\right)^3 d} - \frac{\left((a-b)\left(a^2+4ab+b^2\right)e^{5/2}\right)\text{Subst}\left(\int\frac{1}{-1-x^2}\,dx, x, 1+\frac{\sqrt{2}\sqrt{e\cot(c+dx)}}{\sqrt{e}}\right)}{\sqrt{2}\left(a^2+b^2\right)^3 d}$$

$$= -\frac{\sqrt{a}(a^4+18a^2b^2-15b^4)\,e^{5/2}\arctan\left(\frac{\sqrt{b}\sqrt{e\cot(c+dx)}}{\sqrt{a}\sqrt{e}}\right)}{4b^{3/2}\left(a^2+b^2\right)^3 d} - \frac{(a-b)\left(a^2+4ab+b^2\right)e^{5/2}\arctan\left(1-\frac{\sqrt{2}\sqrt{e\cot(c+dx)}}{\sqrt{e}}\right)}{\sqrt{2}\left(a^2+b^2\right)^3 d} + \frac{(a-b)\left(a^2+4ab+b^2\right)e^{5/2}\arctan\left(1+\frac{\sqrt{2}\sqrt{e\cot(c+dx)}}{\sqrt{e}}\right)}{\sqrt{2}\left(a^2+b^2\right)^3 d} + \frac{a^2e^2\sqrt{e\cot(c+dx)}}{2b\left(a^2+b^2\right)d(a+b\cot(c+dx))^2} - \frac{a(a^2+9b^2)\,e^2\sqrt{e\cot(c+dx)}}{4b\left(a^2+b^2\right)^2 d(a+b\cot(c+dx))} + \frac{(a+b)\left(a^2-4ab+b^2\right)e^{5/2}\log\left(\sqrt{e}+\sqrt{e}\cot(c+dx)-\sqrt{2}\sqrt{e\cot(c+dx)}\right)}{2\sqrt{2}\left(a^2+b^2\right)^3 d} - \frac{(a+b)\left(a^2-4ab+b^2\right)e^{5/2}\log\left(\sqrt{e}+\sqrt{e}\cot(c+dx)+\sqrt{2}\sqrt{e\cot(c+dx)}\right)}{2\sqrt{2}\left(a^2+b^2\right)^3 d}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 6.23 (sec) , antiderivative size = 520, normalized size of antiderivative = 1.11

$$\int\frac{(e\cot(c+dx))^{5/2}}{(a+b\cot(c+dx))^3}\,dx =$$

$$-\frac{(e\cot(c+dx))^{5/2}\left(-\frac{2a^{5/2}(3a^2-b^2)\arctan\left(\frac{\sqrt{b}\sqrt{\cot(c+dx)}}{\sqrt{a}}\right)}{b^{3/2}(a^2+b^2)^3}+\frac{2a^2(3a^2-b^2)\sqrt{\cot(c+dx)}}{b(a^2+b^2)^3}-\frac{2a(3a^2-b^2)\cot^{\frac{3}{2}}(c+dx)}{3(a^2+b^2)^3}+\frac{2b(3a^2-b^2\cdots}{5(a\cdots}\right.}{\cdots}$$

```
[In] Integrate[(e*Cot[c + d*x])^(5/2)/(a + b*Cot[c + d*x])^3,x]
```

```
[Out] -(((e*Cot[c + d*x])^(5/2)*((-2*a^(5/2)*(3*a^2 - b^2)*ArcTan[(Sqrt[b]*Sqrt[C
ot[c + d*x]])/Sqrt[a]])/(b^(3/2)*(a^2 + b^2)^3) + (2*a^2*(3*a^2 - b^2)*Sqrt
[Cot[c + d*x]])/(b*(a^2 + b^2)^3) - (2*a*(3*a^2 - b^2)*Cot[c + d*x]^(3/2))/
(3*(a^2 + b^2)^3) + (2*b*(3*a^2 - b^2)*Cot[c + d*x]^(5/2))/(5*(a^2 + b^2)^3
) + (2*a*(a^2 - 3*b^2)*(Cot[c + d*x]^(3/2) - Cot[c + d*x]^(3/2)*Hypergeomet
ric2F1[3/4, 1, 7/4, -Cot[c + d*x]^2]))/(3*(a^2 + b^2)^3) + (4*b^2*Cot[c + d
*x]^(7/2)*Hypergeometric2F1[2, 7/2, 9/2, -((b*Cot[c + d*x])/a)])/(7*a*(a^2
+ b^2)^2) + (2*b^2*Cot[c + d*x]^(7/2)*Hypergeometric2F1[3, 7/2, 9/2, -((b*C
ot[c + d*x])/a)])/(7*a^3*(a^2 + b^2)) + (b*(3*a^2 - b^2)*(10*Sqrt[2]*ArcTan
[1 - Sqrt[2]*Sqrt[Cot[c + d*x]]] - 10*Sqrt[2]*ArcTan[1 + Sqrt[2]*Sqrt[Cot[c
 + d*x]]] + 40*Sqrt[Cot[c + d*x]] - 8*Cot[c + d*x]^(5/2) + 5*Sqrt[2]*Log[1
- Sqrt[2]*Sqrt[Cot[c + d*x]] + Cot[c + d*x]] - 5*Sqrt[2]*Log[1 + Sqrt[2]*Sq
rt[Cot[c + d*x]] + Cot[c + d*x]]))/(20*(a^2 + b^2)^3)))/(d*Cot[c + d*x]^(5/
2)))
```

Maple [A] (verified)

Time = 1.15 (sec) , antiderivative size = 457, normalized size of antiderivative = 0.97

method	result
derivativedivides	$-\frac{2e^4\left(\frac{a\left(\frac{\left(\frac{1}{8}a^4+\frac{5}{4}a^2b^2+\frac{9}{8}b^4\right)(e\cot(dx+c))^{\frac{3}{2}}-\frac{ae\left(a^4-6a^2b^2-7b^4\right)\sqrt{e\cot(dx+c)}}{8b}}{(e\cot(dx+c)b+ae)^2}+\frac{\left(a^4+18a^2b^2-15b^4\right)\arctan\left(\frac{\sqrt{e\cot(dx+c)}\,b}{\sqrt{aeb}}\right)}{8b\sqrt{aeb}}\right.}{\left(a^2+b^2\right)^3e}\right.}{}$
default	$-\frac{2e^4\left(\frac{a\left(\frac{\left(\frac{1}{8}a^4+\frac{5}{4}a^2b^2+\frac{9}{8}b^4\right)(e\cot(dx+c))^{\frac{3}{2}}-\frac{ae\left(a^4-6a^2b^2-7b^4\right)\sqrt{e\cot(dx+c)}}{8b}}{(e\cot(dx+c)b+ae)^2}+\frac{\left(a^4+18a^2b^2-15b^4\right)\arctan\left(\frac{\sqrt{e\cot(dx+c)}\,b}{\sqrt{aeb}}\right)}{8b\sqrt{aeb}}\right.}{\left(a^2+b^2\right)^3e}\right.}{}$

```
[In] int((e*cot(d*x+c))^(5/2)/(a+b*cot(d*x+c))^3,x,method=_RETURNVERBOSE)
```

```
[Out] -2/d*e^4*(a/(a^2+b^2)^3/e*(((1/8*a^4+5/4*a^2*b^2+9/8*b^4)*(e*cot(d*x+c))^(3
/2)-1/8*a*e*(a^4-6*a^2*b^2-7*b^4)/b*(e*cot(d*x+c))^(1/2))/(e*cot(d*x+c)*b+a
*e)^2+1/8*(a^4+18*a^2*b^2-15*b^4)/b/(a*e*b)^(1/2)*arctan((e*cot(d*x+c))^(1/
2)*b/(a*e*b)^(1/2)))+1/e/(a^2+b^2)^3*(1/8*(-3*a^2*b*e+b^3*e)*(e^2)^(1/4)/e^
2*2^(1/2)*(ln((e*cot(d*x+c)+(e^2)^(1/4)*(e*cot(d*x+c))^(1/2)*2^(1/2)+(e^2)^
(1/2))/(e*cot(d*x+c)-(e^2)^(1/4)*(e*cot(d*x+c))^(1/2)*2^(1/2)+(e^2)^(1/2)))
```

```
+2*arctan(2^(1/2)/(e^2)^(1/4)*(e*cot(d*x+c))^(1/2)+1)-2*arctan(-2^(1/2)/(e^
2)^(1/4)*(e*cot(d*x+c))^(1/2)+1))+1/8*(-a^3+3*a*b^2)/(e^2)^(1/4)*2^(1/2)*(l
n((e*cot(d*x+c)-(e^2)^(1/4)*(e*cot(d*x+c))^(1/2)*2^(1/2)+(e^2)^(1/2))/(e*co
t(d*x+c)+(e^2)^(1/4)*(e*cot(d*x+c))^(1/2)*2^(1/2)+(e^2)^(1/2)))+2*arctan(2^
(1/2)/(e^2)^(1/4)*(e*cot(d*x+c))^(1/2)+1)-2*arctan(-2^(1/2)/(e^2)^(1/4)*(e*
cot(d*x+c))^(1/2)+1)))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 4446 vs. $2(399) = 798$.

Time = 39.25 (sec) , antiderivative size = 8955, normalized size of antiderivative = 19.05

$$\int \frac{(e\cot(c+dx))^{5/2}}{(a+b\cot(c+dx))^3}\,dx = \text{Too large to display}$$

```
[In] integrate((e*cot(d*x+c))^(5/2)/(a+b*cot(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] Too large to include
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(e\cot(c+dx))^{5/2}}{(a+b\cot(c+dx))^3}\,dx = \text{Timed out}$$

```
[In] integrate((e*cot(d*x+c))**(5/2)/(a+b*cot(d*x+c))**3,x)
```

```
[Out] Timed out
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{(e\cot(c+dx))^{5/2}}{(a+b\cot(c+dx))^3}\,dx = \text{Exception raised: ValueError}$$

```
[In] integrate((e*cot(d*x+c))^(5/2)/(a+b*cot(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(e>0)', see `assume?` for more detai
ls)Is e
```

Giac [F]

$$\int \frac{(e\cot(c+dx))^{5/2}}{(a+b\cot(c+dx))^3}\,dx = \int \frac{(e\cot(dx+c))^{\frac{5}{2}}}{(b\cot(dx+c)+a)^3}\,dx$$

```
[In] integrate((e*cot(d*x+c))^(5/2)/(a+b*cot(d*x+c))^3,x, algorithm="giac")
[Out] integrate((e*cot(d*x + c))^(5/2)/(b*cot(d*x + c) + a)^3, x)
```

Mupad [B] (verification not implemented)

Time = 19.08 (sec) , antiderivative size = 19256, normalized size of antiderivative = 40.97

$$\int \frac{(e\cot(c+dx))^{5/2}}{(a+b\cot(c+dx))^3}\,dx = \text{Too large to display}$$

```
[In] int((e*cot(c + d*x))^(5/2)/(a + b*cot(c + d*x))^3,x)
[Out] atan(((((10*a^16*b*d^2*e^18 - 2398*a^2*b^15*d^2*e^18 + 5238*a^4*b^13*d^2*e^
18 + 7386*a^6*b^11*d^2*e^18 - 8322*a^8*b^9*d^2*e^18 - 5498*a^10*b^7*d^2*e^1
8 + 2946*a^12*b^5*d^2*e^18 + 382*a^14*b^3*d^2*e^18)/(b^17*d^5 + a^16*b*d^5
+ 8*a^2*b^15*d^5 + 28*a^4*b^13*d^5 + 56*a^6*b^11*d^5 + 70*a^8*b^9*d^5 + 56*
a^10*b^7*d^5 + 28*a^12*b^5*d^5 + 8*a^14*b^3*d^5) - (((832*a*b^22*d^4*e^13 +
 5952*a^3*b^20*d^4*e^13 + 17664*a^5*b^18*d^4*e^13 + 26880*a^7*b^16*d^4*e^13
 + 18816*a^9*b^14*d^4*e^13 - 2688*a^11*b^12*d^4*e^13 - 16128*a^13*b^10*d^4*
e^13 - 13056*a^15*b^8*d^4*e^13 - 4800*a^17*b^6*d^4*e^13 - 704*a^19*b^4*d^4*
e^13)/(b^17*d^5 + a^16*b*d^5 + 8*a^2*b^15*d^5 + 28*a^4*b^13*d^5 + 56*a^6*b^
11*d^5 + 70*a^8*b^9*d^5 + 56*a^10*b^7*d^5 + 28*a^12*b^5*d^5 + 8*a^14*b^3*d^
5) + ((e*cot(c + d*x))^(1/2)*(-(e^5*1i)/(4*(b^6*d^2 - a^6*d^2 + a*b^5*d^2*6
i + a^5*b*d^2*6i - 15*a^2*b^4*d^2 - a^3*b^3*d^2*20i + 15*a^4*b^2*d^2)))^(1/
2)*(512*b^26*d^4*e^10 + 4608*a^2*b^24*d^4*e^10 + 17920*a^4*b^22*d^4*e^10 +
38400*a^6*b^20*d^4*e^10 + 46080*a^8*b^18*d^4*e^10 + 21504*a^10*b^16*d^4*e^1
0 - 21504*a^12*b^14*d^4*e^10 - 46080*a^14*b^12*d^4*e^10 - 38400*a^16*b^10*d
^4*e^10 - 17920*a^18*b^8*d^4*e^10 - 4608*a^20*b^6*d^4*e^10 - 512*a^22*b^4*d
^4*e^10))/(b^17*d^4 + a^16*b*d^4 + 8*a^2*b^15*d^4 + 28*a^4*b^13*d^4 + 56*a^
6*b^11*d^4 + 70*a^8*b^9*d^4 + 56*a^10*b^7*d^4 + 28*a^12*b^5*d^4 + 8*a^14*b^
3*d^4))*(-(e^5*1i)/(4*(b^6*d^2 - a^6*d^2 + a*b^5*d^2*6i + a^5*b*d^2*6i - 15
*a^2*b^4*d^2 - a^3*b^3*d^2*20i + 15*a^4*b^2*d^2)))^(1/2) - ((e*cot(c + d*x)
)^(1/2)*(8*a^19*b*d^2*e^15 - 1472*a*b^19*d^2*e^15 + 776*a^3*b^17*d^2*e^15 +
 11328*a^5*b^15*d^2*e^15 + 10208*a^7*b^13*d^2*e^15 - 5056*a^9*b^11*d^2*e^15
 - 5328*a^11*b^9*d^2*e^15 + 4032*a^13*b^7*d^2*e^15 + 3552*a^15*b^5*d^2*e^15
 + 384*a^17*b^3*d^2*e^15))/(b^17*d^4 + a^16*b*d^4 + 8*a^2*b^15*d^4 + 28*a^4
*b^13*d^4 + 56*a^6*b^11*d^4 + 70*a^8*b^9*d^4 + 56*a^10*b^7*d^4 + 28*a^12*b^
5*d^4 + 8*a^14*b^3*d^4))*(-(e^5*1i)/(4*(b^6*d^2 - a^6*d^2 + a*b^5*d^2*6i +
```

a^5*b*d^2*6i - 15*a^2*b^4*d^2 - a^3*b^3*d^2*20i + 15*a^4*b^2*d^2)))^(1/2))*
(-(e^5*1i)/(4*(b^6*d^2 - a^6*d^2 + a*b^5*d^2*6i + a^5*b*d^2*6i - 15*a^2*b^4
*d^2 - a^3*b^3*d^2*20i + 15*a^4*b^2*d^2)))^(1/2) + ((e*cot(c + d*x))^(1/2)*
(a^14*e^20 - 32*b^14*e^20 + 97*a^2*b^12*e^20 - 2082*a^4*b^10*e^20 + 3631*a^
6*b^8*e^20 - 2300*a^8*b^6*e^20 + 79*a^10*b^4*e^20 + 30*a^12*b^2*e^20))/(b^1
7*d^4 + a^16*b*d^4 + 8*a^2*b^15*d^4 + 28*a^4*b^13*d^4 + 56*a^6*b^11*d^4 + 7
0*a^8*b^9*d^4 + 56*a^10*b^7*d^4 + 28*a^12*b^5*d^4 + 8*a^14*b^3*d^4))*(-(e^5
1i)/(4(b^6*d^2 - a^6*d^2 + a*b^5*d^2*6i + a^5*b*d^2*6i - 15*a^2*b^4*d^2 -
a^3*b^3*d^2*20i + 15*a^4*b^2*d^2)))^(1/2)*1i - (((10*a^16*b*d^2*e^18 - 239
8*a^2*b^15*d^2*e^18 + 5238*a^4*b^13*d^2*e^18 + 7386*a^6*b^11*d^2*e^18 - 832
2*a^8*b^9*d^2*e^18 - 5498*a^10*b^7*d^2*e^18 + 2946*a^12*b^5*d^2*e^18 + 382*
a^14*b^3*d^2*e^18)/(b^17*d^5 + a^16*b*d^5 + 8*a^2*b^15*d^5 + 28*a^4*b^13*d^
5 + 56*a^6*b^11*d^5 + 70*a^8*b^9*d^5 + 56*a^10*b^7*d^5 + 28*a^12*b^5*d^5 +
8*a^14*b^3*d^5) - (((832*a*b^22*d^4*e^13 + 5952*a^3*b^20*d^4*e^13 + 17664*a
^5*b^18*d^4*e^13 + 26880*a^7*b^16*d^4*e^13 + 18816*a^9*b^14*d^4*e^13 - 2688
*a^11*b^12*d^4*e^13 - 16128*a^13*b^10*d^4*e^13 - 13056*a^15*b^8*d^4*e^13 -
4800*a^17*b^6*d^4*e^13 - 704*a^19*b^4*d^4*e^13)/(b^17*d^5 + a^16*b*d^5 + 8*
a^2*b^15*d^5 + 28*a^4*b^13*d^5 + 56*a^6*b^11*d^5 + 70*a^8*b^9*d^5 + 56*a^10
*b^7*d^5 + 28*a^12*b^5*d^5 + 8*a^14*b^3*d^5) - ((e*cot(c + d*x))^(1/2)*(-(e
^5*1i)/(4*(b^6*d^2 - a^6*d^2 + a*b^5*d^2*6i + a^5*b*d^2*6i - 15*a^2*b^4*d^2
- a^3*b^3*d^2*20i + 15*a^4*b^2*d^2)))^(1/2)*(512*b^26*d^4*e^10 + 4608*a^2*
b^24*d^4*e^10 + 17920*a^4*b^22*d^4*e^10 + 38400*a^6*b^20*d^4*e^10 + 46080*a
^8*b^18*d^4*e^10 + 21504*a^10*b^16*d^4*e^10 - 21504*a^12*b^14*d^4*e^10 - 46
080*a^14*b^12*d^4*e^10 - 38400*a^16*b^10*d^4*e^10 - 17920*a^18*b^8*d^4*e^10
- 4608*a^20*b^6*d^4*e^10 - 512*a^22*b^4*d^4*e^10))/(b^17*d^4 + a^16*b*d^4
+ 8*a^2*b^15*d^4 + 28*a^4*b^13*d^4 + 56*a^6*b^11*d^4 + 70*a^8*b^9*d^4 + 56*
a^10*b^7*d^4 + 28*a^12*b^5*d^4 + 8*a^14*b^3*d^4))*(-(e^5*1i)/(4*(b^6*d^2 -
a^6*d^2 + a*b^5*d^2*6i + a^5*b*d^2*6i - 15*a^2*b^4*d^2 - a^3*b^3*d^2*20i +
15*a^4*b^2*d^2)))^(1/2) + ((e*cot(c + d*x))^(1/2)*(8*a^19*b*d^2*e^15 - 1472
*a*b^19*d^2*e^15 + 776*a^3*b^17*d^2*e^15 + 11328*a^5*b^15*d^2*e^15 + 10208*
a^7*b^13*d^2*e^15 - 5056*a^9*b^11*d^2*e^15 - 5328*a^11*b^9*d^2*e^15 + 4032*
a^13*b^7*d^2*e^15 + 3552*a^15*b^5*d^2*e^15 + 384*a^17*b^3*d^2*e^15))/(b^17*
d^4 + a^16*b*d^4 + 8*a^2*b^15*d^4 + 28*a^4*b^13*d^4 + 56*a^6*b^11*d^4 + 70*
a^8*b^9*d^4 + 56*a^10*b^7*d^4 + 28*a^12*b^5*d^4 + 8*a^14*b^3*d^4))*(-(e^5*1
i)/(4*(b^6*d^2 - a^6*d^2 + a*b^5*d^2*6i + a^5*b*d^2*6i - 15*a^2*b^4*d^2 - a
^3*b^3*d^2*20i + 15*a^4*b^2*d^2)))^(1/2))*(-(e^5*1i)/(4*(b^6*d^2 - a^6*d^2
+ a*b^5*d^2*6i + a^5*b*d^2*6i - 15*a^2*b^4*d^2 - a^3*b^3*d^2*20i + 15*a^4*b
^2*d^2)))^(1/2) - ((e*cot(c + d*x))^(1/2)*(a^14*e^20 - 32*b^14*e^20 + 97*a^
2*b^12*e^20 - 2082*a^4*b^10*e^20 + 3631*a^6*b^8*e^20 - 2300*a^8*b^6*e^20 +
79*a^10*b^4*e^20 + 30*a^12*b^2*e^20))/(b^17*d^4 + a^16*b*d^4 + 8*a^2*b^15*d
^4 + 28*a^4*b^13*d^4 + 56*a^6*b^11*d^4 + 70*a^8*b^9*d^4 + 56*a^10*b^7*d^4 +
28*a^12*b^5*d^4 + 8*a^14*b^3*d^4))*(-(e^5*1i)/(4*(b^6*d^2 - a^6*d^2 + a*b^
5*d^2*6i + a^5*b*d^2*6i - 15*a^2*b^4*d^2 - a^3*b^3*d^2*20i + 15*a^4*b^2*d^2
)))^(1/2)*1i)/((((10*a^16*b*d^2*e^18 - 2398*a^2*b^15*d^2*e^18 + 5238*a^4*b^
13*d^2*e^18 + 7386*a^6*b^11*d^2*e^18 - 8322*a^8*b^9*d^2*e^18 - 5498*a^10*b^

7*d^2*e^18 + 2946*a^12*b^5*d^2*e^18 + 382*a^14*b^3*d^2*e^18)/(b^17*d^5 + a^16*b*d^5 + 8*a^2*b^15*d^5 + 28*a^4*b^13*d^5 + 56*a^6*b^11*d^5 + 70*a^8*b^9*d^5 + 56*a^10*b^7*d^5 + 28*a^12*b^5*d^5 + 8*a^14*b^3*d^5) - (((832*a*b^22*d^4*e^13 + 5952*a^3*b^20*d^4*e^13 + 17664*a^5*b^18*d^4*e^13 + 26880*a^7*b^16*d^4*e^13 + 18816*a^9*b^14*d^4*e^13 - 2688*a^11*b^12*d^4*e^13 - 16128*a^13*b^10*d^4*e^13 - 13056*a^15*b^8*d^4*e^13 - 4800*a^17*b^6*d^4*e^13 - 704*a^19*b^4*d^4*e^13)/(b^17*d^5 + a^16*b*d^5 + 8*a^2*b^15*d^5 + 28*a^4*b^13*d^5 + 56*a^6*b^11*d^5 + 70*a^8*b^9*d^5 + 56*a^10*b^7*d^5 + 28*a^12*b^5*d^5 + 8*a^14*b^3*d^5) + ((e*cot(c + d*x))^(1/2)*(-(e^5*1i)/(4*(b^6*d^2 - a^6*d^2 + a*b^5*d^2*6i + a^5*b*d^2*6i - 15*a^2*b^4*d^2 - a^3*b^3*d^2*20i + 15*a^4*b^2*d^2)))^(1/2)*(512*b^26*d^4*e^10 + 4608*a^2*b^24*d^4*e^10 + 17920*a^4*b^22*d^4*e^10 + 38400*a^6*b^20*d^4*e^10 + 46080*a^8*b^18*d^4*e^10 + 21504*a^10*b^16*d^4*e^10 - 21504*a^12*b^14*d^4*e^10 - 46080*a^14*b^12*d^4*e^10 - 38400*a^16*b^10*d^4*e^10 - 17920*a^18*b^8*d^4*e^10 - 4608*a^20*b^6*d^4*e^10 - 512*a^22*b^4*d^4*e^10))/(b^17*d^4 + a^16*b*d^4 + 8*a^2*b^15*d^4 + 28*a^4*b^13*d^4 + 56*a^6*b^11*d^4 + 70*a^8*b^9*d^4 + 56*a^10*b^7*d^4 + 28*a^12*b^5*d^4 + 8*a^14*b^3*d^4))*(-(e^5*1i)/(4*(b^6*d^2 - a^6*d^2 + a*b^5*d^2*6i + a^5*b*d^2*6i - 15*a^2*b^4*d^2 - a^3*b^3*d^2*20i + 15*a^4*b^2*d^2)))^(1/2) - ((e*cot(c + d*x))^(1/2)*(8*a^19*b*d^2*e^15 - 1472*a*b^19*d^2*e^15 + 776*a^3*b^17*d^2*e^15 + 11328*a^5*b^15*d^2*e^15 + 10208*a^7*b^13*d^2*e^15 - 5056*a^9*b^11*d^2*e^15 - 5328*a^11*b^9*d^2*e^15 + 4032*a^13*b^7*d^2*e^15 + 3552*a^15*b^5*d^2*e^15 + 384*a^17*b^3*d^2*e^15))/(b^17*d^4 + a^16*b*d^4 + 8*a^2*b^15*d^4 + 28*a^4*b^13*d^4 + 56*a^6*b^11*d^4 + 70*a^8*b^9*d^4 + 56*a^10*b^7*d^4 + 28*a^12*b^5*d^4 + 8*a^14*b^3*d^4))*(-(e^5*1i)/(4*(b^6*d^2 - a^6*d^2 + a*b^5*d^2*6i + a^5*b*d^2*6i - 15*a^2*b^4*d^2 - a^3*b^3*d^2*20i + 15*a^4*b^2*d^2)))^(1/2))*(-(e^5*1i)/(4*(b^6*d^2 - a^6*d^2 + a*b^5*d^2*6i + a^5*b*d^2*6i - 15*a^2*b^4*d^2 - a^3*b^3*d^2*20i + 15*a^4*b^2*d^2)))^(1/2) + ((e*cot(c + d*x))^(1/2)*(a^14*e^20 - 32*b^14*e^20 + 97*a^2*b^12*e^20 - 2082*a^4*b^10*e^20 + 3631*a^6*b^8*e^20 - 2300*a^8*b^6*e^20 + 79*a^10*b^4*e^20 + 30*a^12*b^2*e^20))/(b^17*d^4 + a^16*b*d^4 + 8*a^2*b^15*d^4 + 28*a^4*b^13*d^4 + 56*a^6*b^11*d^4 + 70*a^8*b^9*d^4 + 56*a^10*b^7*d^4 + 28*a^12*b^5*d^4 + 8*a^14*b^3*d^4))*(-(e^5*1i)/(4*(b^6*d^2 - a^6*d^2 + a*b^5*d^2*6i + a^5*b*d^2*6i - 15*a^2*b^4*d^2 - a^3*b^3*d^2*20i + 15*a^4*b^2*d^2)))^(1/2) + (((10*a^16*b*d^2*e^18 - 2398*a^2*b^15*d^2*e^18 + 5238*a^4*b^13*d^2*e^18 + 7386*a^6*b^11*d^2*e^18 - 8322*a^8*b^9*d^2*e^18 - 5498*a^10*b^7*d^2*e^18 + 2946*a^12*b^5*d^2*e^18 + 382*a^14*b^3*d^2*e^18)/(b^17*d^5 + a^16*b*d^5 + 8*a^2*b^15*d^5 + 28*a^4*b^13*d^5 + 56*a^6*b^11*d^5 + 70*a^8*b^9*d^5 + 56*a^10*b^7*d^5 + 28*a^12*b^5*d^5 + 8*a^14*b^3*d^5) - (((832*a*b^22*d^4*e^13 + 5952*a^3*b^20*d^4*e^13 + 17664*a^5*b^18*d^4*e^13 + 26880*a^7*b^16*d^4*e^13 + 18816*a^9*b^14*d^4*e^13 - 2688*a^11*b^12*d^4*e^13 - 16128*a^13*b^10*d^4*e^13 - 13056*a^15*b^8*d^4*e^13 - 4800*a^17*b^6*d^4*e^13 - 704*a^19*b^4*d^4*e^13)/(b^17*d^5 + a^16*b*d^5 + 8*a^2*b^15*d^5 + 28*a^4*b^13*d^5 + 56*a^6*b^11*d^5 + 70*a^8*b^9*d^5 + 56*a^10*b^7*d^5 + 28*a^12*b^5*d^5 + 8*a^14*b^3*d^5) - ((e*cot(c + d*x))^(1/2)*(-(e^5*1i)/(4*(b^6*d^2 - a^6*d^2 + a*b^5*d^2*6i + a^5*b*d^2*6i - 15*a^2*b^4*d^2 - a^3*b^3*d^2*20i + 15*a^4*b^2*d^2)))^(1/2)*(512*b^26*d^4*e^10 + 460

8*a^2*b^24*d^4*e^10 + 17920*a^4*b^22*d^4*e^10 + 38400*a^6*b^20*d^4*e^10 + 4
6080*a^8*b^18*d^4*e^10 + 21504*a^10*b^16*d^4*e^10 - 21504*a^12*b^14*d^4*e^1
0 - 46080*a^14*b^12*d^4*e^10 - 38400*a^16*b^10*d^4*e^10 - 17920*a^18*b^8*d^
4*e^10 - 4608*a^20*b^6*d^4*e^10 - 512*a^22*b^4*d^4*e^10))/(b^17*d^4 + a^16*
b*d^4 + 8*a^2*b^15*d^4 + 28*a^4*b^13*d^4 + 56*a^6*b^11*d^4 + 70*a^8*b^9*d^4
 + 56*a^10*b^7*d^4 + 28*a^12*b^5*d^4 + 8*a^14*b^3*d^4))*(-(e^5*1i)/(4*(b^6*
d^2 - a^6*d^2 + a*b^5*d^2*6i + a^5*b*d^2*6i - 15*a^2*b^4*d^2 - a^3*b^3*d^2*
20i + 15*a^4*b^2*d^2)))^(1/2) + ((e*cot(c + d*x))^(1/2)*(8*a^19*b*d^2*e^15
- 1472*a*b^19*d^2*e^15 + 776*a^3*b^17*d^2*e^15 + 11328*a^5*b^15*d^2*e^15 +
10208*a^7*b^13*d^2*e^15 - 5056*a^9*b^11*d^2*e^15 - 5328*a^11*b^9*d^2*e^15 +
 4032*a^13*b^7*d^2*e^15 + 3552*a^15*b^5*d^2*e^15 + 384*a^17*b^3*d^2*e^15))/
(b^17*d^4 + a^16*b*d^4 + 8*a^2*b^15*d^4 + 28*a^4*b^13*d^4 + 56*a^6*b^11*d^4
 + 70*a^8*b^9*d^4 + 56*a^10*b^7*d^4 + 28*a^12*b^5*d^4 + 8*a^14*b^3*d^4))*(-
(e^5*1i)/(4*(b^6*d^2 - a^6*d^2 + a*b^5*d^2*6i + a^5*b*d^2*6i - 15*a^2*b^4*d
^2 - a^3*b^3*d^2*20i + 15*a^4*b^2*d^2)))^(1/2))*(-(e^5*1i)/(4*(b^6*d^2 - a^
6*d^2 + a*b^5*d^2*6i + a^5*b*d^2*6i - 15*a^2*b^4*d^2 - a^3*b^3*d^2*20i + 15
*a^4*b^2*d^2)))^(1/2) - ((e*cot(c + d*x))^(1/2)*(a^14*e^20 - 32*b^14*e^20 +
 97*a^2*b^12*e^20 - 2082*a^4*b^10*e^20 + 3631*a^6*b^8*e^20 - 2300*a^8*b^6*e
^20 + 79*a^10*b^4*e^20 + 30*a^12*b^2*e^20))/(b^17*d^4 + a^16*b*d^4 + 8*a^2*
b^15*d^4 + 28*a^4*b^13*d^4 + 56*a^6*b^11*d^4 + 70*a^8*b^9*d^4 + 56*a^10*b^7
*d^4 + 28*a^12*b^5*d^4 + 8*a^14*b^3*d^4))*(-(e^5*1i)/(4*(b^6*d^2 - a^6*d^2
+ a*b^5*d^2*6i + a^5*b*d^2*6i - 15*a^2*b^4*d^2 - a^3*b^3*d^2*20i + 15*a^4*b
^2*d^2)))^(1/2) + (a^11*e^23 - 120*a*b^10*e^23 + 249*a^3*b^8*e^23 - 388*a^5
*b^6*e^23 + 302*a^7*b^4*e^23 + 36*a^9*b^2*e^23)/(b^17*d^5 + a^16*b*d^5 + 8*
a^2*b^15*d^5 + 28*a^4*b^13*d^5 + 56*a^6*b^11*d^5 + 70*a^8*b^9*d^5 + 56*a^10
*b^7*d^5 + 28*a^12*b^5*d^5 + 8*a^14*b^3*d^5)))*(-(e^5*1i)/(4*(b^6*d^2 - a^6
*d^2 + a*b^5*d^2*6i + a^5*b*d^2*6i - 15*a^2*b^4*d^2 - a^3*b^3*d^2*20i + 15*
a^4*b^2*d^2)))^(1/2)*2i - ((e^3*(e*cot(c + d*x))^(3/2)*(9*a*b^2 + a^3))/(4*
(a^4 + b^4 + 2*a^2*b^2)) - ((e*cot(c + d*x))^(1/2)*(a^4*e^4 - 7*a^2*b^2*e^4
))/(4*b*(a^4 + b^4 + 2*a^2*b^2)))/(a^2*d*e^2 + b^2*d*e^2*cot(c + d*x)^2 + 2
*a*b*d*e^2*cot(c + d*x)) + atan(((((10*a^16*b*d^2*e^18 - 2398*a^2*b^15*d^2*
e^18 + 5238*a^4*b^13*d^2*e^18 + 7386*a^6*b^11*d^2*e^18 - 8322*a^8*b^9*d^2*e
^18 - 5498*a^10*b^7*d^2*e^18 + 2946*a^12*b^5*d^2*e^18 + 382*a^14*b^3*d^2*e^
18)/(b^17*d^5 + a^16*b*d^5 + 8*a^2*b^15*d^5 + 28*a^4*b^13*d^5 + 56*a^6*b^11
*d^5 + 70*a^8*b^9*d^5 + 56*a^10*b^7*d^5 + 28*a^12*b^5*d^5 + 8*a^14*b^3*d^5)
 - (((832*a*b^22*d^4*e^13 + 5952*a^3*b^20*d^4*e^13 + 17664*a^5*b^18*d^4*e^1
3 + 26880*a^7*b^16*d^4*e^13 + 18816*a^9*b^14*d^4*e^13 - 2688*a^11*b^12*d^4*
e^13 - 16128*a^13*b^10*d^4*e^13 - 13056*a^15*b^8*d^4*e^13 - 4800*a^17*b^6*d
^4*e^13 - 704*a^19*b^4*d^4*e^13)/(b^17*d^5 + a^16*b*d^5 + 8*a^2*b^15*d^5 +
28*a^4*b^13*d^5 + 56*a^6*b^11*d^5 + 70*a^8*b^9*d^5 + 56*a^10*b^7*d^5 + 28*a
^12*b^5*d^5 + 8*a^14*b^3*d^5) + ((e*cot(c + d*x))^(1/2)*(-e^5/(4*(b^6*d^2*1
i - a^6*d^2*1i + 6*a*b^5*d^2 + 6*a^5*b*d^2 - a^2*b^4*d^2*15i - 20*a^3*b^3*d
^2 + a^4*b^2*d^2*15i)))^(1/2)*(512*b^26*d^4*e^10 + 4608*a^2*b^24*d^4*e^10 +
 17920*a^4*b^22*d^4*e^10 + 38400*a^6*b^20*d^4*e^10 + 46080*a^8*b^18*d^4*e^1
0 + 21504*a^10*b^16*d^4*e^10 - 21504*a^12*b^14*d^4*e^10 - 46080*a^14*b^12*d

^4*e^10 - 38400*a^16*b^10*d^4*e^10 - 17920*a^18*b^8*d^4*e^10 - 4608*a^20*b^
6*d^4*e^10 - 512*a^22*b^4*d^4*e^10))/(b^17*d^4 + a^16*b*d^4 + 8*a^2*b^15*d^
4 + 28*a^4*b^13*d^4 + 56*a^6*b^11*d^4 + 70*a^8*b^9*d^4 + 56*a^10*b^7*d^4 +
28*a^12*b^5*d^4 + 8*a^14*b^3*d^4))*(-e^5/(4*(b^6*d^2*1i - a^6*d^2*1i + 6*a*
b^5*d^2 + 6*a^5*b*d^2 - a^2*b^4*d^2*15i - 20*a^3*b^3*d^2 + a^4*b^2*d^2*15i)
))^(1/2) - ((e*cot(c + d*x))^(1/2)*(8*a^19*b*d^2*e^15 - 1472*a*b^19*d^2*e^1
5 + 776*a^3*b^17*d^2*e^15 + 11328*a^5*b^15*d^2*e^15 + 10208*a^7*b^13*d^2*e^
15 - 5056*a^9*b^11*d^2*e^15 - 5328*a^11*b^9*d^2*e^15 + 4032*a^13*b^7*d^2*e^
15 + 3552*a^15*b^5*d^2*e^15 + 384*a^17*b^3*d^2*e^15))/(b^17*d^4 + a^16*b*d^
4 + 8*a^2*b^15*d^4 + 28*a^4*b^13*d^4 + 56*a^6*b^11*d^4 + 70*a^8*b^9*d^4 + 5
6*a^10*b^7*d^4 + 28*a^12*b^5*d^4 + 8*a^14*b^3*d^4))*(-e^5/(4*(b^6*d^2*1i -
a^6*d^2*1i + 6*a*b^5*d^2 + 6*a^5*b*d^2 - a^2*b^4*d^2*15i - 20*a^3*b^3*d^2 +
a^4*b^2*d^2*15i)))^(1/2))*(-e^5/(4*(b^6*d^2*1i - a^6*d^2*1i + 6*a*b^5*d^2
+ 6*a^5*b*d^2 - a^2*b^4*d^2*15i - 20*a^3*b^3*d^2 + a^4*b^2*d^2*15i)))^(1/2)
+ ((e*cot(c + d*x))^(1/2)*(a^14*e^20 - 32*b^14*e^20 + 97*a^2*b^12*e^20 - 2
082*a^4*b^10*e^20 + 3631*a^6*b^8*e^20 - 2300*a^8*b^6*e^20 + 79*a^10*b^4*e^2
0 + 30*a^12*b^2*e^20))/(b^17*d^4 + a^16*b*d^4 + 8*a^2*b^15*d^4 + 28*a^4*b^1
3*d^4 + 56*a^6*b^11*d^4 + 70*a^8*b^9*d^4 + 56*a^10*b^7*d^4 + 28*a^12*b^5*d^
4 + 8*a^14*b^3*d^4))*(-e^5/(4*(b^6*d^2*1i - a^6*d^2*1i + 6*a*b^5*d^2 + 6*a^
5*b*d^2 - a^2*b^4*d^2*15i - 20*a^3*b^3*d^2 + a^4*b^2*d^2*15i)))^(1/2)*1i -
(((10*a^16*b*d^2*e^18 - 2398*a^2*b^15*d^2*e^18 + 5238*a^4*b^13*d^2*e^18 + 7
386*a^6*b^11*d^2*e^18 - 8322*a^8*b^9*d^2*e^18 - 5498*a^10*b^7*d^2*e^18 + 29
46*a^12*b^5*d^2*e^18 + 382*a^14*b^3*d^2*e^18)/(b^17*d^5 + a^16*b*d^5 + 8*a^
2*b^15*d^5 + 28*a^4*b^13*d^5 + 56*a^6*b^11*d^5 + 70*a^8*b^9*d^5 + 56*a^10*b
^7*d^5 + 28*a^12*b^5*d^5 + 8*a^14*b^3*d^5) - (((832*a*b^22*d^4*e^13 + 5952*
a^3*b^20*d^4*e^13 + 17664*a^5*b^18*d^4*e^13 + 26880*a^7*b^16*d^4*e^13 + 188
16*a^9*b^14*d^4*e^13 - 2688*a^11*b^12*d^4*e^13 - 16128*a^13*b^10*d^4*e^13 -
13056*a^15*b^8*d^4*e^13 - 4800*a^17*b^6*d^4*e^13 - 704*a^19*b^4*d^4*e^13)/
(b^17*d^5 + a^16*b*d^5 + 8*a^2*b^15*d^5 + 28*a^4*b^13*d^5 + 56*a^6*b^11*d^5
+ 70*a^8*b^9*d^5 + 56*a^10*b^7*d^5 + 28*a^12*b^5*d^5 + 8*a^14*b^3*d^5) - (
(e*cot(c + d*x))^(1/2)*(-e^5/(4*(b^6*d^2*1i - a^6*d^2*1i + 6*a*b^5*d^2 + 6*
a^5*b*d^2 - a^2*b^4*d^2*15i - 20*a^3*b^3*d^2 + a^4*b^2*d^2*15i)))^(1/2)*(51
2*b^26*d^4*e^10 + 4608*a^2*b^24*d^4*e^10 + 17920*a^4*b^22*d^4*e^10 + 38400*
a^6*b^20*d^4*e^10 + 46080*a^8*b^18*d^4*e^10 + 21504*a^10*b^16*d^4*e^10 - 21
504*a^12*b^14*d^4*e^10 - 46080*a^14*b^12*d^4*e^10 - 38400*a^16*b^10*d^4*e^1
0 - 17920*a^18*b^8*d^4*e^10 - 4608*a^20*b^6*d^4*e^10 - 512*a^22*b^4*d^4*e^1
0))/(b^17*d^4 + a^16*b*d^4 + 8*a^2*b^15*d^4 + 28*a^4*b^13*d^4 + 56*a^6*b^11
*d^4 + 70*a^8*b^9*d^4 + 56*a^10*b^7*d^4 + 28*a^12*b^5*d^4 + 8*a^14*b^3*d^4)
)*(-e^5/(4*(b^6*d^2*1i - a^6*d^2*1i + 6*a*b^5*d^2 + 6*a^5*b*d^2 - a^2*b^4*d
^2*15i - 20*a^3*b^3*d^2 + a^4*b^2*d^2*15i)))^(1/2) + ((e*cot(c + d*x))^(1/2
)*(8*a^19*b*d^2*e^15 - 1472*a*b^19*d^2*e^15 + 776*a^3*b^17*d^2*e^15 + 11328
*a^5*b^15*d^2*e^15 + 10208*a^7*b^13*d^2*e^15 - 5056*a^9*b^11*d^2*e^15 - 532
8*a^11*b^9*d^2*e^15 + 4032*a^13*b^7*d^2*e^15 + 3552*a^15*b^5*d^2*e^15 + 384
*a^17*b^3*d^2*e^15))/(b^17*d^4 + a^16*b*d^4 + 8*a^2*b^15*d^4 + 28*a^4*b^13*
d^4 + 56*a^6*b^11*d^4 + 70*a^8*b^9*d^4 + 56*a^10*b^7*d^4 + 28*a^12*b^5*d^4

+ 8*a^14*b^3*d^4))*(-e^5/(4*(b^6*d^2*1i - a^6*d^2*1i + 6*a*b^5*d^2 + 6*a^5*
b*d^2 - a^2*b^4*d^2*15i - 20*a^3*b^3*d^2 + a^4*b^2*d^2*15i)))^(1/2))*(-e^5/
(4*(b^6*d^2*1i - a^6*d^2*1i + 6*a*b^5*d^2 + 6*a^5*b*d^2 - a^2*b^4*d^2*15i -
20*a^3*b^3*d^2 + a^4*b^2*d^2*15i)))^(1/2) - ((e*cot(c + d*x))^(1/2)*(a^14*
e^20 - 32*b^14*e^20 + 97*a^2*b^12*e^20 - 2082*a^4*b^10*e^20 + 3631*a^6*b^8*
e^20 - 2300*a^8*b^6*e^20 + 79*a^10*b^4*e^20 + 30*a^12*b^2*e^20))/(b^17*d^4
+ a^16*b*d^4 + 8*a^2*b^15*d^4 + 28*a^4*b^13*d^4 + 56*a^6*b^11*d^4 + 70*a^8*
b^9*d^4 + 56*a^10*b^7*d^4 + 28*a^12*b^5*d^4 + 8*a^14*b^3*d^4))*(-e^5/(4*(b^
6*d^2*1i - a^6*d^2*1i + 6*a*b^5*d^2 + 6*a^5*b*d^2 - a^2*b^4*d^2*15i - 20*a^
3*b^3*d^2 + a^4*b^2*d^2*15i)))^(1/2)*1i)/((((10*a^16*b*d^2*e^18 - 2398*a^2*
b^15*d^2*e^18 + 5238*a^4*b^13*d^2*e^18 + 7386*a^6*b^11*d^2*e^18 - 8322*a^8*
b^9*d^2*e^18 - 5498*a^10*b^7*d^2*e^18 + 2946*a^12*b^5*d^2*e^18 + 382*a^14*b
^3*d^2*e^18)/(b^17*d^5 + a^16*b*d^5 + 8*a^2*b^15*d^5 + 28*a^4*b^13*d^5 + 56
*a^6*b^11*d^5 + 70*a^8*b^9*d^5 + 56*a^10*b^7*d^5 + 28*a^12*b^5*d^5 + 8*a^14
*b^3*d^5) - (((832*a*b^22*d^4*e^13 + 5952*a^3*b^20*d^4*e^13 + 17664*a^5*b^1
8*d^4*e^13 + 26880*a^7*b^16*d^4*e^13 + 18816*a^9*b^14*d^4*e^13 - 2688*a^11*
b^12*d^4*e^13 - 16128*a^13*b^10*d^4*e^13 - 13056*a^15*b^8*d^4*e^13 - 4800*a
^17*b^6*d^4*e^13 - 704*a^19*b^4*d^4*e^13)/(b^17*d^5 + a^16*b*d^5 + 8*a^2*b^
15*d^5 + 28*a^4*b^13*d^5 + 56*a^6*b^11*d^5 + 70*a^8*b^9*d^5 + 56*a^10*b^7*d
^5 + 28*a^12*b^5*d^5 + 8*a^14*b^3*d^5) + ((e*cot(c + d*x))^(1/2)*(-e^5/(4*(
b^6*d^2*1i - a^6*d^2*1i + 6*a*b^5*d^2 + 6*a^5*b*d^2 - a^2*b^4*d^2*15i - 20*
a^3*b^3*d^2 + a^4*b^2*d^2*15i)))^(1/2)*(512*b^26*d^4*e^10 + 4608*a^2*b^24*d
^4*e^10 + 17920*a^4*b^22*d^4*e^10 + 38400*a^6*b^20*d^4*e^10 + 46080*a^8*b^1
8*d^4*e^10 + 21504*a^10*b^16*d^4*e^10 - 21504*a^12*b^14*d^4*e^10 - 46080*a^
14*b^12*d^4*e^10 - 38400*a^16*b^10*d^4*e^10 - 17920*a^18*b^8*d^4*e^10 - 460
8*a^20*b^6*d^4*e^10 - 512*a^22*b^4*d^4*e^10))/(b^17*d^4 + a^16*b*d^4 + 8*a^
2*b^15*d^4 + 28*a^4*b^13*d^4 + 56*a^6*b^11*d^4 + 70*a^8*b^9*d^4 + 56*a^10*b
^7*d^4 + 28*a^12*b^5*d^4 + 8*a^14*b^3*d^4))*(-e^5/(4*(b^6*d^2*1i - a^6*d^2*
1i + 6*a*b^5*d^2 + 6*a^5*b*d^2 - a^2*b^4*d^2*15i - 20*a^3*b^3*d^2 + a^4*b^2
*d^2*15i)))^(1/2) - ((e*cot(c + d*x))^(1/2)*(8*a^19*b*d^2*e^15 - 1472*a*b^1
9*d^2*e^15 + 776*a^3*b^17*d^2*e^15 + 11328*a^5*b^15*d^2*e^15 + 10208*a^7*b^
13*d^2*e^15 - 5056*a^9*b^11*d^2*e^15 - 5328*a^11*b^9*d^2*e^15 + 4032*a^13*b
^7*d^2*e^15 + 3552*a^15*b^5*d^2*e^15 + 384*a^17*b^3*d^2*e^15))/(b^17*d^4 +
a^16*b*d^4 + 8*a^2*b^15*d^4 + 28*a^4*b^13*d^4 + 56*a^6*b^11*d^4 + 70*a^8*b^
9*d^4 + 56*a^10*b^7*d^4 + 28*a^12*b^5*d^4 + 8*a^14*b^3*d^4))*(-e^5/(4*(b^6*
d^2*1i - a^6*d^2*1i + 6*a*b^5*d^2 + 6*a^5*b*d^2 - a^2*b^4*d^2*15i - 20*a^3*
b^3*d^2 + a^4*b^2*d^2*15i)))^(1/2))*(-e^5/(4*(b^6*d^2*1i - a^6*d^2*1i + 6*a
*b^5*d^2 + 6*a^5*b*d^2 - a^2*b^4*d^2*15i - 20*a^3*b^3*d^2 + a^4*b^2*d^2*15i
)))^(1/2) + ((e*cot(c + d*x))^(1/2)*(a^14*e^20 - 32*b^14*e^20 + 97*a^2*b^12
*e^20 - 2082*a^4*b^10*e^20 + 3631*a^6*b^8*e^20 - 2300*a^8*b^6*e^20 + 79*a^1
0*b^4*e^20 + 30*a^12*b^2*e^20))/(b^17*d^4 + a^16*b*d^4 + 8*a^2*b^15*d^4 + 2
8*a^4*b^13*d^4 + 56*a^6*b^11*d^4 + 70*a^8*b^9*d^4 + 56*a^10*b^7*d^4 + 28*a^
12*b^5*d^4 + 8*a^14*b^3*d^4))*(-e^5/(4*(b^6*d^2*1i - a^6*d^2*1i + 6*a*b^5*d
^2 + 6*a^5*b*d^2 - a^2*b^4*d^2*15i - 20*a^3*b^3*d^2 + a^4*b^2*d^2*15i)))^(1
/2) + (((10*a^16*b*d^2*e^18 - 2398*a^2*b^15*d^2*e^18 + 5238*a^4*b^13*d^2*e^

18 + 7386*a^6*b^11*d^2*e^18 - 8322*a^8*b^9*d^2*e^18 - 5498*a^10*b^7*d^2*e^1
8 + 2946*a^12*b^5*d^2*e^18 + 382*a^14*b^3*d^2*e^18)/(b^17*d^5 + a^16*b*d^5
+ 8*a^2*b^15*d^5 + 28*a^4*b^13*d^5 + 56*a^6*b^11*d^5 + 70*a^8*b^9*d^5 + 56*
a^10*b^7*d^5 + 28*a^12*b^5*d^5 + 8*a^14*b^3*d^5) - (((832*a*b^22*d^4*e^13 +
5952*a^3*b^20*d^4*e^13 + 17664*a^5*b^18*d^4*e^13 + 26880*a^7*b^16*d^4*e^13
+ 18816*a^9*b^14*d^4*e^13 - 2688*a^11*b^12*d^4*e^13 - 16128*a^13*b^10*d^4*
e^13 - 13056*a^15*b^8*d^4*e^13 - 4800*a^17*b^6*d^4*e^13 - 704*a^19*b^4*d^4*
e^13)/(b^17*d^5 + a^16*b*d^5 + 8*a^2*b^15*d^5 + 28*a^4*b^13*d^5 + 56*a^6*b^
11*d^5 + 70*a^8*b^9*d^5 + 56*a^10*b^7*d^5 + 28*a^12*b^5*d^5 + 8*a^14*b^3*d^
5) - ((e*cot(c + d*x))^(1/2)*(-e^5/(4*(b^6*d^2*1i - a^6*d^2*1i + 6*a*b^5*d^
2 + 6*a^5*b*d^2 - a^2*b^4*d^2*15i - 20*a^3*b^3*d^2 + a^4*b^2*d^2*15i)))^(1/
2)*(512*b^26*d^4*e^10 + 4608*a^2*b^24*d^4*e^10 + 17920*a^4*b^22*d^4*e^10 +
38400*a^6*b^20*d^4*e^10 + 46080*a^8*b^18*d^4*e^10 + 21504*a^10*b^16*d^4*e^1
0 - 21504*a^12*b^14*d^4*e^10 - 46080*a^14*b^12*d^4*e^10 - 38400*a^16*b^10*d
^4*e^10 - 17920*a^18*b^8*d^4*e^10 - 4608*a^20*b^6*d^4*e^10 - 512*a^22*b^4*d
^4*e^10))/(b^17*d^4 + a^16*b*d^4 + 8*a^2*b^15*d^4 + 28*a^4*b^13*d^4 + 56*a^
6*b^11*d^4 + 70*a^8*b^9*d^4 + 56*a^10*b^7*d^4 + 28*a^12*b^5*d^4 + 8*a^14*b^
3*d^4))*(-e^5/(4*(b^6*d^2*1i - a^6*d^2*1i + 6*a*b^5*d^2 + 6*a^5*b*d^2 - a^2
*b^4*d^2*15i - 20*a^3*b^3*d^2 + a^4*b^2*d^2*15i)))^(1/2) + ((e*cot(c + d*x)
)^(1/2)*(8*a^19*b*d^2*e^15 - 1472*a*b^19*d^2*e^15 + 776*a^3*b^17*d^2*e^15 +
11328*a^5*b^15*d^2*e^15 + 10208*a^7*b^13*d^2*e^15 - 5056*a^9*b^11*d^2*e^15
- 5328*a^11*b^9*d^2*e^15 + 4032*a^13*b^7*d^2*e^15 + 3552*a^15*b^5*d^2*e^15
+ 384*a^17*b^3*d^2*e^15))/(b^17*d^4 + a^16*b*d^4 + 8*a^2*b^15*d^4 + 28*a^4
*b^13*d^4 + 56*a^6*b^11*d^4 + 70*a^8*b^9*d^4 + 56*a^10*b^7*d^4 + 28*a^12*b^
5*d^4 + 8*a^14*b^3*d^4))*(-e^5/(4*(b^6*d^2*1i - a^6*d^2*1i + 6*a*b^5*d^2 +
6*a^5*b*d^2 - a^2*b^4*d^2*15i - 20*a^3*b^3*d^2 + a^4*b^2*d^2*15i)))^(1/2))*
(-e^5/(4*(b^6*d^2*1i - a^6*d^2*1i + 6*a*b^5*d^2 + 6*a^5*b*d^2 - a^2*b^4*d^2
*15i - 20*a^3*b^3*d^2 + a^4*b^2*d^2*15i)))^(1/2) - ((e*cot(c + d*x))^(1/2)*
(a^14*e^20 - 32*b^14*e^20 + 97*a^2*b^12*e^20 - 2082*a^4*b^10*e^20 + 3631*a^
6*b^8*e^20 - 2300*a^8*b^6*e^20 + 79*a^10*b^4*e^20 + 30*a^12*b^2*e^20))/(b^1
7*d^4 + a^16*b*d^4 + 8*a^2*b^15*d^4 + 28*a^4*b^13*d^4 + 56*a^6*b^11*d^4 + 7
0*a^8*b^9*d^4 + 56*a^10*b^7*d^4 + 28*a^12*b^5*d^4 + 8*a^14*b^3*d^4))*(-e^5/
(4*(b^6*d^2*1i - a^6*d^2*1i + 6*a*b^5*d^2 + 6*a^5*b*d^2 - a^2*b^4*d^2*15i -
20*a^3*b^3*d^2 + a^4*b^2*d^2*15i)))^(1/2) + (a^11*e^23 - 120*a*b^10*e^23 +
249*a^3*b^8*e^23 - 388*a^5*b^6*e^23 + 302*a^7*b^4*e^23 + 36*a^9*b^2*e^23)/
(b^17*d^5 + a^16*b*d^5 + 8*a^2*b^15*d^5 + 28*a^4*b^13*d^5 + 56*a^6*b^11*d^5
+ 70*a^8*b^9*d^5 + 56*a^10*b^7*d^5 + 28*a^12*b^5*d^5 + 8*a^14*b^3*d^5)))*(
-e^5/(4*(b^6*d^2*1i - a^6*d^2*1i + 6*a*b^5*d^2 + 6*a^5*b*d^2 - a^2*b^4*d^2*
15i - 20*a^3*b^3*d^2 + a^4*b^2*d^2*15i)))^(1/2)*2i + (atan((((((e*cot(c + d
x))^(1/2)(a^14*e^20 - 32*b^14*e^20 + 97*a^2*b^12*e^20 - 2082*a^4*b^10*e^2
0 + 3631*a^6*b^8*e^20 - 2300*a^8*b^6*e^20 + 79*a^10*b^4*e^20 + 30*a^12*b^2*
e^20))/(b^17*d^4 + a^16*b*d^4 + 8*a^2*b^15*d^4 + 28*a^4*b^13*d^4 + 56*a^6*b
^11*d^4 + 70*a^8*b^9*d^4 + 56*a^10*b^7*d^4 + 28*a^12*b^5*d^4 + 8*a^14*b^3*d
^4) - (((10*a^16*b*d^2*e^18 - 2398*a^2*b^15*d^2*e^18 + 5238*a^4*b^13*d^2*e^
18 + 7386*a^6*b^11*d^2*e^18 - 8322*a^8*b^9*d^2*e^18 - 5498*a^10*b^7*d^2*e^1

8 + 2946*a^12*b^5*d^2*e^18 + 382*a^14*b^3*d^2*e^18)/(b^17*d^5 + a^16*b*d^5 + 8*a^2*b^15*d^5 + 28*a^4*b^13*d^5 + 56*a^6*b^11*d^5 + 70*a^8*b^9*d^5 + 56*a^10*b^7*d^5 + 28*a^12*b^5*d^5 + 8*a^14*b^3*d^5) - ((((e*cot(c + d*x))^(1/2)*(8*a^19*b*d^2*e^15 - 1472*a*b^19*d^2*e^15 + 776*a^3*b^17*d^2*e^15 + 11328*a^5*b^15*d^2*e^15 + 10208*a^7*b^13*d^2*e^15 - 5056*a^9*b^11*d^2*e^15 - 5328*a^11*b^9*d^2*e^15 + 4032*a^13*b^7*d^2*e^15 + 3552*a^15*b^5*d^2*e^15 + 384*a^17*b^3*d^2*e^15))/(b^17*d^4 + a^16*b*d^4 + 8*a^2*b^15*d^4 + 28*a^4*b^13*d^4 + 56*a^6*b^11*d^4 + 70*a^8*b^9*d^4 + 56*a^10*b^7*d^4 + 28*a^12*b^5*d^4 + 8*a^14*b^3*d^4) + (((832*a*b^22*d^4*e^13 + 5952*a^3*b^20*d^4*e^13 + 17664*a^5*b^18*d^4*e^13 + 26880*a^7*b^16*d^4*e^13 + 18816*a^9*b^14*d^4*e^13 - 2688*a^11*b^12*d^4*e^13 - 16128*a^13*b^10*d^4*e^13 - 13056*a^15*b^8*d^4*e^13 - 4800*a^17*b^6*d^4*e^13 - 704*a^19*b^4*d^4*e^13)/(b^17*d^5 + a^16*b*d^5 + 8*a^2*b^15*d^5 + 28*a^4*b^13*d^5 + 56*a^6*b^11*d^5 + 70*a^8*b^9*d^5 + 56*a^10*b^7*d^5 + 28*a^12*b^5*d^5 + 8*a^14*b^3*d^5) - ((e*cot(c + d*x))^(1/2)*(a^4 - 15*b^4 + 18*a^2*b^2)*(-a*b^3*e^5)^(1/2)*(512*b^26*d^4*e^10 + 4608*a^2*b^24*d^4*e^10 + 17920*a^4*b^22*d^4*e^10 + 38400*a^6*b^20*d^4*e^10 + 46080*a^8*b^18*d^4*e^10 + 21504*a^10*b^16*d^4*e^10 - 21504*a^12*b^14*d^4*e^10 - 46080*a^14*b^12*d^4*e^10 - 38400*a^16*b^10*d^4*e^10 - 17920*a^18*b^8*d^4*e^10 - 4608*a^20*b^6*d^4*e^10 - 512*a^22*b^4*d^4*e^10))/(8*(b^9*d + 3*a^2*b^7*d + 3*a^4*b^5*d + a^6*b^3*d)*(b^17*d^4 + a^16*b*d^4 + 8*a^2*b^15*d^4 + 28*a^4*b^13*d^4 + 56*a^6*b^11*d^4 + 70*a^8*b^9*d^4 + 56*a^10*b^7*d^4 + 28*a^12*b^5*d^4 + 8*a^14*b^3*d^4)))*(a^4 - 15*b^4 + 18*a^2*b^2)*(-a*b^3*e^5)^(1/2))/(8*(b^9*d + 3*a^2*b^7*d + 3*a^4*b^5*d + a^6*b^3*d)))*(a^4 - 15*b^4 + 18*a^2*b^2)*(-a*b^3*e^5)^(1/2))/(8*(b^9*d + 3*a^2*b^7*d + 3*a^4*b^5*d + a^6*b^3*d)))*(a^4 - 15*b^4 + 18*a^2*b^2)*(-a*b^3*e^5)^(1/2))/(8*(b^9*d + 3*a^2*b^7*d + 3*a^4*b^5*d + a^6*b^3*d)))*(a^4 - 15*b^4 + 18*a^2*b^2)*(-a*b^3*e^5)^(1/2)*1i)/(8*(b^9*d + 3*a^2*b^7*d + 3*a^4*b^5*d + a^6*b^3*d)) + ((((e*cot(c + d*x))^(1/2)*(a^14*e^20 - 32*b^14*e^20 + 97*a^2*b^12*e^20 - 2082*a^4*b^10*e^20 + 3631*a^6*b^8*e^20 - 2300*a^8*b^6*e^20 + 79*a^10*b^4*e^20 + 30*a^12*b^2*e^20))/(b^17*d^4 + a^16*b*d^4 + 8*a^2*b^15*d^4 + 28*a^4*b^13*d^4 + 56*a^6*b^11*d^4 + 70*a^8*b^9*d^4 + 56*a^10*b^7*d^4 + 28*a^12*b^5*d^4 + 8*a^14*b^3*d^4) + (((10*a^16*b*d^2*e^18 - 2398*a^2*b^15*d^2*e^18 + 5238*a^4*b^13*d^2*e^18 + 7386*a^6*b^11*d^2*e^18 - 8322*a^8*b^9*d^2*e^18 - 5498*a^10*b^7*d^2*e^18 + 2946*a^12*b^5*d^2*e^18 + 382*a^14*b^3*d^2*e^18)/(b^17*d^5 + a^16*b*d^5 + 8*a^2*b^15*d^5 + 28*a^4*b^13*d^5 + 56*a^6*b^11*d^5 + 70*a^8*b^9*d^5 + 56*a^10*b^7*d^5 + 28*a^12*b^5*d^5 + 8*a^14*b^3*d^5) + ((((e*cot(c + d*x))^(1/2)*(8*a^19*b*d^2*e^15 - 1472*a*b^19*d^2*e^15 + 776*a^3*b^17*d^2*e^15 + 11328*a^5*b^15*d^2*e^15 + 10208*a^7*b^13*d^2*e^15 - 5056*a^9*b^11*d^2*e^15 - 5328*a^11*b^9*d^2*e^15 + 4032*a^13*b^7*d^2*e^15 + 3552*a^15*b^5*d^2*e^15 + 384*a^17*b^3*d^2*e^15))/(b^17*d^4 + a^16*b*d^4 + 8*a^2*b^15*d^4 + 28*a^4*b^13*d^4 + 56*a^6*b^11*d^4 + 70*a^8*b^9*d^4 + 56*a^10*b^7*d^4 + 28*a^12*b^5*d^4 + 8*a^14*b^3*d^4) - (((832*a*b^22*d^4*e^13 + 5952*a^3*b^20*d^4*e^13 + 17664*a^5*b^18*d^4*e^13 + 26880*a^7*b^16*d^4*e^13 + 18816*a^9*b^14*d^4*e^13 - 2688*a^11*b^12*d^4*e^13 - 16128*a^13*b^10*d^4*e^13 - 13056*a^15*b^8*d^4*e^13 - 4800*a^17*b^6*d^4*e^13 - 704*a^19*b^4*d^4*e^13)/(b^17*d^5 + a^16*b*d^5 +

8*a^2*b^15*d^5 + 28*a^4*b^13*d^5 + 56*a^6*b^11*d^5 + 70*a^8*b^9*d^5 + 56*a^10*b^7*d^5 + 28*a^12*b^5*d^5 + 8*a^14*b^3*d^5) + ((e*cot(c + d*x))^(1/2)*(a^4 - 15*b^4 + 18*a^2*b^2)*(-a*b^3*e^5)^(1/2)*(512*b^26*d^4*e^10 + 4608*a^2*b^24*d^4*e^10 + 17920*a^4*b^22*d^4*e^10 + 38400*a^6*b^20*d^4*e^10 + 46080*a^8*b^18*d^4*e^10 + 21504*a^10*b^16*d^4*e^10 - 21504*a^12*b^14*d^4*e^10 - 46080*a^14*b^12*d^4*e^10 - 38400*a^16*b^10*d^4*e^10 - 17920*a^18*b^8*d^4*e^10 - 4608*a^20*b^6*d^4*e^10 - 512*a^22*b^4*d^4*e^10))/(8*(b^9*d + 3*a^2*b^7*d + 3*a^4*b^5*d + a^6*b^3*d)*(b^17*d^4 + a^16*b*d^4 + 8*a^2*b^15*d^4 + 28*a^4*b^13*d^4 + 56*a^6*b^11*d^4 + 70*a^8*b^9*d^4 + 56*a^10*b^7*d^4 + 28*a^12*b^5*d^4 + 8*a^14*b^3*d^4)))*(a^4 - 15*b^4 + 18*a^2*b^2)*(-a*b^3*e^5)^(1/2))/(8*(b^9*d + 3*a^2*b^7*d + 3*a^4*b^5*d + a^6*b^3*d)))*(a^4 - 15*b^4 + 18*a^2*b^2)*(-a*b^3*e^5)^(1/2))/(8*(b^9*d + 3*a^2*b^7*d + 3*a^4*b^5*d + a^6*b^3*d)))*(a^4 - 15*b^4 + 18*a^2*b^2)*(-a*b^3*e^5)^(1/2))/(8*(b^9*d + 3*a^2*b^7*d + 3*a^4*b^5*d + a^6*b^3*d)))*(a^4 - 15*b^4 + 18*a^2*b^2)*(-a*b^3*e^5)^(1/2)*1i)/(8*(b^9*d + 3*a^2*b^7*d + 3*a^4*b^5*d + a^6*b^3*d)))/((a^11*e^23 - 120*a*b^10*e^23 + 249*a^3*b^8*e^23 - 388*a^5*b^6*e^23 + 302*a^7*b^4*e^23 + 36*a^9*b^2*e^23)/(b^17*d^5 + a^16*b*d^5 + 8*a^2*b^15*d^5 + 28*a^4*b^13*d^5 + 56*a^6*b^11*d^5 + 70*a^8*b^9*d^5 + 56*a^10*b^7*d^5 + 28*a^12*b^5*d^5 + 8*a^14*b^3*d^5) - ((((e*cot(c + d*x))^(1/2)*(a^14*e^20 - 32*b^14*e^20 + 97*a^2*b^12*e^20 - 2082*a^4*b^10*e^20 + 3631*a^6*b^8*e^20 - 2300*a^8*b^6*e^20 + 79*a^10*b^4*e^20 + 30*a^12*b^2*e^20))/(b^17*d^4 + a^16*b*d^4 + 8*a^2*b^15*d^4 + 28*a^4*b^13*d^4 + 56*a^6*b^11*d^4 + 70*a^8*b^9*d^4 + 56*a^10*b^7*d^4 + 28*a^12*b^5*d^4 + 8*a^14*b^3*d^4) - (((10*a^16*b*d^2*e^18 - 2398*a^2*b^15*d^2*e^18 + 5238*a^4*b^13*d^2*e^18 + 7386*a^6*b^11*d^2*e^18 - 8322*a^8*b^9*d^2*e^18 - 5498*a^10*b^7*d^2*e^18 + 2946*a^12*b^5*d^2*e^18 + 382*a^14*b^3*d^2*e^18)/(b^17*d^5 + a^16*b*d^5 + 8*a^2*b^15*d^5 + 28*a^4*b^13*d^5 + 56*a^6*b^11*d^5 + 70*a^8*b^9*d^5 + 56*a^10*b^7*d^5 + 28*a^12*b^5*d^5 + 8*a^14*b^3*d^5) - ((((e*cot(c + d*x))^(1/2)*(8*a^19*b*d^2*e^15 - 1472*a*b^19*d^2*e^15 + 776*a^3*b^17*d^2*e^15 + 11328*a^5*b^15*d^2*e^15 + 10208*a^7*b^13*d^2*e^15 - 5056*a^9*b^11*d^2*e^15 - 5328*a^11*b^9*d^2*e^15 + 4032*a^13*b^7*d^2*e^15 + 3552*a^15*b^5*d^2*e^15 + 384*a^17*b^3*d^2*e^15))/(b^17*d^4 + a^16*b*d^4 + 8*a^2*b^15*d^4 + 28*a^4*b^13*d^4 + 56*a^6*b^11*d^4 + 70*a^8*b^9*d^4 + 56*a^10*b^7*d^4 + 28*a^12*b^5*d^4 + 8*a^14*b^3*d^4) + (((832*a*b^22*d^4*e^13 + 5952*a^3*b^20*d^4*e^13 + 17664*a^5*b^18*d^4*e^13 + 26880*a^7*b^16*d^4*e^13 + 18816*a^9*b^14*d^4*e^13 - 2688*a^11*b^12*d^4*e^13 - 16128*a^13*b^10*d^4*e^13 - 13056*a^15*b^8*d^4*e^13 - 4800*a^17*b^6*d^4*e^13 - 704*a^19*b^4*d^4*e^13)/(b^17*d^5 + a^16*b*d^5 + 8*a^2*b^15*d^5 + 28*a^4*b^13*d^5 + 56*a^6*b^11*d^5 + 70*a^8*b^9*d^5 + 56*a^10*b^7*d^5 + 28*a^12*b^5*d^5 + 8*a^14*b^3*d^5) - ((e*cot(c + d*x))^(1/2)*(a^4 - 15*b^4 + 18*a^2*b^2)*(-a*b^3*e^5)^(1/2)*(512*b^26*d^4*e^10 + 4608*a^2*b^24*d^4*e^10 + 17920*a^4*b^22*d^4*e^10 + 38400*a^6*b^20*d^4*e^10 + 46080*a^8*b^18*d^4*e^10 + 21504*a^10*b^16*d^4*e^10 - 21504*a^12*b^14*d^4*e^10 - 46080*a^14*b^12*d^4*e^10 - 38400*a^16*b^10*d^4*e^10 - 17920*a^18*b^8*d^4*e^10 - 4608*a^20*b^6*d^4*e^10 - 512*a^22*b^4*d^4*e^10))/(8*(b^9*d + 3*a^2*b^7*d + 3*a^4*b^5*d + a^6*b^3*d)*(b^17*d^4 + a^16*b*d^4 + 8*a^2*b^15*d^4 + 28*a^4*b^13*d^4 + 56*a^6*b^11*d^4 + 70*a^8*b^9*d^4

+ 56*a^10*b^7*d^4 + 28*a^12*b^5*d^4 + 8*a^14*b^3*d^4)))*(a^4 - 15*b^4 + 18*
a^2*b^2)*(-a*b^3*e^5)^(1/2))/(8*(b^9*d + 3*a^2*b^7*d + 3*a^4*b^5*d + a^6*b^
3*d)))*(a^4 - 15*b^4 + 18*a^2*b^2)*(-a*b^3*e^5)^(1/2))/(8*(b^9*d + 3*a^2*b^
7*d + 3*a^4*b^5*d + a^6*b^3*d)))*(a^4 - 15*b^4 + 18*a^2*b^2)*(-a*b^3*e^5)^(
1/2))/(8*(b^9*d + 3*a^2*b^7*d + 3*a^4*b^5*d + a^6*b^3*d)))*(a^4 - 15*b^4 +
18*a^2*b^2)*(-a*b^3*e^5)^(1/2))/(8*(b^9*d + 3*a^2*b^7*d + 3*a^4*b^5*d + a^6
*b^3*d)) + ((((e*cot(c + d*x))^(1/2)*(a^14*e^20 - 32*b^14*e^20 + 97*a^2*b^1
2*e^20 - 2082*a^4*b^10*e^20 + 3631*a^6*b^8*e^20 - 2300*a^8*b^6*e^20 + 79*a^
10*b^4*e^20 + 30*a^12*b^2*e^20))/(b^17*d^4 + a^16*b*d^4 + 8*a^2*b^15*d^4 +
28*a^4*b^13*d^4 + 56*a^6*b^11*d^4 + 70*a^8*b^9*d^4 + 56*a^10*b^7*d^4 + 28*a
^12*b^5*d^4 + 8*a^14*b^3*d^4) + (((10*a^16*b*d^2*e^18 - 2398*a^2*b^15*d^2*e
^18 + 5238*a^4*b^13*d^2*e^18 + 7386*a^6*b^11*d^2*e^18 - 8322*a^8*b^9*d^2*e^
18 - 5498*a^10*b^7*d^2*e^18 + 2946*a^12*b^5*d^2*e^18 + 382*a^14*b^3*d^2*e^1
8)/(b^17*d^5 + a^16*b*d^5 + 8*a^2*b^15*d^5 + 28*a^4*b^13*d^5 + 56*a^6*b^11*
d^5 + 70*a^8*b^9*d^5 + 56*a^10*b^7*d^5 + 28*a^12*b^5*d^5 + 8*a^14*b^3*d^5)
+ ((((e*cot(c + d*x))^(1/2)*(8*a^19*b*d^2*e^15 - 1472*a*b^19*d^2*e^15 + 776
*a^3*b^17*d^2*e^15 + 11328*a^5*b^15*d^2*e^15 + 10208*a^7*b^13*d^2*e^15 - 50
56*a^9*b^11*d^2*e^15 - 5328*a^11*b^9*d^2*e^15 + 4032*a^13*b^7*d^2*e^15 + 35
52*a^15*b^5*d^2*e^15 + 384*a^17*b^3*d^2*e^15))/(b^17*d^4 + a^16*b*d^4 + 8*a
^2*b^15*d^4 + 28*a^4*b^13*d^4 + 56*a^6*b^11*d^4 + 70*a^8*b^9*d^4 + 56*a^10*
b^7*d^4 + 28*a^12*b^5*d^4 + 8*a^14*b^3*d^4) - (((832*a*b^22*d^4*e^13 + 5952
*a^3*b^20*d^4*e^13 + 17664*a^5*b^18*d^4*e^13 + 26880*a^7*b^16*d^4*e^13 + 18
816*a^9*b^14*d^4*e^13 - 2688*a^11*b^12*d^4*e^13 - 16128*a^13*b^10*d^4*e^13
- 13056*a^15*b^8*d^4*e^13 - 4800*a^17*b^6*d^4*e^13 - 704*a^19*b^4*d^4*e^13)
/(b^17*d^5 + a^16*b*d^5 + 8*a^2*b^15*d^5 + 28*a^4*b^13*d^5 + 56*a^6*b^11*d^
5 + 70*a^8*b^9*d^5 + 56*a^10*b^7*d^5 + 28*a^12*b^5*d^5 + 8*a^14*b^3*d^5) +
((e*cot(c + d*x))^(1/2)*(a^4 - 15*b^4 + 18*a^2*b^2)*(-a*b^3*e^5)^(1/2)*(512
*b^26*d^4*e^10 + 4608*a^2*b^24*d^4*e^10 + 17920*a^4*b^22*d^4*e^10 + 38400*a
^6*b^20*d^4*e^10 + 46080*a^8*b^18*d^4*e^10 + 21504*a^10*b^16*d^4*e^10 - 215
04*a^12*b^14*d^4*e^10 - 46080*a^14*b^12*d^4*e^10 - 38400*a^16*b^10*d^4*e^10
- 17920*a^18*b^8*d^4*e^10 - 4608*a^20*b^6*d^4*e^10 - 512*a^22*b^4*d^4*e^10
))/(8*(b^9*d + 3*a^2*b^7*d + 3*a^4*b^5*d + a^6*b^3*d)*(b^17*d^4 + a^16*b*d^
4 + 8*a^2*b^15*d^4 + 28*a^4*b^13*d^4 + 56*a^6*b^11*d^4 + 70*a^8*b^9*d^4 + 5
6*a^10*b^7*d^4 + 28*a^12*b^5*d^4 + 8*a^14*b^3*d^4)))*(a^4 - 15*b^4 + 18*a^2
b^2)(-a*b^3*e^5)^(1/2))/(8*(b^9*d + 3*a^2*b^7*d + 3*a^4*b^5*d + a^6*b^3*d
)))*(a^4 - 15*b^4 + 18*a^2*b^2)*(-a*b^3*e^5)^(1/2))/(8*(b^9*d + 3*a^2*b^7*d
+ 3*a^4*b^5*d + a^6*b^3*d)))*(a^4 - 15*b^4 + 18*a^2*b^2)*(-a*b^3*e^5)^(1/2
))/(8*(b^9*d + 3*a^2*b^7*d + 3*a^4*b^5*d + a^6*b^3*d)))*(a^4 - 15*b^4 + 18*
a^2*b^2)*(-a*b^3*e^5)^(1/2))/(8*(b^9*d + 3*a^2*b^7*d + 3*a^4*b^5*d + a^6*b^
3*d))))*(a^4 - 15*b^4 + 18*a^2*b^2)*(-a*b^3*e^5)^(1/2)*1i)/(4*(b^9*d + 3*a^
2*b^7*d + 3*a^4*b^5*d + a^6*b^3*d))

3.84 $\int \frac{(e\cot(c+dx))^{3/2}}{(a+b\cot(c+dx))^3}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 461

$$
\begin{aligned}
\int \frac{(e\cot(c+dx))^{3/2}}{(a+b\cot(c+dx))^3}\,dx &= -\frac{(3a^4-26a^2b^2+3b^4)\,e^{3/2}\arctan\left(\frac{\sqrt{b}\sqrt{e\cot(c+dx)}}{\sqrt{a}\sqrt{e}}\right)}{4\sqrt{a}\sqrt{b}\,(a^2+b^2)^3\,d} \\
&- \frac{(a+b)\,(a^2-4ab+b^2)\,e^{3/2}\arctan\left(1-\frac{\sqrt{2}\sqrt{e\cot(c+dx)}}{\sqrt{e}}\right)}{\sqrt{2}\,(a^2+b^2)^3\,d} \\
&+ \frac{(a+b)\,(a^2-4ab+b^2)\,e^{3/2}\arctan\left(1+\frac{\sqrt{2}\sqrt{e\cot(c+dx)}}{\sqrt{e}}\right)}{\sqrt{2}\,(a^2+b^2)^3\,d} \\
&- \frac{ae\sqrt{e\cot(c+dx)}}{2\,(a^2+b^2)\,d(a+b\cot(c+dx))^2} - \frac{(3a^2-5b^2)\,e\sqrt{e\cot(c+dx)}}{4\,(a^2+b^2)^2\,d(a+b\cot(c+dx))} \\
&- \frac{(a-b)\,(a^2+4ab+b^2)\,e^{3/2}\log\left(\sqrt{e}+\sqrt{e}\cot(c+dx)-\sqrt{2}\sqrt{e\cot(c+dx)}\right)}{2\sqrt{2}\,(a^2+b^2)^3\,d} \\
&+ \frac{(a-b)\,(a^2+4ab+b^2)\,e^{3/2}\log\left(\sqrt{e}+\sqrt{e}\cot(c+dx)+\sqrt{2}\sqrt{e\cot(c+dx)}\right)}{2\sqrt{2}\,(a^2+b^2)^3\,d}
\end{aligned}
$$

```
[Out] -1/2*(a+b)*(a^2-4*a*b+b^2)*e^(3/2)*arctan(1-2^(1/2)*(e*cot(d*x+c))^(1/2)/e^
(1/2))/(a^2+b^2)^3/d*2^(1/2)+1/2*(a+b)*(a^2-4*a*b+b^2)*e^(3/2)*arctan(1+2^(
1/2)*(e*cot(d*x+c))^(1/2)/e^(1/2))/(a^2+b^2)^3/d*2^(1/2)-1/4*(a-b)*(a^2+4*a
*b+b^2)*e^(3/2)*ln(e^(1/2)+cot(d*x+c)*e^(1/2)-2^(1/2)*(e*cot(d*x+c))^(1/2))
/(a^2+b^2)^3/d*2^(1/2)+1/4*(a-b)*(a^2+4*a*b+b^2)*e^(3/2)*ln(e^(1/2)+cot(d*x
+c)*e^(1/2)+2^(1/2)*(e*cot(d*x+c))^(1/2))/(a^2+b^2)^3/d*2^(1/2)-1/4*(3*a^4-
26*a^2*b^2+3*b^4)*e^(3/2)*arctan(b^(1/2)*(e*cot(d*x+c))^(1/2)/a^(1/2)/e^(1/
2))/(a^2+b^2)^3/d/a^(1/2)/b^(1/2)-1/2*a*e*(e*cot(d*x+c))^(1/2)/(a^2+b^2)/d/
(a+b*cot(d*x+c))^2-1/4*(3*a^2-5*b^2)*e*(e*cot(d*x+c))^(1/2)/(a^2+b^2)^2/d/(
a+b*cot(d*x+c))
```

Rubi [A] (verified)

Time = 1.30 (sec) , antiderivative size = 461, normalized size of antiderivative = 1.00, number of steps used = 16, number of rules used = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.520, Rules used = {3648, 3730, 3734, 3615, 1182, 1176, 631, 210, 1179, 642, 3715, 65, 211}

$$\int \frac{(e\cot(c+dx))^{3/2}}{(a+b\cot(c+dx))^3}\,dx = -\frac{e^{3/2}(a+b)\left(a^2-4ab+b^2\right)\arctan\left(1-\frac{\sqrt{2}\sqrt{e\cot(c+dx)}}{\sqrt{e}}\right)}{\sqrt{2}d\left(a^2+b^2\right)^3} + \frac{e^{3/2}(a+b)\left(a^2-4ab+b^2\right)\arctan\left(\frac{\sqrt{2}\sqrt{e\cot(c+dx)}}{\sqrt{e}}+1\right)}{\sqrt{2}d\left(a^2+b^2\right)^3} - \frac{e^{3/2}(a-b)\left(a^2+4ab+b^2\right)\log\left(\sqrt{e}\cot(c+dx)-\sqrt{2}\sqrt{e\cot(c+dx)}+\sqrt{e}\right)}{2\sqrt{2}d\left(a^2+b^2\right)^3} + \frac{e^{3/2}(a-b)\left(a^2+4ab+b^2\right)\log\left(\sqrt{e}\cot(c+dx)+\sqrt{2}\sqrt{e\cot(c+dx)}+\sqrt{e}\right)}{2\sqrt{2}d\left(a^2+b^2\right)^3} - \frac{e(3a^2-5b^2)\sqrt{e\cot(c+dx)}}{4d\left(a^2+b^2\right)^2(a+b\cot(c+dx))} - \frac{ae\sqrt{e\cot(c+dx)}}{2d\left(a^2+b^2\right)(a+b\cot(c+dx))^2} - \frac{e^{3/2}\left(3a^4-26a^2b^2+3b^4\right)\arctan\left(\frac{\sqrt{b}\sqrt{e\cot(c+dx)}}{\sqrt{a}\sqrt{e}}\right)}{4\sqrt{a}\sqrt{b}d\left(a^2+b^2\right)^3}$$

```
[In] Int[(e*Cot[c + d*x])^(3/2)/(a + b*Cot[c + d*x])^3,x]
```

```
[Out] -1/4*((3*a^4 - 26*a^2*b^2 + 3*b^4)*e^(3/2)*ArcTan[(Sqrt[b]*Sqrt[e*Cot[c + d
*x]])/(Sqrt[a]*Sqrt[e])])/(Sqrt[a]*Sqrt[b]*(a^2 + b^2)^3*d) - ((a + b)*(a^2
 - 4*a*b + b^2)*e^(3/2)*ArcTan[1 - (Sqrt[2]*Sqrt[e*Cot[c + d*x]])/Sqrt[e]])
/(Sqrt[2]*(a^2 + b^2)^3*d) + ((a + b)*(a^2 - 4*a*b + b^2)*e^(3/2)*ArcTan[1
+ (Sqrt[2]*Sqrt[e*Cot[c + d*x]])/Sqrt[e]])/(Sqrt[2]*(a^2 + b^2)^3*d) - (a*e
*Sqrt[e*Cot[c + d*x]])/(2*(a^2 + b^2)*d*(a + b*Cot[c + d*x])^2) - ((3*a^2 -
 5*b^2)*e*Sqrt[e*Cot[c + d*x]])/(4*(a^2 + b^2)^2*d*(a + b*Cot[c + d*x])) -
((a - b)*(a^2 + 4*a*b + b^2)*e^(3/2)*Log[Sqrt[e] + Sqrt[e]*Cot[c + d*x] - S
qrt[2]*Sqrt[e*Cot[c + d*x]]])/(2*Sqrt[2]*(a^2 + b^2)^3*d) + ((a - b)*(a^2 +
 4*a*b + b^2)*e^(3/2)*Log[Sqrt[e] + Sqrt[e]*Cot[c + d*x] + Sqrt[2]*Sqrt[e*C
ot[c + d*x]]])/(2*Sqrt[2]*(a^2 + b^2)^3*d)
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
```

```
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1179

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 1182

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
a*c, 2]}, Dist[(d*q + a*e)/(2*a*c), Int[(q + c*x^2)/(a + c*x^4), x], x] + D
ist[(d*q - a*e)/(2*a*c), Int[(q - c*x^2)/(a + c*x^4), x], x]] /; FreeQ[{a,
c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0] && NeQ[c*d^2 - a*e^2, 0] && NegQ[(-a)
```

```
*c]
```

Rule 3615

```
Int[((c_) + (d_.)*tan[(e_.) + (f_.)*(x_)])/Sqrt[(b_.)*tan[(e_.) + (f_.)*(x_
)]], x_Symbol] :> Dist[2/f, Subst[Int[(b*c + d*x^2)/(b^2 + x^4), x], x, Sqr
t[b*Tan[e + f*x]]], x] /; FreeQ[{b, c, d, e, f}, x] && NeQ[c^2 - d^2, 0] &&
 NeQ[c^2 + d^2, 0]
```

Rule 3648

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*c - a*d)*(a + b*Tan[e + f*x])^(m +
 1)*((c + d*Tan[e + f*x])^(n - 1)/(f*(m + 1)*(a^2 + b^2))), x] + Dist[1/((m
 + 1)*(a^2 + b^2)), Int[(a + b*Tan[e + f*x])^(m + 1)*(c + d*Tan[e + f*x])^(
n - 2)*Simp[a*c^2*(m + 1) + a*d^2*(n - 1) + b*c*d*(m - n + 2) - (b*c^2 - 2*
a*c*d - b*d^2)*(m + 1)*Tan[e + f*x] - d*(b*c - a*d)*(m + n)*Tan[e + f*x]^2,
 x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2
 + b^2, 0] && NeQ[c^2 + d^2, 0] && LtQ[m, -1] && LtQ[1, n, 2] && IntegerQ[2
*m]
```

Rule 3715

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*tan[(e_.)
+ (f_.)*(x_)])^(n_.)*((A_) + (C_.)*tan[(e_.) + (f_.)*(x_)]^2), x_Symbol] :>
 Dist[A/f, Subst[Int[(a + b*x)^m*(c + d*x)^n, x], x, Tan[e + f*x]], x] /; F
reeQ[{a, b, c, d, e, f, A, C, m, n}, x] && EqQ[A, C]
```

Rule 3730

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*tan[(e_.) + (f_.)*(x_)] + (C_.)*tan[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> Simp[(A*b^2 - a*(b*B - a*C))*(a + b*Tan[e +
 f*x])^(m + 1)*((c + d*Tan[e + f*x])^(n + 1)/(f*(m + 1)*(b*c - a*d)*(a^2 +
b^2))), x] + Dist[1/((m + 1)*(b*c - a*d)*(a^2 + b^2)), Int[(a + b*Tan[e + f
*x])^(m + 1)*(c + d*Tan[e + f*x])^n*Simp[A*(a*(b*c - a*d)*(m + 1) - b^2*d*(
m + n + 2)) + (b*B - a*C)*(b*c*(m + 1) + a*d*(n + 1)) - (m + 1)*(b*c - a*d)
*(A*b - a*B - b*C)*Tan[e + f*x] - d*(A*b^2 - a*(b*B - a*C))*(m + n + 2)*Tan
[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, n}, x] && NeQ[
b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] && LtQ[m, -1] &&  !
(ILtQ[n, -1] && ( !IntegerQ[m] || (EqQ[c, 0] && NeQ[a, 0])))
```

Rule 3734

```
Int[(((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_)*((A_.) + (B_.)*tan[(e_.)
+ (f_.)*(x_)] + (C_.)*tan[(e_.) + (f_.)*(x_)]^2))/((a_.) + (b_.)*tan[(e_.)
```

```
+ (f_.)*(x_)]), x_Symbol] :> Dist[1/(a^2 + b^2), Int[(c + d*Tan[e + f*x])^n
*Simp[b*B + a*(A - C) + (a*B - b*(A - C))*Tan[e + f*x], x], x], x] + Dist[(
A*b^2 - a*b*B + a^2*C)/(a^2 + b^2), Int[(c + d*Tan[e + f*x])^n*((1 + Tan[e
+ f*x]^2)/(a + b*Tan[e + f*x])), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C
, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] &&
 !GtQ[n, 0] &&  !LeQ[n, -1]
```

Rubi steps

$$\text{integral} = -\frac{ae\sqrt{e\cot(c+dx)}}{2(a^2+b^2)d(a+b\cot(c+dx))^2} - \frac{\int \frac{\frac{ae^2}{2}-2be^2\cot(c+dx)-\frac{3}{2}ae^2\cot^2(c+dx)}{\sqrt{e\cot(c+dx)}(a+b\cot(c+dx))^2}\,dx}{2(a^2+b^2)}$$

$$= -\frac{ae\sqrt{e\cot(c+dx)}}{2(a^2+b^2)d(a+b\cot(c+dx))^2} - \frac{(3a^2-5b^2)e\sqrt{e\cot(c+dx)}}{4(a^2+b^2)^2d(a+b\cot(c+dx))} + \frac{\int \frac{-\frac{1}{4}a(5a^2-3b^2)e^3+4a^2be^3\cot(c+dx)+\frac{1}{4}a(3a^2-5b^2)e^3\cot^2(c+dx)}{\sqrt{e\cot(c+dx)}(a+b\cot(c+dx))}\,dx}{2a(a^2+b^2)^2e}$$

$$= -\frac{ae\sqrt{e\cot(c+dx)}}{2(a^2+b^2)d(a+b\cot(c+dx))^2} - \frac{(3a^2-5b^2)e\sqrt{e\cot(c+dx)}}{4(a^2+b^2)^2d(a+b\cot(c+dx))} + \frac{\int \frac{-2a^2(a^2-3b^2)e^3+2ab(3a^2-b^2)e^3\cot(c+dx)}{\sqrt{e\cot(c+dx)}}\,dx}{2a(a^2+b^2)^3e} + \frac{((3a^4-26a^2b^2+3b^4)e^2)\int \frac{1+\cot^2(c+dx)}{\sqrt{e\cot(c+dx)}(a+b\cot(c+dx))}\,dx}{8(a^2+b^2)^3}$$

$$= -\frac{ae\sqrt{e\cot(c+dx)}}{2(a^2+b^2)d(a+b\cot(c+dx))^2} - \frac{(3a^2-5b^2)e\sqrt{e\cot(c+dx)}}{4(a^2+b^2)^2d(a+b\cot(c+dx))} + \frac{\text{Subst}\left(\int \frac{2a^2(a^2-3b^2)e^4-2ab(3a^2-b^2)e^3x^2}{e^2+x^4}\,dx, x, \sqrt{e\cot(c+dx)}\right)}{a(a^2+b^2)^3de} + \frac{((3a^4-26a^2b^2+3b^4)e^2)\,\text{Subst}\left(\int \frac{1}{\sqrt{-ex}(a-bx)}\,dx, x, -\cot(c+dx)\right)}{8(a^2+b^2)^3d}$$

$$= -\frac{ae\sqrt{e\cot(c+dx)}}{2(a^2+b^2)\,d(a+b\cot(c+dx))^2} - \frac{(3a^2-5b^2)\,e\sqrt{e\cot(c+dx)}}{4(a^2+b^2)^2\,d(a+b\cot(c+dx))}$$
$$- \frac{((3a^4-26a^2b^2+3b^4)\,e)\,\text{Subst}\left(\int \frac{1}{a+\frac{bx^2}{e}}\,dx, x, \sqrt{e\cot(c+dx)}\right)}{4(a^2+b^2)^3\,d}$$
$$+ \frac{((a+b)(a^2-4ab+b^2)\,e^2)\,\text{Subst}\left(\int \frac{e+x^2}{e^2+x^4}\,dx, x, \sqrt{e\cot(c+dx)}\right)}{(a^2+b^2)^3\,d}$$
$$+ \frac{((a-b)(a^2+4ab+b^2)\,e^2)\,\text{Subst}\left(\int \frac{e-x^2}{e^2+x^4}\,dx, x, \sqrt{e\cot(c+dx)}\right)}{(a^2+b^2)^3\,d}$$

$$= -\frac{(3a^4-26a^2b^2+3b^4)\,e^{3/2}\arctan\left(\frac{\sqrt{b}\sqrt{e\cot(c+dx)}}{\sqrt{a}\sqrt{e}}\right)}{4\sqrt{a}\sqrt{b}\,(a^2+b^2)^3\,d}$$
$$- \frac{ae\sqrt{e\cot(c+dx)}}{2(a^2+b^2)\,d(a+b\cot(c+dx))^2} - \frac{(3a^2-5b^2)\,e\sqrt{e\cot(c+dx)}}{4(a^2+b^2)^2\,d(a+b\cot(c+dx))}$$
$$- \frac{((a-b)(a^2+4ab+b^2)\,e^{3/2})\,\text{Subst}\left(\int \frac{\sqrt{2}\sqrt{e}+2x}{-e-\sqrt{2}\sqrt{e}x-x^2}\,dx, x, \sqrt{e\cot(c+dx)}\right)}{2\sqrt{2}\,(a^2+b^2)^3\,d}$$
$$- \frac{((a-b)(a^2+4ab+b^2)\,e^{3/2})\,\text{Subst}\left(\int \frac{\sqrt{2}\sqrt{e}-2x}{-e+\sqrt{2}\sqrt{e}x-x^2}\,dx, x, \sqrt{e\cot(c+dx)}\right)}{2\sqrt{2}\,(a^2+b^2)^3\,d}$$
$$+ \frac{((a+b)(a^2-4ab+b^2)\,e^2)\,\text{Subst}\left(\int \frac{1}{e-\sqrt{2}\sqrt{e}x+x^2}\,dx, x, \sqrt{e\cot(c+dx)}\right)}{2(a^2+b^2)^3\,d}$$
$$+ \frac{((a+b)(a^2-4ab+b^2)\,e^2)\,\text{Subst}\left(\int \frac{1}{e+\sqrt{2}\sqrt{e}x+x^2}\,dx, x, \sqrt{e\cot(c+dx)}\right)}{2(a^2+b^2)^3\,d}$$

$$
= -\frac{(3a^4-26a^2b^2+3b^4)\,e^{3/2}\arctan\left(\frac{\sqrt{b}\sqrt{e\cot(c+dx)}}{\sqrt{a}\sqrt{e}}\right)}{4\sqrt{a}\sqrt{b}\,(a^2+b^2)^3\,d}
- \frac{ae\sqrt{e\cot(c+dx)}}{2\,(a^2+b^2)\,d(a+b\cot(c+dx))^2} - \frac{(3a^2-5b^2)\,e\sqrt{e\cot(c+dx)}}{4\,(a^2+b^2)^2\,d(a+b\cot(c+dx))}
- \frac{(a-b)\,(a^2+4ab+b^2)\,e^{3/2}\log\left(\sqrt{e}+\sqrt{e}\cot(c+dx)-\sqrt{2}\sqrt{e\cot(c+dx)}\right)}{2\sqrt{2}\,(a^2+b^2)^3\,d}
+ \frac{(a-b)\,(a^2+4ab+b^2)\,e^{3/2}\log\left(\sqrt{e}+\sqrt{e}\cot(c+dx)+\sqrt{2}\sqrt{e\cot(c+dx)}\right)}{2\sqrt{2}\,(a^2+b^2)^3\,d}
+ \frac{\left((a+b)\,(a^2-4ab+b^2)\,e^{3/2}\right)\operatorname{Subst}\left(\int\frac{1}{-1-x^2}\,dx,x,1-\frac{\sqrt{2}\sqrt{e\cot(c+dx)}}{\sqrt{e}}\right)}{\sqrt{2}\,(a^2+b^2)^3\,d}
- \frac{\left((a+b)\,(a^2-4ab+b^2)\,e^{3/2}\right)\operatorname{Subst}\left(\int\frac{1}{-1-x^2}\,dx,x,1+\frac{\sqrt{2}\sqrt{e\cot(c+dx)}}{\sqrt{e}}\right)}{\sqrt{2}\,(a^2+b^2)^3\,d}
$$

$$
= -\frac{(3a^4-26a^2b^2+3b^4)\,e^{3/2}\arctan\left(\frac{\sqrt{b}\sqrt{e\cot(c+dx)}}{\sqrt{a}\sqrt{e}}\right)}{4\sqrt{a}\sqrt{b}\,(a^2+b^2)^3\,d}
- \frac{(a+b)\,(a^2-4ab+b^2)\,e^{3/2}\arctan\left(1-\frac{\sqrt{2}\sqrt{e\cot(c+dx)}}{\sqrt{e}}\right)}{\sqrt{2}\,(a^2+b^2)^3\,d}
+ \frac{(a+b)\,(a^2-4ab+b^2)\,e^{3/2}\arctan\left(1+\frac{\sqrt{2}\sqrt{e\cot(c+dx)}}{\sqrt{e}}\right)}{\sqrt{2}\,(a^2+b^2)^3\,d}
- \frac{ae\sqrt{e\cot(c+dx)}}{2\,(a^2+b^2)\,d(a+b\cot(c+dx))^2} - \frac{(3a^2-5b^2)\,e\sqrt{e\cot(c+dx)}}{4\,(a^2+b^2)^2\,d(a+b\cot(c+dx))}
- \frac{(a-b)\,(a^2+4ab+b^2)\,e^{3/2}\log\left(\sqrt{e}+\sqrt{e}\cot(c+dx)-\sqrt{2}\sqrt{e\cot(c+dx)}\right)}{2\sqrt{2}\,(a^2+b^2)^3\,d}
+ \frac{(a-b)\,(a^2+4ab+b^2)\,e^{3/2}\log\left(\sqrt{e}+\sqrt{e}\cot(c+dx)+\sqrt{2}\sqrt{e\cot(c+dx)}\right)}{2\sqrt{2}\,(a^2+b^2)^3\,d}
$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 6.20 (sec) , antiderivative size = 535, normalized size of antiderivative = 1.16

$$\int \frac{(e\cot(c+dx))^{3/2}}{(a+b\cot(c+dx))^3}\,dx =$$

$$-\frac{(e\cot(c+dx))^{3/2}\left(\frac{2a^{3/2}(3a^2-b^2)\arctan\left(\frac{\sqrt{b}\sqrt{\cot(c+dx)}}{\sqrt{a}}\right)}{\sqrt{b}(a^2+b^2)^3}-\frac{2a(3a^2-b^2)\sqrt{\cot(c+dx)}}{(a^2+b^2)^3}+\frac{2b(3a^2-b^2)\cot^{\frac{3}{2}}(c+dx)}{3(a^2+b^2)^3}-\frac{-\frac{3\sqrt{b}\arctan\left(\cdots\right.}{\cdots}}{\cdots}\right.}{\cdots}$$

```
[In] Integrate[(e*Cot[c + d*x])^(3/2)/(a + b*Cot[c + d*x])^3,x]
```

```
[Out] -((((e*Cot[c + d*x])^(3/2)*((2*a^(3/2)*(3*a^2 - b^2)*ArcTan[(Sqrt[b]*Sqrt[Co
t[c + d*x]])/Sqrt[a]])/(Sqrt[b]*(a^2 + b^2)^3) - (2*a*(3*a^2 - b^2)*Sqrt[Co
t[c + d*x]])/(a^2 + b^2)^3 + (2*b*(3*a^2 - b^2)*Cot[c + d*x]^(3/2))/(3*(a^2
 + b^2)^3) - ((-3*Sqrt[b]*ArcTan[(Sqrt[b]*Sqrt[Cot[c + d*x]])/Sqrt[a]]*Sqrt
[Cot[c + d*x]])/Sqrt[a] + (2*b^2*Cot[c + d*x]^2)/(a + b*Cot[c + d*x])^2 + (
3*b*Cot[c + d*x])/(a + b*Cot[c + d*x]))/(4*b*(a^2 + b^2)*Sqrt[Cot[c + d*x]]
) - (2*b*(3*a^2 - b^2)*(Cot[c + d*x]^(3/2) - Cot[c + d*x]^(3/2)*Hypergeomet
ric2F1[3/4, 1, 7/4, -Cot[c + d*x]^2]))/(3*(a^2 + b^2)^3) + (4*b^2*Cot[c + d
*x]^(5/2)*Hypergeometric2F1[2, 5/2, 7/2, -((b*Cot[c + d*x])/a)])/(5*a*(a^2
+ b^2)^2) + (a*(a^2 - 3*b^2)*(2*Sqrt[2]*ArcTan[1 - Sqrt[2]*Sqrt[Cot[c + d*x
]]] - 2*Sqrt[2]*ArcTan[1 + Sqrt[2]*Sqrt[Cot[c + d*x]]] + 8*Sqrt[Cot[c + d*x
]] + Sqrt[2]*Log[1 - Sqrt[2]*Sqrt[Cot[c + d*x]] + Cot[c + d*x]] - Sqrt[2]*L
og[1 + Sqrt[2]*Sqrt[Cot[c + d*x]] + Cot[c + d*x]]))/(4*(a^2 + b^2)^3)))/(d*
Cot[c + d*x]^(3/2)))
```

Maple [A] (verified)

Time = 0.05 (sec) , antiderivative size = 456, normalized size of antiderivative = 0.99

method	result
derivativedivides	$-\frac{2e^4\left(\frac{\frac{\left(\frac{3}{8}a^4b-\frac{1}{4}a^2b^3-\frac{5}{8}b^5\right)(e\cot(dx+c))^{\frac{3}{2}}+\frac{ae\left(5a^4+2a^2b^2-3b^4\right)\sqrt{e\cot(dx+c)}}{8}}{(e\cot(dx+c)b+ae)^2}+\frac{\left(3a^4-26a^2b^2+3b^4\right)\arctan\left(\frac{\sqrt{e\cot(dx+c)}\,b}{\sqrt{aeb}}\right)}{8\sqrt{aeb}}}{\left(a^2+b^2\right)^3e^2}\cdots\right.}{\cdots}$
default	$-\frac{2e^4\left(\frac{\frac{\left(\frac{3}{8}a^4b-\frac{1}{4}a^2b^3-\frac{5}{8}b^5\right)(e\cot(dx+c))^{\frac{3}{2}}+\frac{ae\left(5a^4+2a^2b^2-3b^4\right)\sqrt{e\cot(dx+c)}}{8}}{(e\cot(dx+c)b+ae)^2}+\frac{\left(3a^4-26a^2b^2+3b^4\right)\arctan\left(\frac{\sqrt{e\cot(dx+c)}\,b}{\sqrt{aeb}}\right)}{8\sqrt{aeb}}}{\left(a^2+b^2\right)^3e^2}\cdots\right.}{\cdots}$

```
[In] int((e*cot(d*x+c))^(3/2)/(a+b*cot(d*x+c))^3,x,method=_RETURNVERBOSE)
```

```
[Out] -2/d*e^4*(1/(a^2+b^2)^3/e^2*(((3/8*a^4*b-1/4*a^2*b^3-5/8*b^5)*(e*cot(d*x+c)
)^(3/2)+1/8*a*e*(5*a^4+2*a^2*b^2-3*b^4)*(e*cot(d*x+c))^(1/2))/(e*cot(d*x+c)
*b+a*e)^2+1/8*(3*a^4-26*a^2*b^2+3*b^4)/(a*e*b)^(1/2)*arctan((e*cot(d*x+c))^
(1/2)*b/(a*e*b)^(1/2)))+1/(a^2+b^2)^3/e^2*(1/8*(-a^3*e+3*a*b^2*e)*(e^2)^(1/
4)/e^2*2^(1/2)*(ln((e*cot(d*x+c)+(e^2)^(1/4)*(e*cot(d*x+c))^(1/2)*2^(1/2)+(
e^2)^(1/2))/(e*cot(d*x+c)-(e^2)^(1/4)*(e*cot(d*x+c))^(1/2)*2^(1/2)+(e^2)^(1
/2)))+2*arctan(2^(1/2)/(e^2)^(1/4)*(e*cot(d*x+c))^(1/2)+1)-2*arctan(-2^(1/2
)/(e^2)^(1/4)*(e*cot(d*x+c))^(1/2)+1))+1/8*(3*a^2*b-b^3)/(e^2)^(1/4)*2^(1/2
)*(ln((e*cot(d*x+c)-(e^2)^(1/4)*(e*cot(d*x+c))^(1/2)*2^(1/2)+(e^2)^(1/2))/(
e*cot(d*x+c)+(e^2)^(1/4)*(e*cot(d*x+c))^(1/2)*2^(1/2)+(e^2)^(1/2)))+2*arcta
n(2^(1/2)/(e^2)^(1/4)*(e*cot(d*x+c))^(1/2)+1)-2*arctan(-2^(1/2)/(e^2)^(1/4)
*(e*cot(d*x+c))^(1/2)+1))))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 4432 vs. $2(390) = 780$.

Time = 0.78 (sec) , antiderivative size = 8913, normalized size of antiderivative = 19.33

$$\int \frac{(e\cot(c+dx))^{3/2}}{(a+b\cot(c+dx))^3}\,dx = \text{Too large to display}$$

```
[In] integrate((e*cot(d*x+c))^(3/2)/(a+b*cot(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] Too large to include
```

Sympy [F]

$$\int \frac{(e\cot(c+dx))^{3/2}}{(a+b\cot(c+dx))^3}\,dx = \int \frac{(e\cot(c+dx))^{\frac{3}{2}}}{(a+b\cot(c+dx))^3}\,dx$$

```
[In] integrate((e*cot(d*x+c))**(3/2)/(a+b*cot(d*x+c))**3,x)
[Out] Integral((e*cot(c + d*x))**(3/2)/(a + b*cot(c + d*x))**3, x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{(e\cot(c+dx))^{3/2}}{(a+b\cot(c+dx))^3}\,dx = \text{Exception raised: ValueError}$$

```
[In] integrate((e*cot(d*x+c))^(3/2)/(a+b*cot(d*x+c))^3,x, algorithm="maxima")
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(e>0)', see `assume?` for more detai
ls)Is e
```

Giac [F]

$$\int \frac{(e\cot(c+dx))^{3/2}}{(a+b\cot(c+dx))^3}\,dx = \int \frac{(e\cot(dx+c))^{\frac{3}{2}}}{(b\cot(dx+c)+a)^3}\,dx$$

```
[In] integrate((e*cot(d*x+c))^(3/2)/(a+b*cot(d*x+c))^3,x, algorithm="giac")
[Out] integrate((e*cot(d*x + c))^(3/2)/(b*cot(d*x + c) + a)^3, x)
```

Mupad [B] (verification not implemented)

Time = 19.03 (sec) , antiderivative size = 19000, normalized size of antiderivative = 41.21

$$\int \frac{(e\cot(c+dx))^{3/2}}{(a+b\cot(c+dx))^3}\,dx = \text{Too large to display}$$

```
[In] int((e*cot(c + d*x))^(3/2)/(a + b*cot(c + d*x))^3,x)
[Out] atan(((((518*a*b^15*d^2*e^15 - 18*a^15*b*d^2*e^15 - 4494*a^3*b^13*d^2*e^15
+ 3022*a^5*b^11*d^2*e^15 + 17194*a^7*b^9*d^2*e^15 + 5298*a^9*b^7*d^2*e^15 -
```

3338*a^11*b^5*d^2*e^15 + 506*a^13*b^3*d^2*e^15)/(a^16*d^5 + b^16*d^5 + 8*a^2*b^14*d^5 + 28*a^4*b^12*d^5 + 56*a^6*b^10*d^5 + 70*a^8*b^8*d^5 + 56*a^10*b^6*d^5 + 28*a^12*b^4*d^5 + 8*a^14*b^2*d^5) + (((4224*a^4*b^18*d^4*e^12 - 320*a^2*b^20*d^4*e^12 - 192*b^22*d^4*e^12 + 22272*a^6*b^16*d^4*e^12 + 51072*a^8*b^14*d^4*e^12 + 67200*a^10*b^12*d^4*e^12 + 53760*a^12*b^10*d^4*e^12 + 25344*a^14*b^8*d^4*e^12 + 5952*a^16*b^6*d^4*e^12 + 192*a^18*b^4*d^4*e^12 - 128*a^20*b^2*d^4*e^12)/(a^16*d^5 + b^16*d^5 + 8*a^2*b^14*d^5 + 28*a^4*b^12*d^5 + 56*a^6*b^10*d^5 + 70*a^8*b^8*d^5 + 56*a^10*b^6*d^5 + 28*a^12*b^4*d^5 + 8*a^14*b^2*d^5) + ((e*cot(c + d*x))^(1/2)*((e^3*1i)/(4*(b^6*d^2 - a^6*d^2 + a*b^5*d^2*6i + a^5*b*d^2*6i - 15*a^2*b^4*d^2 - a^3*b^3*d^2*20i + 15*a^4*b^2*d^2)))^(1/2)*(512*b^25*d^4*e^10 + 4608*a^2*b^23*d^4*e^10 + 17920*a^4*b^21*d^4*e^10 + 38400*a^6*b^19*d^4*e^10 + 46080*a^8*b^17*d^4*e^10 + 21504*a^10*b^15*d^4*e^10 - 21504*a^12*b^13*d^4*e^10 - 46080*a^14*b^11*d^4*e^10 - 38400*a^16*b^9*d^4*e^10 - 17920*a^18*b^7*d^4*e^10 - 4608*a^20*b^5*d^4*e^10 - 512*a^22*b^3*d^4*e^10))/(a^16*d^4 + b^16*d^4 + 8*a^2*b^14*d^4 + 28*a^4*b^12*d^4 + 56*a^6*b^10*d^4 + 70*a^8*b^8*d^4 + 56*a^10*b^6*d^4 + 28*a^12*b^4*d^4 + 8*a^14*b^2*d^4))*((e^3*1i)/(4*(b^6*d^2 - a^6*d^2 + a*b^5*d^2*6i + a^5*b*d^2*6i - 15*a^2*b^4*d^2 - a^3*b^3*d^2*20i + 15*a^4*b^2*d^2)))^(1/2) - ((e*cot(c + d*x))^(1/2)*(1544*a*b^18*d^2*e^13 + 64*a^3*b^16*d^2*e^13 - 7456*a^5*b^14*d^2*e^13 - 576*a^7*b^12*d^2*e^13 + 19504*a^9*b^10*d^2*e^13 + 18880*a^11*b^8*d^2*e^13 + 3808*a^13*b^6*d^2*e^13 - 960*a^15*b^4*d^2*e^13 + 8*a^17*b^2*d^2*e^13))/(a^16*d^4 + b^16*d^4 + 8*a^2*b^14*d^4 + 28*a^4*b^12*d^4 + 56*a^6*b^10*d^4 + 70*a^8*b^8*d^4 + 56*a^10*b^6*d^4 + 28*a^12*b^4*d^4 + 8*a^14*b^2*d^4))*((e^3*1i)/(4*(b^6*d^2 - a^6*d^2 + a*b^5*d^2*6i + a^5*b*d^2*6i - 15*a^2*b^4*d^2 - a^3*b^3*d^2*20i + 15*a^4*b^2*d^2)))^(1/2))*((e^3*1i)/(4*(b^6*d^2 - a^6*d^2 + a*b^5*d^2*6i + a^5*b*d^2*6i - 15*a^2*b^4*d^2 - a^3*b^3*d^2*20i + 15*a^4*b^2*d^2)))^(1/2) + ((e*cot(c + d*x))^(1/2)*(41*b^13*e^16 + 9*a^12*b*e^16 - 82*a^2*b^11*e^16 + 1831*a^4*b^9*e^16 - 4348*a^6*b^7*e^16 + 1671*a^8*b^5*e^16 - 210*a^10*b^3*e^16))/(a^16*d^4 + b^16*d^4 + 8*a^2*b^14*d^4 + 28*a^4*b^12*d^4 + 56*a^6*b^10*d^4 + 70*a^8*b^8*d^4 + 56*a^10*b^6*d^4 + 28*a^12*b^4*d^4 + 8*a^14*b^2*d^4))*((e^3*1i)/(4*(b^6*d^2 - a^6*d^2 + a*b^5*d^2*6i + a^5*b*d^2*6i - 15*a^2*b^4*d^2 - a^3*b^3*d^2*20i + 15*a^4*b^2*d^2)))^(1/2)*1i - (((518*a*b^15*d^2*e^15 - 18*a^15*b*d^2*e^15 - 4494*a^3*b^13*d^2*e^15 + 3022*a^5*b^11*d^2*e^15 + 17194*a^7*b^9*d^2*e^15 + 5298*a^9*b^7*d^2*e^15 - 3338*a^11*b^5*d^2*e^15 + 506*a^13*b^3*d^2*e^15)/(a^16*d^5 + b^16*d^5 + 8*a^2*b^14*d^5 + 28*a^4*b^12*d^5 + 56*a^6*b^10*d^5 + 70*a^8*b^8*d^5 + 56*a^10*b^6*d^5 + 28*a^12*b^4*d^5 + 8*a^14*b^2*d^5) + (((4224*a^4*b^18*d^4*e^12 - 320*a^2*b^20*d^4*e^12 - 192*b^22*d^4*e^12 + 22272*a^6*b^16*d^4*e^12 + 51072*a^8*b^14*d^4*e^12 + 67200*a^10*b^12*d^4*e^12 + 53760*a^12*b^10*d^4*e^12 + 25344*a^14*b^8*d^4*e^12 + 5952*a^16*b^6*d^4*e^12 + 192*a^18*b^4*d^4*e^12 - 128*a^20*b^2*d^4*e^12)/(a^16*d^5 + b^16*d^5 + 8*a^2*b^14*d^5 + 28*a^4*b^12*d^5 + 56*a^6*b^10*d^5 + 70*a^8*b^8*d^5 + 56*a^10*b^6*d^5 + 28*a^12*b^4*d^5 + 8*a^14*b^2*d^5) - ((e*cot(c + d*x))^(1/2)*((e^3*1i)/(4*(b^6*d^2 - a^6*d^2 + a*b^5*d^2*6i + a^5*b*d^2*6i - 15*a^2*b^4*d^2 - a^3*b^3*d^2*20i + 15*a^4*b^2*d^2)))^(1/2)*(512*b^25*d^4*e^10 + 4608*a^2*b^23*d^4*e^10 + 17920*a^4

*b^21*d^4*e^10 + 38400*a^6*b^19*d^4*e^10 + 46080*a^8*b^17*d^4*e^10 + 21504*
a^10*b^15*d^4*e^10 - 21504*a^12*b^13*d^4*e^10 - 46080*a^14*b^11*d^4*e^10 -
38400*a^16*b^9*d^4*e^10 - 17920*a^18*b^7*d^4*e^10 - 4608*a^20*b^5*d^4*e^10
- 512*a^22*b^3*d^4*e^10))/(a^16*d^4 + b^16*d^4 + 8*a^2*b^14*d^4 + 28*a^4*b^
12*d^4 + 56*a^6*b^10*d^4 + 70*a^8*b^8*d^4 + 56*a^10*b^6*d^4 + 28*a^12*b^4*d
^4 + 8*a^14*b^2*d^4))*((e^3*1i)/(4*(b^6*d^2 - a^6*d^2 + a*b^5*d^2*6i + a^5*
b*d^2*6i - 15*a^2*b^4*d^2 - a^3*b^3*d^2*20i + 15*a^4*b^2*d^2)))^(1/2) + ((e
*cot(c + d*x))^(1/2)*(1544*a*b^18*d^2*e^13 + 64*a^3*b^16*d^2*e^13 - 7456*a^
5*b^14*d^2*e^13 - 576*a^7*b^12*d^2*e^13 + 19504*a^9*b^10*d^2*e^13 + 18880*a
^11*b^8*d^2*e^13 + 3808*a^13*b^6*d^2*e^13 - 960*a^15*b^4*d^2*e^13 + 8*a^17*
b^2*d^2*e^13))/(a^16*d^4 + b^16*d^4 + 8*a^2*b^14*d^4 + 28*a^4*b^12*d^4 + 56
*a^6*b^10*d^4 + 70*a^8*b^8*d^4 + 56*a^10*b^6*d^4 + 28*a^12*b^4*d^4 + 8*a^14
*b^2*d^4))*((e^3*1i)/(4*(b^6*d^2 - a^6*d^2 + a*b^5*d^2*6i + a^5*b*d^2*6i -
15*a^2*b^4*d^2 - a^3*b^3*d^2*20i + 15*a^4*b^2*d^2)))^(1/2))*((e^3*1i)/(4*(b
^6*d^2 - a^6*d^2 + a*b^5*d^2*6i + a^5*b*d^2*6i - 15*a^2*b^4*d^2 - a^3*b^3*d
^2*20i + 15*a^4*b^2*d^2)))^(1/2) - ((e*cot(c + d*x))^(1/2)*(41*b^13*e^16 +
9*a^12*b*e^16 - 82*a^2*b^11*e^16 + 1831*a^4*b^9*e^16 - 4348*a^6*b^7*e^16 +
1671*a^8*b^5*e^16 - 210*a^10*b^3*e^16))/(a^16*d^4 + b^16*d^4 + 8*a^2*b^14*d
^4 + 28*a^4*b^12*d^4 + 56*a^6*b^10*d^4 + 70*a^8*b^8*d^4 + 56*a^10*b^6*d^4 +
28*a^12*b^4*d^4 + 8*a^14*b^2*d^4))*((e^3*1i)/(4*(b^6*d^2 - a^6*d^2 + a*b^5
*d^2*6i + a^5*b*d^2*6i - 15*a^2*b^4*d^2 - a^3*b^3*d^2*20i + 15*a^4*b^2*d^2)
))^(1/2)*1i)/((((518*a*b^15*d^2*e^15 - 18*a^15*b*d^2*e^15 - 4494*a^3*b^13*d
^2*e^15 + 3022*a^5*b^11*d^2*e^15 + 17194*a^7*b^9*d^2*e^15 + 5298*a^9*b^7*d^
2*e^15 - 3338*a^11*b^5*d^2*e^15 + 506*a^13*b^3*d^2*e^15)/(a^16*d^5 + b^16*d
^5 + 8*a^2*b^14*d^5 + 28*a^4*b^12*d^5 + 56*a^6*b^10*d^5 + 70*a^8*b^8*d^5 +
56*a^10*b^6*d^5 + 28*a^12*b^4*d^5 + 8*a^14*b^2*d^5) + (((4224*a^4*b^18*d^4*
e^12 - 320*a^2*b^20*d^4*e^12 - 192*b^22*d^4*e^12 + 22272*a^6*b^16*d^4*e^12
+ 51072*a^8*b^14*d^4*e^12 + 67200*a^10*b^12*d^4*e^12 + 53760*a^12*b^10*d^4*
e^12 + 25344*a^14*b^8*d^4*e^12 + 5952*a^16*b^6*d^4*e^12 + 192*a^18*b^4*d^4*
e^12 - 128*a^20*b^2*d^4*e^12)/(a^16*d^5 + b^16*d^5 + 8*a^2*b^14*d^5 + 28*a^
4*b^12*d^5 + 56*a^6*b^10*d^5 + 70*a^8*b^8*d^5 + 56*a^10*b^6*d^5 + 28*a^12*b
^4*d^5 + 8*a^14*b^2*d^5) + ((e*cot(c + d*x))^(1/2)*((e^3*1i)/(4*(b^6*d^2 -
a^6*d^2 + a*b^5*d^2*6i + a^5*b*d^2*6i - 15*a^2*b^4*d^2 - a^3*b^3*d^2*20i +
15*a^4*b^2*d^2)))^(1/2)*(512*b^25*d^4*e^10 + 4608*a^2*b^23*d^4*e^10 + 17920
*a^4*b^21*d^4*e^10 + 38400*a^6*b^19*d^4*e^10 + 46080*a^8*b^17*d^4*e^10 + 21
504*a^10*b^15*d^4*e^10 - 21504*a^12*b^13*d^4*e^10 - 46080*a^14*b^11*d^4*e^1
0 - 38400*a^16*b^9*d^4*e^10 - 17920*a^18*b^7*d^4*e^10 - 4608*a^20*b^5*d^4*e
^10 - 512*a^22*b^3*d^4*e^10))/(a^16*d^4 + b^16*d^4 + 8*a^2*b^14*d^4 + 28*a^
4*b^12*d^4 + 56*a^6*b^10*d^4 + 70*a^8*b^8*d^4 + 56*a^10*b^6*d^4 + 28*a^12*b
^4*d^4 + 8*a^14*b^2*d^4))*((e^3*1i)/(4*(b^6*d^2 - a^6*d^2 + a*b^5*d^2*6i +
a^5*b*d^2*6i - 15*a^2*b^4*d^2 - a^3*b^3*d^2*20i + 15*a^4*b^2*d^2)))^(1/2) -
((e*cot(c + d*x))^(1/2)*(1544*a*b^18*d^2*e^13 + 64*a^3*b^16*d^2*e^13 - 745
6*a^5*b^14*d^2*e^13 - 576*a^7*b^12*d^2*e^13 + 19504*a^9*b^10*d^2*e^13 + 188
80*a^11*b^8*d^2*e^13 + 3808*a^13*b^6*d^2*e^13 - 960*a^15*b^4*d^2*e^13 + 8*a
^17*b^2*d^2*e^13))/(a^16*d^4 + b^16*d^4 + 8*a^2*b^14*d^4 + 28*a^4*b^12*d^4

+ 56*a^6*b^10*d^4 + 70*a^8*b^8*d^4 + 56*a^10*b^6*d^4 + 28*a^12*b^4*d^4 + 8*
a^14*b^2*d^4))*((e^3*1i)/(4*(b^6*d^2 - a^6*d^2 + a*b^5*d^2*6i + a^5*b*d^2*6
i - 15*a^2*b^4*d^2 - a^3*b^3*d^2*20i + 15*a^4*b^2*d^2)))^(1/2))*((e^3*1i)/(
4*(b^6*d^2 - a^6*d^2 + a*b^5*d^2*6i + a^5*b*d^2*6i - 15*a^2*b^4*d^2 - a^3*b
^3*d^2*20i + 15*a^4*b^2*d^2)))^(1/2) + ((e*cot(c + d*x))^(1/2)*(41*b^13*e^1
6 + 9*a^12*b*e^16 - 82*a^2*b^11*e^16 + 1831*a^4*b^9*e^16 - 4348*a^6*b^7*e^1
6 + 1671*a^8*b^5*e^16 - 210*a^10*b^3*e^16))/(a^16*d^4 + b^16*d^4 + 8*a^2*b^
14*d^4 + 28*a^4*b^12*d^4 + 56*a^6*b^10*d^4 + 70*a^8*b^8*d^4 + 56*a^10*b^6*d
^4 + 28*a^12*b^4*d^4 + 8*a^14*b^2*d^4))*((e^3*1i)/(4*(b^6*d^2 - a^6*d^2 + a
*b^5*d^2*6i + a^5*b*d^2*6i - 15*a^2*b^4*d^2 - a^3*b^3*d^2*20i + 15*a^4*b^2*
d^2)))^(1/2) + (((518*a*b^15*d^2*e^15 - 18*a^15*b*d^2*e^15 - 4494*a^3*b^13*
d^2*e^15 + 3022*a^5*b^11*d^2*e^15 + 17194*a^7*b^9*d^2*e^15 + 5298*a^9*b^7*d
^2*e^15 - 3338*a^11*b^5*d^2*e^15 + 506*a^13*b^3*d^2*e^15)/(a^16*d^5 + b^16*
d^5 + 8*a^2*b^14*d^5 + 28*a^4*b^12*d^5 + 56*a^6*b^10*d^5 + 70*a^8*b^8*d^5 +
56*a^10*b^6*d^5 + 28*a^12*b^4*d^5 + 8*a^14*b^2*d^5) + (((4224*a^4*b^18*d^4
*e^12 - 320*a^2*b^20*d^4*e^12 - 192*b^22*d^4*e^12 + 22272*a^6*b^16*d^4*e^12
+ 51072*a^8*b^14*d^4*e^12 + 67200*a^10*b^12*d^4*e^12 + 53760*a^12*b^10*d^4
*e^12 + 25344*a^14*b^8*d^4*e^12 + 5952*a^16*b^6*d^4*e^12 + 192*a^18*b^4*d^4
*e^12 - 128*a^20*b^2*d^4*e^12)/(a^16*d^5 + b^16*d^5 + 8*a^2*b^14*d^5 + 28*a
^4*b^12*d^5 + 56*a^6*b^10*d^5 + 70*a^8*b^8*d^5 + 56*a^10*b^6*d^5 + 28*a^12*
b^4*d^5 + 8*a^14*b^2*d^5) - ((e*cot(c + d*x))^(1/2)*((e^3*1i)/(4*(b^6*d^2 -
a^6*d^2 + a*b^5*d^2*6i + a^5*b*d^2*6i - 15*a^2*b^4*d^2 - a^3*b^3*d^2*20i +
15*a^4*b^2*d^2)))^(1/2)*(512*b^25*d^4*e^10 + 4608*a^2*b^23*d^4*e^10 + 1792
0*a^4*b^21*d^4*e^10 + 38400*a^6*b^19*d^4*e^10 + 46080*a^8*b^17*d^4*e^10 + 2
1504*a^10*b^15*d^4*e^10 - 21504*a^12*b^13*d^4*e^10 - 46080*a^14*b^11*d^4*e^
10 - 38400*a^16*b^9*d^4*e^10 - 17920*a^18*b^7*d^4*e^10 - 4608*a^20*b^5*d^4*
e^10 - 512*a^22*b^3*d^4*e^10))/(a^16*d^4 + b^16*d^4 + 8*a^2*b^14*d^4 + 28*a
^4*b^12*d^4 + 56*a^6*b^10*d^4 + 70*a^8*b^8*d^4 + 56*a^10*b^6*d^4 + 28*a^12*
b^4*d^4 + 8*a^14*b^2*d^4))*((e^3*1i)/(4*(b^6*d^2 - a^6*d^2 + a*b^5*d^2*6i +
a^5*b*d^2*6i - 15*a^2*b^4*d^2 - a^3*b^3*d^2*20i + 15*a^4*b^2*d^2)))^(1/2)
+ ((e*cot(c + d*x))^(1/2)*(1544*a*b^18*d^2*e^13 + 64*a^3*b^16*d^2*e^13 - 74
56*a^5*b^14*d^2*e^13 - 576*a^7*b^12*d^2*e^13 + 19504*a^9*b^10*d^2*e^13 + 18
880*a^11*b^8*d^2*e^13 + 3808*a^13*b^6*d^2*e^13 - 960*a^15*b^4*d^2*e^13 + 8*
a^17*b^2*d^2*e^13))/(a^16*d^4 + b^16*d^4 + 8*a^2*b^14*d^4 + 28*a^4*b^12*d^4
+ 56*a^6*b^10*d^4 + 70*a^8*b^8*d^4 + 56*a^10*b^6*d^4 + 28*a^12*b^4*d^4 + 8
*a^14*b^2*d^4))*((e^3*1i)/(4*(b^6*d^2 - a^6*d^2 + a*b^5*d^2*6i + a^5*b*d^2*
6i - 15*a^2*b^4*d^2 - a^3*b^3*d^2*20i + 15*a^4*b^2*d^2)))^(1/2))*((e^3*1i)/
(4*(b^6*d^2 - a^6*d^2 + a*b^5*d^2*6i + a^5*b*d^2*6i - 15*a^2*b^4*d^2 - a^3*
b^3*d^2*20i + 15*a^4*b^2*d^2)))^(1/2) - ((e*cot(c + d*x))^(1/2)*(41*b^13*e^
16 + 9*a^12*b*e^16 - 82*a^2*b^11*e^16 + 1831*a^4*b^9*e^16 - 4348*a^6*b^7*e^
16 + 1671*a^8*b^5*e^16 - 210*a^10*b^3*e^16))/(a^16*d^4 + b^16*d^4 + 8*a^2*b
^14*d^4 + 28*a^4*b^12*d^4 + 56*a^6*b^10*d^4 + 70*a^8*b^8*d^4 + 56*a^10*b^6*
d^4 + 28*a^12*b^4*d^4 + 8*a^14*b^2*d^4))*((e^3*1i)/(4*(b^6*d^2 - a^6*d^2 +
a*b^5*d^2*6i + a^5*b*d^2*6i - 15*a^2*b^4*d^2 - a^3*b^3*d^2*20i + 15*a^4*b^2
*d^2)))^(1/2) + (28*a^2*b^8*e^18 - 15*b^10*e^18 + 878*a^4*b^6*e^18 - 180*a^

6*b^4*e^18 + 9*a^8*b^2*e^18)/(a^16*d^5 + b^16*d^5 + 8*a^2*b^14*d^5 + 28*a^4
*b^12*d^5 + 56*a^6*b^10*d^5 + 70*a^8*b^8*d^5 + 56*a^10*b^6*d^5 + 28*a^12*b^
4*d^5 + 8*a^14*b^2*d^5)))*((e^3*1i)/(4*(b^6*d^2 - a^6*d^2 + a*b^5*d^2*6i +
a^5*b*d^2*6i - 15*a^2*b^4*d^2 - a^3*b^3*d^2*20i + 15*a^4*b^2*d^2)))^(1/2)*2
i + atan(((((518*a*b^15*d^2*e^15 - 18*a^15*b*d^2*e^15 - 4494*a^3*b^13*d^2*e
^15 + 3022*a^5*b^11*d^2*e^15 + 17194*a^7*b^9*d^2*e^15 + 5298*a^9*b^7*d^2*e^
15 - 3338*a^11*b^5*d^2*e^15 + 506*a^13*b^3*d^2*e^15)/(a^16*d^5 + b^16*d^5 +
8*a^2*b^14*d^5 + 28*a^4*b^12*d^5 + 56*a^6*b^10*d^5 + 70*a^8*b^8*d^5 + 56*a
^10*b^6*d^5 + 28*a^12*b^4*d^5 + 8*a^14*b^2*d^5) + (((4224*a^4*b^18*d^4*e^12
- 320*a^2*b^20*d^4*e^12 - 192*b^22*d^4*e^12 + 22272*a^6*b^16*d^4*e^12 + 51
072*a^8*b^14*d^4*e^12 + 67200*a^10*b^12*d^4*e^12 + 53760*a^12*b^10*d^4*e^12
+ 25344*a^14*b^8*d^4*e^12 + 5952*a^16*b^6*d^4*e^12 + 192*a^18*b^4*d^4*e^12
- 128*a^20*b^2*d^4*e^12)/(a^16*d^5 + b^16*d^5 + 8*a^2*b^14*d^5 + 28*a^4*b^
12*d^5 + 56*a^6*b^10*d^5 + 70*a^8*b^8*d^5 + 56*a^10*b^6*d^5 + 28*a^12*b^4*d
^5 + 8*a^14*b^2*d^5) + ((e*cot(c + d*x))^(1/2)*(e^3/(4*(b^6*d^2*1i - a^6*d^
2*1i + 6*a*b^5*d^2 + 6*a^5*b*d^2 - a^2*b^4*d^2*15i - 20*a^3*b^3*d^2 + a^4*b
^2*d^2*15i)))^(1/2)*(512*b^25*d^4*e^10 + 4608*a^2*b^23*d^4*e^10 + 17920*a^4
*b^21*d^4*e^10 + 38400*a^6*b^19*d^4*e^10 + 46080*a^8*b^17*d^4*e^10 + 21504*
a^10*b^15*d^4*e^10 - 21504*a^12*b^13*d^4*e^10 - 46080*a^14*b^11*d^4*e^10 -
38400*a^16*b^9*d^4*e^10 - 17920*a^18*b^7*d^4*e^10 - 4608*a^20*b^5*d^4*e^10
- 512*a^22*b^3*d^4*e^10))/(a^16*d^4 + b^16*d^4 + 8*a^2*b^14*d^4 + 28*a^4*b^
12*d^4 + 56*a^6*b^10*d^4 + 70*a^8*b^8*d^4 + 56*a^10*b^6*d^4 + 28*a^12*b^4*d
^4 + 8*a^14*b^2*d^4))*(e^3/(4*(b^6*d^2*1i - a^6*d^2*1i + 6*a*b^5*d^2 + 6*a^
5*b*d^2 - a^2*b^4*d^2*15i - 20*a^3*b^3*d^2 + a^4*b^2*d^2*15i)))^(1/2) - ((e
*cot(c + d*x))^(1/2)*(1544*a*b^18*d^2*e^13 + 64*a^3*b^16*d^2*e^13 - 7456*a^
5*b^14*d^2*e^13 - 576*a^7*b^12*d^2*e^13 + 19504*a^9*b^10*d^2*e^13 + 18880*a
^11*b^8*d^2*e^13 + 3808*a^13*b^6*d^2*e^13 - 960*a^15*b^4*d^2*e^13 + 8*a^17*
b^2*d^2*e^13))/(a^16*d^4 + b^16*d^4 + 8*a^2*b^14*d^4 + 28*a^4*b^12*d^4 + 56
*a^6*b^10*d^4 + 70*a^8*b^8*d^4 + 56*a^10*b^6*d^4 + 28*a^12*b^4*d^4 + 8*a^14
*b^2*d^4))*(e^3/(4*(b^6*d^2*1i - a^6*d^2*1i + 6*a*b^5*d^2 + 6*a^5*b*d^2 - a
^2*b^4*d^2*15i - 20*a^3*b^3*d^2 + a^4*b^2*d^2*15i)))^(1/2))*(e^3/(4*(b^6*d^
2*1i - a^6*d^2*1i + 6*a*b^5*d^2 + 6*a^5*b*d^2 - a^2*b^4*d^2*15i - 20*a^3*b^
3*d^2 + a^4*b^2*d^2*15i)))^(1/2) + ((e*cot(c + d*x))^(1/2)*(41*b^13*e^16 +
9*a^12*b*e^16 - 82*a^2*b^11*e^16 + 1831*a^4*b^9*e^16 - 4348*a^6*b^7*e^16 +
1671*a^8*b^5*e^16 - 210*a^10*b^3*e^16))/(a^16*d^4 + b^16*d^4 + 8*a^2*b^14*d
^4 + 28*a^4*b^12*d^4 + 56*a^6*b^10*d^4 + 70*a^8*b^8*d^4 + 56*a^10*b^6*d^4 +
28*a^12*b^4*d^4 + 8*a^14*b^2*d^4))*(e^3/(4*(b^6*d^2*1i - a^6*d^2*1i + 6*a*
b^5*d^2 + 6*a^5*b*d^2 - a^2*b^4*d^2*15i - 20*a^3*b^3*d^2 + a^4*b^2*d^2*15i)
))^(1/2)*1i - (((518*a*b^15*d^2*e^15 - 18*a^15*b*d^2*e^15 - 4494*a^3*b^13*d
^2*e^15 + 3022*a^5*b^11*d^2*e^15 + 17194*a^7*b^9*d^2*e^15 + 5298*a^9*b^7*d^
2*e^15 - 3338*a^11*b^5*d^2*e^15 + 506*a^13*b^3*d^2*e^15)/(a^16*d^5 + b^16*d
^5 + 8*a^2*b^14*d^5 + 28*a^4*b^12*d^5 + 56*a^6*b^10*d^5 + 70*a^8*b^8*d^5 +
56*a^10*b^6*d^5 + 28*a^12*b^4*d^5 + 8*a^14*b^2*d^5) + (((4224*a^4*b^18*d^4*
e^12 - 320*a^2*b^20*d^4*e^12 - 192*b^22*d^4*e^12 + 22272*a^6*b^16*d^4*e^12
+ 51072*a^8*b^14*d^4*e^12 + 67200*a^10*b^12*d^4*e^12 + 53760*a^12*b^10*d^4*

e^12 + 25344*a^14*b^8*d^4*e^12 + 5952*a^16*b^6*d^4*e^12 + 192*a^18*b^4*d^4*
e^12 - 128*a^20*b^2*d^4*e^12)/(a^16*d^5 + b^16*d^5 + 8*a^2*b^14*d^5 + 28*a^
4*b^12*d^5 + 56*a^6*b^10*d^5 + 70*a^8*b^8*d^5 + 56*a^10*b^6*d^5 + 28*a^12*b
^4*d^5 + 8*a^14*b^2*d^5) - ((e*cot(c + d*x))^(1/2)*(e^3/(4*(b^6*d^2*1i - a^
6*d^2*1i + 6*a*b^5*d^2 + 6*a^5*b*d^2 - a^2*b^4*d^2*15i - 20*a^3*b^3*d^2 + a
^4*b^2*d^2*15i)))^(1/2)*(512*b^25*d^4*e^10 + 4608*a^2*b^23*d^4*e^10 + 17920
*a^4*b^21*d^4*e^10 + 38400*a^6*b^19*d^4*e^10 + 46080*a^8*b^17*d^4*e^10 + 21
504*a^10*b^15*d^4*e^10 - 21504*a^12*b^13*d^4*e^10 - 46080*a^14*b^11*d^4*e^1
0 - 38400*a^16*b^9*d^4*e^10 - 17920*a^18*b^7*d^4*e^10 - 4608*a^20*b^5*d^4*e
^10 - 512*a^22*b^3*d^4*e^10))/(a^16*d^4 + b^16*d^4 + 8*a^2*b^14*d^4 + 28*a^
4*b^12*d^4 + 56*a^6*b^10*d^4 + 70*a^8*b^8*d^4 + 56*a^10*b^6*d^4 + 28*a^12*b
^4*d^4 + 8*a^14*b^2*d^4))*(e^3/(4*(b^6*d^2*1i - a^6*d^2*1i + 6*a*b^5*d^2 +
6*a^5*b*d^2 - a^2*b^4*d^2*15i - 20*a^3*b^3*d^2 + a^4*b^2*d^2*15i)))^(1/2) +
((e*cot(c + d*x))^(1/2)*(1544*a*b^18*d^2*e^13 + 64*a^3*b^16*d^2*e^13 - 745
6*a^5*b^14*d^2*e^13 - 576*a^7*b^12*d^2*e^13 + 19504*a^9*b^10*d^2*e^13 + 188
80*a^11*b^8*d^2*e^13 + 3808*a^13*b^6*d^2*e^13 - 960*a^15*b^4*d^2*e^13 + 8*a
^17*b^2*d^2*e^13))/(a^16*d^4 + b^16*d^4 + 8*a^2*b^14*d^4 + 28*a^4*b^12*d^4
+ 56*a^6*b^10*d^4 + 70*a^8*b^8*d^4 + 56*a^10*b^6*d^4 + 28*a^12*b^4*d^4 + 8*
a^14*b^2*d^4))*(e^3/(4*(b^6*d^2*1i - a^6*d^2*1i + 6*a*b^5*d^2 + 6*a^5*b*d^2
- a^2*b^4*d^2*15i - 20*a^3*b^3*d^2 + a^4*b^2*d^2*15i)))^(1/2))*(e^3/(4*(b^
6*d^2*1i - a^6*d^2*1i + 6*a*b^5*d^2 + 6*a^5*b*d^2 - a^2*b^4*d^2*15i - 20*a^
3*b^3*d^2 + a^4*b^2*d^2*15i)))^(1/2) - ((e*cot(c + d*x))^(1/2)*(41*b^13*e^1
6 + 9*a^12*b*e^16 - 82*a^2*b^11*e^16 + 1831*a^4*b^9*e^16 - 4348*a^6*b^7*e^1
6 + 1671*a^8*b^5*e^16 - 210*a^10*b^3*e^16))/(a^16*d^4 + b^16*d^4 + 8*a^2*b^
14*d^4 + 28*a^4*b^12*d^4 + 56*a^6*b^10*d^4 + 70*a^8*b^8*d^4 + 56*a^10*b^6*d
^4 + 28*a^12*b^4*d^4 + 8*a^14*b^2*d^4))*(e^3/(4*(b^6*d^2*1i - a^6*d^2*1i +
6*a*b^5*d^2 + 6*a^5*b*d^2 - a^2*b^4*d^2*15i - 20*a^3*b^3*d^2 + a^4*b^2*d^2*
15i)))^(1/2)*1i)/((((518*a*b^15*d^2*e^15 - 18*a^15*b*d^2*e^15 - 4494*a^3*b^
13*d^2*e^15 + 3022*a^5*b^11*d^2*e^15 + 17194*a^7*b^9*d^2*e^15 + 5298*a^9*b^
7*d^2*e^15 - 3338*a^11*b^5*d^2*e^15 + 506*a^13*b^3*d^2*e^15)/(a^16*d^5 + b^
16*d^5 + 8*a^2*b^14*d^5 + 28*a^4*b^12*d^5 + 56*a^6*b^10*d^5 + 70*a^8*b^8*d^
5 + 56*a^10*b^6*d^5 + 28*a^12*b^4*d^5 + 8*a^14*b^2*d^5) + (((4224*a^4*b^18*
d^4*e^12 - 320*a^2*b^20*d^4*e^12 - 192*b^22*d^4*e^12 + 22272*a^6*b^16*d^4*e
^12 + 51072*a^8*b^14*d^4*e^12 + 67200*a^10*b^12*d^4*e^12 + 53760*a^12*b^10*
d^4*e^12 + 25344*a^14*b^8*d^4*e^12 + 5952*a^16*b^6*d^4*e^12 + 192*a^18*b^4*
d^4*e^12 - 128*a^20*b^2*d^4*e^12)/(a^16*d^5 + b^16*d^5 + 8*a^2*b^14*d^5 + 2
8*a^4*b^12*d^5 + 56*a^6*b^10*d^5 + 70*a^8*b^8*d^5 + 56*a^10*b^6*d^5 + 28*a^
12*b^4*d^5 + 8*a^14*b^2*d^5) + ((e*cot(c + d*x))^(1/2)*(e^3/(4*(b^6*d^2*1i
- a^6*d^2*1i + 6*a*b^5*d^2 + 6*a^5*b*d^2 - a^2*b^4*d^2*15i - 20*a^3*b^3*d^2
+ a^4*b^2*d^2*15i)))^(1/2)*(512*b^25*d^4*e^10 + 4608*a^2*b^23*d^4*e^10 + 1
7920*a^4*b^21*d^4*e^10 + 38400*a^6*b^19*d^4*e^10 + 46080*a^8*b^17*d^4*e^10
+ 21504*a^10*b^15*d^4*e^10 - 21504*a^12*b^13*d^4*e^10 - 46080*a^14*b^11*d^4
*e^10 - 38400*a^16*b^9*d^4*e^10 - 17920*a^18*b^7*d^4*e^10 - 4608*a^20*b^5*d
^4*e^10 - 512*a^22*b^3*d^4*e^10))/(a^16*d^4 + b^16*d^4 + 8*a^2*b^14*d^4 + 2
8*a^4*b^12*d^4 + 56*a^6*b^10*d^4 + 70*a^8*b^8*d^4 + 56*a^10*b^6*d^4 + 28*a^

12*b^4*d^4 + 8*a^14*b^2*d^4))*(e^3/(4*(b^6*d^2*1i - a^6*d^2*1i + 6*a*b^5*d^2 + 6*a^5*b*d^2 - a^2*b^4*d^2*15i - 20*a^3*b^3*d^2 + a^4*b^2*d^2*15i)))^(1/2) - ((e*cot(c + d*x))^(1/2)*(1544*a*b^18*d^2*e^13 + 64*a^3*b^16*d^2*e^13 - 7456*a^5*b^14*d^2*e^13 - 576*a^7*b^12*d^2*e^13 + 19504*a^9*b^10*d^2*e^13 + 18880*a^11*b^8*d^2*e^13 + 3808*a^13*b^6*d^2*e^13 - 960*a^15*b^4*d^2*e^13 + 8*a^17*b^2*d^2*e^13))/(a^16*d^4 + b^16*d^4 + 8*a^2*b^14*d^4 + 28*a^4*b^12*d^4 + 56*a^6*b^10*d^4 + 70*a^8*b^8*d^4 + 56*a^10*b^6*d^4 + 28*a^12*b^4*d^4 + 8*a^14*b^2*d^4))*(e^3/(4*(b^6*d^2*1i - a^6*d^2*1i + 6*a*b^5*d^2 + 6*a^5*b*d^2 - a^2*b^4*d^2*15i - 20*a^3*b^3*d^2 + a^4*b^2*d^2*15i)))^(1/2))*(e^3/(4*(b^6*d^2*1i - a^6*d^2*1i + 6*a*b^5*d^2 + 6*a^5*b*d^2 - a^2*b^4*d^2*15i - 20*a^3*b^3*d^2 + a^4*b^2*d^2*15i)))^(1/2) + ((e*cot(c + d*x))^(1/2)*(41*b^13*e^16 + 9*a^12*b*e^16 - 82*a^2*b^11*e^16 + 1831*a^4*b^9*e^16 - 4348*a^6*b^7*e^16 + 1671*a^8*b^5*e^16 - 210*a^10*b^3*e^16))/(a^16*d^4 + b^16*d^4 + 8*a^2*b^14*d^4 + 28*a^4*b^12*d^4 + 56*a^6*b^10*d^4 + 70*a^8*b^8*d^4 + 56*a^10*b^6*d^4 + 28*a^12*b^4*d^4 + 8*a^14*b^2*d^4))*(e^3/(4*(b^6*d^2*1i - a^6*d^2*1i + 6*a*b^5*d^2 + 6*a^5*b*d^2 - a^2*b^4*d^2*15i - 20*a^3*b^3*d^2 + a^4*b^2*d^2*15i)))^(1/2) + (((518*a*b^15*d^2*e^15 - 18*a^15*b*d^2*e^15 - 4494*a^3*b^13*d^2*e^15 + 3022*a^5*b^11*d^2*e^15 + 17194*a^7*b^9*d^2*e^15 + 5298*a^9*b^7*d^2*e^15 - 3338*a^11*b^5*d^2*e^15 + 506*a^13*b^3*d^2*e^15)/(a^16*d^5 + b^16*d^5 + 8*a^2*b^14*d^5 + 28*a^4*b^12*d^5 + 56*a^6*b^10*d^5 + 70*a^8*b^8*d^5 + 56*a^10*b^6*d^5 + 28*a^12*b^4*d^5 + 8*a^14*b^2*d^5) + (((4224*a^4*b^18*d^4*e^12 - 320*a^2*b^20*d^4*e^12 - 192*b^22*d^4*e^12 + 22272*a^6*b^16*d^4*e^12 + 51072*a^8*b^14*d^4*e^12 + 67200*a^10*b^12*d^4*e^12 + 53760*a^12*b^10*d^4*e^12 + 25344*a^14*b^8*d^4*e^12 + 5952*a^16*b^6*d^4*e^12 + 192*a^18*b^4*d^4*e^12 - 128*a^20*b^2*d^4*e^12)/(a^16*d^5 + b^16*d^5 + 8*a^2*b^14*d^5 + 28*a^4*b^12*d^5 + 56*a^6*b^10*d^5 + 70*a^8*b^8*d^5 + 56*a^10*b^6*d^5 + 28*a^12*b^4*d^5 + 8*a^14*b^2*d^5) - ((e*cot(c + d*x))^(1/2)*(e^3/(4*(b^6*d^2*1i - a^6*d^2*1i + 6*a*b^5*d^2 + 6*a^5*b*d^2 - a^2*b^4*d^2*15i - 20*a^3*b^3*d^2 + a^4*b^2*d^2*15i)))^(1/2)*(512*b^25*d^4*e^10 + 4608*a^2*b^23*d^4*e^10 + 17920*a^4*b^21*d^4*e^10 + 38400*a^6*b^19*d^4*e^10 + 46080*a^8*b^17*d^4*e^10 + 21504*a^10*b^15*d^4*e^10 - 21504*a^12*b^13*d^4*e^10 - 46080*a^14*b^11*d^4*e^10 - 38400*a^16*b^9*d^4*e^10 - 17920*a^18*b^7*d^4*e^10 - 4608*a^20*b^5*d^4*e^10 - 512*a^22*b^3*d^4*e^10))/(a^16*d^4 + b^16*d^4 + 8*a^2*b^14*d^4 + 28*a^4*b^12*d^4 + 56*a^6*b^10*d^4 + 70*a^8*b^8*d^4 + 56*a^10*b^6*d^4 + 28*a^12*b^4*d^4 + 8*a^14*b^2*d^4))*(e^3/(4*(b^6*d^2*1i - a^6*d^2*1i + 6*a*b^5*d^2 + 6*a^5*b*d^2 - a^2*b^4*d^2*15i - 20*a^3*b^3*d^2 + a^4*b^2*d^2*15i)))^(1/2) + ((e*cot(c + d*x))^(1/2)*(1544*a*b^18*d^2*e^13 + 64*a^3*b^16*d^2*e^13 - 7456*a^5*b^14*d^2*e^13 - 576*a^7*b^12*d^2*e^13 + 19504*a^9*b^10*d^2*e^13 + 18880*a^11*b^8*d^2*e^13 + 3808*a^13*b^6*d^2*e^13 - 960*a^15*b^4*d^2*e^13 + 8*a^17*b^2*d^2*e^13))/(a^16*d^4 + b^16*d^4 + 8*a^2*b^14*d^4 + 28*a^4*b^12*d^4 + 56*a^6*b^10*d^4 + 70*a^8*b^8*d^4 + 56*a^10*b^6*d^4 + 28*a^12*b^4*d^4 + 8*a^14*b^2*d^4))*(e^3/(4*(b^6*d^2*1i - a^6*d^2*1i + 6*a*b^5*d^2 + 6*a^5*b*d^2 - a^2*b^4*d^2*15i - 20*a^3*b^3*d^2 + a^4*b^2*d^2*15i)))^(1/2))*(e^3/(4*(b^6*d^2*1i - a^6*d^2*1i + 6*a*b^5*d^2 + 6*a^5*b*d^2 - a^2*b^4*d^2*15i - 20*a^3*b^3*d^2 + a^4*b^2*d^2*15i)))^(1/2) - ((e*cot(c + d*x))^(1/2)*(41*b^1

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3*e^16 + 9*a^12*b*e^16 - 82*a^2*b^11*e^16 + 1831*a^4*b^9*e^16 - 4348*a^6*b^
7*e^16 + 1671*a^8*b^5*e^16 - 210*a^10*b^3*e^16))/(a^16*d^4 + b^16*d^4 + 8*a
^2*b^14*d^4 + 28*a^4*b^12*d^4 + 56*a^6*b^10*d^4 + 70*a^8*b^8*d^4 + 56*a^10*
b^6*d^4 + 28*a^12*b^4*d^4 + 8*a^14*b^2*d^4))*(e^3/(4*(b^6*d^2*1i - a^6*d^2*
1i + 6*a*b^5*d^2 + 6*a^5*b*d^2 - a^2*b^4*d^2*15i - 20*a^3*b^3*d^2 + a^4*b^2
*d^2*15i)))^(1/2) + (28*a^2*b^8*e^18 - 15*b^10*e^18 + 878*a^4*b^6*e^18 - 18
0*a^6*b^4*e^18 + 9*a^8*b^2*e^18)/(a^16*d^5 + b^16*d^5 + 8*a^2*b^14*d^5 + 28
*a^4*b^12*d^5 + 56*a^6*b^10*d^5 + 70*a^8*b^8*d^5 + 56*a^10*b^6*d^5 + 28*a^1
2*b^4*d^5 + 8*a^14*b^2*d^5)))*(e^3/(4*(b^6*d^2*1i - a^6*d^2*1i + 6*a*b^5*d^
2 + 6*a^5*b*d^2 - a^2*b^4*d^2*15i - 20*a^3*b^3*d^2 + a^4*b^2*d^2*15i)))^(1/
2)*2i - (((e*cot(c + d*x))^(1/2)*(5*a^3*e^3 - 3*a*b^2*e^3))/(4*(a^4 + b^4 +
 2*a^2*b^2)) + (b*e^2*(e*cot(c + d*x))^(3/2)*(3*a^2 - 5*b^2))/(4*(a^4 + b^4
 + 2*a^2*b^2)))/(a^2*d*e^2 + b^2*d*e^2*cot(c + d*x)^2 + 2*a*b*d*e^2*cot(c +
 d*x)) + (atan((((((e*cot(c + d*x))^(1/2)*(41*b^13*e^16 + 9*a^12*b*e^16 - 8
2*a^2*b^11*e^16 + 1831*a^4*b^9*e^16 - 4348*a^6*b^7*e^16 + 1671*a^8*b^5*e^16
 - 210*a^10*b^3*e^16))/(a^16*d^4 + b^16*d^4 + 8*a^2*b^14*d^4 + 28*a^4*b^12*
d^4 + 56*a^6*b^10*d^4 + 70*a^8*b^8*d^4 + 56*a^10*b^6*d^4 + 28*a^12*b^4*d^4
+ 8*a^14*b^2*d^4) - (((518*a*b^15*d^2*e^15 - 18*a^15*b*d^2*e^15 - 4494*a^3*
b^13*d^2*e^15 + 3022*a^5*b^11*d^2*e^15 + 17194*a^7*b^9*d^2*e^15 + 5298*a^9*
b^7*d^2*e^15 - 3338*a^11*b^5*d^2*e^15 + 506*a^13*b^3*d^2*e^15)/(a^16*d^5 +
b^16*d^5 + 8*a^2*b^14*d^5 + 28*a^4*b^12*d^5 + 56*a^6*b^10*d^5 + 70*a^8*b^8*
d^5 + 56*a^10*b^6*d^5 + 28*a^12*b^4*d^5 + 8*a^14*b^2*d^5) + ((((e*cot(c + d
*x))^(1/2)*(1544*a*b^18*d^2*e^13 + 64*a^3*b^16*d^2*e^13 - 7456*a^5*b^14*d^2
*e^13 - 576*a^7*b^12*d^2*e^13 + 19504*a^9*b^10*d^2*e^13 + 18880*a^11*b^8*d^
2*e^13 + 3808*a^13*b^6*d^2*e^13 - 960*a^15*b^4*d^2*e^13 + 8*a^17*b^2*d^2*e^
13))/(a^16*d^4 + b^16*d^4 + 8*a^2*b^14*d^4 + 28*a^4*b^12*d^4 + 56*a^6*b^10*
d^4 + 70*a^8*b^8*d^4 + 56*a^10*b^6*d^4 + 28*a^12*b^4*d^4 + 8*a^14*b^2*d^4)
+ (((4224*a^4*b^18*d^4*e^12 - 320*a^2*b^20*d^4*e^12 - 192*b^22*d^4*e^12 + 2
2272*a^6*b^16*d^4*e^12 + 51072*a^8*b^14*d^4*e^12 + 67200*a^10*b^12*d^4*e^12
 + 53760*a^12*b^10*d^4*e^12 + 25344*a^14*b^8*d^4*e^12 + 5952*a^16*b^6*d^4*e
^12 + 192*a^18*b^4*d^4*e^12 - 128*a^20*b^2*d^4*e^12)/(a^16*d^5 + b^16*d^5 +
 8*a^2*b^14*d^5 + 28*a^4*b^12*d^5 + 56*a^6*b^10*d^5 + 70*a^8*b^8*d^5 + 56*a
^10*b^6*d^5 + 28*a^12*b^4*d^5 + 8*a^14*b^2*d^5) - ((e*cot(c + d*x))^(1/2)*(
3*a^4 + 3*b^4 - 26*a^2*b^2)*(-a*b*e^3)^(1/2)*(512*b^25*d^4*e^10 + 4608*a^2*
b^23*d^4*e^10 + 17920*a^4*b^21*d^4*e^10 + 38400*a^6*b^19*d^4*e^10 + 46080*a
^8*b^17*d^4*e^10 + 21504*a^10*b^15*d^4*e^10 - 21504*a^12*b^13*d^4*e^10 - 46
080*a^14*b^11*d^4*e^10 - 38400*a^16*b^9*d^4*e^10 - 17920*a^18*b^7*d^4*e^10
- 4608*a^20*b^5*d^4*e^10 - 512*a^22*b^3*d^4*e^10))/(8*(3*a^3*b^5*d + 3*a^5*
b^3*d + a*b^7*d + a^7*b*d)*(a^16*d^4 + b^16*d^4 + 8*a^2*b^14*d^4 + 28*a^4*b
^12*d^4 + 56*a^6*b^10*d^4 + 70*a^8*b^8*d^4 + 56*a^10*b^6*d^4 + 28*a^12*b^4*
d^4 + 8*a^14*b^2*d^4)))*(3*a^4 + 3*b^4 - 26*a^2*b^2)*(-a*b*e^3)^(1/2))/(8*(
3*a^3*b^5*d + 3*a^5*b^3*d + a*b^7*d + a^7*b*d)))*(3*a^4 + 3*b^4 - 26*a^2*b^
2)*(-a*b*e^3)^(1/2))/(8*(3*a^3*b^5*d + 3*a^5*b^3*d + a*b^7*d + a^7*b*d)))*(
3*a^4 + 3*b^4 - 26*a^2*b^2)*(-a*b*e^3)^(1/2))/(8*(3*a^3*b^5*d + 3*a^5*b^3*d
 + a*b^7*d + a^7*b*d)))*(3*a^4 + 3*b^4 - 26*a^2*b^2)*(-a*b*e^3)^(1/2)*1i)/(
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8*(3*a^3*b^5*d + 3*a^5*b^3*d + a*b^7*d + a^7*b*d)) + ((((e*cot(c + d*x))^(1/2)*(41*b^13*e^16 + 9*a^12*b*e^16 - 82*a^2*b^11*e^16 + 1831*a^4*b^9*e^16 - 4348*a^6*b^7*e^16 + 1671*a^8*b^5*e^16 - 210*a^10*b^3*e^16))/(a^16*d^4 + b^16*d^4 + 8*a^2*b^14*d^4 + 28*a^4*b^12*d^4 + 56*a^6*b^10*d^4 + 70*a^8*b^8*d^4 + 56*a^10*b^6*d^4 + 28*a^12*b^4*d^4 + 8*a^14*b^2*d^4) + (((518*a*b^15*d^2*e^15 - 18*a^15*b*d^2*e^15 - 4494*a^3*b^13*d^2*e^15 + 3022*a^5*b^11*d^2*e^15 + 17194*a^7*b^9*d^2*e^15 + 5298*a^9*b^7*d^2*e^15 - 3338*a^11*b^5*d^2*e^15 + 506*a^13*b^3*d^2*e^15)/(a^16*d^5 + b^16*d^5 + 8*a^2*b^14*d^5 + 28*a^4*b^12*d^5 + 56*a^6*b^10*d^5 + 70*a^8*b^8*d^5 + 56*a^10*b^6*d^5 + 28*a^12*b^4*d^5 + 8*a^14*b^2*d^5) - ((((e*cot(c + d*x))^(1/2)*(1544*a*b^18*d^2*e^13 + 64*a^3*b^16*d^2*e^13 - 7456*a^5*b^14*d^2*e^13 - 576*a^7*b^12*d^2*e^13 + 19504*a^9*b^10*d^2*e^13 + 18880*a^11*b^8*d^2*e^13 + 3808*a^13*b^6*d^2*e^13 - 960*a^15*b^4*d^2*e^13 + 8*a^17*b^2*d^2*e^13))/(a^16*d^4 + b^16*d^4 + 8*a^2*b^14*d^4 + 28*a^4*b^12*d^4 + 56*a^6*b^10*d^4 + 70*a^8*b^8*d^4 + 56*a^10*b^6*d^4 + 28*a^12*b^4*d^4 + 8*a^14*b^2*d^4) - (((4224*a^4*b^18*d^4*e^12 - 320*a^2*b^20*d^4*e^12 - 192*b^22*d^4*e^12 + 22272*a^6*b^16*d^4*e^12 + 51072*a^8*b^14*d^4*e^12 + 67200*a^10*b^12*d^4*e^12 + 53760*a^12*b^10*d^4*e^12 + 25344*a^14*b^8*d^4*e^12 + 5952*a^16*b^6*d^4*e^12 + 192*a^18*b^4*d^4*e^12 - 128*a^20*b^2*d^4*e^12)/(a^16*d^5 + b^16*d^5 + 8*a^2*b^14*d^5 + 28*a^4*b^12*d^5 + 56*a^6*b^10*d^5 + 70*a^8*b^8*d^5 + 56*a^10*b^6*d^5 + 28*a^12*b^4*d^5 + 8*a^14*b^2*d^5) + ((e*cot(c + d*x))^(1/2)*(3*a^4 + 3*b^4 - 26*a^2*b^2)*(-a*b*e^3)^(1/2)*(512*b^25*d^4*e^10 + 4608*a^2*b^23*d^4*e^10 + 17920*a^4*b^21*d^4*e^10 + 38400*a^6*b^19*d^4*e^10 + 46080*a^8*b^17*d^4*e^10 + 21504*a^10*b^15*d^4*e^10 - 21504*a^12*b^13*d^4*e^10 - 46080*a^14*b^11*d^4*e^10 - 38400*a^16*b^9*d^4*e^10 - 17920*a^18*b^7*d^4*e^10 - 4608*a^20*b^5*d^4*e^10 - 512*a^22*b^3*d^4*e^10))/(8*(3*a^3*b^5*d + 3*a^5*b^3*d + a*b^7*d + a^7*b*d)*(a^16*d^4 + b^16*d^4 + 8*a^2*b^14*d^4 + 28*a^4*b^12*d^4 + 56*a^6*b^10*d^4 + 70*a^8*b^8*d^4 + 56*a^10*b^6*d^4 + 28*a^12*b^4*d^4 + 8*a^14*b^2*d^4)))*(3*a^4 + 3*b^4 - 26*a^2*b^2)*(-a*b*e^3)^(1/2))/(8*(3*a^3*b^5*d + 3*a^5*b^3*d + a*b^7*d + a^7*b*d)))*(3*a^4 + 3*b^4 - 26*a^2*b^2)*(-a*b*e^3)^(1/2))/(8*(3*a^3*b^5*d + 3*a^5*b^3*d + a*b^7*d + a^7*b*d)))*(3*a^4 + 3*b^4 - 26*a^2*b^2)*(-a*b*e^3)^(1/2))/(8*(3*a^3*b^5*d + 3*a^5*b^3*d + a*b^7*d + a^7*b*d)))*(3*a^4 + 3*b^4 - 26*a^2*b^2)*(-a*b*e^3)^(1/2)*1i)/(8*(3*a^3*b^5*d + 3*a^5*b^3*d + a*b^7*d + a^7*b*d)))/((28*a^2*b^8*e^18 - 15*b^10*e^18 + 878*a^4*b^6*e^18 - 180*a^6*b^4*e^18 + 9*a^8*b^2*e^18)/(a^16*d^5 + b^16*d^5 + 8*a^2*b^14*d^5 + 28*a^4*b^12*d^5 + 56*a^6*b^10*d^5 + 70*a^8*b^8*d^5 + 56*a^10*b^6*d^5 + 28*a^12*b^4*d^5 + 8*a^14*b^2*d^5) - ((((e*cot(c + d*x))^(1/2)*(41*b^13*e^16 + 9*a^12*b*e^16 - 82*a^2*b^11*e^16 + 1831*a^4*b^9*e^16 - 4348*a^6*b^7*e^16 + 1671*a^8*b^5*e^16 - 210*a^10*b^3*e^16))/(a^16*d^4 + b^16*d^4 + 8*a^2*b^14*d^4 + 28*a^4*b^12*d^4 + 56*a^6*b^10*d^4 + 70*a^8*b^8*d^4 + 56*a^10*b^6*d^4 + 28*a^12*b^4*d^4 + 8*a^14*b^2*d^4) - (((518*a*b^15*d^2*e^15 - 18*a^15*b*d^2*e^15 - 4494*a^3*b^13*d^2*e^15 + 3022*a^5*b^11*d^2*e^15 + 17194*a^7*b^9*d^2*e^15 + 5298*a^9*b^7*d^2*e^15 - 3338*a^11*b^5*d^2*e^15 + 506*a^13*b^3*d^2*e^15)/(a^16*d^5 + b^16*d^5 + 8*a^2*b^14*d^5 + 28*a^4*b^12*d^5 + 56*a^6*b^10*d^5 + 70*a^8*b^8*d^5 + 56*a^10*b^6*d^5 + 28*a^12*b^4*d^5 + 8*a^14*b^2*d^5) + ((((e*

cot(c + d*x))^(1/2)*(1544*a*b^18*d^2*e^13 + 64*a^3*b^16*d^2*e^13 - 7456*a^5
*b^14*d^2*e^13 - 576*a^7*b^12*d^2*e^13 + 19504*a^9*b^10*d^2*e^13 + 18880*a^
11*b^8*d^2*e^13 + 3808*a^13*b^6*d^2*e^13 - 960*a^15*b^4*d^2*e^13 + 8*a^17*b
^2*d^2*e^13))/(a^16*d^4 + b^16*d^4 + 8*a^2*b^14*d^4 + 28*a^4*b^12*d^4 + 56*
a^6*b^10*d^4 + 70*a^8*b^8*d^4 + 56*a^10*b^6*d^4 + 28*a^12*b^4*d^4 + 8*a^14*
b^2*d^4) + (((4224*a^4*b^18*d^4*e^12 - 320*a^2*b^20*d^4*e^12 - 192*b^22*d^4
*e^12 + 22272*a^6*b^16*d^4*e^12 + 51072*a^8*b^14*d^4*e^12 + 67200*a^10*b^12
*d^4*e^12 + 53760*a^12*b^10*d^4*e^12 + 25344*a^14*b^8*d^4*e^12 + 5952*a^16*
b^6*d^4*e^12 + 192*a^18*b^4*d^4*e^12 - 128*a^20*b^2*d^4*e^12)/(a^16*d^5 + b
^16*d^5 + 8*a^2*b^14*d^5 + 28*a^4*b^12*d^5 + 56*a^6*b^10*d^5 + 70*a^8*b^8*d
^5 + 56*a^10*b^6*d^5 + 28*a^12*b^4*d^5 + 8*a^14*b^2*d^5) - ((e*cot(c + d*x)
)^(1/2)*(3*a^4 + 3*b^4 - 26*a^2*b^2)*(-a*b*e^3)^(1/2)*(512*b^25*d^4*e^10 +
4608*a^2*b^23*d^4*e^10 + 17920*a^4*b^21*d^4*e^10 + 38400*a^6*b^19*d^4*e^10
+ 46080*a^8*b^17*d^4*e^10 + 21504*a^10*b^15*d^4*e^10 - 21504*a^12*b^13*d^4*
e^10 - 46080*a^14*b^11*d^4*e^10 - 38400*a^16*b^9*d^4*e^10 - 17920*a^18*b^7*
d^4*e^10 - 4608*a^20*b^5*d^4*e^10 - 512*a^22*b^3*d^4*e^10))/(8*(3*a^3*b^5*d
 + 3*a^5*b^3*d + a*b^7*d + a^7*b*d)*(a^16*d^4 + b^16*d^4 + 8*a^2*b^14*d^4 +
 28*a^4*b^12*d^4 + 56*a^6*b^10*d^4 + 70*a^8*b^8*d^4 + 56*a^10*b^6*d^4 + 28*
a^12*b^4*d^4 + 8*a^14*b^2*d^4)))*(3*a^4 + 3*b^4 - 26*a^2*b^2)*(-a*b*e^3)^(1
/2))/(8*(3*a^3*b^5*d + 3*a^5*b^3*d + a*b^7*d + a^7*b*d)))*(3*a^4 + 3*b^4 -
26*a^2*b^2)*(-a*b*e^3)^(1/2))/(8*(3*a^3*b^5*d + 3*a^5*b^3*d + a*b^7*d + a^7
*b*d)))*(3*a^4 + 3*b^4 - 26*a^2*b^2)*(-a*b*e^3)^(1/2))/(8*(3*a^3*b^5*d + 3*
a^5*b^3*d + a*b^7*d + a^7*b*d)))*(3*a^4 + 3*b^4 - 26*a^2*b^2)*(-a*b*e^3)^(1
/2))/(8*(3*a^3*b^5*d + 3*a^5*b^3*d + a*b^7*d + a^7*b*d)) + ((((e*cot(c + d*
x))^(1/2)*(41*b^13*e^16 + 9*a^12*b*e^16 - 82*a^2*b^11*e^16 + 1831*a^4*b^9*e
^16 - 4348*a^6*b^7*e^16 + 1671*a^8*b^5*e^16 - 210*a^10*b^3*e^16))/(a^16*d^4
 + b^16*d^4 + 8*a^2*b^14*d^4 + 28*a^4*b^12*d^4 + 56*a^6*b^10*d^4 + 70*a^8*b
^8*d^4 + 56*a^10*b^6*d^4 + 28*a^12*b^4*d^4 + 8*a^14*b^2*d^4) + (((518*a*b^1
5*d^2*e^15 - 18*a^15*b*d^2*e^15 - 4494*a^3*b^13*d^2*e^15 + 3022*a^5*b^11*d^
2*e^15 + 17194*a^7*b^9*d^2*e^15 + 5298*a^9*b^7*d^2*e^15 - 3338*a^11*b^5*d^2
*e^15 + 506*a^13*b^3*d^2*e^15)/(a^16*d^5 + b^16*d^5 + 8*a^2*b^14*d^5 + 28*a
^4*b^12*d^5 + 56*a^6*b^10*d^5 + 70*a^8*b^8*d^5 + 56*a^10*b^6*d^5 + 28*a^12*
b^4*d^5 + 8*a^14*b^2*d^5) - ((((e*cot(c + d*x))^(1/2)*(1544*a*b^18*d^2*e^13
 + 64*a^3*b^16*d^2*e^13 - 7456*a^5*b^14*d^2*e^13 - 576*a^7*b^12*d^2*e^13 +
19504*a^9*b^10*d^2*e^13 + 18880*a^11*b^8*d^2*e^13 + 3808*a^13*b^6*d^2*e^13
- 960*a^15*b^4*d^2*e^13 + 8*a^17*b^2*d^2*e^13))/(a^16*d^4 + b^16*d^4 + 8*a^
2*b^14*d^4 + 28*a^4*b^12*d^4 + 56*a^6*b^10*d^4 + 70*a^8*b^8*d^4 + 56*a^10*b
^6*d^4 + 28*a^12*b^4*d^4 + 8*a^14*b^2*d^4) - (((4224*a^4*b^18*d^4*e^12 - 32
0*a^2*b^20*d^4*e^12 - 192*b^22*d^4*e^12 + 22272*a^6*b^16*d^4*e^12 + 51072*a
^8*b^14*d^4*e^12 + 67200*a^10*b^12*d^4*e^12 + 53760*a^12*b^10*d^4*e^12 + 25
344*a^14*b^8*d^4*e^12 + 5952*a^16*b^6*d^4*e^12 + 192*a^18*b^4*d^4*e^12 - 12
8*a^20*b^2*d^4*e^12)/(a^16*d^5 + b^16*d^5 + 8*a^2*b^14*d^5 + 28*a^4*b^12*d^
5 + 56*a^6*b^10*d^5 + 70*a^8*b^8*d^5 + 56*a^10*b^6*d^5 + 28*a^12*b^4*d^5 +
8*a^14*b^2*d^5) + ((e*cot(c + d*x))^(1/2)*(3*a^4 + 3*b^4 - 26*a^2*b^2)*(-a*
b*e^3)^(1/2)*(512*b^25*d^4*e^10 + 4608*a^2*b^23*d^4*e^10 + 17920*a^4*b^21*d

```
^4*e^10 + 38400*a^6*b^19*d^4*e^10 + 46080*a^8*b^17*d^4*e^10 + 21504*a^10*b^
15*d^4*e^10 - 21504*a^12*b^13*d^4*e^10 - 46080*a^14*b^11*d^4*e^10 - 38400*a
^16*b^9*d^4*e^10 - 17920*a^18*b^7*d^4*e^10 - 4608*a^20*b^5*d^4*e^10 - 512*a
^22*b^3*d^4*e^10))/(8*(3*a^3*b^5*d + 3*a^5*b^3*d + a*b^7*d + a^7*b*d)*(a^16
*d^4 + b^16*d^4 + 8*a^2*b^14*d^4 + 28*a^4*b^12*d^4 + 56*a^6*b^10*d^4 + 70*a
^8*b^8*d^4 + 56*a^10*b^6*d^4 + 28*a^12*b^4*d^4 + 8*a^14*b^2*d^4)))*(3*a^4 +
 3*b^4 - 26*a^2*b^2)*(-a*b*e^3)^(1/2))/(8*(3*a^3*b^5*d + 3*a^5*b^3*d + a*b^
7*d + a^7*b*d)))*(3*a^4 + 3*b^4 - 26*a^2*b^2)*(-a*b*e^3)^(1/2))/(8*(3*a^3*b
^5*d + 3*a^5*b^3*d + a*b^7*d + a^7*b*d)))*(3*a^4 + 3*b^4 - 26*a^2*b^2)*(-a*
b*e^3)^(1/2))/(8*(3*a^3*b^5*d + 3*a^5*b^3*d + a*b^7*d + a^7*b*d)))*(3*a^4 +
 3*b^4 - 26*a^2*b^2)*(-a*b*e^3)^(1/2))/(8*(3*a^3*b^5*d + 3*a^5*b^3*d + a*b^
7*d + a^7*b*d))))*(3*a^4 + 3*b^4 - 26*a^2*b^2)*(-a*b*e^3)^(1/2)*1i)/(4*(3*a
^3*b^5*d + 3*a^5*b^3*d + a*b^7*d + a^7*b*d))
```

3.85 $\int \frac{\sqrt{e\cot(c+dx)}}{(a+b\cot(c+dx))^3}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 463

$$\begin{aligned}
\int \frac{\sqrt{e\cot(c+dx)}}{(a+b\cot(c+dx))^3}\,dx &= \frac{\sqrt{b}(15a^4-18a^2b^2-b^4)\sqrt{e}\arctan\left(\frac{\sqrt{b}\sqrt{e\cot(c+dx)}}{\sqrt{a}\sqrt{e}}\right)}{4a^{3/2}(a^2+b^2)^3 d} \\
&+ \frac{(a-b)(a^2+4ab+b^2)\sqrt{e}\arctan\left(1-\frac{\sqrt{2}\sqrt{e\cot(c+dx)}}{\sqrt{e}}\right)}{\sqrt{2}(a^2+b^2)^3 d} \\
&- \frac{(a-b)(a^2+4ab+b^2)\sqrt{e}\arctan\left(1+\frac{\sqrt{2}\sqrt{e\cot(c+dx)}}{\sqrt{e}}\right)}{\sqrt{2}(a^2+b^2)^3 d} \\
&+ \frac{b\sqrt{e\cot(c+dx)}}{2(a^2+b^2)d(a+b\cot(c+dx))^2} + \frac{b(7a^2-b^2)\sqrt{e\cot(c+dx)}}{4a(a^2+b^2)^2 d(a+b\cot(c+dx))} \\
&- \frac{(a+b)(a^2-4ab+b^2)\sqrt{e}\log\left(\sqrt{e}+\sqrt{e}\cot(c+dx)-\sqrt{2}\sqrt{e\cot(c+dx)}\right)}{2\sqrt{2}(a^2+b^2)^3 d} \\
&+ \frac{(a+b)(a^2-4ab+b^2)\sqrt{e}\log\left(\sqrt{e}+\sqrt{e}\cot(c+dx)+\sqrt{2}\sqrt{e\cot(c+dx)}\right)}{2\sqrt{2}(a^2+b^2)^3 d}
\end{aligned}$$

```
[Out] 1/2*(a-b)*(a^2+4*a*b+b^2)*arctan(1-2^(1/2)*(e*cot(d*x+c))^(1/2)/e^(1/2))*e^
(1/2)/(a^2+b^2)^3/d*2^(1/2)-1/2*(a-b)*(a^2+4*a*b+b^2)*arctan(1+2^(1/2)*(e*c
ot(d*x+c))^(1/2)/e^(1/2))*e^(1/2)/(a^2+b^2)^3/d*2^(1/2)-1/4*(a+b)*(a^2-4*a*
b+b^2)*ln(e^(1/2)+cot(d*x+c)*e^(1/2)-2^(1/2)*(e*cot(d*x+c))^(1/2))*e^(1/2)/
(a^2+b^2)^3/d*2^(1/2)+1/4*(a+b)*(a^2-4*a*b+b^2)*ln(e^(1/2)+cot(d*x+c)*e^(1/
2)+2^(1/2)*(e*cot(d*x+c))^(1/2))*e^(1/2)/(a^2+b^2)^3/d*2^(1/2)+1/4*(15*a^4-
18*a^2*b^2-b^4)*arctan(b^(1/2)*(e*cot(d*x+c))^(1/2)/a^(1/2)/e^(1/2))*b^(1/2
```

```
)*e^(1/2)/a^(3/2)/(a^2+b^2)^3/d+1/2*b*(e*cot(d*x+c))^(1/2)/(a^2+b^2)/d/(a+b
*cot(d*x+c))^2+1/4*b*(7*a^2-b^2)*(e*cot(d*x+c))^(1/2)/a/(a^2+b^2)^2/d/(a+b*
cot(d*x+c))
```

Rubi [A] (verified)

Time = 1.29 (sec) , antiderivative size = 463, normalized size of antiderivative = 1.00, number of steps used = 16, number of rules used = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.520, Rules used = {3649, 3730, 3734, 3615, 1182, 1176, 631, 210, 1179, 642, 3715, 65, 211}

$$\int \frac{\sqrt{e\cot(c+dx)}}{(a+b\cot(c+dx))^3}\,dx$$
$$= \frac{\sqrt{e}(a-b)\left(a^2+4ab+b^2\right)\arctan\left(1-\frac{\sqrt{2}\sqrt{e\cot(c+dx)}}{\sqrt{e}}\right)}{\sqrt{2}d\left(a^2+b^2\right)^3} - \frac{\sqrt{e}(a-b)\left(a^2+4ab+b^2\right)\arctan\left(\frac{\sqrt{2}\sqrt{e\cot(c+dx)}}{\sqrt{e}}+1\right)}{\sqrt{2}d\left(a^2+b^2\right)^3} + \frac{b(7a^2-b^2)\sqrt{e\cot(c+dx)}}{4ad\left(a^2+b^2\right)^2(a+b\cot(c+dx))} + \frac{b\sqrt{e\cot(c+dx)}}{2d\left(a^2+b^2\right)(a+b\cot(c+dx))^2} - \frac{\sqrt{e}(a+b)\left(a^2-4ab+b^2\right)\log\left(\sqrt{e}\cot(c+dx)-\sqrt{2}\sqrt{e\cot(c+dx)}+\sqrt{e}\right)}{2\sqrt{2}d\left(a^2+b^2\right)^3} + \frac{\sqrt{e}(a+b)\left(a^2-4ab+b^2\right)\log\left(\sqrt{e}\cot(c+dx)+\sqrt{2}\sqrt{e\cot(c+dx)}+\sqrt{e}\right)}{2\sqrt{2}d\left(a^2+b^2\right)^3} + \frac{\sqrt{b}\sqrt{e}(15a^4-18a^2b^2-b^4)\arctan\left(\frac{\sqrt{b}\sqrt{e\cot(c+dx)}}{\sqrt{a}\sqrt{e}}\right)}{4a^{3/2}d\left(a^2+b^2\right)^3}$$

```
[In] Int[Sqrt[e*Cot[c + d*x]]/(a + b*Cot[c + d*x])^3,x]
```

```
[Out] (Sqrt[b]*(15*a^4 - 18*a^2*b^2 - b^4)*Sqrt[e]*ArcTan[(Sqrt[b]*Sqrt[e*Cot[c +
 d*x]])/(Sqrt[a]*Sqrt[e])])/(4*a^(3/2)*(a^2 + b^2)^3*d) + ((a - b)*(a^2 + 4
*a*b + b^2)*Sqrt[e]*ArcTan[1 - (Sqrt[2]*Sqrt[e*Cot[c + d*x]])/Sqrt[e]])/(Sq
rt[2]*(a^2 + b^2)^3*d) - ((a - b)*(a^2 + 4*a*b + b^2)*Sqrt[e]*ArcTan[1 + (S
qrt[2]*Sqrt[e*Cot[c + d*x]])/Sqrt[e]])/(Sqrt[2]*(a^2 + b^2)^3*d) + (b*Sqrt[
e*Cot[c + d*x]])/(2*(a^2 + b^2)*d*(a + b*Cot[c + d*x])^2) + (b*(7*a^2 - b^2
)*Sqrt[e*Cot[c + d*x]])/(4*a*(a^2 + b^2)^2*d*(a + b*Cot[c + d*x])) - ((a +
b)*(a^2 - 4*a*b + b^2)*Sqrt[e]*Log[Sqrt[e] + Sqrt[e]*Cot[c + d*x] - Sqrt[2]
*Sqrt[e*Cot[c + d*x]]])/(2*Sqrt[2]*(a^2 + b^2)^3*d) + ((a + b)*(a^2 - 4*a*b
 + b^2)*Sqrt[e]*Log[Sqrt[e] + Sqrt[e]*Cot[c + d*x] + Sqrt[2]*Sqrt[e*Cot[c +
 d*x]]])/(2*Sqrt[2]*(a^2 + b^2)^3*d)
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1179

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 1182

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
a*c, 2]}, Dist[(d*q + a*e)/(2*a*c), Int[(q + c*x^2)/(a + c*x^4), x], x] + D
ist[(d*q - a*e)/(2*a*c), Int[(q - c*x^2)/(a + c*x^4), x], x]] /; FreeQ[{a,
c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0] && NeQ[c*d^2 - a*e^2, 0] && NegQ[(-a)
*c]
```

Rule 3615

```
Int[((c_) + (d_.)*tan[(e_.) + (f_.)*(x_)])/Sqrt[(b_.)*tan[(e_.) + (f_.)*(x_
)]], x_Symbol] :> Dist[2/f, Subst[Int[(b*c + d*x^2)/(b^2 + x^4), x], x, Sqr
t[b*Tan[e + f*x]]], x] /; FreeQ[{b, c, d, e, f}, x] && NeQ[c^2 - d^2, 0] &&
 NeQ[c^2 + d^2, 0]
```

Rule 3649

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)])^(n_), x_Symbol] :> Simp[b*(a + b*Tan[e + f*x])^(m + 1)*((c +
d*Tan[e + f*x])^n/(f*(m + 1)*(a^2 + b^2))), x] + Dist[1/((m + 1)*(a^2 + b^2
)), Int[(a + b*Tan[e + f*x])^(m + 1)*(c + d*Tan[e + f*x])^(n - 1)*Simp[a*c*
(m + 1) - b*d*n - (b*c - a*d)*(m + 1)*Tan[e + f*x] - b*d*(m + n + 1)*Tan[e
+ f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] &
& NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] && LtQ[m, -1] && GtQ[n, 0] && Inte
gerQ[2*m]
```

Rule 3715

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*tan[(e_.)
+ (f_.)*(x_)])^(n_.)*((A_) + (C_.)*tan[(e_.) + (f_.)*(x_)]^2), x_Symbol] :>
 Dist[A/f, Subst[Int[(a + b*x)^m*(c + d*x)^n, x], x, Tan[e + f*x]], x] /; F
reeQ[{a, b, c, d, e, f, A, C, m, n}, x] && EqQ[A, C]
```

Rule 3730

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*tan[(e_.) + (f_.)*(x_)] + (C_.)*tan[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> Simp[(A*b^2 - a*(b*B - a*C))*(a + b*Tan[e +
 f*x])^(m + 1)*((c + d*Tan[e + f*x])^(n + 1)/(f*(m + 1)*(b*c - a*d)*(a^2 +
b^2))), x] + Dist[1/((m + 1)*(b*c - a*d)*(a^2 + b^2)), Int[(a + b*Tan[e + f
*x])^(m + 1)*(c + d*Tan[e + f*x])^n*Simp[A*(a*(b*c - a*d)*(m + 1) - b^2*d*(
m + n + 2)) + (b*B - a*C)*(b*c*(m + 1) + a*d*(n + 1)) - (m + 1)*(b*c - a*d)
*(A*b - a*B - b*C)*Tan[e + f*x] - d*(A*b^2 - a*(b*B - a*C))*(m + n + 2)*Tan
[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, n}, x] && NeQ[
b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] && LtQ[m, -1] &&  !
(ILtQ[n, -1] && ( !IntegerQ[m] || (EqQ[c, 0] && NeQ[a, 0])))
```

Rule 3734

```
Int[(((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_)*((A_.) + (B_.)*tan[(e_.)
+ (f_.)*(x_)] + (C_.)*tan[(e_.) + (f_.)*(x_)]^2))/((a_.) + (b_.)*tan[(e_.)
+ (f_.)*(x_)]), x_Symbol] :> Dist[1/(a^2 + b^2), Int[(c + d*Tan[e + f*x])^n
*Simp[b*B + a*(A - C) + (a*B - b*(A - C))*Tan[e + f*x], x], x], x] + Dist[(
A*b^2 - a*b*B + a^2*C)/(a^2 + b^2), Int[(c + d*Tan[e + f*x])^n*((1 + Tan[e
+ f*x]^2)/(a + b*Tan[e + f*x])), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C
, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] &&
 !GtQ[n, 0] &&  !LeQ[n, -1]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{b\sqrt{e\cot(c+dx)}}{2(a^2+b^2)\,d(a+b\cot(c+dx))^2} - \frac{\int \frac{-\frac{be}{2}-2ae\cot(c+dx)+\frac{3}{2}be\cot^2(c+dx)}{\sqrt{e\cot(c+dx)}(a+b\cot(c+dx))^2}\,dx}{2(a^2+b^2)} \\
&= \frac{b\sqrt{e\cot(c+dx)}}{2(a^2+b^2)\,d(a+b\cot(c+dx))^2} + \frac{b(7a^2-b^2)\sqrt{e\cot(c+dx)}}{4a(a^2+b^2)^2\,d(a+b\cot(c+dx))} \\
&\quad + \frac{\int \frac{\frac{1}{4}b(9a^2+b^2)e^2+2a(a^2-b^2)e^2\cot(c+dx)-\frac{1}{4}b(7a^2-b^2)e^2\cot^2(c+dx)}{\sqrt{e\cot(c+dx)}(a+b\cot(c+dx))}\,dx}{2a(a^2+b^2)^2 e} \\
&= \frac{b\sqrt{e\cot(c+dx)}}{2(a^2+b^2)\,d(a+b\cot(c+dx))^2} + \frac{b(7a^2-b^2)\sqrt{e\cot(c+dx)}}{4a(a^2+b^2)^2\,d(a+b\cot(c+dx))} \\
&\quad + \frac{\int \frac{2ab(3a^2-b^2)e^2+2a^2(a^2-3b^2)e^2\cot(c+dx)}{\sqrt{e\cot(c+dx)}}\,dx}{2a(a^2+b^2)^3 e} \\
&\quad - \frac{(b(15a^4-18a^2b^2-b^4)e)\int \frac{1+\cot^2(c+dx)}{\sqrt{e\cot(c+dx)}(a+b\cot(c+dx))}\,dx}{8a(a^2+b^2)^3} \\
&= \frac{b\sqrt{e\cot(c+dx)}}{2(a^2+b^2)\,d(a+b\cot(c+dx))^2} + \frac{b(7a^2-b^2)\sqrt{e\cot(c+dx)}}{4a(a^2+b^2)^2\,d(a+b\cot(c+dx))} \\
&\quad + \frac{\text{Subst}\left(\int \frac{-2ab(3a^2-b^2)e^3-2a^2(a^2-3b^2)e^2x^2}{e^2+x^4}\,dx, x, \sqrt{e\cot(c+dx)}\right)}{a(a^2+b^2)^3 de} \\
&\quad - \frac{(b(15a^4-18a^2b^2-b^4)e)\,\text{Subst}\left(\int \frac{1}{\sqrt{-ex}(a-bx)}\,dx, x, -\cot(c+dx)\right)}{8a(a^2+b^2)^3 d}
\end{aligned}$$

$$= \frac{b\sqrt{e\cot(c+dx)}}{2(a^2+b^2)d(a+b\cot(c+dx))^2} + \frac{b(7a^2-b^2)\sqrt{e\cot(c+dx)}}{4a(a^2+b^2)^2d(a+b\cot(c+dx))}$$
$$+ \frac{(b(15a^4-18a^2b^2-b^4))\,\mathrm{Subst}\left(\int \frac{1}{a+\frac{bx^2}{e}}\,dx, x, \sqrt{e\cot(c+dx)}\right)}{4a(a^2+b^2)^3d}$$
$$+ \frac{((a+b)(a^2-4ab+b^2)e)\,\mathrm{Subst}\left(\int \frac{e-x^2}{e^2+x^4}\,dx, x, \sqrt{e\cot(c+dx)}\right)}{(a^2+b^2)^3d}$$
$$- \frac{((a-b)(a^2+4ab+b^2)e)\,\mathrm{Subst}\left(\int \frac{e+x^2}{e^2+x^4}\,dx, x, \sqrt{e\cot(c+dx)}\right)}{(a^2+b^2)^3d}$$

$$= \frac{\sqrt{b}(15a^4-18a^2b^2-b^4)\sqrt{e}\arctan\left(\frac{\sqrt{b}\sqrt{e\cot(c+dx)}}{\sqrt{a}\sqrt{e}}\right)}{4a^{3/2}(a^2+b^2)^3d}$$
$$+ \frac{b\sqrt{e\cot(c+dx)}}{2(a^2+b^2)d(a+b\cot(c+dx))^2} + \frac{b(7a^2-b^2)\sqrt{e\cot(c+dx)}}{4a(a^2+b^2)^2d(a+b\cot(c+dx))}$$
$$- \frac{((a+b)(a^2-4ab+b^2)\sqrt{e})\,\mathrm{Subst}\left(\int \frac{\sqrt{2}\sqrt{e}+2x}{-e-\sqrt{2}\sqrt{e}x-x^2}\,dx, x, \sqrt{e\cot(c+dx)}\right)}{2\sqrt{2}(a^2+b^2)^3d}$$
$$- \frac{((a+b)(a^2-4ab+b^2)\sqrt{e})\,\mathrm{Subst}\left(\int \frac{\sqrt{2}\sqrt{e}-2x}{-e+\sqrt{2}\sqrt{e}x-x^2}\,dx, x, \sqrt{e\cot(c+dx)}\right)}{2\sqrt{2}(a^2+b^2)^3d}$$
$$- \frac{((a-b)(a^2+4ab+b^2)e)\,\mathrm{Subst}\left(\int \frac{1}{e-\sqrt{2}\sqrt{e}x+x^2}\,dx, x, \sqrt{e\cot(c+dx)}\right)}{2(a^2+b^2)^3d}$$
$$- \frac{((a-b)(a^2+4ab+b^2)e)\,\mathrm{Subst}\left(\int \frac{1}{e+\sqrt{2}\sqrt{e}x+x^2}\,dx, x, \sqrt{e\cot(c+dx)}\right)}{2(a^2+b^2)^3d}$$

$$= \frac{\sqrt{b}(15a^4-18a^2b^2-b^4)\sqrt{e}\arctan\left(\frac{\sqrt{b}\sqrt{e\cot(c+dx)}}{\sqrt{a}\sqrt{e}}\right)}{4a^{3/2}(a^2+b^2)^3 d} + \frac{b\sqrt{e\cot(c+dx)}}{2(a^2+b^2)d(a+b\cot(c+dx))^2} + \frac{b(7a^2-b^2)\sqrt{e\cot(c+dx)}}{4a(a^2+b^2)^2 d(a+b\cot(c+dx))} - \frac{(a+b)(a^2-4ab+b^2)\sqrt{e}\log\left(\sqrt{e}+\sqrt{e}\cot(c+dx)-\sqrt{2}\sqrt{e\cot(c+dx)}\right)}{2\sqrt{2}(a^2+b^2)^3 d} + \frac{(a+b)(a^2-4ab+b^2)\sqrt{e}\log\left(\sqrt{e}+\sqrt{e}\cot(c+dx)+\sqrt{2}\sqrt{e\cot(c+dx)}\right)}{2\sqrt{2}(a^2+b^2)^3 d} - \frac{\left((a-b)(a^2+4ab+b^2)\sqrt{e}\right)\text{Subst}\left(\int\frac{1}{-1-x^2}dx, x, 1-\frac{\sqrt{2}\sqrt{e\cot(c+dx)}}{\sqrt{e}}\right)}{\sqrt{2}(a^2+b^2)^3 d} + \frac{\left((a-b)(a^2+4ab+b^2)\sqrt{e}\right)\text{Subst}\left(\int\frac{1}{-1-x^2}dx, x, 1+\frac{\sqrt{2}\sqrt{e\cot(c+dx)}}{\sqrt{e}}\right)}{\sqrt{2}(a^2+b^2)^3 d}$$

$$= \frac{\sqrt{b}(15a^4-18a^2b^2-b^4)\sqrt{e}\arctan\left(\frac{\sqrt{b}\sqrt{e\cot(c+dx)}}{\sqrt{a}\sqrt{e}}\right)}{4a^{3/2}(a^2+b^2)^3 d} + \frac{(a-b)(a^2+4ab+b^2)\sqrt{e}\arctan\left(1-\frac{\sqrt{2}\sqrt{e\cot(c+dx)}}{\sqrt{e}}\right)}{\sqrt{2}(a^2+b^2)^3 d} - \frac{(a-b)(a^2+4ab+b^2)\sqrt{e}\arctan\left(1+\frac{\sqrt{2}\sqrt{e\cot(c+dx)}}{\sqrt{e}}\right)}{\sqrt{2}(a^2+b^2)^3 d} + \frac{b\sqrt{e\cot(c+dx)}}{2(a^2+b^2)d(a+b\cot(c+dx))^2} + \frac{b(7a^2-b^2)\sqrt{e\cot(c+dx)}}{4a(a^2+b^2)^2 d(a+b\cot(c+dx))} - \frac{(a+b)(a^2-4ab+b^2)\sqrt{e}\log\left(\sqrt{e}+\sqrt{e}\cot(c+dx)-\sqrt{2}\sqrt{e\cot(c+dx)}\right)}{2\sqrt{2}(a^2+b^2)^3 d} + \frac{(a+b)(a^2-4ab+b^2)\sqrt{e}\log\left(\sqrt{e}+\sqrt{e}\cot(c+dx)+\sqrt{2}\sqrt{e\cot(c+dx)}\right)}{2\sqrt{2}(a^2+b^2)^3 d}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 6.22 (sec) , antiderivative size = 481, normalized size of antiderivative = 1.04

$$\int \frac{\sqrt{e\cot(c+dx)}}{(a+b\cot(c+dx))^3}\,dx = -\frac{\sqrt{e\cot(c+dx)}\left(-\frac{2\sqrt{a}\sqrt{b}(3a^2-b^2)\arctan\left(\frac{\sqrt{b}\sqrt{\cot(c+dx)}}{\sqrt{a}}\right)}{(a^2+b^2)^3} + \frac{2b(3a^2-b^2)\sqrt{\cot(c+dx)}}{(a^2+b^2)^3} - \frac{2\sqrt{a}\sqrt{b}\left(-a\arctan\left(\frac{\sqrt{b}\sqrt{\cot(c+dx)}}{\sqrt{a}}\right)+\sqrt{\cdots}\right.}{(a^2+\cdots}\right.}{\cdots}$$

```
[In] Integrate[Sqrt[e*Cot[c + d*x]]/(a + b*Cot[c + d*x])^3,x]
```

```
[Out] -((Sqrt[e*Cot[c + d*x]]*((-2*Sqrt[a]*Sqrt[b]*(3*a^2 - b^2)*ArcTan[(Sqrt[b]*
Sqrt[Cot[c + d*x]])/Sqrt[a]])/(a^2 + b^2)^3 + (2*b*(3*a^2 - b^2)*Sqrt[Cot[c
 + d*x]])/(a^2 + b^2)^3 - (2*Sqrt[a]*Sqrt[b]*(-(a*ArcTan[(Sqrt[b]*Sqrt[Cot[
c + d*x]])/Sqrt[a]]) + Sqrt[a]*Sqrt[b]*Sqrt[Cot[c + d*x]] - b*ArcTan[(Sqrt[
b]*Sqrt[Cot[c + d*x]])/Sqrt[a]]*Cot[c + d*x]))/((a^2 + b^2)^2*(a + b*Cot[c
+ d*x])) + (2*a*(a^2 - 3*b^2)*Cot[c + d*x]^(3/2)*Hypergeometric2F1[3/4, 1,
7/4, -Cot[c + d*x]^2])/(3*(a^2 + b^2)^3) + (2*b^2*Cot[c + d*x]^(3/2)*Hyperg
eometric2F1[3/2, 3, 5/2, -((b*Cot[c + d*x])/a)])/(3*a^3*(a^2 + b^2)) - (b*(
3*a^2 - b^2)*(2*Sqrt[2]*ArcTan[1 - Sqrt[2]*Sqrt[Cot[c + d*x]]] - 2*Sqrt[2]*
ArcTan[1 + Sqrt[2]*Sqrt[Cot[c + d*x]]] + 8*Sqrt[Cot[c + d*x]] + Sqrt[2]*Log
[1 - Sqrt[2]*Sqrt[Cot[c + d*x]] + Cot[c + d*x]] - Sqrt[2]*Log[1 + Sqrt[2]*S
qrt[Cot[c + d*x]] + Cot[c + d*x]]))/(4*(a^2 + b^2)^3)))/(d*Sqrt[Cot[c + d*x
]]))
```

Maple [A] (verified)

Time = 0.05 (sec) , antiderivative size = 460, normalized size of antiderivative = 0.99

method	result
derivativedivides	$-\frac{2e^4\left(-\frac{b\left(\frac{\frac{b(7a^4+6a^2b^2-b^4)(e\cot(dx+c))^{\frac{3}{2}}}{8a}+\frac{e(9a^4+10a^2b^2+b^4)\sqrt{e\cot(dx+c)}}{8}}{(e\cot(dx+c)b+ae)^2}+\frac{(15a^4-18a^2b^2-b^4)\arctan\left(\frac{\sqrt{e\cot(dx+c)}\,b}{\sqrt{aeb}}\right.}{8a\sqrt{aeb}}\right.}{e^3(a^2+b^2)^3}\right.}{}$
default	$-\frac{2e^4\left(-\frac{b\left(\frac{\frac{b(7a^4+6a^2b^2-b^4)(e\cot(dx+c))^{\frac{3}{2}}}{8a}+\frac{e(9a^4+10a^2b^2+b^4)\sqrt{e\cot(dx+c)}}{8}}{(e\cot(dx+c)b+ae)^2}+\frac{(15a^4-18a^2b^2-b^4)\arctan\left(\frac{\sqrt{e\cot(dx+c)}\,b}{\sqrt{aeb}}\right.}{8a\sqrt{aeb}}\right.}{e^3(a^2+b^2)^3}\right.}{}$

```
[In] int((e*cot(d*x+c))^(1/2)/(a+b*cot(d*x+c))^3,x,method=_RETURNVERBOSE)
```

```
[Out] -2/d*e^4*(-b/e^3/(a^2+b^2)^3*((1/8*b*(7*a^4+6*a^2*b^2-b^4)/a*(e*cot(d*x+c))
^(3/2)+1/8*e*(9*a^4+10*a^2*b^2+b^4)*(e*cot(d*x+c))^(1/2))/(e*cot(d*x+c)*b+a
*e)^2+1/8*(15*a^4-18*a^2*b^2-b^4)/a/(a*e*b)^(1/2)*arctan((e*cot(d*x+c))^(1/
2)*b/(a*e*b)^(1/2)))+1/(a^2+b^2)^3/e^3*(1/8*(3*a^2*b*e-b^3*e)*(e^2)^(1/4)/e
^2*2^(1/2)*(ln((e*cot(d*x+c)+(e^2)^(1/4)*(e*cot(d*x+c))^(1/2)*2^(1/2)+(e^2)
^(1/2))/(e*cot(d*x+c)-(e^2)^(1/4)*(e*cot(d*x+c))^(1/2)*2^(1/2)+(e^2)^(1/2))
)+2*arctan(2^(1/2)/(e^2)^(1/4)*(e*cot(d*x+c))^(1/2)+1)-2*arctan(-2^(1/2)/(e
```

```
^2)^(1/4)*(e*cot(d*x+c))^(1/2)+1))+1/8*(a^3-3*a*b^2)/(e^2)^(1/4)*2^(1/2)*(l
n((e*cot(d*x+c)-(e^2)^(1/4)*(e*cot(d*x+c))^(1/2)*2^(1/2)+(e^2)^(1/2))/(e*co
t(d*x+c)+(e^2)^(1/4)*(e*cot(d*x+c))^(1/2)*2^(1/2)+(e^2)^(1/2)))+2*arctan(2^
(1/2)/(e^2)^(1/4)*(e*cot(d*x+c))^(1/2)+1)-2*arctan(-2^(1/2)/(e^2)^(1/4)*(e*
cot(d*x+c))^(1/2)+1)))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 4405 vs. $2(392) = 784$.

Time = 0.93 (sec) , antiderivative size = 8853, normalized size of antiderivative = 19.12

$$\int \frac{\sqrt{e\cot(c+dx)}}{(a+b\cot(c+dx))^3}\,dx = \text{Too large to display}$$

```
[In] integrate((e*cot(d*x+c))^(1/2)/(a+b*cot(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] Too large to include
```

Sympy [F]

$$\int \frac{\sqrt{e\cot(c+dx)}}{(a+b\cot(c+dx))^3}\,dx = \int \frac{\sqrt{e\cot{(c+dx)}}}{(a+b\cot{(c+dx)})^3}\,dx$$

```
[In] integrate((e*cot(d*x+c))**(1/2)/(a+b*cot(d*x+c))**3,x)
```

```
[Out] Integral(sqrt(e*cot(c + d*x))/(a + b*cot(c + d*x))**3, x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{\sqrt{e\cot(c+dx)}}{(a+b\cot(c+dx))^3}\,dx = \text{Exception raised: ValueError}$$

```
[In] integrate((e*cot(d*x+c))^(1/2)/(a+b*cot(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(e>0)', see `assume?` for more detai
ls)Is e
```

Giac [F]

$$\int \frac{\sqrt{e\cot(c+dx)}}{(a+b\cot(c+dx))^3}\,dx = \int \frac{\sqrt{e\cot(dx+c)}}{(b\cot(dx+c)+a)^3}\,dx$$

```
[In] integrate((e*cot(d*x+c))^(1/2)/(a+b*cot(d*x+c))^3,x, algorithm="giac")
[Out] integrate(sqrt(e*cot(d*x + c))/(b*cot(d*x + c) + a)^3, x)
```

Mupad [B] (verification not implemented)

Time = 18.85 (sec) , antiderivative size = 19534, normalized size of antiderivative = 42.19

$$\int \frac{\sqrt{e\cot(c+dx)}}{(a+b\cot(c+dx))^3}\,dx = \text{Too large to display}$$

```
[In] int((e*cot(c + d*x))^(1/2)/(a + b*cot(c + d*x))^3,x)
[Out] (((e*cot(c + d*x))^(1/2)*(b^3*e^2 + 9*a^2*b*e^2))/(4*(a^4 + b^4 + 2*a^2*b^2
)) + (b^2*e*(e*cot(c + d*x))^(3/2)*(7*a^2 - b^2))/(4*a*(a^4 + b^4 + 2*a^2*b
^2)))/(a^2*d*e^2 + b^2*d*e^2*cot(c + d*x)^2 + 2*a*b*d*e^2*cot(c + d*x)) - a
tan((((((64*a*b^23*d^4*e^11 + 1472*a^3*b^21*d^4*e^11 + 8832*a^5*b^19*d^4*e
^11 + 25344*a^7*b^17*d^4*e^11 + 40320*a^9*b^15*d^4*e^11 + 34944*a^11*b^13*d
^4*e^11 + 10752*a^13*b^11*d^4*e^11 - 8448*a^15*b^9*d^4*e^11 - 10176*a^17*b^
7*d^4*e^11 - 4160*a^19*b^5*d^4*e^11 - 640*a^21*b^3*d^4*e^11)/(a^18*d^5 + a^
2*b^16*d^5 + 8*a^4*b^14*d^5 + 28*a^6*b^12*d^5 + 56*a^8*b^10*d^5 + 70*a^10*b
^8*d^5 + 56*a^12*b^6*d^5 + 28*a^14*b^4*d^5 + 8*a^16*b^2*d^5) + ((e*cot(c +
d*x))^(1/2)*(-e/(4*(b^6*d^2*1i - a^6*d^2*1i + 6*a*b^5*d^2 + 6*a^5*b*d^2 - a
^2*b^4*d^2*15i - 20*a^3*b^3*d^2 + a^4*b^2*d^2*15i)))^(1/2)*(512*a^2*b^25*d^
4*e^10 + 4608*a^4*b^23*d^4*e^10 + 17920*a^6*b^21*d^4*e^10 + 38400*a^8*b^19*
d^4*e^10 + 46080*a^10*b^17*d^4*e^10 + 21504*a^12*b^15*d^4*e^10 - 21504*a^14
*b^13*d^4*e^10 - 46080*a^16*b^11*d^4*e^10 - 38400*a^18*b^9*d^4*e^10 - 17920
*a^20*b^7*d^4*e^10 - 4608*a^22*b^5*d^4*e^10 - 512*a^24*b^3*d^4*e^10))/(a^18
*d^4 + a^2*b^16*d^4 + 8*a^4*b^14*d^4 + 28*a^6*b^12*d^4 + 56*a^8*b^10*d^4 +
70*a^10*b^8*d^4 + 56*a^12*b^6*d^4 + 28*a^14*b^4*d^4 + 8*a^16*b^2*d^4))*(-e/
(4*(b^6*d^2*1i - a^6*d^2*1i + 6*a*b^5*d^2 + 6*a^5*b*d^2 - a^2*b^4*d^2*15i -
 20*a^3*b^3*d^2 + a^4*b^2*d^2*15i)))^(1/2) - ((e*cot(c + d*x))^(1/2)*(8*a*b
^20*d^2*e^11 - 1152*a^3*b^18*d^2*e^11 + 2528*a^5*b^16*d^2*e^11 + 15296*a^7*
b^14*d^2*e^11 + 14128*a^9*b^12*d^2*e^11 - 5056*a^11*b^10*d^2*e^11 - 9248*a^
13*b^8*d^2*e^11 + 64*a^15*b^6*d^2*e^11 + 1800*a^17*b^4*d^2*e^11 + 64*a^19*b
^2*d^2*e^11))/(a^18*d^4 + a^2*b^16*d^4 + 8*a^4*b^14*d^4 + 28*a^6*b^12*d^4 +
 56*a^8*b^10*d^4 + 70*a^10*b^8*d^4 + 56*a^12*b^6*d^4 + 28*a^14*b^4*d^4 + 8*
a^16*b^2*d^4))*(-e/(4*(b^6*d^2*1i - a^6*d^2*1i + 6*a*b^5*d^2 + 6*a^5*b*d^2
- a^2*b^4*d^2*15i - 20*a^3*b^3*d^2 + a^4*b^2*d^2*15i)))^(1/2) - (2*b^18*d^2
```

*e^12 - 138*a^2*b^16*d^2*e^12 - 3046*a^4*b^14*d^2*e^12 + 4862*a^6*b^12*d^2*
e^12 + 9222*a^8*b^10*d^2*e^12 - 5246*a^10*b^8*d^2*e^12 - 4290*a^12*b^6*d^2*
e^12 + 2442*a^14*b^4*d^2*e^12 + 32*a^16*b^2*d^2*e^12)/(a^18*d^5 + a^2*b^16*
d^5 + 8*a^4*b^14*d^5 + 28*a^6*b^12*d^5 + 56*a^8*b^10*d^5 + 70*a^10*b^8*d^5
+ 56*a^12*b^6*d^5 + 28*a^14*b^4*d^5 + 8*a^16*b^2*d^5))*(-e/(4*(b^6*d^2*1i -
a^6*d^2*1i + 6*a*b^5*d^2 + 6*a^5*b*d^2 - a^2*b^4*d^2*15i - 20*a^3*b^3*d^2
+ a^4*b^2*d^2*15i)))^(1/2) + ((e*cot(c + d*x))^(1/2)*(2*a^2*b^13*e^12 - b^1
5*e^12 + 49*a^4*b^11*e^12 + 2460*a^6*b^9*e^12 - 3631*a^8*b^7*e^12 + 1922*a^
10*b^5*e^12 - 225*a^12*b^3*e^12))/(a^18*d^4 + a^2*b^16*d^4 + 8*a^4*b^14*d^4
+ 28*a^6*b^12*d^4 + 56*a^8*b^10*d^4 + 70*a^10*b^8*d^4 + 56*a^12*b^6*d^4 +
28*a^14*b^4*d^4 + 8*a^16*b^2*d^4))*(-e/(4*(b^6*d^2*1i - a^6*d^2*1i + 6*a*b^
5*d^2 + 6*a^5*b*d^2 - a^2*b^4*d^2*15i - 20*a^3*b^3*d^2 + a^4*b^2*d^2*15i)))
^(1/2)*1i - (((((64*a*b^23*d^4*e^11 + 1472*a^3*b^21*d^4*e^11 + 8832*a^5*b^1
9*d^4*e^11 + 25344*a^7*b^17*d^4*e^11 + 40320*a^9*b^15*d^4*e^11 + 34944*a^11
*b^13*d^4*e^11 + 10752*a^13*b^11*d^4*e^11 - 8448*a^15*b^9*d^4*e^11 - 10176*
a^17*b^7*d^4*e^11 - 4160*a^19*b^5*d^4*e^11 - 640*a^21*b^3*d^4*e^11)/(a^18*d
^5 + a^2*b^16*d^5 + 8*a^4*b^14*d^5 + 28*a^6*b^12*d^5 + 56*a^8*b^10*d^5 + 70
*a^10*b^8*d^5 + 56*a^12*b^6*d^5 + 28*a^14*b^4*d^5 + 8*a^16*b^2*d^5) - ((e*c
ot(c + d*x))^(1/2)*(-e/(4*(b^6*d^2*1i - a^6*d^2*1i + 6*a*b^5*d^2 + 6*a^5*b*
d^2 - a^2*b^4*d^2*15i - 20*a^3*b^3*d^2 + a^4*b^2*d^2*15i)))^(1/2)*(512*a^2*
b^25*d^4*e^10 + 4608*a^4*b^23*d^4*e^10 + 17920*a^6*b^21*d^4*e^10 + 38400*a^
8*b^19*d^4*e^10 + 46080*a^10*b^17*d^4*e^10 + 21504*a^12*b^15*d^4*e^10 - 215
04*a^14*b^13*d^4*e^10 - 46080*a^16*b^11*d^4*e^10 - 38400*a^18*b^9*d^4*e^10
- 17920*a^20*b^7*d^4*e^10 - 4608*a^22*b^5*d^4*e^10 - 512*a^24*b^3*d^4*e^10)
)/(a^18*d^4 + a^2*b^16*d^4 + 8*a^4*b^14*d^4 + 28*a^6*b^12*d^4 + 56*a^8*b^10
*d^4 + 70*a^10*b^8*d^4 + 56*a^12*b^6*d^4 + 28*a^14*b^4*d^4 + 8*a^16*b^2*d^4
))*(-e/(4*(b^6*d^2*1i - a^6*d^2*1i + 6*a*b^5*d^2 + 6*a^5*b*d^2 - a^2*b^4*d^
2*15i - 20*a^3*b^3*d^2 + a^4*b^2*d^2*15i)))^(1/2) + ((e*cot(c + d*x))^(1/2)
*(8*a*b^20*d^2*e^11 - 1152*a^3*b^18*d^2*e^11 + 2528*a^5*b^16*d^2*e^11 + 152
96*a^7*b^14*d^2*e^11 + 14128*a^9*b^12*d^2*e^11 - 5056*a^11*b^10*d^2*e^11 -
9248*a^13*b^8*d^2*e^11 + 64*a^15*b^6*d^2*e^11 + 1800*a^17*b^4*d^2*e^11 + 64
*a^19*b^2*d^2*e^11))/(a^18*d^4 + a^2*b^16*d^4 + 8*a^4*b^14*d^4 + 28*a^6*b^1
2*d^4 + 56*a^8*b^10*d^4 + 70*a^10*b^8*d^4 + 56*a^12*b^6*d^4 + 28*a^14*b^4*d
^4 + 8*a^16*b^2*d^4))*(-e/(4*(b^6*d^2*1i - a^6*d^2*1i + 6*a*b^5*d^2 + 6*a^5
*b*d^2 - a^2*b^4*d^2*15i - 20*a^3*b^3*d^2 + a^4*b^2*d^2*15i)))^(1/2) - (2*b
^18*d^2*e^12 - 138*a^2*b^16*d^2*e^12 - 3046*a^4*b^14*d^2*e^12 + 4862*a^6*b^
12*d^2*e^12 + 9222*a^8*b^10*d^2*e^12 - 5246*a^10*b^8*d^2*e^12 - 4290*a^12*b
^6*d^2*e^12 + 2442*a^14*b^4*d^2*e^12 + 32*a^16*b^2*d^2*e^12)/(a^18*d^5 + a^
2*b^16*d^5 + 8*a^4*b^14*d^5 + 28*a^6*b^12*d^5 + 56*a^8*b^10*d^5 + 70*a^10*b
^8*d^5 + 56*a^12*b^6*d^5 + 28*a^14*b^4*d^5 + 8*a^16*b^2*d^5))*(-e/(4*(b^6*d
^2*1i - a^6*d^2*1i + 6*a*b^5*d^2 + 6*a^5*b*d^2 - a^2*b^4*d^2*15i - 20*a^3*b
^3*d^2 + a^4*b^2*d^2*15i)))^(1/2) - ((e*cot(c + d*x))^(1/2)*(2*a^2*b^13*e^1
2 - b^15*e^12 + 49*a^4*b^11*e^12 + 2460*a^6*b^9*e^12 - 3631*a^8*b^7*e^12 +
1922*a^10*b^5*e^12 - 225*a^12*b^3*e^12))/(a^18*d^4 + a^2*b^16*d^4 + 8*a^4*b
^14*d^4 + 28*a^6*b^12*d^4 + 56*a^8*b^10*d^4 + 70*a^10*b^8*d^4 + 56*a^12*b^6

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*d^4 + 28*a^14*b^4*d^4 + 8*a^16*b^2*d^4))*(-e/(4*(b^6*d^2*1i - a^6*d^2*1i +
 6*a*b^5*d^2 + 6*a^5*b*d^2 - a^2*b^4*d^2*15i - 20*a^3*b^3*d^2 + a^4*b^2*d^2
*15i)))^(1/2)*1i)/((7*a*b^11*e^13 + 116*a^3*b^9*e^13 - 270*a^5*b^7*e^13 + 4
20*a^7*b^5*e^13 - 225*a^9*b^3*e^13)/(a^18*d^5 + a^2*b^16*d^5 + 8*a^4*b^14*d
^5 + 28*a^6*b^12*d^5 + 56*a^8*b^10*d^5 + 70*a^10*b^8*d^5 + 56*a^12*b^6*d^5
+ 28*a^14*b^4*d^5 + 8*a^16*b^2*d^5) + (((((64*a*b^23*d^4*e^11 + 1472*a^3*b^
21*d^4*e^11 + 8832*a^5*b^19*d^4*e^11 + 25344*a^7*b^17*d^4*e^11 + 40320*a^9*
b^15*d^4*e^11 + 34944*a^11*b^13*d^4*e^11 + 10752*a^13*b^11*d^4*e^11 - 8448*
a^15*b^9*d^4*e^11 - 10176*a^17*b^7*d^4*e^11 - 4160*a^19*b^5*d^4*e^11 - 640*
a^21*b^3*d^4*e^11)/(a^18*d^5 + a^2*b^16*d^5 + 8*a^4*b^14*d^5 + 28*a^6*b^12*
d^5 + 56*a^8*b^10*d^5 + 70*a^10*b^8*d^5 + 56*a^12*b^6*d^5 + 28*a^14*b^4*d^5
 + 8*a^16*b^2*d^5) + ((e*cot(c + d*x))^(1/2)*(-e/(4*(b^6*d^2*1i - a^6*d^2*1
i + 6*a*b^5*d^2 + 6*a^5*b*d^2 - a^2*b^4*d^2*15i - 20*a^3*b^3*d^2 + a^4*b^2*
d^2*15i)))^(1/2)*(512*a^2*b^25*d^4*e^10 + 4608*a^4*b^23*d^4*e^10 + 17920*a^
6*b^21*d^4*e^10 + 38400*a^8*b^19*d^4*e^10 + 46080*a^10*b^17*d^4*e^10 + 2150
4*a^12*b^15*d^4*e^10 - 21504*a^14*b^13*d^4*e^10 - 46080*a^16*b^11*d^4*e^10
- 38400*a^18*b^9*d^4*e^10 - 17920*a^20*b^7*d^4*e^10 - 4608*a^22*b^5*d^4*e^1
0 - 512*a^24*b^3*d^4*e^10))/(a^18*d^4 + a^2*b^16*d^4 + 8*a^4*b^14*d^4 + 28*
a^6*b^12*d^4 + 56*a^8*b^10*d^4 + 70*a^10*b^8*d^4 + 56*a^12*b^6*d^4 + 28*a^1
4*b^4*d^4 + 8*a^16*b^2*d^4))*(-e/(4*(b^6*d^2*1i - a^6*d^2*1i + 6*a*b^5*d^2
+ 6*a^5*b*d^2 - a^2*b^4*d^2*15i - 20*a^3*b^3*d^2 + a^4*b^2*d^2*15i)))^(1/2)
 - ((e*cot(c + d*x))^(1/2)*(8*a*b^20*d^2*e^11 - 1152*a^3*b^18*d^2*e^11 + 25
28*a^5*b^16*d^2*e^11 + 15296*a^7*b^14*d^2*e^11 + 14128*a^9*b^12*d^2*e^11 -
5056*a^11*b^10*d^2*e^11 - 9248*a^13*b^8*d^2*e^11 + 64*a^15*b^6*d^2*e^11 + 1
800*a^17*b^4*d^2*e^11 + 64*a^19*b^2*d^2*e^11))/(a^18*d^4 + a^2*b^16*d^4 + 8
*a^4*b^14*d^4 + 28*a^6*b^12*d^4 + 56*a^8*b^10*d^4 + 70*a^10*b^8*d^4 + 56*a^
12*b^6*d^4 + 28*a^14*b^4*d^4 + 8*a^16*b^2*d^4))*(-e/(4*(b^6*d^2*1i - a^6*d^
2*1i + 6*a*b^5*d^2 + 6*a^5*b*d^2 - a^2*b^4*d^2*15i - 20*a^3*b^3*d^2 + a^4*b
^2*d^2*15i)))^(1/2) - (2*b^18*d^2*e^12 - 138*a^2*b^16*d^2*e^12 - 3046*a^4*b
^14*d^2*e^12 + 4862*a^6*b^12*d^2*e^12 + 9222*a^8*b^10*d^2*e^12 - 5246*a^10*
b^8*d^2*e^12 - 4290*a^12*b^6*d^2*e^12 + 2442*a^14*b^4*d^2*e^12 + 32*a^16*b^
2*d^2*e^12)/(a^18*d^5 + a^2*b^16*d^5 + 8*a^4*b^14*d^5 + 28*a^6*b^12*d^5 + 5
6*a^8*b^10*d^5 + 70*a^10*b^8*d^5 + 56*a^12*b^6*d^5 + 28*a^14*b^4*d^5 + 8*a^
16*b^2*d^5))*(-e/(4*(b^6*d^2*1i - a^6*d^2*1i + 6*a*b^5*d^2 + 6*a^5*b*d^2 -
a^2*b^4*d^2*15i - 20*a^3*b^3*d^2 + a^4*b^2*d^2*15i)))^(1/2) + ((e*cot(c + d
*x))^(1/2)*(2*a^2*b^13*e^12 - b^15*e^12 + 49*a^4*b^11*e^12 + 2460*a^6*b^9*e
^12 - 3631*a^8*b^7*e^12 + 1922*a^10*b^5*e^12 - 225*a^12*b^3*e^12))/(a^18*d^
4 + a^2*b^16*d^4 + 8*a^4*b^14*d^4 + 28*a^6*b^12*d^4 + 56*a^8*b^10*d^4 + 70*
a^10*b^8*d^4 + 56*a^12*b^6*d^4 + 28*a^14*b^4*d^4 + 8*a^16*b^2*d^4))*(-e/(4*
(b^6*d^2*1i - a^6*d^2*1i + 6*a*b^5*d^2 + 6*a^5*b*d^2 - a^2*b^4*d^2*15i - 20
*a^3*b^3*d^2 + a^4*b^2*d^2*15i)))^(1/2) + (((((64*a*b^23*d^4*e^11 + 1472*a^
3*b^21*d^4*e^11 + 8832*a^5*b^19*d^4*e^11 + 25344*a^7*b^17*d^4*e^11 + 40320*
a^9*b^15*d^4*e^11 + 34944*a^11*b^13*d^4*e^11 + 10752*a^13*b^11*d^4*e^11 - 8
448*a^15*b^9*d^4*e^11 - 10176*a^17*b^7*d^4*e^11 - 4160*a^19*b^5*d^4*e^11 -
640*a^21*b^3*d^4*e^11)/(a^18*d^5 + a^2*b^16*d^5 + 8*a^4*b^14*d^5 + 28*a^6*b
```

^12*d^5 + 56*a^8*b^10*d^5 + 70*a^10*b^8*d^5 + 56*a^12*b^6*d^5 + 28*a^14*b^4
*d^5 + 8*a^16*b^2*d^5) - ((e*cot(c + d*x))^(1/2)*(-e/(4*(b^6*d^2*1i - a^6*d
^2*1i + 6*a*b^5*d^2 + 6*a^5*b*d^2 - a^2*b^4*d^2*15i - 20*a^3*b^3*d^2 + a^4*
b^2*d^2*15i)))^(1/2)*(512*a^2*b^25*d^4*e^10 + 4608*a^4*b^23*d^4*e^10 + 1792
0*a^6*b^21*d^4*e^10 + 38400*a^8*b^19*d^4*e^10 + 46080*a^10*b^17*d^4*e^10 +
21504*a^12*b^15*d^4*e^10 - 21504*a^14*b^13*d^4*e^10 - 46080*a^16*b^11*d^4*e
^10 - 38400*a^18*b^9*d^4*e^10 - 17920*a^20*b^7*d^4*e^10 - 4608*a^22*b^5*d^4
*e^10 - 512*a^24*b^3*d^4*e^10))/(a^18*d^4 + a^2*b^16*d^4 + 8*a^4*b^14*d^4 +
28*a^6*b^12*d^4 + 56*a^8*b^10*d^4 + 70*a^10*b^8*d^4 + 56*a^12*b^6*d^4 + 28
*a^14*b^4*d^4 + 8*a^16*b^2*d^4))*(-e/(4*(b^6*d^2*1i - a^6*d^2*1i + 6*a*b^5*
d^2 + 6*a^5*b*d^2 - a^2*b^4*d^2*15i - 20*a^3*b^3*d^2 + a^4*b^2*d^2*15i)))^(
1/2) + ((e*cot(c + d*x))^(1/2)*(8*a*b^20*d^2*e^11 - 1152*a^3*b^18*d^2*e^11
+ 2528*a^5*b^16*d^2*e^11 + 15296*a^7*b^14*d^2*e^11 + 14128*a^9*b^12*d^2*e^1
1 - 5056*a^11*b^10*d^2*e^11 - 9248*a^13*b^8*d^2*e^11 + 64*a^15*b^6*d^2*e^11
+ 1800*a^17*b^4*d^2*e^11 + 64*a^19*b^2*d^2*e^11))/(a^18*d^4 + a^2*b^16*d^4
+ 8*a^4*b^14*d^4 + 28*a^6*b^12*d^4 + 56*a^8*b^10*d^4 + 70*a^10*b^8*d^4 + 5
6*a^12*b^6*d^4 + 28*a^14*b^4*d^4 + 8*a^16*b^2*d^4))*(-e/(4*(b^6*d^2*1i - a^
6*d^2*1i + 6*a*b^5*d^2 + 6*a^5*b*d^2 - a^2*b^4*d^2*15i - 20*a^3*b^3*d^2 + a
^4*b^2*d^2*15i)))^(1/2) - (2*b^18*d^2*e^12 - 138*a^2*b^16*d^2*e^12 - 3046*a
^4*b^14*d^2*e^12 + 4862*a^6*b^12*d^2*e^12 + 9222*a^8*b^10*d^2*e^12 - 5246*a
^10*b^8*d^2*e^12 - 4290*a^12*b^6*d^2*e^12 + 2442*a^14*b^4*d^2*e^12 + 32*a^1
6*b^2*d^2*e^12)/(a^18*d^5 + a^2*b^16*d^5 + 8*a^4*b^14*d^5 + 28*a^6*b^12*d^5
+ 56*a^8*b^10*d^5 + 70*a^10*b^8*d^5 + 56*a^12*b^6*d^5 + 28*a^14*b^4*d^5 +
8*a^16*b^2*d^5))*(-e/(4*(b^6*d^2*1i - a^6*d^2*1i + 6*a*b^5*d^2 + 6*a^5*b*d^
2 - a^2*b^4*d^2*15i - 20*a^3*b^3*d^2 + a^4*b^2*d^2*15i)))^(1/2) - ((e*cot(c
+ d*x))^(1/2)*(2*a^2*b^13*e^12 - b^15*e^12 + 49*a^4*b^11*e^12 + 2460*a^6*b
^9*e^12 - 3631*a^8*b^7*e^12 + 1922*a^10*b^5*e^12 - 225*a^12*b^3*e^12))/(a^1
8*d^4 + a^2*b^16*d^4 + 8*a^4*b^14*d^4 + 28*a^6*b^12*d^4 + 56*a^8*b^10*d^4 +
70*a^10*b^8*d^4 + 56*a^12*b^6*d^4 + 28*a^14*b^4*d^4 + 8*a^16*b^2*d^4))*(-e
/(4*(b^6*d^2*1i - a^6*d^2*1i + 6*a*b^5*d^2 + 6*a^5*b*d^2 - a^2*b^4*d^2*15i
- 20*a^3*b^3*d^2 + a^4*b^2*d^2*15i)))^(1/2)))*(-e/(4*(b^6*d^2*1i - a^6*d^2*
1i + 6*a*b^5*d^2 + 6*a^5*b*d^2 - a^2*b^4*d^2*15i - 20*a^3*b^3*d^2 + a^4*b^2
*d^2*15i)))^(1/2)*2i - atan((((((64*a*b^23*d^4*e^11 + 1472*a^3*b^21*d^4*e^
11 + 8832*a^5*b^19*d^4*e^11 + 25344*a^7*b^17*d^4*e^11 + 40320*a^9*b^15*d^4*
e^11 + 34944*a^11*b^13*d^4*e^11 + 10752*a^13*b^11*d^4*e^11 - 8448*a^15*b^9*
d^4*e^11 - 10176*a^17*b^7*d^4*e^11 - 4160*a^19*b^5*d^4*e^11 - 640*a^21*b^3*
d^4*e^11)/(a^18*d^5 + a^2*b^16*d^5 + 8*a^4*b^14*d^5 + 28*a^6*b^12*d^5 + 56*
a^8*b^10*d^5 + 70*a^10*b^8*d^5 + 56*a^12*b^6*d^5 + 28*a^14*b^4*d^5 + 8*a^16
*b^2*d^5) + ((e*cot(c + d*x))^(1/2)*(-(e*1i)/(4*(b^6*d^2 - a^6*d^2 + a*b^5*
d^2*6i + a^5*b*d^2*6i - 15*a^2*b^4*d^2 - a^3*b^3*d^2*20i + 15*a^4*b^2*d^2))
)^(1/2)*(512*a^2*b^25*d^4*e^10 + 4608*a^4*b^23*d^4*e^10 + 17920*a^6*b^21*d^
4*e^10 + 38400*a^8*b^19*d^4*e^10 + 46080*a^10*b^17*d^4*e^10 + 21504*a^12*b^
15*d^4*e^10 - 21504*a^14*b^13*d^4*e^10 - 46080*a^16*b^11*d^4*e^10 - 38400*a
^18*b^9*d^4*e^10 - 17920*a^20*b^7*d^4*e^10 - 4608*a^22*b^5*d^4*e^10 - 512*a
^24*b^3*d^4*e^10))/(a^18*d^4 + a^2*b^16*d^4 + 8*a^4*b^14*d^4 + 28*a^6*b^12*

d^4 + 56*a^8*b^10*d^4 + 70*a^10*b^8*d^4 + 56*a^12*b^6*d^4 + 28*a^14*b^4*d^4
 + 8*a^16*b^2*d^4))*(-(e*1i)/(4*(b^6*d^2 - a^6*d^2 + a*b^5*d^2*6i + a^5*b*d
^2*6i - 15*a^2*b^4*d^2 - a^3*b^3*d^2*20i + 15*a^4*b^2*d^2)))^(1/2) - ((e*co
t(c + d*x))^(1/2)*(8*a*b^20*d^2*e^11 - 1152*a^3*b^18*d^2*e^11 + 2528*a^5*b^
16*d^2*e^11 + 15296*a^7*b^14*d^2*e^11 + 14128*a^9*b^12*d^2*e^11 - 5056*a^11
*b^10*d^2*e^11 - 9248*a^13*b^8*d^2*e^11 + 64*a^15*b^6*d^2*e^11 + 1800*a^17*
b^4*d^2*e^11 + 64*a^19*b^2*d^2*e^11))/(a^18*d^4 + a^2*b^16*d^4 + 8*a^4*b^14
*d^4 + 28*a^6*b^12*d^4 + 56*a^8*b^10*d^4 + 70*a^10*b^8*d^4 + 56*a^12*b^6*d^
4 + 28*a^14*b^4*d^4 + 8*a^16*b^2*d^4))*(-(e*1i)/(4*(b^6*d^2 - a^6*d^2 + a*b
^5*d^2*6i + a^5*b*d^2*6i - 15*a^2*b^4*d^2 - a^3*b^3*d^2*20i + 15*a^4*b^2*d^
2)))^(1/2) - (2*b^18*d^2*e^12 - 138*a^2*b^16*d^2*e^12 - 3046*a^4*b^14*d^2*e
^12 + 4862*a^6*b^12*d^2*e^12 + 9222*a^8*b^10*d^2*e^12 - 5246*a^10*b^8*d^2*e
^12 - 4290*a^12*b^6*d^2*e^12 + 2442*a^14*b^4*d^2*e^12 + 32*a^16*b^2*d^2*e^1
2)/(a^18*d^5 + a^2*b^16*d^5 + 8*a^4*b^14*d^5 + 28*a^6*b^12*d^5 + 56*a^8*b^1
0*d^5 + 70*a^10*b^8*d^5 + 56*a^12*b^6*d^5 + 28*a^14*b^4*d^5 + 8*a^16*b^2*d^
5))*(-(e*1i)/(4*(b^6*d^2 - a^6*d^2 + a*b^5*d^2*6i + a^5*b*d^2*6i - 15*a^2*b
^4*d^2 - a^3*b^3*d^2*20i + 15*a^4*b^2*d^2)))^(1/2) + ((e*cot(c + d*x))^(1/2
)*(2*a^2*b^13*e^12 - b^15*e^12 + 49*a^4*b^11*e^12 + 2460*a^6*b^9*e^12 - 363
1*a^8*b^7*e^12 + 1922*a^10*b^5*e^12 - 225*a^12*b^3*e^12))/(a^18*d^4 + a^2*b
^16*d^4 + 8*a^4*b^14*d^4 + 28*a^6*b^12*d^4 + 56*a^8*b^10*d^4 + 70*a^10*b^8*
d^4 + 56*a^12*b^6*d^4 + 28*a^14*b^4*d^4 + 8*a^16*b^2*d^4))*(-(e*1i)/(4*(b^6
*d^2 - a^6*d^2 + a*b^5*d^2*6i + a^5*b*d^2*6i - 15*a^2*b^4*d^2 - a^3*b^3*d^2
*20i + 15*a^4*b^2*d^2)))^(1/2)*1i - (((((64*a*b^23*d^4*e^11 + 1472*a^3*b^21
*d^4*e^11 + 8832*a^5*b^19*d^4*e^11 + 25344*a^7*b^17*d^4*e^11 + 40320*a^9*b^
15*d^4*e^11 + 34944*a^11*b^13*d^4*e^11 + 10752*a^13*b^11*d^4*e^11 - 8448*a^
15*b^9*d^4*e^11 - 10176*a^17*b^7*d^4*e^11 - 4160*a^19*b^5*d^4*e^11 - 640*a^
21*b^3*d^4*e^11)/(a^18*d^5 + a^2*b^16*d^5 + 8*a^4*b^14*d^5 + 28*a^6*b^12*d^
5 + 56*a^8*b^10*d^5 + 70*a^10*b^8*d^5 + 56*a^12*b^6*d^5 + 28*a^14*b^4*d^5 +
 8*a^16*b^2*d^5) - ((e*cot(c + d*x))^(1/2)*(-(e*1i)/(4*(b^6*d^2 - a^6*d^2 +
 a*b^5*d^2*6i + a^5*b*d^2*6i - 15*a^2*b^4*d^2 - a^3*b^3*d^2*20i + 15*a^4*b^
2*d^2)))^(1/2)*(512*a^2*b^25*d^4*e^10 + 4608*a^4*b^23*d^4*e^10 + 17920*a^6*
b^21*d^4*e^10 + 38400*a^8*b^19*d^4*e^10 + 46080*a^10*b^17*d^4*e^10 + 21504*
a^12*b^15*d^4*e^10 - 21504*a^14*b^13*d^4*e^10 - 46080*a^16*b^11*d^4*e^10 -
38400*a^18*b^9*d^4*e^10 - 17920*a^20*b^7*d^4*e^10 - 4608*a^22*b^5*d^4*e^10
- 512*a^24*b^3*d^4*e^10))/(a^18*d^4 + a^2*b^16*d^4 + 8*a^4*b^14*d^4 + 28*a^
6*b^12*d^4 + 56*a^8*b^10*d^4 + 70*a^10*b^8*d^4 + 56*a^12*b^6*d^4 + 28*a^14*
b^4*d^4 + 8*a^16*b^2*d^4))*(-(e*1i)/(4*(b^6*d^2 - a^6*d^2 + a*b^5*d^2*6i +
a^5*b*d^2*6i - 15*a^2*b^4*d^2 - a^3*b^3*d^2*20i + 15*a^4*b^2*d^2)))^(1/2) +
 ((e*cot(c + d*x))^(1/2)*(8*a*b^20*d^2*e^11 - 1152*a^3*b^18*d^2*e^11 + 2528
*a^5*b^16*d^2*e^11 + 15296*a^7*b^14*d^2*e^11 + 14128*a^9*b^12*d^2*e^11 - 50
56*a^11*b^10*d^2*e^11 - 9248*a^13*b^8*d^2*e^11 + 64*a^15*b^6*d^2*e^11 + 180
0*a^17*b^4*d^2*e^11 + 64*a^19*b^2*d^2*e^11))/(a^18*d^4 + a^2*b^16*d^4 + 8*a
^4*b^14*d^4 + 28*a^6*b^12*d^4 + 56*a^8*b^10*d^4 + 70*a^10*b^8*d^4 + 56*a^12
*b^6*d^4 + 28*a^14*b^4*d^4 + 8*a^16*b^2*d^4))*(-(e*1i)/(4*(b^6*d^2 - a^6*d^
2 + a*b^5*d^2*6i + a^5*b*d^2*6i - 15*a^2*b^4*d^2 - a^3*b^3*d^2*20i + 15*a^4

*b^2*d^2)))^(1/2) - (2*b^18*d^2*e^12 - 138*a^2*b^16*d^2*e^12 - 3046*a^4*b^1
4*d^2*e^12 + 4862*a^6*b^12*d^2*e^12 + 9222*a^8*b^10*d^2*e^12 - 5246*a^10*b^
8*d^2*e^12 - 4290*a^12*b^6*d^2*e^12 + 2442*a^14*b^4*d^2*e^12 + 32*a^16*b^2*
d^2*e^12)/(a^18*d^5 + a^2*b^16*d^5 + 8*a^4*b^14*d^5 + 28*a^6*b^12*d^5 + 56*
a^8*b^10*d^5 + 70*a^10*b^8*d^5 + 56*a^12*b^6*d^5 + 28*a^14*b^4*d^5 + 8*a^16
*b^2*d^5))*(-(e*1i)/(4*(b^6*d^2 - a^6*d^2 + a*b^5*d^2*6i + a^5*b*d^2*6i - 1
5*a^2*b^4*d^2 - a^3*b^3*d^2*20i + 15*a^4*b^2*d^2)))^(1/2) - ((e*cot(c + d*x
))^(1/2)*(2*a^2*b^13*e^12 - b^15*e^12 + 49*a^4*b^11*e^12 + 2460*a^6*b^9*e^1
2 - 3631*a^8*b^7*e^12 + 1922*a^10*b^5*e^12 - 225*a^12*b^3*e^12))/(a^18*d^4
+ a^2*b^16*d^4 + 8*a^4*b^14*d^4 + 28*a^6*b^12*d^4 + 56*a^8*b^10*d^4 + 70*a^
10*b^8*d^4 + 56*a^12*b^6*d^4 + 28*a^14*b^4*d^4 + 8*a^16*b^2*d^4))*(-(e*1i)/
(4*(b^6*d^2 - a^6*d^2 + a*b^5*d^2*6i + a^5*b*d^2*6i - 15*a^2*b^4*d^2 - a^3*
b^3*d^2*20i + 15*a^4*b^2*d^2)))^(1/2)*1i)/((7*a*b^11*e^13 + 116*a^3*b^9*e^1
3 - 270*a^5*b^7*e^13 + 420*a^7*b^5*e^13 - 225*a^9*b^3*e^13)/(a^18*d^5 + a^2
*b^16*d^5 + 8*a^4*b^14*d^5 + 28*a^6*b^12*d^5 + 56*a^8*b^10*d^5 + 70*a^10*b^
8*d^5 + 56*a^12*b^6*d^5 + 28*a^14*b^4*d^5 + 8*a^16*b^2*d^5) + (((((64*a*b^2
3*d^4*e^11 + 1472*a^3*b^21*d^4*e^11 + 8832*a^5*b^19*d^4*e^11 + 25344*a^7*b^
17*d^4*e^11 + 40320*a^9*b^15*d^4*e^11 + 34944*a^11*b^13*d^4*e^11 + 10752*a^
13*b^11*d^4*e^11 - 8448*a^15*b^9*d^4*e^11 - 10176*a^17*b^7*d^4*e^11 - 4160*
a^19*b^5*d^4*e^11 - 640*a^21*b^3*d^4*e^11)/(a^18*d^5 + a^2*b^16*d^5 + 8*a^4
*b^14*d^5 + 28*a^6*b^12*d^5 + 56*a^8*b^10*d^5 + 70*a^10*b^8*d^5 + 56*a^12*b
^6*d^5 + 28*a^14*b^4*d^5 + 8*a^16*b^2*d^5) + ((e*cot(c + d*x))^(1/2)*(-(e*1
i)/(4*(b^6*d^2 - a^6*d^2 + a*b^5*d^2*6i + a^5*b*d^2*6i - 15*a^2*b^4*d^2 - a
^3*b^3*d^2*20i + 15*a^4*b^2*d^2)))^(1/2)*(512*a^2*b^25*d^4*e^10 + 4608*a^4*
b^23*d^4*e^10 + 17920*a^6*b^21*d^4*e^10 + 38400*a^8*b^19*d^4*e^10 + 46080*a
^10*b^17*d^4*e^10 + 21504*a^12*b^15*d^4*e^10 - 21504*a^14*b^13*d^4*e^10 - 4
6080*a^16*b^11*d^4*e^10 - 38400*a^18*b^9*d^4*e^10 - 17920*a^20*b^7*d^4*e^10
 - 4608*a^22*b^5*d^4*e^10 - 512*a^24*b^3*d^4*e^10))/(a^18*d^4 + a^2*b^16*d^
4 + 8*a^4*b^14*d^4 + 28*a^6*b^12*d^4 + 56*a^8*b^10*d^4 + 70*a^10*b^8*d^4 +
56*a^12*b^6*d^4 + 28*a^14*b^4*d^4 + 8*a^16*b^2*d^4))*(-(e*1i)/(4*(b^6*d^2 -
 a^6*d^2 + a*b^5*d^2*6i + a^5*b*d^2*6i - 15*a^2*b^4*d^2 - a^3*b^3*d^2*20i +
 15*a^4*b^2*d^2)))^(1/2) - ((e*cot(c + d*x))^(1/2)*(8*a*b^20*d^2*e^11 - 115
2*a^3*b^18*d^2*e^11 + 2528*a^5*b^16*d^2*e^11 + 15296*a^7*b^14*d^2*e^11 + 14
128*a^9*b^12*d^2*e^11 - 5056*a^11*b^10*d^2*e^11 - 9248*a^13*b^8*d^2*e^11 +
64*a^15*b^6*d^2*e^11 + 1800*a^17*b^4*d^2*e^11 + 64*a^19*b^2*d^2*e^11))/(a^1
8*d^4 + a^2*b^16*d^4 + 8*a^4*b^14*d^4 + 28*a^6*b^12*d^4 + 56*a^8*b^10*d^4 +
 70*a^10*b^8*d^4 + 56*a^12*b^6*d^4 + 28*a^14*b^4*d^4 + 8*a^16*b^2*d^4))*(-(
e*1i)/(4*(b^6*d^2 - a^6*d^2 + a*b^5*d^2*6i + a^5*b*d^2*6i - 15*a^2*b^4*d^2
 - a^3*b^3*d^2*20i + 15*a^4*b^2*d^2)))^(1/2) - (2*b^18*d^2*e^12 - 138*a^2*b^
16*d^2*e^12 - 3046*a^4*b^14*d^2*e^12 + 4862*a^6*b^12*d^2*e^12 + 9222*a^8*b^
10*d^2*e^12 - 5246*a^10*b^8*d^2*e^12 - 4290*a^12*b^6*d^2*e^12 + 2442*a^14*b
^4*d^2*e^12 + 32*a^16*b^2*d^2*e^12)/(a^18*d^5 + a^2*b^16*d^5 + 8*a^4*b^14*d
^5 + 28*a^6*b^12*d^5 + 56*a^8*b^10*d^5 + 70*a^10*b^8*d^5 + 56*a^12*b^6*d^5
 + 28*a^14*b^4*d^5 + 8*a^16*b^2*d^5))*(-(e*1i)/(4*(b^6*d^2 - a^6*d^2 + a*b^5
*d^2*6i + a^5*b*d^2*6i - 15*a^2*b^4*d^2 - a^3*b^3*d^2*20i + 15*a^4*b^2*d^2)

))^(1/2) + ((e*cot(c + d*x))^(1/2)*(2*a^2*b^13*e^12 - b^15*e^12 + 49*a^4*b^
11*e^12 + 2460*a^6*b^9*e^12 - 3631*a^8*b^7*e^12 + 1922*a^10*b^5*e^12 - 225*
a^12*b^3*e^12))/(a^18*d^4 + a^2*b^16*d^4 + 8*a^4*b^14*d^4 + 28*a^6*b^12*d^4
 + 56*a^8*b^10*d^4 + 70*a^10*b^8*d^4 + 56*a^12*b^6*d^4 + 28*a^14*b^4*d^4 +
8*a^16*b^2*d^4))*(-(e*1i)/(4*(b^6*d^2 - a^6*d^2 + a*b^5*d^2*6i + a^5*b*d^2*
6i - 15*a^2*b^4*d^2 - a^3*b^3*d^2*20i + 15*a^4*b^2*d^2)))^(1/2) + (((((64*a
*b^23*d^4*e^11 + 1472*a^3*b^21*d^4*e^11 + 8832*a^5*b^19*d^4*e^11 + 25344*a^
7*b^17*d^4*e^11 + 40320*a^9*b^15*d^4*e^11 + 34944*a^11*b^13*d^4*e^11 + 1075
2*a^13*b^11*d^4*e^11 - 8448*a^15*b^9*d^4*e^11 - 10176*a^17*b^7*d^4*e^11 - 4
160*a^19*b^5*d^4*e^11 - 640*a^21*b^3*d^4*e^11)/(a^18*d^5 + a^2*b^16*d^5 + 8
*a^4*b^14*d^5 + 28*a^6*b^12*d^5 + 56*a^8*b^10*d^5 + 70*a^10*b^8*d^5 + 56*a^
12*b^6*d^5 + 28*a^14*b^4*d^5 + 8*a^16*b^2*d^5) - ((e*cot(c + d*x))^(1/2)*(-
(e*1i)/(4*(b^6*d^2 - a^6*d^2 + a*b^5*d^2*6i + a^5*b*d^2*6i - 15*a^2*b^4*d^2
 - a^3*b^3*d^2*20i + 15*a^4*b^2*d^2)))^(1/2)*(512*a^2*b^25*d^4*e^10 + 4608*
a^4*b^23*d^4*e^10 + 17920*a^6*b^21*d^4*e^10 + 38400*a^8*b^19*d^4*e^10 + 460
80*a^10*b^17*d^4*e^10 + 21504*a^12*b^15*d^4*e^10 - 21504*a^14*b^13*d^4*e^10
 - 46080*a^16*b^11*d^4*e^10 - 38400*a^18*b^9*d^4*e^10 - 17920*a^20*b^7*d^4*
e^10 - 4608*a^22*b^5*d^4*e^10 - 512*a^24*b^3*d^4*e^10))/(a^18*d^4 + a^2*b^1
6*d^4 + 8*a^4*b^14*d^4 + 28*a^6*b^12*d^4 + 56*a^8*b^10*d^4 + 70*a^10*b^8*d^
4 + 56*a^12*b^6*d^4 + 28*a^14*b^4*d^4 + 8*a^16*b^2*d^4))*(-(e*1i)/(4*(b^6*d
^2 - a^6*d^2 + a*b^5*d^2*6i + a^5*b*d^2*6i - 15*a^2*b^4*d^2 - a^3*b^3*d^2*2
0i + 15*a^4*b^2*d^2)))^(1/2) + ((e*cot(c + d*x))^(1/2)*(8*a*b^20*d^2*e^11 -
 1152*a^3*b^18*d^2*e^11 + 2528*a^5*b^16*d^2*e^11 + 15296*a^7*b^14*d^2*e^11
+ 14128*a^9*b^12*d^2*e^11 - 5056*a^11*b^10*d^2*e^11 - 9248*a^13*b^8*d^2*e^1
1 + 64*a^15*b^6*d^2*e^11 + 1800*a^17*b^4*d^2*e^11 + 64*a^19*b^2*d^2*e^11))/
(a^18*d^4 + a^2*b^16*d^4 + 8*a^4*b^14*d^4 + 28*a^6*b^12*d^4 + 56*a^8*b^10*d
^4 + 70*a^10*b^8*d^4 + 56*a^12*b^6*d^4 + 28*a^14*b^4*d^4 + 8*a^16*b^2*d^4))
*(-(e*1i)/(4*(b^6*d^2 - a^6*d^2 + a*b^5*d^2*6i + a^5*b*d^2*6i - 15*a^2*b^4*
d^2 - a^3*b^3*d^2*20i + 15*a^4*b^2*d^2)))^(1/2) - (2*b^18*d^2*e^12 - 138*a^
2*b^16*d^2*e^12 - 3046*a^4*b^14*d^2*e^12 + 4862*a^6*b^12*d^2*e^12 + 9222*a^
8*b^10*d^2*e^12 - 5246*a^10*b^8*d^2*e^12 - 4290*a^12*b^6*d^2*e^12 + 2442*a^
14*b^4*d^2*e^12 + 32*a^16*b^2*d^2*e^12)/(a^18*d^5 + a^2*b^16*d^5 + 8*a^4*b^
14*d^5 + 28*a^6*b^12*d^5 + 56*a^8*b^10*d^5 + 70*a^10*b^8*d^5 + 56*a^12*b^6*
d^5 + 28*a^14*b^4*d^5 + 8*a^16*b^2*d^5))*(-(e*1i)/(4*(b^6*d^2 - a^6*d^2 + a
*b^5*d^2*6i + a^5*b*d^2*6i - 15*a^2*b^4*d^2 - a^3*b^3*d^2*20i + 15*a^4*b^2*
d^2)))^(1/2) - ((e*cot(c + d*x))^(1/2)*(2*a^2*b^13*e^12 - b^15*e^12 + 49*a^
4*b^11*e^12 + 2460*a^6*b^9*e^12 - 3631*a^8*b^7*e^12 + 1922*a^10*b^5*e^12 -
225*a^12*b^3*e^12))/(a^18*d^4 + a^2*b^16*d^4 + 8*a^4*b^14*d^4 + 28*a^6*b^12
*d^4 + 56*a^8*b^10*d^4 + 70*a^10*b^8*d^4 + 56*a^12*b^6*d^4 + 28*a^14*b^4*d^
4 + 8*a^16*b^2*d^4))*(-(e*1i)/(4*(b^6*d^2 - a^6*d^2 + a*b^5*d^2*6i + a^5*b*
d^2*6i - 15*a^2*b^4*d^2 - a^3*b^3*d^2*20i + 15*a^4*b^2*d^2)))^(1/2)))*(-(e*
1i)/(4*(b^6*d^2 - a^6*d^2 + a*b^5*d^2*6i + a^5*b*d^2*6i - 15*a^2*b^4*d^2 -
a^3*b^3*d^2*20i + 15*a^4*b^2*d^2)))^(1/2)*2i - (atan(((((((e*cot(c + d*x))^(
1/2)*(2*a^2*b^13*e^12 - b^15*e^12 + 49*a^4*b^11*e^12 + 2460*a^6*b^9*e^12 -
3631*a^8*b^7*e^12 + 1922*a^10*b^5*e^12 - 225*a^12*b^3*e^12))/(a^18*d^4 + a^

2*b^16*d^4 + 8*a^4*b^14*d^4 + 28*a^6*b^12*d^4 + 56*a^8*b^10*d^4 + 70*a^10*b
^8*d^4 + 56*a^12*b^6*d^4 + 28*a^14*b^4*d^4 + 8*a^16*b^2*d^4) + (((2*b^18*d^
2*e^12 - 138*a^2*b^16*d^2*e^12 - 3046*a^4*b^14*d^2*e^12 + 4862*a^6*b^12*d^2
*e^12 + 9222*a^8*b^10*d^2*e^12 - 5246*a^10*b^8*d^2*e^12 - 4290*a^12*b^6*d^2
*e^12 + 2442*a^14*b^4*d^2*e^12 + 32*a^16*b^2*d^2*e^12)/(a^18*d^5 + a^2*b^16
*d^5 + 8*a^4*b^14*d^5 + 28*a^6*b^12*d^5 + 56*a^8*b^10*d^5 + 70*a^10*b^8*d^5
 + 56*a^12*b^6*d^5 + 28*a^14*b^4*d^5 + 8*a^16*b^2*d^5) - ((((e*cot(c + d*x)
)^(1/2)*(8*a*b^20*d^2*e^11 - 1152*a^3*b^18*d^2*e^11 + 2528*a^5*b^16*d^2*e^1
1 + 15296*a^7*b^14*d^2*e^11 + 14128*a^9*b^12*d^2*e^11 - 5056*a^11*b^10*d^2*
e^11 - 9248*a^13*b^8*d^2*e^11 + 64*a^15*b^6*d^2*e^11 + 1800*a^17*b^4*d^2*e^
11 + 64*a^19*b^2*d^2*e^11))/(a^18*d^4 + a^2*b^16*d^4 + 8*a^4*b^14*d^4 + 28*
a^6*b^12*d^4 + 56*a^8*b^10*d^4 + 70*a^10*b^8*d^4 + 56*a^12*b^6*d^4 + 28*a^1
4*b^4*d^4 + 8*a^16*b^2*d^4) + (((64*a*b^23*d^4*e^11 + 1472*a^3*b^21*d^4*e^1
1 + 8832*a^5*b^19*d^4*e^11 + 25344*a^7*b^17*d^4*e^11 + 40320*a^9*b^15*d^4*e
^11 + 34944*a^11*b^13*d^4*e^11 + 10752*a^13*b^11*d^4*e^11 - 8448*a^15*b^9*d
^4*e^11 - 10176*a^17*b^7*d^4*e^11 - 4160*a^19*b^5*d^4*e^11 - 640*a^21*b^3*d
^4*e^11)/(a^18*d^5 + a^2*b^16*d^5 + 8*a^4*b^14*d^5 + 28*a^6*b^12*d^5 + 56*a
^8*b^10*d^5 + 70*a^10*b^8*d^5 + 56*a^12*b^6*d^5 + 28*a^14*b^4*d^5 + 8*a^16*
b^2*d^5) - ((e*cot(c + d*x))^(1/2)*(b^4 - 15*a^4 + 18*a^2*b^2)*(-a^3*b*e)^(
1/2)*(512*a^2*b^25*d^4*e^10 + 4608*a^4*b^23*d^4*e^10 + 17920*a^6*b^21*d^4*e
^10 + 38400*a^8*b^19*d^4*e^10 + 46080*a^10*b^17*d^4*e^10 + 21504*a^12*b^15*
d^4*e^10 - 21504*a^14*b^13*d^4*e^10 - 46080*a^16*b^11*d^4*e^10 - 38400*a^18
*b^9*d^4*e^10 - 17920*a^20*b^7*d^4*e^10 - 4608*a^22*b^5*d^4*e^10 - 512*a^24
*b^3*d^4*e^10))/(8*(a^9*d + a^3*b^6*d + 3*a^5*b^4*d + 3*a^7*b^2*d)*(a^18*d^
4 + a^2*b^16*d^4 + 8*a^4*b^14*d^4 + 28*a^6*b^12*d^4 + 56*a^8*b^10*d^4 + 70*
a^10*b^8*d^4 + 56*a^12*b^6*d^4 + 28*a^14*b^4*d^4 + 8*a^16*b^2*d^4)))*(b^4 -
 15*a^4 + 18*a^2*b^2)*(-a^3*b*e)^(1/2))/(8*(a^9*d + a^3*b^6*d + 3*a^5*b^4*d
 + 3*a^7*b^2*d)))*(b^4 - 15*a^4 + 18*a^2*b^2)*(-a^3*b*e)^(1/2))/(8*(a^9*d +
 a^3*b^6*d + 3*a^5*b^4*d + 3*a^7*b^2*d)))*(b^4 - 15*a^4 + 18*a^2*b^2)*(-a^3
*b*e)^(1/2))/(8*(a^9*d + a^3*b^6*d + 3*a^5*b^4*d + 3*a^7*b^2*d)))*(b^4 - 15
*a^4 + 18*a^2*b^2)*(-a^3*b*e)^(1/2)*1i)/(8*(a^9*d + a^3*b^6*d + 3*a^5*b^4*d
 + 3*a^7*b^2*d)) + ((((e*cot(c + d*x))^(1/2)*(2*a^2*b^13*e^12 - b^15*e^12 +
 49*a^4*b^11*e^12 + 2460*a^6*b^9*e^12 - 3631*a^8*b^7*e^12 + 1922*a^10*b^5*e
^12 - 225*a^12*b^3*e^12))/(a^18*d^4 + a^2*b^16*d^4 + 8*a^4*b^14*d^4 + 28*a^
6*b^12*d^4 + 56*a^8*b^10*d^4 + 70*a^10*b^8*d^4 + 56*a^12*b^6*d^4 + 28*a^14*
b^4*d^4 + 8*a^16*b^2*d^4) - (((2*b^18*d^2*e^12 - 138*a^2*b^16*d^2*e^12 - 30
46*a^4*b^14*d^2*e^12 + 4862*a^6*b^12*d^2*e^12 + 9222*a^8*b^10*d^2*e^12 - 52
46*a^10*b^8*d^2*e^12 - 4290*a^12*b^6*d^2*e^12 + 2442*a^14*b^4*d^2*e^12 + 32
*a^16*b^2*d^2*e^12)/(a^18*d^5 + a^2*b^16*d^5 + 8*a^4*b^14*d^5 + 28*a^6*b^12
*d^5 + 56*a^8*b^10*d^5 + 70*a^10*b^8*d^5 + 56*a^12*b^6*d^5 + 28*a^14*b^4*d^
5 + 8*a^16*b^2*d^5) + ((((e*cot(c + d*x))^(1/2)*(8*a*b^20*d^2*e^11 - 1152*a
^3*b^18*d^2*e^11 + 2528*a^5*b^16*d^2*e^11 + 15296*a^7*b^14*d^2*e^11 + 14128
*a^9*b^12*d^2*e^11 - 5056*a^11*b^10*d^2*e^11 - 9248*a^13*b^8*d^2*e^11 + 64*
a^15*b^6*d^2*e^11 + 1800*a^17*b^4*d^2*e^11 + 64*a^19*b^2*d^2*e^11))/(a^18*d
^4 + a^2*b^16*d^4 + 8*a^4*b^14*d^4 + 28*a^6*b^12*d^4 + 56*a^8*b^10*d^4 + 70

*a^10*b^8*d^4 + 56*a^12*b^6*d^4 + 28*a^14*b^4*d^4 + 8*a^16*b^2*d^4) - (((64
*a*b^23*d^4*e^11 + 1472*a^3*b^21*d^4*e^11 + 8832*a^5*b^19*d^4*e^11 + 25344*
a^7*b^17*d^4*e^11 + 40320*a^9*b^15*d^4*e^11 + 34944*a^11*b^13*d^4*e^11 + 10
752*a^13*b^11*d^4*e^11 - 8448*a^15*b^9*d^4*e^11 - 10176*a^17*b^7*d^4*e^11 -
4160*a^19*b^5*d^4*e^11 - 640*a^21*b^3*d^4*e^11)/(a^18*d^5 + a^2*b^16*d^5 +
8*a^4*b^14*d^5 + 28*a^6*b^12*d^5 + 56*a^8*b^10*d^5 + 70*a^10*b^8*d^5 + 56*
a^12*b^6*d^5 + 28*a^14*b^4*d^5 + 8*a^16*b^2*d^5) + ((e*cot(c + d*x))^(1/2)*
(b^4 - 15*a^4 + 18*a^2*b^2)*(-a^3*b*e)^(1/2)*(512*a^2*b^25*d^4*e^10 + 4608*
a^4*b^23*d^4*e^10 + 17920*a^6*b^21*d^4*e^10 + 38400*a^8*b^19*d^4*e^10 + 460
80*a^10*b^17*d^4*e^10 + 21504*a^12*b^15*d^4*e^10 - 21504*a^14*b^13*d^4*e^10
- 46080*a^16*b^11*d^4*e^10 - 38400*a^18*b^9*d^4*e^10 - 17920*a^20*b^7*d^4*
e^10 - 4608*a^22*b^5*d^4*e^10 - 512*a^24*b^3*d^4*e^10))/(8*(a^9*d + a^3*b^6
*d + 3*a^5*b^4*d + 3*a^7*b^2*d)*(a^18*d^4 + a^2*b^16*d^4 + 8*a^4*b^14*d^4 +
28*a^6*b^12*d^4 + 56*a^8*b^10*d^4 + 70*a^10*b^8*d^4 + 56*a^12*b^6*d^4 + 28
*a^14*b^4*d^4 + 8*a^16*b^2*d^4)))*(b^4 - 15*a^4 + 18*a^2*b^2)*(-a^3*b*e)^(1
/2))/(8*(a^9*d + a^3*b^6*d + 3*a^5*b^4*d + 3*a^7*b^2*d)))*(b^4 - 15*a^4 + 1
8*a^2*b^2)*(-a^3*b*e)^(1/2))/(8*(a^9*d + a^3*b^6*d + 3*a^5*b^4*d + 3*a^7*b^
2*d)))*(b^4 - 15*a^4 + 18*a^2*b^2)*(-a^3*b*e)^(1/2))/(8*(a^9*d + a^3*b^6*d
+ 3*a^5*b^4*d + 3*a^7*b^2*d)))*(b^4 - 15*a^4 + 18*a^2*b^2)*(-a^3*b*e)^(1/2)
1i)/(8(a^9*d + a^3*b^6*d + 3*a^5*b^4*d + 3*a^7*b^2*d)))/((7*a*b^11*e^13 +
116*a^3*b^9*e^13 - 270*a^5*b^7*e^13 + 420*a^7*b^5*e^13 - 225*a^9*b^3*e^13)
/(a^18*d^5 + a^2*b^16*d^5 + 8*a^4*b^14*d^5 + 28*a^6*b^12*d^5 + 56*a^8*b^10*
d^5 + 70*a^10*b^8*d^5 + 56*a^12*b^6*d^5 + 28*a^14*b^4*d^5 + 8*a^16*b^2*d^5)
- ((((e*cot(c + d*x))^(1/2)*(2*a^2*b^13*e^12 - b^15*e^12 + 49*a^4*b^11*e^1
2 + 2460*a^6*b^9*e^12 - 3631*a^8*b^7*e^12 + 1922*a^10*b^5*e^12 - 225*a^12*b
^3*e^12))/(a^18*d^4 + a^2*b^16*d^4 + 8*a^4*b^14*d^4 + 28*a^6*b^12*d^4 + 56*
a^8*b^10*d^4 + 70*a^10*b^8*d^4 + 56*a^12*b^6*d^4 + 28*a^14*b^4*d^4 + 8*a^16
*b^2*d^4) + (((2*b^18*d^2*e^12 - 138*a^2*b^16*d^2*e^12 - 3046*a^4*b^14*d^2*
e^12 + 4862*a^6*b^12*d^2*e^12 + 9222*a^8*b^10*d^2*e^12 - 5246*a^10*b^8*d^2*
e^12 - 4290*a^12*b^6*d^2*e^12 + 2442*a^14*b^4*d^2*e^12 + 32*a^16*b^2*d^2*e^
12)/(a^18*d^5 + a^2*b^16*d^5 + 8*a^4*b^14*d^5 + 28*a^6*b^12*d^5 + 56*a^8*b^
10*d^5 + 70*a^10*b^8*d^5 + 56*a^12*b^6*d^5 + 28*a^14*b^4*d^5 + 8*a^16*b^2*d
^5) - ((((e*cot(c + d*x))^(1/2)*(8*a*b^20*d^2*e^11 - 1152*a^3*b^18*d^2*e^11
+ 2528*a^5*b^16*d^2*e^11 + 15296*a^7*b^14*d^2*e^11 + 14128*a^9*b^12*d^2*e^
11 - 5056*a^11*b^10*d^2*e^11 - 9248*a^13*b^8*d^2*e^11 + 64*a^15*b^6*d^2*e^1
1 + 1800*a^17*b^4*d^2*e^11 + 64*a^19*b^2*d^2*e^11))/(a^18*d^4 + a^2*b^16*d^
4 + 8*a^4*b^14*d^4 + 28*a^6*b^12*d^4 + 56*a^8*b^10*d^4 + 70*a^10*b^8*d^4 +
56*a^12*b^6*d^4 + 28*a^14*b^4*d^4 + 8*a^16*b^2*d^4) + (((64*a*b^23*d^4*e^11
+ 1472*a^3*b^21*d^4*e^11 + 8832*a^5*b^19*d^4*e^11 + 25344*a^7*b^17*d^4*e^1
1 + 40320*a^9*b^15*d^4*e^11 + 34944*a^11*b^13*d^4*e^11 + 10752*a^13*b^11*d^
4*e^11 - 8448*a^15*b^9*d^4*e^11 - 10176*a^17*b^7*d^4*e^11 - 4160*a^19*b^5*d
^4*e^11 - 640*a^21*b^3*d^4*e^11)/(a^18*d^5 + a^2*b^16*d^5 + 8*a^4*b^14*d^5
+ 28*a^6*b^12*d^5 + 56*a^8*b^10*d^5 + 70*a^10*b^8*d^5 + 56*a^12*b^6*d^5 + 2
8*a^14*b^4*d^5 + 8*a^16*b^2*d^5) - ((e*cot(c + d*x))^(1/2)*(b^4 - 15*a^4 +
18*a^2*b^2)*(-a^3*b*e)^(1/2)*(512*a^2*b^25*d^4*e^10 + 4608*a^4*b^23*d^4*e^1

0 + 17920*a^6*b^21*d^4*e^10 + 38400*a^8*b^19*d^4*e^10 + 46080*a^10*b^17*d^4
*e^10 + 21504*a^12*b^15*d^4*e^10 - 21504*a^14*b^13*d^4*e^10 - 46080*a^16*b^
11*d^4*e^10 - 38400*a^18*b^9*d^4*e^10 - 17920*a^20*b^7*d^4*e^10 - 4608*a^22
*b^5*d^4*e^10 - 512*a^24*b^3*d^4*e^10))/(8*(a^9*d + a^3*b^6*d + 3*a^5*b^4*d
 + 3*a^7*b^2*d)*(a^18*d^4 + a^2*b^16*d^4 + 8*a^4*b^14*d^4 + 28*a^6*b^12*d^4
 + 56*a^8*b^10*d^4 + 70*a^10*b^8*d^4 + 56*a^12*b^6*d^4 + 28*a^14*b^4*d^4 +
8*a^16*b^2*d^4)))*(b^4 - 15*a^4 + 18*a^2*b^2)*(-a^3*b*e)^(1/2))/(8*(a^9*d +
 a^3*b^6*d + 3*a^5*b^4*d + 3*a^7*b^2*d)))*(b^4 - 15*a^4 + 18*a^2*b^2)*(-a^3
*b*e)^(1/2))/(8*(a^9*d + a^3*b^6*d + 3*a^5*b^4*d + 3*a^7*b^2*d)))*(b^4 - 15
*a^4 + 18*a^2*b^2)*(-a^3*b*e)^(1/2))/(8*(a^9*d + a^3*b^6*d + 3*a^5*b^4*d +
3*a^7*b^2*d)))*(b^4 - 15*a^4 + 18*a^2*b^2)*(-a^3*b*e)^(1/2))/(8*(a^9*d + a^
3*b^6*d + 3*a^5*b^4*d + 3*a^7*b^2*d)) + ((((e*cot(c + d*x))^(1/2)*(2*a^2*b^
13*e^12 - b^15*e^12 + 49*a^4*b^11*e^12 + 2460*a^6*b^9*e^12 - 3631*a^8*b^7*e
^12 + 1922*a^10*b^5*e^12 - 225*a^12*b^3*e^12))/(a^18*d^4 + a^2*b^16*d^4 + 8
*a^4*b^14*d^4 + 28*a^6*b^12*d^4 + 56*a^8*b^10*d^4 + 70*a^10*b^8*d^4 + 56*a^
12*b^6*d^4 + 28*a^14*b^4*d^4 + 8*a^16*b^2*d^4) - (((2*b^18*d^2*e^12 - 138*a
^2*b^16*d^2*e^12 - 3046*a^4*b^14*d^2*e^12 + 4862*a^6*b^12*d^2*e^12 + 9222*a
^8*b^10*d^2*e^12 - 5246*a^10*b^8*d^2*e^12 - 4290*a^12*b^6*d^2*e^12 + 2442*a
^14*b^4*d^2*e^12 + 32*a^16*b^2*d^2*e^12)/(a^18*d^5 + a^2*b^16*d^5 + 8*a^4*b
^14*d^5 + 28*a^6*b^12*d^5 + 56*a^8*b^10*d^5 + 70*a^10*b^8*d^5 + 56*a^12*b^6
*d^5 + 28*a^14*b^4*d^5 + 8*a^16*b^2*d^5) + ((((e*cot(c + d*x))^(1/2)*(8*a*b
^20*d^2*e^11 - 1152*a^3*b^18*d^2*e^11 + 2528*a^5*b^16*d^2*e^11 + 15296*a^7*
b^14*d^2*e^11 + 14128*a^9*b^12*d^2*e^11 - 5056*a^11*b^10*d^2*e^11 - 9248*a^
13*b^8*d^2*e^11 + 64*a^15*b^6*d^2*e^11 + 1800*a^17*b^4*d^2*e^11 + 64*a^19*b
^2*d^2*e^11))/(a^18*d^4 + a^2*b^16*d^4 + 8*a^4*b^14*d^4 + 28*a^6*b^12*d^4 +
 56*a^8*b^10*d^4 + 70*a^10*b^8*d^4 + 56*a^12*b^6*d^4 + 28*a^14*b^4*d^4 + 8*
a^16*b^2*d^4) - (((64*a*b^23*d^4*e^11 + 1472*a^3*b^21*d^4*e^11 + 8832*a^5*b
^19*d^4*e^11 + 25344*a^7*b^17*d^4*e^11 + 40320*a^9*b^15*d^4*e^11 + 34944*a^
11*b^13*d^4*e^11 + 10752*a^13*b^11*d^4*e^11 - 8448*a^15*b^9*d^4*e^11 - 1017
6*a^17*b^7*d^4*e^11 - 4160*a^19*b^5*d^4*e^11 - 640*a^21*b^3*d^4*e^11)/(a^18
*d^5 + a^2*b^16*d^5 + 8*a^4*b^14*d^5 + 28*a^6*b^12*d^5 + 56*a^8*b^10*d^5 +
70*a^10*b^8*d^5 + 56*a^12*b^6*d^5 + 28*a^14*b^4*d^5 + 8*a^16*b^2*d^5) + ((e
*cot(c + d*x))^(1/2)*(b^4 - 15*a^4 + 18*a^2*b^2)*(-a^3*b*e)^(1/2)*(512*a^2*
b^25*d^4*e^10 + 4608*a^4*b^23*d^4*e^10 + 17920*a^6*b^21*d^4*e^10 + 38400*a^
8*b^19*d^4*e^10 + 46080*a^10*b^17*d^4*e^10 + 21504*a^12*b^15*d^4*e^10 - 215
04*a^14*b^13*d^4*e^10 - 46080*a^16*b^11*d^4*e^10 - 38400*a^18*b^9*d^4*e^10
 - 17920*a^20*b^7*d^4*e^10 - 4608*a^22*b^5*d^4*e^10 - 512*a^24*b^3*d^4*e^10)
)/(8*(a^9*d + a^3*b^6*d + 3*a^5*b^4*d + 3*a^7*b^2*d)*(a^18*d^4 + a^2*b^16*d
^4 + 8*a^4*b^14*d^4 + 28*a^6*b^12*d^4 + 56*a^8*b^10*d^4 + 70*a^10*b^8*d^4 +
 56*a^12*b^6*d^4 + 28*a^14*b^4*d^4 + 8*a^16*b^2*d^4)))*(b^4 - 15*a^4 + 18*a
^2*b^2)*(-a^3*b*e)^(1/2))/(8*(a^9*d + a^3*b^6*d + 3*a^5*b^4*d + 3*a^7*b^2*d
)))*(b^4 - 15*a^4 + 18*a^2*b^2)*(-a^3*b*e)^(1/2))/(8*(a^9*d + a^3*b^6*d + 3
*a^5*b^4*d + 3*a^7*b^2*d)))*(b^4 - 15*a^4 + 18*a^2*b^2)*(-a^3*b*e)^(1/2))/(
8*(a^9*d + a^3*b^6*d + 3*a^5*b^4*d + 3*a^7*b^2*d)))*(b^4 - 15*a^4 + 18*a^2*
b^2)*(-a^3*b*e)^(1/2))/(8*(a^9*d + a^3*b^6*d + 3*a^5*b^4*d + 3*a^7*b^2*d)))

```
)*(b^4 - 15*a^4 + 18*a^2*b^2)*(-a^3*b*e)^(1/2)*1i)/(4*(a^9*d + a^3*b^6*d +
3*a^5*b^4*d + 3*a^7*b^2*d))
```

3.86 $\int \frac{1}{\sqrt{e\cot(c+dx)}(a+b\cot(c+dx))^3}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 476

$$\begin{aligned}\int \frac{1}{\sqrt{e\cot(c+dx)}(a+b\cot(c+dx))^3}\,dx &= -\frac{b^{3/2}(35a^4+6a^2b^2+3b^4)\arctan\left(\frac{\sqrt{b}\sqrt{e\cot(c+dx)}}{\sqrt{a}\sqrt{e}}\right)}{4a^{5/2}(a^2+b^2)^3 d\sqrt{e}} \\ &+ \frac{(a+b)(a^2-4ab+b^2)\arctan\left(1-\frac{\sqrt{2}\sqrt{e\cot(c+dx)}}{\sqrt{e}}\right)}{\sqrt{2}(a^2+b^2)^3 d\sqrt{e}} \\ &- \frac{(a+b)(a^2-4ab+b^2)\arctan\left(1+\frac{\sqrt{2}\sqrt{e\cot(c+dx)}}{\sqrt{e}}\right)}{\sqrt{2}(a^2+b^2)^3 d\sqrt{e}} \\ &- \frac{b^2\sqrt{e\cot(c+dx)}}{2a(a^2+b^2)de(a+b\cot(c+dx))^2} - \frac{b^2(11a^2+3b^2)\sqrt{e\cot(c+dx)}}{4a^2(a^2+b^2)^2 de(a+b\cot(c+dx))} \\ &+ \frac{(a-b)(a^2+4ab+b^2)\log\left(\sqrt{e}+\sqrt{e}\cot(c+dx)-\sqrt{2}\sqrt{e\cot(c+dx)}\right)}{2\sqrt{2}(a^2+b^2)^3 d\sqrt{e}} \\ &- \frac{(a-b)(a^2+4ab+b^2)\log\left(\sqrt{e}+\sqrt{e}\cot(c+dx)+\sqrt{2}\sqrt{e\cot(c+dx)}\right)}{2\sqrt{2}(a^2+b^2)^3 d\sqrt{e}}\end{aligned}$$

```
[Out] -1/4*b^(3/2)*(35*a^4+6*a^2*b^2+3*b^4)*arctan(b^(1/2)*(e*cot(d*x+c))^(1/2)/a
^(1/2)/e^(1/2))/a^(5/2)/(a^2+b^2)^3/d/e^(1/2)+1/2*(a+b)*(a^2-4*a*b+b^2)*arc
tan(1-2^(1/2)*(e*cot(d*x+c))^(1/2)/e^(1/2))/(a^2+b^2)^3/d*2^(1/2)/e^(1/2)-1
/2*(a+b)*(a^2-4*a*b+b^2)*arctan(1+2^(1/2)*(e*cot(d*x+c))^(1/2)/e^(1/2))/(a^
2+b^2)^3/d*2^(1/2)/e^(1/2)+1/4*(a-b)*(a^2+4*a*b+b^2)*ln(e^(1/2)+cot(d*x+c)*
e^(1/2)-2^(1/2)*(e*cot(d*x+c))^(1/2))/(a^2+b^2)^3/d*2^(1/2)/e^(1/2)-1/4*(a-
b)*(a^2+4*a*b+b^2)*ln(e^(1/2)+cot(d*x+c)*e^(1/2)+2^(1/2)*(e*cot(d*x+c))^(1/
```

```
2))/(a^2+b^2)^3/d*2^(1/2)/e^(1/2)-1/2*b^2*(e*cot(d*x+c))^(1/2)/a/(a^2+b^2)/
d/e/(a+b*cot(d*x+c))^2-1/4*b^2*(11*a^2+3*b^2)*(e*cot(d*x+c))^(1/2)/a^2/(a^2
+b^2)^2/d/e/(a+b*cot(d*x+c))
```

Rubi [A] (verified)

Time = 1.35 (sec) , antiderivative size = 476, normalized size of antiderivative = 1.00, number of steps used = 16, number of rules used = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.520$, Rules used = {3650, 3730, 3734, 3615, 1182, 1176, 631, 210, 1179, 642, 3715, 65, 211}

$$\begin{aligned}&\int \frac{1}{\sqrt{e\cot(c+dx)}(a+b\cot(c+dx))^3}\,dx\\&=\frac{(a+b)\left(a^2-4ab+b^2\right)\arctan\left(1-\frac{\sqrt{2}\sqrt{e\cot(c+dx)}}{\sqrt{e}}\right)}{\sqrt{2}d\sqrt{e}\left(a^2+b^2\right)^3}\\&-\frac{(a+b)\left(a^2-4ab+b^2\right)\arctan\left(\frac{\sqrt{2}\sqrt{e\cot(c+dx)}}{\sqrt{e}}+1\right)}{\sqrt{2}d\sqrt{e}\left(a^2+b^2\right)^3}\\&-\frac{b^2(11a^2+3b^2)\sqrt{e\cot(c+dx)}}{4a^2de\left(a^2+b^2\right)^2(a+b\cot(c+dx))}-\frac{b^2\sqrt{e\cot(c+dx)}}{2ade\left(a^2+b^2\right)(a+b\cot(c+dx))^2}\\&+\frac{(a-b)\left(a^2+4ab+b^2\right)\log\left(\sqrt{e}\cot(c+dx)-\sqrt{2}\sqrt{e\cot(c+dx)}+\sqrt{e}\right)}{2\sqrt{2}d\sqrt{e}\left(a^2+b^2\right)^3}\\&-\frac{(a-b)\left(a^2+4ab+b^2\right)\log\left(\sqrt{e}\cot(c+dx)+\sqrt{2}\sqrt{e\cot(c+dx)}+\sqrt{e}\right)}{2\sqrt{2}d\sqrt{e}\left(a^2+b^2\right)^3}\\&-\frac{b^{3/2}(35a^4+6a^2b^2+3b^4)\arctan\left(\frac{\sqrt{b}\sqrt{e\cot(c+dx)}}{\sqrt{a}\sqrt{e}}\right)}{4a^{5/2}d\sqrt{e}\left(a^2+b^2\right)^3}\end{aligned}$$

```
[In] Int[1/(Sqrt[e*Cot[c + d*x]]*(a + b*Cot[c + d*x])^3),x]
```

```
[Out] -1/4*(b^(3/2)*(35*a^4 + 6*a^2*b^2 + 3*b^4)*ArcTan[(Sqrt[b]*Sqrt[e*Cot[c + d
*x]])/(Sqrt[a]*Sqrt[e])])/(a^(5/2)*(a^2 + b^2)^3*d*Sqrt[e]) + ((a + b)*(a^2
 - 4*a*b + b^2)*ArcTan[1 - (Sqrt[2]*Sqrt[e*Cot[c + d*x]])/Sqrt[e]])/(Sqrt[2
]*(a^2 + b^2)^3*d*Sqrt[e]) - ((a + b)*(a^2 - 4*a*b + b^2)*ArcTan[1 + (Sqrt[
2]*Sqrt[e*Cot[c + d*x]])/Sqrt[e]])/(Sqrt[2]*(a^2 + b^2)^3*d*Sqrt[e]) - (b^2
*Sqrt[e*Cot[c + d*x]])/(2*a*(a^2 + b^2)*d*e*(a + b*Cot[c + d*x])^2) - (b^2*
(11*a^2 + 3*b^2)*Sqrt[e*Cot[c + d*x]])/(4*a^2*(a^2 + b^2)^2*d*e*(a + b*Cot[
c + d*x])) + ((a - b)*(a^2 + 4*a*b + b^2)*Log[Sqrt[e] + Sqrt[e]*Cot[c + d*x
] - Sqrt[2]*Sqrt[e*Cot[c + d*x]]])/(2*Sqrt[2]*(a^2 + b^2)^3*d*Sqrt[e]) - ((
a - b)*(a^2 + 4*a*b + b^2)*Log[Sqrt[e] + Sqrt[e]*Cot[c + d*x] + Sqrt[2]*Sqr
t[e*Cot[c + d*x]]])/(2*Sqrt[2]*(a^2 + b^2)^3*d*Sqrt[e])
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1179

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 1182

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
a*c, 2]}, Dist[(d*q + a*e)/(2*a*c), Int[(q + c*x^2)/(a + c*x^4), x], x] + D
ist[(d*q - a*e)/(2*a*c), Int[(q - c*x^2)/(a + c*x^4), x], x]] /; FreeQ[{a,
c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0] && NeQ[c*d^2 - a*e^2, 0] && NegQ[(-a)
*c]
```

Rule 3615

```
Int[((c_) + (d_.)*tan[(e_.) + (f_.)*(x_)])/Sqrt[(b_.)*tan[(e_.) + (f_.)*(x_
)]], x_Symbol] :> Dist[2/f, Subst[Int[(b*c + d*x^2)/(b^2 + x^4), x], x, Sqr
t[b*Tan[e + f*x]]], x] /; FreeQ[{b, c, d, e, f}, x] && NeQ[c^2 - d^2, 0] &&
 NeQ[c^2 + d^2, 0]
```

Rule 3650

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)])^(n_), x_Symbol] :> Simp[b^2*(a + b*Tan[e + f*x])^(m + 1)*((c
+ d*Tan[e + f*x])^(n + 1)/(f*(m + 1)*(a^2 + b^2)*(b*c - a*d))), x] + Dist[1
/((m + 1)*(a^2 + b^2)*(b*c - a*d)), Int[(a + b*Tan[e + f*x])^(m + 1)*(c + d
*Tan[e + f*x])^n*Simp[a*(b*c - a*d)*(m + 1) - b^2*d*(m + n + 2) - b*(b*c -
a*d)*(m + 1)*Tan[e + f*x] - b^2*d*(m + n + 2)*Tan[e + f*x]^2, x], x], x] /;
 FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0]
&& NeQ[c^2 + d^2, 0] && IntegerQ[2*m] && LtQ[m, -1] && (LtQ[n, 0] || Intege
rQ[m]) &&  !(ILtQ[n, -1] && ( !IntegerQ[m] || (EqQ[c, 0] && NeQ[a, 0])))
```

Rule 3715

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*tan[(e_.)
+ (f_.)*(x_)])^(n_.)*((A_) + (C_.)*tan[(e_.) + (f_.)*(x_)]^2), x_Symbol] :>
 Dist[A/f, Subst[Int[(a + b*x)^m*(c + d*x)^n, x], x, Tan[e + f*x]], x] /; F
reeQ[{a, b, c, d, e, f, A, C, m, n}, x] && EqQ[A, C]
```

Rule 3730

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*tan[(e_.) + (f_.)*(x_)] + (C_.)*tan[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> Simp[(A*b^2 - a*(b*B - a*C))*(a + b*Tan[e +
 f*x])^(m + 1)*((c + d*Tan[e + f*x])^(n + 1)/(f*(m + 1)*(b*c - a*d)*(a^2 +
b^2))), x] + Dist[1/((m + 1)*(b*c - a*d)*(a^2 + b^2)), Int[(a + b*Tan[e + f
*x])^(m + 1)*(c + d*Tan[e + f*x])^n*Simp[A*(a*(b*c - a*d)*(m + 1) - b^2*d*(
m + n + 2)) + (b*B - a*C)*(b*c*(m + 1) + a*d*(n + 1)) - (m + 1)*(b*c - a*d)
*(A*b - a*B - b*C)*Tan[e + f*x] - d*(A*b^2 - a*(b*B - a*C))*(m + n + 2)*Tan
[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, n}, x] && NeQ[
b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] && LtQ[m, -1] &&  !
(ILtQ[n, -1] && ( !IntegerQ[m] || (EqQ[c, 0] && NeQ[a, 0])))
```

Rule 3734

```
Int[(((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_)*((A_.) + (B_.)*tan[(e_.)
+ (f_.)*(x_)] + (C_.)*tan[(e_.) + (f_.)*(x_)]^2))/((a_.) + (b_.)*tan[(e_.)
+ (f_.)*(x_)]), x_Symbol] :> Dist[1/(a^2 + b^2), Int[(c + d*Tan[e + f*x])^n
*Simp[b*B + a*(A - C) + (a*B - b*(A - C))*Tan[e + f*x], x], x], x] + Dist[(
A*b^2 - a*b*B + a^2*C)/(a^2 + b^2), Int[(c + d*Tan[e + f*x])^n*((1 + Tan[e
+ f*x]^2)/(a + b*Tan[e + f*x])), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C
, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] &&
 !GtQ[n, 0] &&  !LeQ[n, -1]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\frac{b^2\sqrt{e\cot(c+dx)}}{2a\left(a^2+b^2\right)de(a+b\cot(c+dx))^2} - \frac{\int \frac{-\frac{1}{2}\left(4a^2+3b^2\right)e+2abe\cot(c+dx)-\frac{3}{2}b^2e\cot^2(c+dx)}{\sqrt{e\cot(c+dx)}(a+b\cot(c+dx))^2}\,dx}{2a\left(a^2+b^2\right)e} \\
&= -\frac{b^2\sqrt{e\cot(c+dx)}}{2a\left(a^2+b^2\right)de(a+b\cot(c+dx))^2} - \frac{b^2(11a^2+3b^2)\sqrt{e\cot(c+dx)}}{4a^2\left(a^2+b^2\right)^2de(a+b\cot(c+dx))} \\
&\quad + \frac{\int \frac{\frac{1}{4}\left(8a^4+3a^2b^2+3b^4\right)e^2-4a^3be^2\cot(c+dx)+\frac{1}{4}b^2\left(11a^2+3b^2\right)e^2\cot^2(c+dx)}{\sqrt{e\cot(c+dx)}(a+b\cot(c+dx))}\,dx}{2a^2\left(a^2+b^2\right)^2e^2} \\
&= -\frac{b^2\sqrt{e\cot(c+dx)}}{2a\left(a^2+b^2\right)de(a+b\cot(c+dx))^2} - \frac{b^2(11a^2+3b^2)\sqrt{e\cot(c+dx)}}{4a^2\left(a^2+b^2\right)^2de(a+b\cot(c+dx))} \\
&\quad + \frac{\left(b^2(35a^4+6a^2b^2+3b^4)\right)\int \frac{1+\cot^2(c+dx)}{\sqrt{e\cot(c+dx)}(a+b\cot(c+dx))}\,dx}{8a^2\left(a^2+b^2\right)^3} \\
&\quad + \frac{\int \frac{2a^3\left(a^2-3b^2\right)e^2-2a^2b\left(3a^2-b^2\right)e^2\cot(c+dx)}{\sqrt{e\cot(c+dx)}}\,dx}{2a^2\left(a^2+b^2\right)^3e^2} \\
&= -\frac{b^2\sqrt{e\cot(c+dx)}}{2a\left(a^2+b^2\right)de(a+b\cot(c+dx))^2} - \frac{b^2(11a^2+3b^2)\sqrt{e\cot(c+dx)}}{4a^2\left(a^2+b^2\right)^2de(a+b\cot(c+dx))} \\
&\quad + \frac{\left(b^2(35a^4+6a^2b^2+3b^4)\right)\text{Subst}\left(\int \frac{1}{\sqrt{-ex}(a-bx)}\,dx, x, -\cot(c+dx)\right)}{8a^2\left(a^2+b^2\right)^3d} \\
&\quad + \frac{\text{Subst}\left(\int \frac{-2a^3\left(a^2-3b^2\right)e^3+2a^2b\left(3a^2-b^2\right)e^2x^2}{e^2+x^4}\,dx, x, \sqrt{e\cot(c+dx)}\right)}{a^2\left(a^2+b^2\right)^3de^2}
\end{aligned}$$

$$= -\frac{b^2\sqrt{e\cot(c+dx)}}{2a\,(a^2+b^2)\,de(a+b\cot(c+dx))^2} - \frac{b^2(11a^2+3b^2)\,\sqrt{e\cot(c+dx)}}{4a^2\,(a^2+b^2)^2\,de(a+b\cot(c+dx))}$$

$$-\frac{((a+b)\,(a^2-4ab+b^2))\,\text{Subst}\Big(\int\frac{e+x^2}{e^2+x^4}\,dx,x,\sqrt{e\cot(c+dx)}\Big)}{(a^2+b^2)^3\,d}$$

$$-\frac{((a-b)\,(a^2+4ab+b^2))\,\text{Subst}\Big(\int\frac{e-x^2}{e^2+x^4}\,dx,x,\sqrt{e\cot(c+dx)}\Big)}{(a^2+b^2)^3\,d}$$

$$-\frac{(b^2(35a^4+6a^2b^2+3b^4))\,\text{Subst}\left(\int\frac{1}{a+\frac{bx^2}{e}}\,dx,x,\sqrt{e\cot(c+dx)}\right)}{4a^2\,(a^2+b^2)^3\,de}$$

$$= -\frac{b^{3/2}(35a^4+6a^2b^2+3b^4)\arctan\left(\frac{\sqrt{b}\sqrt{e\cot(c+dx)}}{\sqrt{a}\sqrt{e}}\right)}{4a^{5/2}\,(a^2+b^2)^3\,d\sqrt{e}}$$

$$-\frac{b^2\sqrt{e\cot(c+dx)}}{2a\,(a^2+b^2)\,de(a+b\cot(c+dx))^2} - \frac{b^2(11a^2+3b^2)\,\sqrt{e\cot(c+dx)}}{4a^2\,(a^2+b^2)^2\,de(a+b\cot(c+dx))}$$

$$-\frac{((a+b)\,(a^2-4ab+b^2))\,\text{Subst}\Big(\int\frac{1}{e-\sqrt{2}\sqrt{e}x+x^2}\,dx,x,\sqrt{e\cot(c+dx)}\Big)}{2\,(a^2+b^2)^3\,d}$$

$$-\frac{((a+b)\,(a^2-4ab+b^2))\,\text{Subst}\Big(\int\frac{1}{e+\sqrt{2}\sqrt{e}x+x^2}\,dx,x,\sqrt{e\cot(c+dx)}\Big)}{2\,(a^2+b^2)^3\,d}$$

$$+\frac{((a-b)\,(a^2+4ab+b^2))\,\text{Subst}\Big(\int\frac{\sqrt{2}\sqrt{e}+2x}{-e-\sqrt{2}\sqrt{e}x-x^2}\,dx,x,\sqrt{e\cot(c+dx)}\Big)}{2\sqrt{2}\,(a^2+b^2)^3\,d\sqrt{e}}$$

$$+\frac{((a-b)\,(a^2+4ab+b^2))\,\text{Subst}\Big(\int\frac{\sqrt{2}\sqrt{e}-2x}{-e+\sqrt{2}\sqrt{e}x-x^2}\,dx,x,\sqrt{e\cot(c+dx)}\Big)}{2\sqrt{2}\,(a^2+b^2)^3\,d\sqrt{e}}$$

$$= -\frac{b^{3/2}(35a^4+6a^2b^2+3b^4)\arctan\left(\frac{\sqrt{b}\sqrt{e\cot(c+dx)}}{\sqrt{a}\sqrt{e}}\right)}{4a^{5/2}(a^2+b^2)^3 d\sqrt{e}} - \frac{b^2\sqrt{e\cot(c+dx)}}{2a(a^2+b^2)de(a+b\cot(c+dx))^2} - \frac{b^2(11a^2+3b^2)\sqrt{e\cot(c+dx)}}{4a^2(a^2+b^2)^2 de(a+b\cot(c+dx))} + \frac{(a-b)(a^2+4ab+b^2)\log\left(\sqrt{e}+\sqrt{e}\cot(c+dx)-\sqrt{2}\sqrt{e\cot(c+dx)}\right)}{2\sqrt{2}(a^2+b^2)^3 d\sqrt{e}} - \frac{(a-b)(a^2+4ab+b^2)\log\left(\sqrt{e}+\sqrt{e}\cot(c+dx)+\sqrt{2}\sqrt{e\cot(c+dx)}\right)}{2\sqrt{2}(a^2+b^2)^3 d\sqrt{e}} - \frac{((a+b)(a^2-4ab+b^2))\,\text{Subst}\left(\int\frac{1}{-1-x^2}\,dx,x,1-\frac{\sqrt{2}\sqrt{e\cot(c+dx)}}{\sqrt{e}}\right)}{\sqrt{2}(a^2+b^2)^3 d\sqrt{e}} + \frac{((a+b)(a^2-4ab+b^2))\,\text{Subst}\left(\int\frac{1}{-1-x^2}\,dx,x,1+\frac{\sqrt{2}\sqrt{e\cot(c+dx)}}{\sqrt{e}}\right)}{\sqrt{2}(a^2+b^2)^3 d\sqrt{e}}$$

$$= -\frac{b^{3/2}(35a^4+6a^2b^2+3b^4)\arctan\left(\frac{\sqrt{b}\sqrt{e\cot(c+dx)}}{\sqrt{a}\sqrt{e}}\right)}{4a^{5/2}(a^2+b^2)^3 d\sqrt{e}} + \frac{(a+b)(a^2-4ab+b^2)\arctan\left(1-\frac{\sqrt{2}\sqrt{e\cot(c+dx)}}{\sqrt{e}}\right)}{\sqrt{2}(a^2+b^2)^3 d\sqrt{e}} - \frac{(a+b)(a^2-4ab+b^2)\arctan\left(1+\frac{\sqrt{2}\sqrt{e\cot(c+dx)}}{\sqrt{e}}\right)}{\sqrt{2}(a^2+b^2)^3 d\sqrt{e}} - \frac{b^2\sqrt{e\cot(c+dx)}}{2a(a^2+b^2)de(a+b\cot(c+dx))^2} - \frac{b^2(11a^2+3b^2)\sqrt{e\cot(c+dx)}}{4a^2(a^2+b^2)^2 de(a+b\cot(c+dx))} + \frac{(a-b)(a^2+4ab+b^2)\log\left(\sqrt{e}+\sqrt{e}\cot(c+dx)-\sqrt{2}\sqrt{e\cot(c+dx)}\right)}{2\sqrt{2}(a^2+b^2)^3 d\sqrt{e}} - \frac{(a-b)(a^2+4ab+b^2)\log\left(\sqrt{e}+\sqrt{e}\cot(c+dx)+\sqrt{2}\sqrt{e\cot(c+dx)}\right)}{2\sqrt{2}(a^2+b^2)^3 d\sqrt{e}}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 6.17 (sec) , antiderivative size = 402, normalized size of antiderivative = 0.84

$$\int \frac{1}{\sqrt{e\cot(c+dx)}(a+b\cot(c+dx))^3}\,dx = -\frac{\sqrt{\cot(c+dx)}\left(\frac{2b^{3/2}(3a^2-b^2)\arctan\left(\frac{\sqrt{b}\sqrt{\cot(c+dx)}}{\sqrt{a}}\right)}{\sqrt{a}(a^2+b^2)^3} + \frac{2b^{3/2}\arctan\left(\frac{\sqrt{b}\sqrt{\cot(c+dx)}}{\sqrt{a}}\right)}{\sqrt{a}(a^2+b^2)^2} + \frac{2b^2\sqrt{\cot(c+dx)}}{(a^2+b^2)^2(a+b\cot(c+dx))} + \frac{2b^2\sqrt{\cot(c-}{}\right.}{}$$

[In] Integrate[1/(Sqrt[e*Cot[c + d*x]]*(a + b*Cot[c + d*x])^3),x]

[Out] -((Sqrt[Cot[c + d*x]]*((2*b^(3/2)*(3*a^2 - b^2)*ArcTan[(Sqrt[b]*Sqrt[Cot[c + d*x]])/Sqrt[a]])/(Sqrt[a]*(a^2 + b^2)^3) + (2*b^(3/2)*ArcTan[(Sqrt[b]*Sqrt[Cot[c + d*x]])/Sqrt[a]])/(Sqrt[a]*(a^2 + b^2)^2) + (2*b^2*Sqrt[Cot[c + d*x]])/((a^2 + b^2)^2*(a + b*Cot[c + d*x])) + (2*b^2*Sqrt[Cot[c + d*x]]*Hypergeometric2F1[1/2, 3, 3/2, -((b*Cot[c + d*x])/a)])/(a^3*(a^2 + b^2)) - (2*b*(3*a^2 - b^2)*Cot[c + d*x]^(3/2)*Hypergeometric2F1[3/4, 1, 7/4, -Cot[c + d*x]^2])/(3*(a^2 + b^2)^3) - (a*(a^2 - 3*b^2)*(2*Sqrt[2]*ArcTan[1 - Sqrt[2]*Sqrt[Cot[c + d*x]]] - 2*Sqrt[2]*ArcTan[1 + Sqrt[2]*Sqrt[Cot[c + d*x]]] + Sqrt[2]*Log[1 - Sqrt[2]*Sqrt[Cot[c + d*x]] + Cot[c + d*x]] - Sqrt[2]*Log[1 + Sqrt[2]*Sqrt[Cot[c + d*x]] + Cot[c + d*x]]))/(4*(a^2 + b^2)^3)))/(d*Sqrt[e*Cot[c + d*x]]))

Maple [A] (verified)

Time = 0.05 (sec) , antiderivative size = 465, normalized size of antiderivative = 0.98

method	result
derivativedivides	$-\frac{2e^4\left(\frac{b^2\left(\frac{\frac{b\left(11a^4+14a^2b^2+3b^4\right)(e\cot(dx+c))^{\frac{3}{2}}}{8a^2}+\frac{e\left(13a^4+18a^2b^2+5b^4\right)\sqrt{e\cot(dx+c)}}{8a}}{(e\cot(dx+c)b+ae)^2}+\frac{\left(35a^4+6a^2b^2+3b^4\right)\arctan\left(\frac{\sqrt{e\cot(dx+c)}}{\sqrt{aeb}}\right)}{8a^2\sqrt{aeb}}\right)}{e^4\left(a^2+b^2\right)^3}\right)}{}$
default	$-\frac{2e^4\left(\frac{b^2\left(\frac{\frac{b\left(11a^4+14a^2b^2+3b^4\right)(e\cot(dx+c))^{\frac{3}{2}}}{8a^2}+\frac{e\left(13a^4+18a^2b^2+5b^4\right)\sqrt{e\cot(dx+c)}}{8a}}{(e\cot(dx+c)b+ae)^2}+\frac{\left(35a^4+6a^2b^2+3b^4\right)\arctan\left(\frac{\sqrt{e\cot(dx+c)}}{\sqrt{aeb}}\right)}{8a^2\sqrt{aeb}}\right)}{e^4\left(a^2+b^2\right)^3}\right)}{}$

[In] int(1/(e*cot(d*x+c))^(1/2)/(a+b*cot(d*x+c))^3,x,method=_RETURNVERBOSE)

[Out] -2/d*e^4*(b^2/e^4/(a^2+b^2)^3*((1/8*b*(11*a^4+14*a^2*b^2+3*b^4)/a^2*(e*cot(d*x+c))^(3/2)+1/8*e*(13*a^4+18*a^2*b^2+5*b^4)/a*(e*cot(d*x+c))^(1/2))/(e*cot(d*x+c)*b+a*e)^2+1/8*(35*a^4+6*a^2*b^2+3*b^4)/a^2/(a*e*b)^(1/2)*arctan((e*cot(d*x+c))^(1/2)*b/(a*e*b)^(1/2)))+1/e^4/(a^2+b^2)^3*(1/8*(a^3*e-3*a*b^2*e)*(e^2)^(1/4)/e^2*2^(1/2)*(ln((e*cot(d*x+c)+(e^2)^(1/4)*(e*cot(d*x+c))^(1/2)*2^(1/2)+(e^2)^(1/2))/(e*cot(d*x+c)-(e^2)^(1/4)*(e*cot(d*x+c))^(1/2)*2^(1/2)+(e^2)^(1/2)))+2*arctan(2^(1/2)/(e^2)^(1/4)*(e*cot(d*x+c))^(1/2)+1)-2*arctan(-2^(1/2)/(e^2)^(1/4)*(e*cot(d*x+c))^(1/2)+1))+1/8*(-3*a^2*b+b^3)/(e^2)^(1/4)*2^(1/2)*(ln((e*cot(d*x+c)-(e^2)^(1/4)*(e*cot(d*x+c))^(1/2)*2^(1/2)+(e

```
^2)^(1/2))/(e*cot(d*x+c)+(e^2)^(1/4)*(e*cot(d*x+c))^(1/2)*2^(1/2)+(e^2)^(1/
2)))+2*arctan(2^(1/2)/(e^2)^(1/4)*(e*cot(d*x+c))^(1/2)+1)-2*arctan(-2^(1/2)
/(e^2)^(1/4)*(e*cot(d*x+c))^(1/2)+1))))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 4473 vs. $2(405) = 810$.

Time = 1.58 (sec) , antiderivative size = 8991, normalized size of antiderivative = 18.89

$$\int \frac{1}{\sqrt{e\cot(c+dx)}(a+b\cot(c+dx))^3}\,dx = \text{Too large to display}$$

```
[In] integrate(1/(e*cot(d*x+c))^(1/2)/(a+b*cot(d*x+c))^3,x, algorithm="fricas")
[Out] Too large to include
```

Sympy [F]

$$\int \frac{1}{\sqrt{e\cot(c+dx)}(a+b\cot(c+dx))^3}\,dx = \int \frac{1}{\sqrt{e\cot{(c+dx)}}\,(a+b\cot{(c+dx)})^3}\,dx$$

```
[In] integrate(1/(e*cot(d*x+c))**(1/2)/(a+b*cot(d*x+c))**3,x)
[Out] Integral(1/(sqrt(e*cot(c + d*x))*(a + b*cot(c + d*x))**3), x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{1}{\sqrt{e\cot(c+dx)}(a+b\cot(c+dx))^3}\,dx = \text{Exception raised: ValueError}$$

```
[In] integrate(1/(e*cot(d*x+c))^(1/2)/(a+b*cot(d*x+c))^3,x, algorithm="maxima")
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(e>0)', see `assume?` for more detai
ls)Is e
```

Giac [F]

$$\int \frac{1}{\sqrt{e\cot(c+dx)}(a+b\cot(c+dx))^3}\,dx = \int \frac{1}{\left(b\cot\left(dx+c\right)+a\right)^3\sqrt{e\cot\left(dx+c\right)}}\,dx$$

```
[In] integrate(1/(e*cot(d*x+c))^(1/2)/(a+b*cot(d*x+c))^3,x, algorithm="giac")
[Out] integrate(1/((b*cot(d*x + c) + a)^3*sqrt(e*cot(d*x + c))), x)
```

Mupad [B] (verification not implemented)

Time = 19.45 (sec) , antiderivative size = 20155, normalized size of antiderivative = 42.34

$$\int \frac{1}{\sqrt{e\cot(c+dx)}(a+b\cot(c+dx))^3}\,dx = \text{Too large to display}$$

```
[In] int(1/((e*cot(c + d*x))^(1/2)*(a + b*cot(c + d*x))^3),x)
[Out] atan(((1i/(4*(b^6*d^2*e - a^6*d^2*e - 15*a^2*b^4*d^2*e - a^3*b^3*d^2*e*20i
+ 15*a^4*b^2*d^2*e + a*b^5*d^2*e*6i + a^5*b*d^2*e*6i)))^(1/2)*((1i/(4*(b^6*
d^2*e - a^6*d^2*e - 15*a^2*b^4*d^2*e - a^3*b^3*d^2*e*20i + 15*a^4*b^2*d^2*e
 + a*b^5*d^2*e*6i + a^5*b*d^2*e*6i)))^(1/2)*((1i/(4*(b^6*d^2*e - a^6*d^2*e
- 15*a^2*b^4*d^2*e - a^3*b^3*d^2*e*20i + 15*a^4*b^2*d^2*e + a*b^5*d^2*e*6i
+ a^5*b*d^2*e*6i)))^(1/2)*((1i/(4*(b^6*d^2*e - a^6*d^2*e - 15*a^2*b^4*d^2*e
 - a^3*b^3*d^2*e*20i + 15*a^4*b^2*d^2*e + a*b^5*d^2*e*6i + a^5*b*d^2*e*6i))
)^(1/2)*((192*a^2*b^24*d^4*e^10 + 1728*a^4*b^22*d^4*e^10 + 8320*a^6*b^20*d^
4*e^10 + 27264*a^8*b^18*d^4*e^10 + 62592*a^10*b^16*d^4*e^10 + 99456*a^12*b^
14*d^4*e^10 + 107520*a^14*b^12*d^4*e^10 + 76800*a^16*b^10*d^4*e^10 + 33984*
a^18*b^8*d^4*e^10 + 7872*a^20*b^6*d^4*e^10 + 384*a^22*b^4*d^4*e^10 - 128*a^
24*b^2*d^4*e^10)/(a^20*d^5 + a^4*b^16*d^5 + 8*a^6*b^14*d^5 + 28*a^8*b^12*d^
5 + 56*a^10*b^10*d^5 + 70*a^12*b^8*d^5 + 56*a^14*b^6*d^5 + 28*a^16*b^4*d^5
+ 8*a^18*b^2*d^5) - ((1i/(4*(b^6*d^2*e - a^6*d^2*e - 15*a^2*b^4*d^2*e - a^3
*b^3*d^2*e*20i + 15*a^4*b^2*d^2*e + a*b^5*d^2*e*6i + a^5*b*d^2*e*6i)))^(1/2
)*(e*cot(c + d*x))^(1/2)*(512*a^4*b^25*d^4*e^10 + 4608*a^6*b^23*d^4*e^10 +
17920*a^8*b^21*d^4*e^10 + 38400*a^10*b^19*d^4*e^10 + 46080*a^12*b^17*d^4*e^
10 + 21504*a^14*b^15*d^4*e^10 - 21504*a^16*b^13*d^4*e^10 - 46080*a^18*b^11*
d^4*e^10 - 38400*a^20*b^9*d^4*e^10 - 17920*a^22*b^7*d^4*e^10 - 4608*a^24*b^
5*d^4*e^10 - 512*a^26*b^3*d^4*e^10))/(a^20*d^4 + a^4*b^16*d^4 + 8*a^6*b^14*
d^4 + 28*a^8*b^12*d^4 + 56*a^10*b^10*d^4 + 70*a^12*b^8*d^4 + 56*a^14*b^6*d^
4 + 28*a^16*b^4*d^4 + 8*a^18*b^2*d^4)) + ((e*cot(c + d*x))^(1/2)*(72*a*b^22
*d^2*e^9 + 576*a^3*b^20*d^2*e^9 + 5024*a^5*b^18*d^2*e^9 + 14272*a^7*b^16*d^
2*e^9 + 27824*a^9*b^14*d^2*e^9 + 53184*a^11*b^12*d^2*e^9 + 70240*a^13*b^10*
d^2*e^9 + 47680*a^15*b^8*d^2*e^9 + 12616*a^17*b^6*d^2*e^9 - 64*a^21*b^2*d^2
*e^9))/(a^20*d^4 + a^4*b^16*d^4 + 8*a^6*b^14*d^4 + 28*a^8*b^12*d^4 + 56*a^1
```

0*b^10*d^4 + 70*a^12*b^8*d^4 + 56*a^14*b^6*d^4 + 28*a^16*b^4*d^4 + 8*a^18*b
^2*d^4)) - (90*a*b^19*d^2*e^9 + 846*a^3*b^17*d^2*e^9 + 1714*a^5*b^15*d^2*e^
9 + 3606*a^7*b^13*d^2*e^9 - 14578*a^9*b^11*d^2*e^9 - 34486*a^11*b^9*d^2*e^9
 - 14970*a^13*b^7*d^2*e^9 + 2258*a^15*b^5*d^2*e^9 - 32*a^17*b^3*d^2*e^9)/(a
^20*d^5 + a^4*b^16*d^5 + 8*a^6*b^14*d^5 + 28*a^8*b^12*d^5 + 56*a^10*b^10*d^
5 + 70*a^12*b^8*d^5 + 56*a^14*b^6*d^5 + 28*a^16*b^4*d^5 + 8*a^18*b^2*d^5))
 + ((e*cot(c + d*x))^(1/2)*(18*a^2*b^15*e^8 - 9*b^17*e^8 - 71*a^4*b^13*e^8 +
 892*a^6*b^11*e^8 + 857*a^8*b^9*e^8 + 6802*a^10*b^7*e^8 - 1257*a^12*b^5*e^8
))/(a^20*d^4 + a^4*b^16*d^4 + 8*a^6*b^14*d^4 + 28*a^8*b^12*d^4 + 56*a^10*b^
10*d^4 + 70*a^12*b^8*d^4 + 56*a^14*b^6*d^4 + 28*a^16*b^4*d^4 + 8*a^18*b^2*d
^4))*1i - (1i/(4*(b^6*d^2*e - a^6*d^2*e - 15*a^2*b^4*d^2*e - a^3*b^3*d^2*e*
20i + 15*a^4*b^2*d^2*e + a*b^5*d^2*e*6i + a^5*b*d^2*e*6i)))^(1/2)*((1i/(4*(
b^6*d^2*e - a^6*d^2*e - 15*a^2*b^4*d^2*e - a^3*b^3*d^2*e*20i + 15*a^4*b^2*d
^2*e + a*b^5*d^2*e*6i + a^5*b*d^2*e*6i)))^(1/2)*((1i/(4*(b^6*d^2*e - a^6*d^
2*e - 15*a^2*b^4*d^2*e - a^3*b^3*d^2*e*20i + 15*a^4*b^2*d^2*e + a*b^5*d^2*e
*6i + a^5*b*d^2*e*6i)))^(1/2)*((1i/(4*(b^6*d^2*e - a^6*d^2*e - 15*a^2*b^4*d
^2*e - a^3*b^3*d^2*e*20i + 15*a^4*b^2*d^2*e + a*b^5*d^2*e*6i + a^5*b*d^2*e*
6i)))^(1/2)*((192*a^2*b^24*d^4*e^10 + 1728*a^4*b^22*d^4*e^10 + 8320*a^6*b^2
0*d^4*e^10 + 27264*a^8*b^18*d^4*e^10 + 62592*a^10*b^16*d^4*e^10 + 99456*a^1
2*b^14*d^4*e^10 + 107520*a^14*b^12*d^4*e^10 + 76800*a^16*b^10*d^4*e^10 + 33
984*a^18*b^8*d^4*e^10 + 7872*a^20*b^6*d^4*e^10 + 384*a^22*b^4*d^4*e^10 - 12
8*a^24*b^2*d^4*e^10)/(a^20*d^5 + a^4*b^16*d^5 + 8*a^6*b^14*d^5 + 28*a^8*b^1
2*d^5 + 56*a^10*b^10*d^5 + 70*a^12*b^8*d^5 + 56*a^14*b^6*d^5 + 28*a^16*b^4*
d^5 + 8*a^18*b^2*d^5) + ((1i/(4*(b^6*d^2*e - a^6*d^2*e - 15*a^2*b^4*d^2*e -
 a^3*b^3*d^2*e*20i + 15*a^4*b^2*d^2*e + a*b^5*d^2*e*6i + a^5*b*d^2*e*6i)))^
(1/2)*(e*cot(c + d*x))^(1/2)*(512*a^4*b^25*d^4*e^10 + 4608*a^6*b^23*d^4*e^1
0 + 17920*a^8*b^21*d^4*e^10 + 38400*a^10*b^19*d^4*e^10 + 46080*a^12*b^17*d^
4*e^10 + 21504*a^14*b^15*d^4*e^10 - 21504*a^16*b^13*d^4*e^10 - 46080*a^18*b
^11*d^4*e^10 - 38400*a^20*b^9*d^4*e^10 - 17920*a^22*b^7*d^4*e^10 - 4608*a^2
4*b^5*d^4*e^10 - 512*a^26*b^3*d^4*e^10))/(a^20*d^4 + a^4*b^16*d^4 + 8*a^6*b
^14*d^4 + 28*a^8*b^12*d^4 + 56*a^10*b^10*d^4 + 70*a^12*b^8*d^4 + 56*a^14*b^
6*d^4 + 28*a^16*b^4*d^4 + 8*a^18*b^2*d^4)) - ((e*cot(c + d*x))^(1/2)*(72*a*
b^22*d^2*e^9 + 576*a^3*b^20*d^2*e^9 + 5024*a^5*b^18*d^2*e^9 + 14272*a^7*b^1
6*d^2*e^9 + 27824*a^9*b^14*d^2*e^9 + 53184*a^11*b^12*d^2*e^9 + 70240*a^13*b
^10*d^2*e^9 + 47680*a^15*b^8*d^2*e^9 + 12616*a^17*b^6*d^2*e^9 - 64*a^21*b^2
*d^2*e^9))/(a^20*d^4 + a^4*b^16*d^4 + 8*a^6*b^14*d^4 + 28*a^8*b^12*d^4 + 56
*a^10*b^10*d^4 + 70*a^12*b^8*d^4 + 56*a^14*b^6*d^4 + 28*a^16*b^4*d^4 + 8*a^
18*b^2*d^4)) - (90*a*b^19*d^2*e^9 + 846*a^3*b^17*d^2*e^9 + 1714*a^5*b^15*d^
2*e^9 + 3606*a^7*b^13*d^2*e^9 - 14578*a^9*b^11*d^2*e^9 - 34486*a^11*b^9*d^2
*e^9 - 14970*a^13*b^7*d^2*e^9 + 2258*a^15*b^5*d^2*e^9 - 32*a^17*b^3*d^2*e^9
)/(a^20*d^5 + a^4*b^16*d^5 + 8*a^6*b^14*d^5 + 28*a^8*b^12*d^5 + 56*a^10*b^1
0*d^5 + 70*a^12*b^8*d^5 + 56*a^14*b^6*d^5 + 28*a^16*b^4*d^5 + 8*a^18*b^2*d^
5)) - ((e*cot(c + d*x))^(1/2)*(18*a^2*b^15*e^8 - 9*b^17*e^8 - 71*a^4*b^13*e
^8 + 892*a^6*b^11*e^8 + 857*a^8*b^9*e^8 + 6802*a^10*b^7*e^8 - 1257*a^12*b^5
*e^8))/(a^20*d^4 + a^4*b^16*d^4 + 8*a^6*b^14*d^4 + 28*a^8*b^12*d^4 + 56*a^1

```
0*b^10*d^4 + 70*a^12*b^8*d^4 + 56*a^14*b^6*d^4 + 28*a^16*b^4*d^4 + 8*a^18*b
^2*d^4))*1i)/((9*b^14*e^8 + 60*a^2*b^12*e^8 + 318*a^4*b^10*e^8 + 748*a^6*b^
8*e^8 + 1505*a^8*b^6*e^8)/(a^20*d^5 + a^4*b^16*d^5 + 8*a^6*b^14*d^5 + 28*a^
8*b^12*d^5 + 56*a^10*b^10*d^5 + 70*a^12*b^8*d^5 + 56*a^14*b^6*d^5 + 28*a^16
*b^4*d^5 + 8*a^18*b^2*d^5) + (1i/(4*(b^6*d^2*e - a^6*d^2*e - 15*a^2*b^4*d^2
*e - a^3*b^3*d^2*e*20i + 15*a^4*b^2*d^2*e + a*b^5*d^2*e*6i + a^5*b*d^2*e*6i
)))^(1/2)*((1i/(4*(b^6*d^2*e - a^6*d^2*e - 15*a^2*b^4*d^2*e - a^3*b^3*d^2*e
*20i + 15*a^4*b^2*d^2*e + a*b^5*d^2*e*6i + a^5*b*d^2*e*6i)))^(1/2)*((1i/(4*
(b^6*d^2*e - a^6*d^2*e - 15*a^2*b^4*d^2*e - a^3*b^3*d^2*e*20i + 15*a^4*b^2*
d^2*e + a*b^5*d^2*e*6i + a^5*b*d^2*e*6i)))^(1/2)*((1i/(4*(b^6*d^2*e - a^6*d
^2*e - 15*a^2*b^4*d^2*e - a^3*b^3*d^2*e*20i + 15*a^4*b^2*d^2*e + a*b^5*d^2*
e*6i + a^5*b*d^2*e*6i)))^(1/2)*((192*a^2*b^24*d^4*e^10 + 1728*a^4*b^22*d^4*
e^10 + 8320*a^6*b^20*d^4*e^10 + 27264*a^8*b^18*d^4*e^10 + 62592*a^10*b^16*d
^4*e^10 + 99456*a^12*b^14*d^4*e^10 + 107520*a^14*b^12*d^4*e^10 + 76800*a^16
*b^10*d^4*e^10 + 33984*a^18*b^8*d^4*e^10 + 7872*a^20*b^6*d^4*e^10 + 384*a^2
2*b^4*d^4*e^10 - 128*a^24*b^2*d^4*e^10)/(a^20*d^5 + a^4*b^16*d^5 + 8*a^6*b^
14*d^5 + 28*a^8*b^12*d^5 + 56*a^10*b^10*d^5 + 70*a^12*b^8*d^5 + 56*a^14*b^6
*d^5 + 28*a^16*b^4*d^5 + 8*a^18*b^2*d^5) - ((1i/(4*(b^6*d^2*e - a^6*d^2*e -
 15*a^2*b^4*d^2*e - a^3*b^3*d^2*e*20i + 15*a^4*b^2*d^2*e + a*b^5*d^2*e*6i +
 a^5*b*d^2*e*6i)))^(1/2)*(e*cot(c + d*x))^(1/2)*(512*a^4*b^25*d^4*e^10 + 46
08*a^6*b^23*d^4*e^10 + 17920*a^8*b^21*d^4*e^10 + 38400*a^10*b^19*d^4*e^10 +
 46080*a^12*b^17*d^4*e^10 + 21504*a^14*b^15*d^4*e^10 - 21504*a^16*b^13*d^4*
e^10 - 46080*a^18*b^11*d^4*e^10 - 38400*a^20*b^9*d^4*e^10 - 17920*a^22*b^7*
d^4*e^10 - 4608*a^24*b^5*d^4*e^10 - 512*a^26*b^3*d^4*e^10))/(a^20*d^4 + a^4
*b^16*d^4 + 8*a^6*b^14*d^4 + 28*a^8*b^12*d^4 + 56*a^10*b^10*d^4 + 70*a^12*b
^8*d^4 + 56*a^14*b^6*d^4 + 28*a^16*b^4*d^4 + 8*a^18*b^2*d^4)) + ((e*cot(c +
 d*x))^(1/2)*(72*a*b^22*d^2*e^9 + 576*a^3*b^20*d^2*e^9 + 5024*a^5*b^18*d^2*
e^9 + 14272*a^7*b^16*d^2*e^9 + 27824*a^9*b^14*d^2*e^9 + 53184*a^11*b^12*d^2
*e^9 + 70240*a^13*b^10*d^2*e^9 + 47680*a^15*b^8*d^2*e^9 + 12616*a^17*b^6*d^
2*e^9 - 64*a^21*b^2*d^2*e^9))/(a^20*d^4 + a^4*b^16*d^4 + 8*a^6*b^14*d^4 + 2
8*a^8*b^12*d^4 + 56*a^10*b^10*d^4 + 70*a^12*b^8*d^4 + 56*a^14*b^6*d^4 + 28*
a^16*b^4*d^4 + 8*a^18*b^2*d^4)) - (90*a*b^19*d^2*e^9 + 846*a^3*b^17*d^2*e^9
 + 1714*a^5*b^15*d^2*e^9 + 3606*a^7*b^13*d^2*e^9 - 14578*a^9*b^11*d^2*e^9 -
 34486*a^11*b^9*d^2*e^9 - 14970*a^13*b^7*d^2*e^9 + 2258*a^15*b^5*d^2*e^9 -
32*a^17*b^3*d^2*e^9)/(a^20*d^5 + a^4*b^16*d^5 + 8*a^6*b^14*d^5 + 28*a^8*b^1
2*d^5 + 56*a^10*b^10*d^5 + 70*a^12*b^8*d^5 + 56*a^14*b^6*d^5 + 28*a^16*b^4*
d^5 + 8*a^18*b^2*d^5)) + ((e*cot(c + d*x))^(1/2)*(18*a^2*b^15*e^8 - 9*b^17*
e^8 - 71*a^4*b^13*e^8 + 892*a^6*b^11*e^8 + 857*a^8*b^9*e^8 + 6802*a^10*b^7*
e^8 - 1257*a^12*b^5*e^8))/(a^20*d^4 + a^4*b^16*d^4 + 8*a^6*b^14*d^4 + 28*a^
8*b^12*d^4 + 56*a^10*b^10*d^4 + 70*a^12*b^8*d^4 + 56*a^14*b^6*d^4 + 28*a^16
*b^4*d^4 + 8*a^18*b^2*d^4)) + (1i/(4*(b^6*d^2*e - a^6*d^2*e - 15*a^2*b^4*d^
2*e - a^3*b^3*d^2*e*20i + 15*a^4*b^2*d^2*e + a*b^5*d^2*e*6i + a^5*b*d^2*e*6
i)))^(1/2)*((1i/(4*(b^6*d^2*e - a^6*d^2*e - 15*a^2*b^4*d^2*e - a^3*b^3*d^2*
e*20i + 15*a^4*b^2*d^2*e + a*b^5*d^2*e*6i + a^5*b*d^2*e*6i)))^(1/2)*((1i/(4
*(b^6*d^2*e - a^6*d^2*e - 15*a^2*b^4*d^2*e - a^3*b^3*d^2*e*20i + 15*a^4*b^2
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*d^2*e + a*b^5*d^2*e*6i + a^5*b*d^2*e*6i)))^(1/2)*((1i/(4*(b^6*d^2*e - a^6*
d^2*e - 15*a^2*b^4*d^2*e - a^3*b^3*d^2*e*20i + 15*a^4*b^2*d^2*e + a*b^5*d^2
*e*6i + a^5*b*d^2*e*6i)))^(1/2)*((192*a^2*b^24*d^4*e^10 + 1728*a^4*b^22*d^4
*e^10 + 8320*a^6*b^20*d^4*e^10 + 27264*a^8*b^18*d^4*e^10 + 62592*a^10*b^16*
d^4*e^10 + 99456*a^12*b^14*d^4*e^10 + 107520*a^14*b^12*d^4*e^10 + 76800*a^1
6*b^10*d^4*e^10 + 33984*a^18*b^8*d^4*e^10 + 7872*a^20*b^6*d^4*e^10 + 384*a^
22*b^4*d^4*e^10 - 128*a^24*b^2*d^4*e^10)/(a^20*d^5 + a^4*b^16*d^5 + 8*a^6*b
^14*d^5 + 28*a^8*b^12*d^5 + 56*a^10*b^10*d^5 + 70*a^12*b^8*d^5 + 56*a^14*b^
6*d^5 + 28*a^16*b^4*d^5 + 8*a^18*b^2*d^5) + ((1i/(4*(b^6*d^2*e - a^6*d^2*e
- 15*a^2*b^4*d^2*e - a^3*b^3*d^2*e*20i + 15*a^4*b^2*d^2*e + a*b^5*d^2*e*6i
+ a^5*b*d^2*e*6i)))^(1/2)*(e*cot(c + d*x))^(1/2)*(512*a^4*b^25*d^4*e^10 + 4
608*a^6*b^23*d^4*e^10 + 17920*a^8*b^21*d^4*e^10 + 38400*a^10*b^19*d^4*e^10
+ 46080*a^12*b^17*d^4*e^10 + 21504*a^14*b^15*d^4*e^10 - 21504*a^16*b^13*d^4
*e^10 - 46080*a^18*b^11*d^4*e^10 - 38400*a^20*b^9*d^4*e^10 - 17920*a^22*b^7
*d^4*e^10 - 4608*a^24*b^5*d^4*e^10 - 512*a^26*b^3*d^4*e^10))/(a^20*d^4 + a^
4*b^16*d^4 + 8*a^6*b^14*d^4 + 28*a^8*b^12*d^4 + 56*a^10*b^10*d^4 + 70*a^12*
b^8*d^4 + 56*a^14*b^6*d^4 + 28*a^16*b^4*d^4 + 8*a^18*b^2*d^4)) - ((e*cot(c
+ d*x))^(1/2)*(72*a*b^22*d^2*e^9 + 576*a^3*b^20*d^2*e^9 + 5024*a^5*b^18*d^2
*e^9 + 14272*a^7*b^16*d^2*e^9 + 27824*a^9*b^14*d^2*e^9 + 53184*a^11*b^12*d^
2*e^9 + 70240*a^13*b^10*d^2*e^9 + 47680*a^15*b^8*d^2*e^9 + 12616*a^17*b^6*d
^2*e^9 - 64*a^21*b^2*d^2*e^9))/(a^20*d^4 + a^4*b^16*d^4 + 8*a^6*b^14*d^4 +
28*a^8*b^12*d^4 + 56*a^10*b^10*d^4 + 70*a^12*b^8*d^4 + 56*a^14*b^6*d^4 + 28
*a^16*b^4*d^4 + 8*a^18*b^2*d^4)) - (90*a*b^19*d^2*e^9 + 846*a^3*b^17*d^2*e^
9 + 1714*a^5*b^15*d^2*e^9 + 3606*a^7*b^13*d^2*e^9 - 14578*a^9*b^11*d^2*e^9
- 34486*a^11*b^9*d^2*e^9 - 14970*a^13*b^7*d^2*e^9 + 2258*a^15*b^5*d^2*e^9 -
 32*a^17*b^3*d^2*e^9)/(a^20*d^5 + a^4*b^16*d^5 + 8*a^6*b^14*d^5 + 28*a^8*b^
12*d^5 + 56*a^10*b^10*d^5 + 70*a^12*b^8*d^5 + 56*a^14*b^6*d^5 + 28*a^16*b^4
*d^5 + 8*a^18*b^2*d^5)) - ((e*cot(c + d*x))^(1/2)*(18*a^2*b^15*e^8 - 9*b^17
*e^8 - 71*a^4*b^13*e^8 + 892*a^6*b^11*e^8 + 857*a^8*b^9*e^8 + 6802*a^10*b^7
*e^8 - 1257*a^12*b^5*e^8))/(a^20*d^4 + a^4*b^16*d^4 + 8*a^6*b^14*d^4 + 28*a
^8*b^12*d^4 + 56*a^10*b^10*d^4 + 70*a^12*b^8*d^4 + 56*a^14*b^6*d^4 + 28*a^1
6*b^4*d^4 + 8*a^18*b^2*d^4))))*(1i/(4*(b^6*d^2*e - a^6*d^2*e - 15*a^2*b^4*d
^2*e - a^3*b^3*d^2*e*20i + 15*a^4*b^2*d^2*e + a*b^5*d^2*e*6i + a^5*b*d^2*e*
6i)))^(1/2)*2i - ((b^3*(e*cot(c + d*x))^(3/2)*(11*a^2 + 3*b^2))/(4*a^2*(a^4
 + b^4 + 2*a^2*b^2)) + (b^2*e*(e*cot(c + d*x))^(1/2)*(13*a^2 + 5*b^2))/(4*a
*(a^2 + b^2)^2))/(a^2*d*e^2 + b^2*d*e^2*cot(c + d*x)^2 + 2*a*b*d*e^2*cot(c
+ d*x)) + atan((((((((1/(b^6*d^2*e*1i - a^6*d^2*e*1i - a^2*b^4*d^2*e*15i -
 20*a^3*b^3*d^2*e + a^4*b^2*d^2*e*15i + 6*a*b^5*d^2*e + 6*a^5*b*d^2*e))^(1/
2)*((192*a^2*b^24*d^4*e^10 + 1728*a^4*b^22*d^4*e^10 + 8320*a^6*b^20*d^4*e^1
0 + 27264*a^8*b^18*d^4*e^10 + 62592*a^10*b^16*d^4*e^10 + 99456*a^12*b^14*d^
4*e^10 + 107520*a^14*b^12*d^4*e^10 + 76800*a^16*b^10*d^4*e^10 + 33984*a^18*
b^8*d^4*e^10 + 7872*a^20*b^6*d^4*e^10 + 384*a^22*b^4*d^4*e^10 - 128*a^24*b^
2*d^4*e^10)/(2*(a^20*d^5 + a^4*b^16*d^5 + 8*a^6*b^14*d^5 + 28*a^8*b^12*d^5
+ 56*a^10*b^10*d^5 + 70*a^12*b^8*d^5 + 56*a^14*b^6*d^5 + 28*a^16*b^4*d^5 +
8*a^18*b^2*d^5)) - ((e*cot(c + d*x))^(1/2)*(1/(b^6*d^2*e*1i - a^6*d^2*e*1i
```

- a^2*b^4*d^2*e*15i - 20*a^3*b^3*d^2*e + a^4*b^2*d^2*e*15i + 6*a*b^5*d^2*e
+ 6*a^5*b*d^2*e))^(1/2)*(512*a^4*b^25*d^4*e^10 + 4608*a^6*b^23*d^4*e^10 + 1
7920*a^8*b^21*d^4*e^10 + 38400*a^10*b^19*d^4*e^10 + 46080*a^12*b^17*d^4*e^1
0 + 21504*a^14*b^15*d^4*e^10 - 21504*a^16*b^13*d^4*e^10 - 46080*a^18*b^11*d
^4*e^10 - 38400*a^20*b^9*d^4*e^10 - 17920*a^22*b^7*d^4*e^10 - 4608*a^24*b^5
*d^4*e^10 - 512*a^26*b^3*d^4*e^10))/(4*(a^20*d^4 + a^4*b^16*d^4 + 8*a^6*b^1
4*d^4 + 28*a^8*b^12*d^4 + 56*a^10*b^10*d^4 + 70*a^12*b^8*d^4 + 56*a^14*b^6*
d^4 + 28*a^16*b^4*d^4 + 8*a^18*b^2*d^4))))/2 + ((e*cot(c + d*x))^(1/2)*(72*
a*b^22*d^2*e^9 + 576*a^3*b^20*d^2*e^9 + 5024*a^5*b^18*d^2*e^9 + 14272*a^7*b
^16*d^2*e^9 + 27824*a^9*b^14*d^2*e^9 + 53184*a^11*b^12*d^2*e^9 + 70240*a^13
*b^10*d^2*e^9 + 47680*a^15*b^8*d^2*e^9 + 12616*a^17*b^6*d^2*e^9 - 64*a^21*b
^2*d^2*e^9))/(2*(a^20*d^4 + a^4*b^16*d^4 + 8*a^6*b^14*d^4 + 28*a^8*b^12*d^4
 + 56*a^10*b^10*d^4 + 70*a^12*b^8*d^4 + 56*a^14*b^6*d^4 + 28*a^16*b^4*d^4 +
 8*a^18*b^2*d^4)))*(1/(b^6*d^2*e*1i - a^6*d^2*e*1i - a^2*b^4*d^2*e*15i - 20
*a^3*b^3*d^2*e + a^4*b^2*d^2*e*15i + 6*a*b^5*d^2*e + 6*a^5*b*d^2*e))^(1/2))
/2 - (90*a*b^19*d^2*e^9 + 846*a^3*b^17*d^2*e^9 + 1714*a^5*b^15*d^2*e^9 + 36
06*a^7*b^13*d^2*e^9 - 14578*a^9*b^11*d^2*e^9 - 34486*a^11*b^9*d^2*e^9 - 149
70*a^13*b^7*d^2*e^9 + 2258*a^15*b^5*d^2*e^9 - 32*a^17*b^3*d^2*e^9)/(2*(a^20
*d^5 + a^4*b^16*d^5 + 8*a^6*b^14*d^5 + 28*a^8*b^12*d^5 + 56*a^10*b^10*d^5 +
 70*a^12*b^8*d^5 + 56*a^14*b^6*d^5 + 28*a^16*b^4*d^5 + 8*a^18*b^2*d^5)))*(1
/(b^6*d^2*e*1i - a^6*d^2*e*1i - a^2*b^4*d^2*e*15i - 20*a^3*b^3*d^2*e + a^4*
b^2*d^2*e*15i + 6*a*b^5*d^2*e + 6*a^5*b*d^2*e))^(1/2))/2 + ((e*cot(c + d*x)
)^(1/2)*(18*a^2*b^15*e^8 - 9*b^17*e^8 - 71*a^4*b^13*e^8 + 892*a^6*b^11*e^8
+ 857*a^8*b^9*e^8 + 6802*a^10*b^7*e^8 - 1257*a^12*b^5*e^8))/(2*(a^20*d^4 +
a^4*b^16*d^4 + 8*a^6*b^14*d^4 + 28*a^8*b^12*d^4 + 56*a^10*b^10*d^4 + 70*a^1
2*b^8*d^4 + 56*a^14*b^6*d^4 + 28*a^16*b^4*d^4 + 8*a^18*b^2*d^4)))*(1/(b^6*d
^2*e*1i - a^6*d^2*e*1i - a^2*b^4*d^2*e*15i - 20*a^3*b^3*d^2*e + a^4*b^2*d^2
*e*15i + 6*a*b^5*d^2*e + 6*a^5*b*d^2*e))^(1/2)*1i - (((((((1/(b^6*d^2*e*1i
- a^6*d^2*e*1i - a^2*b^4*d^2*e*15i - 20*a^3*b^3*d^2*e + a^4*b^2*d^2*e*15i +
 6*a*b^5*d^2*e + 6*a^5*b*d^2*e))^(1/2)*((192*a^2*b^24*d^4*e^10 + 1728*a^4*b
^22*d^4*e^10 + 8320*a^6*b^20*d^4*e^10 + 27264*a^8*b^18*d^4*e^10 + 62592*a^1
0*b^16*d^4*e^10 + 99456*a^12*b^14*d^4*e^10 + 107520*a^14*b^12*d^4*e^10 + 76
800*a^16*b^10*d^4*e^10 + 33984*a^18*b^8*d^4*e^10 + 7872*a^20*b^6*d^4*e^10 +
 384*a^22*b^4*d^4*e^10 - 128*a^24*b^2*d^4*e^10)/(2*(a^20*d^5 + a^4*b^16*d^5
 + 8*a^6*b^14*d^5 + 28*a^8*b^12*d^5 + 56*a^10*b^10*d^5 + 70*a^12*b^8*d^5 +
56*a^14*b^6*d^5 + 28*a^16*b^4*d^5 + 8*a^18*b^2*d^5)) + ((e*cot(c + d*x))^(1
/2)*(1/(b^6*d^2*e*1i - a^6*d^2*e*1i - a^2*b^4*d^2*e*15i - 20*a^3*b^3*d^2*e
+ a^4*b^2*d^2*e*15i + 6*a*b^5*d^2*e + 6*a^5*b*d^2*e))^(1/2)*(512*a^4*b^25*d
^4*e^10 + 4608*a^6*b^23*d^4*e^10 + 17920*a^8*b^21*d^4*e^10 + 38400*a^10*b^1
9*d^4*e^10 + 46080*a^12*b^17*d^4*e^10 + 21504*a^14*b^15*d^4*e^10 - 21504*a^
16*b^13*d^4*e^10 - 46080*a^18*b^11*d^4*e^10 - 38400*a^20*b^9*d^4*e^10 - 179
20*a^22*b^7*d^4*e^10 - 4608*a^24*b^5*d^4*e^10 - 512*a^26*b^3*d^4*e^10))/(4*
(a^20*d^4 + a^4*b^16*d^4 + 8*a^6*b^14*d^4 + 28*a^8*b^12*d^4 + 56*a^10*b^10*
d^4 + 70*a^12*b^8*d^4 + 56*a^14*b^6*d^4 + 28*a^16*b^4*d^4 + 8*a^18*b^2*d^4)
)))/2 - ((e*cot(c + d*x))^(1/2)*(72*a*b^22*d^2*e^9 + 576*a^3*b^20*d^2*e^9 +

5024*a^5*b^18*d^2*e^9 + 14272*a^7*b^16*d^2*e^9 + 27824*a^9*b^14*d^2*e^9 + 53184*a^11*b^12*d^2*e^9 + 70240*a^13*b^10*d^2*e^9 + 47680*a^15*b^8*d^2*e^9 + 12616*a^17*b^6*d^2*e^9 - 64*a^21*b^2*d^2*e^9))/(2*(a^20*d^4 + a^4*b^16*d^4 + 8*a^6*b^14*d^4 + 28*a^8*b^12*d^4 + 56*a^10*b^10*d^4 + 70*a^12*b^8*d^4 + 56*a^14*b^6*d^4 + 28*a^16*b^4*d^4 + 8*a^18*b^2*d^4)))*(1/(b^6*d^2*e*1i - a^6*d^2*e*1i - a^2*b^4*d^2*e*15i - 20*a^3*b^3*d^2*e + a^4*b^2*d^2*e*15i + 6*a*b^5*d^2*e + 6*a^5*b*d^2*e))^(1/2))/2 - (90*a*b^19*d^2*e^9 + 846*a^3*b^17*d^2*e^9 + 1714*a^5*b^15*d^2*e^9 + 3606*a^7*b^13*d^2*e^9 - 14578*a^9*b^11*d^2*e^9 - 34486*a^11*b^9*d^2*e^9 - 14970*a^13*b^7*d^2*e^9 + 2258*a^15*b^5*d^2*e^9 - 32*a^17*b^3*d^2*e^9)/(2*(a^20*d^5 + a^4*b^16*d^5 + 8*a^6*b^14*d^5 + 28*a^8*b^12*d^5 + 56*a^10*b^10*d^5 + 70*a^12*b^8*d^5 + 56*a^14*b^6*d^5 + 28*a^16*b^4*d^5 + 8*a^18*b^2*d^5)))*(1/(b^6*d^2*e*1i - a^6*d^2*e*1i - a^2*b^4*d^2*e*15i - 20*a^3*b^3*d^2*e + a^4*b^2*d^2*e*15i + 6*a*b^5*d^2*e + 6*a^5*b*d^2*e))^(1/2))/2 - ((e*cot(c + d*x))^(1/2)*(18*a^2*b^15*e^8 - 9*b^17*e^8 - 71*a^4*b^13*e^8 + 892*a^6*b^11*e^8 + 857*a^8*b^9*e^8 + 6802*a^10*b^7*e^8 - 1257*a^12*b^5*e^8))/(2*(a^20*d^4 + a^4*b^16*d^4 + 8*a^6*b^14*d^4 + 28*a^8*b^12*d^4 + 56*a^10*b^10*d^4 + 70*a^12*b^8*d^4 + 56*a^14*b^6*d^4 + 28*a^16*b^4*d^4 + 8*a^18*b^2*d^4)))*(1/(b^6*d^2*e*1i - a^6*d^2*e*1i - a^2*b^4*d^2*e*15i - 20*a^3*b^3*d^2*e + a^4*b^2*d^2*e*15i + 6*a*b^5*d^2*e + 6*a^5*b*d^2*e))^(1/2)*1i)/((9*b^14*e^8 + 60*a^2*b^12*e^8 + 318*a^4*b^10*e^8 + 748*a^6*b^8*e^8 + 1505*a^8*b^6*e^8)/(a^20*d^5 + a^4*b^16*d^5 + 8*a^6*b^14*d^5 + 28*a^8*b^12*d^5 + 56*a^10*b^10*d^5 + 70*a^12*b^8*d^5 + 56*a^14*b^6*d^5 + 28*a^16*b^4*d^5 + 8*a^18*b^2*d^5) + (((((((1/(b^6*d^2*e*1i - a^6*d^2*e*1i - a^2*b^4*d^2*e*15i - 20*a^3*b^3*d^2*e + a^4*b^2*d^2*e*15i + 6*a*b^5*d^2*e + 6*a^5*b*d^2*e))^(1/2)*((192*a^2*b^24*d^4*e^10 + 1728*a^4*b^22*d^4*e^10 + 8320*a^6*b^20*d^4*e^10 + 27264*a^8*b^18*d^4*e^10 + 62592*a^10*b^16*d^4*e^10 + 99456*a^12*b^14*d^4*e^10 + 107520*a^14*b^12*d^4*e^10 + 76800*a^16*b^10*d^4*e^10 + 33984*a^18*b^8*d^4*e^10 + 7872*a^20*b^6*d^4*e^10 + 384*a^22*b^4*d^4*e^10 - 128*a^24*b^2*d^4*e^10)/(2*(a^20*d^5 + a^4*b^16*d^5 + 8*a^6*b^14*d^5 + 28*a^8*b^12*d^5 + 56*a^10*b^10*d^5 + 70*a^12*b^8*d^5 + 56*a^14*b^6*d^5 + 28*a^16*b^4*d^5 + 8*a^18*b^2*d^5)) - ((e*cot(c + d*x))^(1/2)*(1/(b^6*d^2*e*1i - a^6*d^2*e*1i - a^2*b^4*d^2*e*15i - 20*a^3*b^3*d^2*e + a^4*b^2*d^2*e*15i + 6*a*b^5*d^2*e + 6*a^5*b*d^2*e))^(1/2)*(512*a^4*b^25*d^4*e^10 + 4608*a^6*b^23*d^4*e^10 + 17920*a^8*b^21*d^4*e^10 + 38400*a^10*b^19*d^4*e^10 + 46080*a^12*b^17*d^4*e^10 + 21504*a^14*b^15*d^4*e^10 - 21504*a^16*b^13*d^4*e^10 - 46080*a^18*b^11*d^4*e^10 - 38400*a^20*b^9*d^4*e^10 - 17920*a^22*b^7*d^4*e^10 - 4608*a^24*b^5*d^4*e^10 - 512*a^26*b^3*d^4*e^10))/(4*(a^20*d^4 + a^4*b^16*d^4 + 8*a^6*b^14*d^4 + 28*a^8*b^12*d^4 + 56*a^10*b^10*d^4 + 70*a^12*b^8*d^4 + 56*a^14*b^6*d^4 + 28*a^16*b^4*d^4 + 8*a^18*b^2*d^4))))/2 + ((e*cot(c + d*x))^(1/2)*(72*a*b^22*d^2*e^9 + 576*a^3*b^20*d^2*e^9 + 5024*a^5*b^18*d^2*e^9 + 14272*a^7*b^16*d^2*e^9 + 27824*a^9*b^14*d^2*e^9 + 53184*a^11*b^12*d^2*e^9 + 70240*a^13*b^10*d^2*e^9 + 47680*a^15*b^8*d^2*e^9 + 12616*a^17*b^6*d^2*e^9 - 64*a^21*b^2*d^2*e^9))/(2*(a^20*d^4 + a^4*b^16*d^4 + 8*a^6*b^14*d^4 + 28*a^8*b^12*d^4 + 56*a^10*b^10*d^4 + 70*a^12*b^8*d^4 + 56*a^14*b^6*d^4 + 28*a^16*b^4*d^4 + 8*a^18*b^2*d^4)))*(1/(b^6*d^2*e*1i - a^6*d^2*e*1i - a^2*b^4*d^

2*e*15i - 20*a^3*b^3*d^2*e + a^4*b^2*d^2*e*15i + 6*a*b^5*d^2*e + 6*a^5*b*d^
2*e))^(1/2))/2 - (90*a*b^19*d^2*e^9 + 846*a^3*b^17*d^2*e^9 + 1714*a^5*b^15*
d^2*e^9 + 3606*a^7*b^13*d^2*e^9 - 14578*a^9*b^11*d^2*e^9 - 34486*a^11*b^9*d
^2*e^9 - 14970*a^13*b^7*d^2*e^9 + 2258*a^15*b^5*d^2*e^9 - 32*a^17*b^3*d^2*e
^9)/(2*(a^20*d^5 + a^4*b^16*d^5 + 8*a^6*b^14*d^5 + 28*a^8*b^12*d^5 + 56*a^1
0*b^10*d^5 + 70*a^12*b^8*d^5 + 56*a^14*b^6*d^5 + 28*a^16*b^4*d^5 + 8*a^18*b
^2*d^5)))*(1/(b^6*d^2*e*1i - a^6*d^2*e*1i - a^2*b^4*d^2*e*15i - 20*a^3*b^3*
d^2*e + a^4*b^2*d^2*e*15i + 6*a*b^5*d^2*e + 6*a^5*b*d^2*e))^(1/2))/2 + ((e*
cot(c + d*x))^(1/2)*(18*a^2*b^15*e^8 - 9*b^17*e^8 - 71*a^4*b^13*e^8 + 892*a
^6*b^11*e^8 + 857*a^8*b^9*e^8 + 6802*a^10*b^7*e^8 - 1257*a^12*b^5*e^8))/(2*
(a^20*d^4 + a^4*b^16*d^4 + 8*a^6*b^14*d^4 + 28*a^8*b^12*d^4 + 56*a^10*b^10*
d^4 + 70*a^12*b^8*d^4 + 56*a^14*b^6*d^4 + 28*a^16*b^4*d^4 + 8*a^18*b^2*d^4)
))*(1/(b^6*d^2*e*1i - a^6*d^2*e*1i - a^2*b^4*d^2*e*15i - 20*a^3*b^3*d^2*e +
a^4*b^2*d^2*e*15i + 6*a*b^5*d^2*e + 6*a^5*b*d^2*e))^(1/2) + (((((((1/(b^6*
d^2*e*1i - a^6*d^2*e*1i - a^2*b^4*d^2*e*15i - 20*a^3*b^3*d^2*e + a^4*b^2*d^
2*e*15i + 6*a*b^5*d^2*e + 6*a^5*b*d^2*e))^(1/2)*((192*a^2*b^24*d^4*e^10 + 1
728*a^4*b^22*d^4*e^10 + 8320*a^6*b^20*d^4*e^10 + 27264*a^8*b^18*d^4*e^10 +
62592*a^10*b^16*d^4*e^10 + 99456*a^12*b^14*d^4*e^10 + 107520*a^14*b^12*d^4*
e^10 + 76800*a^16*b^10*d^4*e^10 + 33984*a^18*b^8*d^4*e^10 + 7872*a^20*b^6*d
^4*e^10 + 384*a^22*b^4*d^4*e^10 - 128*a^24*b^2*d^4*e^10)/(2*(a^20*d^5 + a^4
*b^16*d^5 + 8*a^6*b^14*d^5 + 28*a^8*b^12*d^5 + 56*a^10*b^10*d^5 + 70*a^12*b
^8*d^5 + 56*a^14*b^6*d^5 + 28*a^16*b^4*d^5 + 8*a^18*b^2*d^5)) + ((e*cot(c +
d*x))^(1/2)*(1/(b^6*d^2*e*1i - a^6*d^2*e*1i - a^2*b^4*d^2*e*15i - 20*a^3*b
^3*d^2*e + a^4*b^2*d^2*e*15i + 6*a*b^5*d^2*e + 6*a^5*b*d^2*e))^(1/2)*(512*a
^4*b^25*d^4*e^10 + 4608*a^6*b^23*d^4*e^10 + 17920*a^8*b^21*d^4*e^10 + 38400
*a^10*b^19*d^4*e^10 + 46080*a^12*b^17*d^4*e^10 + 21504*a^14*b^15*d^4*e^10 -
21504*a^16*b^13*d^4*e^10 - 46080*a^18*b^11*d^4*e^10 - 38400*a^20*b^9*d^4*e
^10 - 17920*a^22*b^7*d^4*e^10 - 4608*a^24*b^5*d^4*e^10 - 512*a^26*b^3*d^4*e
^10))/(4*(a^20*d^4 + a^4*b^16*d^4 + 8*a^6*b^14*d^4 + 28*a^8*b^12*d^4 + 56*a
^10*b^10*d^4 + 70*a^12*b^8*d^4 + 56*a^14*b^6*d^4 + 28*a^16*b^4*d^4 + 8*a^18
*b^2*d^4))))/2 - ((e*cot(c + d*x))^(1/2)*(72*a*b^22*d^2*e^9 + 576*a^3*b^20*
d^2*e^9 + 5024*a^5*b^18*d^2*e^9 + 14272*a^7*b^16*d^2*e^9 + 27824*a^9*b^14*d
^2*e^9 + 53184*a^11*b^12*d^2*e^9 + 70240*a^13*b^10*d^2*e^9 + 47680*a^15*b^8
*d^2*e^9 + 12616*a^17*b^6*d^2*e^9 - 64*a^21*b^2*d^2*e^9))/(2*(a^20*d^4 + a^
4*b^16*d^4 + 8*a^6*b^14*d^4 + 28*a^8*b^12*d^4 + 56*a^10*b^10*d^4 + 70*a^12*
b^8*d^4 + 56*a^14*b^6*d^4 + 28*a^16*b^4*d^4 + 8*a^18*b^2*d^4)))*(1/(b^6*d^2
*e*1i - a^6*d^2*e*1i - a^2*b^4*d^2*e*15i - 20*a^3*b^3*d^2*e + a^4*b^2*d^2*e
*15i + 6*a*b^5*d^2*e + 6*a^5*b*d^2*e))^(1/2))/2 - (90*a*b^19*d^2*e^9 + 846*
a^3*b^17*d^2*e^9 + 1714*a^5*b^15*d^2*e^9 + 3606*a^7*b^13*d^2*e^9 - 14578*a^
9*b^11*d^2*e^9 - 34486*a^11*b^9*d^2*e^9 - 14970*a^13*b^7*d^2*e^9 + 2258*a^1
5*b^5*d^2*e^9 - 32*a^17*b^3*d^2*e^9)/(2*(a^20*d^5 + a^4*b^16*d^5 + 8*a^6*b^
14*d^5 + 28*a^8*b^12*d^5 + 56*a^10*b^10*d^5 + 70*a^12*b^8*d^5 + 56*a^14*b^6
*d^5 + 28*a^16*b^4*d^5 + 8*a^18*b^2*d^5)))*(1/(b^6*d^2*e*1i - a^6*d^2*e*1i
- a^2*b^4*d^2*e*15i - 20*a^3*b^3*d^2*e + a^4*b^2*d^2*e*15i + 6*a*b^5*d^2*e
+ 6*a^5*b*d^2*e))^(1/2))/2 - ((e*cot(c + d*x))^(1/2)*(18*a^2*b^15*e^8 - 9*b

^17*e^8 - 71*a^4*b^13*e^8 + 892*a^6*b^11*e^8 + 857*a^8*b^9*e^8 + 6802*a^10*
b^7*e^8 - 1257*a^12*b^5*e^8))/(2*(a^20*d^4 + a^4*b^16*d^4 + 8*a^6*b^14*d^4
+ 28*a^8*b^12*d^4 + 56*a^10*b^10*d^4 + 70*a^12*b^8*d^4 + 56*a^14*b^6*d^4 +
28*a^16*b^4*d^4 + 8*a^18*b^2*d^4)))*(1/(b^6*d^2*e*1i - a^6*d^2*e*1i - a^2*b
^4*d^2*e*15i - 20*a^3*b^3*d^2*e + a^4*b^2*d^2*e*15i + 6*a*b^5*d^2*e + 6*a^5
*b*d^2*e))^(1/2)))*(1/(b^6*d^2*e*1i - a^6*d^2*e*1i - a^2*b^4*d^2*e*15i - 20
*a^3*b^3*d^2*e + a^4*b^2*d^2*e*15i + 6*a*b^5*d^2*e + 6*a^5*b*d^2*e))^(1/2)*
1i + (atan((((((e*cot(c + d*x))^(1/2)*(18*a^2*b^15*e^8 - 9*b^17*e^8 - 71*a^
4*b^13*e^8 + 892*a^6*b^11*e^8 + 857*a^8*b^9*e^8 + 6802*a^10*b^7*e^8 - 1257*
a^12*b^5*e^8))/(a^20*d^4 + a^4*b^16*d^4 + 8*a^6*b^14*d^4 + 28*a^8*b^12*d^4
+ 56*a^10*b^10*d^4 + 70*a^12*b^8*d^4 + 56*a^14*b^6*d^4 + 28*a^16*b^4*d^4 +
8*a^18*b^2*d^4) - (((90*a*b^19*d^2*e^9 + 846*a^3*b^17*d^2*e^9 + 1714*a^5*b^
15*d^2*e^9 + 3606*a^7*b^13*d^2*e^9 - 14578*a^9*b^11*d^2*e^9 - 34486*a^11*b^
9*d^2*e^9 - 14970*a^13*b^7*d^2*e^9 + 2258*a^15*b^5*d^2*e^9 - 32*a^17*b^3*d^
2*e^9)/(a^20*d^5 + a^4*b^16*d^5 + 8*a^6*b^14*d^5 + 28*a^8*b^12*d^5 + 56*a^1
0*b^10*d^5 + 70*a^12*b^8*d^5 + 56*a^14*b^6*d^5 + 28*a^16*b^4*d^5 + 8*a^18*b
^2*d^5) - ((((e*cot(c + d*x))^(1/2)*(72*a*b^22*d^2*e^9 + 576*a^3*b^20*d^2*e
^9 + 5024*a^5*b^18*d^2*e^9 + 14272*a^7*b^16*d^2*e^9 + 27824*a^9*b^14*d^2*e^
9 + 53184*a^11*b^12*d^2*e^9 + 70240*a^13*b^10*d^2*e^9 + 47680*a^15*b^8*d^2*
e^9 + 12616*a^17*b^6*d^2*e^9 - 64*a^21*b^2*d^2*e^9))/(a^20*d^4 + a^4*b^16*d
^4 + 8*a^6*b^14*d^4 + 28*a^8*b^12*d^4 + 56*a^10*b^10*d^4 + 70*a^12*b^8*d^4
+ 56*a^14*b^6*d^4 + 28*a^16*b^4*d^4 + 8*a^18*b^2*d^4) + (((192*a^2*b^24*d^4
*e^10 + 1728*a^4*b^22*d^4*e^10 + 8320*a^6*b^20*d^4*e^10 + 27264*a^8*b^18*d^
4*e^10 + 62592*a^10*b^16*d^4*e^10 + 99456*a^12*b^14*d^4*e^10 + 107520*a^14*
b^12*d^4*e^10 + 76800*a^16*b^10*d^4*e^10 + 33984*a^18*b^8*d^4*e^10 + 7872*a
^20*b^6*d^4*e^10 + 384*a^22*b^4*d^4*e^10 - 128*a^24*b^2*d^4*e^10)/(a^20*d^5
 + a^4*b^16*d^5 + 8*a^6*b^14*d^5 + 28*a^8*b^12*d^5 + 56*a^10*b^10*d^5 + 70*
a^12*b^8*d^5 + 56*a^14*b^6*d^5 + 28*a^16*b^4*d^5 + 8*a^18*b^2*d^5) - ((e*co
t(c + d*x))^(1/2)*(-a^5*b^3*e)^(1/2)*(35*a^4 + 3*b^4 + 6*a^2*b^2)*(512*a^4*
b^25*d^4*e^10 + 4608*a^6*b^23*d^4*e^10 + 17920*a^8*b^21*d^4*e^10 + 38400*a^
10*b^19*d^4*e^10 + 46080*a^12*b^17*d^4*e^10 + 21504*a^14*b^15*d^4*e^10 - 21
504*a^16*b^13*d^4*e^10 - 46080*a^18*b^11*d^4*e^10 - 38400*a^20*b^9*d^4*e^10
 - 17920*a^22*b^7*d^4*e^10 - 4608*a^24*b^5*d^4*e^10 - 512*a^26*b^3*d^4*e^10
))/(8*(a^11*d*e + a^5*b^6*d*e + 3*a^7*b^4*d*e + 3*a^9*b^2*d*e)*(a^20*d^4 +
a^4*b^16*d^4 + 8*a^6*b^14*d^4 + 28*a^8*b^12*d^4 + 56*a^10*b^10*d^4 + 70*a^1
2*b^8*d^4 + 56*a^14*b^6*d^4 + 28*a^16*b^4*d^4 + 8*a^18*b^2*d^4)))*(-a^5*b^3
e)^(1/2)(35*a^4 + 3*b^4 + 6*a^2*b^2))/(8*(a^11*d*e + a^5*b^6*d*e + 3*a^7*
b^4*d*e + 3*a^9*b^2*d*e)))*(-a^5*b^3*e)^(1/2)*(35*a^4 + 3*b^4 + 6*a^2*b^2))
/(8*(a^11*d*e + a^5*b^6*d*e + 3*a^7*b^4*d*e + 3*a^9*b^2*d*e)))*(-a^5*b^3*e)
^(1/2)*(35*a^4 + 3*b^4 + 6*a^2*b^2))/(8*(a^11*d*e + a^5*b^6*d*e + 3*a^7*b^4
*d*e + 3*a^9*b^2*d*e)))*(-a^5*b^3*e)^(1/2)*(35*a^4 + 3*b^4 + 6*a^2*b^2)*1i)
/(8*(a^11*d*e + a^5*b^6*d*e + 3*a^7*b^4*d*e + 3*a^9*b^2*d*e)) + ((((e*cot(c
 + d*x))^(1/2)*(18*a^2*b^15*e^8 - 9*b^17*e^8 - 71*a^4*b^13*e^8 + 892*a^6*b^
11*e^8 + 857*a^8*b^9*e^8 + 6802*a^10*b^7*e^8 - 1257*a^12*b^5*e^8))/(a^20*d^
4 + a^4*b^16*d^4 + 8*a^6*b^14*d^4 + 28*a^8*b^12*d^4 + 56*a^10*b^10*d^4 + 70

*a^12*b^8*d^4 + 56*a^14*b^6*d^4 + 28*a^16*b^4*d^4 + 8*a^18*b^2*d^4) + (((90
*a*b^19*d^2*e^9 + 846*a^3*b^17*d^2*e^9 + 1714*a^5*b^15*d^2*e^9 + 3606*a^7*b
^13*d^2*e^9 - 14578*a^9*b^11*d^2*e^9 - 34486*a^11*b^9*d^2*e^9 - 14970*a^13*
b^7*d^2*e^9 + 2258*a^15*b^5*d^2*e^9 - 32*a^17*b^3*d^2*e^9)/(a^20*d^5 + a^4*
b^16*d^5 + 8*a^6*b^14*d^5 + 28*a^8*b^12*d^5 + 56*a^10*b^10*d^5 + 70*a^12*b^
8*d^5 + 56*a^14*b^6*d^5 + 28*a^16*b^4*d^5 + 8*a^18*b^2*d^5) + ((((e*cot(c +
d*x))^(1/2)*(72*a*b^22*d^2*e^9 + 576*a^3*b^20*d^2*e^9 + 5024*a^5*b^18*d^2*
e^9 + 14272*a^7*b^16*d^2*e^9 + 27824*a^9*b^14*d^2*e^9 + 53184*a^11*b^12*d^2
*e^9 + 70240*a^13*b^10*d^2*e^9 + 47680*a^15*b^8*d^2*e^9 + 12616*a^17*b^6*d^
2*e^9 - 64*a^21*b^2*d^2*e^9))/(a^20*d^4 + a^4*b^16*d^4 + 8*a^6*b^14*d^4 + 2
8*a^8*b^12*d^4 + 56*a^10*b^10*d^4 + 70*a^12*b^8*d^4 + 56*a^14*b^6*d^4 + 28*
a^16*b^4*d^4 + 8*a^18*b^2*d^4) - (((192*a^2*b^24*d^4*e^10 + 1728*a^4*b^22*d
^4*e^10 + 8320*a^6*b^20*d^4*e^10 + 27264*a^8*b^18*d^4*e^10 + 62592*a^10*b^1
6*d^4*e^10 + 99456*a^12*b^14*d^4*e^10 + 107520*a^14*b^12*d^4*e^10 + 76800*a
^16*b^10*d^4*e^10 + 33984*a^18*b^8*d^4*e^10 + 7872*a^20*b^6*d^4*e^10 + 384*
a^22*b^4*d^4*e^10 - 128*a^24*b^2*d^4*e^10)/(a^20*d^5 + a^4*b^16*d^5 + 8*a^6
*b^14*d^5 + 28*a^8*b^12*d^5 + 56*a^10*b^10*d^5 + 70*a^12*b^8*d^5 + 56*a^14*
b^6*d^5 + 28*a^16*b^4*d^5 + 8*a^18*b^2*d^5) + ((e*cot(c + d*x))^(1/2)*(-a^5
*b^3*e)^(1/2)*(35*a^4 + 3*b^4 + 6*a^2*b^2)*(512*a^4*b^25*d^4*e^10 + 4608*a^
6*b^23*d^4*e^10 + 17920*a^8*b^21*d^4*e^10 + 38400*a^10*b^19*d^4*e^10 + 4608
0*a^12*b^17*d^4*e^10 + 21504*a^14*b^15*d^4*e^10 - 21504*a^16*b^13*d^4*e^10
- 46080*a^18*b^11*d^4*e^10 - 38400*a^20*b^9*d^4*e^10 - 17920*a^22*b^7*d^4*e
^10 - 4608*a^24*b^5*d^4*e^10 - 512*a^26*b^3*d^4*e^10))/(8*(a^11*d*e + a^5*b
^6*d*e + 3*a^7*b^4*d*e + 3*a^9*b^2*d*e)*(a^20*d^4 + a^4*b^16*d^4 + 8*a^6*b^
14*d^4 + 28*a^8*b^12*d^4 + 56*a^10*b^10*d^4 + 70*a^12*b^8*d^4 + 56*a^14*b^6
*d^4 + 28*a^16*b^4*d^4 + 8*a^18*b^2*d^4)))*(-a^5*b^3*e)^(1/2)*(35*a^4 + 3*b
^4 + 6*a^2*b^2))/(8*(a^11*d*e + a^5*b^6*d*e + 3*a^7*b^4*d*e + 3*a^9*b^2*d*e
)))*(-a^5*b^3*e)^(1/2)*(35*a^4 + 3*b^4 + 6*a^2*b^2))/(8*(a^11*d*e + a^5*b^6
*d*e + 3*a^7*b^4*d*e + 3*a^9*b^2*d*e)))*(-a^5*b^3*e)^(1/2)*(35*a^4 + 3*b^4
+ 6*a^2*b^2))/(8*(a^11*d*e + a^5*b^6*d*e + 3*a^7*b^4*d*e + 3*a^9*b^2*d*e)))
*(-a^5*b^3*e)^(1/2)*(35*a^4 + 3*b^4 + 6*a^2*b^2)*1i)/(8*(a^11*d*e + a^5*b^6
*d*e + 3*a^7*b^4*d*e + 3*a^9*b^2*d*e)))/((9*b^14*e^8 + 60*a^2*b^12*e^8 + 31
8*a^4*b^10*e^8 + 748*a^6*b^8*e^8 + 1505*a^8*b^6*e^8)/(a^20*d^5 + a^4*b^16*d
^5 + 8*a^6*b^14*d^5 + 28*a^8*b^12*d^5 + 56*a^10*b^10*d^5 + 70*a^12*b^8*d^5
+ 56*a^14*b^6*d^5 + 28*a^16*b^4*d^5 + 8*a^18*b^2*d^5) + ((((e*cot(c + d*x))
^(1/2)*(18*a^2*b^15*e^8 - 9*b^17*e^8 - 71*a^4*b^13*e^8 + 892*a^6*b^11*e^8 +
857*a^8*b^9*e^8 + 6802*a^10*b^7*e^8 - 1257*a^12*b^5*e^8))/(a^20*d^4 + a^4*
b^16*d^4 + 8*a^6*b^14*d^4 + 28*a^8*b^12*d^4 + 56*a^10*b^10*d^4 + 70*a^12*b^
8*d^4 + 56*a^14*b^6*d^4 + 28*a^16*b^4*d^4 + 8*a^18*b^2*d^4) - (((90*a*b^19*
d^2*e^9 + 846*a^3*b^17*d^2*e^9 + 1714*a^5*b^15*d^2*e^9 + 3606*a^7*b^13*d^2*
e^9 - 14578*a^9*b^11*d^2*e^9 - 34486*a^11*b^9*d^2*e^9 - 14970*a^13*b^7*d^2*
e^9 + 2258*a^15*b^5*d^2*e^9 - 32*a^17*b^3*d^2*e^9)/(a^20*d^5 + a^4*b^16*d^5
+ 8*a^6*b^14*d^5 + 28*a^8*b^12*d^5 + 56*a^10*b^10*d^5 + 70*a^12*b^8*d^5 +
56*a^14*b^6*d^5 + 28*a^16*b^4*d^5 + 8*a^18*b^2*d^5) - ((((e*cot(c + d*x))^(
1/2)*(72*a*b^22*d^2*e^9 + 576*a^3*b^20*d^2*e^9 + 5024*a^5*b^18*d^2*e^9 + 14

272*a^7*b^16*d^2*e^9 + 27824*a^9*b^14*d^2*e^9 + 53184*a^11*b^12*d^2*e^9 + 7
0240*a^13*b^10*d^2*e^9 + 47680*a^15*b^8*d^2*e^9 + 12616*a^17*b^6*d^2*e^9 -
64*a^21*b^2*d^2*e^9))/(a^20*d^4 + a^4*b^16*d^4 + 8*a^6*b^14*d^4 + 28*a^8*b^
12*d^4 + 56*a^10*b^10*d^4 + 70*a^12*b^8*d^4 + 56*a^14*b^6*d^4 + 28*a^16*b^4
*d^4 + 8*a^18*b^2*d^4) + (((192*a^2*b^24*d^4*e^10 + 1728*a^4*b^22*d^4*e^10
+ 8320*a^6*b^20*d^4*e^10 + 27264*a^8*b^18*d^4*e^10 + 62592*a^10*b^16*d^4*e^
10 + 99456*a^12*b^14*d^4*e^10 + 107520*a^14*b^12*d^4*e^10 + 76800*a^16*b^10
*d^4*e^10 + 33984*a^18*b^8*d^4*e^10 + 7872*a^20*b^6*d^4*e^10 + 384*a^22*b^4
*d^4*e^10 - 128*a^24*b^2*d^4*e^10)/(a^20*d^5 + a^4*b^16*d^5 + 8*a^6*b^14*d^
5 + 28*a^8*b^12*d^5 + 56*a^10*b^10*d^5 + 70*a^12*b^8*d^5 + 56*a^14*b^6*d^5
+ 28*a^16*b^4*d^5 + 8*a^18*b^2*d^5) - ((e*cot(c + d*x))^(1/2)*(-a^5*b^3*e)^
(1/2)*(35*a^4 + 3*b^4 + 6*a^2*b^2)*(512*a^4*b^25*d^4*e^10 + 4608*a^6*b^23*d
^4*e^10 + 17920*a^8*b^21*d^4*e^10 + 38400*a^10*b^19*d^4*e^10 + 46080*a^12*b
^17*d^4*e^10 + 21504*a^14*b^15*d^4*e^10 - 21504*a^16*b^13*d^4*e^10 - 46080*
a^18*b^11*d^4*e^10 - 38400*a^20*b^9*d^4*e^10 - 17920*a^22*b^7*d^4*e^10 - 46
08*a^24*b^5*d^4*e^10 - 512*a^26*b^3*d^4*e^10))/(8*(a^11*d*e + a^5*b^6*d*e +
3*a^7*b^4*d*e + 3*a^9*b^2*d*e)*(a^20*d^4 + a^4*b^16*d^4 + 8*a^6*b^14*d^4 +
28*a^8*b^12*d^4 + 56*a^10*b^10*d^4 + 70*a^12*b^8*d^4 + 56*a^14*b^6*d^4 + 2
8*a^16*b^4*d^4 + 8*a^18*b^2*d^4)))*(-a^5*b^3*e)^(1/2)*(35*a^4 + 3*b^4 + 6*a
^2*b^2))/(8*(a^11*d*e + a^5*b^6*d*e + 3*a^7*b^4*d*e + 3*a^9*b^2*d*e)))*(-a^
5*b^3*e)^(1/2)*(35*a^4 + 3*b^4 + 6*a^2*b^2))/(8*(a^11*d*e + a^5*b^6*d*e + 3
*a^7*b^4*d*e + 3*a^9*b^2*d*e)))*(-a^5*b^3*e)^(1/2)*(35*a^4 + 3*b^4 + 6*a^2*
b^2))/(8*(a^11*d*e + a^5*b^6*d*e + 3*a^7*b^4*d*e + 3*a^9*b^2*d*e)))*(-a^5*b
^3*e)^(1/2)*(35*a^4 + 3*b^4 + 6*a^2*b^2))/(8*(a^11*d*e + a^5*b^6*d*e + 3*a^
7*b^4*d*e + 3*a^9*b^2*d*e)) - ((((e*cot(c + d*x))^(1/2)*(18*a^2*b^15*e^8 -
9*b^17*e^8 - 71*a^4*b^13*e^8 + 892*a^6*b^11*e^8 + 857*a^8*b^9*e^8 + 6802*a^
10*b^7*e^8 - 1257*a^12*b^5*e^8))/(a^20*d^4 + a^4*b^16*d^4 + 8*a^6*b^14*d^4
+ 28*a^8*b^12*d^4 + 56*a^10*b^10*d^4 + 70*a^12*b^8*d^4 + 56*a^14*b^6*d^4 +
28*a^16*b^4*d^4 + 8*a^18*b^2*d^4) + (((90*a*b^19*d^2*e^9 + 846*a^3*b^17*d^2
*e^9 + 1714*a^5*b^15*d^2*e^9 + 3606*a^7*b^13*d^2*e^9 - 14578*a^9*b^11*d^2*e
^9 - 34486*a^11*b^9*d^2*e^9 - 14970*a^13*b^7*d^2*e^9 + 2258*a^15*b^5*d^2*e^
9 - 32*a^17*b^3*d^2*e^9)/(a^20*d^5 + a^4*b^16*d^5 + 8*a^6*b^14*d^5 + 28*a^8
*b^12*d^5 + 56*a^10*b^10*d^5 + 70*a^12*b^8*d^5 + 56*a^14*b^6*d^5 + 28*a^16*
b^4*d^5 + 8*a^18*b^2*d^5) + ((((e*cot(c + d*x))^(1/2)*(72*a*b^22*d^2*e^9 +
576*a^3*b^20*d^2*e^9 + 5024*a^5*b^18*d^2*e^9 + 14272*a^7*b^16*d^2*e^9 + 278
24*a^9*b^14*d^2*e^9 + 53184*a^11*b^12*d^2*e^9 + 70240*a^13*b^10*d^2*e^9 + 4
7680*a^15*b^8*d^2*e^9 + 12616*a^17*b^6*d^2*e^9 - 64*a^21*b^2*d^2*e^9))/(a^2
0*d^4 + a^4*b^16*d^4 + 8*a^6*b^14*d^4 + 28*a^8*b^12*d^4 + 56*a^10*b^10*d^4
+ 70*a^12*b^8*d^4 + 56*a^14*b^6*d^4 + 28*a^16*b^4*d^4 + 8*a^18*b^2*d^4) - (
((192*a^2*b^24*d^4*e^10 + 1728*a^4*b^22*d^4*e^10 + 8320*a^6*b^20*d^4*e^10 +
27264*a^8*b^18*d^4*e^10 + 62592*a^10*b^16*d^4*e^10 + 99456*a^12*b^14*d^4*e
^10 + 107520*a^14*b^12*d^4*e^10 + 76800*a^16*b^10*d^4*e^10 + 33984*a^18*b^8
*d^4*e^10 + 7872*a^20*b^6*d^4*e^10 + 384*a^22*b^4*d^4*e^10 - 128*a^24*b^2*d
^4*e^10)/(a^20*d^5 + a^4*b^16*d^5 + 8*a^6*b^14*d^5 + 28*a^8*b^12*d^5 + 56*a
^10*b^10*d^5 + 70*a^12*b^8*d^5 + 56*a^14*b^6*d^5 + 28*a^16*b^4*d^5 + 8*a^18

```
*b^2*d^5) + ((e*cot(c + d*x))^(1/2)*(-a^5*b^3*e)^(1/2)*(35*a^4 + 3*b^4 + 6*
a^2*b^2)*(512*a^4*b^25*d^4*e^10 + 4608*a^6*b^23*d^4*e^10 + 17920*a^8*b^21*d
^4*e^10 + 38400*a^10*b^19*d^4*e^10 + 46080*a^12*b^17*d^4*e^10 + 21504*a^14*
b^15*d^4*e^10 - 21504*a^16*b^13*d^4*e^10 - 46080*a^18*b^11*d^4*e^10 - 38400
*a^20*b^9*d^4*e^10 - 17920*a^22*b^7*d^4*e^10 - 4608*a^24*b^5*d^4*e^10 - 512
*a^26*b^3*d^4*e^10))/(8*(a^11*d*e + a^5*b^6*d*e + 3*a^7*b^4*d*e + 3*a^9*b^2
*d*e)*(a^20*d^4 + a^4*b^16*d^4 + 8*a^6*b^14*d^4 + 28*a^8*b^12*d^4 + 56*a^10
*b^10*d^4 + 70*a^12*b^8*d^4 + 56*a^14*b^6*d^4 + 28*a^16*b^4*d^4 + 8*a^18*b^
2*d^4)))*(-a^5*b^3*e)^(1/2)*(35*a^4 + 3*b^4 + 6*a^2*b^2))/(8*(a^11*d*e + a^
5*b^6*d*e + 3*a^7*b^4*d*e + 3*a^9*b^2*d*e)))*(-a^5*b^3*e)^(1/2)*(35*a^4 + 3
*b^4 + 6*a^2*b^2))/(8*(a^11*d*e + a^5*b^6*d*e + 3*a^7*b^4*d*e + 3*a^9*b^2*d
*e)))*(-a^5*b^3*e)^(1/2)*(35*a^4 + 3*b^4 + 6*a^2*b^2))/(8*(a^11*d*e + a^5*b
^6*d*e + 3*a^7*b^4*d*e + 3*a^9*b^2*d*e)))*(-a^5*b^3*e)^(1/2)*(35*a^4 + 3*b^
4 + 6*a^2*b^2))/(8*(a^11*d*e + a^5*b^6*d*e + 3*a^7*b^4*d*e + 3*a^9*b^2*d*e)
)))*(-a^5*b^3*e)^(1/2)*(35*a^4 + 3*b^4 + 6*a^2*b^2)*1i)/(4*(a^11*d*e + a^5*
b^6*d*e + 3*a^7*b^4*d*e + 3*a^9*b^2*d*e))
```

3.87 $\int \frac{1}{(e\cot(c+dx))^{3/2}(a+b\cot(c+dx))^3}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 529

$$\begin{aligned}\int \frac{1}{(e\cot(c+dx))^{3/2}(a+b\cot(c+dx))^3}\,dx &= \frac{b^{5/2}(63a^4+46a^2b^2+15b^4)\arctan\left(\frac{\sqrt{b}\sqrt{e\cot(c+dx)}}{\sqrt{a}\sqrt{e}}\right)}{4a^{7/2}\left(a^2+b^2\right)^3de^{3/2}} \\
&- \frac{(a-b)\left(a^2+4ab+b^2\right)\arctan\left(1-\frac{\sqrt{2}\sqrt{e\cot(c+dx)}}{\sqrt{e}}\right)}{\sqrt{2}\left(a^2+b^2\right)^3de^{3/2}} \\
&+ \frac{(a-b)\left(a^2+4ab+b^2\right)\arctan\left(1+\frac{\sqrt{2}\sqrt{e\cot(c+dx)}}{\sqrt{e}}\right)}{\sqrt{2}\left(a^2+b^2\right)^3de^{3/2}} \\
&+ \frac{8a^4+31a^2b^2+15b^4}{4a^3\left(a^2+b^2\right)^2de\sqrt{e\cot(c+dx)}} - \frac{b^2}{2a\left(a^2+b^2\right)de\sqrt{e\cot(c+dx)}(a+b\cot(c+dx))^2} \\
&- \frac{b^2(13a^2+5b^2)}{4a^2\left(a^2+b^2\right)^2de\sqrt{e\cot(c+dx)}(a+b\cot(c+dx))} \\
&+ \frac{(a+b)\left(a^2-4ab+b^2\right)\log\left(\sqrt{e}+\sqrt{e}\cot(c+dx)-\sqrt{2}\sqrt{e\cot(c+dx)}\right)}{2\sqrt{2}\left(a^2+b^2\right)^3de^{3/2}} \\
&- \frac{(a+b)\left(a^2-4ab+b^2\right)\log\left(\sqrt{e}+\sqrt{e}\cot(c+dx)+\sqrt{2}\sqrt{e\cot(c+dx)}\right)}{2\sqrt{2}\left(a^2+b^2\right)^3de^{3/2}}\end{aligned}$$

```
[Out] 1/4*b^(5/2)*(63*a^4+46*a^2*b^2+15*b^4)*arctan(b^(1/2)*(e*cot(d*x+c))^(1/2)/
a^(1/2)/e^(1/2))/a^(7/2)/(a^2+b^2)^3/d/e^(3/2)-1/2*(a-b)*(a^2+4*a*b+b^2)*ar
ctan(1-2^(1/2)*(e*cot(d*x+c))^(1/2)/e^(1/2))/(a^2+b^2)^3/d/e^(3/2)*2^(1/2)+
1/2*(a-b)*(a^2+4*a*b+b^2)*arctan(1+2^(1/2)*(e*cot(d*x+c))^(1/2)/e^(1/2))/(a
^2+b^2)^3/d/e^(3/2)*2^(1/2)+1/4*(a+b)*(a^2-4*a*b+b^2)*ln(e^(1/2)+cot(d*x+c)
*e^(1/2)-2^(1/2)*(e*cot(d*x+c))^(1/2))/(a^2+b^2)^3/d/e^(3/2)*2^(1/2)-1/4*(a
+b)*(a^2-4*a*b+b^2)*ln(e^(1/2)+cot(d*x+c)*e^(1/2)+2^(1/2)*(e*cot(d*x+c))^(1
```

```
/2))/(a^2+b^2)^3/d/e^(3/2)*2^(1/2)+1/4*(8*a^4+31*a^2*b^2+15*b^4)/a^3/(a^2+b
^2)^2/d/e/(e*cot(d*x+c))^(1/2)-1/2*b^2/a/(a^2+b^2)/d/e/(a+b*cot(d*x+c))^2/(
e*cot(d*x+c))^(1/2)-1/4*b^2*(13*a^2+5*b^2)/a^2/(a^2+b^2)^2/d/e/(a+b*cot(d*x
+c))/(e*cot(d*x+c))^(1/2)
```

Rubi [A] (verified)

Time = 1.83 (sec) , antiderivative size = 529, normalized size of antiderivative = 1.00, number of steps used = 17, number of rules used = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.520$, Rules used = {3650, 3730, 3734, 3615, 1182, 1176, 631, 210, 1179, 642, 3715, 65, 211}

$$\int \frac{1}{(e\cot(c+dx))^{3/2}(a+b\cot(c+dx))^3}\,dx = -\frac{(a-b)\left(a^2+4ab+b^2\right)\arctan\left(1-\frac{\sqrt{2}\sqrt{e\cot(c+dx)}}{\sqrt{e}}\right)}{\sqrt{2}de^{3/2}\left(a^2+b^2\right)^3} + \frac{(a-b)\left(a^2+4ab+b^2\right)\arctan\left(\frac{\sqrt{2}\sqrt{e\cot(c+dx)}}{\sqrt{e}}+1\right)}{\sqrt{2}de^{3/2}\left(a^2+b^2\right)^3} + \frac{(a+b)\left(a^2-4ab+b^2\right)\log\left(\sqrt{e}\cot(c+dx)-\sqrt{2}\sqrt{e\cot(c+dx)}+\sqrt{e}\right)}{2\sqrt{2}de^{3/2}\left(a^2+b^2\right)^3} - \frac{(a+b)\left(a^2-4ab+b^2\right)\log\left(\sqrt{e}\cot(c+dx)+\sqrt{2}\sqrt{e\cot(c+dx)}+\sqrt{e}\right)}{2\sqrt{2}de^{3/2}\left(a^2+b^2\right)^3} - \frac{b^2(13a^2+5b^2)}{4a^2de\left(a^2+b^2\right)^2\sqrt{e\cot(c+dx)}(a+b\cot(c+dx))} - \frac{b^2}{2ade\left(a^2+b^2\right)\sqrt{e\cot(c+dx)}(a+b\cot(c+dx))^2} + \frac{b^{5/2}(63a^4+46a^2b^2+15b^4)\arctan\left(\frac{\sqrt{b}\sqrt{e\cot(c+dx)}}{\sqrt{a}\sqrt{e}}\right)}{4a^{7/2}de^{3/2}\left(a^2+b^2\right)^3} + \frac{8a^4+31a^2b^2+15b^4}{4a^3de\left(a^2+b^2\right)^2\sqrt{e\cot(c+dx)}}$$

```
[In] Int[1/((e*Cot[c + d*x])^(3/2)*(a + b*Cot[c + d*x])^3),x]
```

```
[Out] (b^(5/2)*(63*a^4 + 46*a^2*b^2 + 15*b^4)*ArcTan[(Sqrt[b]*Sqrt[e*Cot[c + d*x]
])/(Sqrt[a]*Sqrt[e])])/(4*a^(7/2)*(a^2 + b^2)^3*d*e^(3/2)) - ((a - b)*(a^2
+ 4*a*b + b^2)*ArcTan[1 - (Sqrt[2]*Sqrt[e*Cot[c + d*x]])/Sqrt[e]])/(Sqrt[2]
*(a^2 + b^2)^3*d*e^(3/2)) + ((a - b)*(a^2 + 4*a*b + b^2)*ArcTan[1 + (Sqrt[2
]*Sqrt[e*Cot[c + d*x]])/Sqrt[e]])/(Sqrt[2]*(a^2 + b^2)^3*d*e^(3/2)) + (8*a^
4 + 31*a^2*b^2 + 15*b^4)/(4*a^3*(a^2 + b^2)^2*d*e*Sqrt[e*Cot[c + d*x]]) - b
^2/(2*a*(a^2 + b^2)*d*e*Sqrt[e*Cot[c + d*x]]*(a + b*Cot[c + d*x])^2) - (b^2
```

```
*(13*a^2 + 5*b^2))/(4*a^2*(a^2 + b^2)^2*d*e*Sqrt[e*Cot[c + d*x]]*(a + b*Cot
[c + d*x])) + ((a + b)*(a^2 - 4*a*b + b^2)*Log[Sqrt[e] + Sqrt[e]*Cot[c + d*
x] - Sqrt[2]*Sqrt[e*Cot[c + d*x]]])/(2*Sqrt[2]*(a^2 + b^2)^3*d*e^(3/2)) - (
(a + b)*(a^2 - 4*a*b + b^2)*Log[Sqrt[e] + Sqrt[e]*Cot[c + d*x] + Sqrt[2]*Sq
rt[e*Cot[c + d*x]]])/(2*Sqrt[2]*(a^2 + b^2)^3*d*e^(3/2))
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1179

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 1182

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
a*c, 2]}, Dist[(d*q + a*e)/(2*a*c), Int[(q + c*x^2)/(a + c*x^4), x], x] + D
ist[(d*q - a*e)/(2*a*c), Int[(q - c*x^2)/(a + c*x^4), x], x]] /; FreeQ[{a,
c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0] && NeQ[c*d^2 - a*e^2, 0] && NegQ[(-a)
*c]
```

Rule 3615

```
Int[((c_) + (d_.)*tan[(e_.) + (f_.)*(x_)])/Sqrt[(b_.)*tan[(e_.) + (f_.)*(x_
)]], x_Symbol] :> Dist[2/f, Subst[Int[(b*c + d*x^2)/(b^2 + x^4), x], x, Sqr
t[b*Tan[e + f*x]]], x] /; FreeQ[{b, c, d, e, f}, x] && NeQ[c^2 - d^2, 0] &&
 NeQ[c^2 + d^2, 0]
```

Rule 3650

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)])^(n_), x_Symbol] :> Simp[b^2*(a + b*Tan[e + f*x])^(m + 1)*((c
+ d*Tan[e + f*x])^(n + 1)/(f*(m + 1)*(a^2 + b^2)*(b*c - a*d))), x] + Dist[1
/((m + 1)*(a^2 + b^2)*(b*c - a*d)), Int[(a + b*Tan[e + f*x])^(m + 1)*(c + d
*Tan[e + f*x])^n*Simp[a*(b*c - a*d)*(m + 1) - b^2*d*(m + n + 2) - b*(b*c -
a*d)*(m + 1)*Tan[e + f*x] - b^2*d*(m + n + 2)*Tan[e + f*x]^2, x], x], x] /;
 FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0]
&& NeQ[c^2 + d^2, 0] && IntegerQ[2*m] && LtQ[m, -1] && (LtQ[n, 0] || Intege
rQ[m]) &&  !(ILtQ[n, -1] && ( !IntegerQ[m] || (EqQ[c, 0] && NeQ[a, 0])))
```

Rule 3715

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*tan[(e_.)
+ (f_.)*(x_)])^(n_.)*((A_) + (C_.)*tan[(e_.) + (f_.)*(x_)]^2), x_Symbol] :>
 Dist[A/f, Subst[Int[(a + b*x)^m*(c + d*x)^n, x], x, Tan[e + f*x]], x] /; F
reeQ[{a, b, c, d, e, f, A, C, m, n}, x] && EqQ[A, C]
```

Rule 3730

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*tan[(e_.) + (f_.)*(x_)] + (C_.)*tan[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> Simp[(A*b^2 - a*(b*B - a*C))*(a + b*Tan[e +
 f*x])^(m + 1)*((c + d*Tan[e + f*x])^(n + 1)/(f*(m + 1)*(b*c - a*d)*(a^2 +
b^2))), x] + Dist[1/((m + 1)*(b*c - a*d)*(a^2 + b^2)), Int[(a + b*Tan[e + f
```

```
*x])^(m + 1)*(c + d*Tan[e + f*x])^n*Simp[A*(a*(b*c - a*d)*(m + 1) - b^2*d*(
m + n + 2)) + (b*B - a*C)*(b*c*(m + 1) + a*d*(n + 1)) - (m + 1)*(b*c - a*d)
*(A*b - a*B - b*C)*Tan[e + f*x] - d*(A*b^2 - a*(b*B - a*C))*(m + n + 2)*Tan
[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, n}, x] && NeQ[
b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] && LtQ[m, -1] &&  !
(ILtQ[n, -1] && ( !IntegerQ[m] || (EqQ[c, 0] && NeQ[a, 0])))
```

Rule 3734

```
Int[(((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_)*((A_.) + (B_.)*tan[(e_.)
+ (f_.)*(x_)] + (C_.)*tan[(e_.) + (f_.)*(x_)]^2))/((a_.) + (b_.)*tan[(e_.)
+ (f_.)*(x_)]), x_Symbol] :> Dist[1/(a^2 + b^2), Int[(c + d*Tan[e + f*x])^n
*Simp[b*B + a*(A - C) + (a*B - b*(A - C))*Tan[e + f*x], x], x], x] + Dist[(
A*b^2 - a*b*B + a^2*C)/(a^2 + b^2), Int[(c + d*Tan[e + f*x])^n*((1 + Tan[e
+ f*x]^2)/(a + b*Tan[e + f*x])), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C
, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] &&
 !GtQ[n, 0] &&  !LeQ[n, -1]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\frac{b^2}{2a(a^2+b^2)de\sqrt{e\cot(c+dx)}(a+b\cot(c+dx))^2} - \frac{\int \frac{-\frac{1}{2}(4a^2+5b^2)e+2abe\cot(c+dx)-\frac{5}{2}b^2e\cot^2(c+dx)}{(e\cot(c+dx))^{3/2}(a+b\cot(c+dx))^2}\,dx}{2a(a^2+b^2)e} \\
&= -\frac{b^2}{2a(a^2+b^2)de\sqrt{e\cot(c+dx)}(a+b\cot(c+dx))^2} - \frac{b^2(13a^2+5b^2)}{4a^2(a^2+b^2)^2de\sqrt{e\cot(c+dx)}(a+b\cot(c+dx))} \\
&\quad + \frac{\int \frac{\frac{1}{4}(8a^4+31a^2b^2+15b^4)e^2-4a^3be^2\cot(c+dx)+\frac{3}{4}b^2(13a^2+5b^2)e^2\cot^2(c+dx)}{(e\cot(c+dx))^{3/2}(a+b\cot(c+dx))}\,dx}{2a^2(a^2+b^2)^2e^2} \\
&= \frac{8a^4+31a^2b^2+15b^4}{4a^3(a^2+b^2)^2de\sqrt{e\cot(c+dx)}} - \frac{b^2}{2a(a^2+b^2)de\sqrt{e\cot(c+dx)}(a+b\cot(c+dx))^2} \\
&\quad - \frac{b^2(13a^2+5b^2)}{4a^2(a^2+b^2)^2de\sqrt{e\cot(c+dx)}(a+b\cot(c+dx))} \\
&\quad + \frac{\int \frac{-\frac{1}{8}b(24a^4+31a^2b^2+15b^4)e^4-a^3(a^2-b^2)e^4\cot(c+dx)-\frac{1}{8}b(8a^4+31a^2b^2+15b^4)e^4\cot^2(c+dx)}{\sqrt{e\cot(c+dx)}(a+b\cot(c+dx))}\,dx}{a^3(a^2+b^2)^2e^5}
\end{aligned}$$

$$
\begin{aligned}
&= \frac{8a^4+31a^2b^2+15b^4}{4a^3\left(a^2+b^2\right)^2 de\sqrt{e\cot(c+dx)}} \\
&\quad - \frac{b^2}{2a\left(a^2+b^2\right) de\sqrt{e\cot(c+dx)}(a+b\cot(c+dx))^2} \\
&\quad - \frac{b^2(13a^2+5b^2)}{4a^2\left(a^2+b^2\right)^2 de\sqrt{e\cot(c+dx)}(a+b\cot(c+dx))} \\
&\quad + \frac{\int \frac{-a^3b(3a^2-b^2)e^4-a^4(a^2-3b^2)e^4\cot(c+dx)}{\sqrt{e\cot(c+dx)}}\,dx}{a^3\left(a^2+b^2\right)^3 e^5} \\
&\quad - \frac{(b^3(63a^4+46a^2b^2+15b^4))\int \frac{1+\cot^2(c+dx)}{\sqrt{e\cot(c+dx)}(a+b\cot(c+dx))}\,dx}{8a^3\left(a^2+b^2\right)^3 e}
\end{aligned}
$$

$$
\begin{aligned}
&= \frac{8a^4+31a^2b^2+15b^4}{4a^3\left(a^2+b^2\right)^2 de\sqrt{e\cot(c+dx)}} \\
&\quad - \frac{b^2}{2a\left(a^2+b^2\right) de\sqrt{e\cot(c+dx)}(a+b\cot(c+dx))^2} \\
&\quad - \frac{b^2(13a^2+5b^2)}{4a^2\left(a^2+b^2\right)^2 de\sqrt{e\cot(c+dx)}(a+b\cot(c+dx))} \\
&\quad + \frac{2\mathrm{Subst}\left(\int \frac{a^3b(3a^2-b^2)e^5+a^4(a^2-3b^2)e^4x^2}{e^2+x^4}\,dx, x, \sqrt{e\cot(c+dx)}\right)}{a^3\left(a^2+b^2\right)^3 de^5} \\
&\quad - \frac{(b^3(63a^4+46a^2b^2+15b^4))\,\mathrm{Subst}\left(\int \frac{1}{\sqrt{-ex}(a-bx)}\,dx, x, -\cot(c+dx)\right)}{8a^3\left(a^2+b^2\right)^3 de}
\end{aligned}
$$

$$
\begin{aligned}
&= \frac{8a^4+31a^2b^2+15b^4}{4a^3\left(a^2+b^2\right)^2 de\sqrt{e\cot(c+dx)}} \\
&\quad - \frac{b^2}{2a\left(a^2+b^2\right) de\sqrt{e\cot(c+dx)}(a+b\cot(c+dx))^2} \\
&\quad - \frac{b^2(13a^2+5b^2)}{4a^2\left(a^2+b^2\right)^2 de\sqrt{e\cot(c+dx)}(a+b\cot(c+dx))} \\
&\quad + \frac{(b^3(63a^4+46a^2b^2+15b^4))\,\mathrm{Subst}\left(\int \frac{1}{a+\frac{bx^2}{e}}\,dx, x, \sqrt{e\cot(c+dx)}\right)}{4a^3\left(a^2+b^2\right)^3 de^2} \\
&\quad - \frac{((a+b)\left(a^2-4ab+b^2\right))\,\mathrm{Subst}\left(\int \frac{e-x^2}{e^2+x^4}\,dx, x, \sqrt{e\cot(c+dx)}\right)}{\left(a^2+b^2\right)^3 de} \\
&\quad + \frac{((a-b)\left(a^2+4ab+b^2\right))\,\mathrm{Subst}\left(\int \frac{e+x^2}{e^2+x^4}\,dx, x, \sqrt{e\cot(c+dx)}\right)}{\left(a^2+b^2\right)^3 de}
\end{aligned}
$$

$$
\begin{aligned}
&= \frac{b^{5/2}(63a^4+46a^2b^2+15b^4)\arctan\left(\frac{\sqrt{b}\sqrt{e\cot(c+dx)}}{\sqrt{a}\sqrt{e}}\right)}{4a^{7/2}\left(a^2+b^2\right)^3de^{3/2}} \\
&+ \frac{8a^4+31a^2b^2+15b^4}{4a^3\left(a^2+b^2\right)^2de\sqrt{e\cot(c+dx)}} \\
&- \frac{b^2}{2a\left(a^2+b^2\right)de\sqrt{e\cot(c+dx)}(a+b\cot(c+dx))^2} \\
&- \frac{b^2(13a^2+5b^2)}{4a^2\left(a^2+b^2\right)^2de\sqrt{e\cot(c+dx)}(a+b\cot(c+dx))} \\
&+ \frac{((a+b)\left(a^2-4ab+b^2\right))\,\text{Subst}\left(\int \frac{\sqrt{2}\sqrt{e}+2x}{-e-\sqrt{2}\sqrt{e}x-x^2}\,dx,x,\sqrt{e\cot(c+dx)}\right)}{2\sqrt{2}\left(a^2+b^2\right)^3de^{3/2}} \\
&+ \frac{((a+b)\left(a^2-4ab+b^2\right))\,\text{Subst}\left(\int \frac{\sqrt{2}\sqrt{e}-2x}{-e+\sqrt{2}\sqrt{e}x-x^2}\,dx,x,\sqrt{e\cot(c+dx)}\right)}{2\sqrt{2}\left(a^2+b^2\right)^3de^{3/2}} \\
&+ \frac{((a-b)\left(a^2+4ab+b^2\right))\,\text{Subst}\left(\int \frac{1}{e-\sqrt{2}\sqrt{e}x+x^2}\,dx,x,\sqrt{e\cot(c+dx)}\right)}{2\left(a^2+b^2\right)^3de} \\
&+ \frac{((a-b)\left(a^2+4ab+b^2\right))\,\text{Subst}\left(\int \frac{1}{e+\sqrt{2}\sqrt{e}x+x^2}\,dx,x,\sqrt{e\cot(c+dx)}\right)}{2\left(a^2+b^2\right)^3de}
\end{aligned}
$$

$$
\begin{aligned}
&= \frac{b^{5/2}(63a^4+46a^2b^2+15b^4)\arctan\left(\frac{\sqrt{b}\sqrt{e\cot(c+dx)}}{\sqrt{a}\sqrt{e}}\right)}{4a^{7/2}\left(a^2+b^2\right)^3de^{3/2}} \\
&+ \frac{8a^4+31a^2b^2+15b^4}{4a^3\left(a^2+b^2\right)^2de\sqrt{e\cot(c+dx)}} \\
&- \frac{b^2}{2a\left(a^2+b^2\right)de\sqrt{e\cot(c+dx)}(a+b\cot(c+dx))^2} \\
&- \frac{b^2(13a^2+5b^2)}{4a^2\left(a^2+b^2\right)^2de\sqrt{e\cot(c+dx)}(a+b\cot(c+dx))} \\
&+ \frac{(a+b)\left(a^2-4ab+b^2\right)\log\left(\sqrt{e}+\sqrt{e}\cot(c+dx)-\sqrt{2}\sqrt{e\cot(c+dx)}\right)}{2\sqrt{2}\left(a^2+b^2\right)^3de^{3/2}} \\
&- \frac{(a+b)\left(a^2-4ab+b^2\right)\log\left(\sqrt{e}+\sqrt{e}\cot(c+dx)+\sqrt{2}\sqrt{e\cot(c+dx)}\right)}{2\sqrt{2}\left(a^2+b^2\right)^3de^{3/2}} \\
&+ \frac{((a-b)\left(a^2+4ab+b^2\right))\,\text{Subst}\left(\int \frac{1}{-1-x^2}\,dx,x,1-\frac{\sqrt{2}\sqrt{e\cot(c+dx)}}{\sqrt{e}}\right)}{\sqrt{2}\left(a^2+b^2\right)^3de^{3/2}} \\
&- \frac{((a-b)\left(a^2+4ab+b^2\right))\,\text{Subst}\left(\int \frac{1}{-1-x^2}\,dx,x,1+\frac{\sqrt{2}\sqrt{e\cot(c+dx)}}{\sqrt{e}}\right)}{\sqrt{2}\left(a^2+b^2\right)^3de^{3/2}}
\end{aligned}
$$

$$= \frac{b^{5/2}(63a^4 + 46a^2b^2 + 15b^4)\arctan\left(\frac{\sqrt{b}\sqrt{e\cot(c+dx)}}{\sqrt{a}\sqrt{e}}\right)}{4a^{7/2}(a^2+b^2)^3 de^{3/2}}$$
$$- \frac{(a-b)(a^2+4ab+b^2)\arctan\left(1-\frac{\sqrt{2}\sqrt{e\cot(c+dx)}}{\sqrt{e}}\right)}{\sqrt{2}(a^2+b^2)^3 de^{3/2}}$$
$$+ \frac{(a-b)(a^2+4ab+b^2)\arctan\left(1+\frac{\sqrt{2}\sqrt{e\cot(c+dx)}}{\sqrt{e}}\right)}{\sqrt{2}(a^2+b^2)^3 de^{3/2}}$$
$$+ \frac{8a^4+31a^2b^2+15b^4}{4a^3(a^2+b^2)^2 de\sqrt{e\cot(c+dx)}}$$
$$- \frac{b^2}{2a(a^2+b^2)de\sqrt{e\cot(c+dx)}(a+b\cot(c+dx))^2}$$
$$- \frac{b^2(13a^2+5b^2)}{4a^2(a^2+b^2)^2 de\sqrt{e\cot(c+dx)}(a+b\cot(c+dx))}$$
$$+ \frac{(a+b)(a^2-4ab+b^2)\log\left(\sqrt{e}+\sqrt{e}\cot(c+dx)-\sqrt{2}\sqrt{e\cot(c+dx)}\right)}{2\sqrt{2}(a^2+b^2)^3 de^{3/2}}$$
$$- \frac{(a+b)(a^2-4ab+b^2)\log\left(\sqrt{e}+\sqrt{e}\cot(c+dx)+\sqrt{2}\sqrt{e\cot(c+dx)}\right)}{2\sqrt{2}(a^2+b^2)^3 de^{3/2}}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 1.75 (sec) , antiderivative size = 303, normalized size of antiderivative = 0.57

$$\int \frac{1}{(e\cot(c+dx))^{3/2}(a+b\cot(c+dx))^3}\,dx =$$
$$-\frac{-8a^2b^2(3a^2-b^2)\,\text{Hypergeometric2F1}\left(-\frac{1}{2},1,\frac{1}{2},-\frac{b\cot(c+dx)}{a}\right)-16a^2b^2(a^2+b^2)\,\text{Hypergeometric2F1}\left(-\frac{1}{2},\right.}{}$$

```
[In] Integrate[1/((e*Cot[c + d*x])^(3/2)*(a + b*Cot[c + d*x])^3),x]
```

```
[Out] -1/4*(-8*a^2*b^2*(3*a^2 - b^2)*Hypergeometric2F1[-1/2, 1, 1/2, -((b*Cot[c +
 d*x])/a)] - 16*a^2*b^2*(a^2 + b^2)*Hypergeometric2F1[-1/2, 2, 1/2, -((b*Co
t[c + d*x])/a)] - 8*b^2*(a^2 + b^2)^2*Hypergeometric2F1[-1/2, 3, 1/2, -((b*
Cot[c + d*x])/a)] - 8*a^4*(a^2 - 3*b^2)*Hypergeometric2F1[-1/4, 1, 3/4, -Co
t[c + d*x]^2] + Sqrt[2]*a^3*b*(3*a^2 - b^2)*Sqrt[Cot[c + d*x]]*(2*ArcTan[1
- Sqrt[2]*Sqrt[Cot[c + d*x]]] - 2*ArcTan[1 + Sqrt[2]*Sqrt[Cot[c + d*x]]] +
Log[1 - Sqrt[2]*Sqrt[Cot[c + d*x]] + Cot[c + d*x]] - Log[1 + Sqrt[2]*Sqrt[C
ot[c + d*x]] + Cot[c + d*x]]))/(a^3*(a^2 + b^2)^3*d*e*Sqrt[e*Cot[c + d*x]])
```

Maple [A] (verified)

Time = 0.05 (sec) , antiderivative size = 480, normalized size of antiderivative = 0.91

method	result
derivativedivides	$-\frac{2e^4\left(-\frac{b^3\left(\frac{\left(\frac{15}{8}a^4b+\frac{11}{4}a^2b^3+\frac{7}{8}b^5\right)(e\cot(dx+c))^{\frac{3}{2}}+\frac{ae\left(17a^4+26a^2b^2+9b^4\right)\sqrt{e\cot(dx+c)}}{8}}{(e\cot(dx+c)b+ae)^2}+\frac{\left(63a^4+46a^2b^2+15b^4\right)\arctan\left(\frac{\sqrt{e\cot}}{}\right.}{8\sqrt{aeb}}\right.}{a^3e^5\left(a^2+b^2\right)^3}\right.}{}$
default	$-\frac{2e^4\left(-\frac{b^3\left(\frac{\left(\frac{15}{8}a^4b+\frac{11}{4}a^2b^3+\frac{7}{8}b^5\right)(e\cot(dx+c))^{\frac{3}{2}}+\frac{ae\left(17a^4+26a^2b^2+9b^4\right)\sqrt{e\cot(dx+c)}}{8}}{(e\cot(dx+c)b+ae)^2}+\frac{\left(63a^4+46a^2b^2+15b^4\right)\arctan\left(\frac{\sqrt{e\cot}}{}\right.}{8\sqrt{aeb}}\right.}{a^3e^5\left(a^2+b^2\right)^3}\right.}{}$

```
[In] int(1/(e*cot(d*x+c))^(3/2)/(a+b*cot(d*x+c))^3,x,method=_RETURNVERBOSE)
```

```
[Out] -2/d*e^4*(-b^3/a^3/e^5/(a^2+b^2)^3*(((15/8*a^4*b+11/4*a^2*b^3+7/8*b^5)*(e*c
ot(d*x+c))^(3/2)+1/8*a*e*(17*a^4+26*a^2*b^2+9*b^4)*(e*cot(d*x+c))^(1/2))/(e
*cot(d*x+c)*b+a*e)^2+1/8*(63*a^4+46*a^2*b^2+15*b^4)/(a*e*b)^(1/2)*arctan((e
*cot(d*x+c))^(1/2)*b/(a*e*b)^(1/2)))+1/(a^2+b^2)^3/e^5*(1/8*(-3*a^2*b*e+b^3
*e)*(e^2)^(1/4)/e^2*2^(1/2)*(ln((e*cot(d*x+c)+(e^2)^(1/4)*(e*cot(d*x+c))^(1
/2)*2^(1/2)+(e^2)^(1/2))/(e*cot(d*x+c)-(e^2)^(1/4)*(e*cot(d*x+c))^(1/2)*2^(
1/2)+(e^2)^(1/2)))+2*arctan(2^(1/2)/(e^2)^(1/4)*(e*cot(d*x+c))^(1/2)+1)-2*a
rctan(-2^(1/2)/(e^2)^(1/4)*(e*cot(d*x+c))^(1/2)+1))+1/8*(-a^3+3*a*b^2)/(e^2
)^(1/4)*2^(1/2)*(ln((e*cot(d*x+c)-(e^2)^(1/4)*(e*cot(d*x+c))^(1/2)*2^(1/2)+
(e^2)^(1/2))/(e*cot(d*x+c)+(e^2)^(1/4)*(e*cot(d*x+c))^(1/2)*2^(1/2)+(e^2)^(
1/2)))+2*arctan(2^(1/2)/(e^2)^(1/4)*(e*cot(d*x+c))^(1/2)+1)-2*arctan(-2^(1/
2)/(e^2)^(1/4)*(e*cot(d*x+c))^(1/2)+1)))-1/a^3/e^5/(e*cot(d*x+c))^(1/2))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 5133 vs. $2(454) = 908$.

Time = 2.31 (sec) , antiderivative size = 10308, normalized size of antiderivative = 19.49

$$\int \frac{1}{(e\cot(c+dx))^{3/2}(a+b\cot(c+dx))^3}\,dx = \text{Too large to display}$$

```
[In] integrate(1/(e*cot(d*x+c))^(3/2)/(a+b*cot(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] Too large to include
```

Sympy [F]

$$\int \frac{1}{(e\cot(c+dx))^{3/2}(a+b\cot(c+dx))^3}\,dx = \int \frac{1}{(e\cot(c+dx))^{\frac{3}{2}}(a+b\cot(c+dx))^3}\,dx$$

```
[In] integrate(1/(e*cot(d*x+c))**(3/2)/(a+b*cot(d*x+c))**3,x)
[Out] Integral(1/((e*cot(c + d*x))**(3/2)*(a + b*cot(c + d*x))**3), x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{1}{(e\cot(c+dx))^{3/2}(a+b\cot(c+dx))^3}\,dx = \text{Exception raised: ValueError}$$

```
[In] integrate(1/(e*cot(d*x+c))^(3/2)/(a+b*cot(d*x+c))^3,x, algorithm="maxima")
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(e>0)', see `assume?` for more detai
ls)Is e
```

Giac [F]

$$\int \frac{1}{(e\cot(c+dx))^{3/2}(a+b\cot(c+dx))^3}\,dx = \int \frac{1}{(b\cot(dx+c)+a)^3(e\cot(dx+c))^{\frac{3}{2}}}\,dx$$

```
[In] integrate(1/(e*cot(d*x+c))^(3/2)/(a+b*cot(d*x+c))^3,x, algorithm="giac")
[Out] integrate(1/((b*cot(d*x + c) + a)^3*(e*cot(d*x + c))^(3/2)), x)
```

Mupad [B] (verification not implemented)

Time = 22.78 (sec) , antiderivative size = 21158, normalized size of antiderivative = 40.00

$$\int \frac{1}{(e\cot(c+dx))^{3/2}(a+b\cot(c+dx))^3}\,dx = \text{Too large to display}$$

```
[In] int(1/((e*cot(c + d*x))^(3/2)*(a + b*cot(c + d*x))^3),x)
[Out] ((2*e)/a + (e*cot(c + d*x)*(16*a^4*b + 25*b^5 + 49*a^2*b^3))/(4*a^2*(a^4 +
b^4 + 2*a^2*b^2)) + (b^2*e^2*cot(c + d*x)^2*(8*a^4 + 15*b^4 + 31*a^2*b^2))/
```

(4*a^3*(a^4*e + b^4*e + 2*a^2*b^2*e)))/(b^2*d*(e*cot(c + d*x))^(5/2) + a^2*
d*e^2*(e*cot(c + d*x))^(1/2) + 2*a*b*d*e*(e*cot(c + d*x))^(3/2)) + atan((((
-1i/(4*(b^6*d^2*e^3 - a^6*d^2*e^3 + a*b^5*d^2*e^3*6i + a^5*b*d^2*e^3*6i - 1
5*a^2*b^4*d^2*e^3 - a^3*b^3*d^2*e^3*20i + 15*a^4*b^2*d^2*e^3)))^(1/2)*(((e*
cot(c + d*x))^(1/2)*(471859200*a^22*b^44*d^7*e^16 + 9500098560*a^24*b^42*d^
7*e^16 + 91857354752*a^26*b^40*d^7*e^16 + 564502986752*a^28*b^38*d^7*e^16 +
 2464648527872*a^30*b^36*d^7*e^16 + 8104469069824*a^32*b^34*d^7*e^16 + 2076
9933361152*a^34*b^32*d^7*e^16 + 42351565209600*a^36*b^30*d^7*e^16 + 6953494
5902592*a^38*b^28*d^7*e^16 + 92434029608960*a^40*b^26*d^7*e^16 + 9950871735
5008*a^42*b^24*d^7*e^16 + 86342935511040*a^44*b^22*d^7*e^16 + 5976709555814
4*a^46*b^20*d^7*e^16 + 32432589897728*a^48*b^18*d^7*e^16 + 13411815522304*a
^50*b^16*d^7*e^16 + 4030457708544*a^52*b^14*d^7*e^16 + 805425905664*a^54*b^
12*d^7*e^16 + 86608183296*a^56*b^10*d^7*e^16 + 1612709888*a^58*b^8*d^7*e^16
 + 16777216*a^60*b^6*d^7*e^16 + 167772160*a^62*b^4*d^7*e^16 + 16777216*a^64
*b^2*d^7*e^16) + (-1i/(4*(b^6*d^2*e^3 - a^6*d^2*e^3 + a*b^5*d^2*e^3*6i + a^
5*b*d^2*e^3*6i - 15*a^2*b^4*d^2*e^3 - a^3*b^3*d^2*e^3*20i + 15*a^4*b^2*d^2*
e^3)))^(1/2)*(251658240*a^24*b^45*d^8*e^18 - (e*cot(c + d*x))^(1/2)*(-1i/(4
*(b^6*d^2*e^3 - a^6*d^2*e^3 + a*b^5*d^2*e^3*6i + a^5*b*d^2*e^3*6i - 15*a^2*
b^4*d^2*e^3 - a^3*b^3*d^2*e^3*20i + 15*a^4*b^2*d^2*e^3)))^(1/2)*(134217728*
a^27*b^45*d^9*e^19 + 2550136832*a^29*b^43*d^9*e^19 + 22817013760*a^31*b^41*
d^9*e^19 + 127506841600*a^33*b^39*d^9*e^19 + 497276682240*a^35*b^37*d^9*e^1
9 + 1430626762752*a^37*b^35*d^9*e^19 + 3121367482368*a^39*b^33*d^9*e^19 + 5
202279137280*a^41*b^31*d^9*e^19 + 6502848921600*a^43*b^29*d^9*e^19 + 563580
2398720*a^45*b^27*d^9*e^19 + 2254320959488*a^47*b^25*d^9*e^19 - 22543209594
88*a^49*b^23*d^9*e^19 - 5635802398720*a^51*b^21*d^9*e^19 - 6502848921600*a^
53*b^19*d^9*e^19 - 5202279137280*a^55*b^17*d^9*e^19 - 3121367482368*a^57*b^
15*d^9*e^19 - 1430626762752*a^59*b^13*d^9*e^19 - 497276682240*a^61*b^11*d^9
*e^19 - 127506841600*a^63*b^9*d^9*e^19 - 22817013760*a^65*b^7*d^9*e^19 - 25
50136832*a^67*b^5*d^9*e^19 - 134217728*a^69*b^3*d^9*e^19) + 5049942016*a^26
*b^43*d^8*e^18 + 48368713728*a^28*b^41*d^8*e^18 + 293819383808*a^30*b^39*d^
8*e^18 + 1268458192896*a^32*b^37*d^8*e^18 + 4132731617280*a^34*b^35*d^8*e^1
8 + 10531192700928*a^36*b^33*d^8*e^18 + 21462823993344*a^38*b^31*d^8*e^18 +
 35469618315264*a^40*b^29*d^8*e^18 + 47896904859648*a^42*b^27*d^8*e^18 + 52
983958077440*a^44*b^25*d^8*e^18 + 47896904859648*a^46*b^23*d^8*e^18 + 35090
285461504*a^48*b^21*d^8*e^18 + 20487396655104*a^50*b^19*d^8*e^18 + 92306229
16608*a^52*b^17*d^8*e^18 + 2994733056000*a^54*b^15*d^8*e^18 + 565576728576*
a^56*b^13*d^8*e^18 - 18572378112*a^58*b^11*d^8*e^18 - 50281316352*a^60*b^9*
d^8*e^18 - 16089350144*a^62*b^7*d^8*e^18 - 2516582400*a^64*b^5*d^8*e^18 - 1
67772160*a^66*b^3*d^8*e^18))*(-1i/(4*(b^6*d^2*e^3 - a^6*d^2*e^3 + a*b^5*d^2
*e^3*6i + a^5*b*d^2*e^3*6i - 15*a^2*b^4*d^2*e^3 - a^3*b^3*d^2*e^3*20i + 15*
a^4*b^2*d^2*e^3)))^(1/2) - 117964800*a^21*b^42*d^6*e^15 - 841482240*a^23*b^
40*d^6*e^15 + 3829399552*a^25*b^38*d^6*e^15 + 78068580352*a^27*b^36*d^6*e^1
5 + 497438162944*a^29*b^34*d^6*e^15 + 1899895980032*a^31*b^32*d^6*e^15 + 49
72695519232*a^33*b^30*d^6*e^15 + 9371195015168*a^35*b^28*d^6*e^15 + 1289072
0436224*a^37*b^26*d^6*e^15 + 12726089809920*a^39*b^24*d^6*e^15 + 8366961197

056*a^41*b^22*d^6*e^15 + 2597662490624*a^43*b^20*d^6*e^15 - 1171836108800*a
^45*b^18*d^6*e^15 - 1986881650688*a^47*b^16*d^6*e^15 - 1237583921152*a^49*b
^14*d^6*e^15 - 449507753984*a^51*b^12*d^6*e^15 - 97476149248*a^53*b^10*d^6*
e^15 - 11931222016*a^55*b^8*d^6*e^15 - 1006632960*a^57*b^6*d^6*e^15 - 13421
7728*a^59*b^4*d^6*e^15 - 8388608*a^61*b^2*d^6*e^15) - (e*cot(c + d*x))^(1/2
)*(7610564608*a^27*b^33*d^5*e^13 - 597688320*a^23*b^37*d^5*e^13 - 167143014
4*a^25*b^35*d^5*e^13 - 58982400*a^21*b^39*d^5*e^13 + 85774565376*a^29*b^31*
d^5*e^13 + 385487994880*a^31*b^29*d^5*e^13 + 1104303620096*a^33*b^27*d^5*e^
13 + 2240523796480*a^35*b^25*d^5*e^13 + 3345249468416*a^37*b^23*d^5*e^13 +
3717287903232*a^39*b^21*d^5*e^13 + 3053967114240*a^41*b^19*d^5*e^13 + 18074
74491392*a^43*b^17*d^5*e^13 + 726513221632*a^45*b^15*d^5*e^13 + 17076899020
8*a^47*b^13*d^5*e^13 + 10492051456*a^49*b^11*d^5*e^13 - 4917821440*a^51*b^9
*d^5*e^13 - 923009024*a^53*b^7*d^5*e^13 + 8388608*a^55*b^5*d^5*e^13))*(-1i/
(4*(b^6*d^2*e^3 - a^6*d^2*e^3 + a*b^5*d^2*e^3*6i + a^5*b*d^2*e^3*6i - 15*a^
2*b^4*d^2*e^3 - a^3*b^3*d^2*e^3*20i + 15*a^4*b^2*d^2*e^3)))^(1/2)*1i + ((-1
i/(4*(b^6*d^2*e^3 - a^6*d^2*e^3 + a*b^5*d^2*e^3*6i + a^5*b*d^2*e^3*6i - 15*
a^2*b^4*d^2*e^3 - a^3*b^3*d^2*e^3*20i + 15*a^4*b^2*d^2*e^3)))^(1/2)*(((e*co
t(c + d*x))^(1/2)*(471859200*a^22*b^44*d^7*e^16 + 9500098560*a^24*b^42*d^7*
e^16 + 91857354752*a^26*b^40*d^7*e^16 + 564502986752*a^28*b^38*d^7*e^16 + 2
464648527872*a^30*b^36*d^7*e^16 + 8104469069824*a^32*b^34*d^7*e^16 + 207699
33361152*a^34*b^32*d^7*e^16 + 42351565209600*a^36*b^30*d^7*e^16 + 695349459
02592*a^38*b^28*d^7*e^16 + 92434029608960*a^40*b^26*d^7*e^16 + 995087173550
08*a^42*b^24*d^7*e^16 + 86342935511040*a^44*b^22*d^7*e^16 + 59767095558144*
a^46*b^20*d^7*e^16 + 32432589897728*a^48*b^18*d^7*e^16 + 13411815522304*a^5
0*b^16*d^7*e^16 + 4030457708544*a^52*b^14*d^7*e^16 + 805425905664*a^54*b^12
*d^7*e^16 + 86608183296*a^56*b^10*d^7*e^16 + 1612709888*a^58*b^8*d^7*e^16 +
16777216*a^60*b^6*d^7*e^16 + 167772160*a^62*b^4*d^7*e^16 + 16777216*a^64*b
^2*d^7*e^16) - (-1i/(4*(b^6*d^2*e^3 - a^6*d^2*e^3 + a*b^5*d^2*e^3*6i + a^5*
b*d^2*e^3*6i - 15*a^2*b^4*d^2*e^3 - a^3*b^3*d^2*e^3*20i + 15*a^4*b^2*d^2*e^
3)))^(1/2)*((e*cot(c + d*x))^(1/2)*(-1i/(4*(b^6*d^2*e^3 - a^6*d^2*e^3 + a*b
^5*d^2*e^3*6i + a^5*b*d^2*e^3*6i - 15*a^2*b^4*d^2*e^3 - a^3*b^3*d^2*e^3*20i
+ 15*a^4*b^2*d^2*e^3)))^(1/2)*(134217728*a^27*b^45*d^9*e^19 + 2550136832*a
^29*b^43*d^9*e^19 + 22817013760*a^31*b^41*d^9*e^19 + 127506841600*a^33*b^39
*d^9*e^19 + 497276682240*a^35*b^37*d^9*e^19 + 1430626762752*a^37*b^35*d^9*e
^19 + 3121367482368*a^39*b^33*d^9*e^19 + 5202279137280*a^41*b^31*d^9*e^19 +
6502848921600*a^43*b^29*d^9*e^19 + 5635802398720*a^45*b^27*d^9*e^19 + 2254
320959488*a^47*b^25*d^9*e^19 - 2254320959488*a^49*b^23*d^9*e^19 - 563580239
8720*a^51*b^21*d^9*e^19 - 6502848921600*a^53*b^19*d^9*e^19 - 5202279137280*
a^55*b^17*d^9*e^19 - 3121367482368*a^57*b^15*d^9*e^19 - 1430626762752*a^59*
b^13*d^9*e^19 - 497276682240*a^61*b^11*d^9*e^19 - 127506841600*a^63*b^9*d^9
*e^19 - 22817013760*a^65*b^7*d^9*e^19 - 2550136832*a^67*b^5*d^9*e^19 - 1342
17728*a^69*b^3*d^9*e^19) + 251658240*a^24*b^45*d^8*e^18 + 5049942016*a^26*b
^43*d^8*e^18 + 48368713728*a^28*b^41*d^8*e^18 + 293819383808*a^30*b^39*d^8*
e^18 + 1268458192896*a^32*b^37*d^8*e^18 + 4132731617280*a^34*b^35*d^8*e^18
+ 10531192700928*a^36*b^33*d^8*e^18 + 21462823993344*a^38*b^31*d^8*e^18 + 3

5469618315264*a^40*b^29*d^8*e^18 + 47896904859648*a^42*b^27*d^8*e^18 + 5298
3958077440*a^44*b^25*d^8*e^18 + 47896904859648*a^46*b^23*d^8*e^18 + 3509028
5461504*a^48*b^21*d^8*e^18 + 20487396655104*a^50*b^19*d^8*e^18 + 9230622916
608*a^52*b^17*d^8*e^18 + 2994733056000*a^54*b^15*d^8*e^18 + 565576728576*a^
56*b^13*d^8*e^18 - 18572378112*a^58*b^11*d^8*e^18 - 50281316352*a^60*b^9*d^
8*e^18 - 16089350144*a^62*b^7*d^8*e^18 - 2516582400*a^64*b^5*d^8*e^18 - 167
772160*a^66*b^3*d^8*e^18))*(-1i/(4*(b^6*d^2*e^3 - a^6*d^2*e^3 + a*b^5*d^2*e
^3*6i + a^5*b*d^2*e^3*6i - 15*a^2*b^4*d^2*e^3 - a^3*b^3*d^2*e^3*20i + 15*a^
4*b^2*d^2*e^3)))^(1/2) + 117964800*a^21*b^42*d^6*e^15 + 841482240*a^23*b^40
*d^6*e^15 - 3829399552*a^25*b^38*d^6*e^15 - 78068580352*a^27*b^36*d^6*e^15
- 497438162944*a^29*b^34*d^6*e^15 - 1899895980032*a^31*b^32*d^6*e^15 - 4972
695519232*a^33*b^30*d^6*e^15 - 9371195015168*a^35*b^28*d^6*e^15 - 128907204
36224*a^37*b^26*d^6*e^15 - 12726089809920*a^39*b^24*d^6*e^15 - 836696119705
6*a^41*b^22*d^6*e^15 - 2597662490624*a^43*b^20*d^6*e^15 + 1171836108800*a^4
5*b^18*d^6*e^15 + 1986881650688*a^47*b^16*d^6*e^15 + 1237583921152*a^49*b^1
4*d^6*e^15 + 449507753984*a^51*b^12*d^6*e^15 + 97476149248*a^53*b^10*d^6*e^
15 + 11931222016*a^55*b^8*d^6*e^15 + 1006632960*a^57*b^6*d^6*e^15 + 1342177
28*a^59*b^4*d^6*e^15 + 8388608*a^61*b^2*d^6*e^15) - (e*cot(c + d*x))^(1/2)*
(7610564608*a^27*b^33*d^5*e^13 - 597688320*a^23*b^37*d^5*e^13 - 1671430144*
a^25*b^35*d^5*e^13 - 58982400*a^21*b^39*d^5*e^13 + 85774565376*a^29*b^31*d^
5*e^13 + 385487994880*a^31*b^29*d^5*e^13 + 1104303620096*a^33*b^27*d^5*e^13
 + 2240523796480*a^35*b^25*d^5*e^13 + 3345249468416*a^37*b^23*d^5*e^13 + 37
17287903232*a^39*b^21*d^5*e^13 + 3053967114240*a^41*b^19*d^5*e^13 + 1807474
491392*a^43*b^17*d^5*e^13 + 726513221632*a^45*b^15*d^5*e^13 + 170768990208*
a^47*b^13*d^5*e^13 + 10492051456*a^49*b^11*d^5*e^13 - 4917821440*a^51*b^9*d
^5*e^13 - 923009024*a^53*b^7*d^5*e^13 + 8388608*a^55*b^5*d^5*e^13))*(-1i/(4
*(b^6*d^2*e^3 - a^6*d^2*e^3 + a*b^5*d^2*e^3*6i + a^5*b*d^2*e^3*6i - 15*a^2*
b^4*d^2*e^3 - a^3*b^3*d^2*e^3*20i + 15*a^4*b^2*d^2*e^3)))^(1/2)*1i)/(((-1i/
(4*(b^6*d^2*e^3 - a^6*d^2*e^3 + a*b^5*d^2*e^3*6i + a^5*b*d^2*e^3*6i - 15*a^
2*b^4*d^2*e^3 - a^3*b^3*d^2*e^3*20i + 15*a^4*b^2*d^2*e^3)))^(1/2)*(((e*cot(
c + d*x))^(1/2)*(471859200*a^22*b^44*d^7*e^16 + 9500098560*a^24*b^42*d^7*e^
16 + 91857354752*a^26*b^40*d^7*e^16 + 564502986752*a^28*b^38*d^7*e^16 + 246
4648527872*a^30*b^36*d^7*e^16 + 8104469069824*a^32*b^34*d^7*e^16 + 20769933
361152*a^34*b^32*d^7*e^16 + 42351565209600*a^36*b^30*d^7*e^16 + 69534945902
592*a^38*b^28*d^7*e^16 + 92434029608960*a^40*b^26*d^7*e^16 + 99508717355008
*a^42*b^24*d^7*e^16 + 86342935511040*a^44*b^22*d^7*e^16 + 59767095558144*a^
46*b^20*d^7*e^16 + 32432589897728*a^48*b^18*d^7*e^16 + 13411815522304*a^50*
b^16*d^7*e^16 + 4030457708544*a^52*b^14*d^7*e^16 + 805425905664*a^54*b^12*d
^7*e^16 + 86608183296*a^56*b^10*d^7*e^16 + 1612709888*a^58*b^8*d^7*e^16 + 1
6777216*a^60*b^6*d^7*e^16 + 167772160*a^62*b^4*d^7*e^16 + 16777216*a^64*b^2
*d^7*e^16) - (-1i/(4*(b^6*d^2*e^3 - a^6*d^2*e^3 + a*b^5*d^2*e^3*6i + a^5*b*
d^2*e^3*6i - 15*a^2*b^4*d^2*e^3 - a^3*b^3*d^2*e^3*20i + 15*a^4*b^2*d^2*e^3)
))^(1/2)*((e*cot(c + d*x))^(1/2)*(-1i/(4*(b^6*d^2*e^3 - a^6*d^2*e^3 + a*b^5
*d^2*e^3*6i + a^5*b*d^2*e^3*6i - 15*a^2*b^4*d^2*e^3 - a^3*b^3*d^2*e^3*20i +
 15*a^4*b^2*d^2*e^3)))^(1/2)*(134217728*a^27*b^45*d^9*e^19 + 2550136832*a^2

9*b^43*d^9*e^19 + 22817013760*a^31*b^41*d^9*e^19 + 127506841600*a^33*b^39*d
^9*e^19 + 497276682240*a^35*b^37*d^9*e^19 + 1430626762752*a^37*b^35*d^9*e^1
9 + 3121367482368*a^39*b^33*d^9*e^19 + 5202279137280*a^41*b^31*d^9*e^19 + 6
502848921600*a^43*b^29*d^9*e^19 + 5635802398720*a^45*b^27*d^9*e^19 + 225432
0959488*a^47*b^25*d^9*e^19 - 2254320959488*a^49*b^23*d^9*e^19 - 56358023987
20*a^51*b^21*d^9*e^19 - 6502848921600*a^53*b^19*d^9*e^19 - 5202279137280*a^
55*b^17*d^9*e^19 - 3121367482368*a^57*b^15*d^9*e^19 - 1430626762752*a^59*b^
13*d^9*e^19 - 497276682240*a^61*b^11*d^9*e^19 - 127506841600*a^63*b^9*d^9*e
^19 - 22817013760*a^65*b^7*d^9*e^19 - 2550136832*a^67*b^5*d^9*e^19 - 134217
728*a^69*b^3*d^9*e^19) + 251658240*a^24*b^45*d^8*e^18 + 5049942016*a^26*b^4
3*d^8*e^18 + 48368713728*a^28*b^41*d^8*e^18 + 293819383808*a^30*b^39*d^8*e^
18 + 1268458192896*a^32*b^37*d^8*e^18 + 4132731617280*a^34*b^35*d^8*e^18 +
10531192700928*a^36*b^33*d^8*e^18 + 21462823993344*a^38*b^31*d^8*e^18 + 354
69618315264*a^40*b^29*d^8*e^18 + 47896904859648*a^42*b^27*d^8*e^18 + 529839
58077440*a^44*b^25*d^8*e^18 + 47896904859648*a^46*b^23*d^8*e^18 + 350902854
61504*a^48*b^21*d^8*e^18 + 20487396655104*a^50*b^19*d^8*e^18 + 923062291660
8*a^52*b^17*d^8*e^18 + 2994733056000*a^54*b^15*d^8*e^18 + 565576728576*a^56
*b^13*d^8*e^18 - 18572378112*a^58*b^11*d^8*e^18 - 50281316352*a^60*b^9*d^8*
e^18 - 16089350144*a^62*b^7*d^8*e^18 - 2516582400*a^64*b^5*d^8*e^18 - 16777
2160*a^66*b^3*d^8*e^18))*(-1i/(4*(b^6*d^2*e^3 - a^6*d^2*e^3 + a*b^5*d^2*e^3
*6i + a^5*b*d^2*e^3*6i - 15*a^2*b^4*d^2*e^3 - a^3*b^3*d^2*e^3*20i + 15*a^4*
b^2*d^2*e^3)))^(1/2) + 117964800*a^21*b^42*d^6*e^15 + 841482240*a^23*b^40*d
^6*e^15 - 3829399552*a^25*b^38*d^6*e^15 - 78068580352*a^27*b^36*d^6*e^15 -
497438162944*a^29*b^34*d^6*e^15 - 1899895980032*a^31*b^32*d^6*e^15 - 497269
5519232*a^33*b^30*d^6*e^15 - 9371195015168*a^35*b^28*d^6*e^15 - 12890720436
224*a^37*b^26*d^6*e^15 - 12726089809920*a^39*b^24*d^6*e^15 - 8366961197056*
a^41*b^22*d^6*e^15 - 2597662490624*a^43*b^20*d^6*e^15 + 1171836108800*a^45*
b^18*d^6*e^15 + 1986881650688*a^47*b^16*d^6*e^15 + 1237583921152*a^49*b^14*
d^6*e^15 + 449507753984*a^51*b^12*d^6*e^15 + 97476149248*a^53*b^10*d^6*e^15
 + 11931222016*a^55*b^8*d^6*e^15 + 1006632960*a^57*b^6*d^6*e^15 + 134217728
*a^59*b^4*d^6*e^15 + 8388608*a^61*b^2*d^6*e^15) - (e*cot(c + d*x))^(1/2)*(7
610564608*a^27*b^33*d^5*e^13 - 597688320*a^23*b^37*d^5*e^13 - 1671430144*a^
25*b^35*d^5*e^13 - 58982400*a^21*b^39*d^5*e^13 + 85774565376*a^29*b^31*d^5*
e^13 + 385487994880*a^31*b^29*d^5*e^13 + 1104303620096*a^33*b^27*d^5*e^13 +
 2240523796480*a^35*b^25*d^5*e^13 + 3345249468416*a^37*b^23*d^5*e^13 + 3717
287903232*a^39*b^21*d^5*e^13 + 3053967114240*a^41*b^19*d^5*e^13 + 180747449
1392*a^43*b^17*d^5*e^13 + 726513221632*a^45*b^15*d^5*e^13 + 170768990208*a^
47*b^13*d^5*e^13 + 10492051456*a^49*b^11*d^5*e^13 - 4917821440*a^51*b^9*d^5
*e^13 - 923009024*a^53*b^7*d^5*e^13 + 8388608*a^55*b^5*d^5*e^13))*(-1i/(4*(
b^6*d^2*e^3 - a^6*d^2*e^3 + a*b^5*d^2*e^3*6i + a^5*b*d^2*e^3*6i - 15*a^2*b^
4*d^2*e^3 - a^3*b^3*d^2*e^3*20i + 15*a^4*b^2*d^2*e^3)))^(1/2) - ((-1i/(4*(b
^6*d^2*e^3 - a^6*d^2*e^3 + a*b^5*d^2*e^3*6i + a^5*b*d^2*e^3*6i - 15*a^2*b^4
*d^2*e^3 - a^3*b^3*d^2*e^3*20i + 15*a^4*b^2*d^2*e^3)))^(1/2)*(((e*cot(c + d
x))^(1/2)(471859200*a^22*b^44*d^7*e^16 + 9500098560*a^24*b^42*d^7*e^16 +
91857354752*a^26*b^40*d^7*e^16 + 564502986752*a^28*b^38*d^7*e^16 + 24646485

27872*a^30*b^36*d^7*e^16 + 8104469069824*a^32*b^34*d^7*e^16 + 2076993336115
2*a^34*b^32*d^7*e^16 + 42351565209600*a^36*b^30*d^7*e^16 + 69534945902592*a
^38*b^28*d^7*e^16 + 92434029608960*a^40*b^26*d^7*e^16 + 99508717355008*a^42
*b^24*d^7*e^16 + 86342935511040*a^44*b^22*d^7*e^16 + 59767095558144*a^46*b^
20*d^7*e^16 + 32432589897728*a^48*b^18*d^7*e^16 + 13411815522304*a^50*b^16*
d^7*e^16 + 4030457708544*a^52*b^14*d^7*e^16 + 805425905664*a^54*b^12*d^7*e^
16 + 86608183296*a^56*b^10*d^7*e^16 + 1612709888*a^58*b^8*d^7*e^16 + 167772
16*a^60*b^6*d^7*e^16 + 167772160*a^62*b^4*d^7*e^16 + 16777216*a^64*b^2*d^7*
e^16) + (-1i/(4*(b^6*d^2*e^3 - a^6*d^2*e^3 + a*b^5*d^2*e^3*6i + a^5*b*d^2*e
^3*6i - 15*a^2*b^4*d^2*e^3 - a^3*b^3*d^2*e^3*20i + 15*a^4*b^2*d^2*e^3)))^(1
/2)*(251658240*a^24*b^45*d^8*e^18 - (e*cot(c + d*x))^(1/2)*(-1i/(4*(b^6*d^2
*e^3 - a^6*d^2*e^3 + a*b^5*d^2*e^3*6i + a^5*b*d^2*e^3*6i - 15*a^2*b^4*d^2*e
^3 - a^3*b^3*d^2*e^3*20i + 15*a^4*b^2*d^2*e^3)))^(1/2)*(134217728*a^27*b^45
*d^9*e^19 + 2550136832*a^29*b^43*d^9*e^19 + 22817013760*a^31*b^41*d^9*e^19
+ 127506841600*a^33*b^39*d^9*e^19 + 497276682240*a^35*b^37*d^9*e^19 + 14306
26762752*a^37*b^35*d^9*e^19 + 3121367482368*a^39*b^33*d^9*e^19 + 5202279137
280*a^41*b^31*d^9*e^19 + 6502848921600*a^43*b^29*d^9*e^19 + 5635802398720*a
^45*b^27*d^9*e^19 + 2254320959488*a^47*b^25*d^9*e^19 - 2254320959488*a^49*b
^23*d^9*e^19 - 5635802398720*a^51*b^21*d^9*e^19 - 6502848921600*a^53*b^19*d
^9*e^19 - 5202279137280*a^55*b^17*d^9*e^19 - 3121367482368*a^57*b^15*d^9*e^
19 - 1430626762752*a^59*b^13*d^9*e^19 - 497276682240*a^61*b^11*d^9*e^19 - 1
27506841600*a^63*b^9*d^9*e^19 - 22817013760*a^65*b^7*d^9*e^19 - 2550136832*
a^67*b^5*d^9*e^19 - 134217728*a^69*b^3*d^9*e^19) + 5049942016*a^26*b^43*d^8
*e^18 + 48368713728*a^28*b^41*d^8*e^18 + 293819383808*a^30*b^39*d^8*e^18 +
1268458192896*a^32*b^37*d^8*e^18 + 4132731617280*a^34*b^35*d^8*e^18 + 10531
192700928*a^36*b^33*d^8*e^18 + 21462823993344*a^38*b^31*d^8*e^18 + 35469618
315264*a^40*b^29*d^8*e^18 + 47896904859648*a^42*b^27*d^8*e^18 + 52983958077
440*a^44*b^25*d^8*e^18 + 47896904859648*a^46*b^23*d^8*e^18 + 35090285461504
*a^48*b^21*d^8*e^18 + 20487396655104*a^50*b^19*d^8*e^18 + 9230622916608*a^5
2*b^17*d^8*e^18 + 2994733056000*a^54*b^15*d^8*e^18 + 565576728576*a^56*b^13
*d^8*e^18 - 18572378112*a^58*b^11*d^8*e^18 - 50281316352*a^60*b^9*d^8*e^18
- 16089350144*a^62*b^7*d^8*e^18 - 2516582400*a^64*b^5*d^8*e^18 - 167772160*
a^66*b^3*d^8*e^18))*(-1i/(4*(b^6*d^2*e^3 - a^6*d^2*e^3 + a*b^5*d^2*e^3*6i +
 a^5*b*d^2*e^3*6i - 15*a^2*b^4*d^2*e^3 - a^3*b^3*d^2*e^3*20i + 15*a^4*b^2*d
^2*e^3)))^(1/2) - 117964800*a^21*b^42*d^6*e^15 - 841482240*a^23*b^40*d^6*e^
15 + 3829399552*a^25*b^38*d^6*e^15 + 78068580352*a^27*b^36*d^6*e^15 + 49743
8162944*a^29*b^34*d^6*e^15 + 1899895980032*a^31*b^32*d^6*e^15 + 49726955192
32*a^33*b^30*d^6*e^15 + 9371195015168*a^35*b^28*d^6*e^15 + 12890720436224*a
^37*b^26*d^6*e^15 + 12726089809920*a^39*b^24*d^6*e^15 + 8366961197056*a^41*
b^22*d^6*e^15 + 2597662490624*a^43*b^20*d^6*e^15 - 1171836108800*a^45*b^18*
d^6*e^15 - 1986881650688*a^47*b^16*d^6*e^15 - 1237583921152*a^49*b^14*d^6*e
^15 - 449507753984*a^51*b^12*d^6*e^15 - 97476149248*a^53*b^10*d^6*e^15 - 11
931222016*a^55*b^8*d^6*e^15 - 1006632960*a^57*b^6*d^6*e^15 - 134217728*a^59
*b^4*d^6*e^15 - 8388608*a^61*b^2*d^6*e^15) - (e*cot(c + d*x))^(1/2)*(761056
4608*a^27*b^33*d^5*e^13 - 597688320*a^23*b^37*d^5*e^13 - 1671430144*a^25*b^

35*d^5*e^13 - 58982400*a^21*b^39*d^5*e^13 + 85774565376*a^29*b^31*d^5*e^13
+ 385487994880*a^31*b^29*d^5*e^13 + 1104303620096*a^33*b^27*d^5*e^13 + 2240
523796480*a^35*b^25*d^5*e^13 + 3345249468416*a^37*b^23*d^5*e^13 + 371728790
3232*a^39*b^21*d^5*e^13 + 3053967114240*a^41*b^19*d^5*e^13 + 1807474491392*
a^43*b^17*d^5*e^13 + 726513221632*a^45*b^15*d^5*e^13 + 170768990208*a^47*b^
13*d^5*e^13 + 10492051456*a^49*b^11*d^5*e^13 - 4917821440*a^51*b^9*d^5*e^13
 - 923009024*a^53*b^7*d^5*e^13 + 8388608*a^55*b^5*d^5*e^13))*(-1i/(4*(b^6*d
^2*e^3 - a^6*d^2*e^3 + a*b^5*d^2*e^3*6i + a^5*b*d^2*e^3*6i - 15*a^2*b^4*d^2
*e^3 - a^3*b^3*d^2*e^3*20i + 15*a^4*b^2*d^2*e^3)))^(1/2) + 58982400*a^22*b^
35*d^4*e^12 + 920125440*a^24*b^33*d^4*e^12 + 6879444992*a^26*b^31*d^4*e^12
+ 32454475776*a^28*b^29*d^4*e^12 + 107338792960*a^30*b^27*d^4*e^12 + 262062
735360*a^32*b^25*d^4*e^12 + 485059461120*a^34*b^23*d^4*e^12 + 688908140544*
a^36*b^21*d^4*e^12 + 751987064832*a^38*b^19*d^4*e^12 + 626086379520*a^40*b^
17*d^4*e^12 + 390506741760*a^42*b^15*d^4*e^12 + 176637870080*a^44*b^13*d^4*
e^12 + 54704996352*a^46*b^11*d^4*e^12 + 10374086656*a^48*b^9*d^4*e^12 + 908
328960*a^50*b^7*d^4*e^12))*(-1i/(4*(b^6*d^2*e^3 - a^6*d^2*e^3 + a*b^5*d^2*e
^3*6i + a^5*b*d^2*e^3*6i - 15*a^2*b^4*d^2*e^3 - a^3*b^3*d^2*e^3*20i + 15*a^
4*b^2*d^2*e^3)))^(1/2)*2i + (log(((((-1/(b^6*d^2*e^3*1i - a^6*d^2*e^3*1i +
6*a*b^5*d^2*e^3 + 6*a^5*b*d^2*e^3 - a^2*b^4*d^2*e^3*15i - 20*a^3*b^3*d^2*e^
3 + a^4*b^2*d^2*e^3*15i))^(1/2)*((((e*cot(c + d*x))^(1/2)*(471859200*a^22*b
^44*d^7*e^16 + 9500098560*a^24*b^42*d^7*e^16 + 91857354752*a^26*b^40*d^7*e^
16 + 564502986752*a^28*b^38*d^7*e^16 + 2464648527872*a^30*b^36*d^7*e^16 + 8
104469069824*a^32*b^34*d^7*e^16 + 20769933361152*a^34*b^32*d^7*e^16 + 42351
565209600*a^36*b^30*d^7*e^16 + 69534945902592*a^38*b^28*d^7*e^16 + 92434029
608960*a^40*b^26*d^7*e^16 + 99508717355008*a^42*b^24*d^7*e^16 + 86342935511
040*a^44*b^22*d^7*e^16 + 59767095558144*a^46*b^20*d^7*e^16 + 32432589897728
*a^48*b^18*d^7*e^16 + 13411815522304*a^50*b^16*d^7*e^16 + 4030457708544*a^5
2*b^14*d^7*e^16 + 805425905664*a^54*b^12*d^7*e^16 + 86608183296*a^56*b^10*d
^7*e^16 + 1612709888*a^58*b^8*d^7*e^16 + 16777216*a^60*b^6*d^7*e^16 + 16777
2160*a^62*b^4*d^7*e^16 + 16777216*a^64*b^2*d^7*e^16) + ((-1/(b^6*d^2*e^3*1i
 - a^6*d^2*e^3*1i + 6*a*b^5*d^2*e^3 + 6*a^5*b*d^2*e^3 - a^2*b^4*d^2*e^3*15i
 - 20*a^3*b^3*d^2*e^3 + a^4*b^2*d^2*e^3*15i))^(1/2)*(251658240*a^24*b^45*d^
8*e^18 - ((e*cot(c + d*x))^(1/2)*(-1/(b^6*d^2*e^3*1i - a^6*d^2*e^3*1i + 6*a
*b^5*d^2*e^3 + 6*a^5*b*d^2*e^3 - a^2*b^4*d^2*e^3*15i - 20*a^3*b^3*d^2*e^3 +
 a^4*b^2*d^2*e^3*15i))^(1/2)*(134217728*a^27*b^45*d^9*e^19 + 2550136832*a^2
9*b^43*d^9*e^19 + 22817013760*a^31*b^41*d^9*e^19 + 127506841600*a^33*b^39*d
^9*e^19 + 497276682240*a^35*b^37*d^9*e^19 + 1430626762752*a^37*b^35*d^9*e^1
9 + 3121367482368*a^39*b^33*d^9*e^19 + 5202279137280*a^41*b^31*d^9*e^19 + 6
502848921600*a^43*b^29*d^9*e^19 + 5635802398720*a^45*b^27*d^9*e^19 + 225432
0959488*a^47*b^25*d^9*e^19 - 2254320959488*a^49*b^23*d^9*e^19 - 56358023987
20*a^51*b^21*d^9*e^19 - 6502848921600*a^53*b^19*d^9*e^19 - 5202279137280*a^
55*b^17*d^9*e^19 - 3121367482368*a^57*b^15*d^9*e^19 - 1430626762752*a^59*b^
13*d^9*e^19 - 497276682240*a^61*b^11*d^9*e^19 - 127506841600*a^63*b^9*d^9*e
^19 - 22817013760*a^65*b^7*d^9*e^19 - 2550136832*a^67*b^5*d^9*e^19 - 134217
728*a^69*b^3*d^9*e^19))/2 + 5049942016*a^26*b^43*d^8*e^18 + 48368713728*a^2

8*b^41*d^8*e^18 + 293819383808*a^30*b^39*d^8*e^18 + 1268458192896*a^32*b^37*d^8*e^18 + 4132731617280*a^34*b^35*d^8*e^18 + 10531192700928*a^36*b^33*d^8*e^18 + 21462823993344*a^38*b^31*d^8*e^18 + 35469618315264*a^40*b^29*d^8*e^18 + 47896904859648*a^42*b^27*d^8*e^18 + 52983958077440*a^44*b^25*d^8*e^18 + 47896904859648*a^46*b^23*d^8*e^18 + 35090285461504*a^48*b^21*d^8*e^18 + 20487396655104*a^50*b^19*d^8*e^18 + 9230622916608*a^52*b^17*d^8*e^18 + 2994733056000*a^54*b^15*d^8*e^18 + 565576728576*a^56*b^13*d^8*e^18 - 18572378112*a^58*b^11*d^8*e^18 - 50281316352*a^60*b^9*d^8*e^18 - 16089350144*a^62*b^7*d^8*e^18 - 2516582400*a^64*b^5*d^8*e^18 - 167772160*a^66*b^3*d^8*e^18))/2)*(-1/(b^6*d^2*e^3*1i - a^6*d^2*e^3*1i + 6*a*b^5*d^2*e^3 + 6*a^5*b*d^2*e^3 - a^2*b^4*d^2*e^3*15i - 20*a^3*b^3*d^2*e^3 + a^4*b^2*d^2*e^3*15i))^(1/2))/2 - 117964800*a^21*b^42*d^6*e^15 - 841482240*a^23*b^40*d^6*e^15 + 3829399552*a^25*b^38*d^6*e^15 + 78068580352*a^27*b^36*d^6*e^15 + 497438162944*a^29*b^34*d^6*e^15 + 1899895980032*a^31*b^32*d^6*e^15 + 4972695519232*a^33*b^30*d^6*e^15 + 9371195015168*a^35*b^28*d^6*e^15 + 12890720436224*a^37*b^26*d^6*e^15 + 12726089809920*a^39*b^24*d^6*e^15 + 8366961197056*a^41*b^22*d^6*e^15 + 2597662490624*a^43*b^20*d^6*e^15 - 1171836108800*a^45*b^18*d^6*e^15 - 1986881650688*a^47*b^16*d^6*e^15 - 1237583921152*a^49*b^14*d^6*e^15 - 449507753984*a^51*b^12*d^6*e^15 - 97476149248*a^53*b^10*d^6*e^15 - 11931222016*a^55*b^8*d^6*e^15 - 1006632960*a^57*b^6*d^6*e^15 - 134217728*a^59*b^4*d^6*e^15 - 8388608*a^61*b^2*d^6*e^15))/2 - (e*cot(c + d*x))^(1/2)*(7610564608*a^27*b^33*d^5*e^13 - 597688320*a^23*b^37*d^5*e^13 - 1671430144*a^25*b^35*d^5*e^13 - 58982400*a^21*b^39*d^5*e^13 + 85774565376*a^29*b^31*d^5*e^13 + 385487994880*a^31*b^29*d^5*e^13 + 1104303620096*a^33*b^27*d^5*e^13 + 2240523796480*a^35*b^25*d^5*e^13 + 3345249468416*a^37*b^23*d^5*e^13 + 3717287903232*a^39*b^21*d^5*e^13 + 3053967114240*a^41*b^19*d^5*e^13 + 1807474491392*a^43*b^17*d^5*e^13 + 726513221632*a^45*b^15*d^5*e^13 + 170768990208*a^47*b^13*d^5*e^13 + 10492051456*a^49*b^11*d^5*e^13 - 4917821440*a^51*b^9*d^5*e^13 - 923009024*a^53*b^7*d^5*e^13 + 8388608*a^55*b^5*d^5*e^13))*(-1/(b^6*d^2*e^3*1i - a^6*d^2*e^3*1i + 6*a*b^5*d^2*e^3 + 6*a^5*b*d^2*e^3 - a^2*b^4*d^2*e^3*15i - 20*a^3*b^3*d^2*e^3 + a^4*b^2*d^2*e^3*15i))^(1/2))/2 - 29491200*a^22*b^35*d^4*e^12 - 460062720*a^24*b^33*d^4*e^12 - 3439722496*a^26*b^31*d^4*e^12 - 16227237888*a^28*b^29*d^4*e^12 - 53669396480*a^30*b^27*d^4*e^12 - 131031367680*a^32*b^25*d^4*e^12 - 242529730560*a^34*b^23*d^4*e^12 - 344454070272*a^36*b^21*d^4*e^12 - 375993532416*a^38*b^19*d^4*e^12 - 313043189760*a^40*b^17*d^4*e^12 - 195253370880*a^42*b^15*d^4*e^12 - 88318935040*a^44*b^13*d^4*e^12 - 27352498176*a^46*b^11*d^4*e^12 - 5187043328*a^48*b^9*d^4*e^12 - 454164480*a^50*b^7*d^4*e^12)*(-1/(b^6*d^2*e^3*1i - a^6*d^2*e^3*1i + 6*a*b^5*d^2*e^3 + 6*a^5*b*d^2*e^3 - a^2*b^4*d^2*e^3*15i - 20*a^3*b^3*d^2*e^3 + a^4*b^2*d^2*e^3*15i))^(1/2))/2 - log(- ((-1/(4*(b^6*d^2*e^3*1i - a^6*d^2*e^3*1i + 6*a*b^5*d^2*e^3 + 6*a^5*b*d^2*e^3 - a^2*b^4*d^2*e^3*15i - 20*a^3*b^3*d^2*e^3 + a^4*b^2*d^2*e^3*15i)))^(1/2)*(((e*cot(c + d*x))^(1/2)*(471859200*a^22*b^44*d^7*e^16 + 9500098560*a^24*b^42*d^7*e^16 + 91857354752*a^26*b^40*d^7*e^16 + 564502986752*a^28*b^38*d^7*e^16 + 2464648527872*a^30*b^36*d^7*e^16 + 8104469069824*a^32*b^34*d^7*e^16 + 20769933361152*a^34*b^32*d^7*e^16 + 42351565209600*a^3

6*b^30*d^7*e^16 + 69534945902592*a^38*b^28*d^7*e^16 + 92434029608960*a^40*b
^26*d^7*e^16 + 99508717355008*a^42*b^24*d^7*e^16 + 86342935511040*a^44*b^22
*d^7*e^16 + 59767095558144*a^46*b^20*d^7*e^16 + 32432589897728*a^48*b^18*d^
7*e^16 + 13411815522304*a^50*b^16*d^7*e^16 + 4030457708544*a^52*b^14*d^7*e^
16 + 805425905664*a^54*b^12*d^7*e^16 + 86608183296*a^56*b^10*d^7*e^16 + 161
2709888*a^58*b^8*d^7*e^16 + 16777216*a^60*b^6*d^7*e^16 + 167772160*a^62*b^4
*d^7*e^16 + 16777216*a^64*b^2*d^7*e^16) - (-1/(4*(b^6*d^2*e^3*1i - a^6*d^2*
e^3*1i + 6*a*b^5*d^2*e^3 + 6*a^5*b*d^2*e^3 - a^2*b^4*d^2*e^3*15i - 20*a^3*b
^3*d^2*e^3 + a^4*b^2*d^2*e^3*15i)))^(1/2)*((e*cot(c + d*x))^(1/2)*(-1/(4*(b
^6*d^2*e^3*1i - a^6*d^2*e^3*1i + 6*a*b^5*d^2*e^3 + 6*a^5*b*d^2*e^3 - a^2*b^
4*d^2*e^3*15i - 20*a^3*b^3*d^2*e^3 + a^4*b^2*d^2*e^3*15i)))^(1/2)*(13421772
8*a^27*b^45*d^9*e^19 + 2550136832*a^29*b^43*d^9*e^19 + 22817013760*a^31*b^4
1*d^9*e^19 + 127506841600*a^33*b^39*d^9*e^19 + 497276682240*a^35*b^37*d^9*e
^19 + 1430626762752*a^37*b^35*d^9*e^19 + 3121367482368*a^39*b^33*d^9*e^19 +
 5202279137280*a^41*b^31*d^9*e^19 + 6502848921600*a^43*b^29*d^9*e^19 + 5635
802398720*a^45*b^27*d^9*e^19 + 2254320959488*a^47*b^25*d^9*e^19 - 225432095
9488*a^49*b^23*d^9*e^19 - 5635802398720*a^51*b^21*d^9*e^19 - 6502848921600*
a^53*b^19*d^9*e^19 - 5202279137280*a^55*b^17*d^9*e^19 - 3121367482368*a^57*
b^15*d^9*e^19 - 1430626762752*a^59*b^13*d^9*e^19 - 497276682240*a^61*b^11*d
^9*e^19 - 127506841600*a^63*b^9*d^9*e^19 - 22817013760*a^65*b^7*d^9*e^19 -
2550136832*a^67*b^5*d^9*e^19 - 134217728*a^69*b^3*d^9*e^19) + 251658240*a^2
4*b^45*d^8*e^18 + 5049942016*a^26*b^43*d^8*e^18 + 48368713728*a^28*b^41*d^8
*e^18 + 293819383808*a^30*b^39*d^8*e^18 + 1268458192896*a^32*b^37*d^8*e^18
+ 4132731617280*a^34*b^35*d^8*e^18 + 10531192700928*a^36*b^33*d^8*e^18 + 21
462823993344*a^38*b^31*d^8*e^18 + 35469618315264*a^40*b^29*d^8*e^18 + 47896
904859648*a^42*b^27*d^8*e^18 + 52983958077440*a^44*b^25*d^8*e^18 + 47896904
859648*a^46*b^23*d^8*e^18 + 35090285461504*a^48*b^21*d^8*e^18 + 20487396655
104*a^50*b^19*d^8*e^18 + 9230622916608*a^52*b^17*d^8*e^18 + 2994733056000*a
^54*b^15*d^8*e^18 + 565576728576*a^56*b^13*d^8*e^18 - 18572378112*a^58*b^11
*d^8*e^18 - 50281316352*a^60*b^9*d^8*e^18 - 16089350144*a^62*b^7*d^8*e^18 -
 2516582400*a^64*b^5*d^8*e^18 - 167772160*a^66*b^3*d^8*e^18))*(-1/(4*(b^6*d
^2*e^3*1i - a^6*d^2*e^3*1i + 6*a*b^5*d^2*e^3 + 6*a^5*b*d^2*e^3 - a^2*b^4*d^
2*e^3*15i - 20*a^3*b^3*d^2*e^3 + a^4*b^2*d^2*e^3*15i)))^(1/2) + 117964800*a
^21*b^42*d^6*e^15 + 841482240*a^23*b^40*d^6*e^15 - 3829399552*a^25*b^38*d^6
*e^15 - 78068580352*a^27*b^36*d^6*e^15 - 497438162944*a^29*b^34*d^6*e^15 -
1899895980032*a^31*b^32*d^6*e^15 - 4972695519232*a^33*b^30*d^6*e^15 - 93711
95015168*a^35*b^28*d^6*e^15 - 12890720436224*a^37*b^26*d^6*e^15 - 127260898
09920*a^39*b^24*d^6*e^15 - 8366961197056*a^41*b^22*d^6*e^15 - 2597662490624
*a^43*b^20*d^6*e^15 + 1171836108800*a^45*b^18*d^6*e^15 + 1986881650688*a^47
*b^16*d^6*e^15 + 1237583921152*a^49*b^14*d^6*e^15 + 449507753984*a^51*b^12*
d^6*e^15 + 97476149248*a^53*b^10*d^6*e^15 + 11931222016*a^55*b^8*d^6*e^15 +
 1006632960*a^57*b^6*d^6*e^15 + 134217728*a^59*b^4*d^6*e^15 + 8388608*a^61*
b^2*d^6*e^15) - (e*cot(c + d*x))^(1/2)*(7610564608*a^27*b^33*d^5*e^13 - 597
688320*a^23*b^37*d^5*e^13 - 1671430144*a^25*b^35*d^5*e^13 - 58982400*a^21*b
^39*d^5*e^13 + 85774565376*a^29*b^31*d^5*e^13 + 385487994880*a^31*b^29*d^5*

e^13 + 1104303620096*a^33*b^27*d^5*e^13 + 2240523796480*a^35*b^25*d^5*e^13 + 3345249468416*a^37*b^23*d^5*e^13 + 3717287903232*a^39*b^21*d^5*e^13 + 305 3967114240*a^41*b^19*d^5*e^13 + 1807474491392*a^43*b^17*d^5*e^13 + 72651322 1632*a^45*b^15*d^5*e^13 + 170768990208*a^47*b^13*d^5*e^13 + 10492051456*a^4 9*b^11*d^5*e^13 - 4917821440*a^51*b^9*d^5*e^13 - 923009024*a^53*b^7*d^5*e^1 3 + 8388608*a^55*b^5*d^5*e^13))*(-1/(4*(b^6*d^2*e^3*1i - a^6*d^2*e^3*1i + 6 *a*b^5*d^2*e^3 + 6*a^5*b*d^2*e^3 - a^2*b^4*d^2*e^3*15i - 20*a^3*b^3*d^2*e^3 + a^4*b^2*d^2*e^3*15i)))^(1/2) - 29491200*a^22*b^35*d^4*e^12 - 460062720*a ^24*b^33*d^4*e^12 - 3439722496*a^26*b^31*d^4*e^12 - 16227237888*a^28*b^29*d ^4*e^12 - 53669396480*a^30*b^27*d^4*e^12 - 131031367680*a^32*b^25*d^4*e^12 - 242529730560*a^34*b^23*d^4*e^12 - 344454070272*a^36*b^21*d^4*e^12 - 37599 3532416*a^38*b^19*d^4*e^12 - 313043189760*a^40*b^17*d^4*e^12 - 195253370880 *a^42*b^15*d^4*e^12 - 88318935040*a^44*b^13*d^4*e^12 - 27352498176*a^46*b^1 1*d^4*e^12 - 5187043328*a^48*b^9*d^4*e^12 - 454164480*a^50*b^7*d^4*e^12)*(- 1/(4*(b^6*d^2*e^3*1i - a^6*d^2*e^3*1i + 6*a*b^5*d^2*e^3 + 6*a^5*b*d^2*e^3 - a^2*b^4*d^2*e^3*15i - 20*a^3*b^3*d^2*e^3 + a^4*b^2*d^2*e^3*15i)))^(1/2) - (atan(((((e*cot(c + d*x))^(1/2)*(7610564608*a^27*b^33*d^5*e^13 - 597688320* a^23*b^37*d^5*e^13 - 1671430144*a^25*b^35*d^5*e^13 - 58982400*a^21*b^39*d^5 *e^13 + 85774565376*a^29*b^31*d^5*e^13 + 385487994880*a^31*b^29*d^5*e^13 + 1104303620096*a^33*b^27*d^5*e^13 + 2240523796480*a^35*b^25*d^5*e^13 + 33452 49468416*a^37*b^23*d^5*e^13 + 3717287903232*a^39*b^21*d^5*e^13 + 3053967114 240*a^41*b^19*d^5*e^13 + 1807474491392*a^43*b^17*d^5*e^13 + 726513221632*a^ 45*b^15*d^5*e^13 + 170768990208*a^47*b^13*d^5*e^13 + 10492051456*a^49*b^11* d^5*e^13 - 4917821440*a^51*b^9*d^5*e^13 - 923009024*a^53*b^7*d^5*e^13 + 838 8608*a^55*b^5*d^5*e^13) - ((63*a^4 + 15*b^4 + 46*a^2*b^2)*(-a^7*b^5*e^3)^(1 /2)*((((e*cot(c + d*x))^(1/2)*(471859200*a^22*b^44*d^7*e^16 + 9500098560*a^ 24*b^42*d^7*e^16 + 91857354752*a^26*b^40*d^7*e^16 + 564502986752*a^28*b^38* d^7*e^16 + 2464648527872*a^30*b^36*d^7*e^16 + 8104469069824*a^32*b^34*d^7*e ^16 + 20769933361152*a^34*b^32*d^7*e^16 + 42351565209600*a^36*b^30*d^7*e^16 + 69534945902592*a^38*b^28*d^7*e^16 + 92434029608960*a^40*b^26*d^7*e^16 + 99508717355008*a^42*b^24*d^7*e^16 + 86342935511040*a^44*b^22*d^7*e^16 + 597 67095558144*a^46*b^20*d^7*e^16 + 32432589897728*a^48*b^18*d^7*e^16 + 134118 15522304*a^50*b^16*d^7*e^16 + 4030457708544*a^52*b^14*d^7*e^16 + 8054259056 64*a^54*b^12*d^7*e^16 + 86608183296*a^56*b^10*d^7*e^16 + 1612709888*a^58*b^ 8*d^7*e^16 + 16777216*a^60*b^6*d^7*e^16 + 167772160*a^62*b^4*d^7*e^16 + 167 77216*a^64*b^2*d^7*e^16) + ((63*a^4 + 15*b^4 + 46*a^2*b^2)*(-a^7*b^5*e^3)^(1/2)*(251658240*a^24*b^45*d^8*e^18 + 5049942016*a^26*b^43*d^8*e^18 + 483687 13728*a^28*b^41*d^8*e^18 + 293819383808*a^30*b^39*d^8*e^18 + 1268458192896* a^32*b^37*d^8*e^18 + 4132731617280*a^34*b^35*d^8*e^18 + 10531192700928*a^36 *b^33*d^8*e^18 + 21462823993344*a^38*b^31*d^8*e^18 + 35469618315264*a^40*b^ 29*d^8*e^18 + 47896904859648*a^42*b^27*d^8*e^18 + 52983958077440*a^44*b^25* d^8*e^18 + 47896904859648*a^46*b^23*d^8*e^18 + 35090285461504*a^48*b^21*d^8 *e^18 + 20487396655104*a^50*b^19*d^8*e^18 + 9230622916608*a^52*b^17*d^8*e^1 8 + 2994733056000*a^54*b^15*d^8*e^18 + 565576728576*a^56*b^13*d^8*e^18 - 18 572378112*a^58*b^11*d^8*e^18 - 50281316352*a^60*b^9*d^8*e^18 - 16089350144*

a^62*b^7*d^8*e^18 - 2516582400*a^64*b^5*d^8*e^18 - 167772160*a^66*b^3*d^8*e
^18 - ((e*cot(c + d*x))^(1/2)*(63*a^4 + 15*b^4 + 46*a^2*b^2)*(-a^7*b^5*e^3)
^(1/2)*(134217728*a^27*b^45*d^9*e^19 + 2550136832*a^29*b^43*d^9*e^19 + 2281
7013760*a^31*b^41*d^9*e^19 + 127506841600*a^33*b^39*d^9*e^19 + 497276682240
*a^35*b^37*d^9*e^19 + 1430626762752*a^37*b^35*d^9*e^19 + 3121367482368*a^39
*b^33*d^9*e^19 + 5202279137280*a^41*b^31*d^9*e^19 + 6502848921600*a^43*b^29
*d^9*e^19 + 5635802398720*a^45*b^27*d^9*e^19 + 2254320959488*a^47*b^25*d^9*
e^19 - 2254320959488*a^49*b^23*d^9*e^19 - 5635802398720*a^51*b^21*d^9*e^19
- 6502848921600*a^53*b^19*d^9*e^19 - 5202279137280*a^55*b^17*d^9*e^19 - 312
1367482368*a^57*b^15*d^9*e^19 - 1430626762752*a^59*b^13*d^9*e^19 - 49727668
2240*a^61*b^11*d^9*e^19 - 127506841600*a^63*b^9*d^9*e^19 - 22817013760*a^65
*b^7*d^9*e^19 - 2550136832*a^67*b^5*d^9*e^19 - 134217728*a^69*b^3*d^9*e^19)
)/(8*(a^13*d*e^3 + a^7*b^6*d*e^3 + 3*a^9*b^4*d*e^3 + 3*a^11*b^2*d*e^3))))/(
8*(a^13*d*e^3 + a^7*b^6*d*e^3 + 3*a^9*b^4*d*e^3 + 3*a^11*b^2*d*e^3)))*(63*a
^4 + 15*b^4 + 46*a^2*b^2)*(-a^7*b^5*e^3)^(1/2))/(8*(a^13*d*e^3 + a^7*b^6*d*
e^3 + 3*a^9*b^4*d*e^3 + 3*a^11*b^2*d*e^3)) - 117964800*a^21*b^42*d^6*e^15 -
841482240*a^23*b^40*d^6*e^15 + 3829399552*a^25*b^38*d^6*e^15 + 78068580352
*a^27*b^36*d^6*e^15 + 497438162944*a^29*b^34*d^6*e^15 + 1899895980032*a^31*
b^32*d^6*e^15 + 4972695519232*a^33*b^30*d^6*e^15 + 9371195015168*a^35*b^28*
d^6*e^15 + 12890720436224*a^37*b^26*d^6*e^15 + 12726089809920*a^39*b^24*d^6
*e^15 + 8366961197056*a^41*b^22*d^6*e^15 + 2597662490624*a^43*b^20*d^6*e^15
- 1171836108800*a^45*b^18*d^6*e^15 - 1986881650688*a^47*b^16*d^6*e^15 - 12
37583921152*a^49*b^14*d^6*e^15 - 449507753984*a^51*b^12*d^6*e^15 - 97476149
248*a^53*b^10*d^6*e^15 - 11931222016*a^55*b^8*d^6*e^15 - 1006632960*a^57*b^
6*d^6*e^15 - 134217728*a^59*b^4*d^6*e^15 - 8388608*a^61*b^2*d^6*e^15))/(8*(
a^13*d*e^3 + a^7*b^6*d*e^3 + 3*a^9*b^4*d*e^3 + 3*a^11*b^2*d*e^3)))*(63*a^4
+ 15*b^4 + 46*a^2*b^2)*(-a^7*b^5*e^3)^(1/2)*1i)/(8*(a^13*d*e^3 + a^7*b^6*d*
e^3 + 3*a^9*b^4*d*e^3 + 3*a^11*b^2*d*e^3)) + (((e*cot(c + d*x))^(1/2)*(7610
564608*a^27*b^33*d^5*e^13 - 597688320*a^23*b^37*d^5*e^13 - 1671430144*a^25*
b^35*d^5*e^13 - 58982400*a^21*b^39*d^5*e^13 + 85774565376*a^29*b^31*d^5*e^1
3 + 385487994880*a^31*b^29*d^5*e^13 + 1104303620096*a^33*b^27*d^5*e^13 + 22
40523796480*a^35*b^25*d^5*e^13 + 3345249468416*a^37*b^23*d^5*e^13 + 3717287
903232*a^39*b^21*d^5*e^13 + 3053967114240*a^41*b^19*d^5*e^13 + 180747449139
2*a^43*b^17*d^5*e^13 + 726513221632*a^45*b^15*d^5*e^13 + 170768990208*a^47*
b^13*d^5*e^13 + 10492051456*a^49*b^11*d^5*e^13 - 4917821440*a^51*b^9*d^5*e^
13 - 923009024*a^53*b^7*d^5*e^13 + 8388608*a^55*b^5*d^5*e^13) - ((63*a^4 +
15*b^4 + 46*a^2*b^2)*(-a^7*b^5*e^3)^(1/2)*((((e*cot(c + d*x))^(1/2)*(471859
200*a^22*b^44*d^7*e^16 + 9500098560*a^24*b^42*d^7*e^16 + 91857354752*a^26*b
^40*d^7*e^16 + 564502986752*a^28*b^38*d^7*e^16 + 2464648527872*a^30*b^36*d^
7*e^16 + 8104469069824*a^32*b^34*d^7*e^16 + 20769933361152*a^34*b^32*d^7*e^
16 + 42351565209600*a^36*b^30*d^7*e^16 + 69534945902592*a^38*b^28*d^7*e^16
+ 92434029608960*a^40*b^26*d^7*e^16 + 99508717355008*a^42*b^24*d^7*e^16 + 8
6342935511040*a^44*b^22*d^7*e^16 + 59767095558144*a^46*b^20*d^7*e^16 + 3243
2589897728*a^48*b^18*d^7*e^16 + 13411815522304*a^50*b^16*d^7*e^16 + 4030457
708544*a^52*b^14*d^7*e^16 + 805425905664*a^54*b^12*d^7*e^16 + 86608183296*a

^56*b^10*d^7*e^16 + 1612709888*a^58*b^8*d^7*e^16 + 16777216*a^60*b^6*d^7*e^16 + 167772160*a^62*b^4*d^7*e^16 + 16777216*a^64*b^2*d^7*e^16) - ((63*a^4 + 15*b^4 + 46*a^2*b^2)*(-a^7*b^5*e^3)^(1/2)*(251658240*a^24*b^45*d^8*e^18 + 5049942016*a^26*b^43*d^8*e^18 + 48368713728*a^28*b^41*d^8*e^18 + 293819383808*a^30*b^39*d^8*e^18 + 1268458192896*a^32*b^37*d^8*e^18 + 4132731617280*a^34*b^35*d^8*e^18 + 10531192700928*a^36*b^33*d^8*e^18 + 21462823993344*a^38*b^31*d^8*e^18 + 35469618315264*a^40*b^29*d^8*e^18 + 47896904859648*a^42*b^27*d^8*e^18 + 52983958077440*a^44*b^25*d^8*e^18 + 47896904859648*a^46*b^23*d^8*e^18 + 35090285461504*a^48*b^21*d^8*e^18 + 20487396655104*a^50*b^19*d^8*e^18 + 9230622916608*a^52*b^17*d^8*e^18 + 2994733056000*a^54*b^15*d^8*e^18 + 565576728576*a^56*b^13*d^8*e^18 - 18572378112*a^58*b^11*d^8*e^18 - 50281316352*a^60*b^9*d^8*e^18 - 16089350144*a^62*b^7*d^8*e^18 - 2516582400*a^64*b^5*d^8*e^18 - 167772160*a^66*b^3*d^8*e^18 + ((e*cot(c + d*x))^(1/2)*(63*a^4 + 15*b^4 + 46*a^2*b^2)*(-a^7*b^5*e^3)^(1/2)*(134217728*a^27*b^45*d^9*e^19 + 2550136832*a^29*b^43*d^9*e^19 + 22817013760*a^31*b^41*d^9*e^19 + 127506841600*a^33*b^39*d^9*e^19 + 497276682240*a^35*b^37*d^9*e^19 + 1430626762752*a^37*b^35*d^9*e^19 + 3121367482368*a^39*b^33*d^9*e^19 + 5202279137280*a^41*b^31*d^9*e^19 + 6502848921600*a^43*b^29*d^9*e^19 + 5635802398720*a^45*b^27*d^9*e^19 + 2254320959488*a^47*b^25*d^9*e^19 - 2254320959488*a^49*b^23*d^9*e^19 - 5635802398720*a^51*b^21*d^9*e^19 - 6502848921600*a^53*b^19*d^9*e^19 - 5202279137280*a^55*b^17*d^9*e^19 - 3121367482368*a^57*b^15*d^9*e^19 - 1430626762752*a^59*b^13*d^9*e^19 - 497276682240*a^61*b^11*d^9*e^19 - 127506841600*a^63*b^9*d^9*e^19 - 22817013760*a^65*b^7*d^9*e^19 - 2550136832*a^67*b^5*d^9*e^19 - 134217728*a^69*b^3*d^9*e^19))/(8*(a^13*d*e^3 + a^7*b^6*d*e^3 + 3*a^9*b^4*d*e^3 + 3*a^11*b^2*d*e^3))))/(8*(a^13*d*e^3 + a^7*b^6*d*e^3 + 3*a^9*b^4*d*e^3 + 3*a^11*b^2*d*e^3)))*(63*a^4 + 15*b^4 + 46*a^2*b^2)*(-a^7*b^5*e^3)^(1/2))/(8*(a^13*d*e^3 + a^7*b^6*d*e^3 + 3*a^9*b^4*d*e^3 + 3*a^11*b^2*d*e^3)) + 117964800*a^21*b^42*d^6*e^15 + 841482240*a^23*b^40*d^6*e^15 - 3829399552*a^25*b^38*d^6*e^15 - 78068580352*a^27*b^36*d^6*e^15 - 497438162944*a^29*b^34*d^6*e^15 - 1899895980032*a^31*b^32*d^6*e^15 - 4972695519232*a^33*b^30*d^6*e^15 - 9371195015168*a^35*b^28*d^6*e^15 - 12890720436224*a^37*b^26*d^6*e^15 - 12726089809920*a^39*b^24*d^6*e^15 - 8366961197056*a^41*b^22*d^6*e^15 - 2597662490624*a^43*b^20*d^6*e^15 + 1171836108800*a^45*b^18*d^6*e^15 + 1986881650688*a^47*b^16*d^6*e^15 + 1237583921152*a^49*b^14*d^6*e^15 + 449507753984*a^51*b^12*d^6*e^15 + 97476149248*a^53*b^10*d^6*e^15 + 11931222016*a^55*b^8*d^6*e^15 + 1006632960*a^57*b^6*d^6*e^15 + 134217728*a^59*b^4*d^6*e^15 + 8388608*a^61*b^2*d^6*e^15))/(8*(a^13*d*e^3 + a^7*b^6*d*e^3 + 3*a^9*b^4*d*e^3 + 3*a^11*b^2*d*e^3)))*(63*a^4 + 15*b^4 + 46*a^2*b^2)*(-a^7*b^5*e^3)^(1/2)*1i)/(8*(a^13*d*e^3 + a^7*b^6*d*e^3 + 3*a^9*b^4*d*e^3 + 3*a^11*b^2*d*e^3)))/(58982400*a^22*b^35*d^4*e^12 + 920125440*a^24*b^33*d^4*e^12 + 6879444992*a^26*b^31*d^4*e^12 + 32454475776*a^28*b^29*d^4*e^12 + 107338792960*a^30*b^27*d^4*e^12 + 262062735360*a^32*b^25*d^4*e^12 + 485059461120*a^34*b^23*d^4*e^12 + 688908140544*a^36*b^21*d^4*e^12 + 751987064832*a^38*b^19*d^4*e^12 + 626086379520*a^40*b^17*d^4*e^12 + 390506741760*a^42*b^15*d^4*e^12 + 176637870080*a^44*b^13*d^4*e^12 + 54704996352*a^46*b^11*d^4*e^12 + 10374086656

*a^48*b^9*d^4*e^12 + 908328960*a^50*b^7*d^4*e^12 + (((e*cot(c + d*x))^(1/2)
*(7610564608*a^27*b^33*d^5*e^13 - 597688320*a^23*b^37*d^5*e^13 - 1671430144
*a^25*b^35*d^5*e^13 - 58982400*a^21*b^39*d^5*e^13 + 85774565376*a^29*b^31*d
^5*e^13 + 385487994880*a^31*b^29*d^5*e^13 + 1104303620096*a^33*b^27*d^5*e^1
3 + 2240523796480*a^35*b^25*d^5*e^13 + 3345249468416*a^37*b^23*d^5*e^13 + 3
717287903232*a^39*b^21*d^5*e^13 + 3053967114240*a^41*b^19*d^5*e^13 + 180747
4491392*a^43*b^17*d^5*e^13 + 726513221632*a^45*b^15*d^5*e^13 + 170768990208
*a^47*b^13*d^5*e^13 + 10492051456*a^49*b^11*d^5*e^13 - 4917821440*a^51*b^9*
d^5*e^13 - 923009024*a^53*b^7*d^5*e^13 + 8388608*a^55*b^5*d^5*e^13) - ((63*
a^4 + 15*b^4 + 46*a^2*b^2)*(-a^7*b^5*e^3)^(1/2)*((((e*cot(c + d*x))^(1/2)*(
471859200*a^22*b^44*d^7*e^16 + 9500098560*a^24*b^42*d^7*e^16 + 91857354752*
a^26*b^40*d^7*e^16 + 564502986752*a^28*b^38*d^7*e^16 + 2464648527872*a^30*b
^36*d^7*e^16 + 8104469069824*a^32*b^34*d^7*e^16 + 20769933361152*a^34*b^32*
d^7*e^16 + 42351565209600*a^36*b^30*d^7*e^16 + 69534945902592*a^38*b^28*d^7
*e^16 + 92434029608960*a^40*b^26*d^7*e^16 + 99508717355008*a^42*b^24*d^7*e^
16 + 86342935511040*a^44*b^22*d^7*e^16 + 59767095558144*a^46*b^20*d^7*e^16
+ 32432589897728*a^48*b^18*d^7*e^16 + 13411815522304*a^50*b^16*d^7*e^16 + 4
030457708544*a^52*b^14*d^7*e^16 + 805425905664*a^54*b^12*d^7*e^16 + 8660818
3296*a^56*b^10*d^7*e^16 + 1612709888*a^58*b^8*d^7*e^16 + 16777216*a^60*b^6*
d^7*e^16 + 167772160*a^62*b^4*d^7*e^16 + 16777216*a^64*b^2*d^7*e^16) + ((63
*a^4 + 15*b^4 + 46*a^2*b^2)*(-a^7*b^5*e^3)^(1/2)*(251658240*a^24*b^45*d^8*e
^18 + 5049942016*a^26*b^43*d^8*e^18 + 48368713728*a^28*b^41*d^8*e^18 + 2938
19383808*a^30*b^39*d^8*e^18 + 1268458192896*a^32*b^37*d^8*e^18 + 4132731617
280*a^34*b^35*d^8*e^18 + 10531192700928*a^36*b^33*d^8*e^18 + 21462823993344
*a^38*b^31*d^8*e^18 + 35469618315264*a^40*b^29*d^8*e^18 + 47896904859648*a^
42*b^27*d^8*e^18 + 52983958077440*a^44*b^25*d^8*e^18 + 47896904859648*a^46*
b^23*d^8*e^18 + 35090285461504*a^48*b^21*d^8*e^18 + 20487396655104*a^50*b^1
9*d^8*e^18 + 9230622916608*a^52*b^17*d^8*e^18 + 2994733056000*a^54*b^15*d^8
*e^18 + 565576728576*a^56*b^13*d^8*e^18 - 18572378112*a^58*b^11*d^8*e^18 -
50281316352*a^60*b^9*d^8*e^18 - 16089350144*a^62*b^7*d^8*e^18 - 2516582400*
a^64*b^5*d^8*e^18 - 167772160*a^66*b^3*d^8*e^18 - ((e*cot(c + d*x))^(1/2)*(
63*a^4 + 15*b^4 + 46*a^2*b^2)*(-a^7*b^5*e^3)^(1/2)*(134217728*a^27*b^45*d^9
*e^19 + 2550136832*a^29*b^43*d^9*e^19 + 22817013760*a^31*b^41*d^9*e^19 + 12
7506841600*a^33*b^39*d^9*e^19 + 497276682240*a^35*b^37*d^9*e^19 + 143062676
2752*a^37*b^35*d^9*e^19 + 3121367482368*a^39*b^33*d^9*e^19 + 5202279137280*
a^41*b^31*d^9*e^19 + 6502848921600*a^43*b^29*d^9*e^19 + 5635802398720*a^45*
b^27*d^9*e^19 + 2254320959488*a^47*b^25*d^9*e^19 - 2254320959488*a^49*b^23*
d^9*e^19 - 5635802398720*a^51*b^21*d^9*e^19 - 6502848921600*a^53*b^19*d^9*e
^19 - 5202279137280*a^55*b^17*d^9*e^19 - 3121367482368*a^57*b^15*d^9*e^19 -
1430626762752*a^59*b^13*d^9*e^19 - 497276682240*a^61*b^11*d^9*e^19 - 12750
6841600*a^63*b^9*d^9*e^19 - 22817013760*a^65*b^7*d^9*e^19 - 2550136832*a^67
*b^5*d^9*e^19 - 134217728*a^69*b^3*d^9*e^19))/(8*(a^13*d*e^3 + a^7*b^6*d*e^
3 + 3*a^9*b^4*d*e^3 + 3*a^11*b^2*d*e^3))))/(8*(a^13*d*e^3 + a^7*b^6*d*e^3 +
3*a^9*b^4*d*e^3 + 3*a^11*b^2*d*e^3)))*(63*a^4 + 15*b^4 + 46*a^2*b^2)*(-a^7
*b^5*e^3)^(1/2))/(8*(a^13*d*e^3 + a^7*b^6*d*e^3 + 3*a^9*b^4*d*e^3 + 3*a^11*

b^2*d*e^3)) - 117964800*a^21*b^42*d^6*e^15 - 841482240*a^23*b^40*d^6*e^15 + 3829399552*a^25*b^38*d^6*e^15 + 78068580352*a^27*b^36*d^6*e^15 + 497438162 944*a^29*b^34*d^6*e^15 + 1899895980032*a^31*b^32*d^6*e^15 + 4972695519232*a ^33*b^30*d^6*e^15 + 9371195015168*a^35*b^28*d^6*e^15 + 12890720436224*a^37* b^26*d^6*e^15 + 12726089809920*a^39*b^24*d^6*e^15 + 8366961197056*a^41*b^22 *d^6*e^15 + 2597662490624*a^43*b^20*d^6*e^15 - 1171836108800*a^45*b^18*d^6* e^15 - 1986881650688*a^47*b^16*d^6*e^15 - 1237583921152*a^49*b^14*d^6*e^15 - 449507753984*a^51*b^12*d^6*e^15 - 97476149248*a^53*b^10*d^6*e^15 - 119312 22016*a^55*b^8*d^6*e^15 - 1006632960*a^57*b^6*d^6*e^15 - 134217728*a^59*b^4 *d^6*e^15 - 8388608*a^61*b^2*d^6*e^15))/(8*(a^13*d*e^3 + a^7*b^6*d*e^3 + 3* a^9*b^4*d*e^3 + 3*a^11*b^2*d*e^3)))*(63*a^4 + 15*b^4 + 46*a^2*b^2)*(-a^7*b^ 5*e^3)^(1/2))/(8*(a^13*d*e^3 + a^7*b^6*d*e^3 + 3*a^9*b^4*d*e^3 + 3*a^11*b^2 *d*e^3)) - (((e*cot(c + d*x))^(1/2)*(7610564608*a^27*b^33*d^5*e^13 - 597688 320*a^23*b^37*d^5*e^13 - 1671430144*a^25*b^35*d^5*e^13 - 58982400*a^21*b^39 *d^5*e^13 + 85774565376*a^29*b^31*d^5*e^13 + 385487994880*a^31*b^29*d^5*e^1 3 + 1104303620096*a^33*b^27*d^5*e^13 + 2240523796480*a^35*b^25*d^5*e^13 + 3 345249468416*a^37*b^23*d^5*e^13 + 3717287903232*a^39*b^21*d^5*e^13 + 305396 7114240*a^41*b^19*d^5*e^13 + 1807474491392*a^43*b^17*d^5*e^13 + 72651322163 2*a^45*b^15*d^5*e^13 + 170768990208*a^47*b^13*d^5*e^13 + 10492051456*a^49*b ^11*d^5*e^13 - 4917821440*a^51*b^9*d^5*e^13 - 923009024*a^53*b^7*d^5*e^13 + 8388608*a^55*b^5*d^5*e^13) - ((63*a^4 + 15*b^4 + 46*a^2*b^2)*(-a^7*b^5*e^3)^(1/2)*((((e*cot(c + d*x))^(1/2)*(471859200*a^22*b^44*d^7*e^16 + 950009856 0*a^24*b^42*d^7*e^16 + 91857354752*a^26*b^40*d^7*e^16 + 564502986752*a^28*b ^38*d^7*e^16 + 2464648527872*a^30*b^36*d^7*e^16 + 8104469069824*a^32*b^34*d ^7*e^16 + 20769933361152*a^34*b^32*d^7*e^16 + 42351565209600*a^36*b^30*d^7* e^16 + 69534945902592*a^38*b^28*d^7*e^16 + 92434029608960*a^40*b^26*d^7*e^1 6 + 99508717355008*a^42*b^24*d^7*e^16 + 86342935511040*a^44*b^22*d^7*e^16 + 59767095558144*a^46*b^20*d^7*e^16 + 32432589897728*a^48*b^18*d^7*e^16 + 13 411815522304*a^50*b^16*d^7*e^16 + 4030457708544*a^52*b^14*d^7*e^16 + 805425 905664*a^54*b^12*d^7*e^16 + 86608183296*a^56*b^10*d^7*e^16 + 1612709888*a^5 8*b^8*d^7*e^16 + 16777216*a^60*b^6*d^7*e^16 + 167772160*a^62*b^4*d^7*e^16 + 16777216*a^64*b^2*d^7*e^16) - ((63*a^4 + 15*b^4 + 46*a^2*b^2)*(-a^7*b^5*e^ 3)^(1/2)*(251658240*a^24*b^45*d^8*e^18 + 5049942016*a^26*b^43*d^8*e^18 + 48 368713728*a^28*b^41*d^8*e^18 + 293819383808*a^30*b^39*d^8*e^18 + 1268458192 896*a^32*b^37*d^8*e^18 + 4132731617280*a^34*b^35*d^8*e^18 + 10531192700928* a^36*b^33*d^8*e^18 + 21462823993344*a^38*b^31*d^8*e^18 + 35469618315264*a^4 0*b^29*d^8*e^18 + 47896904859648*a^42*b^27*d^8*e^18 + 52983958077440*a^44*b ^25*d^8*e^18 + 47896904859648*a^46*b^23*d^8*e^18 + 35090285461504*a^48*b^21 *d^8*e^18 + 20487396655104*a^50*b^19*d^8*e^18 + 9230622916608*a^52*b^17*d^8 *e^18 + 2994733056000*a^54*b^15*d^8*e^18 + 565576728576*a^56*b^13*d^8*e^18 - 18572378112*a^58*b^11*d^8*e^18 - 50281316352*a^60*b^9*d^8*e^18 - 16089350 144*a^62*b^7*d^8*e^18 - 2516582400*a^64*b^5*d^8*e^18 - 167772160*a^66*b^3*d ^8*e^18 + ((e*cot(c + d*x))^(1/2)*(63*a^4 + 15*b^4 + 46*a^2*b^2)*(-a^7*b^5* e^3)^(1/2)*(134217728*a^27*b^45*d^9*e^19 + 2550136832*a^29*b^43*d^9*e^19 + 22817013760*a^31*b^41*d^9*e^19 + 127506841600*a^33*b^39*d^9*e^19 + 49727668

2240*a^35*b^37*d^9*e^19 + 1430626762752*a^37*b^35*d^9*e^19 + 3121367482368*a^39*b^33*d^9*e^19 + 5202279137280*a^41*b^31*d^9*e^19 + 6502848921600*a^43*b^29*d^9*e^19 + 5635802398720*a^45*b^27*d^9*e^19 + 2254320959488*a^47*b^25*d^9*e^19 - 2254320959488*a^49*b^23*d^9*e^19 - 5635802398720*a^51*b^21*d^9*e^19 - 6502848921600*a^53*b^19*d^9*e^19 - 5202279137280*a^55*b^17*d^9*e^19 - 3121367482368*a^57*b^15*d^9*e^19 - 1430626762752*a^59*b^13*d^9*e^19 - 497276682240*a^61*b^11*d^9*e^19 - 127506841600*a^63*b^9*d^9*e^19 - 22817013760*a^65*b^7*d^9*e^19 - 2550136832*a^67*b^5*d^9*e^19 - 134217728*a^69*b^3*d^9*e^19))/(8*(a^13*d*e^3 + a^7*b^6*d*e^3 + 3*a^9*b^4*d*e^3 + 3*a^11*b^2*d*e^3))))/(8*(a^13*d*e^3 + a^7*b^6*d*e^3 + 3*a^9*b^4*d*e^3 + 3*a^11*b^2*d*e^3)))*(63*a^4 + 15*b^4 + 46*a^2*b^2)*(-a^7*b^5*e^3)^(1/2))/(8*(a^13*d*e^3 + a^7*b^6*d*e^3 + 3*a^9*b^4*d*e^3 + 3*a^11*b^2*d*e^3)) + 117964800*a^21*b^42*d^6*e^15 + 841482240*a^23*b^40*d^6*e^15 - 3829399552*a^25*b^38*d^6*e^15 - 78068580352*a^27*b^36*d^6*e^15 - 497438162944*a^29*b^34*d^6*e^15 - 1899895980032*a^31*b^32*d^6*e^15 - 4972695519232*a^33*b^30*d^6*e^15 - 9371195015168*a^35*b^28*d^6*e^15 - 12890720436224*a^37*b^26*d^6*e^15 - 12726089809920*a^39*b^24*d^6*e^15 - 8366961197056*a^41*b^22*d^6*e^15 - 2597662490624*a^43*b^20*d^6*e^15 + 1171836108800*a^45*b^18*d^6*e^15 + 1986881650688*a^47*b^16*d^6*e^15 + 1237583921152*a^49*b^14*d^6*e^15 + 449507753984*a^51*b^12*d^6*e^15 + 97476149248*a^53*b^10*d^6*e^15 + 11931222016*a^55*b^8*d^6*e^15 + 1006632960*a^57*b^6*d^6*e^15 + 134217728*a^59*b^4*d^6*e^15 + 8388608*a^61*b^2*d^6*e^15))/(8*(a^13*d*e^3 + a^7*b^6*d*e^3 + 3*a^9*b^4*d*e^3 + 3*a^11*b^2*d*e^3)))*(63*a^4 + 15*b^4 + 46*a^2*b^2)*(-a^7*b^5*e^3)^(1/2))/(8*(a^13*d*e^3 + a^7*b^6*d*e^3 + 3*a^9*b^4*d*e^3 + 3*a^11*b^2*d*e^3))))*(63*a^4 + 15*b^4 + 46*a^2*b^2)*(-a^7*b^5*e^3)^(1/2)*1i)/(4*(a^13*d*e^3 + a^7*b^6*d*e^3 + 3*a^9*b^4*d*e^3 + 3*a^11*b^2*d*e^3))

3.88 $\int (a+b\cot(c+dx))^n\,dx$

Optimal result

Integrand size = 12, antiderivative size = 167

$$\int (a+b\cot(c+dx))^n\,dx = -\frac{b(a+b\cot(c+dx))^{1+n}\operatorname{Hypergeometric2F1}\left(1,1+n,2+n,\frac{a+b\cot(c+dx)}{a-\sqrt{-b^2}}\right)}{2\sqrt{-b^2}\left(a-\sqrt{-b^2}\right)d(1+n)} + \frac{b(a+b\cot(c+dx))^{1+n}\operatorname{Hypergeometric2F1}\left(1,1+n,2+n,\frac{a+b\cot(c+dx)}{a+\sqrt{-b^2}}\right)}{2\sqrt{-b^2}\left(a+\sqrt{-b^2}\right)d(1+n)}$$

```
[Out] -1/2*b*(a+b*cot(d*x+c))^(1+n)*hypergeom([1, 1+n],[2+n],(a+b*cot(d*x+c))/(a-
(-b^2)^(1/2)))/d/(1+n)/(a-(-b^2)^(1/2))/(-b^2)^(1/2)+1/2*b*(a+b*cot(d*x+c))
^(1+n)*hypergeom([1, 1+n],[2+n],(a+b*cot(d*x+c))/(a+(-b^2)^(1/2)))/d/(1+n)/
(-b^2)^(1/2)/(a+(-b^2)^(1/2))
```

Rubi [A] (verified)

Time = 0.32 (sec) , antiderivative size = 167, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {3566, 726, 70}

$$\int (a+b\cot(c+dx))^n\,dx = \frac{b(a+b\cot(c+dx))^{n+1}\operatorname{Hypergeometric2F1}\left(1,n+1,n+2,\frac{a+b\cot(c+dx)}{a+\sqrt{-b^2}}\right)}{2\sqrt{-b^2}d(n+1)\left(a+\sqrt{-b^2}\right)} - \frac{b(a+b\cot(c+dx))^{n+1}\operatorname{Hypergeometric2F1}\left(1,n+1,n+2,\frac{a+b\cot(c+dx)}{a-\sqrt{-b^2}}\right)}{2\sqrt{-b^2}d(n+1)\left(a-\sqrt{-b^2}\right)}$$

```
[In] Int[(a + b*Cot[c + d*x])^n,x]
```

```
[Out] -1/2*(b*(a + b*Cot[c + d*x])^(1 + n)*Hypergeometric2F1[1, 1 + n, 2 + n, (a
+ b*Cot[c + d*x])/(a - Sqrt[-b^2])])/(Sqrt[-b^2]*(a - Sqrt[-b^2])*d*(1 + n)
) + (b*(a + b*Cot[c + d*x])^(1 + n)*Hypergeometric2F1[1, 1 + n, 2 + n, (a +
 b*Cot[c + d*x])/(a + Sqrt[-b^2])])/(2*Sqrt[-b^2]*(a + Sqrt[-b^2])*d*(1 + n
))
```

Rule 70

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[(b
*c - a*d)^n*((a + b*x)^(m + 1)/(b^(n + 1)*(m + 1)))*Hypergeometric2F1[-n, m
 + 1, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m}, x]
 && NeQ[b*c - a*d, 0] &&  !IntegerQ[m] && IntegerQ[n]
```

Rule 726

```
Int[((d_) + (e_.)*(x_))^(m_)/((a_) + (c_.)*(x_)^2), x_Symbol] :> Int[Expand
Integrand[(d + e*x)^m, 1/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e, m}, x] &
& NeQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[m]
```

Rule 3566

```
Int[((a_) + (b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[b/d, Sub
st[Int[(a + x)^n/(b^2 + x^2), x], x, b*Tan[c + d*x]], x] /; FreeQ[{a, b, c,
 d, n}, x] && NeQ[a^2 + b^2, 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\frac{b\text{Subst}\left(\int \frac{(a+x)^n}{b^2+x^2}\,dx, x, b\cot(c+dx)\right)}{d}\\
&= -\frac{b\text{Subst}\left(\int \left(\frac{\sqrt{-b^2}(a+x)^n}{2b^2\left(\sqrt{-b^2}-x\right)}+\frac{\sqrt{-b^2}(a+x)^n}{2b^2\left(\sqrt{-b^2}+x\right)}\right)\,dx, x, b\cot(c+dx)\right)}{d}\\
&= \frac{b\text{Subst}\left(\int \frac{(a+x)^n}{\sqrt{-b^2}-x}\,dx, x, b\cot(c+dx)\right)}{2\sqrt{-b^2}d}+\frac{b\text{Subst}\left(\int \frac{(a+x)^n}{\sqrt{-b^2}+x}\,dx, x, b\cot(c+dx)\right)}{2\sqrt{-b^2}d}\\
&= -\frac{b(a+b\cot(c+dx))^{1+n}\,\text{Hypergeometric2F1}\left(1,1+n,2+n,\frac{a+b\cot(c+dx)}{a-\sqrt{-b^2}}\right)}{2\sqrt{-b^2}\left(a-\sqrt{-b^2}\right)d(1+n)}\\
&\quad+\frac{b(a+b\cot(c+dx))^{1+n}\,\text{Hypergeometric2F1}\left(1,1+n,2+n,\frac{a+b\cot(c+dx)}{a+\sqrt{-b^2}}\right)}{2\sqrt{-b^2}\left(a+\sqrt{-b^2}\right)d(1+n)}
\end{aligned}$$

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.36 (sec) , antiderivative size = 118, normalized size of antiderivative = 0.71

$$\int (a+b\cot(c+dx))^n\,dx = \frac{(a+b\cot(c+dx))^{1+n}\left((a+ib)\,\text{Hypergeometric2F1}\left(1,1+n,2+n,\frac{a+b\cot(c+dx)}{a-ib}\right)-(a-ib)\,\text{Hypergeome}\right.}{2(a-ib)(-ia+b)d(1+n)}$$

```
[In] Integrate[(a + b*Cot[c + d*x])^n,x]
```

```
[Out] ((a + b*Cot[c + d*x])^(1 + n)*((a + I*b)*Hypergeometric2F1[1, 1 + n, 2 + n,
 (a + b*Cot[c + d*x])/(a - I*b)] - (a - I*b)*Hypergeometric2F1[1, 1 + n, 2
+ n, (a + b*Cot[c + d*x])/(a + I*b)]))/(2*(a - I*b)*((-I)*a + b)*d*(1 + n))
```

Maple [F]

$$\int (a+b\cot(dx+c))^n\,dx$$

```
[In] int((a+b*cot(d*x+c))^n,x)
```

```
[Out] int((a+b*cot(d*x+c))^n,x)
```

Fricas [F]

$$\int (a+b\cot(c+dx))^n\,dx = \int (b\cot(dx+c)+a)^n\,dx$$

```
[In] integrate((a+b*cot(d*x+c))^n,x, algorithm="fricas")
```

```
[Out] integral((b*cot(d*x + c) + a)^n, x)
```

Sympy [F]

$$\int (a+b\cot(c+dx))^n\,dx = \int (a+b\cot(c+dx))^n\,dx$$

```
[In] integrate((a+b*cot(d*x+c))**n,x)
```

```
[Out] Integral((a + b*cot(c + d*x))**n, x)
```

Maxima [F]

$$\int (a + b\cot(c + dx))^n \, dx = \int (b\cot(dx + c) + a)^n \, dx$$

```
[In] integrate((a+b*cot(d*x+c))^n,x, algorithm="maxima")
[Out] integrate((b*cot(d*x + c) + a)^n, x)
```

Giac [F]

$$\int (a + b\cot(c + dx))^n \, dx = \int (b\cot(dx + c) + a)^n \, dx$$

```
[In] integrate((a+b*cot(d*x+c))^n,x, algorithm="giac")
[Out] integrate((b*cot(d*x + c) + a)^n, x)
```

Mupad [F(-1)]

Timed out.

$$\int (a + b\cot(c + dx))^n \, dx = \int (a + b\cot(c + d\,x))^n \, dx$$

```
[In] int((a + b*cot(c + d*x))^n,x)
[Out] int((a + b*cot(c + d*x))^n, x)
```

3.89 $\int (a+b\cot(e+fx))^m(d\tan(e+fx))^n\,dx$

Optimal result

Integrand size = 23, antiderivative size = 193

$$\int (a+b\cot(e+fx))^m(d\tan(e+fx))^n\,dx = -\frac{\text{AppellF1}\left(1-n,-m,1,2-n,-\frac{b\cot(e+fx)}{a},-i\cot(e+fx)\right)\cot(e+fx)(a+b\cot(e+fx))^m\left(1+\frac{b\cot(e+fx)}{a}\right.}{2f(1-n)} - \frac{\text{AppellF1}\left(1-n,-m,1,2-n,-\frac{b\cot(e+fx)}{a},i\cot(e+fx)\right)\cot(e+fx)(a+b\cot(e+fx))^m\left(1+\frac{b\cot(e+f}{a}\right.}{2f(1-n)}$$

```
[Out] -1/2*AppellF1(1-n,1,-m,2-n,-I*cot(f*x+e),-b*cot(f*x+e)/a)*cot(f*x+e)*(a+b*c
ot(f*x+e))^m*(d*tan(f*x+e))^n/f/(1-n)/((1+b*cot(f*x+e)/a)^m)-1/2*AppellF1(1
-n,1,-m,2-n,I*cot(f*x+e),-b*cot(f*x+e)/a)*cot(f*x+e)*(a+b*cot(f*x+e))^m*(d*
tan(f*x+e))^n/f/(1-n)/((1+b*cot(f*x+e)/a)^m)
```

Rubi [A] (verified)

Time = 0.33 (sec) , antiderivative size = 193, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.217$, Rules used = {4327, 3656, 926, 140, 138}

$$\int (a+b\cot(e+fx))^m(d\tan(e+fx))^n\,dx = -\frac{\cot(e+fx)(d\tan(e+fx))^n(a+b\cot(e+fx))^m\left(\frac{b\cot(e+fx)}{a}+1\right)^{-m}\text{AppellF1}\left(1-n,-m,1,2-n,-\frac{b}{}\right.}{2f(1-n)} - \frac{\cot(e+fx)(d\tan(e+fx))^n(a+b\cot(e+fx))^m\left(\frac{b\cot(e+fx)}{a}+1\right)^{-m}\text{AppellF1}\left(1-n,-m,1,2-n,-\frac{b}{}\right.}{2f(1-n)}$$

```
[In] Int[(a + b*Cot[e + f*x])^m*(d*Tan[e + f*x])^n,x]
```

```
[Out] -1/2*(AppellF1[1 - n, -m, 1, 2 - n, -((b*Cot[e + f*x])/a), (-I)*Cot[e + f*x
]]*Cot[e + f*x]*(a + b*Cot[e + f*x])^m*(d*Tan[e + f*x])^n)/(f*(1 - n)*(1 +
(b*Cot[e + f*x])/a)^m) - (AppellF1[1 - n, -m, 1, 2 - n, -((b*Cot[e + f*x])/
a), I*Cot[e + f*x]]*Cot[e + f*x]*(a + b*Cot[e + f*x])^m*(d*Tan[e + f*x])^n)
/(2*f*(1 - n)*(1 + (b*Cot[e + f*x])/a)^m)
```

Rule 138

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_)*((e_) + (f_.)*(x_))^(p_), x_
Symbol] :> Simp[c^n*e^p*((b*x)^(m + 1)/(b*(m + 1)))*AppellF1[m + 1, -n, -p,
 m + 2, (-d)*(x/c), (-f)*(x/e)], x] /; FreeQ[{b, c, d, e, f, m, n, p}, x] &
&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[c, 0] && (IntegerQ[p] || GtQ[e, 0])
```

Rule 140

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_)*((e_) + (f_.)*(x_))^(p_), x_
Symbol] :> Dist[c^IntPart[n]*((c + d*x)^FracPart[n]/(1 + d*(x/c))^FracPart[
n]), Int[(b*x)^m*(1 + d*(x/c))^n*(e + f*x)^p, x], x] /; FreeQ[{b, c, d, e,
f, m, n, p}, x] &&  !IntegerQ[m] &&  !IntegerQ[n] &&  !GtQ[c, 0]
```

Rule 926

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_))/((a_) + (c_.)*(x_
)^2), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g*x)^n, 1/(a + c*x^
2), x], x] /; FreeQ[{a, c, d, e, f, g, m, n}, x] && NeQ[c*d^2 + a*e^2, 0] &
&  !IntegerQ[m] &&  !IntegerQ[n]
```

Rule 3656

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*tan[(e_.) +
(f_.)*(x_)])^(n_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, D
ist[ff/f, Subst[Int[(a + b*ff*x)^m*((c + d*ff*x)^n/(1 + ff^2*x^2)), x], x,
Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && NeQ[b*c - a*
d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0]
```

Rule 4327

```
Int[(u_)*((c_.)*tan[(a_.) + (b_.)*(x_)])^(m_.), x_Symbol] :> Dist[(c*Cot[a
+ b*x])^m*(c*Tan[a + b*x])^m, Int[ActivateTrig[u]/(c*Cot[a + b*x])^m, x], x
] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownCotangentIntegrandQ[u,
 x]
```

Rubi steps

$$\text{integral} = ((d\cot(e+fx))^n(d\tan(e+fx))^n)\int(d\cot(e+fx))^{-n}(a+b\cot(e+fx))^m\,dx$$

$$= -\frac{((d\cot(e+fx))^n(d\tan(e+fx))^n)\,\text{Subst}\left(\int \frac{(dx)^{-n}(a+bx)^m}{1+x^2}\,dx, x, \cot(e+fx)\right)}{f}$$

$$= -\frac{((d\cot(e+fx))^n(d\tan(e+fx))^n)\,\text{Subst}\left(\int \left(\frac{i(dx)^{-n}(a+bx)^m}{2(i-x)}+\frac{i(dx)^{-n}(a+bx)^m}{2(i+x)}\right)dx, x, \cot(e+fx)\right)}{f}$$

$$= -\frac{(i(d\cot(e+fx))^n(d\tan(e+fx))^n)\,\text{Subst}\left(\int \frac{(dx)^{-n}(a+bx)^m}{i-x}\,dx, x, \cot(e+fx)\right)}{2f} - \frac{(i(d\cot(e+fx))^n(d\tan(e+fx))^n)\,\text{Subst}\left(\int \frac{(dx)^{-n}(a+bx)^m}{i+x}\,dx, x, \cot(e+fx)\right)}{2f}$$

$$= -\frac{\left(i(d\cot(e+fx))^n(a+b\cot(e+fx))^m\left(1+\frac{b\cot(e+fx)}{a}\right)^{-m}(d\tan(e+fx))^n\right)\text{Subst}\left(\int \frac{(dx)^{-n}\left(1+\right.}{i-x}\right.}{2f} - \frac{\left(i(d\cot(e+fx))^n(a+b\cot(e+fx))^m\left(1+\frac{b\cot(e+fx)}{a}\right)^{-m}(d\tan(e+fx))^n\right)\text{Subst}\left(\int \frac{(dx)^{-n}\left(1+\right.}{i+x}\right.}{2f}$$

$$= -\frac{\text{AppellF1}\left(1-n,-m,1,2-n,-\frac{b\cot(e+fx)}{a},-i\cot(e+fx)\right)\cot(e+fx)(a+b\cot(e+fx))^m\left(1\right.}{2f(1-n)} - \frac{\text{AppellF1}\left(1-n,-m,1,2-n,-\frac{b\cot(e+fx)}{a},i\cot(e+fx)\right)\cot(e+fx)(a+b\cot(e+fx))^m\left(1+\right.}{2f(1-n)}$$

Mathematica [F]

$$\int (a+b\cot(e+fx))^m(d\tan(e+fx))^n\,dx = \int (a+b\cot(e+fx))^m(d\tan(e+fx))^n\,dx$$

```
[In] Integrate[(a + b*Cot[e + f*x])^m*(d*Tan[e + f*x])^n,x]
[Out] Integrate[(a + b*Cot[e + f*x])^m*(d*Tan[e + f*x])^n, x]
```

Maple [F]

$$\int \left(a+b\cot\left(fx+e\right)\right)^{m} \left(d\tan\left(fx+e\right)\right)^{n} dx$$

```
[In] int((a+b*cot(f*x+e))^m*(d*tan(f*x+e))^n,x)
[Out] int((a+b*cot(f*x+e))^m*(d*tan(f*x+e))^n,x)
```

Fricas [F]

$$\int (a+b\cot(e+fx))^m(d\tan(e+fx))^n\,dx = \int \left(b\cot\left(fx+e\right)+a\right)^m\left(d\tan\left(fx+e\right)\right)^n\,dx$$

```
[In] integrate((a+b*cot(f*x+e))^m*(d*tan(f*x+e))^n,x, algorithm="fricas")
[Out] integral((b*cot(f*x + e) + a)^m*(d*tan(f*x + e))^n, x)
```

Sympy [F]

$$\int (a+b\cot(e+fx))^m(d\tan(e+fx))^n\,dx = \int \left(d\tan\left(e+fx\right)\right)^n\left(a+b\cot\left(e+fx\right)\right)^m\,dx$$

```
[In] integrate((a+b*cot(f*x+e))**m*(d*tan(f*x+e))**n,x)
[Out] Integral((d*tan(e + f*x))**n*(a + b*cot(e + f*x))**m, x)
```

Maxima [F(-2)]

Exception generated.

$$\int (a+b\cot(e+fx))^m(d\tan(e+fx))^n\,dx = \text{Exception raised: RuntimeError}$$

```
[In] integrate((a+b*cot(f*x+e))^m*(d*tan(f*x+e))^n,x, algorithm="maxima")
[Out] Exception raised: RuntimeError >> ECL says: THROW: The catch RAT-ERR is und
efined.
```

Giac [F]

$$\int (a+b\cot(e+fx))^m(d\tan(e+fx))^n\,dx=\int (b\cot(fx+e)+a)^m(d\tan(fx+e))^n\,dx$$

```
[In] integrate((a+b*cot(f*x+e))^m*(d*tan(f*x+e))^n,x, algorithm="giac")
[Out] integrate((b*cot(f*x + e) + a)^m*(d*tan(f*x + e))^n, x)
```

Mupad [F(-1)]

Timed out.

$$\int (a+b\cot(e+fx))^m(d\tan(e+fx))^n\,dx=\int (d\tan(e+f\,x))^n\,(a+b\cot(e+f\,x))^m\,dx$$

```
[In] int((d*tan(e + f*x))^n*(a + b*cot(e + f*x))^m,x)
[Out] int((d*tan(e + f*x))^n*(a + b*cot(e + f*x))^m, x)
```

3.90 $\int \frac{1+i\cot(c+dx)}{\sqrt{a+b\cot(c+dx)}}\,dx$

Optimal result

Integrand size = 27, antiderivative size = 45

$$\int \frac{1+i\cot(c+dx)}{\sqrt{a+b\cot(c+dx)}}\,dx=\frac{2i\text{arctanh}\left(\frac{\sqrt{a+b\cot(c+dx)}}{\sqrt{a-ib}}\right)}{\sqrt{a-ib}d}$$

```
[Out] 2*I*arctanh((a+b*cot(d*x+c))^(1/2)/(a-I*b)^(1/2))/d/(a-I*b)^(1/2)
```

Rubi [A] (verified)

Time = 0.08 (sec) , antiderivative size = 45, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {3618, 65, 214}

$$\int \frac{1+i\cot(c+dx)}{\sqrt{a+b\cot(c+dx)}}\,dx=\frac{2i\text{arctanh}\left(\frac{\sqrt{a+b\cot(c+dx)}}{\sqrt{a-ib}}\right)}{d\sqrt{a-ib}}$$

```
[In] Int[(1 + I*Cot[c + d*x])/Sqrt[a + b*Cot[c + d*x]],x]
```

```
[Out] ((2*I)*ArcTanh[Sqrt[a + b*Cot[c + d*x]]/Sqrt[a - I*b]])/(Sqrt[a - I*b]*d)
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 3618

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*tan[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Dist[c*(d/f), Subst[Int[(a + (b/d)*x)^m/(d^2 + c
*x), x], x, d*Tan[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b
*c - a*d, 0] && NeQ[a^2 + b^2, 0] && EqQ[c^2 + d^2, 0]
```

Rubi steps

$$\text{integral} = -\frac{i\text{Subst}\left(\int \frac{1}{(-1+x)\sqrt{a-ibx}}\,dx, x, i\cot(c+dx)\right)}{d}$$

$$= \frac{2\text{Subst}\left(\int \frac{1}{-1-\frac{ia}{b}+\frac{ix^2}{b}}\,dx, x, \sqrt{a+b\cot(c+dx)}\right)}{bd}$$

$$= \frac{2i\text{arctanh}\left(\frac{\sqrt{a+b\cot(c+dx)}}{\sqrt{a-ib}}\right)}{\sqrt{a-ib}d}$$

Mathematica [A] (verified)

Time = 0.10 (sec) , antiderivative size = 45, normalized size of antiderivative = 1.00

$$\int \frac{1+i\cot(c+dx)}{\sqrt{a+b\cot(c+dx)}}\,dx = \frac{2i\text{arctanh}\left(\frac{\sqrt{a+b\cot(c+dx)}}{\sqrt{a-ib}}\right)}{\sqrt{a-ib}d}$$

```
[In] Integrate[(1 + I*Cot[c + d*x])/Sqrt[a + b*Cot[c + d*x]],x]
[Out] ((2*I)*ArcTanh[Sqrt[a + b*Cot[c + d*x]]/Sqrt[a - I*b]])/(Sqrt[a - I*b]*d)
```

Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 733 vs. 2(36) = 72.

Time = 0.42 (sec) , antiderivative size = 734, normalized size of antiderivative = 16.31

method	result
derivativedivides	$-\frac{\frac{\left(2i\sqrt{a^2+b^2}\,a^2+i\sqrt{a^2+b^2}\,b^2+2ia^3+2ia\,b^2-\sqrt{a^2+b^2}\,ab-a^2b-b^3\right)\ln\left(b\cot(dx+c)+a-\sqrt{a+b\cot(dx+c)}\,\sqrt{2\sqrt{a^2+b^2}+2a}+\sqrt{a^2+b^2}\right.}{2}}{}$
default	$-\frac{\frac{\left(2i\sqrt{a^2+b^2}\,a^2+i\sqrt{a^2+b^2}\,b^2+2ia^3+2ia\,b^2-\sqrt{a^2+b^2}\,ab-a^2b-b^3\right)\ln\left(b\cot(dx+c)+a-\sqrt{a+b\cot(dx+c)}\,\sqrt{2\sqrt{a^2+b^2}+2a}+\sqrt{a^2+b^2}\right.}{2}}{}$
parts	Expression too large to display

```
[In] int((1+I*cot(d*x+c))/(a+b*cot(d*x+c))^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(-1/(2*(a^2+b^2)^(1/2)+2*a)^(1/2)/(a^2+b^2)^(1/2)/((a^2+b^2)^(1/2)*a+a^
2+b^2)*(1/2*(2*I*(a^2+b^2)^(1/2)*a^2+I*(a^2+b^2)^(1/2)*b^2+2*I*a^3+2*I*a*b^
2-(a^2+b^2)^(1/2)*a*b-a^2*b-b^3)*ln(b*cot(d*x+c)+a-(a+b*cot(d*x+c))^(1/2)*(
2*(a^2+b^2)^(1/2)+2*a)^(1/2)+(a^2+b^2)^(1/2))+2*(-I*(a^2+b^2)^(1/2)*(2*(a^2
+b^2)^(1/2)+2*a)^(1/2)*a^2-I*(2*(a^2+b^2)^(1/2)+2*a)^(1/2)*a^3-I*(2*(a^2+b^
2)^(1/2)+2*a)^(1/2)*a*b^2+(a^2+b^2)^(1/2)*(2*(a^2+b^2)^(1/2)+2*a)^(1/2)*a*b
+(2*(a^2+b^2)^(1/2)+2*a)^(1/2)*a^2*b+(2*(a^2+b^2)^(1/2)+2*a)^(1/2)*b^3+1/2*
(2*I*(a^2+b^2)^(1/2)*a^2+I*(a^2+b^2)^(1/2)*b^2+2*I*a^3+2*I*a*b^2-(a^2+b^2)^
(1/2)*a*b-a^2*b-b^3)*(2*(a^2+b^2)^(1/2)+2*a)^(1/2))/(2*(a^2+b^2)^(1/2)-2*a)
^(1/2)*arctan((2*(a+b*cot(d*x+c))^(1/2)-(2*(a^2+b^2)^(1/2)+2*a)^(1/2))/(2*(
a^2+b^2)^(1/2)-2*a)^(1/2)))-1/(2*(a^2+b^2)^(1/2)+2*a)^(1/2)/(a^2+b^2)^(1/2)
*(1/2*(-I*(a^2+b^2)^(1/2)-I*a+b)*ln(b*cot(d*x+c)+a+(a+b*cot(d*x+c))^(1/2)*(
2*(a^2+b^2)^(1/2)+2*a)^(1/2)+(a^2+b^2)^(1/2))+2*(-I*(2*(a^2+b^2)^(1/2)+2*a)
^(1/2)*a+(2*(a^2+b^2)^(1/2)+2*a)^(1/2)*b-1/2*(-I*(a^2+b^2)^(1/2)-I*a+b)*(2*
(a^2+b^2)^(1/2)+2*a)^(1/2))/(2*(a^2+b^2)^(1/2)-2*a)^(1/2)*arctan((2*(a+b*co
t(d*x+c))^(1/2)+(2*(a^2+b^2)^(1/2)+2*a)^(1/2))/(2*(a^2+b^2)^(1/2)-2*a)^(1/2
))))
```

Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 159 vs. $2(33) = 66$.

Time = 0.30 (sec) , antiderivative size = 159, normalized size of antiderivative = 3.53

$$\int \frac{1+i\cot(c+dx)}{\sqrt{a+b\cot(c+dx)}}\,dx = -\frac{1}{2}\sqrt{-\frac{4i}{(i\,a+b)d^2}}\log\left(\frac{1}{2}\,(i\,a+b)d\sqrt{-\frac{4i}{(i\,a+b)d^2}} + \sqrt{\frac{(a+i\,b)e^{(2i\,dx+2i\,c)}-a+i\,b}{e^{(2i\,dx+2i\,c)}-1}}\right) + \frac{1}{2}\sqrt{-\frac{4i}{(i\,a+b)d^2}}\log\left(\frac{1}{2}\,(-i\,a-b)d\sqrt{-\frac{4i}{(i\,a+b)d^2}} + \sqrt{\frac{(a+i\,b)e^{(2i\,dx+2i\,c)}-a+i\,b}{e^{(2i\,dx+2i\,c)}-1}}\right)$$

```
[In] integrate((1+I*cot(d*x+c))/(a+b*cot(d*x+c))^(1/2),x, algorithm="fricas")
[Out] -1/2*sqrt(-4*I/((I*a + b)*d^2))*log(1/2*(I*a + b)*d*sqrt(-4*I/((I*a + b)*d^
2)) + sqrt(((a + I*b)*e^(2*I*d*x + 2*I*c) - a + I*b)/(e^(2*I*d*x + 2*I*c) -
 1))) + 1/2*sqrt(-4*I/((I*a + b)*d^2))*log(1/2*(-I*a - b)*d*sqrt(-4*I/((I*a
 + b)*d^2)) + sqrt(((a + I*b)*e^(2*I*d*x + 2*I*c) - a + I*b)/(e^(2*I*d*x +
2*I*c) - 1)))
```

Sympy [F]

$$\int \frac{1+i\cot(c+dx)}{\sqrt{a+b\cot(c+dx)}}\,dx = i\left(\int\left(-\frac{i}{\sqrt{a+b\cot(c+dx)}}\right)dx + \int\frac{\cot(c+dx)}{\sqrt{a+b\cot(c+dx)}}\,dx\right)$$

```
[In] integrate((1+I*cot(d*x+c))/(a+b*cot(d*x+c))**(1/2),x)
[Out] I*(Integral(-I/sqrt(a + b*cot(c + d*x)), x) + Integral(cot(c + d*x)/sqrt(a
+ b*cot(c + d*x)), x))
```

Maxima [F]

$$\int \frac{1+i\cot(c+dx)}{\sqrt{a+b\cot(c+dx)}}\,dx = \int \frac{i\,\cot(dx+c)+1}{\sqrt{b\cot(dx+c)+a}}\,dx$$

```
[In] integrate((1+I*cot(d*x+c))/(a+b*cot(d*x+c))^(1/2),x, algorithm="maxima")
[Out] integrate((I*cot(d*x + c) + 1)/sqrt(b*cot(d*x + c) + a), x)
```

Giac [F]

$$\int \frac{1+i\cot(c+dx)}{\sqrt{a+b\cot(c+dx)}}\,dx = \int \frac{i\,\cot(dx+c)+1}{\sqrt{b\cot(dx+c)+a}}\,dx$$

```
[In] integrate((1+I*cot(d*x+c))/(a+b*cot(d*x+c))^(1/2),x, algorithm="giac")
[Out] integrate((I*cot(d*x + c) + 1)/sqrt(b*cot(d*x + c) + a), x)
```

Mupad [B] (verification not implemented)

Time = 15.16 (sec) , antiderivative size = 1410, normalized size of antiderivative = 31.33

$$\int \frac{1+i\cot(c+dx)}{\sqrt{a+b\cot(c+dx)}}\,dx = \text{Too large to display}$$

```
[In] int((cot(c + d*x)*1i + 1)/(a + b*cot(c + d*x))^(1/2),x)
[Out] (log(d*(-1/(d^2*(a - b*1i)))^(1/2)*(a + b*cot(c + d*x))^(1/2) + 1i)*(-1/(a*
d^2 - b*d^2*1i))^(1/2))/2 - log(d*(-1/(d^2*(a - b*1i)))^(1/2)*(a + b*cot(c
+ d*x))^(1/2)*1i + 1)*(-1/(4*(a*d^2 - b*d^2*1i)))^(1/2) + (log(16*b^3*d*(-1
/(d^2*(a - b*1i)))^(1/2) - 16*b^2*(a + b*cot(c + d*x))^(1/2) + (16*a*b^2*(a
 + b*cot(c + d*x))^(1/2))/(a - b*1i))*(-1/(a*d^2 - b*d^2*1i))^(1/2))/2 - lo
g(16*b^2*(a + b*cot(c + d*x))^(1/2) + 16*b^3*d*(-1/(d^2*(a - b*1i)))^(1/2)
- (16*a*b^2*(a + b*cot(c + d*x))^(1/2))/(a - b*1i))*(-1/(4*(a*d^2 - b*d^2*1
i)))^(1/2) - 2*atanh((32*b^2*(a + b*cot(c + d*x))^(1/2)*((b*1i)/(4*a^2*d^2
+ 4*b^2*d^2) - a/(4*a^2*d^2 + 4*b^2*d^2))^(1/2))/((b^4*d^2*64i)/(4*a^2*d^3
+ 4*b^2*d^3) - (64*a*b^3*d^2)/(4*a^2*d^3 + 4*b^2*d^3)) + (a*b^3*(a + b*cot(
c + d*x))^(1/2)*((b*1i)/(4*a^2*d^2 + 4*b^2*d^2) - a/(4*a^2*d^2 + 4*b^2*d^2)
)^(1/2)*128i)/((b^6*d^2*256i)/(4*a^2*d^3 + 4*b^2*d^3) + (a^2*b^4*d^2*256i)/
(4*a^2*d^3 + 4*b^2*d^3) - (256*a^3*b^3*d^2)/(4*a^2*d^3 + 4*b^2*d^3) - (256*
a*b^5*d^2)/(4*a^2*d^3 + 4*b^2*d^3)) - (128*a^2*b^2*(a + b*cot(c + d*x))^(1/
2)*((b*1i)/(4*a^2*d^2 + 4*b^2*d^2) - a/(4*a^2*d^2 + 4*b^2*d^2))^(1/2))/((b^
6*d^2*256i)/(4*a^2*d^3 + 4*b^2*d^3) + (a^2*b^4*d^2*256i)/(4*a^2*d^3 + 4*b^2
*d^3) - (256*a^3*b^3*d^2)/(4*a^2*d^3 + 4*b^2*d^3) - (256*a*b^5*d^2)/(4*a^2*
d^3 + 4*b^2*d^3)))*(-(a - b*1i)/(4*a^2*d^2 + 4*b^2*d^2))^(1/2) - 2*atanh((3
2*b^2*(a + b*cot(c + d*x))^(1/2)*((b*1i)/(4*a^2*d^2 + 4*b^2*d^2) - a/(4*a^2
*d^2 + 4*b^2*d^2))^(1/2))/((a^2*b^2*d^2*64i)/(4*a^2*d^3 + 4*b^2*d^3) - (b^2
*16i)/d + (64*a*b^3*d^2)/(4*a^2*d^3 + 4*b^2*d^3)) - (128*a^2*b^2*(a + b*cot
(c + d*x))^(1/2)*((b*1i)/(4*a^2*d^2 + 4*b^2*d^2) - a/(4*a^2*d^2 + 4*b^2*d^2
))^(1/2))/((a^2*b^4*d^2*256i)/(4*a^2*d^3 + 4*b^2*d^3) - (a^2*b^2*64i)/d - (
b^4*64i)/d + (256*a^3*b^3*d^2)/(4*a^2*d^3 + 4*b^2*d^3) + (a^4*b^2*d^2*256i)
/(4*a^2*d^3 + 4*b^2*d^3) + (256*a*b^5*d^2)/(4*a^2*d^3 + 4*b^2*d^3)) + (a*b^
3*(a + b*cot(c + d*x))^(1/2)*((b*1i)/(4*a^2*d^2 + 4*b^2*d^2) - a/(4*a^2*d^2
```

```
 + 4*b^2*d^2))^(1/2)*128i)/((a^2*b^4*d^2*256i)/(4*a^2*d^3 + 4*b^2*d^3) - (a
^2*b^2*64i)/d - (b^4*64i)/d + (256*a^3*b^3*d^2)/(4*a^2*d^3 + 4*b^2*d^3) + (
a^4*b^2*d^2*256i)/(4*a^2*d^3 + 4*b^2*d^3) + (256*a*b^5*d^2)/(4*a^2*d^3 + 4*
b^2*d^3)))*(-(a - b*1i)/(4*a^2*d^2 + 4*b^2*d^2))^(1/2)
```

3.91 $\int \frac{1-i\cot(c+dx)}{\sqrt{a+b\cot(c+dx)}}\,dx$

Optimal result

Integrand size = 27, antiderivative size = 45

$$\int \frac{1-i\cot(c+dx)}{\sqrt{a+b\cot(c+dx)}}\,dx = -\frac{2i\operatorname{arctanh}\left(\frac{\sqrt{a+b\cot(c+dx)}}{\sqrt{a+ib}}\right)}{\sqrt{a+ib}d}$$

```
[Out] -2*I*arctanh((a+b*cot(d*x+c))^(1/2)/(a+I*b)^(1/2))/d/(a+I*b)^(1/2)
```

Rubi [A] (verified)

Time = 0.07 (sec) , antiderivative size = 45, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {3618, 65, 214}

$$\int \frac{1-i\cot(c+dx)}{\sqrt{a+b\cot(c+dx)}}\,dx = -\frac{2i\operatorname{arctanh}\left(\frac{\sqrt{a+b\cot(c+dx)}}{\sqrt{a+ib}}\right)}{d\sqrt{a+ib}}$$

```
[In] Int[(1 - I*Cot[c + d*x])/Sqrt[a + b*Cot[c + d*x]],x]
[Out] ((-2*I)*ArcTanh[Sqrt[a + b*Cot[c + d*x]]/Sqrt[a + I*b]])/(Sqrt[a + I*b]*d)
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 3618

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*tan[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Dist[c*(d/f), Subst[Int[(a + (b/d)*x)^m/(d^2 + c
*x), x], x, d*Tan[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b
*c - a*d, 0] && NeQ[a^2 + b^2, 0] && EqQ[c^2 + d^2, 0]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \frac{i\text{Subst}\left(\int \frac{1}{(-1+x)\sqrt{a+ibx}}\,dx, x, -i\cot(c+dx)\right)}{d}\\ &= \frac{2\text{Subst}\left(\int \frac{1}{-1+\frac{ia}{b}-\frac{ix^2}{b}}\,dx, x, \sqrt{a+b\cot(c+dx)}\right)}{bd}\\ &= -\frac{2i\text{arctanh}\left(\frac{\sqrt{a+b\cot(c+dx)}}{\sqrt{a+ib}}\right)}{\sqrt{a+ib}d}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.48 (sec) , antiderivative size = 45, normalized size of antiderivative = 1.00

$$\int \frac{1-i\cot(c+dx)}{\sqrt{a+b\cot(c+dx)}}\,dx = -\frac{2i\text{arctanh}\left(\frac{\sqrt{a+b\cot(c+dx)}}{\sqrt{a+ib}}\right)}{\sqrt{a+ib}d}$$

```
[In] Integrate[(1 - I*Cot[c + d*x])/Sqrt[a + b*Cot[c + d*x]],x]
[Out] ((-2*I)*ArcTanh[Sqrt[a + b*Cot[c + d*x]]/Sqrt[a + I*b]])/(Sqrt[a + I*b]*d)
```

Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 738 vs. 2(36) = 72.

Time = 0.08 (sec) , antiderivative size = 739, normalized size of antiderivative = 16.42

method	result
derivativedivides	$-\frac{\left(-2i\sqrt{a^2+b^2}\,a^2-i\sqrt{a^2+b^2}\,b^2-2ia^3-2ia\,b^2-\sqrt{a^2+b^2}\,ab-a^2b-b^3\right)\ln\left(\sqrt{a+b\cot(dx+c)}\,\sqrt{2\sqrt{a^2+b^2}+2a}-b\cot(dx+c)-\sqrt{a^2+b^2}-\right.}{2}$
default	$-\frac{\left(-2i\sqrt{a^2+b^2}\,a^2-i\sqrt{a^2+b^2}\,b^2-2ia^3-2ia\,b^2-\sqrt{a^2+b^2}\,ab-a^2b-b^3\right)\ln\left(\sqrt{a+b\cot(dx+c)}\,\sqrt{2\sqrt{a^2+b^2}+2a}-b\cot(dx+c)-\sqrt{a^2+b^2}-\right.}{2}$
parts	Expression too large to display

```
[In] int((1-I*cot(d*x+c))/(a+b*cot(d*x+c))^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(1/(2*(a^2+b^2)^(1/2)+2*a)^(1/2)/(a^2+b^2)^(1/2)/((a^2+b^2)^(1/2)*a+a^2
+b^2)*(-1/2*(-2*I*(a^2+b^2)^(1/2)*a^2-I*(a^2+b^2)^(1/2)*b^2-2*I*a^3-2*I*a*b
^2-(a^2+b^2)^(1/2)*a*b-a^2*b-b^3)*ln((a+b*cot(d*x+c))^(1/2)*(2*(a^2+b^2)^(1
/2)+2*a)^(1/2)-b*cot(d*x+c)-(a^2+b^2)^(1/2)-a)+2*(I*(2*(a^2+b^2)^(1/2)+2*a)
^(1/2)*(a^2+b^2)^(1/2)*a^2+I*(2*(a^2+b^2)^(1/2)+2*a)^(1/2)*a^3+I*(2*(a^2+b^
2)^(1/2)+2*a)^(1/2)*a*b^2+(a^2+b^2)^(1/2)*(2*(a^2+b^2)^(1/2)+2*a)^(1/2)*a*b
+(2*(a^2+b^2)^(1/2)+2*a)^(1/2)*a^2*b+(2*(a^2+b^2)^(1/2)+2*a)^(1/2)*b^3+1/2*
(-2*I*(a^2+b^2)^(1/2)*a^2-I*(a^2+b^2)^(1/2)*b^2-2*I*a^3-2*I*a*b^2-(a^2+b^2)
^(1/2)*a*b-a^2*b-b^3)*(2*(a^2+b^2)^(1/2)+2*a)^(1/2))/(2*(a^2+b^2)^(1/2)-2*a
)^(1/2)*arctan((-2*(a+b*cot(d*x+c))^(1/2)+(2*(a^2+b^2)^(1/2)+2*a)^(1/2))/(2
*(a^2+b^2)^(1/2)-2*a)^(1/2)))+1/(2*(a^2+b^2)^(1/2)+2*a)^(1/2)/(a^2+b^2)^(1/
2)*(1/2*(-I*(a^2+b^2)^(1/2)-I*a-b)*ln(b*cot(d*x+c)+a+(a+b*cot(d*x+c))^(1/2)
*(2*(a^2+b^2)^(1/2)+2*a)^(1/2)+(a^2+b^2)^(1/2))+2*(-I*(2*(a^2+b^2)^(1/2)+2*
a)^(1/2)*a-(2*(a^2+b^2)^(1/2)+2*a)^(1/2)*b-1/2*(-I*(a^2+b^2)^(1/2)-I*a-b)*(
2*(a^2+b^2)^(1/2)+2*a)^(1/2))/(2*(a^2+b^2)^(1/2)-2*a)^(1/2)*arctan((2*(a+b*
cot(d*x+c))^(1/2)+(2*(a^2+b^2)^(1/2)+2*a)^(1/2))/(2*(a^2+b^2)^(1/2)-2*a)^(1
/2))))
```

Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 159 vs. $2(33) = 66$.

Time = 0.28 (sec) , antiderivative size = 159, normalized size of antiderivative = 3.53

$$\int \frac{1-i\cot(c+dx)}{\sqrt{a+b\cot(c+dx)}}\,dx = \frac{1}{2}\sqrt{\frac{4i}{(-i\,a+b)d^2}}\log\left(\frac{1}{2}\,(i\,a-b)d\sqrt{\frac{4i}{(-i\,a+b)d^2}} + \sqrt{\frac{(a+i\,b)e^{(2i\,dx+2i\,c)}-a+i\,b}{e^{(2i\,dx+2i\,c)}-1}}\right) - \frac{1}{2}\sqrt{\frac{4i}{(-i\,a+b)d^2}}\log\left(\frac{1}{2}\,(-i\,a+b)d\sqrt{\frac{4i}{(-i\,a+b)d^2}} + \sqrt{\frac{(a+i\,b)e^{(2i\,dx+2i\,c)}-a+i\,b}{e^{(2i\,dx+2i\,c)}-1}}\right)$$

```
[In] integrate((1-I*cot(d*x+c))/(a+b*cot(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] 1/2*sqrt(4*I/((-I*a + b)*d^2))*log(1/2*(I*a - b)*d*sqrt(4*I/((-I*a + b)*d^2
)) + sqrt(((a + I*b)*e^(2*I*d*x + 2*I*c) - a + I*b)/(e^(2*I*d*x + 2*I*c) -
1))) - 1/2*sqrt(4*I/((-I*a + b)*d^2))*log(1/2*(-I*a + b)*d*sqrt(4*I/((-I*a
+ b)*d^2)) + sqrt(((a + I*b)*e^(2*I*d*x + 2*I*c) - a + I*b)/(e^(2*I*d*x + 2
*I*c) - 1)))
```

Sympy [F]

$$\int \frac{1-i\cot(c+dx)}{\sqrt{a+b\cot(c+dx)}}\,dx = -i\left(\int \frac{i}{\sqrt{a+b\cot{(c+dx)}}}\,dx + \int \frac{\cot{(c+dx)}}{\sqrt{a+b\cot{(c+dx)}}}\,dx\right)$$

```
[In] integrate((1-I*cot(d*x+c))/(a+b*cot(d*x+c))**(1/2),x)
```

```
[Out] -I*(Integral(I/sqrt(a + b*cot(c + d*x)), x) + Integral(cot(c + d*x)/sqrt(a
+ b*cot(c + d*x)), x))
```

Maxima [F]

$$\int \frac{1-i\cot(c+dx)}{\sqrt{a+b\cot(c+dx)}}\,dx = \int \frac{-i\,\cot{(dx+c)}+1}{\sqrt{b\cot{(dx+c)}+a}}\,dx$$

```
[In] integrate((1-I*cot(d*x+c))/(a+b*cot(d*x+c))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((-I*cot(d*x + c) + 1)/sqrt(b*cot(d*x + c) + a), x)
```

Giac [F]

$$\int \frac{1 - i\cot(c+dx)}{\sqrt{a+b\cot(c+dx)}}\,dx = \int \frac{-i\cot(dx+c)+1}{\sqrt{b\cot(dx+c)+a}}\,dx$$

```
[In] integrate((1-I*cot(d*x+c))/(a+b*cot(d*x+c))^(1/2),x, algorithm="giac")
[Out] integrate((-I*cot(d*x + c) + 1)/sqrt(b*cot(d*x + c) + a), x)
```

Mupad [B] (verification not implemented)

Time = 14.07 (sec) , antiderivative size = 1410, normalized size of antiderivative = 31.33

$$\int \frac{1 - i\cot(c+dx)}{\sqrt{a+b\cot(c+dx)}}\,dx = \text{Too large to display}$$

```
[In] int(-(cot(c + d*x)*1i - 1)/(a + b*cot(c + d*x))^(1/2),x)
[Out] (log(d*(-1/(d^2*(a - b*1i)))^(1/2)*(a + b*cot(c + d*x))^(1/2)*1i + 1)*(-1/(
a*d^2 - b*d^2*1i))^(1/2))/2 - log(d*(-1/(d^2*(a - b*1i)))^(1/2)*(a + b*cot(
c + d*x))^(1/2) + 1i)*(-1/(4*(a*d^2 - b*d^2*1i)))^(1/2) + (log(16*b^3*d*(-1
/(d^2*(a - b*1i)))^(1/2) - 16*b^2*(a + b*cot(c + d*x))^(1/2) + (16*a*b^2*(a
 + b*cot(c + d*x))^(1/2))/(a - b*1i))*(-1/(a*d^2 - b*d^2*1i))^(1/2))/2 - lo
g(16*b^2*(a + b*cot(c + d*x))^(1/2) + 16*b^3*d*(-1/(d^2*(a - b*1i)))^(1/2)
- (16*a*b^2*(a + b*cot(c + d*x))^(1/2))/(a - b*1i))*(-1/(4*(a*d^2 - b*d^2*1
i)))^(1/2) - 2*atanh((32*b^2*(a + b*cot(c + d*x))^(1/2)*((b*1i)/(4*a^2*d^2
+ 4*b^2*d^2) - a/(4*a^2*d^2 + 4*b^2*d^2))^(1/2))/((b^4*d^2*64i)/(4*a^2*d^3
+ 4*b^2*d^3) - (64*a*b^3*d^2)/(4*a^2*d^3 + 4*b^2*d^3)) + (a*b^3*(a + b*cot(
c + d*x))^(1/2)*((b*1i)/(4*a^2*d^2 + 4*b^2*d^2) - a/(4*a^2*d^2 + 4*b^2*d^2)
)^(1/2)*128i)/((b^6*d^2*256i)/(4*a^2*d^3 + 4*b^2*d^3) + (a^2*b^4*d^2*256i)/
(4*a^2*d^3 + 4*b^2*d^3) - (256*a^3*b^3*d^2)/(4*a^2*d^3 + 4*b^2*d^3) - (256*
a*b^5*d^2)/(4*a^2*d^3 + 4*b^2*d^3)) - (128*a^2*b^2*(a + b*cot(c + d*x))^(1/
2)*((b*1i)/(4*a^2*d^2 + 4*b^2*d^2) - a/(4*a^2*d^2 + 4*b^2*d^2))^(1/2))/((b^
6*d^2*256i)/(4*a^2*d^3 + 4*b^2*d^3) + (a^2*b^4*d^2*256i)/(4*a^2*d^3 + 4*b^2
*d^3) - (256*a^3*b^3*d^2)/(4*a^2*d^3 + 4*b^2*d^3) - (256*a*b^5*d^2)/(4*a^2*
d^3 + 4*b^2*d^3)))*(-(a - b*1i)/(4*a^2*d^2 + 4*b^2*d^2))^(1/2) + 2*atanh((3
2*b^2*(a + b*cot(c + d*x))^(1/2)*((b*1i)/(4*a^2*d^2 + 4*b^2*d^2) - a/(4*a^2
*d^2 + 4*b^2*d^2))^(1/2))/((a^2*b^2*d^2*64i)/(4*a^2*d^3 + 4*b^2*d^3) - (b^2
*16i)/d + (64*a*b^3*d^2)/(4*a^2*d^3 + 4*b^2*d^3)) - (128*a^2*b^2*(a + b*cot
(c + d*x))^(1/2)*((b*1i)/(4*a^2*d^2 + 4*b^2*d^2) - a/(4*a^2*d^2 + 4*b^2*d^2
))^(1/2))/((a^2*b^4*d^2*256i)/(4*a^2*d^3 + 4*b^2*d^3) - (a^2*b^2*64i)/d - (
b^4*64i)/d + (256*a^3*b^3*d^2)/(4*a^2*d^3 + 4*b^2*d^3) + (a^4*b^2*d^2*256i)
/(4*a^2*d^3 + 4*b^2*d^3) + (256*a*b^5*d^2)/(4*a^2*d^3 + 4*b^2*d^3)) + (a*b^
3*(a + b*cot(c + d*x))^(1/2)*((b*1i)/(4*a^2*d^2 + 4*b^2*d^2) - a/(4*a^2*d^2
```

```
 + 4*b^2*d^2))^(1/2)*128i)/((a^2*b^4*d^2*256i)/(4*a^2*d^3 + 4*b^2*d^3) - (a
^2*b^2*64i)/d - (b^4*64i)/d + (256*a^3*b^3*d^2)/(4*a^2*d^3 + 4*b^2*d^3) + (
a^4*b^2*d^2*256i)/(4*a^2*d^3 + 4*b^2*d^3) + (256*a*b^5*d^2)/(4*a^2*d^3 + 4*
b^2*d^3)))*(-(a - b*1i)/(4*a^2*d^2 + 4*b^2*d^2))^(1/2)
```

3.92 $\int \frac{A+B\cot(c+dx)}{a+b\cot(c+dx)}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 59

$$\int \frac{A+B\cot(c+dx)}{a+b\cot(c+dx)}\,dx=\frac{(aA+bB)x}{a^2+b^2}-\frac{(Ab-aB)\log(b\cos(c+dx)+a\sin(c+dx))}{(a^2+b^2)\,d}$$

```
[Out] (A*a+B*b)*x/(a^2+b^2)-(A*b-B*a)*ln(b*cos(d*x+c)+a*sin(d*x+c))/(a^2+b^2)/d
```

Rubi [A] (verified)

Time = 0.09 (sec) , antiderivative size = 59, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.087, Rules used = {3612, 3611}

$$\int \frac{A+B\cot(c+dx)}{a+b\cot(c+dx)}\,dx=\frac{x(aA+bB)}{a^2+b^2}-\frac{(Ab-aB)\log(a\sin(c+dx)+b\cos(c+dx))}{d\,(a^2+b^2)}$$

```
[In] Int[(A + B*Cot[c + d*x])/(a + b*Cot[c + d*x]),x]
```

```
[Out] ((a*A + b*B)*x)/(a^2 + b^2) - ((A*b - a*B)*Log[b*Cos[c + d*x] + a*Sin[c + d
*x]])/((a^2 + b^2)*d)
```

Rule 3611

```
Int[((c_) + (d_.)*tan[(e_.) + (f_.)*(x_)])/((a_) + (b_.)*tan[(e_.) + (f_.)*
(x_)]), x_Symbol] :> Simp[(c/(b*f))*Log[RemoveContent[a*Cos[e + f*x] + b*Si
n[e + f*x], x]], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] &&
 NeQ[a^2 + b^2, 0] && EqQ[a*c + b*d, 0]
```

Rule 3612

```
Int[((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])/((a_.) + (b_.)*tan[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[(a*c + b*d)*(x/(a^2 + b^2)), x] + Dist[(b*c - a
*d)/(a^2 + b^2), Int[(b - a*Tan[e + f*x])/(a + b*Tan[e + f*x]), x], x] /; F
reeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && Ne
Q[a*c + b*d, 0]
```

Rubi steps

$$\text{integral} = \frac{(aA+bB)x}{a^2+b^2} - \frac{(Ab-aB)\int \frac{-b+a\cot(c+dx)}{a+b\cot(c+dx)}\,dx}{a^2+b^2}$$
$$= \frac{(aA+bB)x}{a^2+b^2} - \frac{(Ab-aB)\log(b\cos(c+dx)+a\sin(c+dx))}{(a^2+b^2)\,d}$$

Mathematica [A] (verified)

Time = 0.18 (sec) , antiderivative size = 67, normalized size of antiderivative = 1.14

$$\int \frac{A+B\cot(c+dx)}{a+b\cot(c+dx)}\,dx = -\frac{2(aA+bB)\arctan(\cot(c+dx))+(Ab-aB)\left(2\log(a+b\cot(c+dx))-\log\left(\csc^2(c+dx)\right)\right)}{2\left(a^2+b^2\right)d}$$

```
[In] Integrate[(A + B*Cot[c + d*x])/(a + b*Cot[c + d*x]),x]
[Out] -1/2*(2*(a*A + b*B)*ArcTan[Cot[c + d*x]] + (A*b - a*B)*(2*Log[a + b*Cot[c +
 d*x]] - Log[Csc[c + d*x]^2]))/((a^2 + b^2)*d)
```

Maple [A] (verified)

Time = 0.08 (sec) , antiderivative size = 66, normalized size of antiderivative = 1.12

method	result
parallelrisch	$\frac{(-2Ab+2Ba)\ln(a\tan(dx+c)+b)+(Ab-Ba)\ln\left(\sec(dx+c)^2\right)+2dx(Aa+Bb)}{2d(a^2+b^2)}$
norman	$\frac{(Aa+Bb)x}{a^2+b^2}+\frac{(Ab-Ba)\ln\left(1+\tan(dx+c)^2\right)}{2d(a^2+b^2)}-\frac{(Ab-Ba)\ln(a\tan(dx+c)+b)}{d(a^2+b^2)}$
derivativedivides	$\frac{\frac{\frac{(Ab-Ba)\ln\left(\cot(dx+c)^2+1\right)}{2}+(-Aa-Bb)\left(\frac{\pi}{2}-\operatorname{arccot}(\cot(dx+c))\right)}{a^2+b^2}-\frac{(Ab-Ba)\ln(a+b\cot(dx+c))}{a^2+b^2}}{d}$
default	$\frac{\frac{\frac{(Ab-Ba)\ln\left(\cot(dx+c)^2+1\right)}{2}+(-Aa-Bb)\left(\frac{\pi}{2}-\operatorname{arccot}(\cot(dx+c))\right)}{a^2+b^2}-\frac{(Ab-Ba)\ln(a+b\cot(dx+c))}{a^2+b^2}}{d}$
risch	$\frac{ixB}{ib+a}+\frac{xA}{ib+a}+\frac{2iAbx}{a^2+b^2}-\frac{2iBax}{a^2+b^2}+\frac{2iAbc}{d(a^2+b^2)}-\frac{2iBac}{d(a^2+b^2)}-\frac{\ln\left(e^{2i(dx+c)}+\frac{ib-a}{ib+a}\right)Ab}{d(a^2+b^2)}+\frac{\ln\left(e^{2i(dx+c)}+\frac{ib-a}{ib+a}\right)}{d(a^2+b^2)}$

```
[In] int((A+B*cot(d*x+c))/(a+b*cot(d*x+c)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/2*((-2*A*b+2*B*a)*ln(a*tan(d*x+c)+b)+(A*b-B*a)*ln(sec(d*x+c)^2)+2*d*x*(A*
a+B*b))/d/(a^2+b^2)
```

Fricas [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 79, normalized size of antiderivative = 1.34

$$\int \frac{A+B\cot(c+dx)}{a+b\cot(c+dx)}\,dx = \frac{2\,(Aa+Bb)dx+(Ba-Ab)\log\left(ab\sin\left(2\,dx+2\,c\right)+\frac{1}{2}\,a^2+\frac{1}{2}\,b^2-\frac{1}{2}\,(a^2-b^2)\cos\left(2\,dx+2\,c\right)\right)}{2\,(a^2+b^2)d}$$

```
[In] integrate((A+B*cot(d*x+c))/(a+b*cot(d*x+c)),x, algorithm="fricas")
```

```
[Out] 1/2*(2*(A*a + B*b)*d*x + (B*a - A*b)*log(a*b*sin(2*d*x + 2*c) + 1/2*a^2 + 1
/2*b^2 - 1/2*(a^2 - b^2)*cos(2*d*x + 2*c)))/((a^2 + b^2)*d)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.53 (sec) , antiderivative size = 524, normalized size of antiderivative = 8.88

$$\int \frac{A+B\cot(c+dx)}{a+b\cot(c+dx)}\,dx$$
$$= \begin{cases} \frac{\tilde{\infty}x(A+B\cot(c))}{\cot(c)} \\ \frac{\frac{A\log\left(\tan^2(c+dx)+1\right)}{2d}+Bx}{b} \\ \frac{iAdx\cot(c+dx)}{2bd\cot(c+dx)-2ibd}+\frac{Adx}{2bd\cot(c+dx)-2ibd}-\frac{iA}{2bd\cot(c+dx)-2ibd}+\frac{Bdx\cot(c+dx)}{2bd\cot(c+dx)-2ibd}-\frac{iBdx}{2bd\cot(c+dx)-2ibd}+\frac{B}{2bd\cot(c+dx)-2\ldots} \\ -\frac{iAdx\cot(c+dx)}{2bd\cot(c+dx)+2ibd}+\frac{Adx}{2bd\cot(c+dx)+2ibd}+\frac{iA}{2bd\cot(c+dx)+2ibd}+\frac{Bdx\cot(c+dx)}{2bd\cot(c+dx)+2ibd}+\frac{iBdx}{2bd\cot(c+dx)+2ibd}+\frac{B}{2bd\cot(c+dx)\ldots} \\ \frac{x(A+B\cot(c))}{a+b\cot(c)} \\ \frac{2Aadx}{2a^2d+2b^2d}-\frac{2Ab\log\left(\tan(c+dx)+\frac{b}{a}\right)}{2a^2d+2b^2d}+\frac{Ab\log\left(\tan^2(c+dx)+1\right)}{2a^2d+2b^2d}+\frac{2Ba\log\left(\tan(c+dx)+\frac{b}{a}\right)}{2a^2d+2b^2d}-\frac{Ba\log\left(\tan^2(c+dx)+1\right)}{2a^2d+2b^2d}+\frac{2Bbdx}{2a^2d+2b^2\ldots} \end{cases}$$

```
[In] integrate((A+B*cot(d*x+c))/(a+b*cot(d*x+c)),x)
```

```
[Out] Piecewise((zoo*x*(A + B*cot(c))/cot(c), Eq(a, 0) & Eq(b, 0) & Eq(d, 0)), ((
A*log(tan(c + d*x)**2 + 1)/(2*d) + B*x)/b, Eq(a, 0)), (I*A*d*x*cot(c + d*x)
/(2*b*d*cot(c + d*x) - 2*I*b*d) + A*d*x/(2*b*d*cot(c + d*x) - 2*I*b*d) - I*
A/(2*b*d*cot(c + d*x) - 2*I*b*d) + B*d*x*cot(c + d*x)/(2*b*d*cot(c + d*x) -
 2*I*b*d) - I*B*d*x/(2*b*d*cot(c + d*x) - 2*I*b*d) + B/(2*b*d*cot(c + d*x)
- 2*I*b*d), Eq(a, -I*b)), (-I*A*d*x*cot(c + d*x)/(2*b*d*cot(c + d*x) + 2*I*
b*d) + A*d*x/(2*b*d*cot(c + d*x) + 2*I*b*d) + I*A/(2*b*d*cot(c + d*x) + 2*I
*b*d) + B*d*x*cot(c + d*x)/(2*b*d*cot(c + d*x) + 2*I*b*d) + I*B*d*x/(2*b*d*
cot(c + d*x) + 2*I*b*d) + B/(2*b*d*cot(c + d*x) + 2*I*b*d), Eq(a, I*b)), (x
*(A + B*cot(c))/(a + b*cot(c)), Eq(d, 0)), (2*A*a*d*x/(2*a**2*d + 2*b**2*d)
 - 2*A*b*log(tan(c + d*x) + b/a)/(2*a**2*d + 2*b**2*d) + A*b*log(tan(c + d*
x)**2 + 1)/(2*a**2*d + 2*b**2*d) + 2*B*a*log(tan(c + d*x) + b/a)/(2*a**2*d
+ 2*b**2*d) - B*a*log(tan(c + d*x)**2 + 1)/(2*a**2*d + 2*b**2*d) + 2*B*b*d*
x/(2*a**2*d + 2*b**2*d), True))
```

Maxima [A] (verification not implemented)

none

Time = 0.43 (sec) , antiderivative size = 89, normalized size of antiderivative = 1.51

$$\int \frac{A+B\cot(c+dx)}{a+b\cot(c+dx)}\,dx = \frac{\frac{2\,(Aa+Bb)(dx+c)}{a^2+b^2}+\frac{2\,(Ba-Ab)\log(a\tan(dx+c)+b)}{a^2+b^2}-\frac{(Ba-Ab)\log\left(\tan(dx+c)^2+1\right)}{a^2+b^2}}{2\,d}$$

```
[In] integrate((A+B*cot(d*x+c))/(a+b*cot(d*x+c)),x, algorithm="maxima")
```

```
[Out] 1/2*(2*(A*a + B*b)*(d*x + c)/(a^2 + b^2) + 2*(B*a - A*b)*log(a*tan(d*x + c)
 + b)/(a^2 + b^2) - (B*a - A*b)*log(tan(d*x + c)^2 + 1)/(a^2 + b^2))/d
```

Giac [A] (verification not implemented)

none

Time = 0.31 (sec) , antiderivative size = 95, normalized size of antiderivative = 1.61

$$\int \frac{A+B\cot(c+dx)}{a+b\cot(c+dx)}\,dx = \frac{\frac{2\,(Aa+Bb)(dx+c)}{a^2+b^2} - \frac{(Ba-Ab)\log\left(\tan(dx+c)^2+1\right)}{a^2+b^2} + \frac{2\,(Ba^2-Aab)\log(|a\tan(dx+c)+b|)}{a^3+ab^2}}{2\,d}$$

```
[In] integrate((A+B*cot(d*x+c))/(a+b*cot(d*x+c)),x, algorithm="giac")
```

```
[Out] 1/2*(2*(A*a + B*b)*(d*x + c)/(a^2 + b^2) - (B*a - A*b)*log(tan(d*x + c)^2 +
 1)/(a^2 + b^2) + 2*(B*a^2 - A*a*b)*log(abs(a*tan(d*x + c) + b))/(a^3 + a*b
^2))/d
```

Mupad [B] (verification not implemented)

Time = 0.80 (sec) , antiderivative size = 155, normalized size of antiderivative = 2.63

$$\begin{aligned}\int \frac{A+B\cot(c+dx)}{a+b\cot(c+dx)}\,dx = {} & \frac{A\,\ln\left(\cot(c+d\,x)+1\mathrm{i}\right)}{2\,\left(b\,d+a\,d\,1\mathrm{i}\right)} - \frac{B\,\ln\left(\cot(c+d\,x)+1\mathrm{i}\right)}{2\,\left(a\,d-b\,d\,1\mathrm{i}\right)} \\ & - \frac{A\,b\,\ln\left(a+b\cot(c+d\,x)\right)}{d\,\left(a^2+b^2\right)} + \frac{B\,a\,\ln\left(a+b\cot(c+d\,x)\right)}{d\,\left(a^2+b^2\right)} \\ & + \frac{A\,\ln\left(\cot(c+d\,x)-\mathrm{i}\right)\,1\mathrm{i}}{2\,\left(a\,d+b\,d\,1\mathrm{i}\right)} - \frac{B\,\ln\left(\cot(c+d\,x)-\mathrm{i}\right)\,1\mathrm{i}}{2\,\left(-b\,d+a\,d\,1\mathrm{i}\right)}\end{aligned}$$

```
[In] int((A + B*cot(c + d*x))/(a + b*cot(c + d*x)),x)
```

```
[Out] (A*log(cot(c + d*x) - 1i)*1i)/(2*(a*d + b*d*1i)) + (A*log(cot(c + d*x) + 1i
))/(2*(a*d*1i + b*d)) - (B*log(cot(c + d*x) + 1i))/(2*(a*d - b*d*1i)) - (B*
log(cot(c + d*x) - 1i)*1i)/(2*(a*d*1i - b*d)) - (A*b*log(a + b*cot(c + d*x)
))/(d*(a^2 + b^2)) + (B*a*log(a + b*cot(c + d*x)))/(d*(a^2 + b^2))
```

3.93 $\int \frac{A+B\cot(c+dx)}{(a+b\cot(c+dx))^2}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 111

$$\int \frac{A+B\cot(c+dx)}{(a+b\cot(c+dx))^2}\,dx=\frac{(a^2A-Ab^2+2abB)\,x}{(a^2+b^2)^2}+\frac{Ab-aB}{(a^2+b^2)\,d(a+b\cot(c+dx))}-\frac{(2aAb-a^2B+b^2B)\log(b\cos(c+dx)+a\sin(c+dx))}{(a^2+b^2)^2\,d}$$

```
[Out] (A*a^2-A*b^2+2*B*a*b)*x/(a^2+b^2)^2+(A*b-B*a)/(a^2+b^2)/d/(a+b*cot(d*x+c))-
(2*A*a*b-B*a^2+B*b^2)*ln(b*cos(d*x+c)+a*sin(d*x+c))/(a^2+b^2)^2/d
```

Rubi [A] (verified)

Time = 0.18 (sec) , antiderivative size = 111, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.130, Rules used = {3610, 3612, 3611}

$$\int \frac{A+B\cot(c+dx)}{(a+b\cot(c+dx))^2}\,dx=\frac{Ab-aB}{d\,(a^2+b^2)\,(a+b\cot(c+dx))}-\frac{(a^2(-B)+2aAb+b^2B)\log(a\sin(c+dx)+b\cos(c+dx))}{d\,(a^2+b^2)^2}+\frac{x(a^2A+2abB-Ab^2)}{(a^2+b^2)^2}$$

```
[In] Int[(A + B*Cot[c + d*x])/(a + b*Cot[c + d*x])^2,x]
```

```
[Out] ((a^2*A - A*b^2 + 2*a*b*B)*x)/(a^2 + b^2)^2 + (A*b - a*B)/((a^2 + b^2)*d*(a
 + b*Cot[c + d*x])) - ((2*a*A*b - a^2*B + b^2*B)*Log[b*Cos[c + d*x] + a*Sin
[c + d*x]])/((a^2 + b^2)^2*d)
```

Rule 3610

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)]), x_Symbol] :> Simp[(b*c - a*d)*((a + b*Tan[e + f*x])^(m + 1)/
(f*(m + 1)*(a^2 + b^2))), x] + Dist[1/(a^2 + b^2), Int[(a + b*Tan[e + f*x])
^(m + 1)*Simp[a*c + b*d - (b*c - a*d)*Tan[e + f*x], x], x], x] /; FreeQ[{a,
 b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && LtQ[m, -1]
```

Rule 3611

```
Int[((c_) + (d_.)*tan[(e_.) + (f_.)*(x_)])/((a_) + (b_.)*tan[(e_.) + (f_.)*
(x_)]), x_Symbol] :> Simp[(c/(b*f))*Log[RemoveContent[a*Cos[e + f*x] + b*Si
n[e + f*x], x]], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] &&
 NeQ[a^2 + b^2, 0] && EqQ[a*c + b*d, 0]
```

Rule 3612

```
Int[((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])/((a_.) + (b_.)*tan[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[(a*c + b*d)*(x/(a^2 + b^2)), x] + Dist[(b*c - a
*d)/(a^2 + b^2), Int[(b - a*Tan[e + f*x])/(a + b*Tan[e + f*x]), x], x] /; F
reeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && Ne
Q[a*c + b*d, 0]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{Ab-aB}{(a^2+b^2)\,d(a+b\cot(c+dx))}+\frac{\int \frac{aA+bB-(Ab-aB)\cot(c+dx)}{a+b\cot(c+dx)}\,dx}{a^2+b^2}\\
&= \frac{(a^2A-Ab^2+2abB)\,x}{(a^2+b^2)^2}+\frac{Ab-aB}{(a^2+b^2)\,d(a+b\cot(c+dx))}\\
&\quad-\frac{(2aAb-a^2B+b^2B)\int \frac{-b+a\cot(c+dx)}{a+b\cot(c+dx)}\,dx}{(a^2+b^2)^2}\\
&= \frac{(a^2A-Ab^2+2abB)\,x}{(a^2+b^2)^2}+\frac{Ab-aB}{(a^2+b^2)\,d(a+b\cot(c+dx))}\\
&\quad-\frac{(2aAb-a^2B+b^2B)\log(b\cos(c+dx)+a\sin(c+dx))}{(a^2+b^2)^2\,d}
\end{aligned}
$$

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 2.13 (sec) , antiderivative size = 144, normalized size of antiderivative = 1.30

$$\int \frac{A+B\cot(c+dx)}{(a+b\cot(c+dx))^2}\,dx = \frac{-\frac{(iA+B)\log(i-\tan(c+dx))}{(a-ib)^2}+\frac{i(A+iB)\log(i+\tan(c+dx))}{(a+ib)^2}+\frac{2(-2aAb+a^2B-b^2B)\log(b+a\tan(c+dx))}{(a^2+b^2)^2}+\frac{2b(-Ab+aB)}{a(a^2+b^2)(b+a\tan(c+dx))}}{2d}$$

```
[In] Integrate[(A + B*Cot[c + d*x])/(a + b*Cot[c + d*x])^2,x]
```

```
[Out] (-(((I*A + B)*Log[I - Tan[c + d*x]])/(a - I*b)^2) + (I*(A + I*B)*Log[I + Ta
n[c + d*x]])/(a + I*b)^2 + (2*(-2*a*A*b + a^2*B - b^2*B)*Log[b + a*Tan[c +
d*x]])/(a^2 + b^2)^2 + (2*b*(-(A*b) + a*B))/(a*(a^2 + b^2)*(b + a*Tan[c + d
*x])))/(2*d)
```

Maple [A] (verified)

Time = 0.12 (sec) , antiderivative size = 147, normalized size of antiderivative = 1.32

method	result
derivativedivides	$\frac{\frac{\frac{(2Aab-Ba^2+Bb^2)\ln(\cot(dx+c)^2+1)}{2}+(-Aa^2+Ab^2-2Bab)(\frac{\pi}{2}-\operatorname{arccot}(\cot(dx+c)))}{(a^2+b^2)^2}+\frac{Ab-Ba}{(a^2+b^2)(a+b\cot(dx+c))}-\frac{(2Aab-Ba^2+\cdots}{\cdots}}{d}$
default	$\frac{\frac{\frac{(2Aab-Ba^2+Bb^2)\ln(\cot(dx+c)^2+1)}{2}+(-Aa^2+Ab^2-2Bab)(\frac{\pi}{2}-\operatorname{arccot}(\cot(dx+c)))}{(a^2+b^2)^2}+\frac{Ab-Ba}{(a^2+b^2)(a+b\cot(dx+c))}-\frac{(2Aab-Ba^2+\cdots}{\cdots}}{d}$
parallelrisch	$\frac{-2(Aab-\frac{1}{2}Ba^2+\frac{1}{2}Bb^2)a(a\tan(dx+c)+b)\ln(a\tan(dx+c)+b)+(Aab-\frac{1}{2}Ba^2+\frac{1}{2}Bb^2)a(a\tan(dx+c)+b)\ln(\sec(dx+c)^2\cdots}{(a\tan(dx+c)+b)d(a^2+b^2)^2\cdots}$
norman	$\frac{\frac{b(Aa^2-Ab^2+2Bab)x}{a^4+2a^2b^2+b^4}+\frac{a(Aa^2-Ab^2+2Bab)x\tan(dx+c)}{a^4+2a^2b^2+b^4}-\frac{(Ab-Ba)b}{ad(a^2+b^2)}}{a\tan(dx+c)+b}+\frac{(2Aab-Ba^2+Bb^2)\ln(1+\tan(dx+c)^2)}{2d(a^4+2a^2b^2+b^4)}-\frac{(2Aa\cdots}{\cdots}$
risch	$\frac{ixB}{2iab+a^2-b^2}+\frac{xA}{2iab+a^2-b^2}+\frac{4iAabx}{a^4+2a^2b^2+b^4}-\frac{2iBa^2x}{a^4+2a^2b^2+b^4}+\frac{2iBb^2x}{a^4+2a^2b^2+b^4}+\frac{4iAabc}{d(a^4+2a^2b^2+b^4)}-\frac{2iB\cdots}{d(a^4+2a\cdots}$

```
[In] int((A+B*cot(d*x+c))/(a+b*cot(d*x+c))^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(1/(a^2+b^2)^2*(1/2*(2*A*a*b-B*a^2+B*b^2)*ln(cot(d*x+c)^2+1)+(-A*a^2+A*
b^2-2*B*a*b)*(1/2*Pi-arccot(cot(d*x+c))))+(A*b-B*a)/(a^2+b^2)/(a+b*cot(d*x+
c))-(2*A*a*b-B*a^2+B*b^2)/(a^2+b^2)^2*ln(a+b*cot(d*x+c)))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 340 vs. $2(111) = 222$.

Time = 0.30 (sec) , antiderivative size = 340, normalized size of antiderivative = 3.06

$$\int \frac{A+B\cot(c+dx)}{(a+b\cot(c+dx))^2}\,dx = \frac{2\,Ba^2b - 2\,Aab^2 + 2\,(Aa^2b + 2\,Bab^2 - Ab^3)dx + 2\,(Ba^2b - Aab^2 + (Aa^2b + 2\,Bab^2 - Ab^3)dx)\cos\left(2\,dx + \right.}{}$$

```
[In] integrate((A+B*cot(d*x+c))/(a+b*cot(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] 1/2*(2*B*a^2*b - 2*A*a*b^2 + 2*(A*a^2*b + 2*B*a*b^2 - A*b^3)*d*x + 2*(B*a^2
*b - A*a*b^2 + (A*a^2*b + 2*B*a*b^2 - A*b^3)*d*x)*cos(2*d*x + 2*c) + (B*a^2
*b - 2*A*a*b^2 - B*b^3 + (B*a^2*b - 2*A*a*b^2 - B*b^3)*cos(2*d*x + 2*c) + (
B*a^3 - 2*A*a^2*b - B*a*b^2)*sin(2*d*x + 2*c))*log(a*b*sin(2*d*x + 2*c) + 1
/2*a^2 + 1/2*b^2 - 1/2*(a^2 - b^2)*cos(2*d*x + 2*c)) - 2*(B*a*b^2 - A*b^3 -
 (A*a^3 + 2*B*a^2*b - A*a*b^2)*d*x)*sin(2*d*x + 2*c))/((a^4*b + 2*a^2*b^3 +
 b^5)*d*cos(2*d*x + 2*c) + (a^5 + 2*a^3*b^2 + a*b^4)*d*sin(2*d*x + 2*c) + (
a^4*b + 2*a^2*b^3 + b^5)*d)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 1.62 (sec) , antiderivative size = 3964, normalized size of antiderivative = 35.71

$$\int \frac{A+B\cot(c+dx)}{(a+b\cot(c+dx))^2}\,dx = \text{Too large to display}$$

```
[In] integrate((A+B*cot(d*x+c))/(a+b*cot(d*x+c))**2,x)
```

```
[Out] Piecewise((zoo*x*(A + B*cot(c))/cot(c)**2, Eq(a, 0) & Eq(b, 0) & Eq(d, 0)),
 ((-A*x + A*tan(c + d*x)/d + B*log(tan(c + d*x)**2 + 1)/(2*d))/b**2, Eq(a,
0)), (A*d*x*cot(c + d*x)**2/(4*a**2*d*cot(c + d*x)**2 + 8*I*a**2*d*cot(c +
d*x) - 4*a**2*d) + 2*I*A*d*x*cot(c + d*x)/(4*a**2*d*cot(c + d*x)**2 + 8*I*a
**2*d*cot(c + d*x) - 4*a**2*d) - A*d*x/(4*a**2*d*cot(c + d*x)**2 + 8*I*a**2
*d*cot(c + d*x) - 4*a**2*d) - A*cot(c + d*x)/(4*a**2*d*cot(c + d*x)**2 + 8*
I*a**2*d*cot(c + d*x) - 4*a**2*d) - 2*I*A/(4*a**2*d*cot(c + d*x)**2 + 8*I*a
**2*d*cot(c + d*x) - 4*a**2*d) + I*B*d*x*cot(c + d*x)**2/(4*a**2*d*cot(c +
d*x)**2 + 8*I*a**2*d*cot(c + d*x) - 4*a**2*d) - 2*B*d*x*cot(c + d*x)/(4*a**
2*d*cot(c + d*x)**2 + 8*I*a**2*d*cot(c + d*x) - 4*a**2*d) - I*B*d*x/(4*a**2
*d*cot(c + d*x)**2 + 8*I*a**2*d*cot(c + d*x) - 4*a**2*d) - I*B*cot(c + d*x)
/(4*a**2*d*cot(c + d*x)**2 + 8*I*a**2*d*cot(c + d*x) - 4*a**2*d), Eq(b, -I*
a)), (A*d*x*cot(c + d*x)**2/(4*a**2*d*cot(c + d*x)**2 - 8*I*a**2*d*cot(c +
```

d*x) - 4*a**2*d) - 2*I*A*d*x*cot(c + d*x)/(4*a**2*d*cot(c + d*x)**2 - 8*I*a **2*d*cot(c + d*x) - 4*a**2*d) - A*d*x/(4*a**2*d*cot(c + d*x)**2 - 8*I*a**2 *d*cot(c + d*x) - 4*a**2*d) - A*cot(c + d*x)/(4*a**2*d*cot(c + d*x)**2 - 8* I*a**2*d*cot(c + d*x) - 4*a**2*d) + 2*I*A/(4*a**2*d*cot(c + d*x)**2 - 8*I*a **2*d*cot(c + d*x) - 4*a**2*d) - I*B*d*x*cot(c + d*x)**2/(4*a**2*d*cot(c + d*x)**2 - 8*I*a**2*d*cot(c + d*x) - 4*a**2*d) - 2*B*d*x*cot(c + d*x)/(4*a** 2*d*cot(c + d*x)**2 - 8*I*a**2*d*cot(c + d*x) - 4*a**2*d) + I*B*d*x/(4*a**2 *d*cot(c + d*x)**2 - 8*I*a**2*d*cot(c + d*x) - 4*a**2*d) + I*B*cot(c + d*x) /(4*a**2*d*cot(c + d*x)**2 - 8*I*a**2*d*cot(c + d*x) - 4*a**2*d), Eq(b, I*a)), (3*A*d*x*tan(c + d*x)**2*cot(c + d*x)**2/(8*a**2*d*tan(c + d*x)**2*cot(c + d*x)**2 - 16*a**2*d*tan(c + d*x)*cot(c + d*x) + 8*a**2*d) + A*d*x*tan(c + d*x)**2/(8*a**2*d*tan(c + d*x)**2*cot(c + d*x)**2 - 16*a**2*d*tan(c + d* x)*cot(c + d*x) + 8*a**2*d) - 4*A*d*x*tan(c + d*x)*cot(c + d*x)/(8*a**2*d*t an(c + d*x)**2*cot(c + d*x)**2 - 16*a**2*d*tan(c + d*x)*cot(c + d*x) + 8*a* *2*d) + A*d*x*cot(c + d*x)**2/(8*a**2*d*tan(c + d*x)**2*cot(c + d*x)**2 - 1 6*a**2*d*tan(c + d*x)*cot(c + d*x) + 8*a**2*d) + 3*A*d*x/(8*a**2*d*tan(c + d*x)**2*cot(c + d*x)**2 - 16*a**2*d*tan(c + d*x)*cot(c + d*x) + 8*a**2*d) - 3*A*tan(c + d*x)**2*cot(c + d*x)/(8*a**2*d*tan(c + d*x)**2*cot(c + d*x)**2 - 16*a**2*d*tan(c + d*x)*cot(c + d*x) + 8*a**2*d) + 4*A*tan(c + d*x)/(8*a* *2*d*tan(c + d*x)**2*cot(c + d*x)**2 - 16*a**2*d*tan(c + d*x)*cot(c + d*x) + 8*a**2*d) - A*cot(c + d*x)/(8*a**2*d*tan(c + d*x)**2*cot(c + d*x)**2 - 16 *a**2*d*tan(c + d*x)*cot(c + d*x) + 8*a**2*d) - 2*B*d*x*tan(c + d*x)**2*cot (c + d*x)/(8*a**2*d*tan(c + d*x)**2*cot(c + d*x)**2 - 16*a**2*d*tan(c + d*x)*cot(c + d*x) + 8*a**2*d) - 2*B*d*x*tan(c + d*x)*cot(c + d*x)**2/(8*a**2*d *tan(c + d*x)**2*cot(c + d*x)**2 - 16*a**2*d*tan(c + d*x)*cot(c + d*x) + 8* a**2*d) + 2*B*d*x*tan(c + d*x)/(8*a**2*d*tan(c + d*x)**2*cot(c + d*x)**2 - 16*a**2*d*tan(c + d*x)*cot(c + d*x) + 8*a**2*d) + 2*B*d*x*cot(c + d*x)/(8*a **2*d*tan(c + d*x)**2*cot(c + d*x)**2 - 16*a**2*d*tan(c + d*x)*cot(c + d*x) + 8*a**2*d) + B*tan(c + d*x)**2/(8*a**2*d*tan(c + d*x)**2*cot(c + d*x)**2 - 16*a**2*d*tan(c + d*x)*cot(c + d*x) + 8*a**2*d) - B*cot(c + d*x)**2/(8*a* *2*d*tan(c + d*x)**2*cot(c + d*x)**2 - 16*a**2*d*tan(c + d*x)*cot(c + d*x) + 8*a**2*d) + 2*B/(8*a**2*d*tan(c + d*x)**2*cot(c + d*x)**2 - 16*a**2*d*tan (c + d*x)*cot(c + d*x) + 8*a**2*d), Eq(b, -a*tan(c + d*x))), (x*(A + B*cot(c))/(a + b*cot(c))**2, Eq(d, 0)), (2*A*a**4*d*x*tan(c + d*x)/(2*a**6*d*tan(c + d*x) + 2*a**5*b*d + 4*a**4*b**2*d*tan(c + d*x) + 4*a**3*b**3*d + 2*a**2 *b**4*d*tan(c + d*x) + 2*a*b**5*d) + 2*A*a**3*b*d*x/(2*a**6*d*tan(c + d*x) + 2*a**5*b*d + 4*a**4*b**2*d*tan(c + d*x) + 4*a**3*b**3*d + 2*a**2*b**4*d*t an(c + d*x) + 2*a*b**5*d) - 4*A*a**3*b*log(tan(c + d*x) + b/a)*tan(c + d*x) /(2*a**6*d*tan(c + d*x) + 2*a**5*b*d + 4*a**4*b**2*d*tan(c + d*x) + 4*a**3* b**3*d + 2*a**2*b**4*d*tan(c + d*x) + 2*a*b**5*d) + 2*A*a**3*b*log(tan(c + d*x)**2 + 1)*tan(c + d*x)/(2*a**6*d*tan(c + d*x) + 2*a**5*b*d + 4*a**4*b**2 *d*tan(c + d*x) + 4*a**3*b**3*d + 2*a**2*b**4*d*tan(c + d*x) + 2*a*b**5*d) - 2*A*a**2*b**2*d*x*tan(c + d*x)/(2*a**6*d*tan(c + d*x) + 2*a**5*b*d + 4*a* *4*b**2*d*tan(c + d*x) + 4*a**3*b**3*d + 2*a**2*b**4*d*tan(c + d*x) + 2*a*b **5*d) - 4*A*a**2*b**2*log(tan(c + d*x) + b/a)/(2*a**6*d*tan(c + d*x) + 2*a

```
**5*b*d + 4*a**4*b**2*d*tan(c + d*x) + 4*a**3*b**3*d + 2*a**2*b**4*d*tan(c
+ d*x) + 2*a*b**5*d) + 2*A*a**2*b**2*log(tan(c + d*x)**2 + 1)/(2*a**6*d*tan
(c + d*x) + 2*a**5*b*d + 4*a**4*b**2*d*tan(c + d*x) + 4*a**3*b**3*d + 2*a**
2*b**4*d*tan(c + d*x) + 2*a*b**5*d) - 2*A*a**2*b**2/(2*a**6*d*tan(c + d*x)
+ 2*a**5*b*d + 4*a**4*b**2*d*tan(c + d*x) + 4*a**3*b**3*d + 2*a**2*b**4*d*t
an(c + d*x) + 2*a*b**5*d) - 2*A*a*b**3*d*x/(2*a**6*d*tan(c + d*x) + 2*a**5*
b*d + 4*a**4*b**2*d*tan(c + d*x) + 4*a**3*b**3*d + 2*a**2*b**4*d*tan(c + d*
x) + 2*a*b**5*d) - 2*A*b**4/(2*a**6*d*tan(c + d*x) + 2*a**5*b*d + 4*a**4*b*
*2*d*tan(c + d*x) + 4*a**3*b**3*d + 2*a**2*b**4*d*tan(c + d*x) + 2*a*b**5*d
) + 2*B*a**4*log(tan(c + d*x) + b/a)*tan(c + d*x)/(2*a**6*d*tan(c + d*x) +
2*a**5*b*d + 4*a**4*b**2*d*tan(c + d*x) + 4*a**3*b**3*d + 2*a**2*b**4*d*tan
(c + d*x) + 2*a*b**5*d) - B*a**4*log(tan(c + d*x)**2 + 1)*tan(c + d*x)/(2*a
**6*d*tan(c + d*x) + 2*a**5*b*d + 4*a**4*b**2*d*tan(c + d*x) + 4*a**3*b**3*
d + 2*a**2*b**4*d*tan(c + d*x) + 2*a*b**5*d) + 4*B*a**3*b*d*x*tan(c + d*x)/
(2*a**6*d*tan(c + d*x) + 2*a**5*b*d + 4*a**4*b**2*d*tan(c + d*x) + 4*a**3*b
**3*d + 2*a**2*b**4*d*tan(c + d*x) + 2*a*b**5*d) + 2*B*a**3*b*log(tan(c + d
*x) + b/a)/(2*a**6*d*tan(c + d*x) + 2*a**5*b*d + 4*a**4*b**2*d*tan(c + d*x)
 + 4*a**3*b**3*d + 2*a**2*b**4*d*tan(c + d*x) + 2*a*b**5*d) - B*a**3*b*log(
tan(c + d*x)**2 + 1)/(2*a**6*d*tan(c + d*x) + 2*a**5*b*d + 4*a**4*b**2*d*ta
n(c + d*x) + 4*a**3*b**3*d + 2*a**2*b**4*d*tan(c + d*x) + 2*a*b**5*d) + 2*B
*a**3*b/(2*a**6*d*tan(c + d*x) + 2*a**5*b*d + 4*a**4*b**2*d*tan(c + d*x) +
4*a**3*b**3*d + 2*a**2*b**4*d*tan(c + d*x) + 2*a*b**5*d) + 4*B*a**2*b**2*d*
x/(2*a**6*d*tan(c + d*x) + 2*a**5*b*d + 4*a**4*b**2*d*tan(c + d*x) + 4*a**3
*b**3*d + 2*a**2*b**4*d*tan(c + d*x) + 2*a*b**5*d) - 2*B*a**2*b**2*log(tan(
c + d*x) + b/a)*tan(c + d*x)/(2*a**6*d*tan(c + d*x) + 2*a**5*b*d + 4*a**4*b
**2*d*tan(c + d*x) + 4*a**3*b**3*d + 2*a**2*b**4*d*tan(c + d*x) + 2*a*b**5*
d) + B*a**2*b**2*log(tan(c + d*x)**2 + 1)*tan(c + d*x)/(2*a**6*d*tan(c + d*
x) + 2*a**5*b*d + 4*a**4*b**2*d*tan(c + d*x) + 4*a**3*b**3*d + 2*a**2*b**4*
d*tan(c + d*x) + 2*a*b**5*d) - 2*B*a*b**3*log(tan(c + d*x) + b/a)/(2*a**6*d
*tan(c + d*x) + 2*a**5*b*d + 4*a**4*b**2*d*tan(c + d*x) + 4*a**3*b**3*d + 2
*a**2*b**4*d*tan(c + d*x) + 2*a*b**5*d) + B*a*b**3*log(tan(c + d*x)**2 + 1)
/(2*a**6*d*tan(c + d*x) + 2*a**5*b*d + 4*a**4*b**2*d*tan(c + d*x) + 4*a**3*
b**3*d + 2*a**2*b**4*d*tan(c + d*x) + 2*a*b**5*d) + 2*B*a*b**3/(2*a**6*d*ta
n(c + d*x) + 2*a**5*b*d + 4*a**4*b**2*d*tan(c + d*x) + 4*a**3*b**3*d + 2*a*
*2*b**4*d*tan(c + d*x) + 2*a*b**5*d), True))
```

Maxima [A] (verification not implemented)

none

Time = 0.44 (sec) , antiderivative size = 185, normalized size of antiderivative = 1.67

$$\int \frac{A + B\cot(c + dx)}{(a + b\cot(c + dx))^2}\,dx = \frac{\frac{2\,(Aa^2+2\,Bab-Ab^2)(dx+c)}{a^4+2\,a^2b^2+b^4} + \frac{2\,(Ba^2-2\,Aab-Bb^2)\log(a\tan(dx+c)+b)}{a^4+2\,a^2b^2+b^4} - \frac{(Ba^2-2\,Aab-Bb^2)\log\left(\tan(dx+c)^2+1\right)}{a^4+2\,a^2b^2+b^4} + \frac{2\,(Bab-Ab^2}{a^3b+ab^3+(a^4+a^2b^2)}}{2\,d}$$

[In] integrate((A+B*cot(d*x+c))/(a+b*cot(d*x+c))^2,x, algorithm="maxima")

[Out] 1/2*(2*(A*a^2 + 2*B*a*b - A*b^2)*(d*x + c)/(a^4 + 2*a^2*b^2 + b^4) + 2*(B*a^2 - 2*A*a*b - B*b^2)*log(a*tan(d*x + c) + b)/(a^4 + 2*a^2*b^2 + b^4) - (B*a^2 - 2*A*a*b - B*b^2)*log(tan(d*x + c)^2 + 1)/(a^4 + 2*a^2*b^2 + b^4) + 2*(B*a*b - A*b^2)/(a^3*b + a*b^3 + (a^4 + a^2*b^2)*tan(d*x + c)))/d

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 241 vs. 2(111) = 222.

Time = 0.35 (sec) , antiderivative size = 241, normalized size of antiderivative = 2.17

$$\int \frac{A + B\cot(c + dx)}{(a + b\cot(c + dx))^2}\,dx = \frac{\frac{2\,(Aa^2+2\,Bab-Ab^2)(dx+c)}{a^4+2\,a^2b^2+b^4} - \frac{(Ba^2-2\,Aab-Bb^2)\log\left(\tan(dx+c)^2+1\right)}{a^4+2\,a^2b^2+b^4} + \frac{2\,(Ba^3-2\,Aa^2b-Bab^2)\log(|a\tan(dx+c)+b|)}{a^5+2\,a^3b^2+ab^4} - \frac{2\,(Ba^4\tan(dx+c)}{}}{2\,d}$$

[In] integrate((A+B*cot(d*x+c))/(a+b*cot(d*x+c))^2,x, algorithm="giac")

[Out] 1/2*(2*(A*a^2 + 2*B*a*b - A*b^2)*(d*x + c)/(a^4 + 2*a^2*b^2 + b^4) - (B*a^2 - 2*A*a*b - B*b^2)*log(tan(d*x + c)^2 + 1)/(a^4 + 2*a^2*b^2 + b^4) + 2*(B*a^3 - 2*A*a^2*b - B*a*b^2)*log(abs(a*tan(d*x + c) + b))/(a^5 + 2*a^3*b^2 + a*b^4) - 2*(B*a^4*tan(d*x + c) - 2*A*a^3*b*tan(d*x + c) - B*a^2*b^2*tan(d*x + c) - A*a^2*b^2 - 2*B*a*b^3 + A*b^4)/((a^5 + 2*a^3*b^2 + a*b^4)*(a*tan(d*x + c) + b)))/d

Mupad [B] (verification not implemented)

Time = 14.09 (sec) , antiderivative size = 268, normalized size of antiderivative = 2.41

$$\int \frac{A+B\cot(c+dx)}{(a+b\cot(c+dx))^2}\,dx = \ln\left(a+b\cot(c+d\,x)\right)\,\left(\frac{B}{d\,\left(a^2+b^2\right)}-\frac{2\,B\,b^2}{d\,{\left(a^2+b^2\right)}^2}\right) + \frac{A\,\ln\left(\cot(c+d\,x)-\mathrm{i}\right)}{2\,\left(-1\mathrm{i}\,d\,a^2+2\,d\,a\,b+1\mathrm{i}\,d\,b^2\right)} - \frac{B\,\ln\left(\cot(c+d\,x)-\mathrm{i}\right)}{2\,\left(d\,a^2+2\mathrm{i}\,d\,a\,b-d\,b^2\right)} + \frac{A\,b}{\left(a\,d+b\,d\cot\left(c+d\,x\right)\right)\,\left(a^2+b^2\right)} - \frac{B\,a}{\left(a\,d+b\,d\cot\left(c+d\,x\right)\right)\,\left(a^2+b^2\right)} - \frac{2\,A\,a\,b\,\ln\left(a+b\cot(c+d\,x)\right)}{d\,{\left(a^2+b^2\right)}^2} + \frac{A\,\ln\left(\cot(c+d\,x)+1\mathrm{i}\right)\,1\mathrm{i}}{2\,\left(-d\,a^2+2\mathrm{i}\,d\,a\,b+d\,b^2\right)} - \frac{B\,\ln\left(\cot(c+d\,x)+1\mathrm{i}\right)\,1\mathrm{i}}{2\,\left(1\mathrm{i}\,d\,a^2+2\,d\,a\,b-1\mathrm{i}\,d\,b^2\right)}$$

```
[In] int((A + B*cot(c + d*x))/(a + b*cot(c + d*x))^2,x)
```

```
[Out] log(a + b*cot(c + d*x))*(B/(d*(a^2 + b^2)) - (2*B*b^2)/(d*(a^2 + b^2)^2)) +
 (A*log(cot(c + d*x) + 1i)*1i)/(2*(b^2*d - a^2*d + a*b*d*2i)) + (A*log(cot(
c + d*x) - 1i))/(2*(b^2*d*1i - a^2*d*1i + 2*a*b*d)) - (B*log(cot(c + d*x) -
 1i))/(2*(a^2*d - b^2*d + a*b*d*2i)) - (B*log(cot(c + d*x) + 1i)*1i)/(2*(a^
2*d*1i - b^2*d*1i + 2*a*b*d)) + (A*b)/((a*d + b*d*cot(c + d*x))*(a^2 + b^2)
) - (B*a)/((a*d + b*d*cot(c + d*x))*(a^2 + b^2)) - (2*A*a*b*log(a + b*cot(c
 + d*x)))/(d*(a^2 + b^2)^2)
```

3.94 $\int \frac{A+B\cot(c+dx)}{(a+b\cot(c+dx))^3}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 175

$$\begin{aligned}
&\int \frac{A+B\cot(c+dx)}{(a+b\cot(c+dx))^3}\,dx\\
&=\frac{(a^3A-3aAb^2+3a^2bB-b^3B)\,x}{(a^2+b^2)^3}\\
&\quad+\frac{Ab-aB}{2\,(a^2+b^2)\,d(a+b\cot(c+dx))^2}+\frac{2aAb-a^2B+b^2B}{(a^2+b^2)^2\,d(a+b\cot(c+dx))}\\
&\quad-\frac{(3a^2Ab-Ab^3-a^3B+3ab^2B)\log(b\cos(c+dx)+a\sin(c+dx))}{(a^2+b^2)^3\,d}
\end{aligned}$$

```
[Out] (A*a^3-3*A*a*b^2+3*B*a^2*b-B*b^3)*x/(a^2+b^2)^3+1/2*(A*b-B*a)/(a^2+b^2)/d/(
a+b*cot(d*x+c))^2+(2*A*a*b-B*a^2+B*b^2)/(a^2+b^2)^2/d/(a+b*cot(d*x+c))-(3*A
*a^2*b-A*b^3-B*a^3+3*B*a*b^2)*ln(b*cos(d*x+c)+a*sin(d*x+c))/(a^2+b^2)^3/d
```

Rubi [A] (verified)

Time = 0.33 (sec) , antiderivative size = 175, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.130, Rules used = {3610, 3612, 3611}

$$\int \frac{A+B\cot(c+dx)}{(a+b\cot(c+dx))^3}\,dx = \frac{Ab-aB}{2d\left(a^2+b^2\right)(a+b\cot(c+dx))^2}+\frac{a^2(-B)+2aAb+b^2B}{d\left(a^2+b^2\right)^2(a+b\cot(c+dx))} - \frac{\left(a^3(-B)+3a^2Ab+3ab^2B-Ab^3\right)\log(a\sin(c+dx)+b\cos(c+dx))}{d\left(a^2+b^2\right)^3} + \frac{x\left(a^3A+3a^2bB-3aAb^2-b^3B\right)}{\left(a^2+b^2\right)^3}$$

```
[In] Int[(A + B*Cot[c + d*x])/(a + b*Cot[c + d*x])^3,x]
```

```
[Out] ((a^3*A - 3*a*A*b^2 + 3*a^2*b*B - b^3*B)*x)/(a^2 + b^2)^3 + (A*b - a*B)/(2*
(a^2 + b^2)*d*(a + b*Cot[c + d*x])^2) + (2*a*A*b - a^2*B + b^2*B)/((a^2 + b
^2)^2*d*(a + b*Cot[c + d*x])) - ((3*a^2*A*b - A*b^3 - a^3*B + 3*a*b^2*B)*Lo
g[b*Cos[c + d*x] + a*Sin[c + d*x]])/((a^2 + b^2)^3*d)
```

Rule 3610

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)]), x_Symbol] :> Simp[(b*c - a*d)*((a + b*Tan[e + f*x])^(m + 1)/
(f*(m + 1)*(a^2 + b^2))), x] + Dist[1/(a^2 + b^2), Int[(a + b*Tan[e + f*x])
^(m + 1)*Simp[a*c + b*d - (b*c - a*d)*Tan[e + f*x], x], x], x] /; FreeQ[{a,
 b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && LtQ[m, -1]
```

Rule 3611

```
Int[((c_) + (d_.)*tan[(e_.) + (f_.)*(x_)])/((a_) + (b_.)*tan[(e_.) + (f_.)*
(x_)]), x_Symbol] :> Simp[(c/(b*f))*Log[RemoveContent[a*Cos[e + f*x] + b*Si
n[e + f*x], x]], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] &&
 NeQ[a^2 + b^2, 0] && EqQ[a*c + b*d, 0]
```

Rule 3612

```
Int[((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])/((a_.) + (b_.)*tan[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[(a*c + b*d)*(x/(a^2 + b^2)), x] + Dist[(b*c - a
*d)/(a^2 + b^2), Int[(b - a*Tan[e + f*x])/(a + b*Tan[e + f*x]), x], x] /; F
reeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && Ne
Q[a*c + b*d, 0]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{Ab-aB}{2\left(a^2+b^2\right)d(a+b\cot(c+dx))^2}+\frac{\int \frac{aA+bB-(Ab-aB)\cot(c+dx)}{(a+b\cot(c+dx))^2}\,dx}{a^2+b^2} \\
&= \frac{Ab-aB}{2\left(a^2+b^2\right)d(a+b\cot(c+dx))^2}+\frac{2aAb-a^2B+b^2B}{\left(a^2+b^2\right)^2d(a+b\cot(c+dx))} \\
&\quad+\frac{\int \frac{a^2A-Ab^2+2abB-\left(2aAb-a^2B+b^2B\right)\cot(c+dx)}{a+b\cot(c+dx)}\,dx}{\left(a^2+b^2\right)^2} \\
&= \frac{\left(a^3A-3aAb^2+3a^2bB-b^3B\right)x}{\left(a^2+b^2\right)^3}+\frac{Ab-aB}{2\left(a^2+b^2\right)d(a+b\cot(c+dx))^2} \\
&\quad+\frac{2aAb-a^2B+b^2B}{\left(a^2+b^2\right)^2d(a+b\cot(c+dx))} \\
&\quad-\frac{\left(3a^2Ab-Ab^3-a^3B+3ab^2B\right)\int \frac{-b+a\cot(c+dx)}{a+b\cot(c+dx)}\,dx}{\left(a^2+b^2\right)^3} \\
&= \frac{\left(a^3A-3aAb^2+3a^2bB-b^3B\right)x}{\left(a^2+b^2\right)^3}+\frac{Ab-aB}{2\left(a^2+b^2\right)d(a+b\cot(c+dx))^2} \\
&\quad+\frac{2aAb-a^2B+b^2B}{\left(a^2+b^2\right)^2d(a+b\cot(c+dx))} \\
&\quad-\frac{\left(3a^2Ab-Ab^3-a^3B+3ab^2B\right)\log(b\cos(c+dx)+a\sin(c+dx))}{\left(a^2+b^2\right)^3d}
\end{aligned}
$$

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 5.32 (sec) , antiderivative size = 202, normalized size of antiderivative = 1.15

$$
\int \frac{A+B\cot(c+dx)}{(a+b\cot(c+dx))^3}\,dx
$$

$$
=\frac{-\frac{i(A-iB)\log(i-\tan(c+dx))}{(a-ib)^3}+\frac{i(A+iB)\log(i+\tan(c+dx))}{(a+ib)^3}+\frac{2\left(-3a^2Ab+Ab^3+a^3B-3ab^2B\right)\log(b+a\tan(c+dx))-\frac{b\left(a^2+b^2\right)\left(b\left(5a^2Ab+Ab^3-3\right.\right.}{}}{\left(a^2+b^2\right)^3}}{2d}
$$

```
[In] Integrate[(A + B*Cot[c + d*x])/(a + b*Cot[c + d*x])^3,x]
```

```
[Out] (((-I)*(A - I*B)*Log[I - Tan[c + d*x]])/(a - I*b)^3 + (I*(A + I*B)*Log[I +
Tan[c + d*x]])/(a + I*b)^3 + (2*(-3*a^2*A*b + A*b^3 + a^3*B - 3*a*b^2*B)*Lo
g[b + a*Tan[c + d*x]] - (b*(a^2 + b^2)*(b*(5*a^2*A*b + A*b^3 - 3*a^3*B + a*
b^2*B) + (6*a^3*A*b + 2*a*A*b^3 - 4*a^4*B)*Tan[c + d*x]))/(a^2*(b + a*Tan[c
 + d*x])^2))/(a^2 + b^2)^3)/(2*d)
```

Maple [A] (verified)

Time = 0.25 (sec) , antiderivative size = 216, normalized size of antiderivative = 1.23

method	result
derivativedivides	$\frac{\frac{\frac{(3A a^2 b - A b^3 - B a^3 + 3B a b^2)\ln(\cot(dx+c)^2+1)}{2} + (-A a^3 + 3A a b^2 - 3B a^2 b + B b^3)(\frac{\pi}{2} - \operatorname{arccot}(\cot(dx+c)))}{(a^2+b^2)^3} - \frac{(3A a^2 b - A b^3 - B a^3 \cdots}{(\cdots}}{d}$
default	$\frac{\frac{\frac{(3A a^2 b - A b^3 - B a^3 + 3B a b^2)\ln(\cot(dx+c)^2+1)}{2} + (-A a^3 + 3A a b^2 - 3B a^2 b + B b^3)(\frac{\pi}{2} - \operatorname{arccot}(\cot(dx+c)))}{(a^2+b^2)^3} - \frac{(3A a^2 b - A b^3 - B a^3 \cdots}{(\cdots}}{d}$
parallelrisch	$\frac{-6a^2(A a^2 b - \frac{1}{3}A b^3 - \frac{1}{3}B a^3 + B a b^2)(a\tan(dx+c)+b)^2\ln(a\tan(dx+c)+b) + 3a^2(A a^2 b - \frac{1}{3}A b^3 - \frac{1}{3}B a^3 + B a b^2)(a\tan(d \cdots}{\cdots}$
norman	$\frac{\frac{b^2(A a^3 - 3A a b^2 + 3B a^2 b - B b^3)x}{(a^4+2a^2b^2+b^4)(a^2+b^2)} + \frac{(A a^3 - 3A a b^2 + 3B a^2 b - B b^3)a^2 x\tan(dx+c)^2}{(a^4+2a^2b^2+b^4)(a^2+b^2)} - \frac{b^2(5A a^2 b + A b^3 - 3B a^3 + B a b^2)}{2d a^2(a^4+2a^2b^2+b^4)} - \frac{b(3A a^2 b + \cdots}{da \cdots}}{(a\tan(dx+c)+b)^2}$
risch	$\frac{ixB}{3ia^2b - ib^3 + a^3 - 3a b^2} + \frac{xA}{3ia^2b - ib^3 + a^3 - 3a b^2} + \frac{6iA a^2 bx}{a^6+3b^2a^4+3b^4a^2+b^6} - \frac{2iA b^3 x}{a^6+3b^2a^4+3b^4a^2+b^6} - \frac{2iB a^3 x}{a^6+3b^2a^4+3b^4 \cdots}$

[In] int((A+B*cot(d*x+c))/(a+b*cot(d*x+c))^3,x,method=_RETURNVERBOSE)

[Out] 1/d*(1/(a^2+b^2)^3*(1/2*(3*A*a^2*b-A*b^3-B*a^3+3*B*a*b^2)*ln(cot(d*x+c)^2+1
)+(-A*a^3+3*A*a*b^2-3*B*a^2*b+B*b^3)*(1/2*Pi-arccot(cot(d*x+c))))-(3*A*a^2*
b-A*b^3-B*a^3+3*B*a*b^2)/(a^2+b^2)^3*ln(a+b*cot(d*x+c))+1/2*(A*b-B*a)/(a^2+
b^2)/(a+b*cot(d*x+c))^2+(2*A*a*b-B*a^2+B*b^2)/(a^2+b^2)^2/(a+b*cot(d*x+c)))

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 549 vs. $2(171) = 342$.

Time = 0.34 (sec) , antiderivative size = 549, normalized size of antiderivative = 3.14

$$\int \frac{A + B\cot(c+dx)}{(a+b\cot(c+dx))^3}\,dx = \frac{2\,Ba^3b^2 - 2\,Aa^2b^3 + 2\,Bab^4 - 2\,Ab^5 - 2\,(Aa^5 + 3\,Ba^4b - 2\,Aa^3b^2 + 2\,Ba^2b^3 - 3\,Aab^4 - Bb^5)dx - 2\,(4\,B \cdots}{\cdots}$$

[In] integrate((A+B*cot(d*x+c))/(a+b*cot(d*x+c))^3,x, algorithm="fricas")

[Out] 1/2*(2*B*a^3*b^2 - 2*A*a^2*b^3 + 2*B*a*b^4 - 2*A*b^5 - 2*(A*a^5 + 3*B*a^4*b
 - 2*A*a^3*b^2 + 2*B*a^2*b^3 - 3*A*a*b^4 - B*b^5)*d*x - 2*(4*B*a^3*b^2 - 6*
A*a^2*b^3 - 2*B*a*b^4 - (A*a^5 + 3*B*a^4*b - 4*A*a^3*b^2 - 4*B*a^2*b^3 + 3*
A*a*b^4 + B*b^5)*d*x)*cos(2*d*x + 2*c) - (B*a^5 - 3*A*a^4*b - 2*B*a^3*b^2 -
 2*A*a^2*b^3 - 3*B*a*b^4 + A*b^5 - (B*a^5 - 3*A*a^4*b - 4*B*a^3*b^2 + 4*A*a
^2*b^3 + 3*B*a*b^4 - A*b^5)*cos(2*d*x + 2*c) + 2*(B*a^4*b - 3*A*a^3*b^2 - 3

```
*B*a^2*b^3 + A*a*b^4)*sin(2*d*x + 2*c))*log(a*b*sin(2*d*x + 2*c) + 1/2*a^2
+ 1/2*b^2 - 1/2*(a^2 - b^2)*cos(2*d*x + 2*c)) - 2*(2*B*a^4*b - 3*A*a^3*b^2
- 3*B*a^2*b^3 + 3*A*a*b^4 + B*b^5 + 2*(A*a^4*b + 3*B*a^3*b^2 - 3*A*a^2*b^3
- B*a*b^4)*d*x)*sin(2*d*x + 2*c))/((a^8 + 2*a^6*b^2 - 2*a^2*b^6 - b^8)*d*co
s(2*d*x + 2*c) - 2*(a^7*b + 3*a^5*b^3 + 3*a^3*b^5 + a*b^7)*d*sin(2*d*x + 2*
c) - (a^8 + 4*a^6*b^2 + 6*a^4*b^4 + 4*a^2*b^6 + b^8)*d)
```

Sympy [F(-2)]

Exception generated.

$$\int \frac{A+B\cot(c+dx)}{(a+b\cot(c+dx))^3}\,dx = \text{Exception raised: AttributeError}$$

```
[In] integrate((A+B*cot(d*x+c))/(a+b*cot(d*x+c))**3,x)
```

```
[Out] Exception raised: AttributeError >> 'NoneType' object has no attribute 'pri
mitive'
```

Maxima [A] (verification not implemented)

none

Time = 0.40 (sec) , antiderivative size = 337, normalized size of antiderivative = 1.93

$$\int \frac{A+B\cot(c+dx)}{(a+b\cot(c+dx))^3}\,dx = \frac{\frac{2\,(Aa^3+3\,Ba^2b-3\,Aab^2-Bb^3)(dx+c)}{a^6+3\,a^4b^2+3\,a^2b^4+b^6} + \frac{2\,(Ba^3-3\,Aa^2b-3\,Bab^2+Ab^3)\log(a\tan(dx+c)+b)}{a^6+3\,a^4b^2+3\,a^2b^4+b^6} - \frac{(Ba^3-3\,Aa^2b-3\,Bab^2+Ab^3)\log\left(\tan(dx+c)^2+\right.}{a^6+3\,a^4b^2+3\,a^2b^4+b^6}}{2\,d}$$

```
[In] integrate((A+B*cot(d*x+c))/(a+b*cot(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] 1/2*(2*(A*a^3 + 3*B*a^2*b - 3*A*a*b^2 - B*b^3)*(d*x + c)/(a^6 + 3*a^4*b^2 +
 3*a^2*b^4 + b^6) + 2*(B*a^3 - 3*A*a^2*b - 3*B*a*b^2 + A*b^3)*log(a*tan(d*x
 + c) + b)/(a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6) - (B*a^3 - 3*A*a^2*b - 3*B*a
*b^2 + A*b^3)*log(tan(d*x + c)^2 + 1)/(a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6) +
 (3*B*a^3*b^2 - 5*A*a^2*b^3 - B*a*b^4 - A*b^5 + 2*(2*B*a^4*b - 3*A*a^3*b^2
- A*a*b^4)*tan(d*x + c))/(a^6*b^2 + 2*a^4*b^4 + a^2*b^6 + (a^8 + 2*a^6*b^2
+ a^4*b^4)*tan(d*x + c)^2 + 2*(a^7*b + 2*a^5*b^3 + a^3*b^5)*tan(d*x + c)))/
d
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 412 vs. $2(171) = 342$.

Time = 0.43 (sec) , antiderivative size = 412, normalized size of antiderivative = 2.35

$$\int \frac{A+B\cot(c+dx)}{(a+b\cot(c+dx))^3}\,dx = \frac{\frac{2\left(Aa^3+3\,Ba^2b-3\,Aab^2-Bb^3\right)(dx+c)}{a^6+3\,a^4b^2+3\,a^2b^4+b^6} - \frac{\left(Ba^3-3\,Aa^2b-3\,Bab^2+Ab^3\right)\log\left(\tan(dx+c)^2+1\right)}{a^6+3\,a^4b^2+3\,a^2b^4+b^6} + \frac{2\left(Ba^4-3\,Aa^3b-3\,Ba^2b^2+Aab^3\right)\log(|a\tan(dx}{a^7+3\,a^5b^2+3\,a^3b^4+ab^6}}{}$$

```
[In] integrate((A+B*cot(d*x+c))/(a+b*cot(d*x+c))^3,x, algorithm="giac")
[Out] 1/2*(2*(A*a^3 + 3*B*a^2*b - 3*A*a*b^2 - B*b^3)*(d*x + c)/(a^6 + 3*a^4*b^2 +
 3*a^2*b^4 + b^6) - (B*a^3 - 3*A*a^2*b - 3*B*a*b^2 + A*b^3)*log(tan(d*x + c
)^2 + 1)/(a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6) + 2*(B*a^4 - 3*A*a^3*b - 3*B*a
^2*b^2 + A*a*b^3)*log(abs(a*tan(d*x + c) + b))/(a^7 + 3*a^5*b^2 + 3*a^3*b^4
 + a*b^6) - (3*B*a^7*tan(d*x + c)^2 - 9*A*a^6*b*tan(d*x + c)^2 - 9*B*a^5*b^
2*tan(d*x + c)^2 + 3*A*a^4*b^3*tan(d*x + c)^2 + 2*B*a^6*b*tan(d*x + c) - 12
*A*a^5*b^2*tan(d*x + c) - 22*B*a^4*b^3*tan(d*x + c) + 14*A*a^3*b^4*tan(d*x
+ c) + 2*A*a*b^6*tan(d*x + c) - 4*A*a^4*b^3 - 11*B*a^3*b^4 + 9*A*a^2*b^5 +
B*a*b^6 + A*b^7)/((a^8 + 3*a^6*b^2 + 3*a^4*b^4 + a^2*b^6)*(a*tan(d*x + c) +
 b)^2))/d
```

Mupad [B] (verification not implemented)

Time = 15.12 (sec) , antiderivative size = 481, normalized size of antiderivative = 2.75

$$\begin{aligned}\int \frac{A+B\cot(c+dx)}{(a+b\cot(c+dx))^3}\,dx = {} & \frac{\frac{5\,A\,a^2\,b+A\,b^3}{2\,(a^4+2\,a^2\,b^2+b^4)}+\frac{2\,A\,a\,b^2\cot(c+d\,x)}{a^4+2\,a^2\,b^2+b^4}}{d\,a^2+2\,d\,a\,b\cot\left(c+d\,x\right)+d\,b^2\cot\left(c+d\,x\right)^2} \\ & -\ln\left(a+b\cot(c+d\,x)\right)\left(\frac{3\,A\,b}{d\,\left(a^2+b^2\right)^2}-\frac{4\,A\,b^3}{d\,\left(a^2+b^2\right)^3}\right) \\ & -\frac{\frac{3\,B\,a^3-B\,a\,b^2}{2\,(a^4+2\,a^2\,b^2+b^4)}-\frac{\cot(c+d\,x)\,(B\,b^3-B\,a^2\,b)}{a^4+2\,a^2\,b^2+b^4}}{d\,a^2+2\,d\,a\,b\cot\left(c+d\,x\right)+d\,b^2\cot\left(c+d\,x\right)^2} \\ & +\ln\left(a+b\cot(c+d\,x)\right)\left(\frac{B\,a}{d\,\left(a^2+b^2\right)^2}-\frac{4\,B\,a\,b^2}{d\,\left(a^2+b^2\right)^3}\right) \\ & +\frac{A\,\ln\left(\cot(c+d\,x)-\mathrm{i}\right)\,1\mathrm{i}}{2\,\left(d\,a^3+3\mathrm{i}\,d\,a^2\,b-3\,d\,a\,b^2-1\mathrm{i}\,d\,b^3\right)} \\ & +\frac{A\,\ln\left(\cot(c+d\,x)+1\mathrm{i}\right)}{2\,\left(1\mathrm{i}\,d\,a^3+3\,d\,a^2\,b-3\mathrm{i}\,d\,a\,b^2-d\,b^3\right)} \\ & -\frac{B\,\ln\left(\cot(c+d\,x)-\mathrm{i}\right)\,1\mathrm{i}}{2\,\left(1\mathrm{i}\,d\,a^3-3\,d\,a^2\,b-3\mathrm{i}\,d\,a\,b^2+d\,b^3\right)} \\ & -\frac{B\,\ln\left(\cot(c+d\,x)+1\mathrm{i}\right)}{2\,\left(d\,a^3-3\mathrm{i}\,d\,a^2\,b-3\,d\,a\,b^2+1\mathrm{i}\,d\,b^3\right)}\end{aligned}$$

```
[In] int((A + B*cot(c + d*x))/(a + b*cot(c + d*x))^3,x)
```

```
[Out] ((A*b^3 + 5*A*a^2*b)/(2*(a^4 + b^4 + 2*a^2*b^2)) + (2*A*a*b^2*cot(c + d*x))
/(a^4 + b^4 + 2*a^2*b^2))/(a^2*d + b^2*d*cot(c + d*x)^2 + 2*a*b*d*cot(c + d
*x)) - log(a + b*cot(c + d*x))*((3*A*b)/(d*(a^2 + b^2)^2) - (4*A*b^3)/(d*(a
^2 + b^2)^3)) - ((3*B*a^3 - B*a*b^2)/(2*(a^4 + b^4 + 2*a^2*b^2)) - (cot(c +
 d*x)*(B*b^3 - B*a^2*b))/(a^4 + b^4 + 2*a^2*b^2))/(a^2*d + b^2*d*cot(c + d*
x)^2 + 2*a*b*d*cot(c + d*x)) + log(a + b*cot(c + d*x))*((B*a)/(d*(a^2 + b^2
)^2) - (4*B*a*b^2)/(d*(a^2 + b^2)^3)) + (A*log(cot(c + d*x) - 1i)*1i)/(2*(a
^3*d - b^3*d*1i - 3*a*b^2*d + a^2*b*d*3i)) + (A*log(cot(c + d*x) + 1i))/(2*
(a^3*d*1i - b^3*d - a*b^2*d*3i + 3*a^2*b*d)) - (B*log(cot(c + d*x) - 1i)*1i
)/(2*(a^3*d*1i + b^3*d - a*b^2*d*3i - 3*a^2*b*d)) - (B*log(cot(c + d*x) + 1
i))/(2*(a^3*d + b^3*d*1i - 3*a*b^2*d - a^2*b*d*3i))
```

3.95 $\int (a+b\cot(c+dx))^{5/2}(A+B\cot(c+dx))\,dx$

Optimal result

Integrand size = 25, antiderivative size = 188

$$\int (a+b\cot(c+dx))^{5/2}(A+B\cot(c+dx))\,dx = \frac{(a-ib)^{5/2}(iA+B)\operatorname{arctanh}\left(\frac{\sqrt{a+b\cot(c+dx)}}{\sqrt{a-ib}}\right)}{d} - \frac{(a+ib)^{5/2}(iA-B)\operatorname{arctanh}\left(\frac{\sqrt{a+b\cot(c+dx)}}{\sqrt{a+ib}}\right)}{d} - \frac{2(2aAb+a^2B-b^2B)\sqrt{a+b\cot(c+dx)}}{d} - \frac{2(Ab+aB)(a+b\cot(c+dx))^{3/2}}{3d} - \frac{2B(a+b\cot(c+dx))^{5/2}}{5d}$$

```
[Out] (a-I*b)^(5/2)*(I*A+B)*arctanh((a+b*cot(d*x+c))^(1/2)/(a-I*b)^(1/2))/d-(a+I*
b)^(5/2)*(I*A-B)*arctanh((a+b*cot(d*x+c))^(1/2)/(a+I*b)^(1/2))/d-2/3*(A*b+B
*a)*(a+b*cot(d*x+c))^(3/2)/d-2/5*B*(a+b*cot(d*x+c))^(5/2)/d-2*(2*A*a*b+B*a^
2-B*b^2)*(a+b*cot(d*x+c))^(1/2)/d
```

Rubi [A] (verified)

Time = 0.55 (sec) , antiderivative size = 188, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used

$= \{3609, 3620, 3618, 65, 214\}$

$$\int (a+b\cot(c+dx))^{5/2}(A+B\cot(c+dx))\,dx = -\frac{2(a^2B+2aAb-b^2B)\sqrt{a+b\cot(c+dx)}}{d} + \frac{(a-ib)^{5/2}(B+iA)\mathrm{arctanh}\left(\frac{\sqrt{a+b\cot(c+dx)}}{\sqrt{a-ib}}\right)}{d} - \frac{(a+ib)^{5/2}(-B+iA)\mathrm{arctanh}\left(\frac{\sqrt{a+b\cot(c+dx)}}{\sqrt{a+ib}}\right)}{d} - \frac{2(aB+Ab)(a+b\cot(c+dx))^{3/2}}{3d} - \frac{2B(a+b\cot(c+dx))^{5/2}}{5d}$$

```
[In] Int[(a + b*Cot[c + d*x])^(5/2)*(A + B*Cot[c + d*x]),x]
```

```
[Out] ((a - I*b)^(5/2)*(I*A + B)*ArcTanh[Sqrt[a + b*Cot[c + d*x]]/Sqrt[a - I*b]])
/d - ((a + I*b)^(5/2)*(I*A - B)*ArcTanh[Sqrt[a + b*Cot[c + d*x]]/Sqrt[a + I
*b]])/d - (2*(2*a*A*b + a^2*B - b^2*B)*Sqrt[a + b*Cot[c + d*x]])/d - (2*(A*
b + a*B)*(a + b*Cot[c + d*x])^(3/2))/(3*d) - (2*B*(a + b*Cot[c + d*x])^(5/2
))/(5*d)
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 3609

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)]), x_Symbol] :> Simp[d*((a + b*Tan[e + f*x])^m/(f*m)), x] + Int
[(a + b*Tan[e + f*x])^(m - 1)*Simp[a*c - b*d + (b*c + a*d)*Tan[e + f*x], x]
, x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2,
 0] && GtQ[m, 0]
```

Rule 3618

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*tan[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Dist[c*(d/f), Subst[Int[(a + (b/d)*x)^m/(d^2 + c
```

```
*x), x], x, d*Tan[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b
*c - a*d, 0] && NeQ[a^2 + b^2, 0] && EqQ[c^2 + d^2, 0]
```

Rule 3620

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)]), x_Symbol] :> Dist[(c + I*d)/2, Int[(a + b*Tan[e + f*x])^m*(1
 - I*Tan[e + f*x]), x], x] + Dist[(c - I*d)/2, Int[(a + b*Tan[e + f*x])^m*(
1 + I*Tan[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b*c -
 a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] &&  !IntegerQ[m]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= -\frac{2B(a+b\cot(c+dx))^{5/2}}{5d} \\
&\quad + \int (a+b\cot(c+dx))^{3/2}(aA-bB+(Ab+aB)\cot(c+dx))\,dx \\
&= -\frac{2(Ab+aB)(a+b\cot(c+dx))^{3/2}}{3d} - \frac{2B(a+b\cot(c+dx))^{5/2}}{5d} \\
&\quad + \int \sqrt{a+b\cot(c+dx)}\left(a^2A-Ab^2-2abB+\left(2aAb+a^2B-b^2B\right)\cot(c+dx)\right)\,dx \\
&= -\frac{2(2aAb+a^2B-b^2B)\sqrt{a+b\cot(c+dx)}}{d} \\
&\quad - \frac{2(Ab+aB)(a+b\cot(c+dx))^{3/2}}{3d} - \frac{2B(a+b\cot(c+dx))^{5/2}}{5d} \\
&\quad + \int \frac{a^3A-3aAb^2-3a^2bB+b^3B+(3a^2Ab-Ab^3+a^3B-3ab^2B)\cot(c+dx)}{\sqrt{a+b\cot(c+dx)}}\,dx \\
&= -\frac{2(2aAb+a^2B-b^2B)\sqrt{a+b\cot(c+dx)}}{d} - \frac{2(Ab+aB)(a+b\cot(c+dx))^{3/2}}{3d} \\
&\quad - \frac{2B(a+b\cot(c+dx))^{5/2}}{5d} + \frac{1}{2}\left((a-ib)^3(A-iB)\right)\int \frac{1+i\cot(c+dx)}{\sqrt{a+b\cot(c+dx)}}\,dx \\
&\quad + \frac{1}{2}\left((a+ib)^3(A+iB)\right)\int \frac{1-i\cot(c+dx)}{\sqrt{a+b\cot(c+dx)}}\,dx \\
&= -\frac{2(2aAb+a^2B-b^2B)\sqrt{a+b\cot(c+dx)}}{d} \\
&\quad - \frac{2(Ab+aB)(a+b\cot(c+dx))^{3/2}}{3d} - \frac{2B(a+b\cot(c+dx))^{5/2}}{5d} \\
&\quad + \frac{\left(i(a+ib)^3(A+iB)\right)\operatorname{Subst}\left(\int \frac{1}{(-1+x)\sqrt{a+ibx}}\,dx, x, -i\cot(c+dx)\right)}{2d} \\
&\quad - \frac{\left((a-ib)^3(iA+B)\right)\operatorname{Subst}\left(\int \frac{1}{(-1+x)\sqrt{a-ibx}}\,dx, x, i\cot(c+dx)\right)}{2d}
\end{aligned}
$$

$$= -\frac{2(2aAb+a^2B-b^2B)\sqrt{a+b\cot(c+dx)}}{d} - \frac{2(Ab+aB)(a+b\cot(c+dx))^{3/2}}{3d} - \frac{2B(a+b\cot(c+dx))^{5/2}}{5d} + \frac{((a-ib)^3(A-iB))\operatorname{Subst}\left(\int \frac{1}{-1-\frac{ia}{b}+\frac{ix^2}{b}}\,dx, x, \sqrt{a+b\cot(c+dx)}\right)}{bd} + \frac{((a+ib)^3(A+iB))\operatorname{Subst}\left(\int \frac{1}{-1+\frac{ia}{b}-\frac{ix^2}{b}}\,dx, x, \sqrt{a+b\cot(c+dx)}\right)}{bd}$$

$$= \frac{(a-ib)^{5/2}(iA+B)\operatorname{arctanh}\left(\frac{\sqrt{a+b\cot(c+dx)}}{\sqrt{a-ib}}\right)}{d} - \frac{(a+ib)^{5/2}(iA-B)\operatorname{arctanh}\left(\frac{\sqrt{a+b\cot(c+dx)}}{\sqrt{a+ib}}\right)}{d} - \frac{2(2aAb+a^2B-b^2B)\sqrt{a+b\cot(c+dx)}}{d} - \frac{2(Ab+aB)(a+b\cot(c+dx))^{3/2}}{3d} - \frac{2B(a+b\cot(c+dx))^{5/2}}{5d}$$

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 379 vs. 2(188) = 376.

Time = 1.91 (sec) , antiderivative size = 379, normalized size of antiderivative = 2.02

$$\int (a+b\cot(c+dx))^{5/2}(A+B\cot(c+dx))\,dx =$$

$$-\frac{2\left(\frac{\sqrt{a-\sqrt{-b^2}}\left(-3a^2b\left(A\sqrt{-b^2}+bB\right)+b^3\left(A\sqrt{-b^2}+bB\right)+a^3\left(Ab-\sqrt{-b^2}B\right)+3ab^2\left(-Ab+\sqrt{-b^2}B\right)\right)\operatorname{arctanh}\left(\frac{\sqrt{a+b\cot(c+dx)}}{\sqrt{a-\sqrt{-b^2}}}\right)}{2\left(b^2+a\sqrt{-b^2}\right)} + \frac{\left(b^3\left(A\sqrt{-\right.\right.}{}\right.}{}$$

```
[In] Integrate[(a + b*Cot[c + d*x])^(5/2)*(A + B*Cot[c + d*x]),x]
```

```
[Out] (-2*((Sqrt[a - Sqrt[-b^2]]*(-3*a^2*b*(A*Sqrt[-b^2] + b*B) + b^3*(A*Sqrt[-b^
2] + b*B) + a^3*(A*b - Sqrt[-b^2]*B) + 3*a*b^2*(-(A*b) + Sqrt[-b^2]*B))*Arc
Tanh[Sqrt[a + b*Cot[c + d*x]]/Sqrt[a - Sqrt[-b^2]]])/(2*(b^2 + a*Sqrt[-b^2]
)) + ((b^3*(A*Sqrt[-b^2] - b*B) + 3*a^2*b*(-(A*Sqrt[-b^2]) + b*B) - a^3*(A*
b + Sqrt[-b^2]*B) + 3*a*b^2*(A*b + Sqrt[-b^2]*B))*ArcTanh[Sqrt[a + b*Cot[c
+ d*x]]/Sqrt[a + Sqrt[-b^2]]])/(2*Sqrt[-b^2]*Sqrt[a + Sqrt[-b^2]]) + (2*a*A
*b + a^2*B - b^2*B)*Sqrt[a + b*Cot[c + d*x]] + ((A*b + a*B)*(a + b*Cot[c +
d*x])^(3/2))/3 + (B*(a + b*Cot[c + d*x])^(5/2))/5))/d
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 2391 vs. 2(160) = 320.

Time = 0.17 (sec) , antiderivative size = 2392, normalized size of antiderivative = 12.72

method	result	size
parts	Expression too large to display	2392
derivativedivides	Expression too large to display	2405
default	Expression too large to display	2405

```
[In] int((a+b*cot(d*x+c))^(5/2)*(A+B*cot(d*x+c)),x,method=_RETURNVERBOSE)
```

```
[Out] A*(-2/3*b*(a+b*cot(d*x+c))^(3/2)/d-4/d*b*(a+b*cot(d*x+c))^(1/2)*a+1/4/d/b*l
n(b*cot(d*x+c)+a+(a+b*cot(d*x+c))^(1/2)*(2*(a^2+b^2)^(1/2)+2*a)^(1/2)+(a^2+
b^2)^(1/2))*(2*(a^2+b^2)^(1/2)+2*a)^(1/2)*(a^2+b^2)^(1/2)*a^2-1/4/d*b*ln(b*
cot(d*x+c)+a+(a+b*cot(d*x+c))^(1/2)*(2*(a^2+b^2)^(1/2)+2*a)^(1/2)+(a^2+b^2)
^(1/2))*(2*(a^2+b^2)^(1/2)+2*a)^(1/2)*(a^2+b^2)^(1/2)-1/4/d/b*ln(b*cot(d*x+
c)+a+(a+b*cot(d*x+c))^(1/2)*(2*(a^2+b^2)^(1/2)+2*a)^(1/2)+(a^2+b^2)^(1/2))*
(2*(a^2+b^2)^(1/2)+2*a)^(1/2)*a^3+3/4/d*b*ln(b*cot(d*x+c)+a+(a+b*cot(d*x+c)
)^(1/2)*(2*(a^2+b^2)^(1/2)+2*a)^(1/2)+(a^2+b^2)^(1/2))*(2*(a^2+b^2)^(1/2)+2
*a)^(1/2)*a+2/d*b/(2*(a^2+b^2)^(1/2)-2*a)^(1/2)*arctan((2*(a+b*cot(d*x+c))^
(1/2)+(2*(a^2+b^2)^(1/2)+2*a)^(1/2))/(2*(a^2+b^2)^(1/2)-2*a)^(1/2))*(a^2+b^
2)^(1/2)*a-3/d*b/(2*(a^2+b^2)^(1/2)-2*a)^(1/2)*arctan((2*(a+b*cot(d*x+c))^(
1/2)+(2*(a^2+b^2)^(1/2)+2*a)^(1/2))/(2*(a^2+b^2)^(1/2)-2*a)^(1/2))*a^2+1/d*
b^3/(2*(a^2+b^2)^(1/2)-2*a)^(1/2)*arctan((2*(a+b*cot(d*x+c))^(1/2)+(2*(a^2+
b^2)^(1/2)+2*a)^(1/2))/(2*(a^2+b^2)^(1/2)-2*a)^(1/2))-1/4/d/b*ln((a+b*cot(d
*x+c))^(1/2)*(2*(a^2+b^2)^(1/2)+2*a)^(1/2)-b*cot(d*x+c)-(a^2+b^2)^(1/2)-a)*
(2*(a^2+b^2)^(1/2)+2*a)^(1/2)*(a^2+b^2)^(1/2)*a^2+1/4/d*b*ln((a+b*cot(d*x+c
))^(1/2)*(2*(a^2+b^2)^(1/2)+2*a)^(1/2)-b*cot(d*x+c)-(a^2+b^2)^(1/2)-a)*(2*(
a^2+b^2)^(1/2)+2*a)^(1/2)*(a^2+b^2)^(1/2)+1/4/d/b*ln((a+b*cot(d*x+c))^(1/2)
*(2*(a^2+b^2)^(1/2)+2*a)^(1/2)-b*cot(d*x+c)-(a^2+b^2)^(1/2)-a)*(2*(a^2+b^2)
^(1/2)+2*a)^(1/2)*a^3-3/4/d*b*ln((a+b*cot(d*x+c))^(1/2)*(2*(a^2+b^2)^(1/2)+
2*a)^(1/2)-b*cot(d*x+c)-(a^2+b^2)^(1/2)-a)*(2*(a^2+b^2)^(1/2)+2*a)^(1/2)*a-
2/d*b/(2*(a^2+b^2)^(1/2)-2*a)^(1/2)*arctan((-2*(a+b*cot(d*x+c))^(1/2)+(2*(a
^2+b^2)^(1/2)+2*a)^(1/2))/(2*(a^2+b^2)^(1/2)-2*a)^(1/2))*(a^2+b^2)^(1/2)*a+
3/d*b/(2*(a^2+b^2)^(1/2)-2*a)^(1/2)*arctan((-2*(a+b*cot(d*x+c))^(1/2)+(2*(a
^2+b^2)^(1/2)+2*a)^(1/2))/(2*(a^2+b^2)^(1/2)-2*a)^(1/2))*a^2-1/d*b^3/(2*(a^
2+b^2)^(1/2)-2*a)^(1/2)*arctan((-2*(a+b*cot(d*x+c))^(1/2)+(2*(a^2+b^2)^(1/2
)+2*a)^(1/2))/(2*(a^2+b^2)^(1/2)-2*a)^(1/2)))-1/2/d*ln(b*cot(d*x+c)+a+(a+b*
cot(d*x+c))^(1/2)*(2*(a^2+b^2)^(1/2)+2*a)^(1/2)+(a^2+b^2)^(1/2))*B*(2*(a^2+
b^2)^(1/2)+2*a)^(1/2)*(a^2+b^2)^(1/2)*a+1/d/(2*(a^2+b^2)^(1/2)-2*a)^(1/2)*a
rctan((2*(a+b*cot(d*x+c))^(1/2)+(2*(a^2+b^2)^(1/2)+2*a)^(1/2))/(2*(a^2+b^2)
^(1/2)-2*a)^(1/2))*B*(a^2+b^2)^(1/2)*a^2-3/4/d*ln((a+b*cot(d*x+c))^(1/2)*(2
*(a^2+b^2)^(1/2)+2*a)^(1/2)-b*cot(d*x+c)-(a^2+b^2)^(1/2)-a)*B*(2*(a^2+b^2)^
(1/2)+2*a)^(1/2)*a^2+1/4/d*b^2*ln((a+b*cot(d*x+c))^(1/2)*(2*(a^2+b^2)^(1/2)
```

```
+2*a)^(1/2)-b*cot(d*x+c)-(a^2+b^2)^(1/2)-a)*B*(2*(a^2+b^2)^(1/2)+2*a)^(1/2)
-1/d/(2*(a^2+b^2)^(1/2)-2*a)^(1/2)*arctan((2*(a+b*cot(d*x+c))^(1/2)+(2*(a^2
+b^2)^(1/2)+2*a)^(1/2))/(2*(a^2+b^2)^(1/2)-2*a)^(1/2))*B*a^3+1/d/(2*(a^2+b^
2)^(1/2)-2*a)^(1/2)*arctan((-2*(a+b*cot(d*x+c))^(1/2)+(2*(a^2+b^2)^(1/2)+2*
a)^(1/2))/(2*(a^2+b^2)^(1/2)-2*a)^(1/2))*B*a^3+3/4/d*ln(b*cot(d*x+c)+a+(a+b
*cot(d*x+c))^(1/2)*(2*(a^2+b^2)^(1/2)+2*a)^(1/2)+(a^2+b^2)^(1/2))*B*(2*(a^2
+b^2)^(1/2)+2*a)^(1/2)*a^2-1/4/d*b^2*ln(b*cot(d*x+c)+a+(a+b*cot(d*x+c))^(1/
2)*(2*(a^2+b^2)^(1/2)+2*a)^(1/2)+(a^2+b^2)^(1/2))*B*(2*(a^2+b^2)^(1/2)+2*a)
^(1/2)-1/d*b^2/(2*(a^2+b^2)^(1/2)-2*a)^(1/2)*arctan((2*(a+b*cot(d*x+c))^(1/
2)+(2*(a^2+b^2)^(1/2)+2*a)^(1/2))/(2*(a^2+b^2)^(1/2)-2*a)^(1/2))*B*(a^2+b^2
)^(1/2)+1/2/d*ln((a+b*cot(d*x+c))^(1/2)*(2*(a^2+b^2)^(1/2)+2*a)^(1/2)-b*cot
(d*x+c)-(a^2+b^2)^(1/2)-a)*B*(2*(a^2+b^2)^(1/2)+2*a)^(1/2)*(a^2+b^2)^(1/2)*
a-1/d/(2*(a^2+b^2)^(1/2)-2*a)^(1/2)*arctan((-2*(a+b*cot(d*x+c))^(1/2)+(2*(a
^2+b^2)^(1/2)+2*a)^(1/2))/(2*(a^2+b^2)^(1/2)-2*a)^(1/2))*B*(a^2+b^2)^(1/2)*
a^2+1/d*b^2/(2*(a^2+b^2)^(1/2)-2*a)^(1/2)*arctan((-2*(a+b*cot(d*x+c))^(1/2)
+(2*(a^2+b^2)^(1/2)+2*a)^(1/2))/(2*(a^2+b^2)^(1/2)-2*a)^(1/2))*B*(a^2+b^2)^
(1/2)+3/d*b^2/(2*(a^2+b^2)^(1/2)-2*a)^(1/2)*arctan((2*(a+b*cot(d*x+c))^(1/2
)+(2*(a^2+b^2)^(1/2)+2*a)^(1/2))/(2*(a^2+b^2)^(1/2)-2*a)^(1/2))*B*a-3/d*b^2
/(2*(a^2+b^2)^(1/2)-2*a)^(1/2)*arctan((-2*(a+b*cot(d*x+c))^(1/2)+(2*(a^2+b^
2)^(1/2)+2*a)^(1/2))/(2*(a^2+b^2)^(1/2)-2*a)^(1/2))*B*a-2/5*B*(a+b*cot(d*x+
c))^(5/2)/d-2/3/d*B*a*(a+b*cot(d*x+c))^(3/2)-2/d*(a+b*cot(d*x+c))^(1/2)*B*a
^2+2/d*(a+b*cot(d*x+c))^(1/2)*B*b^2
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 5073 vs. $2(154) = 308$.

Time = 1.22 (sec) , antiderivative size = 5073, normalized size of antiderivative = 26.98

$$\int (a+b\cot(c+dx))^{5/2}(A+B\cot(c+dx))\,dx = \text{Too large to display}$$

```
[In] integrate((a+b*cot(d*x+c))^(5/2)*(A+B*cot(d*x+c)),x, algorithm="fricas")
[Out] Too large to include
```

Sympy [F]

$$\int (a+b\cot(c+dx))^{5/2}(A+B\cot(c+dx))\,dx = \int (A+B\cot(c+dx))\,(a+b\cot(c+dx))^{\frac{5}{2}}\,dx$$

```
[In] integrate((a+b*cot(d*x+c))**(5/2)*(A+B*cot(d*x+c)),x)
[Out] Integral((A + B*cot(c + d*x))*(a + b*cot(c + d*x))**(5/2), x)
```

Maxima [F]

$$\int (a+b\cot(c+dx))^{5/2}(A+B\cot(c+dx))\,dx = \int (B\cot(dx+c)+A)(b\cot(dx+c)+a)^{\frac{5}{2}}\,dx$$

```
[In] integrate((a+b*cot(d*x+c))^(5/2)*(A+B*cot(d*x+c)),x, algorithm="maxima")
[Out] integrate((B*cot(d*x + c) + A)*(b*cot(d*x + c) + a)^(5/2), x)
```

Giac [F]

$$\int (a+b\cot(c+dx))^{5/2}(A+B\cot(c+dx))\,dx = \int (B\cot(dx+c)+A)(b\cot(dx+c)+a)^{\frac{5}{2}}\,dx$$

```
[In] integrate((a+b*cot(d*x+c))^(5/2)*(A+B*cot(d*x+c)),x, algorithm="giac")
[Out] integrate((B*cot(d*x + c) + A)*(b*cot(d*x + c) + a)^(5/2), x)
```

Mupad [B] (verification not implemented)

Time = 45.24 (sec) , antiderivative size = 3864, normalized size of antiderivative = 20.55

$$\int (a+b\cot(c+dx))^{5/2}(A+B\cot(c+dx))\,dx = \text{Too large to display}$$

```
[In] int((A + B*cot(c + d*x))*(a + b*cot(c + d*x))^(5/2),x)
[Out] log((8*B^3*a*b^2*(a^2 - 3*b^2)*(a^2 + b^2)^3)/d^3 - ((((((-B^4*b^2*d^4*(5*a
^4 + b^4 - 10*a^2*b^2)^2)^(1/2) + B^2*a^5*d^2 - 10*B^2*a^3*b^2*d^2 + 5*B^2*
a*b^4*d^2)/d^4)^(1/2)*(32*B*a^4*b^2 - 32*B*b^6 + 32*a*b^2*d*(((-B^4*b^2*d^4
*(5*a^4 + b^4 - 10*a^2*b^2)^2)^(1/2) + B^2*a^5*d^2 - 10*B^2*a^3*b^2*d^2 + 5
*B^2*a*b^4*d^2)/d^4)^(1/2)*(a + b*cot(c + d*x))^(1/2)))/(2*d) - (16*B^2*b^2
*(a + b*cot(c + d*x))^(1/2)*(a^6 - b^6 + 15*a^2*b^4 - 15*a^4*b^2))/d^2)*(((
-B^4*b^2*d^4*(5*a^4 + b^4 - 10*a^2*b^2)^2)^(1/2) + B^2*a^5*d^2 - 10*B^2*a^3
*b^2*d^2 + 5*B^2*a*b^4*d^2)/d^4)^(1/2))/2)*((20*B^4*a^2*b^8*d^4 - B^4*b^10*
d^4 - 110*B^4*a^4*b^6*d^4 + 100*B^4*a^6*b^4*d^4 - 25*B^4*a^8*b^2*d^4)^(1/2)
/(4*d^4) + (B^2*a^5)/(4*d^2) - (5*B^2*a^3*b^2)/(2*d^2) + (5*B^2*a*b^4)/(4*d
^2))^(1/2) - log((((((-B^4*b^2*d^4*(5*a^4 + b^4 - 10*a^2*b^2)^2)^(1/2) + B
^2*a^5*d^2 - 10*B^2*a^3*b^2*d^2 + 5*B^2*a*b^4*d^2)/d^4)^(1/2)*(32*B*b^6 - 3
2*B*a^4*b^2 + 32*a*b^2*d*(((-B^4*b^2*d^4*(5*a^4 + b^4 - 10*a^2*b^2)^2)^(1/2
) + B^2*a^5*d^2 - 10*B^2*a^3*b^2*d^2 + 5*B^2*a*b^4*d^2)/d^4)^(1/2)*(a + b*c
ot(c + d*x))^(1/2)))/(2*d) - (16*B^2*b^2*(a + b*cot(c + d*x))^(1/2)*(a^6 -
b^6 + 15*a^2*b^4 - 15*a^4*b^2))/d^2)*(((-B^4*b^2*d^4*(5*a^4 + b^4 - 10*a^2*
```

b^2)^2)^(1/2) + B^2*a^5*d^2 - 10*B^2*a^3*b^2*d^2 + 5*B^2*a*b^4*d^2)/d^4)^(1/2))/2 + (8*B^3*a*b^2*(a^2 - 3*b^2)*(a^2 + b^2)^3)/d^3)*(((20*B^4*a^2*b^8*d^4 - B^4*b^10*d^4 - 110*B^4*a^4*b^6*d^4 + 100*B^4*a^6*b^4*d^4 - 25*B^4*a^8*b^2*d^4)^(1/2) + B^2*a^5*d^2 - 10*B^2*a^3*b^2*d^2 + 5*B^2*a*b^4*d^2)/(4*d^4))^(1/2) - log(((((-((-B^4*b^2*d^4*(5*a^4 + b^4 - 10*a^2*b^2)^2)^(1/2) - B^2*a^5*d^2 + 10*B^2*a^3*b^2*d^2 - 5*B^2*a*b^4*d^2)/d^4)^(1/2)*(32*B*b^6 - 32*B*a^4*b^2 + 32*a*b^2*d*(-((-B^4*b^2*d^4*(5*a^4 + b^4 - 10*a^2*b^2)^2)^(1/2) - B^2*a^5*d^2 + 10*B^2*a^3*b^2*d^2 - 5*B^2*a*b^4*d^2)/d^4)^(1/2)*(a + b*cot(c + d*x))^(1/2)))/(2*d) - (16*B^2*b^2*(a + b*cot(c + d*x))^(1/2)*(a^6 - b^6 + 15*a^2*b^4 - 15*a^4*b^2))/d^2)*(-((-B^4*b^2*d^4*(5*a^4 + b^4 - 10*a^2*b^2)^2)^(1/2) - B^2*a^5*d^2 + 10*B^2*a^3*b^2*d^2 - 5*B^2*a*b^4*d^2)/d^4)^(1/2))/2 + (8*B^3*a*b^2*(a^2 - 3*b^2)*(a^2 + b^2)^3)/d^3)*(-((20*B^4*a^2*b^8*d^4 - B^4*b^10*d^4 - 110*B^4*a^4*b^6*d^4 + 100*B^4*a^6*b^4*d^4 - 25*B^4*a^8*b^2*d^4)^(1/2) - B^2*a^5*d^2 + 10*B^2*a^3*b^2*d^2 - 5*B^2*a*b^4*d^2)/(4*d^4))^(1/2) + log((8*B^3*a*b^2*(a^2 - 3*b^2)*(a^2 + b^2)^3)/d^3 - ((((-((-B^4*b^2*d^4*(5*a^4 + b^4 - 10*a^2*b^2)^2)^(1/2) - B^2*a^5*d^2 + 10*B^2*a^3*b^2*d^2 - 5*B^2*a*b^4*d^2)/d^4)^(1/2)*(32*B*a^4*b^2 - 32*B*b^6 + 32*a*b^2*d*(-((-B^4*b^2*d^4*(5*a^4 + b^4 - 10*a^2*b^2)^2)^(1/2) - B^2*a^5*d^2 + 10*B^2*a^3*b^2*d^2 - 5*B^2*a*b^4*d^2)/d^4)^(1/2)*(a + b*cot(c + d*x))^(1/2)))/(2*d) - (16*B^2*b^2*(a + b*cot(c + d*x))^(1/2)*(a^6 - b^6 + 15*a^2*b^4 - 15*a^4*b^2))/d^2)*(-((-B^4*b^2*d^4*(5*a^4 + b^4 - 10*a^2*b^2)^2)^(1/2) - B^2*a^5*d^2 + 10*B^2*a^3*b^2*d^2 - 5*B^2*a*b^4*d^2)/d^4)^(1/2))/2)*((B^2*a^5)/(4*d^2) - (20*B^4*a^2*b^8*d^4 - B^4*b^10*d^4 - 110*B^4*a^4*b^6*d^4 + 100*B^4*a^6*b^4*d^4 - 25*B^4*a^8*b^2*d^4)^(1/2)/(4*d^4) - (5*B^2*a^3*b^2)/(2*d^2) + (5*B^2*a*b^4)/(4*d^2))^(1/2) - ((4*B*a^2)/d - (2*B*(a^2 + b^2))/d)*(a + b*cot(c + d*x))^(1/2) - log((8*A^3*b^3*(3*a^2 - b^2)*(a^2 + b^2)^3)/d^3 - ((-((-A^4*b^2*d^4*(5*a^4 + b^4 - 10*a^2*b^2)^2)^(1/2) + A^2*a^5*d^2 - 10*A^2*a^3*b^2*d^2 + 5*A^2*a*b^4*d^2)/d^4)^(1/2)*(((-((-A^4*b^2*d^4*(5*a^4 + b^4 - 10*a^2*b^2)^2)^(1/2) + A^2*a^5*d^2 - 10*A^2*a^3*b^2*d^2 + 5*A^2*a*b^4*d^2)/d^4)^(1/2)*(64*A*a^3*b^3 + 64*A*a*b^5 - 32*a*b^2*d*(-((-A^4*b^2*d^4*(5*a^4 + b^4 - 10*a^2*b^2)^2)^(1/2) + A^2*a^5*d^2 - 10*A^2*a^3*b^2*d^2 + 5*A^2*a*b^4*d^2)/d^4)^(1/2)*(a + b*cot(c + d*x))^(1/2)))/(2*d) - (16*A^2*b^2*(a + b*cot(c + d*x))^(1/2)*(a^6 - b^6 + 15*a^2*b^4 - 15*a^4*b^2))/d^2))/2)*(-((20*A^4*a^2*b^8*d^4 - A^4*b^10*d^4 - 110*A^4*a^4*b^6*d^4 + 100*A^4*a^6*b^4*d^4 - 25*A^4*a^8*b^2*d^4)^(1/2) + A^2*a^5*d^2 - 10*A^2*a^3*b^2*d^2 + 5*A^2*a*b^4*d^2)/(4*d^4))^(1/2) - log((8*A^3*b^3*(3*a^2 - b^2)*(a^2 + b^2)^3)/d^3 - ((((-A^4*b^2*d^4*(5*a^4 + b^4 - 10*a^2*b^2)^2)^(1/2) - A^2*a^5*d^2 + 10*A^2*a^3*b^2*d^2 - 5*A^2*a*b^4*d^2)/d^4)^(1/2)*(((((-A^4*b^2*d^4*(5*a^4 + b^4 - 10*a^2*b^2)^2)^(1/2) - A^2*a^5*d^2 + 10*A^2*a^3*b^2*d^2 - 5*A^2*a*b^4*d^2)/d^4)^(1/2)*(64*A*a^3*b^3 + 64*A*a*b^5 - 32*a*b^2*d*(((-A^4*b^2*d^4*(5*a^4 + b^4 - 10*a^2*b^2)^2)^(1/2) - A^2*a^5*d^2 + 10*A^2*a^3*b^2*d^2 - 5*A^2*a*b^4*d^2)/d^4)^(1/2)*(a + b*cot(c + d*x))^(1/2)))/(2*d) - (16*A^2*b^2*(a + b*cot(c + d*x))^(1/2)*(a^6 - b^6 + 15*a^2*b^4 - 15*a^4*b^2))/d^2))/2)*(((20*A^4*a^2*b^8*d^4 - A^4*b^10*d^4 - 110*A^4*a^4*b^6*d^4 + 100*A^4*a^6*b^4*d^4 - 25*A^4*a^8*b^2*d^4)^(1/2) - A^2*a^5*d^2 + 10*A^2*a^3*b^2*d^2 - 5*A^2*a*b^4*d^2

```
)/(4*d^4))^(1/2) + log((8*A^3*b^3*(3*a^2 - b^2)*(a^2 + b^2)^3)/d^3 - ((((-A
^4*b^2*d^4*(5*a^4 + b^4 - 10*a^2*b^2)^2)^(1/2) - A^2*a^5*d^2 + 10*A^2*a^3*b
^2*d^2 - 5*A^2*a*b^4*d^2)/d^4)^(1/2)*(((((-A^4*b^2*d^4*(5*a^4 + b^4 - 10*a^
2*b^2)^2)^(1/2) - A^2*a^5*d^2 + 10*A^2*a^3*b^2*d^2 - 5*A^2*a*b^4*d^2)/d^4)^
(1/2)*(64*A*a^3*b^3 + 64*A*a*b^5 + 32*a*b^2*d*(((-A^4*b^2*d^4*(5*a^4 + b^4
- 10*a^2*b^2)^2)^(1/2) - A^2*a^5*d^2 + 10*A^2*a^3*b^2*d^2 - 5*A^2*a*b^4*d^2
)/d^4)^(1/2)*(a + b*cot(c + d*x))^(1/2)))/(2*d) + (16*A^2*b^2*(a + b*cot(c
+ d*x))^(1/2)*(a^6 - b^6 + 15*a^2*b^4 - 15*a^4*b^2))/d^2))/2)*((20*A^4*a^2*
b^8*d^4 - A^4*b^10*d^4 - 110*A^4*a^4*b^6*d^4 + 100*A^4*a^6*b^4*d^4 - 25*A^4
*a^8*b^2*d^4)^(1/2)/(4*d^4) - (A^2*a^5)/(4*d^2) + (5*A^2*a^3*b^2)/(2*d^2) -
 (5*A^2*a*b^4)/(4*d^2))^(1/2) + log((8*A^3*b^3*(3*a^2 - b^2)*(a^2 + b^2)^3)
/d^3 - ((-((-A^4*b^2*d^4*(5*a^4 + b^4 - 10*a^2*b^2)^2)^(1/2) + A^2*a^5*d^2
- 10*A^2*a^3*b^2*d^2 + 5*A^2*a*b^4*d^2)/d^4)^(1/2)*(((-((-A^4*b^2*d^4*(5*a^
4 + b^4 - 10*a^2*b^2)^2)^(1/2) + A^2*a^5*d^2 - 10*A^2*a^3*b^2*d^2 + 5*A^2*a
*b^4*d^2)/d^4)^(1/2)*(64*A*a^3*b^3 + 64*A*a*b^5 + 32*a*b^2*d*(-((-A^4*b^2*d
^4*(5*a^4 + b^4 - 10*a^2*b^2)^2)^(1/2) + A^2*a^5*d^2 - 10*A^2*a^3*b^2*d^2 +
 5*A^2*a*b^4*d^2)/d^4)^(1/2)*(a + b*cot(c + d*x))^(1/2)))/(2*d) + (16*A^2*b
^2*(a + b*cot(c + d*x))^(1/2)*(a^6 - b^6 + 15*a^2*b^4 - 15*a^4*b^2))/d^2))/
2)*((5*A^2*a^3*b^2)/(2*d^2) - (A^2*a^5)/(4*d^2) - (20*A^4*a^2*b^8*d^4 - A^4
*b^10*d^4 - 110*A^4*a^4*b^6*d^4 + 100*A^4*a^6*b^4*d^4 - 25*A^4*a^8*b^2*d^4)
^(1/2)/(4*d^4) - (5*A^2*a*b^4)/(4*d^2))^(1/2) - (2*B*(a + b*cot(c + d*x))^(
5/2))/(5*d) - (2*A*b*(a + b*cot(c + d*x))^(3/2))/(3*d) - (2*B*a*(a + b*cot(
c + d*x))^(3/2))/(3*d) - (4*A*a*b*(a + b*cot(c + d*x))^(1/2))/d
```

3.96 $\int (a+b\cot(c+dx))^{3/2}(A+B\cot(c+dx))\,dx$

Optimal result

Integrand size = 25, antiderivative size = 150

$$\int (a+b\cot(c+dx))^{3/2}(A+B\cot(c+dx))\,dx = \frac{(a-ib)^{3/2}(iA+B)\operatorname{arctanh}\left(\frac{\sqrt{a+b\cot(c+dx)}}{\sqrt{a-ib}}\right)}{d} - \frac{(a+ib)^{3/2}(iA-B)\operatorname{arctanh}\left(\frac{\sqrt{a+b\cot(c+dx)}}{\sqrt{a+ib}}\right)}{d} - \frac{2(Ab+aB)\sqrt{a+b\cot(c+dx)}}{d} - \frac{2B(a+b\cot(c+dx))^{3/2}}{3d}$$

```
[Out] (a-I*b)^(3/2)*(I*A+B)*arctanh((a+b*cot(d*x+c))^(1/2)/(a-I*b)^(1/2))/d-(a+I*
b)^(3/2)*(I*A-B)*arctanh((a+b*cot(d*x+c))^(1/2)/(a+I*b)^(1/2))/d-2/3*B*(a+b
*cot(d*x+c))^(3/2)/d-2*(A*b+B*a)*(a+b*cot(d*x+c))^(1/2)/d
```

Rubi [A] (verified)

Time = 0.38 (sec) , antiderivative size = 150, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {3609, 3620, 3618, 65, 214}

$$\int (a+b\cot(c+dx))^{3/2}(A+B\cot(c+dx))\,dx = \frac{(a-ib)^{3/2}(B+iA)\text{arctanh}\left(\frac{\sqrt{a+b\cot(c+dx)}}{\sqrt{a-ib}}\right)}{d} - \frac{(a+ib)^{3/2}(-B+iA)\text{arctanh}\left(\frac{\sqrt{a+b\cot(c+dx)}}{\sqrt{a+ib}}\right)}{d} - \frac{2(aB+Ab)\sqrt{a+b\cot(c+dx)}}{d} - \frac{2B(a+b\cot(c+dx))^{3/2}}{3d}$$

```
[In] Int[(a + b*Cot[c + d*x])^(3/2)*(A + B*Cot[c + d*x]),x]
```

```
[Out] ((a - I*b)^(3/2)*(I*A + B)*ArcTanh[Sqrt[a + b*Cot[c + d*x]]/Sqrt[a - I*b]])
/d - ((a + I*b)^(3/2)*(I*A - B)*ArcTanh[Sqrt[a + b*Cot[c + d*x]]/Sqrt[a + I
*b]])/d - (2*(A*b + a*B)*Sqrt[a + b*Cot[c + d*x]])/d - (2*B*(a + b*Cot[c +
d*x])^(3/2))/(3*d)
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 3609

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)]), x_Symbol] :> Simp[d*((a + b*Tan[e + f*x])^m/(f*m)), x] + Int
[(a + b*Tan[e + f*x])^(m - 1)*Simp[a*c - b*d + (b*c + a*d)*Tan[e + f*x], x]
, x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2,
 0] && GtQ[m, 0]
```

Rule 3618

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*tan[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Dist[c*(d/f), Subst[Int[(a + (b/d)*x)^m/(d^2 + c
```

```
*x), x], x, d*Tan[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b
*c - a*d, 0] && NeQ[a^2 + b^2, 0] && EqQ[c^2 + d^2, 0]
```

Rule 3620

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)]), x_Symbol] :> Dist[(c + I*d)/2, Int[(a + b*Tan[e + f*x])^m*(1
 - I*Tan[e + f*x]), x], x] + Dist[(c - I*d)/2, Int[(a + b*Tan[e + f*x])^m*(
1 + I*Tan[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b*c -
 a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] &&  !IntegerQ[m]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\frac{2B(a+b\cot(c+dx))^{3/2}}{3d} \\
&\quad + \int \sqrt{a+b\cot(c+dx)}(aA-bB+(Ab+aB)\cot(c+dx))\,dx \\
&= -\frac{2(Ab+aB)\sqrt{a+b\cot(c+dx)}}{d} - \frac{2B(a+b\cot(c+dx))^{3/2}}{3d} \\
&\quad + \int \frac{a^2A-Ab^2-2abB+(2aAb+a^2B-b^2B)\cot(c+dx)}{\sqrt{a+b\cot(c+dx)}}\,dx \\
&= -\frac{2(Ab+aB)\sqrt{a+b\cot(c+dx)}}{d} - \frac{2B(a+b\cot(c+dx))^{3/2}}{3d} \\
&\quad + \frac{1}{2}\left((a-ib)^2(A-iB)\right)\int \frac{1+i\cot(c+dx)}{\sqrt{a+b\cot(c+dx)}}\,dx \\
&\quad + \frac{1}{2}\left((a+ib)^2(A+iB)\right)\int \frac{1-i\cot(c+dx)}{\sqrt{a+b\cot(c+dx)}}\,dx \\
&= -\frac{2(Ab+aB)\sqrt{a+b\cot(c+dx)}}{d} - \frac{2B(a+b\cot(c+dx))^{3/2}}{3d} \\
&\quad + \frac{\left((a+ib)^2(iA-B)\right)\text{Subst}\left(\int \frac{1}{(-1+x)\sqrt{a+ibx}}\,dx, x, -i\cot(c+dx)\right)}{2d} \\
&\quad - \frac{\left((a-ib)^2(iA+B)\right)\text{Subst}\left(\int \frac{1}{(-1+x)\sqrt{a-ibx}}\,dx, x, i\cot(c+dx)\right)}{2d} \\
&= -\frac{2(Ab+aB)\sqrt{a+b\cot(c+dx)}}{d} - \frac{2B(a+b\cot(c+dx))^{3/2}}{3d} \\
&\quad + \frac{\left((a-ib)^2(A-iB)\right)\text{Subst}\left(\int \frac{1}{-1-\frac{ia}{b}+\frac{ix^2}{b}}\,dx, x, \sqrt{a+b\cot(c+dx)}\right)}{bd} \\
&\quad + \frac{\left((a+ib)^2(A+iB)\right)\text{Subst}\left(\int \frac{1}{-1+\frac{ia}{b}-\frac{ix^2}{b}}\,dx, x, \sqrt{a+b\cot(c+dx)}\right)}{bd}
\end{aligned}$$

$$= \frac{(a-ib)^{3/2}(iA+B)\operatorname{arctanh}\left(\frac{\sqrt{a+b\cot(c+dx)}}{\sqrt{a-ib}}\right)}{d} - \frac{(a+ib)^{3/2}(iA-B)\operatorname{arctanh}\left(\frac{\sqrt{a+b\cot(c+dx)}}{\sqrt{a+ib}}\right)}{d} - \frac{2(Ab+aB)\sqrt{a+b\cot(c+dx)}}{d} - \frac{2B(a+b\cot(c+dx))^{3/2}}{3d}$$

Mathematica [A] (verified)

Time = 1.02 (sec) , antiderivative size = 294, normalized size of antiderivative = 1.96

$$\int (a+b\cot(c+dx))^{3/2}(A+B\cot(c+dx))\,dx =$$

$$-\frac{\frac{3\sqrt{a-\sqrt{-b^2}}\left(-2ab\left(A\sqrt{-b^2}+bB\right)+a^2\left(Ab-\sqrt{-b^2}B\right)+b^2\left(-Ab+\sqrt{-b^2}B\right)\right)\operatorname{arctanh}\left(\frac{\sqrt{a+b\cot(c+dx)}}{\sqrt{a-\sqrt{-b^2}}}\right)}{b^2+a\sqrt{-b^2}} + \frac{3\left(2ab\left(-A\sqrt{-b^2}+bB\right)-a^2\left(Ab+\right.\right.}{}}{3}$$

[In] Integrate[(a + b*Cot[c + d*x])^(3/2)*(A + B*Cot[c + d*x]),x]

[Out] -1/3*((3*Sqrt[a - Sqrt[-b^2]]*(-2*a*b*(A*Sqrt[-b^2] + b*B) + a^2*(A*b - Sqr
t[-b^2]*B) + b^2*(-(A*b) + Sqrt[-b^2]*B))*ArcTanh[Sqrt[a + b*Cot[c + d*x]]/
Sqrt[a - Sqrt[-b^2]]])/(b^2 + a*Sqrt[-b^2]) + (3*(2*a*b*(-(A*Sqrt[-b^2]) +
b*B) - a^2*(A*b + Sqrt[-b^2]*B) + b^2*(A*b + Sqrt[-b^2]*B))*ArcTanh[Sqrt[a
+ b*Cot[c + d*x]]/Sqrt[a + Sqrt[-b^2]]])/(Sqrt[-b^2]*Sqrt[a + Sqrt[-b^2]])
+ 6*(A*b + a*B)*Sqrt[a + b*Cot[c + d*x]] + 2*B*(a + b*Cot[c + d*x])^(3/2))/
d

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 1656 vs. 2(126) = 252.

Time = 0.12 (sec) , antiderivative size = 1657, normalized size of antiderivative = 11.05

method	result	size
parts	Expression too large to display	1657
derivativedivides	Expression too large to display	1665
default	Expression too large to display	1665

[In] int((a+b*cot(d*x+c))^(3/2)*(A+B*cot(d*x+c)),x,method=_RETURNVERBOSE)

[Out] A*(-2*b*(a+b*cot(d*x+c))^(1/2)/d+1/4/d/b*ln(b*cot(d*x+c)+a+(a+b*cot(d*x+c))
^(1/2)*(2*(a^2+b^2)^(1/2)+2*a)^(1/2)+(a^2+b^2)^(1/2))*(2*(a^2+b^2)^(1/2)+2*
a)^(1/2)*(a^2+b^2)^(1/2)*a-1/4/d/b*ln(b*cot(d*x+c)+a+(a+b*cot(d*x+c))^(1/2)
(2(a^2+b^2)^(1/2)+2*a)^(1/2)+(a^2+b^2)^(1/2))*(2*(a^2+b^2)^(1/2)+2*a)^(1/

```
2)*a^2+1/4/d*b*ln(b*cot(d*x+c)+a+(a+b*cot(d*x+c))^(1/2)*(2*(a^2+b^2)^(1/2)+
2*a)^(1/2)+(a^2+b^2)^(1/2))*(2*(a^2+b^2)^(1/2)+2*a)^(1/2)+1/d*b/(2*(a^2+b^2
)^(1/2)-2*a)^(1/2)*arctan((2*(a+b*cot(d*x+c))^(1/2)+(2*(a^2+b^2)^(1/2)+2*a)
^(1/2))/(2*(a^2+b^2)^(1/2)-2*a)^(1/2))*(a^2+b^2)^(1/2)-2/d*b/(2*(a^2+b^2)^(
1/2)-2*a)^(1/2)*arctan((2*(a+b*cot(d*x+c))^(1/2)+(2*(a^2+b^2)^(1/2)+2*a)^(1
/2))/(2*(a^2+b^2)^(1/2)-2*a)^(1/2))*a-1/4/d/b*ln((a+b*cot(d*x+c))^(1/2)*(2*
(a^2+b^2)^(1/2)+2*a)^(1/2)-b*cot(d*x+c)-(a^2+b^2)^(1/2)-a)*(2*(a^2+b^2)^(1/
2)+2*a)^(1/2)*(a^2+b^2)^(1/2)*a+1/4/d/b*ln((a+b*cot(d*x+c))^(1/2)*(2*(a^2+b
^2)^(1/2)+2*a)^(1/2)-b*cot(d*x+c)-(a^2+b^2)^(1/2)-a)*(2*(a^2+b^2)^(1/2)+2*a
)^(1/2)*a^2-1/4/d*b*ln((a+b*cot(d*x+c))^(1/2)*(2*(a^2+b^2)^(1/2)+2*a)^(1/2)
-b*cot(d*x+c)-(a^2+b^2)^(1/2)-a)*(2*(a^2+b^2)^(1/2)+2*a)^(1/2)-1/d*b/(2*(a^
2+b^2)^(1/2)-2*a)^(1/2)*arctan((-2*(a+b*cot(d*x+c))^(1/2)+(2*(a^2+b^2)^(1/2
)+2*a)^(1/2))/(2*(a^2+b^2)^(1/2)-2*a)^(1/2))*(a^2+b^2)^(1/2)+2/d*b/(2*(a^2+
b^2)^(1/2)-2*a)^(1/2)*arctan((-2*(a+b*cot(d*x+c))^(1/2)+(2*(a^2+b^2)^(1/2)+
2*a)^(1/2))/(2*(a^2+b^2)^(1/2)-2*a)^(1/2))*a)+B*(-2/3/d*(a+b*cot(d*x+c))^(3
/2)-2/d*(a+b*cot(d*x+c))^(1/2)*a-1/4/d*ln(b*cot(d*x+c)+a+(a+b*cot(d*x+c))^(
1/2)*(2*(a^2+b^2)^(1/2)+2*a)^(1/2)+(a^2+b^2)^(1/2))*(2*(a^2+b^2)^(1/2)+2*a)
^(1/2)*(a^2+b^2)^(1/2)+1/2/d*ln(b*cot(d*x+c)+a+(a+b*cot(d*x+c))^(1/2)*(2*(a
^2+b^2)^(1/2)+2*a)^(1/2)+(a^2+b^2)^(1/2))*(2*(a^2+b^2)^(1/2)+2*a)^(1/2)*a+1
/d/(2*(a^2+b^2)^(1/2)-2*a)^(1/2)*arctan((2*(a+b*cot(d*x+c))^(1/2)+(2*(a^2+b
^2)^(1/2)+2*a)^(1/2))/(2*(a^2+b^2)^(1/2)-2*a)^(1/2))*(a^2+b^2)+1/d/(2*(a^2+
b^2)^(1/2)-2*a)^(1/2)*arctan((2*(a+b*cot(d*x+c))^(1/2)+(2*(a^2+b^2)^(1/2)+2
*a)^(1/2))/(2*(a^2+b^2)^(1/2)-2*a)^(1/2))*(a^2+b^2)^(1/2)*a-2/d/(2*(a^2+b^2
)^(1/2)-2*a)^(1/2)*arctan((2*(a+b*cot(d*x+c))^(1/2)+(2*(a^2+b^2)^(1/2)+2*a)
^(1/2))/(2*(a^2+b^2)^(1/2)-2*a)^(1/2))*a^2+1/4/d*ln((a+b*cot(d*x+c))^(1/2)*
(2*(a^2+b^2)^(1/2)+2*a)^(1/2)-b*cot(d*x+c)-(a^2+b^2)^(1/2)-a)*(2*(a^2+b^2)^
(1/2)+2*a)^(1/2)*(a^2+b^2)^(1/2)-1/2/d*ln((a+b*cot(d*x+c))^(1/2)*(2*(a^2+b^
2)^(1/2)+2*a)^(1/2)-b*cot(d*x+c)-(a^2+b^2)^(1/2)-a)*(2*(a^2+b^2)^(1/2)+2*a)
^(1/2)*a-1/d/(2*(a^2+b^2)^(1/2)-2*a)^(1/2)*arctan((-2*(a+b*cot(d*x+c))^(1/2
)+(2*(a^2+b^2)^(1/2)+2*a)^(1/2))/(2*(a^2+b^2)^(1/2)-2*a)^(1/2))*(a^2+b^2)-1
/d/(2*(a^2+b^2)^(1/2)-2*a)^(1/2)*arctan((-2*(a+b*cot(d*x+c))^(1/2)+(2*(a^2+
b^2)^(1/2)+2*a)^(1/2))/(2*(a^2+b^2)^(1/2)-2*a)^(1/2))*(a^2+b^2)^(1/2)*a+2/d
/(2*(a^2+b^2)^(1/2)-2*a)^(1/2)*arctan((-2*(a+b*cot(d*x+c))^(1/2)+(2*(a^2+b^
2)^(1/2)+2*a)^(1/2))/(2*(a^2+b^2)^(1/2)-2*a)^(1/2))*a^2)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 3252 vs. $2(120) = 240$.

Time = 0.63 (sec) , antiderivative size = 3252, normalized size of antiderivative = 21.68

$$\int (a+b\cot(c+dx))^{3/2}(A+B\cot(c+dx))\,dx = \text{Too large to display}$$

```
[In] integrate((a+b*cot(d*x+c))^(3/2)*(A+B*cot(d*x+c)),x, algorithm="fricas")
```

```
[Out] -1/6*(3*d*sqrt((6*A*B*a^2*b - 2*A*B*b^3 - (A^2 - B^2)*a^3 + 3*(A^2 - B^2)*a
*b^2 + d^2*sqrt(-(4*A^2*B^2*a^6 + 12*(A^3*B - A*B^3)*a^5*b + 3*(3*A^4 - 14*
A^2*B^2 + 3*B^4)*a^4*b^2 - 40*(A^3*B - A*B^3)*a^3*b^3 - 6*(A^4 - 8*A^2*B^2
+ B^4)*a^2*b^4 + 12*(A^3*B - A*B^3)*a*b^5 + (A^4 - 2*A^2*B^2 + B^4)*b^6)/d^
4))/d^2)*log((2*(A^3*B + A*B^3)*a^5 + 3*(A^4 - B^4)*a^4*b - 4*(A^3*B + A*B^
3)*a^3*b^2 + 2*(A^4 - B^4)*a^2*b^3 - 6*(A^3*B + A*B^3)*a*b^4 - (A^4 - B^4)*
b^5)*sqrt((b*cos(2*d*x + 2*c) + a*sin(2*d*x + 2*c) + b)/sin(2*d*x + 2*c)) +
 ((A*a - B*b)*d^3*sqrt(-(4*A^2*B^2*a^6 + 12*(A^3*B - A*B^3)*a^5*b + 3*(3*A^
4 - 14*A^2*B^2 + 3*B^4)*a^4*b^2 - 40*(A^3*B - A*B^3)*a^3*b^3 - 6*(A^4 - 8*A
^2*B^2 + B^4)*a^2*b^4 + 12*(A^3*B - A*B^3)*a*b^5 + (A^4 - 2*A^2*B^2 + B^4)*
b^6)/d^4) - (2*A*B^2*a^4 + (5*A^2*B - 3*B^3)*a^3*b + 3*(A^3 - 3*A*B^2)*a^2*
b^2 - (7*A^2*B - B^3)*a*b^3 - (A^3 - A*B^2)*b^4)*d)*sqrt((6*A*B*a^2*b - 2*A
*B*b^3 - (A^2 - B^2)*a^3 + 3*(A^2 - B^2)*a*b^2 + d^2*sqrt(-(4*A^2*B^2*a^6 +
 12*(A^3*B - A*B^3)*a^5*b + 3*(3*A^4 - 14*A^2*B^2 + 3*B^4)*a^4*b^2 - 40*(A^
3*B - A*B^3)*a^3*b^3 - 6*(A^4 - 8*A^2*B^2 + B^4)*a^2*b^4 + 12*(A^3*B - A*B^
3)*a*b^5 + (A^4 - 2*A^2*B^2 + B^4)*b^6)/d^4))/d^2))*sin(2*d*x + 2*c) - 3*d*
sqrt((6*A*B*a^2*b - 2*A*B*b^3 - (A^2 - B^2)*a^3 + 3*(A^2 - B^2)*a*b^2 + d^2
*sqrt(-(4*A^2*B^2*a^6 + 12*(A^3*B - A*B^3)*a^5*b + 3*(3*A^4 - 14*A^2*B^2 +
3*B^4)*a^4*b^2 - 40*(A^3*B - A*B^3)*a^3*b^3 - 6*(A^4 - 8*A^2*B^2 + B^4)*a^2
*b^4 + 12*(A^3*B - A*B^3)*a*b^5 + (A^4 - 2*A^2*B^2 + B^4)*b^6)/d^4))/d^2)*l
og((2*(A^3*B + A*B^3)*a^5 + 3*(A^4 - B^4)*a^4*b - 4*(A^3*B + A*B^3)*a^3*b^2
 + 2*(A^4 - B^4)*a^2*b^3 - 6*(A^3*B + A*B^3)*a*b^4 - (A^4 - B^4)*b^5)*sqrt(
(b*cos(2*d*x + 2*c) + a*sin(2*d*x + 2*c) + b)/sin(2*d*x + 2*c)) - ((A*a - B
*b)*d^3*sqrt(-(4*A^2*B^2*a^6 + 12*(A^3*B - A*B^3)*a^5*b + 3*(3*A^4 - 14*A^2
*B^2 + 3*B^4)*a^4*b^2 - 40*(A^3*B - A*B^3)*a^3*b^3 - 6*(A^4 - 8*A^2*B^2 + B
^4)*a^2*b^4 + 12*(A^3*B - A*B^3)*a*b^5 + (A^4 - 2*A^2*B^2 + B^4)*b^6)/d^4)
- (2*A*B^2*a^4 + (5*A^2*B - 3*B^3)*a^3*b + 3*(A^3 - 3*A*B^2)*a^2*b^2 - (7*A
^2*B - B^3)*a*b^3 - (A^3 - A*B^2)*b^4)*d)*sqrt((6*A*B*a^2*b - 2*A*B*b^3 - (
A^2 - B^2)*a^3 + 3*(A^2 - B^2)*a*b^2 + d^2*sqrt(-(4*A^2*B^2*a^6 + 12*(A^3*B
 - A*B^3)*a^5*b + 3*(3*A^4 - 14*A^2*B^2 + 3*B^4)*a^4*b^2 - 40*(A^3*B - A*B^
3)*a^3*b^3 - 6*(A^4 - 8*A^2*B^2 + B^4)*a^2*b^4 + 12*(A^3*B - A*B^3)*a*b^5 +
 (A^4 - 2*A^2*B^2 + B^4)*b^6)/d^4))/d^2))*sin(2*d*x + 2*c) - 3*d*sqrt((6*A*
B*a^2*b - 2*A*B*b^3 - (A^2 - B^2)*a^3 + 3*(A^2 - B^2)*a*b^2 - d^2*sqrt(-(4*
A^2*B^2*a^6 + 12*(A^3*B - A*B^3)*a^5*b + 3*(3*A^4 - 14*A^2*B^2 + 3*B^4)*a^4
*b^2 - 40*(A^3*B - A*B^3)*a^3*b^3 - 6*(A^4 - 8*A^2*B^2 + B^4)*a^2*b^4 + 12*
(A^3*B - A*B^3)*a*b^5 + (A^4 - 2*A^2*B^2 + B^4)*b^6)/d^4))/d^2)*log((2*(A^3
*B + A*B^3)*a^5 + 3*(A^4 - B^4)*a^4*b - 4*(A^3*B + A*B^3)*a^3*b^2 + 2*(A^4
- B^4)*a^2*b^3 - 6*(A^3*B + A*B^3)*a*b^4 - (A^4 - B^4)*b^5)*sqrt((b*cos(2*d
*x + 2*c) + a*sin(2*d*x + 2*c) + b)/sin(2*d*x + 2*c)) + ((A*a - B*b)*d^3*sq
rt(-(4*A^2*B^2*a^6 + 12*(A^3*B - A*B^3)*a^5*b + 3*(3*A^4 - 14*A^2*B^2 + 3*B
^4)*a^4*b^2 - 40*(A^3*B - A*B^3)*a^3*b^3 - 6*(A^4 - 8*A^2*B^2 + B^4)*a^2*b^
4 + 12*(A^3*B - A*B^3)*a*b^5 + (A^4 - 2*A^2*B^2 + B^4)*b^6)/d^4) + (2*A*B^2
*a^4 + (5*A^2*B - 3*B^3)*a^3*b + 3*(A^3 - 3*A*B^2)*a^2*b^2 - (7*A^2*B - B^3
)*a*b^3 - (A^3 - A*B^2)*b^4)*d)*sqrt((6*A*B*a^2*b - 2*A*B*b^3 - (A^2 - B^2)
*a^3 + 3*(A^2 - B^2)*a*b^2 - d^2*sqrt(-(4*A^2*B^2*a^6 + 12*(A^3*B - A*B^3)*
```

```
a^5*b + 3*(3*A^4 - 14*A^2*B^2 + 3*B^4)*a^4*b^2 - 40*(A^3*B - A*B^3)*a^3*b^3
 - 6*(A^4 - 8*A^2*B^2 + B^4)*a^2*b^4 + 12*(A^3*B - A*B^3)*a*b^5 + (A^4 - 2*
A^2*B^2 + B^4)*b^6)/d^4))/d^2))*sin(2*d*x + 2*c) + 3*d*sqrt((6*A*B*a^2*b -
2*A*B*b^3 - (A^2 - B^2)*a^3 + 3*(A^2 - B^2)*a*b^2 - d^2*sqrt(-(4*A^2*B^2*a^
6 + 12*(A^3*B - A*B^3)*a^5*b + 3*(3*A^4 - 14*A^2*B^2 + 3*B^4)*a^4*b^2 - 40*
(A^3*B - A*B^3)*a^3*b^3 - 6*(A^4 - 8*A^2*B^2 + B^4)*a^2*b^4 + 12*(A^3*B - A
*B^3)*a*b^5 + (A^4 - 2*A^2*B^2 + B^4)*b^6)/d^4))/d^2)*log((2*(A^3*B + A*B^3
)*a^5 + 3*(A^4 - B^4)*a^4*b - 4*(A^3*B + A*B^3)*a^3*b^2 + 2*(A^4 - B^4)*a^2
*b^3 - 6*(A^3*B + A*B^3)*a*b^4 - (A^4 - B^4)*b^5)*sqrt((b*cos(2*d*x + 2*c)
+ a*sin(2*d*x + 2*c) + b)/sin(2*d*x + 2*c)) - ((A*a - B*b)*d^3*sqrt(-(4*A^2
*B^2*a^6 + 12*(A^3*B - A*B^3)*a^5*b + 3*(3*A^4 - 14*A^2*B^2 + 3*B^4)*a^4*b^
2 - 40*(A^3*B - A*B^3)*a^3*b^3 - 6*(A^4 - 8*A^2*B^2 + B^4)*a^2*b^4 + 12*(A^
3*B - A*B^3)*a*b^5 + (A^4 - 2*A^2*B^2 + B^4)*b^6)/d^4) + (2*A*B^2*a^4 + (5*
A^2*B - 3*B^3)*a^3*b + 3*(A^3 - 3*A*B^2)*a^2*b^2 - (7*A^2*B - B^3)*a*b^3 -
(A^3 - A*B^2)*b^4)*d)*sqrt((6*A*B*a^2*b - 2*A*B*b^3 - (A^2 - B^2)*a^3 + 3*(
A^2 - B^2)*a*b^2 - d^2*sqrt(-(4*A^2*B^2*a^6 + 12*(A^3*B - A*B^3)*a^5*b + 3*
(3*A^4 - 14*A^2*B^2 + 3*B^4)*a^4*b^2 - 40*(A^3*B - A*B^3)*a^3*b^3 - 6*(A^4
- 8*A^2*B^2 + B^4)*a^2*b^4 + 12*(A^3*B - A*B^3)*a*b^5 + (A^4 - 2*A^2*B^2 +
B^4)*b^6)/d^4))/d^2))*sin(2*d*x + 2*c) + 4*(B*b*cos(2*d*x + 2*c) + B*b + (4
*B*a + 3*A*b)*sin(2*d*x + 2*c))*sqrt((b*cos(2*d*x + 2*c) + a*sin(2*d*x + 2*
c) + b)/sin(2*d*x + 2*c)))/(d*sin(2*d*x + 2*c))
```

Sympy [F]

$$\int (a+b\cot(c+dx))^{3/2}(A+B\cot(c+dx))\,dx=\int (A+B\cot{(c+dx)})\,(a+b\cot{(c+dx)})^{\frac{3}{2}}\,dx$$

[In] integrate((a+b*cot(d*x+c))**(3/2)*(A+B*cot(d*x+c)),x)

[Out] Integral((A + B*cot(c + d*x))*(a + b*cot(c + d*x))**(3/2), x)

Maxima [F]

$$\int (a+b\cot(c+dx))^{3/2}(A+B\cot(c+dx))\,dx=\int (B\cot{(dx+c)}+A)(b\cot{(dx+c)}+a)^{\frac{3}{2}}\,dx$$

[In] integrate((a+b*cot(d*x+c))^(3/2)*(A+B*cot(d*x+c)),x, algorithm="maxima")

[Out] integrate((B*cot(d*x + c) + A)*(b*cot(d*x + c) + a)^(3/2), x)

Giac [F]

$$\int (a+b\cot(c+dx))^{3/2}(A+B\cot(c+dx))\,dx=\int (B\cot(dx+c)+A)(b\cot(dx+c)+a)^{\frac{3}{2}}\,dx$$

```
[In] integrate((a+b*cot(d*x+c))^(3/2)*(A+B*cot(d*x+c)),x, algorithm="giac")
[Out] integrate((B*cot(d*x + c) + A)*(b*cot(d*x + c) + a)^(3/2), x)
```

Mupad [B] (verification not implemented)

Time = 26.83 (sec) , antiderivative size = 2823, normalized size of antiderivative = 18.82

$$\int (a+b\cot(c+dx))^{3/2}(A+B\cot(c+dx))\,dx = \text{Too large to display}$$

```
[In] int((A + B*cot(c + d*x))*(a + b*cot(c + d*x))^(3/2),x)
[Out] log((16*A^3*a*b^3*(a^2 + b^2)^2)/d^3 - (((16*b^2*(((-A^4*b^2*d^4*(3*a^2 - b
^2)^2)^(1/2) - A^2*a^3*d^2 + 3*A^2*a*b^2*d^2)/d^4)^(1/2)*(A*b^3 + A*a^2*b +
 a*d*(((-A^4*b^2*d^4*(3*a^2 - b^2)^2)^(1/2) - A^2*a^3*d^2 + 3*A^2*a*b^2*d^2
)/d^4)^(1/2)*(a + b*cot(c + d*x))^(1/2)))/d + (16*A^2*b^2*(a + b*cot(c + d*
x))^(1/2)*(a^4 + b^4 - 6*a^2*b^2))/d^2)*(((-A^4*b^2*d^4*(3*a^2 - b^2)^2)^(1
/2) - A^2*a^3*d^2 + 3*A^2*a*b^2*d^2)/d^4)^(1/2))/2)*((6*A^4*a^2*b^4*d^4 - A
^4*b^6*d^4 - 9*A^4*a^4*b^2*d^4)^(1/2)/(4*d^4) - (A^2*a^3)/(4*d^2) + (3*A^2*
a*b^2)/(4*d^2))^(1/2) - log((16*A^3*a*b^3*(a^2 + b^2)^2)/d^3 - (((16*b^2*((
(-A^4*b^2*d^4*(3*a^2 - b^2)^2)^(1/2) - A^2*a^3*d^2 + 3*A^2*a*b^2*d^2)/d^4)^
(1/2)*(A*b^3 + A*a^2*b - a*d*(((-A^4*b^2*d^4*(3*a^2 - b^2)^2)^(1/2) - A^2*a
^3*d^2 + 3*A^2*a*b^2*d^2)/d^4)^(1/2)*(a + b*cot(c + d*x))^(1/2)))/d - (16*A
^2*b^2*(a + b*cot(c + d*x))^(1/2)*(a^4 + b^4 - 6*a^2*b^2))/d^2)*(((-A^4*b^2
*d^4*(3*a^2 - b^2)^2)^(1/2) - A^2*a^3*d^2 + 3*A^2*a*b^2*d^2)/d^4)^(1/2))/2)
*(((6*A^4*a^2*b^4*d^4 - A^4*b^6*d^4 - 9*A^4*a^4*b^2*d^4)^(1/2) - A^2*a^3*d^
2 + 3*A^2*a*b^2*d^2)/(4*d^4))^(1/2) - log((16*A^3*a*b^3*(a^2 + b^2)^2)/d^3
- (((16*b^2*(-((-A^4*b^2*d^4*(3*a^2 - b^2)^2)^(1/2) + A^2*a^3*d^2 - 3*A^2*a
*b^2*d^2)/d^4)^(1/2)*(A*b^3 + A*a^2*b - a*d*(-((-A^4*b^2*d^4*(3*a^2 - b^2)^
2)^(1/2) + A^2*a^3*d^2 - 3*A^2*a*b^2*d^2)/d^4)^(1/2)*(a + b*cot(c + d*x))^(
1/2)))/d - (16*A^2*b^2*(a + b*cot(c + d*x))^(1/2)*(a^4 + b^4 - 6*a^2*b^2))/
d^2)*(-((-A^4*b^2*d^4*(3*a^2 - b^2)^2)^(1/2) + A^2*a^3*d^2 - 3*A^2*a*b^2*d^
2)/d^4)^(1/2))/2)*(-((6*A^4*a^2*b^4*d^4 - A^4*b^6*d^4 - 9*A^4*a^4*b^2*d^4)^
(1/2) + A^2*a^3*d^2 - 3*A^2*a*b^2*d^2)/(4*d^4))^(1/2) + log((16*A^3*a*b^3*(
a^2 + b^2)^2)/d^3 - (((16*b^2*(-((-A^4*b^2*d^4*(3*a^2 - b^2)^2)^(1/2) + A^2
*a^3*d^2 - 3*A^2*a*b^2*d^2)/d^4)^(1/2)*(A*b^3 + A*a^2*b + a*d*(-((-A^4*b^2*
d^4*(3*a^2 - b^2)^2)^(1/2) + A^2*a^3*d^2 - 3*A^2*a*b^2*d^2)/d^4)^(1/2)*(a +
 b*cot(c + d*x))^(1/2)))/d + (16*A^2*b^2*(a + b*cot(c + d*x))^(1/2)*(a^4 +
```

b^4 - 6*a^2*b^2))/d^2)*(-((-A^4*b^2*d^4*(3*a^2 - b^2)^2)^(1/2) + A^2*a^3*d^
2 - 3*A^2*a*b^2*d^2)/d^4)^(1/2))/2)*((3*A^2*a*b^2)/(4*d^2) - (A^2*a^3)/(4*d
^2) - (6*A^4*a^2*b^4*d^4 - A^4*b^6*d^4 - 9*A^4*a^4*b^2*d^4)^(1/2)/(4*d^4))^
(1/2) - log((8*B^3*b^2*(a^2 - b^2)*(a^2 + b^2)^2)/d^3 - (((16*B^2*b^2*(a +
b*cot(c + d*x))^(1/2)*(a^4 + b^4 - 6*a^2*b^2))/d^2 + (16*a*b^2*(((-B^4*b^2*
d^4*(3*a^2 - b^2)^2)^(1/2) + B^2*a^3*d^2 - 3*B^2*a*b^2*d^2)/d^4)^(1/2)*(B*a
^2 + B*b^2 - d*(((-B^4*b^2*d^4*(3*a^2 - b^2)^2)^(1/2) + B^2*a^3*d^2 - 3*B^2
*a*b^2*d^2)/d^4)^(1/2)*(a + b*cot(c + d*x))^(1/2)))/d)*(((-B^4*b^2*d^4*(3*a
^2 - b^2)^2)^(1/2) + B^2*a^3*d^2 - 3*B^2*a*b^2*d^2)/d^4)^(1/2))/2)*(((6*B^4
*a^2*b^4*d^4 - B^4*b^6*d^4 - 9*B^4*a^4*b^2*d^4)^(1/2) + B^2*a^3*d^2 - 3*B^2
*a*b^2*d^2)/(4*d^4))^(1/2) - log((8*B^3*b^2*(a^2 - b^2)*(a^2 + b^2)^2)/d^3
- (((16*B^2*b^2*(a + b*cot(c + d*x))^(1/2)*(a^4 + b^4 - 6*a^2*b^2))/d^2 + (
16*a*b^2*(-((-B^4*b^2*d^4*(3*a^2 - b^2)^2)^(1/2) - B^2*a^3*d^2 + 3*B^2*a*b^
2*d^2)/d^4)^(1/2)*(B*a^2 + B*b^2 - d*(-((-B^4*b^2*d^4*(3*a^2 - b^2)^2)^(1/2
) - B^2*a^3*d^2 + 3*B^2*a*b^2*d^2)/d^4)^(1/2)*(a + b*cot(c + d*x))^(1/2)))/
d)*(-((-B^4*b^2*d^4*(3*a^2 - b^2)^2)^(1/2) - B^2*a^3*d^2 + 3*B^2*a*b^2*d^2)
/d^4)^(1/2))/2)*(-((6*B^4*a^2*b^4*d^4 - B^4*b^6*d^4 - 9*B^4*a^4*b^2*d^4)^(1
/2) - B^2*a^3*d^2 + 3*B^2*a*b^2*d^2)/(4*d^4))^(1/2) + log((((16*B^2*b^2*(a
+ b*cot(c + d*x))^(1/2)*(a^4 + b^4 - 6*a^2*b^2))/d^2 - (16*a*b^2*(((-B^4*b^
2*d^4*(3*a^2 - b^2)^2)^(1/2) + B^2*a^3*d^2 - 3*B^2*a*b^2*d^2)/d^4)^(1/2)*(B
*a^2 + B*b^2 + d*(((-B^4*b^2*d^4*(3*a^2 - b^2)^2)^(1/2) + B^2*a^3*d^2 - 3*B
^2*a*b^2*d^2)/d^4)^(1/2)*(a + b*cot(c + d*x))^(1/2)))/d)*(((-B^4*b^2*d^4*(3
*a^2 - b^2)^2)^(1/2) + B^2*a^3*d^2 - 3*B^2*a*b^2*d^2)/d^4)^(1/2))/2 + (8*B^
3*b^2*(a^2 - b^2)*(a^2 + b^2)^2)/d^3)*((6*B^4*a^2*b^4*d^4 - B^4*b^6*d^4 - 9
*B^4*a^4*b^2*d^4)^(1/2)/(4*d^4) + (B^2*a^3)/(4*d^2) - (3*B^2*a*b^2)/(4*d^2)
)^(1/2) + log((((16*B^2*b^2*(a + b*cot(c + d*x))^(1/2)*(a^4 + b^4 - 6*a^2*b
^2))/d^2 - (16*a*b^2*(-((-B^4*b^2*d^4*(3*a^2 - b^2)^2)^(1/2) - B^2*a^3*d^2
+ 3*B^2*a*b^2*d^2)/d^4)^(1/2)*(B*a^2 + B*b^2 + d*(-((-B^4*b^2*d^4*(3*a^2 -
b^2)^2)^(1/2) - B^2*a^3*d^2 + 3*B^2*a*b^2*d^2)/d^4)^(1/2)*(a + b*cot(c + d*
x))^(1/2)))/d)*(-((-B^4*b^2*d^4*(3*a^2 - b^2)^2)^(1/2) - B^2*a^3*d^2 + 3*B^
2*a*b^2*d^2)/d^4)^(1/2))/2 + (8*B^3*b^2*(a^2 - b^2)*(a^2 + b^2)^2)/d^3)*((B
^2*a^3)/(4*d^2) - (6*B^4*a^2*b^4*d^4 - B^4*b^6*d^4 - 9*B^4*a^4*b^2*d^4)^(1/
2)/(4*d^4) - (3*B^2*a*b^2)/(4*d^2))^(1/2) - (2*B*(a + b*cot(c + d*x))^(3/2)
)/(3*d) - (2*A*b*(a + b*cot(c + d*x))^(1/2))/d - (2*B*a*(a + b*cot(c + d*x)
)^(1/2))/d

3.97 $\int \sqrt{a+b\cot(c+dx)}(A+B\cot(c+dx))\,dx$

Optimal result

Integrand size = 25, antiderivative size = 122

$$\int \sqrt{a+b\cot(c+dx)}(A+B\cot(c+dx))\,dx = \frac{\sqrt{a-ib}(iA+B)\operatorname{arctanh}\left(\frac{\sqrt{a+b\cot(c+dx)}}{\sqrt{a-ib}}\right)}{d} - \frac{\sqrt{a+ib}(iA-B)\operatorname{arctanh}\left(\frac{\sqrt{a+b\cot(c+dx)}}{\sqrt{a+ib}}\right)}{d} - \frac{2B\sqrt{a+b\cot(c+dx)}}{d}$$

```
[Out] (I*A+B)*arctanh((a+b*cot(d*x+c))^(1/2)/(a-I*b)^(1/2))*(a-I*b)^(1/2)/d-(I*A-
B)*arctanh((a+b*cot(d*x+c))^(1/2)/(a+I*b)^(1/2))*(a+I*b)^(1/2)/d-2*B*(a+b*c
ot(d*x+c))^(1/2)/d
```

Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 122, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {3609, 3620, 3618, 65, 214}

$$\int \sqrt{a+b\cot(c+dx)}(A+B\cot(c+dx))\,dx = \frac{\sqrt{a-ib}(B+iA)\operatorname{arctanh}\left(\frac{\sqrt{a+b\cot(c+dx)}}{\sqrt{a-ib}}\right)}{d} - \frac{\sqrt{a+ib}(-B+iA)\operatorname{arctanh}\left(\frac{\sqrt{a+b\cot(c+dx)}}{\sqrt{a+ib}}\right)}{d} - \frac{2B\sqrt{a+b\cot(c+dx)}}{d}$$

```
[In] Int[Sqrt[a + b*Cot[c + d*x]]*(A + B*Cot[c + d*x]),x]
```

```
[Out] (Sqrt[a - I*b]*(I*A + B)*ArcTanh[Sqrt[a + b*Cot[c + d*x]]/Sqrt[a - I*b]])/d
 - (Sqrt[a + I*b]*(I*A - B)*ArcTanh[Sqrt[a + b*Cot[c + d*x]]/Sqrt[a + I*b]]
)/d - (2*B*Sqrt[a + b*Cot[c + d*x]])/d
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 3609

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)]), x_Symbol] :> Simp[d*((a + b*Tan[e + f*x])^m/(f*m)), x] + Int
[(a + b*Tan[e + f*x])^(m - 1)*Simp[a*c - b*d + (b*c + a*d)*Tan[e + f*x], x]
, x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2,
 0] && GtQ[m, 0]
```

Rule 3618

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*tan[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Dist[c*(d/f), Subst[Int[(a + (b/d)*x)^m/(d^2 + c
*x), x], x, d*Tan[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b
*c - a*d, 0] && NeQ[a^2 + b^2, 0] && EqQ[c^2 + d^2, 0]
```

Rule 3620

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)]), x_Symbol] :> Dist[(c + I*d)/2, Int[(a + b*Tan[e + f*x])^m*(1
 - I*Tan[e + f*x]), x], x] + Dist[(c - I*d)/2, Int[(a + b*Tan[e + f*x])^m*(
1 + I*Tan[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b*c -
 a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] &&  !IntegerQ[m]
```

Rubi steps

$$\text{integral} = -\frac{2B\sqrt{a+b\cot(c+dx)}}{d} + \int \frac{aA-bB+(Ab+aB)\cot(c+dx)}{\sqrt{a+b\cot(c+dx)}}\,dx$$

$$= -\frac{2B\sqrt{a+b\cot(c+dx)}}{d} + \frac{1}{2}((a-ib)(A-iB))\int \frac{1+i\cot(c+dx)}{\sqrt{a+b\cot(c+dx)}}\,dx + \frac{1}{2}((a+ib)(A+iB))\int \frac{1-i\cot(c+dx)}{\sqrt{a+b\cot(c+dx)}}\,dx$$

$$= -\frac{2B\sqrt{a+b\cot(c+dx)}}{d} - \frac{(i(a-ib)(A-iB))\text{Subst}\left(\int \frac{1}{(-1+x)\sqrt{a-ibx}}\,dx, x, i\cot(c+dx)\right)}{2d} + \frac{((ia-b)(A+iB))\text{Subst}\left(\int \frac{1}{(-1+x)\sqrt{a+ibx}}\,dx, x, -i\cot(c+dx)\right)}{2d}$$

$$= -\frac{2B\sqrt{a+b\cot(c+dx)}}{d} + \frac{((a-ib)(A-iB))\text{Subst}\left(\int \frac{1}{-1-\frac{ia}{b}+\frac{ix^2}{b}}\,dx, x, \sqrt{a+b\cot(c+dx)}\right)}{bd} + \frac{((a+ib)(A+iB))\text{Subst}\left(\int \frac{1}{-1+\frac{ia}{b}-\frac{ix^2}{b}}\,dx, x, \sqrt{a+b\cot(c+dx)}\right)}{bd}$$

$$= \frac{\sqrt{a-ib}(iA+B)\text{arctanh}\left(\frac{\sqrt{a+b\cot(c+dx)}}{\sqrt{a-ib}}\right)}{d} - \frac{\sqrt{a+ib}(iA-B)\text{arctanh}\left(\frac{\sqrt{a+b\cot(c+dx)}}{\sqrt{a+ib}}\right)}{d} - \frac{2B\sqrt{a+b\cot(c+dx)}}{d}$$

Mathematica [A] (verified)

Time = 0.66 (sec) , antiderivative size = 212, normalized size of antiderivative = 1.74

$$\int \sqrt{a+b\cot(c+dx)}(A+B\cot(c+dx))\,dx = -\frac{\frac{\left(aAb-Ab\sqrt{-b^2}-b^2B-a\sqrt{-b^2}B\right)\text{arctanh}\left(\frac{\sqrt{a+b\cot(c+dx)}}{\sqrt{a-\sqrt{-b^2}}}\right)}{\sqrt{-b^2}\sqrt{a-\sqrt{-b^2}}} - \frac{\left(aAb+Ab\sqrt{-b^2}-b^2B+a\sqrt{-b^2}B\right)\text{arctanh}\left(\frac{\sqrt{a+b\cot(c+dx)}}{\sqrt{a+\sqrt{-b^2}}}\right)}{\sqrt{-b^2}\sqrt{a+\sqrt{-b^2}}} + 2B}{d}$$

```
[In] Integrate[Sqrt[a + b*Cot[c + d*x]]*(A + B*Cot[c + d*x]),x]
```

```
[Out] -(((((a*A*b - A*b*Sqrt[-b^2] - b^2*B - a*Sqrt[-b^2]*B)*ArcTanh[Sqrt[a + b*Co
t[c + d*x]]/Sqrt[a - Sqrt[-b^2]]])/(Sqrt[-b^2]*Sqrt[a - Sqrt[-b^2]]) - ((a*
A*b + A*b*Sqrt[-b^2] - b^2*B + a*Sqrt[-b^2]*B)*ArcTanh[Sqrt[a + b*Cot[c + d
*x]]/Sqrt[a + Sqrt[-b^2]]])/(Sqrt[-b^2]*Sqrt[a + Sqrt[-b^2]]) + 2*B*Sqrt[a
+ b*Cot[c + d*x]])/d)
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 813 vs. $2(102) = 204$.

Time = 0.15 (sec) , antiderivative size = 814, normalized size of antiderivative = 6.67

method	result
parts	$-\frac{\ln\left(b\cot(dx+c)+a+\sqrt{a+b\cot(dx+c)}\,\sqrt{2\sqrt{a^2+b^2}+2a}+\sqrt{a^2+b^2}\right)A\sqrt{2\sqrt{a^2+b^2}+2a}\,a}{4db}+\frac{\ln\left(b\cot(dx+c)+a+\sqrt{a+b\cot\right.}{}$
derivativedivides	$-\frac{2B\sqrt{a+b\cot(dx+c)}}{d}-\frac{\ln\left(\sqrt{a+b\cot(dx+c)}\,\sqrt{2\sqrt{a^2+b^2}+2a}-b\cot(dx+c)-\sqrt{a^2+b^2}-a\right)A\sqrt{2\sqrt{a^2+b^2}+2a}\,\sqrt{a^2+b^2}}{4db}$
default	$-\frac{2B\sqrt{a+b\cot(dx+c)}}{d}-\frac{\ln\left(\sqrt{a+b\cot(dx+c)}\,\sqrt{2\sqrt{a^2+b^2}+2a}-b\cot(dx+c)-\sqrt{a^2+b^2}-a\right)A\sqrt{2\sqrt{a^2+b^2}+2a}\,\sqrt{a^2+b^2}}{4db}$

```
[In] int((a+b*cot(d*x+c))^(1/2)*(A+B*cot(d*x+c)),x,method=_RETURNVERBOSE)
```

```
[Out] -1/4/d/b*ln(b*cot(d*x+c)+a+(a+b*cot(d*x+c))^(1/2)*(2*(a^2+b^2)^(1/2)+2*a)^(
1/2)+(a^2+b^2)^(1/2))*A*(2*(a^2+b^2)^(1/2)+2*a)^(1/2)*a+1/4/d/b*ln(b*cot(d*
x+c)+a+(a+b*cot(d*x+c))^(1/2)*(2*(a^2+b^2)^(1/2)+2*a)^(1/2)+(a^2+b^2)^(1/2)
)*A*(2*(a^2+b^2)^(1/2)+2*a)^(1/2)*(a^2+b^2)^(1/2)-1/d*b/(2*(a^2+b^2)^(1/2)-
2*a)^(1/2)*arctan((2*(a+b*cot(d*x+c))^(1/2)+(2*(a^2+b^2)^(1/2)+2*a)^(1/2))/
(2*(a^2+b^2)^(1/2)-2*a)^(1/2))*A+1/4/d/b*ln((a+b*cot(d*x+c))^(1/2)*(2*(a^2+
b^2)^(1/2)+2*a)^(1/2)-b*cot(d*x+c)-(a^2+b^2)^(1/2)-a)*A*(2*(a^2+b^2)^(1/2)+
2*a)^(1/2)*a-1/4/d/b*ln((a+b*cot(d*x+c))^(1/2)*(2*(a^2+b^2)^(1/2)+2*a)^(1/2
)-b*cot(d*x+c)-(a^2+b^2)^(1/2)-a)*A*(2*(a^2+b^2)^(1/2)+2*a)^(1/2)*(a^2+b^2)
^(1/2)+1/d*b/(2*(a^2+b^2)^(1/2)-2*a)^(1/2)*arctan((-2*(a+b*cot(d*x+c))^(1/2
)+(2*(a^2+b^2)^(1/2)+2*a)^(1/2))/(2*(a^2+b^2)^(1/2)-2*a)^(1/2))*A+B/d*(-2*(
a+b*cot(d*x+c))^(1/2)+1/4*(2*(a^2+b^2)^(1/2)+2*a)^(1/2)*ln(b*cot(d*x+c)+a+(
a+b*cot(d*x+c))^(1/2)*(2*(a^2+b^2)^(1/2)+2*a)^(1/2)+(a^2+b^2)^(1/2))-(a-(a^
2+b^2)^(1/2))/(2*(a^2+b^2)^(1/2)-2*a)^(1/2)*arctan((2*(a+b*cot(d*x+c))^(1/2
)+(2*(a^2+b^2)^(1/2)+2*a)^(1/2))/(2*(a^2+b^2)^(1/2)-2*a)^(1/2))-1/4*(2*(a^2
+b^2)^(1/2)+2*a)^(1/2)*ln((a+b*cot(d*x+c))^(1/2)*(2*(a^2+b^2)^(1/2)+2*a)^(1
/2)-b*cot(d*x+c)-(a^2+b^2)^(1/2)-a)-((a^2+b^2)^(1/2)-a)/(2*(a^2+b^2)^(1/2)-
2*a)^(1/2)*arctan((-2*(a+b*cot(d*x+c))^(1/2)+(2*(a^2+b^2)^(1/2)+2*a)^(1/2))
/(2*(a^2+b^2)^(1/2)-2*a)^(1/2)))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1329 vs. $2(96) = 192$.

Time = 0.33 (sec) , antiderivative size = 1329, normalized size of antiderivative = 10.89

$$\int \sqrt{a + b\cot(c + dx)}(A + B\cot(c + dx))\, dx = \text{Too large to display}$$

```
[In] integrate((a+b*cot(d*x+c))^(1/2)*(A+B*cot(d*x+c)),x, algorithm="fricas")
```

```
[Out] 1/2*(d*sqrt((2*A*B*b + d^2*sqrt(-(4*A^2*B^2*a^2 + 4*(A^3*B - A*B^3)*a*b + (
A^4 - 2*A^2*B^2 + B^4)*b^2)/d^4) - (A^2 - B^2)*a)/d^2)*log(-(2*(A^3*B + A*B
^3)*a + (A^4 - B^4)*b)*sqrt((b*cos(2*d*x + 2*c) + a*sin(2*d*x + 2*c) + b)/s
in(2*d*x + 2*c)) + (A*d^3*sqrt(-(4*A^2*B^2*a^2 + 4*(A^3*B - A*B^3)*a*b + (A
^4 - 2*A^2*B^2 + B^4)*b^2)/d^4) - (2*A*B^2*a + (A^2*B - B^3)*b)*d)*sqrt((2*
A*B*b + d^2*sqrt(-(4*A^2*B^2*a^2 + 4*(A^3*B - A*B^3)*a*b + (A^4 - 2*A^2*B^2
 + B^4)*b^2)/d^4) - (A^2 - B^2)*a)/d^2)) - d*sqrt((2*A*B*b + d^2*sqrt(-(4*A
^2*B^2*a^2 + 4*(A^3*B - A*B^3)*a*b + (A^4 - 2*A^2*B^2 + B^4)*b^2)/d^4) - (A
^2 - B^2)*a)/d^2)*log(-(2*(A^3*B + A*B^3)*a + (A^4 - B^4)*b)*sqrt((b*cos(2*
d*x + 2*c) + a*sin(2*d*x + 2*c) + b)/sin(2*d*x + 2*c)) - (A*d^3*sqrt(-(4*A^
2*B^2*a^2 + 4*(A^3*B - A*B^3)*a*b + (A^4 - 2*A^2*B^2 + B^4)*b^2)/d^4) - (2*
A*B^2*a + (A^2*B - B^3)*b)*d)*sqrt((2*A*B*b + d^2*sqrt(-(4*A^2*B^2*a^2 + 4*
(A^3*B - A*B^3)*a*b + (A^4 - 2*A^2*B^2 + B^4)*b^2)/d^4) - (A^2 - B^2)*a)/d^
2)) - d*sqrt((2*A*B*b - d^2*sqrt(-(4*A^2*B^2*a^2 + 4*(A^3*B - A*B^3)*a*b +
(A^4 - 2*A^2*B^2 + B^4)*b^2)/d^4) - (A^2 - B^2)*a)/d^2)*log(-(2*(A^3*B + A*
B^3)*a + (A^4 - B^4)*b)*sqrt((b*cos(2*d*x + 2*c) + a*sin(2*d*x + 2*c) + b)/
sin(2*d*x + 2*c)) + (A*d^3*sqrt(-(4*A^2*B^2*a^2 + 4*(A^3*B - A*B^3)*a*b + (
A^4 - 2*A^2*B^2 + B^4)*b^2)/d^4) + (2*A*B^2*a + (A^2*B - B^3)*b)*d)*sqrt((2
*A*B*b - d^2*sqrt(-(4*A^2*B^2*a^2 + 4*(A^3*B - A*B^3)*a*b + (A^4 - 2*A^2*B^
2 + B^4)*b^2)/d^4) - (A^2 - B^2)*a)/d^2)) + d*sqrt((2*A*B*b - d^2*sqrt(-(4*
A^2*B^2*a^2 + 4*(A^3*B - A*B^3)*a*b + (A^4 - 2*A^2*B^2 + B^4)*b^2)/d^4) - (
A^2 - B^2)*a)/d^2)*log(-(2*(A^3*B + A*B^3)*a + (A^4 - B^4)*b)*sqrt((b*cos(2
*d*x + 2*c) + a*sin(2*d*x + 2*c) + b)/sin(2*d*x + 2*c)) - (A*d^3*sqrt(-(4*A
^2*B^2*a^2 + 4*(A^3*B - A*B^3)*a*b + (A^4 - 2*A^2*B^2 + B^4)*b^2)/d^4) + (2
*A*B^2*a + (A^2*B - B^3)*b)*d)*sqrt((2*A*B*b - d^2*sqrt(-(4*A^2*B^2*a^2 + 4
*(A^3*B - A*B^3)*a*b + (A^4 - 2*A^2*B^2 + B^4)*b^2)/d^4) - (A^2 - B^2)*a)/d
^2)) - 4*B*sqrt((b*cos(2*d*x + 2*c) + a*sin(2*d*x + 2*c) + b)/sin(2*d*x + 2
*c)))/d
```

Sympy [F]

$$\int \sqrt{a+b\cot(c+dx)}(A+B\cot(c+dx))\,dx = \int (A+B\cot{(c+dx)})\sqrt{a+b\cot{(c+dx)}}\,dx$$

```
[In] integrate((a+b*cot(d*x+c))**(1/2)*(A+B*cot(d*x+c)),x)
[Out] Integral((A + B*cot(c + d*x))*sqrt(a + b*cot(c + d*x)), x)
```

Maxima [F]

$$\int \sqrt{a+b\cot(c+dx)}(A+B\cot(c+dx))\,dx = \int (B\cot{(dx+c)}+A)\sqrt{b\cot{(dx+c)}+a}\,dx$$

```
[In] integrate((a+b*cot(d*x+c))^(1/2)*(A+B*cot(d*x+c)),x, algorithm="maxima")
[Out] integrate((B*cot(d*x + c) + A)*sqrt(b*cot(d*x + c) + a), x)
```

Giac [F]

$$\int \sqrt{a+b\cot(c+dx)}(A+B\cot(c+dx))\,dx = \int (B\cot{(dx+c)}+A)\sqrt{b\cot{(dx+c)}+a}\,dx$$

```
[In] integrate((a+b*cot(d*x+c))^(1/2)*(A+B*cot(d*x+c)),x, algorithm="giac")
[Out] integrate((B*cot(d*x + c) + A)*sqrt(b*cot(d*x + c) + a), x)
```

Mupad [B] (verification not implemented)

Time = 15.52 (sec) , antiderivative size = 843, normalized size of antiderivative = 6.91

$$\int \sqrt{a+b\cot(c+dx)}(A+B\cot(c+dx))\,dx$$
$$=\operatorname{atanh}\left(\frac{d^3\left(\frac{16\,(A^2\,b^4-A^2\,a^2\,b^2)\,\sqrt{a+b\cot(c+d\,x)}}{d^2}+\frac{16\,a\,b^2\,\left(\sqrt{-A^4\,b^2\,d^4}+A^2\,a\,d^2\right)\sqrt{a+b\cot(c+d\,x)}}{d^4}\right)\sqrt{-\frac{\sqrt{-A^4\,b^2\,d^4}+A^2\,a\,d^2}{d^4}}}{16\,\left(A^3\,a^2\,b^3+A^3\,b^5\right)}\right)$$
$$+\operatorname{atanh}\left(\frac{d^3\left(\frac{16\,(A^2\,b^4-A^2\,a^2\,b^2)\,\sqrt{a+b\cot(c+d\,x)}}{d^2}-\frac{16\,a\,b^2\,\left(\sqrt{-A^4\,b^2\,d^4}-A^2\,a\,d^2\right)\sqrt{a+b\cot(c+d\,x)}}{d^4}\right)\sqrt{\frac{\sqrt{-A^4\,b^2\,d^4}-A^2\,a\,d^2}{d^4}}}{16\,\left(A^3\,a^2\,b^3+A^3\,b^5\right)}\right)$$
$$+2\operatorname{atanh}\left(\frac{32\,B^2\,b^4\sqrt{\frac{\sqrt{-B^4\,b^2\,d^4}}{4\,d^4}+\frac{B^2\,a}{4\,d^2}}\sqrt{a+b\cot\left(c+d\,x\right)}}{\frac{16\,B\,b^4\sqrt{-B^4\,b^2\,d^4}}{d^3}+\frac{16\,B\,a^2\,b^2\sqrt{-B^4\,b^2\,d^4}}{d^3}}\right.$$
$$\left.+\frac{32\,a\,b^2\sqrt{\frac{\sqrt{-B^4\,b^2\,d^4}}{4\,d^4}+\frac{B^2\,a}{4\,d^2}}\sqrt{a+b\cot\left(c+d\,x\right)}\sqrt{-B^4\,b^2\,d^4}}{\frac{16\,B\,b^4\sqrt{-B^4\,b^2\,d^4}}{d}+\frac{16\,B\,a^2\,b^2\sqrt{-B^4\,b^2\,d^4}}{d}}\right)\sqrt{\frac{\sqrt{-B^4\,b^2\,d^4}+B^2\,a\,d^2}{4\,d^4}}$$
$$-2\operatorname{atanh}\left(\frac{32\,B^2\,b^4\sqrt{\frac{B^2\,a}{4\,d^2}-\frac{\sqrt{-B^4\,b^2\,d^4}}{4\,d^4}}\sqrt{a+b\cot\left(c+d\,x\right)}}{\frac{16\,B\,b^4\sqrt{-B^4\,b^2\,d^4}}{d^3}+\frac{16\,B\,a^2\,b^2\sqrt{-B^4\,b^2\,d^4}}{d^3}}\right.$$
$$\left.-\frac{32\,a\,b^2\sqrt{\frac{B^2\,a}{4\,d^2}-\frac{\sqrt{-B^4\,b^2\,d^4}}{4\,d^4}}\sqrt{a+b\cot\left(c+d\,x\right)}\sqrt{-B^4\,b^2\,d^4}}{\frac{16\,B\,b^4\sqrt{-B^4\,b^2\,d^4}}{d}+\frac{16\,B\,a^2\,b^2\sqrt{-B^4\,b^2\,d^4}}{d}}\right)\sqrt{-\frac{\sqrt{-B^4\,b^2\,d^4}-B^2\,a\,d^2}{4\,d^4}}$$
$$-\frac{2\,B\sqrt{a+b\cot\left(c+d\,x\right)}}{d}$$

```
[In] int((A + B*cot(c + d*x))*(a + b*cot(c + d*x))^(1/2),x)
```

```
[Out] atanh((d^3*((16*(A^2*b^4 - A^2*a^2*b^2)*(a + b*cot(c + d*x))^(1/2))/d^2 + (
16*a*b^2*((-A^4*b^2*d^4)^(1/2) + A^2*a*d^2)*(a + b*cot(c + d*x))^(1/2))/d^4
)*(-((-A^4*b^2*d^4)^(1/2) + A^2*a*d^2)/d^4)^(1/2))/(16*(A^3*b^5 + A^3*a^2*b
^3)))*(-((-A^4*b^2*d^4)^(1/2) + A^2*a*d^2)/d^4)^(1/2) + atanh((d^3*((16*(A^
2*b^4 - A^2*a^2*b^2)*(a + b*cot(c + d*x))^(1/2))/d^2 - (16*a*b^2*((-A^4*b^2
*d^4)^(1/2) - A^2*a*d^2)*(a + b*cot(c + d*x))^(1/2))/d^4)*(((-A^4*b^2*d^4)^
(1/2) - A^2*a*d^2)/d^4)^(1/2))/(16*(A^3*b^5 + A^3*a^2*b^3)))*(((-A^4*b^2*d^
4)^(1/2) - A^2*a*d^2)/d^4)^(1/2) + 2*atanh((32*B^2*b^4*((-B^4*b^2*d^4)^(1/2
)/(4*d^4) + (B^2*a)/(4*d^2))^(1/2)*(a + b*cot(c + d*x))^(1/2))/((16*B*b^4*(
-B^4*b^2*d^4)^(1/2))/d^3 + (16*B*a^2*b^2*(-B^4*b^2*d^4)^(1/2))/d^3) + (32*a
*b^2*((-B^4*b^2*d^4)^(1/2)/(4*d^4) + (B^2*a)/(4*d^2))^(1/2)*(a + b*cot(c +
d*x))^(1/2)*(-B^4*b^2*d^4)^(1/2))/((16*B*b^4*(-B^4*b^2*d^4)^(1/2))/d + (16*
```

```
B*a^2*b^2*(-B^4*b^2*d^4)^(1/2))/d))*(((-B^4*b^2*d^4)^(1/2) + B^2*a*d^2)/(4*
d^4))^(1/2) - 2*atanh((32*B^2*b^4*((B^2*a)/(4*d^2) - (-B^4*b^2*d^4)^(1/2)/(
4*d^4))^(1/2)*(a + b*cot(c + d*x))^(1/2))/((16*B*b^4*(-B^4*b^2*d^4)^(1/2))/
d^3 + (16*B*a^2*b^2*(-B^4*b^2*d^4)^(1/2))/d^3) - (32*a*b^2*((B^2*a)/(4*d^2)
 - (-B^4*b^2*d^4)^(1/2)/(4*d^4))^(1/2)*(a + b*cot(c + d*x))^(1/2)*(-B^4*b^2
*d^4)^(1/2))/((16*B*b^4*(-B^4*b^2*d^4)^(1/2))/d + (16*B*a^2*b^2*(-B^4*b^2*d
^4)^(1/2))/d))*(-((-B^4*b^2*d^4)^(1/2) - B^2*a*d^2)/(4*d^4))^(1/2) - (2*B*(
a + b*cot(c + d*x))^(1/2))/d
```

3.98 $\int (-a+b\cot(c+dx))(a+b\cot(c+dx))^{5/2}\,dx$

Optimal result

Integrand size = 27, antiderivative size = 151

$$\int (-a+b\cot(c+dx))(a+b\cot(c+dx))^{5/2}\,dx = -\frac{(ia-b)(a-ib)^{5/2}\text{arctanh}\left(\frac{\sqrt{a+b\cot(c+dx)}}{\sqrt{a-ib}}\right)}{d} + \frac{(a+ib)^{5/2}(ia+b)\text{arctanh}\left(\frac{\sqrt{a+b\cot(c+dx)}}{\sqrt{a+ib}}\right)}{d} + \frac{2b(a^2+b^2)\sqrt{a+b\cot(c+dx)}}{d} - \frac{2b(a+b\cot(c+dx))^{5/2}}{5d}$$

```
[Out] -(I*a-b)*(a-I*b)^(5/2)*arctanh((a+b*cot(d*x+c))^(1/2)/(a-I*b)^(1/2))/d+(a+I
*b)^(5/2)*(I*a+b)*arctanh((a+b*cot(d*x+c))^(1/2)/(a+I*b)^(1/2))/d-2/5*b*(a+
b*cot(d*x+c))^(5/2)/d+2*b*(a^2+b^2)*(a+b*cot(d*x+c))^(1/2)/d
```

Rubi [A] (verified)

Time = 0.31 (sec) , antiderivative size = 151, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.259, Rules used = {3609, 12, 3563, 3620, 3618, 65, 214}

$$\int (-a+b\cot(c+dx))(a+b\cot(c+dx))^{5/2}\,dx = \frac{2b(a^2+b^2)\sqrt{a+b\cot(c+dx)}}{d} - \frac{(-b+ia)(a-ib)^{5/2}\text{arctanh}\left(\frac{\sqrt{a+b\cot(c+dx)}}{\sqrt{a-ib}}\right)}{d} + \frac{(a+ib)^{5/2}(b+ia)\text{arctanh}\left(\frac{\sqrt{a+b\cot(c+dx)}}{\sqrt{a+ib}}\right)}{d} - \frac{2b(a+b\cot(c+dx))^{5/2}}{5d}$$

```
[In] Int[(-a + b*Cot[c + d*x])*(a + b*Cot[c + d*x])^(5/2),x]
```

```
[Out] -(((I*a - b)*(a - I*b)^(5/2)*ArcTanh[Sqrt[a + b*Cot[c + d*x]]/Sqrt[a - I*b]
])/d) + ((a + I*b)^(5/2)*(I*a + b)*ArcTanh[Sqrt[a + b*Cot[c + d*x]]/Sqrt[a
+ I*b]])/d + (2*b*(a^2 + b^2)*Sqrt[a + b*Cot[c + d*x]])/d - (2*b*(a + b*Cot
[c + d*x])^(5/2))/(5*d)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 3563

```
Int[((a_) + (b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[b*((a +
b*Tan[c + d*x])^(n - 1)/(d*(n - 1))), x] + Int[(a^2 - b^2 + 2*a*b*Tan[c + d
*x])*(a + b*Tan[c + d*x])^(n - 2), x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2
+ b^2, 0] && GtQ[n, 1]
```

Rule 3609

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)]), x_Symbol] :> Simp[d*((a + b*Tan[e + f*x])^m/(f*m)), x] + Int
[(a + b*Tan[e + f*x])^(m - 1)*Simp[a*c - b*d + (b*c + a*d)*Tan[e + f*x], x]
, x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2,
 0] && GtQ[m, 0]
```

Rule 3618

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*tan[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Dist[c*(d/f), Subst[Int[(a + (b/d)*x)^m/(d^2 + c
*x), x], x, d*Tan[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b
*c - a*d, 0] && NeQ[a^2 + b^2, 0] && EqQ[c^2 + d^2, 0]
```

Rule 3620

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)]), x_Symbol] :> Dist[(c + I*d)/2, Int[(a + b*Tan[e + f*x])^m*(1
 - I*Tan[e + f*x]), x], x] + Dist[(c - I*d)/2, Int[(a + b*Tan[e + f*x])^m*(
1 + I*Tan[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b*c -
 a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] &&  !IntegerQ[m]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\frac{2b(a+b\cot(c+dx))^{5/2}}{5d}+\int\left(-a^2-b^2\right)(a+b\cot(c+dx))^{3/2}\,dx\\
&= -\frac{2b(a+b\cot(c+dx))^{5/2}}{5d}+\left(-a^2-b^2\right)\int(a+b\cot(c+dx))^{3/2}\,dx\\
&= \frac{2b(a^2+b^2)\sqrt{a+b\cot(c+dx)}}{d}-\frac{2b(a+b\cot(c+dx))^{5/2}}{5d}\\
&\quad+\left(-a^2-b^2\right)\int\frac{a^2-b^2+2ab\cot(c+dx)}{\sqrt{a+b\cot(c+dx)}}\,dx\\
&= \frac{2b(a^2+b^2)\sqrt{a+b\cot(c+dx)}}{d}-\frac{2b(a+b\cot(c+dx))^{5/2}}{5d}\\
&\quad-\frac{1}{2}\left((a-ib)^2\left(a^2+b^2\right)\right)\int\frac{1+i\cot(c+dx)}{\sqrt{a+b\cot(c+dx)}}\,dx\\
&\quad-\frac{1}{2}\left((a+ib)^2\left(a^2+b^2\right)\right)\int\frac{1-i\cot(c+dx)}{\sqrt{a+b\cot(c+dx)}}\,dx\\
&= \frac{2b(a^2+b^2)\sqrt{a+b\cot(c+dx)}}{d}-\frac{2b(a+b\cot(c+dx))^{5/2}}{5d}\\
&\quad-\frac{((a+ib)^3(ia+b))\,\text{Subst}\left(\int\frac{1}{(-1+x)\sqrt{a+ibx}}\,dx,x,-i\cot(c+dx)\right)}{2d}\\
&\quad-\frac{((a+ib)(ia+b)^3)\,\text{Subst}\left(\int\frac{1}{(-1+x)\sqrt{a-ibx}}\,dx,x,i\cot(c+dx)\right)}{2d}\\
&= \frac{2b(a^2+b^2)\sqrt{a+b\cot(c+dx)}}{d}-\frac{2b(a+b\cot(c+dx))^{5/2}}{5d}\\
&\quad-\frac{((a-ib)^3(a+ib))\,\text{Subst}\left(\int\frac{1}{-1-\frac{ia}{b}+\frac{ix^2}{b}}\,dx,x,\sqrt{a+b\cot(c+dx)}\right)}{bd}\\
&\quad-\frac{((a-ib)(a+ib)^3)\,\text{Subst}\left(\int\frac{1}{-1+\frac{ia}{b}-\frac{ix^2}{b}}\,dx,x,\sqrt{a+b\cot(c+dx)}\right)}{bd}
\end{aligned}$$

$$= -\frac{(ia-b)(a-ib)^{5/2}\operatorname{arctanh}\left(\frac{\sqrt{a+b\cot(c+dx)}}{\sqrt{a-ib}}\right)}{d} + \frac{(a+ib)^{5/2}(ia+b)\operatorname{arctanh}\left(\frac{\sqrt{a+b\cot(c+dx)}}{\sqrt{a+ib}}\right)}{d} + \frac{2b(a^2+b^2)\sqrt{a+b\cot(c+dx)}}{d} - \frac{2b(a+b\cot(c+dx))^{5/2}}{5d}$$

Mathematica [A] (verified)

Time = 1.21 (sec) , antiderivative size = 222, normalized size of antiderivative = 1.47

$$\int(-a+b\cot(c+dx))(a+b\cot(c+dx))^{5/2}\,dx = \frac{b\left(\frac{5(a^2+b^2)\left(a^2-b^2-2a\sqrt{-b^2}\right)\operatorname{arctanh}\left(\frac{\sqrt{a+b\cot(c+dx)}}{\sqrt{a-\sqrt{-b^2}}}\right)}{\sqrt{-b^2}\sqrt{a-\sqrt{-b^2}}} - \frac{5(a^2+b^2)\left(a^2-b^2+2a\sqrt{-b^2}\right)\operatorname{arctanh}\left(\frac{\sqrt{a+b\cot(c+dx)}}{\sqrt{a+\sqrt{-b^2}}}\right)}{\sqrt{-b^2}\sqrt{a+\sqrt{-b^2}}} + \right.}{5d}$$

```
[In] Integrate[(-a + b*Cot[c + d*x])*(a + b*Cot[c + d*x])^(5/2),x]
```

```
[Out] (b*((5*(a^2 + b^2)*(a^2 - b^2 - 2*a*Sqrt[-b^2])*ArcTanh[Sqrt[a + b*Cot[c +
d*x]]/Sqrt[a - Sqrt[-b^2]]])/(Sqrt[-b^2]*Sqrt[a - Sqrt[-b^2]]) - (5*(a^2 +
b^2)*(a^2 - b^2 + 2*a*Sqrt[-b^2])*ArcTanh[Sqrt[a + b*Cot[c + d*x]]/Sqrt[a +
 Sqrt[-b^2]]])/(Sqrt[-b^2]*Sqrt[a + Sqrt[-b^2]]) + 10*(a^2 + b^2)*Sqrt[a +
b*Cot[c + d*x]] - 2*(a + b*Cot[c + d*x])^(5/2)))/(5*d)
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 1374 vs. 2(127) = 254.

Time = 0.08 (sec) , antiderivative size = 1375, normalized size of antiderivative = 9.11

method	result	size
derivativedivides	Expression too large to display	1375
default	Expression too large to display	1375
parts	Expression too large to display	2386

```
[In] int((-a+b*cot(d*x+c))*(a+b*cot(d*x+c))^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] -2/5*b*(a+b*cot(d*x+c))^(5/2)/d+2/d*b*(a+b*cot(d*x+c))^(1/2)*a^2+2/d*b^3*(a
+b*cot(d*x+c))^(1/2)+1/4/d/b*ln((a+b*cot(d*x+c))^(1/2)*(2*(a^2+b^2)^(1/2)+2
*a)^(1/2)-b*cot(d*x+c)-(a^2+b^2)^(1/2)-a)*(2*(a^2+b^2)^(1/2)+2*a)^(1/2)*(a^
2+b^2)^(1/2)*a^3+1/4/d*b*ln((a+b*cot(d*x+c))^(1/2)*(2*(a^2+b^2)^(1/2)+2*a)^
(1/2)-b*cot(d*x+c)-(a^2+b^2)^(1/2)-a)*(2*(a^2+b^2)^(1/2)+2*a)^(1/2)*(a^2+b^
```

```
2)^(1/2)*a-1/4/d/b*ln((a+b*cot(d*x+c))^(1/2)*(2*(a^2+b^2)^(1/2)+2*a)^(1/2)-
b*cot(d*x+c)-(a^2+b^2)^(1/2)-a)*(2*(a^2+b^2)^(1/2)+2*a)^(1/2)*a^4+1/4/d*b^3
*ln((a+b*cot(d*x+c))^(1/2)*(2*(a^2+b^2)^(1/2)+2*a)^(1/2)-b*cot(d*x+c)-(a^2+
b^2)^(1/2)-a)*(2*(a^2+b^2)^(1/2)+2*a)^(1/2)+1/d*b/(2*(a^2+b^2)^(1/2)-2*a)^(
1/2)*arctan((-2*(a+b*cot(d*x+c))^(1/2)+(2*(a^2+b^2)^(1/2)+2*a)^(1/2))/(2*(a
^2+b^2)^(1/2)-2*a)^(1/2))*(a^2+b^2)^(1/2)*a^2+1/d*b^3/(2*(a^2+b^2)^(1/2)-2*
a)^(1/2)*arctan((-2*(a+b*cot(d*x+c))^(1/2)+(2*(a^2+b^2)^(1/2)+2*a)^(1/2))/(
2*(a^2+b^2)^(1/2)-2*a)^(1/2))*(a^2+b^2)^(1/2)-2/d*b/(2*(a^2+b^2)^(1/2)-2*a)
^(1/2)*arctan((-2*(a+b*cot(d*x+c))^(1/2)+(2*(a^2+b^2)^(1/2)+2*a)^(1/2))/(2*
(a^2+b^2)^(1/2)-2*a)^(1/2))*a^3-2/d*b^3/(2*(a^2+b^2)^(1/2)-2*a)^(1/2)*arcta
n((-2*(a+b*cot(d*x+c))^(1/2)+(2*(a^2+b^2)^(1/2)+2*a)^(1/2))/(2*(a^2+b^2)^(1
/2)-2*a)^(1/2))*a-1/4/d/b*ln(b*cot(d*x+c)+a+(a+b*cot(d*x+c))^(1/2)*(2*(a^2+
b^2)^(1/2)+2*a)^(1/2)+(a^2+b^2)^(1/2))*(2*(a^2+b^2)^(1/2)+2*a)^(1/2)*(a^2+b
^2)^(1/2)*a^3-1/4/d*b*ln(b*cot(d*x+c)+a+(a+b*cot(d*x+c))^(1/2)*(2*(a^2+b^2)
^(1/2)+2*a)^(1/2)+(a^2+b^2)^(1/2))*(2*(a^2+b^2)^(1/2)+2*a)^(1/2)*(a^2+b^2)^
(1/2)*a+1/4/d/b*ln(b*cot(d*x+c)+a+(a+b*cot(d*x+c))^(1/2)*(2*(a^2+b^2)^(1/2)
+2*a)^(1/2)+(a^2+b^2)^(1/2))*(2*(a^2+b^2)^(1/2)+2*a)^(1/2)*a^4-1/4/d*b^3*ln
(b*cot(d*x+c)+a+(a+b*cot(d*x+c))^(1/2)*(2*(a^2+b^2)^(1/2)+2*a)^(1/2)+(a^2+b
^2)^(1/2))*(2*(a^2+b^2)^(1/2)+2*a)^(1/2)-1/d*b/(2*(a^2+b^2)^(1/2)-2*a)^(1/2
)*arctan((2*(a+b*cot(d*x+c))^(1/2)+(2*(a^2+b^2)^(1/2)+2*a)^(1/2))/(2*(a^2+b
^2)^(1/2)-2*a)^(1/2))*(a^2+b^2)^(1/2)*a^2-1/d*b^3/(2*(a^2+b^2)^(1/2)-2*a)^(
1/2)*arctan((2*(a+b*cot(d*x+c))^(1/2)+(2*(a^2+b^2)^(1/2)+2*a)^(1/2))/(2*(a^
2+b^2)^(1/2)-2*a)^(1/2))*(a^2+b^2)^(1/2)+2/d*b/(2*(a^2+b^2)^(1/2)-2*a)^(1/2
)*arctan((2*(a+b*cot(d*x+c))^(1/2)+(2*(a^2+b^2)^(1/2)+2*a)^(1/2))/(2*(a^2+b
^2)^(1/2)-2*a)^(1/2))*a^3+2/d*b^3/(2*(a^2+b^2)^(1/2)-2*a)^(1/2)*arctan((2*(
a+b*cot(d*x+c))^(1/2)+(2*(a^2+b^2)^(1/2)+2*a)^(1/2))/(2*(a^2+b^2)^(1/2)-2*a
)^(1/2))*a
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1684 vs. $2(118) = 236$.

Time = 0.31 (sec) , antiderivative size = 1684, normalized size of antiderivative = 11.15

$$\int (-a + b\cot(c + dx))(a + b\cot(c + dx))^{5/2}\,dx = \text{Too large to display}$$

```
[In] integrate((-a+b*cot(d*x+c))*(a+b*cot(d*x+c))^(5/2),x, algorithm="fricas")
```

```
[Out] 1/10*(5*(d*cos(2*d*x + 2*c) - d)*sqrt(-(a^7 - a^5*b^2 - 5*a^3*b^4 - 3*a*b^6
 + d^2*sqrt(-(9*a^12*b^2 + 30*a^10*b^4 + 31*a^8*b^6 + 4*a^6*b^8 - 9*a^4*b^1
0 - 2*a^2*b^12 + b^14)/d^4))/d^2)*log(-(3*a^10*b + 11*a^8*b^3 + 14*a^6*b^5
+ 6*a^4*b^7 - a^2*b^9 - b^11)*sqrt((b*cos(2*d*x + 2*c) + a*sin(2*d*x + 2*c)
 + b)/sin(2*d*x + 2*c)) + (a*d^3*sqrt(-(9*a^12*b^2 + 30*a^10*b^4 + 31*a^8*b
^6 + 4*a^6*b^8 - 9*a^4*b^10 - 2*a^2*b^12 + b^14)/d^4) + (3*a^6*b^2 + 5*a^4*
b^4 + a^2*b^6 - b^8)*d)*sqrt(-(a^7 - a^5*b^2 - 5*a^3*b^4 - 3*a*b^6 + d^2*sq
```

```
rt(-(9*a^12*b^2 + 30*a^10*b^4 + 31*a^8*b^6 + 4*a^6*b^8 - 9*a^4*b^10 - 2*a^2
*b^12 + b^14)/d^4))/d^2)) - 5*(d*cos(2*d*x + 2*c) - d)*sqrt(-(a^7 - a^5*b^2
 - 5*a^3*b^4 - 3*a*b^6 + d^2*sqrt(-(9*a^12*b^2 + 30*a^10*b^4 + 31*a^8*b^6 +
 4*a^6*b^8 - 9*a^4*b^10 - 2*a^2*b^12 + b^14)/d^4))/d^2)*log(-(3*a^10*b + 11
*a^8*b^3 + 14*a^6*b^5 + 6*a^4*b^7 - a^2*b^9 - b^11)*sqrt((b*cos(2*d*x + 2*c
) + a*sin(2*d*x + 2*c) + b)/sin(2*d*x + 2*c)) - (a*d^3*sqrt(-(9*a^12*b^2 +
30*a^10*b^4 + 31*a^8*b^6 + 4*a^6*b^8 - 9*a^4*b^10 - 2*a^2*b^12 + b^14)/d^4)
 + (3*a^6*b^2 + 5*a^4*b^4 + a^2*b^6 - b^8)*d)*sqrt(-(a^7 - a^5*b^2 - 5*a^3*
b^4 - 3*a*b^6 + d^2*sqrt(-(9*a^12*b^2 + 30*a^10*b^4 + 31*a^8*b^6 + 4*a^6*b^
8 - 9*a^4*b^10 - 2*a^2*b^12 + b^14)/d^4))/d^2)) - 5*(d*cos(2*d*x + 2*c) - d
)*sqrt(-(a^7 - a^5*b^2 - 5*a^3*b^4 - 3*a*b^6 - d^2*sqrt(-(9*a^12*b^2 + 30*a
^10*b^4 + 31*a^8*b^6 + 4*a^6*b^8 - 9*a^4*b^10 - 2*a^2*b^12 + b^14)/d^4))/d^
2)*log(-(3*a^10*b + 11*a^8*b^3 + 14*a^6*b^5 + 6*a^4*b^7 - a^2*b^9 - b^11)*s
qrt((b*cos(2*d*x + 2*c) + a*sin(2*d*x + 2*c) + b)/sin(2*d*x + 2*c)) + (a*d^
3*sqrt(-(9*a^12*b^2 + 30*a^10*b^4 + 31*a^8*b^6 + 4*a^6*b^8 - 9*a^4*b^10 - 2
*a^2*b^12 + b^14)/d^4) - (3*a^6*b^2 + 5*a^4*b^4 + a^2*b^6 - b^8)*d)*sqrt(-(
a^7 - a^5*b^2 - 5*a^3*b^4 - 3*a*b^6 - d^2*sqrt(-(9*a^12*b^2 + 30*a^10*b^4 +
 31*a^8*b^6 + 4*a^6*b^8 - 9*a^4*b^10 - 2*a^2*b^12 + b^14)/d^4))/d^2)) + 5*(
d*cos(2*d*x + 2*c) - d)*sqrt(-(a^7 - a^5*b^2 - 5*a^3*b^4 - 3*a*b^6 - d^2*sq
rt(-(9*a^12*b^2 + 30*a^10*b^4 + 31*a^8*b^6 + 4*a^6*b^8 - 9*a^4*b^10 - 2*a^2
*b^12 + b^14)/d^4))/d^2)*log(-(3*a^10*b + 11*a^8*b^3 + 14*a^6*b^5 + 6*a^4*b
^7 - a^2*b^9 - b^11)*sqrt((b*cos(2*d*x + 2*c) + a*sin(2*d*x + 2*c) + b)/sin
(2*d*x + 2*c)) - (a*d^3*sqrt(-(9*a^12*b^2 + 30*a^10*b^4 + 31*a^8*b^6 + 4*a^
6*b^8 - 9*a^4*b^10 - 2*a^2*b^12 + b^14)/d^4) - (3*a^6*b^2 + 5*a^4*b^4 + a^2
*b^6 - b^8)*d)*sqrt(-(a^7 - a^5*b^2 - 5*a^3*b^4 - 3*a*b^6 - d^2*sqrt(-(9*a^
12*b^2 + 30*a^10*b^4 + 31*a^8*b^6 + 4*a^6*b^8 - 9*a^4*b^10 - 2*a^2*b^12 + b
^14)/d^4))/d^2)) + 8*(a*b^2*sin(2*d*x + 2*c) - 2*a^2*b - 2*b^3 + (2*a^2*b +
 3*b^3)*cos(2*d*x + 2*c))*sqrt((b*cos(2*d*x + 2*c) + a*sin(2*d*x + 2*c) + b
)/sin(2*d*x + 2*c)))/(d*cos(2*d*x + 2*c) - d)
```

Sympy [F]

$$\begin{aligned}\int (-a+b\cot(c+dx))(a+b\cot(c+dx))^{5/2}\,dx =& \\ -\int a^3\sqrt{a+b\cot{(c+dx)}}\,dx - \int \left(-b^3\sqrt{a+b\cot{(c+dx)}}\cot^3{(c+dx)}\right)\,dx& \\ -\int \left(-ab^2\sqrt{a+b\cot{(c+dx)}}\cot^2{(c+dx)}\right)\,dx& \\ -\int a^2b\sqrt{a+b\cot{(c+dx)}}\cot{(c+dx)}\,dx&\end{aligned}$$

```
[In] integrate((-a+b*cot(d*x+c))*(a+b*cot(d*x+c))**(5/2),x)
```

```
[Out] -Integral(a**3*sqrt(a + b*cot(c + d*x)), x) - Integral(-b**3*sqrt(a + b*cot
(c + d*x))*cot(c + d*x)**3, x) - Integral(-a*b**2*sqrt(a + b*cot(c + d*x))*
cot(c + d*x)**2, x) - Integral(a**2*b*sqrt(a + b*cot(c + d*x))*cot(c + d*x)
, x)
```

Maxima [F]

$$\int (-a+b\cot(c+dx))(a+b\cot(c+dx))^{5/2}\,dx=\int (b\cot(dx+c)+a)^{\frac{5}{2}}(b\cot(dx+c)-a)\,dx$$

```
[In] integrate((-a+b*cot(d*x+c))*(a+b*cot(d*x+c))^(5/2),x, algorithm="maxima")
[Out] integrate((b*cot(d*x + c) + a)^(5/2)*(b*cot(d*x + c) - a), x)
```

Giac [F]

$$\int (-a+b\cot(c+dx))(a+b\cot(c+dx))^{5/2}\,dx=\int (b\cot(dx+c)+a)^{\frac{5}{2}}(b\cot(dx+c)-a)\,dx$$

```
[In] integrate((-a+b*cot(d*x+c))*(a+b*cot(d*x+c))^(5/2),x, algorithm="giac")
[Out] integrate((b*cot(d*x + c) + a)^(5/2)*(b*cot(d*x + c) - a), x)
```

Mupad [B] (verification not implemented)

Time = 40.09 (sec) , antiderivative size = 3442, normalized size of antiderivative = 22.79

$$\int (-a+b\cot(c+dx))(a+b\cot(c+dx))^{5/2}\,dx=\text{Too large to display}$$

```
[In] int(-(a + b*cot(c + d*x))^(5/2)*(a - b*cot(c + d*x)),x)
[Out] log(((((-a^4*b^2*d^4*(5*a^4 + b^4 - 10*a^2*b^2)^2)^(1/2) - a^7*d^2 - 5*a^3*
b^4*d^2 + 10*a^5*b^2*d^2)/d^4)^(1/2)*(((((-a^4*b^2*d^4*(5*a^4 + b^4 - 10*a^
2*b^2)^2)^(1/2) - a^7*d^2 - 5*a^3*b^4*d^2 + 10*a^5*b^2*d^2)/d^4)^(1/2)*(64*
a^2*b^5 + 64*a^4*b^3 - 32*a*b^2*d*(((-a^4*b^2*d^4*(5*a^4 + b^4 - 10*a^2*b^2
)^2)^(1/2) - a^7*d^2 - 5*a^3*b^4*d^2 + 10*a^5*b^2*d^2)/d^4)^(1/2)*(a + b*co
t(c + d*x))^(1/2)))/(2*d) - (16*a^2*b^2*(a + b*cot(c + d*x))^(1/2)*(a^6 - b
^6 + 15*a^2*b^4 - 15*a^4*b^2))/d^2))/2 - (8*a^3*b^3*(3*a^2 - b^2)*(a^2 + b^
2)^3)/d^3)*((20*a^6*b^8*d^4 - a^4*b^10*d^4 - 110*a^8*b^6*d^4 + 100*a^10*b^4
*d^4 - 25*a^12*b^2*d^4)^(1/2)/(4*d^4) - a^7/(4*d^2) - (5*a^3*b^4)/(4*d^2) +
 (5*a^5*b^2)/(2*d^2))^(1/2) - log(((-((-a^4*b^2*d^4*(5*a^4 + b^4 - 10*a^2*b
^2)^2)^(1/2) + a^7*d^2 + 5*a^3*b^4*d^2 - 10*a^5*b^2*d^2)/d^4)^(1/2)*(((-((-
```

a^4*b^2*d^4*(5*a^4 + b^4 - 10*a^2*b^2)^2)^(1/2) + a^7*d^2 + 5*a^3*b^4*d^2 - 10*a^5*b^2*d^2)/d^4)^(1/2)*(64*a^2*b^5 + 64*a^4*b^3 + 32*a*b^2*d*(-((-a^4*b^2*d^4*(5*a^4 + b^4 - 10*a^2*b^2)^2)^(1/2) + a^7*d^2 + 5*a^3*b^4*d^2 - 10*a^5*b^2*d^2)/d^4)^(1/2)*(a + b*cot(c + d*x))^(1/2)))/(2*d) + (16*a^2*b^2*(a + b*cot(c + d*x))^(1/2)*(a^6 - b^6 + 15*a^2*b^4 - 15*a^4*b^2))/d^2))/2 - (8*a^3*b^3*(3*a^2 - b^2)*(a^2 + b^2)^3)/d^3)*(-(a^7*d^2 + (20*a^6*b^8*d^4 - a^4*b^10*d^4 - 110*a^8*b^6*d^4 + 100*a^10*b^4*d^4 - 25*a^12*b^2*d^4)^(1/2) + 5*a^3*b^4*d^2 - 10*a^5*b^2*d^2)/(4*d^4))^(1/2) + log(((-((-a^4*b^2*d^4*(5*a^4 + b^4 - 10*a^2*b^2)^2)^(1/2) + a^7*d^2 + 5*a^3*b^4*d^2 - 10*a^5*b^2*d^2)/d^4)^(1/2)*(((-((-a^4*b^2*d^4*(5*a^4 + b^4 - 10*a^2*b^2)^2)^(1/2) + a^7*d^2 + 5*a^3*b^4*d^2 - 10*a^5*b^2*d^2)/d^4)^(1/2)*(64*a^2*b^5 + 64*a^4*b^3 - 32*a*b^2*d*(-((-a^4*b^2*d^4*(5*a^4 + b^4 - 10*a^2*b^2)^2)^(1/2) + a^7*d^2 + 5*a^3*b^4*d^2 - 10*a^5*b^2*d^2)/d^4)^(1/2)*(a + b*cot(c + d*x))^(1/2)))/(2*d) - (16*a^2*b^2*(a + b*cot(c + d*x))^(1/2)*(a^6 - b^6 + 15*a^2*b^4 - 15*a^4*b^2))/d^2))/2 - (8*a^3*b^3*(3*a^2 - b^2)*(a^2 + b^2)^3)/d^3)*((5*a^5*b^2)/(2*d^2) - a^7/(4*d^2) - (5*a^3*b^4)/(4*d^2) - (20*a^6*b^8*d^4 - a^4*b^10*d^4 - 110*a^8*b^6*d^4 + 100*a^10*b^4*d^4 - 25*a^12*b^2*d^4)^(1/2)/(4*d^4))^(1/2) - ((4*a^2*b)/d - (2*b*(a^2 + b^2))/d)*(a + b*cot(c + d*x))^(1/2) - log((((((-b^6*d^4*(5*a^4 + b^4 - 10*a^2*b^2)^2)^(1/2) + 5*a*b^6*d^2 - 10*a^3*b^4*d^2 + a^5*b^2*d^2)/d^4)^(1/2)*(32*b^7 - 32*a^4*b^3 + 32*a*b^2*d*(((-b^6*d^4*(5*a^4 + b^4 - 10*a^2*b^2)^2)^(1/2) + 5*a*b^6*d^2 - 10*a^3*b^4*d^2 + a^5*b^2*d^2)/d^4)^(1/2)*(a + b*cot(c + d*x))^(1/2)))/(2*d) + (16*(a + b*cot(c + d*x))^(1/2)*(b^10 - 15*a^2*b^8 + 15*a^4*b^6 - a^6*b^4))/d^2)*(((-b^6*d^4*(5*a^4 + b^4 - 10*a^2*b^2)^2)^(1/2) + 5*a*b^6*d^2 - 10*a^3*b^4*d^2 + a^5*b^2*d^2)/d^4)^(1/2))/2 + (8*a*b^5*(a^2 - 3*b^2)*(a^2 + b^2)^3)/d^3)*(((20*a^2*b^12*d^4 - b^14*d^4 - 110*a^4*b^10*d^4 + 100*a^6*b^8*d^4 - 25*a^8*b^6*d^4)^(1/2) + 5*a*b^6*d^2 - 10*a^3*b^4*d^2 + a^5*b^2*d^2)/(4*d^4))^(1/2) + log((8*a*b^5*(a^2 - 3*b^2)*(a^2 + b^2)^3)/d^3 - ((((((-b^6*d^4*(5*a^4 + b^4 - 10*a^2*b^2)^2)^(1/2) + 5*a*b^6*d^2 - 10*a^3*b^4*d^2 + a^5*b^2*d^2)/d^4)^(1/2)*(32*a^4*b^3 - 32*b^7 + 32*a*b^2*d*(((-b^6*d^4*(5*a^4 + b^4 - 10*a^2*b^2)^2)^(1/2) + 5*a*b^6*d^2 - 10*a^3*b^4*d^2 + a^5*b^2*d^2)/d^4)^(1/2)*(a + b*cot(c + d*x))^(1/2)))/(2*d) + (16*(a + b*cot(c + d*x))^(1/2)*(b^10 - 15*a^2*b^8 + 15*a^4*b^6 - a^6*b^4))/d^2)*(((-b^6*d^4*(5*a^4 + b^4 - 10*a^2*b^2)^2)^(1/2) + 5*a*b^6*d^2 - 10*a^3*b^4*d^2 + a^5*b^2*d^2)/d^4)^(1/2))/2)*((20*a^2*b^12*d^4 - b^14*d^4 - 110*a^4*b^10*d^4 + 100*a^6*b^8*d^4 - 25*a^8*b^6*d^4)^(1/2)/(4*d^4) + (5*a*b^6)/(4*d^2) - (5*a^3*b^4)/(2*d^2) + (a^5*b^2)/(4*d^2))^(1/2) - log(((((-((-b^6*d^4*(5*a^4 + b^4 - 10*a^2*b^2)^2)^(1/2) - 5*a*b^6*d^2 + 10*a^3*b^4*d^2 - a^5*b^2*d^2)/d^4)^(1/2)*(32*b^7 - 32*a^4*b^3 + 32*a*b^2*d*(-((-b^6*d^4*(5*a^4 + b^4 - 10*a^2*b^2)^2)^(1/2) - 5*a*b^6*d^2 + 10*a^3*b^4*d^2 - a^5*b^2*d^2)/d^4)^(1/2)*(a + b*cot(c + d*x))^(1/2)))/(2*d) + (16*(a + b*cot(c + d*x))^(1/2)*(b^10 - 15*a^2*b^8 + 15*a^4*b^6 - a^6*b^4))/d^2)*(-((-b^6*d^4*(5*a^4 + b^4 - 10*a^2*b^2)^2)^(1/2) - 5*a*b^6*d^2 + 10*a^3*b^4*d^2 - a^5*b^2*d^2)/d^4)^(1/2))/2 + (8*a*b^5*(a^2 - 3*b^2)*(a^2 + b^2)^3)/d^3)*(-((20*a^2*b^12*d^4 - b^14*d^4 - 110*a^4*b^10*d^4 + 100*a^6*b^8*d^4 - 25*a^8*b^6*d^4)^(1/2) - 5*a*b^6*d^2 + 10*a^3*b^4*d^2 - a^5*b^2*d^2)

```
/(4*d^4))^(1/2) + log((8*a*b^5*(a^2 - 3*b^2)*(a^2 + b^2)^3)/d^3 - ((((-((-b
^6*d^4*(5*a^4 + b^4 - 10*a^2*b^2)^2)^(1/2) - 5*a*b^6*d^2 + 10*a^3*b^4*d^2 -
 a^5*b^2*d^2)/d^4)^(1/2)*(32*a^4*b^3 - 32*b^7 + 32*a*b^2*d*(-((-b^6*d^4*(5*
a^4 + b^4 - 10*a^2*b^2)^2)^(1/2) - 5*a*b^6*d^2 + 10*a^3*b^4*d^2 - a^5*b^2*d
^2)/d^4)^(1/2)*(a + b*cot(c + d*x))^(1/2)))/(2*d) + (16*(a + b*cot(c + d*x)
)^(1/2)*(b^10 - 15*a^2*b^8 + 15*a^4*b^6 - a^6*b^4))/d^2)*(-((-b^6*d^4*(5*a^
4 + b^4 - 10*a^2*b^2)^2)^(1/2) - 5*a*b^6*d^2 + 10*a^3*b^4*d^2 - a^5*b^2*d^2
)/d^4)^(1/2))/2)*((5*a*b^6)/(4*d^2) - (20*a^2*b^12*d^4 - b^14*d^4 - 110*a^4
*b^10*d^4 + 100*a^6*b^8*d^4 - 25*a^8*b^6*d^4)^(1/2)/(4*d^4) - (5*a^3*b^4)/(
2*d^2) + (a^5*b^2)/(4*d^2))^(1/2) - log(((((-a^4*b^2*d^4*(5*a^4 + b^4 - 10*
a^2*b^2)^2)^(1/2) - a^7*d^2 - 5*a^3*b^4*d^2 + 10*a^5*b^2*d^2)/d^4)^(1/2)*((
(((-a^4*b^2*d^4*(5*a^4 + b^4 - 10*a^2*b^2)^2)^(1/2) - a^7*d^2 - 5*a^3*b^4*d
^2 + 10*a^5*b^2*d^2)/d^4)^(1/2)*(64*a^2*b^5 + 64*a^4*b^3 + 32*a*b^2*d*(((-a
^4*b^2*d^4*(5*a^4 + b^4 - 10*a^2*b^2)^2)^(1/2) - a^7*d^2 - 5*a^3*b^4*d^2 +
10*a^5*b^2*d^2)/d^4)^(1/2)*(a + b*cot(c + d*x))^(1/2)))/(2*d) + (16*a^2*b^2
*(a + b*cot(c + d*x))^(1/2)*(a^6 - b^6 + 15*a^2*b^4 - 15*a^4*b^2))/d^2))/2
- (8*a^3*b^3*(3*a^2 - b^2)*(a^2 + b^2)^3)/d^3)*(-(a^7*d^2 - (20*a^6*b^8*d^4
 - a^4*b^10*d^4 - 110*a^8*b^6*d^4 + 100*a^10*b^4*d^4 - 25*a^12*b^2*d^4)^(1/
2) + 5*a^3*b^4*d^2 - 10*a^5*b^2*d^2)/(4*d^4))^(1/2) - (2*b*(a + b*cot(c + d
*x))^(5/2))/(5*d) + (4*a^2*b*(a + b*cot(c + d*x))^(1/2))/d
```

3.99 $\int (-a+b\cot(c+dx))(a+b\cot(c+dx))^{3/2}\,dx$

Optimal result

Integrand size = 27, antiderivative size = 408

$$\int (-a+b\cot(c+dx))(a+b\cot(c+dx))^{3/2}\,dx = \frac{b(a^2+b^2)\operatorname{arctanh}\left(\frac{\sqrt{a+\sqrt{a^2+b^2}}-\sqrt{2}\sqrt{a+b\cot(c+dx)}}{\sqrt{a-\sqrt{a^2+b^2}}}\right)}{\sqrt{2}\sqrt{a-\sqrt{a^2+b^2}}\,d} - \frac{b(a^2+b^2)\operatorname{arctanh}\left(\frac{\sqrt{a+\sqrt{a^2+b^2}}+\sqrt{2}\sqrt{a+b\cot(c+dx)}}{\sqrt{a-\sqrt{a^2+b^2}}}\right)}{\sqrt{2}\sqrt{a-\sqrt{a^2+b^2}}\,d} - \frac{2b(a+b\cot(c+dx))^{3/2}}{3d} + \frac{b(a^2+b^2)\log\left(a+\sqrt{a^2+b^2}+b\cot(c+dx)-\sqrt{2}\sqrt{a+\sqrt{a^2+b^2}}\sqrt{a+b\cot(c+dx)}\right)}{2\sqrt{2}\sqrt{a+\sqrt{a^2+b^2}}\,d} - \frac{b(a^2+b^2)\log\left(a+\sqrt{a^2+b^2}+b\cot(c+dx)+\sqrt{2}\sqrt{a+\sqrt{a^2+b^2}}\sqrt{a+b\cot(c+dx)}\right)}{2\sqrt{2}\sqrt{a+\sqrt{a^2+b^2}}\,d}$$

```
[Out] -2/3*b*(a+b*cot(d*x+c))^(3/2)/d+1/2*b*(a^2+b^2)*arctanh((-2^(1/2)*(a+b*cot(
d*x+c))^(1/2)+(a+(a^2+b^2)^(1/2))^(1/2))/(a-(a^2+b^2)^(1/2))^(1/2))/d*2^(1/
2)/(a-(a^2+b^2)^(1/2))^(1/2)-1/2*b*(a^2+b^2)*arctanh((2^(1/2)*(a+b*cot(d*x+
c))^(1/2)+(a+(a^2+b^2)^(1/2))^(1/2))/(a-(a^2+b^2)^(1/2))^(1/2))/d*2^(1/2)/(
a-(a^2+b^2)^(1/2))^(1/2)+1/4*b*(a^2+b^2)*ln(a+b*cot(d*x+c)+(a^2+b^2)^(1/2)-
2^(1/2)*(a+b*cot(d*x+c))^(1/2)*(a+(a^2+b^2)^(1/2))^(1/2))/d*2^(1/2)/(a+(a^2
+b^2)^(1/2))^(1/2)-1/4*b*(a^2+b^2)*ln(a+b*cot(d*x+c)+(a^2+b^2)^(1/2)+2^(1/2
)*(a+b*cot(d*x+c))^(1/2)*(a+(a^2+b^2)^(1/2))^(1/2))/d*2^(1/2)/(a+(a^2+b^2)^
(1/2))^(1/2)
```

Rubi [A] (verified)

Time = 0.61 (sec) , antiderivative size = 408, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {3609, 12, 3566, 714, 1143, 648, 632, 212, 642}

$$\int (-a+b\cot(c+dx))(a+b\cot(c+dx))^{3/2}\,dx = \frac{b(a^2+b^2)\operatorname{arctanh}\left(\frac{\sqrt{\sqrt{a^2+b^2}+a}-\sqrt{2}\sqrt{a+b\cot(c+dx)}}{\sqrt{a-\sqrt{a^2+b^2}}}\right)}{\sqrt{2}d\sqrt{a-\sqrt{a^2+b^2}}} - \frac{b(a^2+b^2)\operatorname{arctanh}\left(\frac{\sqrt{\sqrt{a^2+b^2}+a}+\sqrt{2}\sqrt{a+b\cot(c+dx)}}{\sqrt{a-\sqrt{a^2+b^2}}}\right)}{\sqrt{2}d\sqrt{a-\sqrt{a^2+b^2}}} + \frac{b(a^2+b^2)\log\left(-\sqrt{2}\sqrt{\sqrt{a^2+b^2}+a}\sqrt{a+b\cot(c+dx)}+\sqrt{a^2+b^2}+a+b\cot(c+dx)\right)}{2\sqrt{2}d\sqrt{\sqrt{a^2+b^2}+a}} - \frac{b(a^2+b^2)\log\left(\sqrt{2}\sqrt{\sqrt{a^2+b^2}+a}\sqrt{a+b\cot(c+dx)}+\sqrt{a^2+b^2}+a+b\cot(c+dx)\right)}{2\sqrt{2}d\sqrt{\sqrt{a^2+b^2}+a}} - \frac{2b(a+b\cot(c+dx))^{3/2}}{3d}$$

```
[In] Int[(-a + b*Cot[c + d*x])*(a + b*Cot[c + d*x])^(3/2),x]
```

```
[Out] (b*(a^2 + b^2)*ArcTanh[(Sqrt[a + Sqrt[a^2 + b^2]] - Sqrt[2]*Sqrt[a + b*Cot[
c + d*x]])/Sqrt[a - Sqrt[a^2 + b^2]]])/(Sqrt[2]*Sqrt[a - Sqrt[a^2 + b^2]]*d
) - (b*(a^2 + b^2)*ArcTanh[(Sqrt[a + Sqrt[a^2 + b^2]] + Sqrt[2]*Sqrt[a + b*
Cot[c + d*x]])/Sqrt[a - Sqrt[a^2 + b^2]]])/(Sqrt[2]*Sqrt[a - Sqrt[a^2 + b^2
]]*d) - (2*b*(a + b*Cot[c + d*x])^(3/2))/(3*d) + (b*(a^2 + b^2)*Log[a + Sqr
t[a^2 + b^2] + b*Cot[c + d*x] - Sqrt[2]*Sqrt[a + Sqrt[a^2 + b^2]]*Sqrt[a +
b*Cot[c + d*x]]])/(2*Sqrt[2]*Sqrt[a + Sqrt[a^2 + b^2]]*d) - (b*(a^2 + b^2)*
Log[a + Sqrt[a^2 + b^2] + b*Cot[c + d*x] + Sqrt[2]*Sqrt[a + Sqrt[a^2 + b^2]
]*Sqrt[a + b*Cot[c + d*x]]])/(2*Sqrt[2]*Sqrt[a + Sqrt[a^2 + b^2]]*d)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 714

```
Int[Sqrt[(d_) + (e_.)*(x_)]/((a_) + (c_.)*(x_)^2), x_Symbol] :> Dist[2*e, S
ubst[Int[x^2/(c*d^2 + a*e^2 - 2*c*d*x^2 + c*x^4), x], x, Sqrt[d + e*x]], x]
 /; FreeQ[{a, c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0]
```

Rule 1143

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :> With[{q =
 Rt[a/c, 2]}, With[{r = Rt[2*q - b/c, 2]}, Dist[1/(2*c*r), Int[x^(m - 1)/(q
 - r*x + x^2), x], x] - Dist[1/(2*c*r), Int[x^(m - 1)/(q + r*x + x^2), x],
x]]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && GeQ[m, 1] && LtQ[m, 3
] && NegQ[b^2 - 4*a*c]
```

Rule 3566

```
Int[((a_) + (b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[b/d, Sub
st[Int[(a + x)^n/(b^2 + x^2), x], x, b*Tan[c + d*x]], x] /; FreeQ[{a, b, c,
 d, n}, x] && NeQ[a^2 + b^2, 0]
```

Rule 3609

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)]), x_Symbol] :> Simp[d*((a + b*Tan[e + f*x])^m/(f*m)), x] + Int
[(a + b*Tan[e + f*x])^(m - 1)*Simp[a*c - b*d + (b*c + a*d)*Tan[e + f*x], x]
, x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2,
 0] && GtQ[m, 0]
```

Rubi steps

$$\text{integral} = -\frac{2b(a+b\cot(c+dx))^{3/2}}{3d} + \int (-a^2-b^2)\sqrt{a+b\cot(c+dx)}\,dx$$

$$= -\frac{2b(a+b\cot(c+dx))^{3/2}}{3d} + (-a^2-b^2)\int \sqrt{a+b\cot(c+dx)}\,dx$$

$$= -\frac{2b(a+b\cot(c+dx))^{3/2}}{3d} + \frac{(b(a^2+b^2))\,\text{Subst}\left(\int \frac{\sqrt{a+x}}{b^2+x^2}\,dx, x, b\cot(c+dx)\right)}{d}$$

$$= -\frac{2b(a+b\cot(c+dx))^{3/2}}{3d} + \frac{(2b(a^2+b^2))\,\text{Subst}\left(\int \frac{x^2}{a^2+b^2-2ax^2+x^4}\,dx, x, \sqrt{a+b\cot(c+dx)}\right)}{d}$$

$$= -\frac{2b(a+b\cot(c+dx))^{3/2}}{3d} + \frac{(b(a^2+b^2))\,\text{Subst}\left(\int \frac{x}{\sqrt{a^2+b^2}-\sqrt{2}\sqrt{a+\sqrt{a^2+b^2}}x+x^2}\,dx, x, \sqrt{a+b\cot(c+dx)}\right)}{\sqrt{2}\sqrt{a+\sqrt{a^2+b^2}}d} - \frac{(b(a^2+b^2))\,\text{Subst}\left(\int \frac{x}{\sqrt{a^2+b^2}+\sqrt{2}\sqrt{a+\sqrt{a^2+b^2}}x+x^2}\,dx, x, \sqrt{a+b\cot(c+dx)}\right)}{\sqrt{2}\sqrt{a+\sqrt{a^2+b^2}}d}$$

$$= -\frac{2b(a+b\cot(c+dx))^{3/2}}{3d} + \frac{(b(a^2+b^2))\,\text{Subst}\left(\int \frac{1}{\sqrt{a^2+b^2}-\sqrt{2}\sqrt{a+\sqrt{a^2+b^2}}x+x^2}\,dx, x, \sqrt{a+b\cot(c+dx)}\right)}{2d} + \frac{(b(a^2+b^2))\,\text{Subst}\left(\int \frac{1}{\sqrt{a^2+b^2}+\sqrt{2}\sqrt{a+\sqrt{a^2+b^2}}x+x^2}\,dx, x, \sqrt{a+b\cot(c+dx)}\right)}{2d} + \frac{(b(a^2+b^2))\,\text{Subst}\left(\int \frac{-\sqrt{2}\sqrt{a+\sqrt{a^2+b^2}}+2x}{\sqrt{a^2+b^2}-\sqrt{2}\sqrt{a+\sqrt{a^2+b^2}}x+x^2}\,dx, x, \sqrt{a+b\cot(c+dx)}\right)}{2\sqrt{2}\sqrt{a+\sqrt{a^2+b^2}}d} - \frac{(b(a^2+b^2))\,\text{Subst}\left(\int \frac{\sqrt{2}\sqrt{a+\sqrt{a^2+b^2}}+2x}{\sqrt{a^2+b^2}+\sqrt{2}\sqrt{a+\sqrt{a^2+b^2}}x+x^2}\,dx, x, \sqrt{a+b\cot(c+dx)}\right)}{2\sqrt{2}\sqrt{a+\sqrt{a^2+b^2}}d}$$

$$= -\frac{2b(a+b\cot(c+dx))^{3/2}}{3d} + \frac{b(a^2+b^2)\log\left(a+\sqrt{a^2+b^2}+b\cot(c+dx)-\sqrt{2}\sqrt{a+\sqrt{a^2+b^2}}\sqrt{a+b\cot(c+dx)}\right)}{2\sqrt{2}\sqrt{a+\sqrt{a^2+b^2}}d} - \frac{b(a^2+b^2)\log\left(a+\sqrt{a^2+b^2}+b\cot(c+dx)+\sqrt{2}\sqrt{a+\sqrt{a^2+b^2}}\sqrt{a+b\cot(c+dx)}\right)}{2\sqrt{2}\sqrt{a+\sqrt{a^2+b^2}}d} - \frac{(b(a^2+b^2))\,\text{Subst}\left(\int \frac{1}{2\left(a-\sqrt{a^2+b^2}\right)-x^2}\,dx, x, -\sqrt{2}\sqrt{a+\sqrt{a^2+b^2}}+2\sqrt{a+b\cot(c+dx)}\right)}{d} - \frac{(b(a^2+b^2))\,\text{Subst}\left(\int \frac{1}{2\left(a-\sqrt{a^2+b^2}\right)-x^2}\,dx, x, \sqrt{2}\sqrt{a+\sqrt{a^2+b^2}}+2\sqrt{a+b\cot(c+dx)}\right)}{d}$$

$$= \frac{b(a^2+b^2)\,\text{arctanh}\left(\frac{\sqrt{a+\sqrt{a^2+b^2}}-\sqrt{2}\sqrt{a+b\cot(c+dx)}}{\sqrt{a-\sqrt{a^2+b^2}}}\right)}{\sqrt{2}\sqrt{a-\sqrt{a^2+b^2}}d} - \frac{b(a^2+b^2)\,\text{arctanh}\left(\frac{\sqrt{a+\sqrt{a^2+b^2}}+\sqrt{2}\sqrt{a+b\cot(c+dx)}}{\sqrt{a-\sqrt{a^2+b^2}}}\right)}{\sqrt{2}\sqrt{a-\sqrt{a^2+b^2}}d} - \frac{2b(a+b\cot(c+dx))^{3/2}}{3d} + \frac{b(a^2+b^2)\log\left(a+\sqrt{a^2+b^2}+b\cot(c+dx)-\sqrt{2}\sqrt{a+\sqrt{a^2+b^2}}\sqrt{a+b\cot(c+dx)}\right)}{2\sqrt{2}\sqrt{a+\sqrt{a^2+b^2}}d} - \frac{b(a^2+b^2)\log\left(a+\sqrt{a^2+b^2}+b\cot(c+dx)+\sqrt{2}\sqrt{a+\sqrt{a^2+b^2}}\sqrt{a+b\cot(c+dx)}\right)}{2\sqrt{2}\sqrt{a+\sqrt{a^2+b^2}}d}$$

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 6.62 (sec) , antiderivative size = 178, normalized size of antiderivative = 0.44

$$\int (-a+b\cot(c+dx))(a+b\cot(c+dx))^{3/2}\,dx = \frac{(-a+b\cot(c+dx))(a+b\cot(c+dx))\left(3i\sqrt{a-ib}(a^2+b^2)\,\text{arctanh}\left(\frac{\sqrt{a+b\cot(c+dx)}}{\sqrt{a-ib}}\right)-3i\sqrt{a+\cdots}\right.}{-3b^2d\cos^2(c+dx)+3a^2d\sin\cdots}$$

```
[In] Integrate[(-a + b*Cot[c + d*x])*(a + b*Cot[c + d*x])^(3/2),x]
```

```
[Out] ((-a + b*Cot[c + d*x])*(a + b*Cot[c + d*x])*((3*I)*Sqrt[a - I*b]*(a^2 + b^2
)*ArcTanh[Sqrt[a + b*Cot[c + d*x]]/Sqrt[a - I*b]] - (3*I)*Sqrt[a + I*b]*(a^
2 + b^2)*ArcTanh[Sqrt[a + b*Cot[c + d*x]]/Sqrt[a + I*b]] + 2*b*(a + b*Cot[c
 + d*x])^(3/2))*Sin[c + d*x]^2)/(-3*b^2*d*Cos[c + d*x]^2 + 3*a^2*d*Sin[c +
d*x]^2)
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 985 vs. $2(333) = 666$.

Time = 0.07 (sec) , antiderivative size = 986, normalized size of antiderivative = 2.42

method	result
derivativedivides	$-\frac{2b(a+b\cot(dx+c))^{\frac{3}{2}}}{3d}+\frac{\ln\left(\sqrt{a+b\cot(dx+c)}\,\sqrt{2\sqrt{a^2+b^2}+2a}-b\cot(dx+c)-\sqrt{a^2+b^2}-a\right)\sqrt{2\sqrt{a^2+b^2}+2a}\,\sqrt{a^2+b^2}}{4db}$
default	$-\frac{2b(a+b\cot(dx+c))^{\frac{3}{2}}}{3d}+\frac{\ln\left(\sqrt{a+b\cot(dx+c)}\,\sqrt{2\sqrt{a^2+b^2}+2a}-b\cot(dx+c)-\sqrt{a^2+b^2}-a\right)\sqrt{2\sqrt{a^2+b^2}+2a}\,\sqrt{a^2+b^2}}{4db}$
parts	Expression too large to display

```
[In] int((-a+b*cot(d*x+c))*(a+b*cot(d*x+c))^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] -2/3*b*(a+b*cot(d*x+c))^(3/2)/d+1/4/d/b*ln((a+b*cot(d*x+c))^(1/2)*(2*(a^2+b
^2)^(1/2)+2*a)^(1/2)-b*cot(d*x+c)-(a^2+b^2)^(1/2)-a)*(2*(a^2+b^2)^(1/2)+2*a
)^(1/2)*(a^2+b^2)^(1/2)*a^2+1/4/d*b*ln((a+b*cot(d*x+c))^(1/2)*(2*(a^2+b^2)^
(1/2)+2*a)^(1/2)-b*cot(d*x+c)-(a^2+b^2)^(1/2)-a)*(2*(a^2+b^2)^(1/2)+2*a)^(1
/2)*(a^2+b^2)^(1/2)-1/d*b/(2*(a^2+b^2)^(1/2)-2*a)^(1/2)*arctan((-2*(a+b*cot
(d*x+c))^(1/2)+(2*(a^2+b^2)^(1/2)+2*a)^(1/2))/(2*(a^2+b^2)^(1/2)-2*a)^(1/2)
)*a^2-1/d*b^3/(2*(a^2+b^2)^(1/2)-2*a)^(1/2)*arctan((-2*(a+b*cot(d*x+c))^(1/
2)+(2*(a^2+b^2)^(1/2)+2*a)^(1/2))/(2*(a^2+b^2)^(1/2)-2*a)^(1/2))-1/4/d/b*ln
((a+b*cot(d*x+c))^(1/2)*(2*(a^2+b^2)^(1/2)+2*a)^(1/2)-b*cot(d*x+c)-(a^2+b^2
)^(1/2)-a)*(2*(a^2+b^2)^(1/2)+2*a)^(1/2)*a^3-1/4/d*b*ln((a+b*cot(d*x+c))^(1
/2)*(2*(a^2+b^2)^(1/2)+2*a)^(1/2)-b*cot(d*x+c)-(a^2+b^2)^(1/2)-a)*(2*(a^2+b
^2)^(1/2)+2*a)^(1/2)*a-1/4/d/b*ln(b*cot(d*x+c)+a+(a+b*cot(d*x+c))^(1/2)*(2*
(a^2+b^2)^(1/2)+2*a)^(1/2)+(a^2+b^2)^(1/2))*(2*(a^2+b^2)^(1/2)+2*a)^(1/2)*(
a^2+b^2)^(1/2)*a^2-1/4/d*b*ln(b*cot(d*x+c)+a+(a+b*cot(d*x+c))^(1/2)*(2*(a^2
+b^2)^(1/2)+2*a)^(1/2)+(a^2+b^2)^(1/2))*(2*(a^2+b^2)^(1/2)+2*a)^(1/2)*(a^2+
b^2)^(1/2)+1/d*b/(2*(a^2+b^2)^(1/2)-2*a)^(1/2)*arctan((2*(a+b*cot(d*x+c))^(
1/2)+(2*(a^2+b^2)^(1/2)+2*a)^(1/2))/(2*(a^2+b^2)^(1/2)-2*a)^(1/2))*a^2+1/d*
b^3/(2*(a^2+b^2)^(1/2)-2*a)^(1/2)*arctan((2*(a+b*cot(d*x+c))^(1/2)+(2*(a^2+
b^2)^(1/2)+2*a)^(1/2))/(2*(a^2+b^2)^(1/2)-2*a)^(1/2))+1/4/d/b*ln(b*cot(d*x+
c)+a+(a+b*cot(d*x+c))^(1/2)*(2*(a^2+b^2)^(1/2)+2*a)^(1/2)+(a^2+b^2)^(1/2))*
(2*(a^2+b^2)^(1/2)+2*a)^(1/2)*a^3+1/4/d*b*ln(b*cot(d*x+c)+a+(a+b*cot(d*x+c)
)^(1/2)*(2*(a^2+b^2)^(1/2)+2*a)^(1/2)+(a^2+b^2)^(1/2))*(2*(a^2+b^2)^(1/2)+2
*a)^(1/2)*a
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1148 vs. $2(335) = 670$.

Time = 0.29 (sec) , antiderivative size = 1148, normalized size of antiderivative = 2.81

$$\int (-a + b\cot(c + dx))(a + b\cot(c + dx))^{3/2}\,dx = \text{Too large to display}$$

```
[In] integrate((-a+b*cot(d*x+c))*(a+b*cot(d*x+c))^(3/2),x, algorithm="fricas")
```

```
[Out] -1/6*(3*d*sqrt(-(a^5 + 2*a^3*b^2 + a*b^4 + d^2*sqrt(-(a^8*b^2 + 4*a^6*b^4 +
 6*a^4*b^6 + 4*a^2*b^8 + b^10)/d^4))/d^2)*log(d^3*sqrt(-(a^5 + 2*a^3*b^2 +
a*b^4 + d^2*sqrt(-(a^8*b^2 + 4*a^6*b^4 + 6*a^4*b^6 + 4*a^2*b^8 + b^10)/d^4)
)/d^2)*sqrt(-(a^8*b^2 + 4*a^6*b^4 + 6*a^4*b^6 + 4*a^2*b^8 + b^10)/d^4) + (a
^6*b + 3*a^4*b^3 + 3*a^2*b^5 + b^7)*sqrt((b*cos(2*d*x + 2*c) + a*sin(2*d*x
+ 2*c) + b)/sin(2*d*x + 2*c)))*sin(2*d*x + 2*c) - 3*d*sqrt(-(a^5 + 2*a^3*b^
2 + a*b^4 + d^2*sqrt(-(a^8*b^2 + 4*a^6*b^4 + 6*a^4*b^6 + 4*a^2*b^8 + b^10)/
d^4))/d^2)*log(-d^3*sqrt(-(a^5 + 2*a^3*b^2 + a*b^4 + d^2*sqrt(-(a^8*b^2 + 4
*a^6*b^4 + 6*a^4*b^6 + 4*a^2*b^8 + b^10)/d^4))/d^2)*sqrt(-(a^8*b^2 + 4*a^6*
b^4 + 6*a^4*b^6 + 4*a^2*b^8 + b^10)/d^4) + (a^6*b + 3*a^4*b^3 + 3*a^2*b^5 +
 b^7)*sqrt((b*cos(2*d*x + 2*c) + a*sin(2*d*x + 2*c) + b)/sin(2*d*x + 2*c)))
*sin(2*d*x + 2*c) - 3*d*sqrt(-(a^5 + 2*a^3*b^2 + a*b^4 - d^2*sqrt(-(a^8*b^2
 + 4*a^6*b^4 + 6*a^4*b^6 + 4*a^2*b^8 + b^10)/d^4))/d^2)*log(d^3*sqrt(-(a^5
+ 2*a^3*b^2 + a*b^4 - d^2*sqrt(-(a^8*b^2 + 4*a^6*b^4 + 6*a^4*b^6 + 4*a^2*b^
8 + b^10)/d^4))/d^2)*sqrt(-(a^8*b^2 + 4*a^6*b^4 + 6*a^4*b^6 + 4*a^2*b^8 + b
^10)/d^4) + (a^6*b + 3*a^4*b^3 + 3*a^2*b^5 + b^7)*sqrt((b*cos(2*d*x + 2*c)
+ a*sin(2*d*x + 2*c) + b)/sin(2*d*x + 2*c)))*sin(2*d*x + 2*c) + 3*d*sqrt(-(
a^5 + 2*a^3*b^2 + a*b^4 - d^2*sqrt(-(a^8*b^2 + 4*a^6*b^4 + 6*a^4*b^6 + 4*a^
2*b^8 + b^10)/d^4))/d^2)*log(-d^3*sqrt(-(a^5 + 2*a^3*b^2 + a*b^4 - d^2*sqrt
(-(a^8*b^2 + 4*a^6*b^4 + 6*a^4*b^6 + 4*a^2*b^8 + b^10)/d^4))/d^2)*sqrt(-(a^
8*b^2 + 4*a^6*b^4 + 6*a^4*b^6 + 4*a^2*b^8 + b^10)/d^4) + (a^6*b + 3*a^4*b^3
 + 3*a^2*b^5 + b^7)*sqrt((b*cos(2*d*x + 2*c) + a*sin(2*d*x + 2*c) + b)/sin(
2*d*x + 2*c)))*sin(2*d*x + 2*c) + 4*(b^2*cos(2*d*x + 2*c) + a*b*sin(2*d*x +
 2*c) + b^2)*sqrt((b*cos(2*d*x + 2*c) + a*sin(2*d*x + 2*c) + b)/sin(2*d*x +
 2*c)))/(d*sin(2*d*x + 2*c))
```

Sympy [F]

$$\int (-a + b\cot(c + dx))(a + b\cot(c + dx))^{3/2}\,dx = -\int a^2\sqrt{a + b\cot(c + dx)}\,dx - \int \left(-b^2\sqrt{a + b\cot(c + dx)}\cot^2(c + dx)\right)\,dx$$

```
[In] integrate((-a+b*cot(d*x+c))*(a+b*cot(d*x+c))**(3/2),x)
```

```
[Out] -Integral(a**2*sqrt(a + b*cot(c + d*x)), x) - Integral(-b**2*sqrt(a + b*cot
(c + d*x))*cot(c + d*x)**2, x)
```

Maxima [F]

$$\int (-a+b\cot(c+dx))(a+b\cot(c+dx))^{3/2}\,dx = \int (b\cot(dx+c)+a)^{\frac{3}{2}}(b\cot(dx+c)-a)\,dx$$

```
[In] integrate((-a+b*cot(d*x+c))*(a+b*cot(d*x+c))^(3/2),x, algorithm="maxima")
[Out] integrate((b*cot(d*x + c) + a)^(3/2)*(b*cot(d*x + c) - a), x)
```

Giac [F]

$$\int (-a+b\cot(c+dx))(a+b\cot(c+dx))^{3/2}\,dx = \int (b\cot(dx+c)+a)^{\frac{3}{2}}(b\cot(dx+c)-a)\,dx$$

```
[In] integrate((-a+b*cot(d*x+c))*(a+b*cot(d*x+c))^(3/2),x, algorithm="giac")
[Out] integrate((b*cot(d*x + c) + a)^(3/2)*(b*cot(d*x + c) - a), x)
```

Mupad [B] (verification not implemented)

Time = 25.17 (sec) , antiderivative size = 2529, normalized size of antiderivative = 6.20

$$\int (-a+b\cot(c+dx))(a+b\cot(c+dx))^{3/2}\,dx = \text{Too large to display}$$

```
[In] int(-(a + b*cot(c + d*x))^(3/2)*(a - b*cot(c + d*x)),x)
[Out] log((((16*b^4*(a + b*cot(c + d*x))^(1/2)*(a^4 + b^4 - 6*a^2*b^2))/d^2 - (16
*a*b^2*(((-b^6*d^4*(3*a^2 - b^2)^2)^(1/2) - 3*a*b^4*d^2 + a^3*b^2*d^2)/d^4)
^(1/2)*(a^2*b + b^3 + d*(((-b^6*d^4*(3*a^2 - b^2)^2)^(1/2) - 3*a*b^4*d^2 +
a^3*b^2*d^2)/d^4)^(1/2)*(a + b*cot(c + d*x))^(1/2)))/d)*(((-b^6*d^4*(3*a^2
- b^2)^2)^(1/2) - 3*a*b^4*d^2 + a^3*b^2*d^2)/d^4)^(1/2))/2 + (8*b^5*(a^2 -
b^2)*(a^2 + b^2)^2)/d^3)*((6*a^2*b^8*d^4 - b^10*d^4 - 9*a^4*b^6*d^4)^(1/2)/
(4*d^4) - (3*a*b^4)/(4*d^2) + (a^3*b^2)/(4*d^2))^(1/2) - log((8*b^5*(a^2 -
b^2)*(a^2 + b^2)^2)/d^3 - (((16*b^4*(a + b*cot(c + d*x))^(1/2)*(a^4 + b^4 -
 6*a^2*b^2))/d^2 + (16*a*b^2*(-((-b^6*d^4*(3*a^2 - b^2)^2)^(1/2) + 3*a*b^4*
d^2 - a^3*b^2*d^2)/d^4)^(1/2)*(a^2*b + b^3 - d*(-((-b^6*d^4*(3*a^2 - b^2)^2
)^(1/2) + 3*a*b^4*d^2 - a^3*b^2*d^2)/d^4)^(1/2)*(a + b*cot(c + d*x))^(1/2))
)/d)*(-((-b^6*d^4*(3*a^2 - b^2)^2)^(1/2) + 3*a*b^4*d^2 - a^3*b^2*d^2)/d^4)^
(1/2))/2)*(-((6*a^2*b^8*d^4 - b^10*d^4 - 9*a^4*b^6*d^4)^(1/2) + 3*a*b^4*d^2
```

```
 - a^3*b^2*d^2)/(4*d^4))^(1/2) - log((8*b^5*(a^2 - b^2)*(a^2 + b^2)^2)/d^3
- (((16*b^4*(a + b*cot(c + d*x))^(1/2)*(a^4 + b^4 - 6*a^2*b^2))/d^2 + (16*a
*b^2*(((-b^6*d^4*(3*a^2 - b^2)^2)^(1/2) - 3*a*b^4*d^2 + a^3*b^2*d^2)/d^4)^(
1/2)*(a^2*b + b^3 - d*(((-b^6*d^4*(3*a^2 - b^2)^2)^(1/2) - 3*a*b^4*d^2 + a^
3*b^2*d^2)/d^4)^(1/2)*(a + b*cot(c + d*x))^(1/2)))/d)*(((-b^6*d^4*(3*a^2 -
b^2)^2)^(1/2) - 3*a*b^4*d^2 + a^3*b^2*d^2)/d^4)^(1/2))/2)*(((6*a^2*b^8*d^4
- b^10*d^4 - 9*a^4*b^6*d^4)^(1/2) - 3*a*b^4*d^2 + a^3*b^2*d^2)/(4*d^4))^(1/
2) + log((((16*b^4*(a + b*cot(c + d*x))^(1/2)*(a^4 + b^4 - 6*a^2*b^2))/d^2
- (16*a*b^2*(-((-b^6*d^4*(3*a^2 - b^2)^2)^(1/2) + 3*a*b^4*d^2 - a^3*b^2*d^2
)/d^4)^(1/2)*(a^2*b + b^3 + d*(-((-b^6*d^4*(3*a^2 - b^2)^2)^(1/2) + 3*a*b^4
*d^2 - a^3*b^2*d^2)/d^4)^(1/2)*(a + b*cot(c + d*x))^(1/2)))/d)*(-((-b^6*d^4
*(3*a^2 - b^2)^2)^(1/2) + 3*a*b^4*d^2 - a^3*b^2*d^2)/d^4)^(1/2))/2 + (8*b^5
*(a^2 - b^2)*(a^2 + b^2)^2)/d^3)*((a^3*b^2)/(4*d^2) - (3*a*b^4)/(4*d^2) - (
6*a^2*b^8*d^4 - b^10*d^4 - 9*a^4*b^6*d^4)^(1/2)/(4*d^4))^(1/2) - log(((-((-
a^4*b^2*d^4*(3*a^2 - b^2)^2)^(1/2) + a^5*d^2 - 3*a^3*b^2*d^2)/d^4)^(1/2)*((
16*a^2*b^2*(a + b*cot(c + d*x))^(1/2)*(a^4 + b^4 - 6*a^2*b^2))/d^2 + (16*a*
b^2*(-((-a^4*b^2*d^4*(3*a^2 - b^2)^2)^(1/2) + a^5*d^2 - 3*a^3*b^2*d^2)/d^4)
^(1/2)*(a^2*b + b^3 + d*(-((-a^4*b^2*d^4*(3*a^2 - b^2)^2)^(1/2) + a^5*d^2 -
 3*a^3*b^2*d^2)/d^4)^(1/2)*(a + b*cot(c + d*x))^(1/2)))/d))/2 - (16*a^4*b^3
*(a^2 + b^2)^2)/d^3)*(-((6*a^6*b^4*d^4 - a^4*b^6*d^4 - 9*a^8*b^2*d^4)^(1/2)
 + a^5*d^2 - 3*a^3*b^2*d^2)/(4*d^4))^(1/2) - log(((((-a^4*b^2*d^4*(3*a^2 -
b^2)^2)^(1/2) - a^5*d^2 + 3*a^3*b^2*d^2)/d^4)^(1/2)*((16*a^2*b^2*(a + b*cot
(c + d*x))^(1/2)*(a^4 + b^4 - 6*a^2*b^2))/d^2 + (16*a*b^2*(((-a^4*b^2*d^4*(
3*a^2 - b^2)^2)^(1/2) - a^5*d^2 + 3*a^3*b^2*d^2)/d^4)^(1/2)*(a^2*b + b^3 +
d*(((-a^4*b^2*d^4*(3*a^2 - b^2)^2)^(1/2) - a^5*d^2 + 3*a^3*b^2*d^2)/d^4)^(1
/2)*(a + b*cot(c + d*x))^(1/2)))/d))/2 - (16*a^4*b^3*(a^2 + b^2)^2)/d^3)*((
(6*a^6*b^4*d^4 - a^4*b^6*d^4 - 9*a^8*b^2*d^4)^(1/2) - a^5*d^2 + 3*a^3*b^2*d
^2)/(4*d^4))^(1/2) + log(- ((((-a^4*b^2*d^4*(3*a^2 - b^2)^2)^(1/2) - a^5*d^
2 + 3*a^3*b^2*d^2)/d^4)^(1/2)*((16*a^2*b^2*(a + b*cot(c + d*x))^(1/2)*(a^4
+ b^4 - 6*a^2*b^2))/d^2 - (16*a*b^2*(((-a^4*b^2*d^4*(3*a^2 - b^2)^2)^(1/2)
- a^5*d^2 + 3*a^3*b^2*d^2)/d^4)^(1/2)*(a^2*b + b^3 - d*(((-a^4*b^2*d^4*(3*a
^2 - b^2)^2)^(1/2) - a^5*d^2 + 3*a^3*b^2*d^2)/d^4)^(1/2)*(a + b*cot(c + d*x
))^(1/2)))/d))/2 - (16*a^4*b^3*(a^2 + b^2)^2)/d^3)*((6*a^6*b^4*d^4 - a^4*b^
6*d^4 - 9*a^8*b^2*d^4)^(1/2)/(4*d^4) - a^5/(4*d^2) + (3*a^3*b^2)/(4*d^2))^(
1/2) + log(- ((-((-a^4*b^2*d^4*(3*a^2 - b^2)^2)^(1/2) + a^5*d^2 - 3*a^3*b^2
*d^2)/d^4)^(1/2)*((16*a^2*b^2*(a + b*cot(c + d*x))^(1/2)*(a^4 + b^4 - 6*a^2
*b^2))/d^2 - (16*a*b^2*(-((-a^4*b^2*d^4*(3*a^2 - b^2)^2)^(1/2) + a^5*d^2 -
3*a^3*b^2*d^2)/d^4)^(1/2)*(a^2*b + b^3 - d*(-((-a^4*b^2*d^4*(3*a^2 - b^2)^2
)^(1/2) + a^5*d^2 - 3*a^3*b^2*d^2)/d^4)^(1/2)*(a + b*cot(c + d*x))^(1/2)))/
d))/2 - (16*a^4*b^3*(a^2 + b^2)^2)/d^3)*((3*a^3*b^2)/(4*d^2) - a^5/(4*d^2)
- (6*a^6*b^4*d^4 - a^4*b^6*d^4 - 9*a^8*b^2*d^4)^(1/2)/(4*d^4))^(1/2) - (2*b
*(a + b*cot(c + d*x))^(3/2))/(3*d)
```

3.100 $\int (-a+b\cot(c+dx))\sqrt{a+b\cot(c+dx)}\,dx$

Optimal result

Integrand size = 27, antiderivative size = 422

$$\int (-a+b\cot(c+dx))\sqrt{a+b\cot(c+dx)}\,dx = \frac{b\sqrt{a^2+b^2}\operatorname{arctanh}\left(\frac{\sqrt{a+\sqrt{a^2+b^2}}-\sqrt{2}\sqrt{a+b\cot(c+dx)}}{\sqrt{a-\sqrt{a^2+b^2}}}\right)}{\sqrt{2}\sqrt{a-\sqrt{a^2+b^2}}d} - \frac{b\sqrt{a^2+b^2}\operatorname{arctanh}\left(\frac{\sqrt{a+\sqrt{a^2+b^2}}+\sqrt{2}\sqrt{a+b\cot(c+dx)}}{\sqrt{a-\sqrt{a^2+b^2}}}\right)}{\sqrt{2}\sqrt{a-\sqrt{a^2+b^2}}d} - \frac{2b\sqrt{a+b\cot(c+dx)}}{d} - \frac{b\sqrt{a^2+b^2}\log\left(a+\sqrt{a^2+b^2}+b\cot(c+dx)-\sqrt{2}\sqrt{a+\sqrt{a^2+b^2}}\sqrt{a+b\cot(c+dx)}\right)}{2\sqrt{2}\sqrt{a+\sqrt{a^2+b^2}}d} + \frac{b\sqrt{a^2+b^2}\log\left(a+\sqrt{a^2+b^2}+b\cot(c+dx)+\sqrt{2}\sqrt{a+\sqrt{a^2+b^2}}\sqrt{a+b\cot(c+dx)}\right)}{2\sqrt{2}\sqrt{a+\sqrt{a^2+b^2}}d}$$

```
[Out] -2*b*(a+b*cot(d*x+c))^(1/2)/d+1/2*b*arctanh((-2^(1/2)*(a+b*cot(d*x+c))^(1/2
)+(a+(a^2+b^2)^(1/2))^(1/2))/(a-(a^2+b^2)^(1/2))^(1/2))*(a^2+b^2)^(1/2)/d*2
^(1/2)/(a-(a^2+b^2)^(1/2))^(1/2)-1/2*b*arctanh((2^(1/2)*(a+b*cot(d*x+c))^(1
/2)+(a+(a^2+b^2)^(1/2))^(1/2))/(a-(a^2+b^2)^(1/2))^(1/2))*(a^2+b^2)^(1/2)/d
*2^(1/2)/(a-(a^2+b^2)^(1/2))^(1/2)-1/4*b*ln(a+b*cot(d*x+c)+(a^2+b^2)^(1/2)-
2^(1/2)*(a+b*cot(d*x+c))^(1/2)*(a+(a^2+b^2)^(1/2))^(1/2))*(a^2+b^2)^(1/2)/d
*2^(1/2)/(a+(a^2+b^2)^(1/2))^(1/2)+1/4*b*ln(a+b*cot(d*x+c)+(a^2+b^2)^(1/2)+
2^(1/2)*(a+b*cot(d*x+c))^(1/2)*(a+(a^2+b^2)^(1/2))^(1/2))*(a^2+b^2)^(1/2)/d
*2^(1/2)/(a+(a^2+b^2)^(1/2))^(1/2)
```

Rubi [A] (verified)

Time = 0.61 (sec) , antiderivative size = 422, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {3609, 12, 3566, 722, 1108, 648, 632, 212, 642}

$$\int (-a+b\cot(c+dx))\sqrt{a+b\cot(c+dx)}\,dx$$

$$= \frac{b\sqrt{a^2+b^2}\text{arctanh}\left(\frac{\sqrt{\sqrt{a^2+b^2}+a}-\sqrt{2}\sqrt{a+b\cot(c+dx)}}{\sqrt{a-\sqrt{a^2+b^2}}}\right)}{\sqrt{2}d\sqrt{a-\sqrt{a^2+b^2}}} - \frac{b\sqrt{a^2+b^2}\text{arctanh}\left(\frac{\sqrt{\sqrt{a^2+b^2}+a}+\sqrt{2}\sqrt{a+b\cot(c+dx)}}{\sqrt{a-\sqrt{a^2+b^2}}}\right)}{\sqrt{2}d\sqrt{a-\sqrt{a^2+b^2}}} - \frac{b\sqrt{a^2+b^2}\log\left(-\sqrt{2}\sqrt{\sqrt{a^2+b^2}+a}\sqrt{a+b\cot(c+dx)}+\sqrt{a^2+b^2}+a+b\cot(c+dx)\right)}{2\sqrt{2}d\sqrt{\sqrt{a^2+b^2}+a}} + \frac{b\sqrt{a^2+b^2}\log\left(\sqrt{2}\sqrt{\sqrt{a^2+b^2}+a}\sqrt{a+b\cot(c+dx)}+\sqrt{a^2+b^2}+a+b\cot(c+dx)\right)}{2\sqrt{2}d\sqrt{\sqrt{a^2+b^2}+a}} - \frac{2b\sqrt{a+b\cot(c+dx)}}{d}$$

```
[In] Int[(-a + b*Cot[c + d*x])*Sqrt[a + b*Cot[c + d*x]],x]
```

```
[Out] (b*Sqrt[a^2 + b^2]*ArcTanh[(Sqrt[a + Sqrt[a^2 + b^2]] - Sqrt[2]*Sqrt[a + b*
Cot[c + d*x]])/Sqrt[a - Sqrt[a^2 + b^2]]])/(Sqrt[2]*Sqrt[a - Sqrt[a^2 + b^2
]]*d) - (b*Sqrt[a^2 + b^2]*ArcTanh[(Sqrt[a + Sqrt[a^2 + b^2]] + Sqrt[2]*Sqr
t[a + b*Cot[c + d*x]])/Sqrt[a - Sqrt[a^2 + b^2]]])/(Sqrt[2]*Sqrt[a - Sqrt[a
^2 + b^2]]*d) - (2*b*Sqrt[a + b*Cot[c + d*x]])/d - (b*Sqrt[a^2 + b^2]*Log[a
 + Sqrt[a^2 + b^2] + b*Cot[c + d*x] - Sqrt[2]*Sqrt[a + Sqrt[a^2 + b^2]]*Sqr
t[a + b*Cot[c + d*x]]])/(2*Sqrt[2]*Sqrt[a + Sqrt[a^2 + b^2]]*d) + (b*Sqrt[a
^2 + b^2]*Log[a + Sqrt[a^2 + b^2] + b*Cot[c + d*x] + Sqrt[2]*Sqrt[a + Sqrt[
a^2 + b^2]]*Sqrt[a + b*Cot[c + d*x]]])/(2*Sqrt[2]*Sqrt[a + Sqrt[a^2 + b^2]]
*d)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 722

```
Int[1/(Sqrt[(d_) + (e_.)*(x_)]*((a_) + (c_.)*(x_)^2)), x_Symbol] :> Dist[2*
e, Subst[Int[1/(c*d^2 + a*e^2 - 2*c*d*x^2 + c*x^4), x], x, Sqrt[d + e*x]],
x] /; FreeQ[{a, c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0]
```

Rule 1108

```
Int[((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(-1), x_Symbol] :> With[{q = Rt[a/
c, 2]}, With[{r = Rt[2*q - b/c, 2]}, Dist[1/(2*c*q*r), Int[(r - x)/(q - r*x
 + x^2), x], x] + Dist[1/(2*c*q*r), Int[(r + x)/(q + r*x + x^2), x], x]]] /
; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && NegQ[b^2 - 4*a*c]
```

Rule 3566

```
Int[((a_) + (b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[b/d, Sub
st[Int[(a + x)^n/(b^2 + x^2), x], x, b*Tan[c + d*x]], x] /; FreeQ[{a, b, c,
 d, n}, x] && NeQ[a^2 + b^2, 0]
```

Rule 3609

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)]), x_Symbol] :> Simp[d*((a + b*Tan[e + f*x])^m/(f*m)), x] + Int
[(a + b*Tan[e + f*x])^(m - 1)*Simp[a*c - b*d + (b*c + a*d)*Tan[e + f*x], x]
, x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2,
 0] && GtQ[m, 0]
```

Rubi steps

$$\text{integral} = -\frac{2b\sqrt{a+b\cot(c+dx)}}{d} + \int \frac{-a^2-b^2}{\sqrt{a+b\cot(c+dx)}}\,dx$$

$$= -\frac{2b\sqrt{a+b\cot(c+dx)}}{d} + \left(-a^2-b^2\right)\int \frac{1}{\sqrt{a+b\cot(c+dx)}}\,dx$$

$$= -\frac{2b\sqrt{a+b\cot(c+dx)}}{d} + \frac{(b(a^2+b^2))\,\text{Subst}\left(\int \frac{1}{\sqrt{a+x}(b^2+x^2)}\,dx, x, b\cot(c+dx)\right)}{d}$$

$$= -\frac{2b\sqrt{a+b\cot(c+dx)}}{d} + \frac{(2b(a^2+b^2))\,\text{Subst}\left(\int \frac{1}{a^2+b^2-2ax^2+x^4}\,dx, x, \sqrt{a+b\cot(c+dx)}\right)}{d}$$

$$= -\frac{2b\sqrt{a+b\cot(c+dx)}}{d} + \frac{\left(b\sqrt{a^2+b^2}\right)\text{Subst}\left(\int \frac{\sqrt{2}\sqrt{a+\sqrt{a^2+b^2}}-x}{\sqrt{a^2+b^2}-\sqrt{2}\sqrt{a+\sqrt{a^2+b^2}}x+x^2}\,dx, x, \sqrt{a+b\cot(c+dx)}\right)}{\sqrt{2}\sqrt{a+\sqrt{a^2+b^2}}d} + \frac{\left(b\sqrt{a^2+b^2}\right)\text{Subst}\left(\int \frac{\sqrt{2}\sqrt{a+\sqrt{a^2+b^2}}+x}{\sqrt{a^2+b^2}+\sqrt{2}\sqrt{a+\sqrt{a^2+b^2}}x+x^2}\,dx, x, \sqrt{a+b\cot(c+dx)}\right)}{\sqrt{2}\sqrt{a+\sqrt{a^2+b^2}}d}$$

$$= -\frac{2b\sqrt{a+b\cot(c+dx)}}{d} + \frac{\left(b\sqrt{a^2+b^2}\right)\text{Subst}\left(\int \frac{1}{\sqrt{a^2+b^2}-\sqrt{2}\sqrt{a+\sqrt{a^2+b^2}}x+x^2}\,dx, x, \sqrt{a+b\cot(c+dx)}\right)}{2d} + \frac{\left(b\sqrt{a^2+b^2}\right)\text{Subst}\left(\int \frac{1}{\sqrt{a^2+b^2}+\sqrt{2}\sqrt{a+\sqrt{a^2+b^2}}x+x^2}\,dx, x, \sqrt{a+b\cot(c+dx)}\right)}{2d} - \frac{\left(b\sqrt{a^2+b^2}\right)\text{Subst}\left(\int \frac{-\sqrt{2}\sqrt{a+\sqrt{a^2+b^2}}+2x}{\sqrt{a^2+b^2}-\sqrt{2}\sqrt{a+\sqrt{a^2+b^2}}x+x^2}\,dx, x, \sqrt{a+b\cot(c+dx)}\right)}{2\sqrt{2}\sqrt{a+\sqrt{a^2+b^2}}d} + \frac{\left(b\sqrt{a^2+b^2}\right)\text{Subst}\left(\int \frac{\sqrt{2}\sqrt{a+\sqrt{a^2+b^2}}+2x}{\sqrt{a^2+b^2}+\sqrt{2}\sqrt{a+\sqrt{a^2+b^2}}x+x^2}\,dx, x, \sqrt{a+b\cot(c+dx)}\right)}{2\sqrt{2}\sqrt{a+\sqrt{a^2+b^2}}d}$$

$$= -\frac{2b\sqrt{a+b\cot(c+dx)}}{d} - \frac{b\sqrt{a^2+b^2}\log\left(a+\sqrt{a^2+b^2}+b\cot(c+dx)-\sqrt{2}\sqrt{a+\sqrt{a^2+b^2}}\sqrt{a+b\cot(c+dx)}\right)}{2\sqrt{2}\sqrt{a+\sqrt{a^2+b^2}}d} + \frac{b\sqrt{a^2+b^2}\log\left(a+\sqrt{a^2+b^2}+b\cot(c+dx)+\sqrt{2}\sqrt{a+\sqrt{a^2+b^2}}\sqrt{a+b\cot(c+dx)}\right)}{2\sqrt{2}\sqrt{a+\sqrt{a^2+b^2}}d} - \frac{\left(b\sqrt{a^2+b^2}\right)\text{Subst}\left(\int\frac{1}{2\left(a-\sqrt{a^2+b^2}\right)-x^2}\,dx, x, -\sqrt{2}\sqrt{a+\sqrt{a^2+b^2}}+2\sqrt{a+b\cot(c+dx)}\right)}{d} - \frac{\left(b\sqrt{a^2+b^2}\right)\text{Subst}\left(\int\frac{1}{2\left(a-\sqrt{a^2+b^2}\right)-x^2}\,dx, x, \sqrt{2}\sqrt{a+\sqrt{a^2+b^2}}+2\sqrt{a+b\cot(c+dx)}\right)}{d}$$

$$= \frac{b\sqrt{a^2+b^2}\text{arctanh}\left(\frac{\sqrt{a+\sqrt{a^2+b^2}}-\sqrt{2}\sqrt{a+b\cot(c+dx)}}{\sqrt{a-\sqrt{a^2+b^2}}}\right)}{\sqrt{2}\sqrt{a-\sqrt{a^2+b^2}}d} - \frac{b\sqrt{a^2+b^2}\text{arctanh}\left(\frac{\sqrt{a+\sqrt{a^2+b^2}}+\sqrt{2}\sqrt{a+b\cot(c+dx)}}{\sqrt{a-\sqrt{a^2+b^2}}}\right)}{\sqrt{2}\sqrt{a-\sqrt{a^2+b^2}}d} - \frac{2b\sqrt{a+b\cot(c+dx)}}{d} - \frac{b\sqrt{a^2+b^2}\log\left(a+\sqrt{a^2+b^2}+b\cot(c+dx)-\sqrt{2}\sqrt{a+\sqrt{a^2+b^2}}\sqrt{a+b\cot(c+dx)}\right)}{2\sqrt{2}\sqrt{a+\sqrt{a^2+b^2}}d} + \frac{b\sqrt{a^2+b^2}\log\left(a+\sqrt{a^2+b^2}+b\cot(c+dx)+\sqrt{2}\sqrt{a+\sqrt{a^2+b^2}}\sqrt{a+b\cot(c+dx)}\right)}{2\sqrt{2}\sqrt{a+\sqrt{a^2+b^2}}d}$$

Mathematica [A] (verified)

Time = 0.75 (sec) , antiderivative size = 153, normalized size of antiderivative = 0.36

$$\int(-a+b\cot(c+dx))\sqrt{a+b\cot(c+dx)}\,dx = \frac{b\left(\frac{(a^2+b^2)\text{arctanh}\left(\frac{\sqrt{a+b\cot(c+dx)}}{\sqrt{a-\sqrt{-b^2}}}\right)}{\sqrt{-b^2}\sqrt{a-\sqrt{-b^2}}} - \frac{(a^2+b^2)\text{arctanh}\left(\frac{\sqrt{a+b\cot(c+dx)}}{\sqrt{a+\sqrt{-b^2}}}\right)}{\sqrt{-b^2}\sqrt{a+\sqrt{-b^2}}} - 2\sqrt{a+b\cot(c+dx)}\right)}{d}$$

```
[In] Integrate[(-a + b*Cot[c + d*x])*Sqrt[a + b*Cot[c + d*x]],x]
[Out] (b*(((a^2 + b^2)*ArcTanh[Sqrt[a + b*Cot[c + d*x]]/Sqrt[a - Sqrt[-b^2]]])/(S
qrt[-b^2]*Sqrt[a - Sqrt[-b^2]]) - ((a^2 + b^2)*ArcTanh[Sqrt[a + b*Cot[c + d
*x]]/Sqrt[a + Sqrt[-b^2]]])/(Sqrt[-b^2]*Sqrt[a + Sqrt[-b^2]]) - 2*Sqrt[a +
b*Cot[c + d*x]]))/d
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 815 vs. 2(341) = 682.

Time = 0.06 (sec) , antiderivative size = 816, normalized size of antiderivative = 1.93

method	result
parts	$\frac{b\left(-2\sqrt{a+b\cot(dx+c)}+\frac{\sqrt{2\sqrt{a^2+b^2}+2a}\,\ln\left(b\cot(dx+c)+a+\sqrt{a+b\cot(dx+c)}\,\sqrt{2\sqrt{a^2+b^2}+2a}+\sqrt{a^2+b^2}\right)}{4}-\frac{\left(a-\sqrt{a^2+b^2}\right)\arctan\left(\right.}{}\right.}{}$
derivativedivides	Expression too large to display
default	Expression too large to display

```
[In] int((-a+b*cot(d*x+c))*(a+b*cot(d*x+c))^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] b/d*(-2*(a+b*cot(d*x+c))^(1/2)+1/4*(2*(a^2+b^2)^(1/2)+2*a)^(1/2)*ln(b*cot(d
*x+c)+a+(a+b*cot(d*x+c))^(1/2)*(2*(a^2+b^2)^(1/2)+2*a)^(1/2)+(a^2+b^2)^(1/2
))-(a-(a^2+b^2)^(1/2))/(2*(a^2+b^2)^(1/2)-2*a)^(1/2)*arctan((2*(a+b*cot(d*x
+c))^(1/2)+(2*(a^2+b^2)^(1/2)+2*a)^(1/2))/(2*(a^2+b^2)^(1/2)-2*a)^(1/2))-1/
4*(2*(a^2+b^2)^(1/2)+2*a)^(1/2)*ln((a+b*cot(d*x+c))^(1/2)*(2*(a^2+b^2)^(1/2
)+2*a)^(1/2)-b*cot(d*x+c)-(a^2+b^2)^(1/2)-a)-((a^2+b^2)^(1/2)-a)/(2*(a^2+b^
2)^(1/2)-2*a)^(1/2)*arctan((-2*(a+b*cot(d*x+c))^(1/2)+(2*(a^2+b^2)^(1/2)+2*
a)^(1/2))/(2*(a^2+b^2)^(1/2)-2*a)^(1/2)))+1/4/d/b*ln(b*cot(d*x+c)+a+(a+b*co
t(d*x+c))^(1/2)*(2*(a^2+b^2)^(1/2)+2*a)^(1/2)+(a^2+b^2)^(1/2))*(2*(a^2+b^2)
^(1/2)+2*a)^(1/2)*a^2-1/4/d/b*ln(b*cot(d*x+c)+a+(a+b*cot(d*x+c))^(1/2)*(2*(
a^2+b^2)^(1/2)+2*a)^(1/2)+(a^2+b^2)^(1/2))*(2*(a^2+b^2)^(1/2)+2*a)^(1/2)*(a
^2+b^2)^(1/2)*a+1/d*b/(2*(a^2+b^2)^(1/2)-2*a)^(1/2)*arctan((2*(a+b*cot(d*x+
c))^(1/2)+(2*(a^2+b^2)^(1/2)+2*a)^(1/2))/(2*(a^2+b^2)^(1/2)-2*a)^(1/2))*a-1
/4/d/b*ln((a+b*cot(d*x+c))^(1/2)*(2*(a^2+b^2)^(1/2)+2*a)^(1/2)-b*cot(d*x+c)
-(a^2+b^2)^(1/2)-a)*(2*(a^2+b^2)^(1/2)+2*a)^(1/2)*a^2+1/4/d/b*ln((a+b*cot(d
*x+c))^(1/2)*(2*(a^2+b^2)^(1/2)+2*a)^(1/2)-b*cot(d*x+c)-(a^2+b^2)^(1/2)-a)*
(2*(a^2+b^2)^(1/2)+2*a)^(1/2)*(a^2+b^2)^(1/2)*a-1/d*b/(2*(a^2+b^2)^(1/2)-2*
a)^(1/2)*arctan((-2*(a+b*cot(d*x+c))^(1/2)+(2*(a^2+b^2)^(1/2)+2*a)^(1/2))/(
2*(a^2+b^2)^(1/2)-2*a)^(1/2))*a
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 849 vs. 2(343) = 686.

Time = 0.31 (sec) , antiderivative size = 849, normalized size of antiderivative = 2.01

$$\int(-a+b\cot(c+dx))\sqrt{a+b\cot(c+dx)}\,dx$$

$$=\frac{d\sqrt{-\frac{a^3+ab^2+d^2\sqrt{-\frac{a^4b^2+2\,a^2b^4+b^6}{d^4}}}{d^2}}\log\left(\left(a^4b+2\,a^2b^3+b^5\right)\sqrt{\frac{b\cos(2\,dx+2\,c)+a\sin(2\,dx+2\,c)+b}{\sin(2\,dx+2\,c)}}+\left(ad^3\sqrt{-\frac{a^4b^2+2\,a^2b^4+b^6}{d^4}}\right.\right.}{}$$

```
[In] integrate((-a+b*cot(d*x+c))*(a+b*cot(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] 1/2*(d*sqrt(-(a^3 + a*b^2 + d^2*sqrt(-(a^4*b^2 + 2*a^2*b^4 + b^6)/d^4))/d^2
)*log((a^4*b + 2*a^2*b^3 + b^5)*sqrt((b*cos(2*d*x + 2*c) + a*sin(2*d*x + 2*
c) + b)/sin(2*d*x + 2*c)) + (a*d^3*sqrt(-(a^4*b^2 + 2*a^2*b^4 + b^6)/d^4) +
 (a^2*b^2 + b^4)*d)*sqrt(-(a^3 + a*b^2 + d^2*sqrt(-(a^4*b^2 + 2*a^2*b^4 + b
^6)/d^4))/d^2)) - d*sqrt(-(a^3 + a*b^2 + d^2*sqrt(-(a^4*b^2 + 2*a^2*b^4 + b
^6)/d^4))/d^2)*log((a^4*b + 2*a^2*b^3 + b^5)*sqrt((b*cos(2*d*x + 2*c) + a*s
in(2*d*x + 2*c) + b)/sin(2*d*x + 2*c)) - (a*d^3*sqrt(-(a^4*b^2 + 2*a^2*b^4
+ b^6)/d^4) + (a^2*b^2 + b^4)*d)*sqrt(-(a^3 + a*b^2 + d^2*sqrt(-(a^4*b^2 +
2*a^2*b^4 + b^6)/d^4))/d^2)) - d*sqrt(-(a^3 + a*b^2 - d^2*sqrt(-(a^4*b^2 +
2*a^2*b^4 + b^6)/d^4))/d^2)*log((a^4*b + 2*a^2*b^3 + b^5)*sqrt((b*cos(2*d*x
 + 2*c) + a*sin(2*d*x + 2*c) + b)/sin(2*d*x + 2*c)) + (a*d^3*sqrt(-(a^4*b^2
 + 2*a^2*b^4 + b^6)/d^4) - (a^2*b^2 + b^4)*d)*sqrt(-(a^3 + a*b^2 - d^2*sqrt
(-(a^4*b^2 + 2*a^2*b^4 + b^6)/d^4))/d^2)) + d*sqrt(-(a^3 + a*b^2 - d^2*sqrt
(-(a^4*b^2 + 2*a^2*b^4 + b^6)/d^4))/d^2)*log((a^4*b + 2*a^2*b^3 + b^5)*sqrt
((b*cos(2*d*x + 2*c) + a*sin(2*d*x + 2*c) + b)/sin(2*d*x + 2*c)) - (a*d^3*s
qrt(-(a^4*b^2 + 2*a^2*b^4 + b^6)/d^4) - (a^2*b^2 + b^4)*d)*sqrt(-(a^3 + a*b
^2 - d^2*sqrt(-(a^4*b^2 + 2*a^2*b^4 + b^6)/d^4))/d^2)) - 4*b*sqrt((b*cos(2*
d*x + 2*c) + a*sin(2*d*x + 2*c) + b)/sin(2*d*x + 2*c)))/d
```

Sympy [F]

$$\int (-a+b\cot(c+dx))\sqrt{a+b\cot(c+dx)}\,dx = -\int a\sqrt{a+b\cot{(c+dx)}}\,dx - \int \left(-b\sqrt{a+b\cot{(c+dx)}}\cot{(c+dx)}\right)\,dx$$

```
[In] integrate((-a+b*cot(d*x+c))*(a+b*cot(d*x+c))**(1/2),x)
```

```
[Out] -Integral(a*sqrt(a + b*cot(c + d*x)), x) - Integral(-b*sqrt(a + b*cot(c + d
*x))*cot(c + d*x), x)
```

Maxima [F]

$$\int (-a+b\cot(c+dx))\sqrt{a+b\cot(c+dx)}\,dx = \int \sqrt{b\cot{(dx+c)}+a}(b\cot{(dx+c)}-a)\,dx$$

```
[In] integrate((-a+b*cot(d*x+c))*(a+b*cot(d*x+c))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(b*cot(d*x + c) + a)*(b*cot(d*x + c) - a), x)
```

Giac [F]

$$\int (-a+b\cot(c+dx))\sqrt{a+b\cot(c+dx)}\,dx=\int \sqrt{b\cot(dx+c)+a}(b\cot(dx+c)-a)\,dx$$

[In] integrate((-a+b*cot(d*x+c))*(a+b*cot(d*x+c))^(1/2),x, algorithm="giac")

[Out] integrate(sqrt(b*cot(d*x + c) + a)*(b*cot(d*x + c) - a), x)

Mupad [B] (verification not implemented)

Time = 14.20 (sec) , antiderivative size = 583, normalized size of antiderivative = 1.38

$$\int (-a+b\cot(c+dx))\sqrt{a+b\cot(c+dx)}\,dx=-\operatorname{atanh}\left(\frac{d^3\left(\frac{16\,(a^2\,b^4-a^4\,b^2)\,\sqrt{a+b\cot(c+d\,x)}}{d^2}+\frac{16\,a\,b^2\,(a^3+1\mathrm{i}\,b\,a^2)\,\sqrt{a+b\cot(c+d\,x)}}{d^2}\right)\sqrt{-\frac{a^3+1\mathrm{i}\,b\,a^2}{d^2}}}{16\,(a^5\,b^3+a^3\,b^5)}\right)\sqrt{-\frac{a^3+1\mathrm{i}\,b\,a^2}{d^2}}-\operatorname{atanh}\left(\frac{d^3\,\sqrt{\frac{-a^3+a^2\,b\,1\mathrm{i}}{d^2}}\left(\frac{16\,(a^2\,b^4-a^4\,b^2)\,\sqrt{a+b\cot(c+d\,x)}}{d^2}-\frac{16\,a\,b^2\,(-a^3+a^2\,b\,1\mathrm{i})\,\sqrt{a+b\cot(c+d\,x)}}{d^2}\right)}{16\,(a^5\,b^3+a^3\,b^5)}\right)\sqrt{\frac{-a^3+a^2\,b\,1\mathrm{i}}{d^2}}-\frac{2\,b\,\sqrt{a+b\cot(c+d\,x)}}{d}+\operatorname{atan}\left(\frac{b^6\,\sqrt{\frac{a\,b^2}{4\,d^2}-\frac{b^3\,1\mathrm{i}}{4\,d^2}}\,\sqrt{a+b\cot(c+d\,x)}\,32\mathrm{i}}{\frac{b^8\,16\mathrm{i}}{d}+\frac{a^2\,b^6\,16\mathrm{i}}{d}}+\frac{32\,a\,b^5\,\sqrt{\frac{a\,b^2}{4\,d^2}-\frac{b^3\,1\mathrm{i}}{4\,d^2}}\,\sqrt{a+b\cot(c+d\,x)}}{\frac{b^8\,16\mathrm{i}}{d}+\frac{a^2\,b^6\,16\mathrm{i}}{d}}\right)\sqrt{\frac{a\,b^2-b^3\,1\mathrm{i}}{4\,d^2}}\,2\mathrm{i}-\operatorname{atan}\left(\frac{b^6\,\sqrt{\frac{a\,b^2}{4\,d^2}+\frac{b^3\,1\mathrm{i}}{4\,d^2}}\,\sqrt{a+b\cot(c+d\,x)}\,32\mathrm{i}}{\frac{b^8\,16\mathrm{i}}{d}+\frac{a^2\,b^6\,16\mathrm{i}}{d}}-\frac{32\,a\,b^5\,\sqrt{\frac{a\,b^2}{4\,d^2}+\frac{b^3\,1\mathrm{i}}{4\,d^2}}\,\sqrt{a+b\cot(c+d\,x)}}{\frac{b^8\,16\mathrm{i}}{d}+\frac{a^2\,b^6\,16\mathrm{i}}{d}}\right)\sqrt{\frac{b^3\,1\mathrm{i}+a\,b^2}{4\,d^2}}\,2\mathrm{i}$$

[In] int(-(a + b*cot(c + d*x))^(1/2)*(a - b*cot(c + d*x)),x)

[Out] atan((b^6*((a*b^2)/(4*d^2) - (b^3*1i)/(4*d^2))^(1/2)*(a + b*cot(c + d*x))^(1/2)*32i)/((b^8*16i)/d + (a^2*b^6*16i)/d) + (32*a*b^5*((a*b^2)/(4*d^2) - (b^3*1i)/(4*d^2))^(1/2)*(a + b*cot(c + d*x))^(1/2))/((b^8*16i)/d + (a^2*b^6*16i)/d))*((a*b^2 - b^3*1i)/(4*d^2))^(1/2)*2i - atan((b^6*((b^3*1i)/(4*d^2) + (a*b^2)/(4*d^2))^(1/2)*(a + b*cot(c + d*x))^(1/2)*32i)/((b^8*16i)/d + (a^2*b^6*16i)/d) - (32*a*b^5*((b^3*1i)/(4*d^2) + (a*b^2)/(4*d^2))^(1/2)*(a + b*

```
cot(c + d*x))^(1/2))/((b^8*16i)/d + (a^2*b^6*16i)/d))*((a*b^2 + b^3*1i)/(4*
d^2))^(1/2)*2i - atanh((d^3*((16*(a^2*b^4 - a^4*b^2)*(a + b*cot(c + d*x))^(
1/2))/d^2 + (16*a*b^2*(a^2*b*1i + a^3)*(a + b*cot(c + d*x))^(1/2))/d^2)*(-(
a^2*b*1i + a^3)/d^2)^(1/2))/(16*(a^3*b^5 + a^5*b^3)))*(-(a^2*b*1i + a^3)/d^
2)^(1/2) - atanh((d^3*((a^2*b*1i - a^3)/d^2)^(1/2)*((16*(a^2*b^4 - a^4*b^2)
*(a + b*cot(c + d*x))^(1/2))/d^2 - (16*a*b^2*(a^2*b*1i - a^3)*(a + b*cot(c
+ d*x))^(1/2))/d^2))/(16*(a^3*b^5 + a^5*b^3)))*((a^2*b*1i - a^3)/d^2)^(1/2)
 - (2*b*(a + b*cot(c + d*x))^(1/2))/d
```

3.101 $\int \frac{A+B\cot(c+dx)}{\sqrt{a+b\cot(c+dx)}}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 102

$$\int \frac{A+B\cot(c+dx)}{\sqrt{a+b\cot(c+dx)}}\,dx = \frac{(iA+B)\text{arctanh}\left(\frac{\sqrt{a+b\cot(c+dx)}}{\sqrt{a-ib}}\right)}{\sqrt{a-ib}d} - \frac{(iA-B)\text{arctanh}\left(\frac{\sqrt{a+b\cot(c+dx)}}{\sqrt{a+ib}}\right)}{\sqrt{a+ib}d}$$

```
[Out] (I*A+B)*arctanh((a+b*cot(d*x+c))^(1/2)/(a-I*b)^(1/2))/d/(a-I*b)^(1/2)-(I*A-
B)*arctanh((a+b*cot(d*x+c))^(1/2)/(a+I*b)^(1/2))/d/(a+I*b)^(1/2)
```

Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 102, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.160, Rules used = {3620, 3618, 65, 214}

$$\int \frac{A+B\cot(c+dx)}{\sqrt{a+b\cot(c+dx)}}\,dx = \frac{(B+iA)\text{arctanh}\left(\frac{\sqrt{a+b\cot(c+dx)}}{\sqrt{a-ib}}\right)}{d\sqrt{a-ib}} - \frac{(-B+iA)\text{arctanh}\left(\frac{\sqrt{a+b\cot(c+dx)}}{\sqrt{a+ib}}\right)}{d\sqrt{a+ib}}$$

```
[In] Int[(A + B*Cot[c + d*x])/Sqrt[a + b*Cot[c + d*x]],x]
```

```
[Out] ((I*A + B)*ArcTanh[Sqrt[a + b*Cot[c + d*x]]/Sqrt[a - I*b]])/(Sqrt[a - I*b]*
d) - ((I*A - B)*ArcTanh[Sqrt[a + b*Cot[c + d*x]]/Sqrt[a + I*b]])/(Sqrt[a +
I*b]*d)
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 3618

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*tan[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Dist[c*(d/f), Subst[Int[(a + (b/d)*x)^m/(d^2 + c
*x), x], x, d*Tan[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b
*c - a*d, 0] && NeQ[a^2 + b^2, 0] && EqQ[c^2 + d^2, 0]
```

Rule 3620

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)]), x_Symbol] :> Dist[(c + I*d)/2, Int[(a + b*Tan[e + f*x])^m*(1
 - I*Tan[e + f*x]), x], x] + Dist[(c - I*d)/2, Int[(a + b*Tan[e + f*x])^m*(
1 + I*Tan[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b*c -
 a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] &&  !IntegerQ[m]
```

Rubi steps

$$\text{integral} = \frac{1}{2}(A-iB)\int \frac{1+i\cot(c+dx)}{\sqrt{a+b\cot(c+dx)}}\,dx + \frac{1}{2}(A+iB)\int \frac{1-i\cot(c+dx)}{\sqrt{a+b\cot(c+dx)}}\,dx$$

$$= \frac{(iA-B)\text{Subst}\left(\int \frac{1}{(-1+x)\sqrt{a+ibx}}\,dx, x, -i\cot(c+dx)\right)}{2d} - \frac{(iA+B)\text{Subst}\left(\int \frac{1}{(-1+x)\sqrt{a-ibx}}\,dx, x, i\cot(c+dx)\right)}{2d}$$

$$= \frac{(A-iB)\text{Subst}\left(\int \frac{1}{-1-\frac{ia}{b}+\frac{ix^2}{b}}\,dx, x, \sqrt{a+b\cot(c+dx)}\right)}{bd} + \frac{(A+iB)\text{Subst}\left(\int \frac{1}{-1+\frac{ia}{b}-\frac{ix^2}{b}}\,dx, x, \sqrt{a+b\cot(c+dx)}\right)}{bd}$$

$$= \frac{(iA+B)\text{arctanh}\left(\frac{\sqrt{a+b\cot(c+dx)}}{\sqrt{a-ib}}\right)}{\sqrt{a-ib}d} - \frac{(iA-B)\text{arctanh}\left(\frac{\sqrt{a+b\cot(c+dx)}}{\sqrt{a+ib}}\right)}{\sqrt{a+ib}d}$$

Mathematica [A] (verified)

Time = 2.15 (sec) , antiderivative size = 154, normalized size of antiderivative = 1.51

$$\int \frac{A+B\cot(c+dx)}{\sqrt{a+b\cot(c+dx)}}\,dx = \frac{\left(\sqrt{a+ib}(iA+B)\operatorname{arctanh}\left(\frac{\sqrt{a+b\cot(c+dx)}}{\sqrt{a-ib}}\right)+\sqrt{a-ib}(-iA+B)\operatorname{arctanh}\left(\frac{\sqrt{a+b\cot(c+dx)}}{\sqrt{a+ib}}\right)\right)(A+B\cot(c+}{\sqrt{a-ib}\sqrt{a+ib}d(B\cos(c+dx)+A\sin(c+dx))}$$

```
[In] Integrate[(A + B*Cot[c + d*x])/Sqrt[a + b*Cot[c + d*x]],x]
```

```
[Out] ((Sqrt[a + I*b]*(I*A + B)*ArcTanh[Sqrt[a + b*Cot[c + d*x]]/Sqrt[a - I*b]] +
 Sqrt[a - I*b]*((-I)*A + B)*ArcTanh[Sqrt[a + b*Cot[c + d*x]]/Sqrt[a + I*b]]
)*(A + B*Cot[c + d*x])*Sin[c + d*x])/(Sqrt[a - I*b]*Sqrt[a + I*b]*d*(B*Cos[
c + d*x] + A*Sin[c + d*x]))
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 1889 vs. 2(84) = 168.

Time = 0.09 (sec) , antiderivative size = 1890, normalized size of antiderivative = 18.53

method	result	size
parts	Expression too large to display	1890
derivativedivides	Expression too large to display	3976
default	Expression too large to display	3976

```
[In] int((A+B*cot(d*x+c))/(a+b*cot(d*x+c))^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] A*(-1/4/d/b/(a^2+b^2)*ln(b*cot(d*x+c)+a+(a+b*cot(d*x+c))^(1/2)*(2*(a^2+b^2)
^(1/2)+2*a)^(1/2)+(a^2+b^2)^(1/2))*(2*(a^2+b^2)^(1/2)+2*a)^(1/2)*a^2-1/4/d*
b/(a^2+b^2)*ln(b*cot(d*x+c)+a+(a+b*cot(d*x+c))^(1/2)*(2*(a^2+b^2)^(1/2)+2*a
)^(1/2)+(a^2+b^2)^(1/2))*(2*(a^2+b^2)^(1/2)+2*a)^(1/2)+1/4/d/b/(a^2+b^2)^(3
/2)*ln(b*cot(d*x+c)+a+(a+b*cot(d*x+c))^(1/2)*(2*(a^2+b^2)^(1/2)+2*a)^(1/2)+
(a^2+b^2)^(1/2))*(2*(a^2+b^2)^(1/2)+2*a)^(1/2)*a^3+1/4/d*b/(a^2+b^2)^(3/2)*
ln(b*cot(d*x+c)+a+(a+b*cot(d*x+c))^(1/2)*(2*(a^2+b^2)^(1/2)+2*a)^(1/2)+(a^2
+b^2)^(1/2))*(2*(a^2+b^2)^(1/2)+2*a)^(1/2)*a+1/d/b/(a^2+b^2)^(1/2)/(2*(a^2+
b^2)^(1/2)-2*a)^(1/2)*arctan((2*(a+b*cot(d*x+c))^(1/2)+(2*(a^2+b^2)^(1/2)+2
*a)^(1/2))/(2*(a^2+b^2)^(1/2)-2*a)^(1/2))*a^2+1/d*b/(a^2+b^2)^(1/2)/(2*(a^2
+b^2)^(1/2)-2*a)^(1/2)*arctan((2*(a+b*cot(d*x+c))^(1/2)+(2*(a^2+b^2)^(1/2)+
2*a)^(1/2))/(2*(a^2+b^2)^(1/2)-2*a)^(1/2))-1/d/b/(a^2+b^2)^(3/2)/(2*(a^2+b^
2)^(1/2)-2*a)^(1/2)*arctan((2*(a+b*cot(d*x+c))^(1/2)+(2*(a^2+b^2)^(1/2)+2*a
)^(1/2))/(2*(a^2+b^2)^(1/2)-2*a)^(1/2))*a^4-3/d*b/(a^2+b^2)^(3/2)/(2*(a^2+b
^2)^(1/2)-2*a)^(1/2)*arctan((2*(a+b*cot(d*x+c))^(1/2)+(2*(a^2+b^2)^(1/2)+2*
```

```
a)^(1/2))/(2*(a^2+b^2)^(1/2)-2*a)^(1/2))*a^2-2/d*b^3/(a^2+b^2)^(3/2)/(2*(a^
2+b^2)^(1/2)-2*a)^(1/2)*arctan((2*(a+b*cot(d*x+c))^(1/2)+(2*(a^2+b^2)^(1/2)
+2*a)^(1/2))/(2*(a^2+b^2)^(1/2)-2*a)^(1/2))+1/4/d/b/(a^2+b^2)*ln(b*cot(d*x+
c)+a-(a+b*cot(d*x+c))^(1/2)*(2*(a^2+b^2)^(1/2)+2*a)^(1/2)+(a^2+b^2)^(1/2))*
(2*(a^2+b^2)^(1/2)+2*a)^(1/2)*a^2+1/4/d*b/(a^2+b^2)*ln(b*cot(d*x+c)+a-(a+b*
cot(d*x+c))^(1/2)*(2*(a^2+b^2)^(1/2)+2*a)^(1/2)+(a^2+b^2)^(1/2))*(2*(a^2+b^
2)^(1/2)+2*a)^(1/2)-1/4/d/b/(a^2+b^2)^(3/2)*ln(b*cot(d*x+c)+a-(a+b*cot(d*x+
c))^(1/2)*(2*(a^2+b^2)^(1/2)+2*a)^(1/2)+(a^2+b^2)^(1/2))*(2*(a^2+b^2)^(1/2)
+2*a)^(1/2)*a^3-1/4/d*b/(a^2+b^2)^(3/2)*ln(b*cot(d*x+c)+a-(a+b*cot(d*x+c))^
(1/2)*(2*(a^2+b^2)^(1/2)+2*a)^(1/2)+(a^2+b^2)^(1/2))*(2*(a^2+b^2)^(1/2)+2*a
)^(1/2)*a+1/d/b/(a^2+b^2)^(1/2)/(2*(a^2+b^2)^(1/2)-2*a)^(1/2)*arctan((2*(a+
b*cot(d*x+c))^(1/2)-(2*(a^2+b^2)^(1/2)+2*a)^(1/2))/(2*(a^2+b^2)^(1/2)-2*a)^
(1/2))*a^2+1/d*b/(a^2+b^2)^(1/2)/(2*(a^2+b^2)^(1/2)-2*a)^(1/2)*arctan((2*(a
+b*cot(d*x+c))^(1/2)-(2*(a^2+b^2)^(1/2)+2*a)^(1/2))/(2*(a^2+b^2)^(1/2)-2*a)
^(1/2))-1/d/b/(a^2+b^2)^(3/2)/(2*(a^2+b^2)^(1/2)-2*a)^(1/2)*arctan((2*(a+b*
cot(d*x+c))^(1/2)-(2*(a^2+b^2)^(1/2)+2*a)^(1/2))/(2*(a^2+b^2)^(1/2)-2*a)^(1
/2))*a^4-3/d*b/(a^2+b^2)^(3/2)/(2*(a^2+b^2)^(1/2)-2*a)^(1/2)*arctan((2*(a+b
*cot(d*x+c))^(1/2)-(2*(a^2+b^2)^(1/2)+2*a)^(1/2))/(2*(a^2+b^2)^(1/2)-2*a)^(
1/2))*a^2-2/d*b^3/(a^2+b^2)^(3/2)/(2*(a^2+b^2)^(1/2)-2*a)^(1/2)*arctan((2*(
a+b*cot(d*x+c))^(1/2)-(2*(a^2+b^2)^(1/2)+2*a)^(1/2))/(2*(a^2+b^2)^(1/2)-2*a
)^(1/2)))+B/d*(-1/2/(a^2+b^2)^(1/2)*(-1/2*(2*(a^2+b^2)^(1/2)+2*a)^(1/2)*ln(
b*cot(d*x+c)+a+(a+b*cot(d*x+c))^(1/2)*(2*(a^2+b^2)^(1/2)+2*a)^(1/2)+(a^2+b^
2)^(1/2))+2*((a^2+b^2)^(1/2)-a)/(2*(a^2+b^2)^(1/2)-2*a)^(1/2)*arctan((2*(a+
b*cot(d*x+c))^(1/2)+(2*(a^2+b^2)^(1/2)+2*a)^(1/2))/(2*(a^2+b^2)^(1/2)-2*a)^
(1/2)))-1/2/(a^2+b^2)^(1/2)*(1/2*(2*(a^2+b^2)^(1/2)+2*a)^(1/2)*ln(b*cot(d*x
+c)+a-(a+b*cot(d*x+c))^(1/2)*(2*(a^2+b^2)^(1/2)+2*a)^(1/2)+(a^2+b^2)^(1/2))
+2*((a^2+b^2)^(1/2)-a)/(2*(a^2+b^2)^(1/2)-2*a)^(1/2)*arctan((2*(a+b*cot(d*x
+c))^(1/2)-(2*(a^2+b^2)^(1/2)+2*a)^(1/2))/(2*(a^2+b^2)^(1/2)-2*a)^(1/2))))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1773 vs. $2(78) = 156$.

Time = 0.32 (sec) , antiderivative size = 1773, normalized size of antiderivative = 17.38

$$\int \frac{A + B\cot(c + dx)}{\sqrt{a + b\cot(c + dx)}}\, dx = \text{Too large to display}$$

```
[In] integrate((A+B*cot(d*x+c))/(a+b*cot(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] 1/2*sqrt(-((a^2 + b^2)*d^2*sqrt(-(4*A^2*B^2*a^2 - 4*(A^3*B - A*B^3)*a*b + (
A^4 - 2*A^2*B^2 + B^4)*b^2)/((a^4 + 2*a^2*b^2 + b^4)*d^4)) + 2*A*B*b + (A^2
 - B^2)*a)/((a^2 + b^2)*d^2))*log((2*(A^3*B + A*B^3)*a - (A^4 - B^4)*b)*sqr
t((b*cos(2*d*x + 2*c) + a*sin(2*d*x + 2*c) + b)/sin(2*d*x + 2*c)) + ((A*a^3
 + B*a^2*b + A*a*b^2 + B*b^3)*d^3*sqrt(-(4*A^2*B^2*a^2 - 4*(A^3*B - A*B^3)*
```

```
a*b + (A^4 - 2*A^2*B^2 + B^4)*b^2)/((a^4 + 2*a^2*b^2 + b^4)*d^4)) + (2*A*B^
2*a^2 - (3*A^2*B - B^3)*a*b + (A^3 - A*B^2)*b^2)*d)*sqrt(-((a^2 + b^2)*d^2*
sqrt(-(4*A^2*B^2*a^2 - 4*(A^3*B - A*B^3)*a*b + (A^4 - 2*A^2*B^2 + B^4)*b^2)
/((a^4 + 2*a^2*b^2 + b^4)*d^4)) + 2*A*B*b + (A^2 - B^2)*a)/((a^2 + b^2)*d^2
))) - 1/2*sqrt(-((a^2 + b^2)*d^2*sqrt(-(4*A^2*B^2*a^2 - 4*(A^3*B - A*B^3)*a
*b + (A^4 - 2*A^2*B^2 + B^4)*b^2)/((a^4 + 2*a^2*b^2 + b^4)*d^4)) + 2*A*B*b
+ (A^2 - B^2)*a)/((a^2 + b^2)*d^2))*log((2*(A^3*B + A*B^3)*a - (A^4 - B^4)*
b)*sqrt((b*cos(2*d*x + 2*c) + a*sin(2*d*x + 2*c) + b)/sin(2*d*x + 2*c)) - (
(A*a^3 + B*a^2*b + A*a*b^2 + B*b^3)*d^3*sqrt(-(4*A^2*B^2*a^2 - 4*(A^3*B - A
*B^3)*a*b + (A^4 - 2*A^2*B^2 + B^4)*b^2)/((a^4 + 2*a^2*b^2 + b^4)*d^4)) + (
2*A*B^2*a^2 - (3*A^2*B - B^3)*a*b + (A^3 - A*B^2)*b^2)*d)*sqrt(-((a^2 + b^2
)*d^2*sqrt(-(4*A^2*B^2*a^2 - 4*(A^3*B - A*B^3)*a*b + (A^4 - 2*A^2*B^2 + B^4
)*b^2)/((a^4 + 2*a^2*b^2 + b^4)*d^4)) + 2*A*B*b + (A^2 - B^2)*a)/((a^2 + b^
2)*d^2))) - 1/2*sqrt(((a^2 + b^2)*d^2*sqrt(-(4*A^2*B^2*a^2 - 4*(A^3*B - A*B
^3)*a*b + (A^4 - 2*A^2*B^2 + B^4)*b^2)/((a^4 + 2*a^2*b^2 + b^4)*d^4)) - 2*A
*B*b - (A^2 - B^2)*a)/((a^2 + b^2)*d^2))*log((2*(A^3*B + A*B^3)*a - (A^4 -
B^4)*b)*sqrt((b*cos(2*d*x + 2*c) + a*sin(2*d*x + 2*c) + b)/sin(2*d*x + 2*c)
) + ((A*a^3 + B*a^2*b + A*a*b^2 + B*b^3)*d^3*sqrt(-(4*A^2*B^2*a^2 - 4*(A^3*
B - A*B^3)*a*b + (A^4 - 2*A^2*B^2 + B^4)*b^2)/((a^4 + 2*a^2*b^2 + b^4)*d^4)
) - (2*A*B^2*a^2 - (3*A^2*B - B^3)*a*b + (A^3 - A*B^2)*b^2)*d)*sqrt(((a^2 +
 b^2)*d^2*sqrt(-(4*A^2*B^2*a^2 - 4*(A^3*B - A*B^3)*a*b + (A^4 - 2*A^2*B^2 +
 B^4)*b^2)/((a^4 + 2*a^2*b^2 + b^4)*d^4)) - 2*A*B*b - (A^2 - B^2)*a)/((a^2
+ b^2)*d^2))) + 1/2*sqrt(((a^2 + b^2)*d^2*sqrt(-(4*A^2*B^2*a^2 - 4*(A^3*B -
 A*B^3)*a*b + (A^4 - 2*A^2*B^2 + B^4)*b^2)/((a^4 + 2*a^2*b^2 + b^4)*d^4)) -
 2*A*B*b - (A^2 - B^2)*a)/((a^2 + b^2)*d^2))*log((2*(A^3*B + A*B^3)*a - (A^
4 - B^4)*b)*sqrt((b*cos(2*d*x + 2*c) + a*sin(2*d*x + 2*c) + b)/sin(2*d*x +
2*c)) - ((A*a^3 + B*a^2*b + A*a*b^2 + B*b^3)*d^3*sqrt(-(4*A^2*B^2*a^2 - 4*(
A^3*B - A*B^3)*a*b + (A^4 - 2*A^2*B^2 + B^4)*b^2)/((a^4 + 2*a^2*b^2 + b^4)*
d^4)) - (2*A*B^2*a^2 - (3*A^2*B - B^3)*a*b + (A^3 - A*B^2)*b^2)*d)*sqrt(((a
^2 + b^2)*d^2*sqrt(-(4*A^2*B^2*a^2 - 4*(A^3*B - A*B^3)*a*b + (A^4 - 2*A^2*B
^2 + B^4)*b^2)/((a^4 + 2*a^2*b^2 + b^4)*d^4)) - 2*A*B*b - (A^2 - B^2)*a)/((
a^2 + b^2)*d^2)))
```

Sympy [F]

$$\int \frac{A + B\cot(c + dx)}{\sqrt{a + b\cot(c + dx)}}\, dx = \int \frac{A + B\cot\left(c + dx\right)}{\sqrt{a + b\cot\left(c + dx\right)}}\, dx$$

```
[In] integrate((A+B*cot(d*x+c))/(a+b*cot(d*x+c))**(1/2),x)
```

```
[Out] Integral((A + B*cot(c + d*x))/sqrt(a + b*cot(c + d*x)), x)
```

Maxima [F]

$$\int \frac{A+B\cot(c+dx)}{\sqrt{a+b\cot(c+dx)}}\,dx=\int \frac{B\cot\left(dx+c\right)+A}{\sqrt{b\cot\left(dx+c\right)+a}}\,dx$$

```
[In] integrate((A+B*cot(d*x+c))/(a+b*cot(d*x+c))^(1/2),x, algorithm="maxima")
[Out] integrate((B*cot(d*x + c) + A)/sqrt(b*cot(d*x + c) + a), x)
```

Giac [F]

$$\int \frac{A+B\cot(c+dx)}{\sqrt{a+b\cot(c+dx)}}\,dx=\int \frac{B\cot\left(dx+c\right)+A}{\sqrt{b\cot\left(dx+c\right)+a}}\,dx$$

```
[In] integrate((A+B*cot(d*x+c))/(a+b*cot(d*x+c))^(1/2),x, algorithm="giac")
[Out] integrate((B*cot(d*x + c) + A)/sqrt(b*cot(d*x + c) + a), x)
```

Mupad [B] (verification not implemented)

Time = 15.29 (sec) , antiderivative size = 2909, normalized size of antiderivative = 28.52

$$\int \frac{A+B\cot(c+dx)}{\sqrt{a+b\cot(c+dx)}}\,dx=\text{Too large to display}$$

```
[In] int((A + B*cot(c + d*x))/(a + b*cot(c + d*x))^(1/2),x)
[Out] 2*atanh((32*B^2*b^2*((B^2*a*d^2)/(4*(a^2*d^4 + b^2*d^4)) - (-16*B^4*b^2*d^4
)^(1/2)/(16*(a^2*d^4 + b^2*d^4)))^(1/2)*(a + b*cot(c + d*x))^(1/2))/((16*B^
3*b^2)/d - (16*B^3*a^2*b^2*d^3)/(a^2*d^4 + b^2*d^4) + (4*B*a*b^2*d^2*(-16*B
^4*b^2*d^4)^(1/2))/(a^2*d^5 + b^2*d^5)) + (8*a*b^2*((B^2*a*d^2)/(4*(a^2*d^4
 + b^2*d^4)) - (-16*B^4*b^2*d^4)^(1/2)/(16*(a^2*d^4 + b^2*d^4)))^(1/2)*(a +
 b*cot(c + d*x))^(1/2)*(-16*B^4*b^2*d^4)^(1/2))/(16*B^3*b^4*d + 16*B^3*a^2*
b^2*d - (16*B^3*a^2*b^4*d^5)/(a^2*d^4 + b^2*d^4) - (16*B^3*a^4*b^2*d^5)/(a^
2*d^4 + b^2*d^4) + (4*B*a^3*b^2*d^4*(-16*B^4*b^2*d^4)^(1/2))/(a^2*d^5 + b^2
*d^5) + (4*B*a*b^4*d^4*(-16*B^4*b^2*d^4)^(1/2))/(a^2*d^5 + b^2*d^5)) - (32*
B^2*a^2*b^2*d^2*((B^2*a*d^2)/(4*(a^2*d^4 + b^2*d^4)) - (-16*B^4*b^2*d^4)^(1
/2)/(16*(a^2*d^4 + b^2*d^4)))^(1/2)*(a + b*cot(c + d*x))^(1/2))/(16*B^3*b^4
*d + 16*B^3*a^2*b^2*d - (16*B^3*a^2*b^4*d^5)/(a^2*d^4 + b^2*d^4) - (16*B^3*
a^4*b^2*d^5)/(a^2*d^4 + b^2*d^4) + (4*B*a^3*b^2*d^4*(-16*B^4*b^2*d^4)^(1/2)
)/(a^2*d^5 + b^2*d^5) + (4*B*a*b^4*d^4*(-16*B^4*b^2*d^4)^(1/2))/(a^2*d^5 +
b^2*d^5)))*((B^2*a*d^2)/(4*(a^2*d^4 + b^2*d^4)) - (-16*B^4*b^2*d^4)^(1/2)/(
```

16*(a^2*d^4 + b^2*d^4)))^(1/2) + 2*atanh((8*a*b^2*((-16*B^4*b^2*d^4)^(1/2)/
(16*(a^2*d^4 + b^2*d^4)) + (B^2*a*d^2)/(4*(a^2*d^4 + b^2*d^4)))^(1/2)*(a +
b*cot(c + d*x))^(1/2)*(-16*B^4*b^2*d^4)^(1/2))/((16*B^3*a^2*b^4*d^5)/(a^2*d
^4 + b^2*d^4) - 16*B^3*a^2*b^2*d - 16*B^3*b^4*d + (16*B^3*a^4*b^2*d^5)/(a^2
*d^4 + b^2*d^4) + (4*B*a^3*b^2*d^4*(-16*B^4*b^2*d^4)^(1/2))/(a^2*d^5 + b^2*
d^5) + (4*B*a*b^4*d^4*(-16*B^4*b^2*d^4)^(1/2))/(a^2*d^5 + b^2*d^5)) - (32*B
^2*b^2*((-16*B^4*b^2*d^4)^(1/2)/(16*(a^2*d^4 + b^2*d^4)) + (B^2*a*d^2)/(4*(
a^2*d^4 + b^2*d^4)))^(1/2)*(a + b*cot(c + d*x))^(1/2))/((16*B^3*a^2*b^2*d^3
)/(a^2*d^4 + b^2*d^4) - (16*B^3*b^2)/d + (4*B*a*b^2*d^2*(-16*B^4*b^2*d^4)^(
1/2))/(a^2*d^5 + b^2*d^5)) + (32*B^2*a^2*b^2*d^2*((-16*B^4*b^2*d^4)^(1/2)/(
16*(a^2*d^4 + b^2*d^4)) + (B^2*a*d^2)/(4*(a^2*d^4 + b^2*d^4)))^(1/2)*(a + b
*cot(c + d*x))^(1/2))/((16*B^3*a^2*b^4*d^5)/(a^2*d^4 + b^2*d^4) - 16*B^3*a^
2*b^2*d - 16*B^3*b^4*d + (16*B^3*a^4*b^2*d^5)/(a^2*d^4 + b^2*d^4) + (4*B*a^
3*b^2*d^4*(-16*B^4*b^2*d^4)^(1/2))/(a^2*d^5 + b^2*d^5) + (4*B*a*b^4*d^4*(-1
6*B^4*b^2*d^4)^(1/2))/(a^2*d^5 + b^2*d^5)))*((-16*B^4*b^2*d^4)^(1/2)/(16*(a
^2*d^4 + b^2*d^4)) + (B^2*a*d^2)/(4*(a^2*d^4 + b^2*d^4)))^(1/2) + 2*atanh((
32*A^2*b^2*((-16*A^4*b^2*d^4)^(1/2)/(16*(a^2*d^4 + b^2*d^4)) - (A^2*a*d^2)/
(4*(a^2*d^4 + b^2*d^4)))^(1/2)*(a + b*cot(c + d*x))^(1/2))/((16*A^3*a*b^3*d
^3)/(a^2*d^4 + b^2*d^4) - (4*A*b^3*d^2*(-16*A^4*b^2*d^4)^(1/2))/(a^2*d^5 +
b^2*d^5)) + (8*a*b^2*((-16*A^4*b^2*d^4)^(1/2)/(16*(a^2*d^4 + b^2*d^4)) - (A
^2*a*d^2)/(4*(a^2*d^4 + b^2*d^4)))^(1/2)*(a + b*cot(c + d*x))^(1/2)*(-16*A^
4*b^2*d^4)^(1/2))/((16*A^3*a*b^5*d^5)/(a^2*d^4 + b^2*d^4) - (4*A*b^5*d^4*(-
16*A^4*b^2*d^4)^(1/2))/(a^2*d^5 + b^2*d^5) + (16*A^3*a^3*b^3*d^5)/(a^2*d^4
+ b^2*d^4) - (4*A*a^2*b^3*d^4*(-16*A^4*b^2*d^4)^(1/2))/(a^2*d^5 + b^2*d^5))
 - (32*A^2*a^2*b^2*d^2*((-16*A^4*b^2*d^4)^(1/2)/(16*(a^2*d^4 + b^2*d^4)) -
(A^2*a*d^2)/(4*(a^2*d^4 + b^2*d^4)))^(1/2)*(a + b*cot(c + d*x))^(1/2))/((16
*A^3*a*b^5*d^5)/(a^2*d^4 + b^2*d^4) - (4*A*b^5*d^4*(-16*A^4*b^2*d^4)^(1/2))
/(a^2*d^5 + b^2*d^5) + (16*A^3*a^3*b^3*d^5)/(a^2*d^4 + b^2*d^4) - (4*A*a^2*
b^3*d^4*(-16*A^4*b^2*d^4)^(1/2))/(a^2*d^5 + b^2*d^5)))*((-16*A^4*b^2*d^4)^(
1/2)/(16*(a^2*d^4 + b^2*d^4)) - (A^2*a*d^2)/(4*(a^2*d^4 + b^2*d^4)))^(1/2)
- 2*atanh((8*a*b^2*(- (-16*A^4*b^2*d^4)^(1/2)/(16*(a^2*d^4 + b^2*d^4)) - (A
^2*a*d^2)/(4*(a^2*d^4 + b^2*d^4)))^(1/2)*(a + b*cot(c + d*x))^(1/2)*(-16*A^
4*b^2*d^4)^(1/2))/((16*A^3*a*b^5*d^5)/(a^2*d^4 + b^2*d^4) + (4*A*b^5*d^4*(-
16*A^4*b^2*d^4)^(1/2))/(a^2*d^5 + b^2*d^5) + (16*A^3*a^3*b^3*d^5)/(a^2*d^4
+ b^2*d^4) + (4*A*a^2*b^3*d^4*(-16*A^4*b^2*d^4)^(1/2))/(a^2*d^5 + b^2*d^5))
 - (32*A^2*b^2*(- (-16*A^4*b^2*d^4)^(1/2)/(16*(a^2*d^4 + b^2*d^4)) - (A^2*a
d^2)/(4(a^2*d^4 + b^2*d^4)))^(1/2)*(a + b*cot(c + d*x))^(1/2))/((16*A^3*a
*b^3*d^3)/(a^2*d^4 + b^2*d^4) + (4*A*b^3*d^2*(-16*A^4*b^2*d^4)^(1/2))/(a^2*
d^5 + b^2*d^5)) + (32*A^2*a^2*b^2*d^2*(- (-16*A^4*b^2*d^4)^(1/2)/(16*(a^2*d
^4 + b^2*d^4)) - (A^2*a*d^2)/(4*(a^2*d^4 + b^2*d^4)))^(1/2)*(a + b*cot(c +
d*x))^(1/2))/((16*A^3*a*b^5*d^5)/(a^2*d^4 + b^2*d^4) + (4*A*b^5*d^4*(-16*A^
4*b^2*d^4)^(1/2))/(a^2*d^5 + b^2*d^5) + (16*A^3*a^3*b^3*d^5)/(a^2*d^4 + b^2
*d^4) + (4*A*a^2*b^3*d^4*(-16*A^4*b^2*d^4)^(1/2))/(a^2*d^5 + b^2*d^5)))*(-
(-16*A^4*b^2*d^4)^(1/2)/(16*(a^2*d^4 + b^2*d^4)) - (A^2*a*d^2)/(4*(a^2*d^4
+ b^2*d^4)))^(1/2)

3.102 $\int \frac{A+B\cot(c+dx)}{(a+b\cot(c+dx))^{3/2}}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 138

$$\int \frac{A+B\cot(c+dx)}{(a+b\cot(c+dx))^{3/2}}\,dx=\frac{(iA+B)\text{arctanh}\left(\frac{\sqrt{a+b\cot(c+dx)}}{\sqrt{a-ib}}\right)}{(a-ib)^{3/2}d}-\frac{(iA-B)\text{arctanh}\left(\frac{\sqrt{a+b\cot(c+dx)}}{\sqrt{a+ib}}\right)}{(a+ib)^{3/2}d}+\frac{2(Ab-aB)}{(a^2+b^2)\,d\sqrt{a+b\cot(c+dx)}}$$

```
[Out] (I*A+B)*arctanh((a+b*cot(d*x+c))^(1/2)/(a-I*b)^(1/2))/(a-I*b)^(3/2)/d-(I*A-
B)*arctanh((a+b*cot(d*x+c))^(1/2)/(a+I*b)^(1/2))/(a+I*b)^(3/2)/d+2*(A*b-B*a
)/(a^2+b^2)/d/(a+b*cot(d*x+c))^(1/2)
```

Rubi [A] (verified)

Time = 0.33 (sec) , antiderivative size = 138, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {3610, 3620, 3618, 65, 214}

$$\int \frac{A+B\cot(c+dx)}{(a+b\cot(c+dx))^{3/2}}\,dx=\frac{2(Ab-aB)}{d\,(a^2+b^2)\,\sqrt{a+b\cot(c+dx)}}+\frac{(B+iA)\text{arctanh}\left(\frac{\sqrt{a+b\cot(c+dx)}}{\sqrt{a-ib}}\right)}{d(a-ib)^{3/2}}-\frac{(-B+iA)\text{arctanh}\left(\frac{\sqrt{a+b\cot(c+dx)}}{\sqrt{a+ib}}\right)}{d(a+ib)^{3/2}}$$

```
[In] Int[(A + B*Cot[c + d*x])/(a + b*Cot[c + d*x])^(3/2),x]
```

```
[Out] ((I*A + B)*ArcTanh[Sqrt[a + b*Cot[c + d*x]]/Sqrt[a - I*b]])/((a - I*b)^(3/2
)*d) - ((I*A - B)*ArcTanh[Sqrt[a + b*Cot[c + d*x]]/Sqrt[a + I*b]])/((a + I*
b)^(3/2)*d) + (2*(A*b - a*B))/((a^2 + b^2)*d*Sqrt[a + b*Cot[c + d*x]])
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 3610

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)]), x_Symbol] :> Simp[(b*c - a*d)*((a + b*Tan[e + f*x])^(m + 1)/
(f*(m + 1)*(a^2 + b^2))), x] + Dist[1/(a^2 + b^2), Int[(a + b*Tan[e + f*x])
^(m + 1)*Simp[a*c + b*d - (b*c - a*d)*Tan[e + f*x], x], x], x] /; FreeQ[{a,
 b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && LtQ[m, -1]
```

Rule 3618

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*tan[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Dist[c*(d/f), Subst[Int[(a + (b/d)*x)^m/(d^2 + c
*x), x], x, d*Tan[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b
*c - a*d, 0] && NeQ[a^2 + b^2, 0] && EqQ[c^2 + d^2, 0]
```

Rule 3620

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)]), x_Symbol] :> Dist[(c + I*d)/2, Int[(a + b*Tan[e + f*x])^m*(1
 - I*Tan[e + f*x]), x], x] + Dist[(c - I*d)/2, Int[(a + b*Tan[e + f*x])^m*(
1 + I*Tan[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b*c -
 a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] &&  !IntegerQ[m]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \frac{2(Ab-aB)}{(a^2+b^2)\,d\sqrt{a+b\cot(c+dx)}}+\frac{\int \frac{aA+bB-(Ab-aB)\cot(c+dx)}{\sqrt{a+b\cot(c+dx)}}\,dx}{a^2+b^2}\\ &= \frac{2(Ab-aB)}{(a^2+b^2)\,d\sqrt{a+b\cot(c+dx)}}+\frac{(A-iB)\int \frac{1+i\cot(c+dx)}{\sqrt{a+b\cot(c+dx)}}\,dx}{2(a-ib)}+\frac{(A+iB)\int \frac{1-i\cot(c+dx)}{\sqrt{a+b\cot(c+dx)}}\,dx}{2(a+ib)}\end{aligned}$$

$$= \frac{2(Ab-aB)}{(a^2+b^2)\,d\sqrt{a+b\cot(c+dx)}} + \frac{(i(A+iB))\text{Subst}\left(\int \frac{1}{(-1+x)\sqrt{a+ibx}}\,dx, x, -i\cot(c+dx)\right)}{2(a+ib)d} - \frac{(iA+B)\text{Subst}\left(\int \frac{1}{(-1+x)\sqrt{a-ibx}}\,dx, x, i\cot(c+dx)\right)}{2(a-ib)d}$$

$$= \frac{2(Ab-aB)}{(a^2+b^2)\,d\sqrt{a+b\cot(c+dx)}} + \frac{(A-iB)\text{Subst}\left(\int \frac{1}{-1-\frac{ia}{b}+\frac{ix^2}{b}}\,dx, x, \sqrt{a+b\cot(c+dx)}\right)}{(a-ib)bd} + \frac{(A+iB)\text{Subst}\left(\int \frac{1}{-1+\frac{ia}{b}-\frac{ix^2}{b}}\,dx, x, \sqrt{a+b\cot(c+dx)}\right)}{(a+ib)bd}$$

$$= \frac{(iA+B)\text{arctanh}\left(\frac{\sqrt{a+b\cot(c+dx)}}{\sqrt{a-ib}}\right)}{(a-ib)^{3/2}d} - \frac{(iA-B)\text{arctanh}\left(\frac{\sqrt{a+b\cot(c+dx)}}{\sqrt{a+ib}}\right)}{(a+ib)^{3/2}d} + \frac{2(Ab-aB)}{(a^2+b^2)\,d\sqrt{a+b\cot(c+dx)}}$$

Mathematica [A] (verified)

Time = 1.81 (sec) , antiderivative size = 226, normalized size of antiderivative = 1.64

$$\int \frac{A+B\cot(c+dx)}{(a+b\cot(c+dx))^{3/2}}\,dx = -\frac{\frac{\left(aAb+Ab\sqrt{-b^2}+b^2B-a\sqrt{-b^2}B\right)\text{arctanh}\left(\frac{\sqrt{a+b\cot(c+dx)}}{\sqrt{a-\sqrt{-b^2}}}\right)}{\sqrt{-b^2}\sqrt{a-\sqrt{-b^2}}} - \frac{\left(aAb-Ab\sqrt{-b^2}+b^2B+a\sqrt{-b^2}B\right)\text{arctanh}\left(\frac{\sqrt{a+b\cot(c+dx)}}{\sqrt{a+\sqrt{-b^2}}}\right)}{\sqrt{-b^2}\sqrt{a+\sqrt{-b^2}}} + \frac{2(-\cdots}{\sqrt{a+b\cdots}}}{(a^2+b^2)\,d}$$

```
[In] Integrate[(A + B*Cot[c + d*x])/(a + b*Cot[c + d*x])^(3/2),x]
```

```
[Out] -((((a*A*b + A*b*Sqrt[-b^2] + b^2*B - a*Sqrt[-b^2]*B)*ArcTanh[Sqrt[a + b*Co
t[c + d*x]]/Sqrt[a - Sqrt[-b^2]]])/(Sqrt[-b^2]*Sqrt[a - Sqrt[-b^2]]) - ((a*
A*b - A*b*Sqrt[-b^2] + b^2*B + a*Sqrt[-b^2]*B)*ArcTanh[Sqrt[a + b*Cot[c + d
*x]]/Sqrt[a + Sqrt[-b^2]]])/(Sqrt[-b^2]*Sqrt[a + Sqrt[-b^2]]) + (2*(-(A*b)
+ a*B))/Sqrt[a + b*Cot[c + d*x]])/((a^2 + b^2)*d))
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 3682 vs. 2(118) = 236.

Time = 0.15 (sec) , antiderivative size = 3683, normalized size of antiderivative = 26.69

method	result	size
parts	Expression too large to display	3683
derivativedivides	Expression too large to display	7951
default	Expression too large to display	7951

```
[In] int((A+B*cot(d*x+c))/(a+b*cot(d*x+c))^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] A*(-1/4/d/b/(a^2+b^2)^2*ln(b*cot(d*x+c)+a+(a+b*cot(d*x+c))^(1/2)*(2*(a^2+b^
2)^(1/2)+2*a)^(1/2)+(a^2+b^2)^(1/2))*(2*(a^2+b^2)^(1/2)+2*a)^(1/2)*a^3-1/4/
d*b/(a^2+b^2)^2*ln(b*cot(d*x+c)+a+(a+b*cot(d*x+c))^(1/2)*(2*(a^2+b^2)^(1/2)
+2*a)^(1/2)+(a^2+b^2)^(1/2))*(2*(a^2+b^2)^(1/2)+2*a)^(1/2)*a+1/4/d/b/(a^2+b
^2)^(5/2)*ln(b*cot(d*x+c)+a+(a+b*cot(d*x+c))^(1/2)*(2*(a^2+b^2)^(1/2)+2*a)^
(1/2)+(a^2+b^2)^(1/2))*(2*(a^2+b^2)^(1/2)+2*a)^(1/2)*a^4-1/4/d*b^3/(a^2+b^2
)^(5/2)*ln(b*cot(d*x+c)+a+(a+b*cot(d*x+c))^(1/2)*(2*(a^2+b^2)^(1/2)+2*a)^(1
/2)+(a^2+b^2)^(1/2))*(2*(a^2+b^2)^(1/2)+2*a)^(1/2)+1/d/b/(a^2+b^2)^(3/2)/(2
*(a^2+b^2)^(1/2)-2*a)^(1/2)*arctan((2*(a+b*cot(d*x+c))^(1/2)+(2*(a^2+b^2)^(
1/2)+2*a)^(1/2))/(2*(a^2+b^2)^(1/2)-2*a)^(1/2))*a^3+1/d*b/(a^2+b^2)^(3/2)/(
2*(a^2+b^2)^(1/2)-2*a)^(1/2)*arctan((2*(a+b*cot(d*x+c))^(1/2)+(2*(a^2+b^2)^
(1/2)+2*a)^(1/2))/(2*(a^2+b^2)^(1/2)-2*a)^(1/2))*a+1/d*b/(a^2+b^2)^2/(2*(a^
2+b^2)^(1/2)-2*a)^(1/2)*arctan((2*(a+b*cot(d*x+c))^(1/2)+(2*(a^2+b^2)^(1/2)
+2*a)^(1/2))/(2*(a^2+b^2)^(1/2)-2*a)^(1/2))*a^2-1/d/b/(a^2+b^2)^(5/2)/(2*(a
^2+b^2)^(1/2)-2*a)^(1/2)*arctan((2*(a+b*cot(d*x+c))^(1/2)+(2*(a^2+b^2)^(1/2
)+2*a)^(1/2))/(2*(a^2+b^2)^(1/2)-2*a)^(1/2))*a^5+1/d*b^3/(a^2+b^2)^2/(2*(a^
2+b^2)^(1/2)-2*a)^(1/2)*arctan((2*(a+b*cot(d*x+c))^(1/2)+(2*(a^2+b^2)^(1/2)
+2*a)^(1/2))/(2*(a^2+b^2)^(1/2)-2*a)^(1/2))-3/d*b^3/(a^2+b^2)^(5/2)/(2*(a^2
+b^2)^(1/2)-2*a)^(1/2)*arctan((2*(a+b*cot(d*x+c))^(1/2)+(2*(a^2+b^2)^(1/2)+
2*a)^(1/2))/(2*(a^2+b^2)^(1/2)-2*a)^(1/2))*a-4/d*b/(a^2+b^2)^(5/2)/(2*(a^2+
b^2)^(1/2)-2*a)^(1/2)*arctan((2*(a+b*cot(d*x+c))^(1/2)+(2*(a^2+b^2)^(1/2)+2
*a)^(1/2))/(2*(a^2+b^2)^(1/2)-2*a)^(1/2))*a^3+1/4/d/b/(a^2+b^2)^2*ln((a+b*c
ot(d*x+c))^(1/2)*(2*(a^2+b^2)^(1/2)+2*a)^(1/2)-b*cot(d*x+c)-(a^2+b^2)^(1/2)
-a)*(2*(a^2+b^2)^(1/2)+2*a)^(1/2)*a^3+1/4/d*b/(a^2+b^2)^2*ln((a+b*cot(d*x+c
))^(1/2)*(2*(a^2+b^2)^(1/2)+2*a)^(1/2)-b*cot(d*x+c)-(a^2+b^2)^(1/2)-a)*(2*(
a^2+b^2)^(1/2)+2*a)^(1/2)*a-1/4/d/b/(a^2+b^2)^(5/2)*ln((a+b*cot(d*x+c))^(1/
2)*(2*(a^2+b^2)^(1/2)+2*a)^(1/2)-b*cot(d*x+c)-(a^2+b^2)^(1/2)-a)*(2*(a^2+b^
2)^(1/2)+2*a)^(1/2)*a^4+1/4/d*b^3/(a^2+b^2)^(5/2)*ln((a+b*cot(d*x+c))^(1/2)
*(2*(a^2+b^2)^(1/2)+2*a)^(1/2)-b*cot(d*x+c)-(a^2+b^2)^(1/2)-a)*(2*(a^2+b^2)
^(1/2)+2*a)^(1/2)-1/d/b/(a^2+b^2)^(3/2)/(2*(a^2+b^2)^(1/2)-2*a)^(1/2)*arcta
n((-2*(a+b*cot(d*x+c))^(1/2)+(2*(a^2+b^2)^(1/2)+2*a)^(1/2))/(2*(a^2+b^2)^(1
/2)-2*a)^(1/2))*a^3-1/d*b/(a^2+b^2)^(3/2)/(2*(a^2+b^2)^(1/2)-2*a)^(1/2)*arc
tan((-2*(a+b*cot(d*x+c))^(1/2)+(2*(a^2+b^2)^(1/2)+2*a)^(1/2))/(2*(a^2+b^2)^
```

(1/2)-2*a)^(1/2))*a-1/d*b/(a^2+b^2)^2/(2*(a^2+b^2)^(1/2)-2*a)^(1/2)*arctan(
(-2*(a+b*cot(d*x+c))^(1/2)+(2*(a^2+b^2)^(1/2)+2*a)^(1/2))/(2*(a^2+b^2)^(1/2
)-2*a)^(1/2))*a^2+1/d/b/(a^2+b^2)^(5/2)/(2*(a^2+b^2)^(1/2)-2*a)^(1/2)*arcta
n((-2*(a+b*cot(d*x+c))^(1/2)+(2*(a^2+b^2)^(1/2)+2*a)^(1/2))/(2*(a^2+b^2)^(1
/2)-2*a)^(1/2))*a^5-1/d*b^3/(a^2+b^2)^2/(2*(a^2+b^2)^(1/2)-2*a)^(1/2)*arcta
n((-2*(a+b*cot(d*x+c))^(1/2)+(2*(a^2+b^2)^(1/2)+2*a)^(1/2))/(2*(a^2+b^2)^(1
/2)-2*a)^(1/2))+3/d*b^3/(a^2+b^2)^(5/2)/(2*(a^2+b^2)^(1/2)-2*a)^(1/2)*arcta
n((-2*(a+b*cot(d*x+c))^(1/2)+(2*(a^2+b^2)^(1/2)+2*a)^(1/2))/(2*(a^2+b^2)^(1
/2)-2*a)^(1/2))*a+4/d*b/(a^2+b^2)^(5/2)/(2*(a^2+b^2)^(1/2)-2*a)^(1/2)*arcta
n((-2*(a+b*cot(d*x+c))^(1/2)+(2*(a^2+b^2)^(1/2)+2*a)^(1/2))/(2*(a^2+b^2)^(1
/2)-2*a)^(1/2))*a^3+2/d*b/(a^2+b^2)/(a+b*cot(d*x+c))^(1/2))+B*(-1/4/d/(a^2+
b^2)^2*ln(b*cot(d*x+c)+a+(a+b*cot(d*x+c))^(1/2)*(2*(a^2+b^2)^(1/2)+2*a)^(1/
2)+(a^2+b^2)^(1/2))*(2*(a^2+b^2)^(1/2)+2*a)^(1/2)*a^2-1/4/d/(a^2+b^2)^2*ln(
b*cot(d*x+c)+a+(a+b*cot(d*x+c))^(1/2)*(2*(a^2+b^2)^(1/2)+2*a)^(1/2)+(a^2+b^
2)^(1/2))*(2*(a^2+b^2)^(1/2)+2*a)^(1/2)*b^2+1/2/d/(a^2+b^2)^(5/2)*ln(b*cot(
d*x+c)+a+(a+b*cot(d*x+c))^(1/2)*(2*(a^2+b^2)^(1/2)+2*a)^(1/2)+(a^2+b^2)^(1/
2))*(2*(a^2+b^2)^(1/2)+2*a)^(1/2)*a^3+1/2/d/(a^2+b^2)^(5/2)*ln(b*cot(d*x+c)
+a+(a+b*cot(d*x+c))^(1/2)*(2*(a^2+b^2)^(1/2)+2*a)^(1/2)+(a^2+b^2)^(1/2))*(2
*(a^2+b^2)^(1/2)+2*a)^(1/2)*a*b^2+1/d/(a^2+b^2)^(3/2)/(2*(a^2+b^2)^(1/2)-2*
a)^(1/2)*arctan((2*(a+b*cot(d*x+c))^(1/2)+(2*(a^2+b^2)^(1/2)+2*a)^(1/2))/(2
*(a^2+b^2)^(1/2)-2*a)^(1/2))*a^2-1/d/(a^2+b^2)^2/(2*(a^2+b^2)^(1/2)-2*a)^(1
/2)*arctan((2*(a+b*cot(d*x+c))^(1/2)+(2*(a^2+b^2)^(1/2)+2*a)^(1/2))/(2*(a^2
+b^2)^(1/2)-2*a)^(1/2))*a^3+1/d/(a^2+b^2)^(3/2)/(2*(a^2+b^2)^(1/2)-2*a)^(1/
2)*arctan((2*(a+b*cot(d*x+c))^(1/2)+(2*(a^2+b^2)^(1/2)+2*a)^(1/2))/(2*(a^2+
b^2)^(1/2)-2*a)^(1/2))*b^2-1/d/(a^2+b^2)^2/(2*(a^2+b^2)^(1/2)-2*a)^(1/2)*ar
ctan((2*(a+b*cot(d*x+c))^(1/2)+(2*(a^2+b^2)^(1/2)+2*a)^(1/2))/(2*(a^2+b^2)^
(1/2)-2*a)^(1/2))*b^2*a-2/d/(a^2+b^2)^(5/2)/(2*(a^2+b^2)^(1/2)-2*a)^(1/2)*a
rctan((2*(a+b*cot(d*x+c))^(1/2)+(2*(a^2+b^2)^(1/2)+2*a)^(1/2))/(2*(a^2+b^2)
^(1/2)-2*a)^(1/2))*a^2*b^2-2/d/(a^2+b^2)^(5/2)/(2*(a^2+b^2)^(1/2)-2*a)^(1/2
)*arctan((2*(a+b*cot(d*x+c))^(1/2)+(2*(a^2+b^2)^(1/2)+2*a)^(1/2))/(2*(a^2+b
^2)^(1/2)-2*a)^(1/2))*b^4+1/4/d/(a^2+b^2)^2*ln((a+b*cot(d*x+c))^(1/2)*(2*(a
^2+b^2)^(1/2)+2*a)^(1/2)-b*cot(d*x+c)-(a^2+b^2)^(1/2)-a)*(2*(a^2+b^2)^(1/2)
+2*a)^(1/2)*a^2+1/4/d/(a^2+b^2)^2*ln((a+b*cot(d*x+c))^(1/2)*(2*(a^2+b^2)^(1
/2)+2*a)^(1/2)-b*cot(d*x+c)-(a^2+b^2)^(1/2)-a)*(2*(a^2+b^2)^(1/2)+2*a)^(1/2
)*b^2-1/2/d/(a^2+b^2)^(5/2)*ln((a+b*cot(d*x+c))^(1/2)*(2*(a^2+b^2)^(1/2)+2*
a)^(1/2)-b*cot(d*x+c)-(a^2+b^2)^(1/2)-a)*(2*(a^2+b^2)^(1/2)+2*a)^(1/2)*a^3-
1/2/d/(a^2+b^2)^(5/2)*ln((a+b*cot(d*x+c))^(1/2)*(2*(a^2+b^2)^(1/2)+2*a)^(1/
2)-b*cot(d*x+c)-(a^2+b^2)^(1/2)-a)*(2*(a^2+b^2)^(1/2)+2*a)^(1/2)*a*b^2-1/d/
(a^2+b^2)^(3/2)/(2*(a^2+b^2)^(1/2)-2*a)^(1/2)*arctan((-2*(a+b*cot(d*x+c))^(
1/2)+(2*(a^2+b^2)^(1/2)+2*a)^(1/2))/(2*(a^2+b^2)^(1/2)-2*a)^(1/2))*a^2+1/d/
(a^2+b^2)^2/(2*(a^2+b^2)^(1/2)-2*a)^(1/2)*arctan((-2*(a+b*cot(d*x+c))^(1/2)
+(2*(a^2+b^2)^(1/2)+2*a)^(1/2))/(2*(a^2+b^2)^(1/2)-2*a)^(1/2))*a^3-1/d/(a^2
+b^2)^(3/2)/(2*(a^2+b^2)^(1/2)-2*a)^(1/2)*arctan((-2*(a+b*cot(d*x+c))^(1/2)
+(2*(a^2+b^2)^(1/2)+2*a)^(1/2))/(2*(a^2+b^2)^(1/2)-2*a)^(1/2))*b^2+1/d/(a^2
+b^2)^2/(2*(a^2+b^2)^(1/2)-2*a)^(1/2)*arctan((-2*(a+b*cot(d*x+c))^(1/2)+(2*

```
(a^2+b^2)^(1/2)+2*a)^(1/2))/(2*(a^2+b^2)^(1/2)-2*a)^(1/2))*b^2*a+2/d/(a^2+b
^2)^(5/2)/(2*(a^2+b^2)^(1/2)-2*a)^(1/2)*arctan((-2*(a+b*cot(d*x+c))^(1/2)+(
2*(a^2+b^2)^(1/2)+2*a)^(1/2))/(2*(a^2+b^2)^(1/2)-2*a)^(1/2))*a^2*b^2+2/d/(a
^2+b^2)^(5/2)/(2*(a^2+b^2)^(1/2)-2*a)^(1/2)*arctan((-2*(a+b*cot(d*x+c))^(1/
2)+(2*(a^2+b^2)^(1/2)+2*a)^(1/2))/(2*(a^2+b^2)^(1/2)-2*a)^(1/2))*b^4-2/d*a/
(a^2+b^2)/(a+b*cot(d*x+c))^(1/2))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 4572 vs. $2(112) = 224$.

Time = 0.80 (sec) , antiderivative size = 4572, normalized size of antiderivative = 33.13

$$\int \frac{A + B\cot(c+dx)}{(a+b\cot(c+dx))^{3/2}}\,dx = \text{Too large to display}$$

```
[In] integrate((A+B*cot(d*x+c))/(a+b*cot(d*x+c))^(3/2),x, algorithm="fricas")
```

```
[Out] -1/2*(((a^2*b + b^3)*d*cos(2*d*x + 2*c) + (a^3 + a*b^2)*d*sin(2*d*x + 2*c)
+ (a^2*b + b^3)*d)*sqrt(-(6*A*B*a^2*b - 2*A*B*b^3 + (A^2 - B^2)*a^3 - 3*(A^
2 - B^2)*a*b^2 + (a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6)*d^2*sqrt(-(4*A^2*B^2*a
^6 - 12*(A^3*B - A*B^3)*a^5*b + 3*(3*A^4 - 14*A^2*B^2 + 3*B^4)*a^4*b^2 + 40
*(A^3*B - A*B^3)*a^3*b^3 - 6*(A^4 - 8*A^2*B^2 + B^4)*a^2*b^4 - 12*(A^3*B -
A*B^3)*a*b^5 + (A^4 - 2*A^2*B^2 + B^4)*b^6)/((a^12 + 6*a^10*b^2 + 15*a^8*b^
4 + 20*a^6*b^6 + 15*a^4*b^8 + 6*a^2*b^10 + b^12)*d^4)))/((a^6 + 3*a^4*b^2 +
 3*a^2*b^4 + b^6)*d^2))*log(-(2*(A^3*B + A*B^3)*a^3 - 3*(A^4 - B^4)*a^2*b -
 6*(A^3*B + A*B^3)*a*b^2 + (A^4 - B^4)*b^3)*sqrt((b*cos(2*d*x + 2*c) + a*si
n(2*d*x + 2*c) + b)/sin(2*d*x + 2*c)) + ((A*a^8 + 2*B*a^7*b + 2*A*a^6*b^2 +
 6*B*a^5*b^3 + 6*B*a^3*b^5 - 2*A*a^2*b^6 + 2*B*a*b^7 - A*b^8)*d^3*sqrt(-(4*
A^2*B^2*a^6 - 12*(A^3*B - A*B^3)*a^5*b + 3*(3*A^4 - 14*A^2*B^2 + 3*B^4)*a^4
*b^2 + 40*(A^3*B - A*B^3)*a^3*b^3 - 6*(A^4 - 8*A^2*B^2 + B^4)*a^2*b^4 - 12*
(A^3*B - A*B^3)*a*b^5 + (A^4 - 2*A^2*B^2 + B^4)*b^6)/((a^12 + 6*a^10*b^2 +
15*a^8*b^4 + 20*a^6*b^6 + 15*a^4*b^8 + 6*a^2*b^10 + b^12)*d^4)) + (2*A*B^2*
a^5 - (7*A^2*B - 3*B^3)*a^4*b + 2*(3*A^3 - 7*A*B^2)*a^3*b^2 + 4*(4*A^2*B -
B^3)*a^2*b^3 - 2*(A^3 - 4*A*B^2)*a*b^4 - (A^2*B - B^3)*b^5)*d)*sqrt(-(6*A*B
*a^2*b - 2*A*B*b^3 + (A^2 - B^2)*a^3 - 3*(A^2 - B^2)*a*b^2 + (a^6 + 3*a^4*b
^2 + 3*a^2*b^4 + b^6)*d^2*sqrt(-(4*A^2*B^2*a^6 - 12*(A^3*B - A*B^3)*a^5*b +
 3*(3*A^4 - 14*A^2*B^2 + 3*B^4)*a^4*b^2 + 40*(A^3*B - A*B^3)*a^3*b^3 - 6*(A
^4 - 8*A^2*B^2 + B^4)*a^2*b^4 - 12*(A^3*B - A*B^3)*a*b^5 + (A^4 - 2*A^2*B^2
 + B^4)*b^6)/((a^12 + 6*a^10*b^2 + 15*a^8*b^4 + 20*a^6*b^6 + 15*a^4*b^8 + 6
*a^2*b^10 + b^12)*d^4)))/((a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6)*d^2))) - ((a^
2*b + b^3)*d*cos(2*d*x + 2*c) + (a^3 + a*b^2)*d*sin(2*d*x + 2*c) + (a^2*b +
 b^3)*d)*sqrt(-(6*A*B*a^2*b - 2*A*B*b^3 + (A^2 - B^2)*a^3 - 3*(A^2 - B^2)*a
*b^2 + (a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6)*d^2*sqrt(-(4*A^2*B^2*a^6 - 12*(A
^3*B - A*B^3)*a^5*b + 3*(3*A^4 - 14*A^2*B^2 + 3*B^4)*a^4*b^2 + 40*(A^3*B -
A*B^3)*a^3*b^3 - 6*(A^4 - 8*A^2*B^2 + B^4)*a^2*b^4 - 12*(A^3*B - A*B^3)*a*b
```

```
^5 + (A^4 - 2*A^2*B^2 + B^4)*b^6)/((a^12 + 6*a^10*b^2 + 15*a^8*b^4 + 20*a^6
*b^6 + 15*a^4*b^8 + 6*a^2*b^10 + b^12)*d^4)))/((a^6 + 3*a^4*b^2 + 3*a^2*b^4
 + b^6)*d^2))*log(-(2*(A^3*B + A*B^3)*a^3 - 3*(A^4 - B^4)*a^2*b - 6*(A^3*B
+ A*B^3)*a*b^2 + (A^4 - B^4)*b^3)*sqrt((b*cos(2*d*x + 2*c) + a*sin(2*d*x +
2*c) + b)/sin(2*d*x + 2*c)) - ((A*a^8 + 2*B*a^7*b + 2*A*a^6*b^2 + 6*B*a^5*b
^3 + 6*B*a^3*b^5 - 2*A*a^2*b^6 + 2*B*a*b^7 - A*b^8)*d^3*sqrt(-(4*A^2*B^2*a^
6 - 12*(A^3*B - A*B^3)*a^5*b + 3*(3*A^4 - 14*A^2*B^2 + 3*B^4)*a^4*b^2 + 40*
(A^3*B - A*B^3)*a^3*b^3 - 6*(A^4 - 8*A^2*B^2 + B^4)*a^2*b^4 - 12*(A^3*B - A
*B^3)*a*b^5 + (A^4 - 2*A^2*B^2 + B^4)*b^6)/((a^12 + 6*a^10*b^2 + 15*a^8*b^4
 + 20*a^6*b^6 + 15*a^4*b^8 + 6*a^2*b^10 + b^12)*d^4)) + (2*A*B^2*a^5 - (7*A
^2*B - 3*B^3)*a^4*b + 2*(3*A^3 - 7*A*B^2)*a^3*b^2 + 4*(4*A^2*B - B^3)*a^2*b
^3 - 2*(A^3 - 4*A*B^2)*a*b^4 - (A^2*B - B^3)*b^5)*d)*sqrt(-(6*A*B*a^2*b - 2
*A*B*b^3 + (A^2 - B^2)*a^3 - 3*(A^2 - B^2)*a*b^2 + (a^6 + 3*a^4*b^2 + 3*a^2
*b^4 + b^6)*d^2*sqrt(-(4*A^2*B^2*a^6 - 12*(A^3*B - A*B^3)*a^5*b + 3*(3*A^4
- 14*A^2*B^2 + 3*B^4)*a^4*b^2 + 40*(A^3*B - A*B^3)*a^3*b^3 - 6*(A^4 - 8*A^2
*B^2 + B^4)*a^2*b^4 - 12*(A^3*B - A*B^3)*a*b^5 + (A^4 - 2*A^2*B^2 + B^4)*b^
6)/((a^12 + 6*a^10*b^2 + 15*a^8*b^4 + 20*a^6*b^6 + 15*a^4*b^8 + 6*a^2*b^10
+ b^12)*d^4)))/((a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6)*d^2))) - ((a^2*b + b^3)
*d*cos(2*d*x + 2*c) + (a^3 + a*b^2)*d*sin(2*d*x + 2*c) + (a^2*b + b^3)*d)*s
qrt(-(6*A*B*a^2*b - 2*A*B*b^3 + (A^2 - B^2)*a^3 - 3*(A^2 - B^2)*a*b^2 - (a^
6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6)*d^2*sqrt(-(4*A^2*B^2*a^6 - 12*(A^3*B - A*B
^3)*a^5*b + 3*(3*A^4 - 14*A^2*B^2 + 3*B^4)*a^4*b^2 + 40*(A^3*B - A*B^3)*a^3
*b^3 - 6*(A^4 - 8*A^2*B^2 + B^4)*a^2*b^4 - 12*(A^3*B - A*B^3)*a*b^5 + (A^4
- 2*A^2*B^2 + B^4)*b^6)/((a^12 + 6*a^10*b^2 + 15*a^8*b^4 + 20*a^6*b^6 + 15*
a^4*b^8 + 6*a^2*b^10 + b^12)*d^4)))/((a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6)*d^
2))*log(-(2*(A^3*B + A*B^3)*a^3 - 3*(A^4 - B^4)*a^2*b - 6*(A^3*B + A*B^3)*a
*b^2 + (A^4 - B^4)*b^3)*sqrt((b*cos(2*d*x + 2*c) + a*sin(2*d*x + 2*c) + b)/
sin(2*d*x + 2*c)) + ((A*a^8 + 2*B*a^7*b + 2*A*a^6*b^2 + 6*B*a^5*b^3 + 6*B*a
^3*b^5 - 2*A*a^2*b^6 + 2*B*a*b^7 - A*b^8)*d^3*sqrt(-(4*A^2*B^2*a^6 - 12*(A^
3*B - A*B^3)*a^5*b + 3*(3*A^4 - 14*A^2*B^2 + 3*B^4)*a^4*b^2 + 40*(A^3*B - A
*B^3)*a^3*b^3 - 6*(A^4 - 8*A^2*B^2 + B^4)*a^2*b^4 - 12*(A^3*B - A*B^3)*a*b^
5 + (A^4 - 2*A^2*B^2 + B^4)*b^6)/((a^12 + 6*a^10*b^2 + 15*a^8*b^4 + 20*a^6*
b^6 + 15*a^4*b^8 + 6*a^2*b^10 + b^12)*d^4)) - (2*A*B^2*a^5 - (7*A^2*B - 3*B
^3)*a^4*b + 2*(3*A^3 - 7*A*B^2)*a^3*b^2 + 4*(4*A^2*B - B^3)*a^2*b^3 - 2*(A^
3 - 4*A*B^2)*a*b^4 - (A^2*B - B^3)*b^5)*d)*sqrt(-(6*A*B*a^2*b - 2*A*B*b^3 +
 (A^2 - B^2)*a^3 - 3*(A^2 - B^2)*a*b^2 - (a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6
)*d^2*sqrt(-(4*A^2*B^2*a^6 - 12*(A^3*B - A*B^3)*a^5*b + 3*(3*A^4 - 14*A^2*B
^2 + 3*B^4)*a^4*b^2 + 40*(A^3*B - A*B^3)*a^3*b^3 - 6*(A^4 - 8*A^2*B^2 + B^4
)*a^2*b^4 - 12*(A^3*B - A*B^3)*a*b^5 + (A^4 - 2*A^2*B^2 + B^4)*b^6)/((a^12
+ 6*a^10*b^2 + 15*a^8*b^4 + 20*a^6*b^6 + 15*a^4*b^8 + 6*a^2*b^10 + b^12)*d^
4)))/((a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6)*d^2))) + ((a^2*b + b^3)*d*cos(2*d
*x + 2*c) + (a^3 + a*b^2)*d*sin(2*d*x + 2*c) + (a^2*b + b^3)*d)*sqrt(-(6*A*
B*a^2*b - 2*A*B*b^3 + (A^2 - B^2)*a^3 - 3*(A^2 - B^2)*a*b^2 - (a^6 + 3*a^4*
b^2 + 3*a^2*b^4 + b^6)*d^2*sqrt(-(4*A^2*B^2*a^6 - 12*(A^3*B - A*B^3)*a^5*b
+ 3*(3*A^4 - 14*A^2*B^2 + 3*B^4)*a^4*b^2 + 40*(A^3*B - A*B^3)*a^3*b^3 - 6*(
```

```
A^4 - 8*A^2*B^2 + B^4)*a^2*b^4 - 12*(A^3*B - A*B^3)*a*b^5 + (A^4 - 2*A^2*B^
2 + B^4)*b^6)/((a^12 + 6*a^10*b^2 + 15*a^8*b^4 + 20*a^6*b^6 + 15*a^4*b^8 +
6*a^2*b^10 + b^12)*d^4)))/((a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6)*d^2))*log(-(
2*(A^3*B + A*B^3)*a^3 - 3*(A^4 - B^4)*a^2*b - 6*(A^3*B + A*B^3)*a*b^2 + (A^
4 - B^4)*b^3)*sqrt((b*cos(2*d*x + 2*c) + a*sin(2*d*x + 2*c) + b)/sin(2*d*x
+ 2*c)) - ((A*a^8 + 2*B*a^7*b + 2*A*a^6*b^2 + 6*B*a^5*b^3 + 6*B*a^3*b^5 - 2
*A*a^2*b^6 + 2*B*a*b^7 - A*b^8)*d^3*sqrt(-(4*A^2*B^2*a^6 - 12*(A^3*B - A*B^
3)*a^5*b + 3*(3*A^4 - 14*A^2*B^2 + 3*B^4)*a^4*b^2 + 40*(A^3*B - A*B^3)*a^3*
b^3 - 6*(A^4 - 8*A^2*B^2 + B^4)*a^2*b^4 - 12*(A^3*B - A*B^3)*a*b^5 + (A^4 -
 2*A^2*B^2 + B^4)*b^6)/((a^12 + 6*a^10*b^2 + 15*a^8*b^4 + 20*a^6*b^6 + 15*a
^4*b^8 + 6*a^2*b^10 + b^12)*d^4)) - (2*A*B^2*a^5 - (7*A^2*B - 3*B^3)*a^4*b
+ 2*(3*A^3 - 7*A*B^2)*a^3*b^2 + 4*(4*A^2*B - B^3)*a^2*b^3 - 2*(A^3 - 4*A*B^
2)*a*b^4 - (A^2*B - B^3)*b^5)*d)*sqrt(-(6*A*B*a^2*b - 2*A*B*b^3 + (A^2 - B^
2)*a^3 - 3*(A^2 - B^2)*a*b^2 - (a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6)*d^2*sqrt
(-(4*A^2*B^2*a^6 - 12*(A^3*B - A*B^3)*a^5*b + 3*(3*A^4 - 14*A^2*B^2 + 3*B^4
)*a^4*b^2 + 40*(A^3*B - A*B^3)*a^3*b^3 - 6*(A^4 - 8*A^2*B^2 + B^4)*a^2*b^4
- 12*(A^3*B - A*B^3)*a*b^5 + (A^4 - 2*A^2*B^2 + B^4)*b^6)/((a^12 + 6*a^10*b
^2 + 15*a^8*b^4 + 20*a^6*b^6 + 15*a^4*b^8 + 6*a^2*b^10 + b^12)*d^4)))/((a^6
 + 3*a^4*b^2 + 3*a^2*b^4 + b^6)*d^2))) + 4*(B*a - A*b)*sqrt((b*cos(2*d*x +
2*c) + a*sin(2*d*x + 2*c) + b)/sin(2*d*x + 2*c))*sin(2*d*x + 2*c))/((a^2*b
+ b^3)*d*cos(2*d*x + 2*c) + (a^3 + a*b^2)*d*sin(2*d*x + 2*c) + (a^2*b + b^3
)*d)
```

Sympy [F]

$$\int \frac{A+B\cot(c+dx)}{(a+b\cot(c+dx))^{3/2}}\,dx = \int \frac{A+B\cot\left(c+dx\right)}{\left(a+b\cot\left(c+dx\right)\right)^{\frac{3}{2}}}\,dx$$

```
[In] integrate((A+B*cot(d*x+c))/(a+b*cot(d*x+c))**(3/2),x)
[Out] Integral((A + B*cot(c + d*x))/(a + b*cot(c + d*x))**(3/2), x)
```

Maxima [F]

$$\int \frac{A+B\cot(c+dx)}{(a+b\cot(c+dx))^{3/2}}\,dx = \int \frac{B\cot\left(dx+c\right)+A}{\left(b\cot\left(dx+c\right)+a\right)^{\frac{3}{2}}}\,dx$$

```
[In] integrate((A+B*cot(d*x+c))/(a+b*cot(d*x+c))^(3/2),x, algorithm="maxima")
[Out] integrate((B*cot(d*x + c) + A)/(b*cot(d*x + c) + a)^(3/2), x)
```

Giac [F]

$$\int \frac{A+B\cot(c+dx)}{(a+b\cot(c+dx))^{3/2}}\,dx=\int \frac{B\cot(dx+c)+A}{(b\cot(dx+c)+a)^{\frac{3}{2}}}\,dx$$

```
[In] integrate((A+B*cot(d*x+c))/(a+b*cot(d*x+c))^(3/2),x, algorithm="giac")
[Out] integrate((B*cot(d*x + c) + A)/(b*cot(d*x + c) + a)^(3/2), x)
```

Mupad [B] (verification not implemented)

Time = 19.29 (sec) , antiderivative size = 5737, normalized size of antiderivative = 41.57

$$\int \frac{A+B\cot(c+dx)}{(a+b\cot(c+dx))^{3/2}}\,dx=\text{Too large to display}$$

```
[In] int((A + B*cot(c + d*x))/(a + b*cot(c + d*x))^(3/2),x)
[Out] (log((((a + b*cot(c + d*x))^(1/2)*(16*A^2*b^10*d^3 + 32*A^2*a^2*b^8*d^3 - 3
2*A^2*a^6*b^4*d^3 - 16*A^2*a^8*b^2*d^3) + ((((96*A^4*a^2*b^4*d^4 - 16*A^4*b
^6*d^4 - 144*A^4*a^4*b^2*d^4)^(1/2) - 4*A^2*a^3*d^2 + 12*A^2*a*b^2*d^2)/(a^
6*d^4 + b^6*d^4 + 3*a^2*b^4*d^4 + 3*a^4*b^2*d^4))^(1/2)*(64*A*a*b^11*d^4 -
((((96*A^4*a^2*b^4*d^4 - 16*A^4*b^6*d^4 - 144*A^4*a^4*b^2*d^4)^(1/2) - 4*A^
2*a^3*d^2 + 12*A^2*a*b^2*d^2)/(a^6*d^4 + b^6*d^4 + 3*a^2*b^4*d^4 + 3*a^4*b^
2*d^4))^(1/2)*(a + b*cot(c + d*x))^(1/2)*(64*a*b^12*d^5 + 320*a^3*b^10*d^5
+ 640*a^5*b^8*d^5 + 640*a^7*b^6*d^5 + 320*a^9*b^4*d^5 + 64*a^11*b^2*d^5))/4
 + 256*A*a^3*b^9*d^4 + 384*A*a^5*b^7*d^4 + 256*A*a^7*b^5*d^4 + 64*A*a^9*b^3
*d^4))/4)*(((96*A^4*a^2*b^4*d^4 - 16*A^4*b^6*d^4 - 144*A^4*a^4*b^2*d^4)^(1/
2) - 4*A^2*a^3*d^2 + 12*A^2*a*b^2*d^2)/(a^6*d^4 + b^6*d^4 + 3*a^2*b^4*d^4 +
 3*a^4*b^2*d^4))^(1/2))/4 - 8*A^3*b^9*d^2 - 24*A^3*a^2*b^7*d^2 - 24*A^3*a^4
*b^5*d^2 - 8*A^3*a^6*b^3*d^2)*(((96*A^4*a^2*b^4*d^4 - 16*A^4*b^6*d^4 - 144*
A^4*a^4*b^2*d^4)^(1/2) - 4*A^2*a^3*d^2 + 12*A^2*a*b^2*d^2)/(a^6*d^4 + b^6*d
^4 + 3*a^2*b^4*d^4 + 3*a^4*b^2*d^4))^(1/2))/4 + (log((((a + b*cot(c + d*x))
^(1/2)*(16*A^2*b^10*d^3 + 32*A^2*a^2*b^8*d^3 - 32*A^2*a^6*b^4*d^3 - 16*A^2*
a^8*b^2*d^3) + ((-((96*A^4*a^2*b^4*d^4 - 16*A^4*b^6*d^4 - 144*A^4*a^4*b^2*d
^4)^(1/2) + 4*A^2*a^3*d^2 - 12*A^2*a*b^2*d^2)/(a^6*d^4 + b^6*d^4 + 3*a^2*b^
4*d^4 + 3*a^4*b^2*d^4))^(1/2)*(64*A*a*b^11*d^4 - ((-((96*A^4*a^2*b^4*d^4 -
16*A^4*b^6*d^4 - 144*A^4*a^4*b^2*d^4)^(1/2) + 4*A^2*a^3*d^2 - 12*A^2*a*b^2*
d^2)/(a^6*d^4 + b^6*d^4 + 3*a^2*b^4*d^4 + 3*a^4*b^2*d^4))^(1/2)*(a + b*cot(
c + d*x))^(1/2)*(64*a*b^12*d^5 + 320*a^3*b^10*d^5 + 640*a^5*b^8*d^5 + 640*a
^7*b^6*d^5 + 320*a^9*b^4*d^5 + 64*a^11*b^2*d^5))/4 + 256*A*a^3*b^9*d^4 + 38
4*A*a^5*b^7*d^4 + 256*A*a^7*b^5*d^4 + 64*A*a^9*b^3*d^4))/4)*(-((96*A^4*a^2*
b^4*d^4 - 16*A^4*b^6*d^4 - 144*A^4*a^4*b^2*d^4)^(1/2) + 4*A^2*a^3*d^2 - 12*
A^2*a*b^2*d^2)/(a^6*d^4 + b^6*d^4 + 3*a^2*b^4*d^4 + 3*a^4*b^2*d^4))^(1/2))/
```

4 - 8*A^3*b^9*d^2 - 24*A^3*a^2*b^7*d^2 - 24*A^3*a^4*b^5*d^2 - 8*A^3*a^6*b^3
d^2)(-((96*A^4*a^2*b^4*d^4 - 16*A^4*b^6*d^4 - 144*A^4*a^4*b^2*d^4)^(1/2)
+ 4*A^2*a^3*d^2 - 12*A^2*a*b^2*d^2)/(a^6*d^4 + b^6*d^4 + 3*a^2*b^4*d^4 + 3*
a^4*b^2*d^4))^(1/2))/4 - log(- ((a + b*cot(c + d*x))^(1/2)*(16*A^2*b^10*d^3
 + 32*A^2*a^2*b^8*d^3 - 32*A^2*a^6*b^4*d^3 - 16*A^2*a^8*b^2*d^3) - (((96*A^
4*a^2*b^4*d^4 - 16*A^4*b^6*d^4 - 144*A^4*a^4*b^2*d^4)^(1/2) - 4*A^2*a^3*d^2
 + 12*A^2*a*b^2*d^2)/(16*a^6*d^4 + 16*b^6*d^4 + 48*a^2*b^4*d^4 + 48*a^4*b^2
d^4))^(1/2)((((96*A^4*a^2*b^4*d^4 - 16*A^4*b^6*d^4 - 144*A^4*a^4*b^2*d^4)
^(1/2) - 4*A^2*a^3*d^2 + 12*A^2*a*b^2*d^2)/(16*a^6*d^4 + 16*b^6*d^4 + 48*a^
2*b^4*d^4 + 48*a^4*b^2*d^4))^(1/2)*(a + b*cot(c + d*x))^(1/2)*(64*a*b^12*d^
5 + 320*a^3*b^10*d^5 + 640*a^5*b^8*d^5 + 640*a^7*b^6*d^5 + 320*a^9*b^4*d^5
+ 64*a^11*b^2*d^5) + 64*A*a*b^11*d^4 + 256*A*a^3*b^9*d^4 + 384*A*a^5*b^7*d^
4 + 256*A*a^7*b^5*d^4 + 64*A*a^9*b^3*d^4))*(((96*A^4*a^2*b^4*d^4 - 16*A^4*b
^6*d^4 - 144*A^4*a^4*b^2*d^4)^(1/2) - 4*A^2*a^3*d^2 + 12*A^2*a*b^2*d^2)/(16
*a^6*d^4 + 16*b^6*d^4 + 48*a^2*b^4*d^4 + 48*a^4*b^2*d^4))^(1/2) - 8*A^3*b^9
*d^2 - 24*A^3*a^2*b^7*d^2 - 24*A^3*a^4*b^5*d^2 - 8*A^3*a^6*b^3*d^2)*(((96*A
^4*a^2*b^4*d^4 - 16*A^4*b^6*d^4 - 144*A^4*a^4*b^2*d^4)^(1/2) - 4*A^2*a^3*d^
2 + 12*A^2*a*b^2*d^2)/(16*a^6*d^4 + 16*b^6*d^4 + 48*a^2*b^4*d^4 + 48*a^4*b^
2*d^4))^(1/2) - log(- ((a + b*cot(c + d*x))^(1/2)*(16*A^2*b^10*d^3 + 32*A^2
*a^2*b^8*d^3 - 32*A^2*a^6*b^4*d^3 - 16*A^2*a^8*b^2*d^3) - (-((96*A^4*a^2*b^
4*d^4 - 16*A^4*b^6*d^4 - 144*A^4*a^4*b^2*d^4)^(1/2) + 4*A^2*a^3*d^2 - 12*A^
2*a*b^2*d^2)/(16*a^6*d^4 + 16*b^6*d^4 + 48*a^2*b^4*d^4 + 48*a^4*b^2*d^4))^(
1/2)*((-((96*A^4*a^2*b^4*d^4 - 16*A^4*b^6*d^4 - 144*A^4*a^4*b^2*d^4)^(1/2)
+ 4*A^2*a^3*d^2 - 12*A^2*a*b^2*d^2)/(16*a^6*d^4 + 16*b^6*d^4 + 48*a^2*b^4*d
^4 + 48*a^4*b^2*d^4))^(1/2)*(a + b*cot(c + d*x))^(1/2)*(64*a*b^12*d^5 + 320
*a^3*b^10*d^5 + 640*a^5*b^8*d^5 + 640*a^7*b^6*d^5 + 320*a^9*b^4*d^5 + 64*a^
11*b^2*d^5) + 64*A*a*b^11*d^4 + 256*A*a^3*b^9*d^4 + 384*A*a^5*b^7*d^4 + 256
*A*a^7*b^5*d^4 + 64*A*a^9*b^3*d^4))*(-((96*A^4*a^2*b^4*d^4 - 16*A^4*b^6*d^4
 - 144*A^4*a^4*b^2*d^4)^(1/2) + 4*A^2*a^3*d^2 - 12*A^2*a*b^2*d^2)/(16*a^6*d
^4 + 16*b^6*d^4 + 48*a^2*b^4*d^4 + 48*a^4*b^2*d^4))^(1/2) - 8*A^3*b^9*d^2 -
 24*A^3*a^2*b^7*d^2 - 24*A^3*a^4*b^5*d^2 - 8*A^3*a^6*b^3*d^2)*(-((96*A^4*a^
2*b^4*d^4 - 16*A^4*b^6*d^4 - 144*A^4*a^4*b^2*d^4)^(1/2) + 4*A^2*a^3*d^2 - 1
2*A^2*a*b^2*d^2)/(16*a^6*d^4 + 16*b^6*d^4 + 48*a^2*b^4*d^4 + 48*a^4*b^2*d^4
))^(1/2) + (log(24*B^3*a^3*b^6*d^2 - ((((96*B^4*a^2*b^4*d^4 - 16*B^4*b^6*d^
4 - 144*B^4*a^4*b^2*d^4)^(1/2) + 4*B^2*a^3*d^2 - 12*B^2*a*b^2*d^2)/(a^6*d^4
 + b^6*d^4 + 3*a^2*b^4*d^4 + 3*a^4*b^2*d^4))^(1/2)*(((((96*B^4*a^2*b^4*d^4
- 16*B^4*b^6*d^4 - 144*B^4*a^4*b^2*d^4)^(1/2) + 4*B^2*a^3*d^2 - 12*B^2*a*b^
2*d^2)/(a^6*d^4 + b^6*d^4 + 3*a^2*b^4*d^4 + 3*a^4*b^2*d^4))^(1/2)*(((((96*B
^4*a^2*b^4*d^4 - 16*B^4*b^6*d^4 - 144*B^4*a^4*b^2*d^4)^(1/2) + 4*B^2*a^3*d^
2 - 12*B^2*a*b^2*d^2)/(a^6*d^4 + b^6*d^4 + 3*a^2*b^4*d^4 + 3*a^4*b^2*d^4))^
(1/2)*(a + b*cot(c + d*x))^(1/2)*(64*a*b^12*d^5 + 320*a^3*b^10*d^5 + 640*a^
5*b^8*d^5 + 640*a^7*b^6*d^5 + 320*a^9*b^4*d^5 + 64*a^11*b^2*d^5))/4 - 32*B*
b^12*d^4 - 96*B*a^2*b^10*d^4 - 64*B*a^4*b^8*d^4 + 64*B*a^6*b^6*d^4 + 96*B*a
^8*b^4*d^4 + 32*B*a^10*b^2*d^4))/4 + (a + b*cot(c + d*x))^(1/2)*(16*B^2*b^1
0*d^3 + 32*B^2*a^2*b^8*d^3 - 32*B^2*a^6*b^4*d^3 - 16*B^2*a^8*b^2*d^3)))/4 +

24*B^3*a^5*b^4*d^2 + 8*B^3*a^7*b^2*d^2 + 8*B^3*a*b^8*d^2)*(((96*B^4*a^2*b^
4*d^4 - 16*B^4*b^6*d^4 - 144*B^4*a^4*b^2*d^4)^(1/2) + 4*B^2*a^3*d^2 - 12*B^
2*a*b^2*d^2)/(a^6*d^4 + b^6*d^4 + 3*a^2*b^4*d^4 + 3*a^4*b^2*d^4))^(1/2))/4
+ (log(24*B^3*a^3*b^6*d^2 - ((-((96*B^4*a^2*b^4*d^4 - 16*B^4*b^6*d^4 - 144*
B^4*a^4*b^2*d^4)^(1/2) - 4*B^2*a^3*d^2 + 12*B^2*a*b^2*d^2)/(a^6*d^4 + b^6*d
^4 + 3*a^2*b^4*d^4 + 3*a^4*b^2*d^4))^(1/2)*(((-((96*B^4*a^2*b^4*d^4 - 16*B^
4*b^6*d^4 - 144*B^4*a^4*b^2*d^4)^(1/2) - 4*B^2*a^3*d^2 + 12*B^2*a*b^2*d^2)/
(a^6*d^4 + b^6*d^4 + 3*a^2*b^4*d^4 + 3*a^4*b^2*d^4))^(1/2)*(((-((96*B^4*a^2
*b^4*d^4 - 16*B^4*b^6*d^4 - 144*B^4*a^4*b^2*d^4)^(1/2) - 4*B^2*a^3*d^2 + 12
*B^2*a*b^2*d^2)/(a^6*d^4 + b^6*d^4 + 3*a^2*b^4*d^4 + 3*a^4*b^2*d^4))^(1/2)*
(a + b*cot(c + d*x))^(1/2)*(64*a*b^12*d^5 + 320*a^3*b^10*d^5 + 640*a^5*b^8*
d^5 + 640*a^7*b^6*d^5 + 320*a^9*b^4*d^5 + 64*a^11*b^2*d^5))/4 - 32*B*b^12*d
^4 - 96*B*a^2*b^10*d^4 - 64*B*a^4*b^8*d^4 + 64*B*a^6*b^6*d^4 + 96*B*a^8*b^4
*d^4 + 32*B*a^10*b^2*d^4))/4 + (a + b*cot(c + d*x))^(1/2)*(16*B^2*b^10*d^3
+ 32*B^2*a^2*b^8*d^3 - 32*B^2*a^6*b^4*d^3 - 16*B^2*a^8*b^2*d^3)))/4 + 24*B^
3*a^5*b^4*d^2 + 8*B^3*a^7*b^2*d^2 + 8*B^3*a*b^8*d^2)*(-((96*B^4*a^2*b^4*d^4
 - 16*B^4*b^6*d^4 - 144*B^4*a^4*b^2*d^4)^(1/2) - 4*B^2*a^3*d^2 + 12*B^2*a*b
^2*d^2)/(a^6*d^4 + b^6*d^4 + 3*a^2*b^4*d^4 + 3*a^4*b^2*d^4))^(1/2))/4 - log
((((96*B^4*a^2*b^4*d^4 - 16*B^4*b^6*d^4 - 144*B^4*a^4*b^2*d^4)^(1/2) + 4*B^
2*a^3*d^2 - 12*B^2*a*b^2*d^2)/(16*a^6*d^4 + 16*b^6*d^4 + 48*a^2*b^4*d^4 + 4
8*a^4*b^2*d^4))^(1/2)*((((96*B^4*a^2*b^4*d^4 - 16*B^4*b^6*d^4 - 144*B^4*a^4
*b^2*d^4)^(1/2) + 4*B^2*a^3*d^2 - 12*B^2*a*b^2*d^2)/(16*a^6*d^4 + 16*b^6*d^
4 + 48*a^2*b^4*d^4 + 48*a^4*b^2*d^4))^(1/2)*(32*B*b^12*d^4 + (((96*B^4*a^2*
b^4*d^4 - 16*B^4*b^6*d^4 - 144*B^4*a^4*b^2*d^4)^(1/2) + 4*B^2*a^3*d^2 - 12*
B^2*a*b^2*d^2)/(16*a^6*d^4 + 16*b^6*d^4 + 48*a^2*b^4*d^4 + 48*a^4*b^2*d^4))
^(1/2)*(a + b*cot(c + d*x))^(1/2)*(64*a*b^12*d^5 + 320*a^3*b^10*d^5 + 640*a
^5*b^8*d^5 + 640*a^7*b^6*d^5 + 320*a^9*b^4*d^5 + 64*a^11*b^2*d^5) + 96*B*a^
2*b^10*d^4 + 64*B*a^4*b^8*d^4 - 64*B*a^6*b^6*d^4 - 96*B*a^8*b^4*d^4 - 32*B*
a^10*b^2*d^4) + (a + b*cot(c + d*x))^(1/2)*(16*B^2*b^10*d^3 + 32*B^2*a^2*b^
8*d^3 - 32*B^2*a^6*b^4*d^3 - 16*B^2*a^8*b^2*d^3)) + 24*B^3*a^3*b^6*d^2 + 24
*B^3*a^5*b^4*d^2 + 8*B^3*a^7*b^2*d^2 + 8*B^3*a*b^8*d^2)*(((96*B^4*a^2*b^4*d
^4 - 16*B^4*b^6*d^4 - 144*B^4*a^4*b^2*d^4)^(1/2) + 4*B^2*a^3*d^2 - 12*B^2*a
*b^2*d^2)/(16*a^6*d^4 + 16*b^6*d^4 + 48*a^2*b^4*d^4 + 48*a^4*b^2*d^4))^(1/2
) - log((-((96*B^4*a^2*b^4*d^4 - 16*B^4*b^6*d^4 - 144*B^4*a^4*b^2*d^4)^(1/2
) - 4*B^2*a^3*d^2 + 12*B^2*a*b^2*d^2)/(16*a^6*d^4 + 16*b^6*d^4 + 48*a^2*b^4
*d^4 + 48*a^4*b^2*d^4))^(1/2)*((-((96*B^4*a^2*b^4*d^4 - 16*B^4*b^6*d^4 - 14
4*B^4*a^4*b^2*d^4)^(1/2) - 4*B^2*a^3*d^2 + 12*B^2*a*b^2*d^2)/(16*a^6*d^4 +
16*b^6*d^4 + 48*a^2*b^4*d^4 + 48*a^4*b^2*d^4))^(1/2)*(32*B*b^12*d^4 + (-((9
6*B^4*a^2*b^4*d^4 - 16*B^4*b^6*d^4 - 144*B^4*a^4*b^2*d^4)^(1/2) - 4*B^2*a^3
*d^2 + 12*B^2*a*b^2*d^2)/(16*a^6*d^4 + 16*b^6*d^4 + 48*a^2*b^4*d^4 + 48*a^4
*b^2*d^4))^(1/2)*(a + b*cot(c + d*x))^(1/2)*(64*a*b^12*d^5 + 320*a^3*b^10*d
^5 + 640*a^5*b^8*d^5 + 640*a^7*b^6*d^5 + 320*a^9*b^4*d^5 + 64*a^11*b^2*d^5)
 + 96*B*a^2*b^10*d^4 + 64*B*a^4*b^8*d^4 - 64*B*a^6*b^6*d^4 - 96*B*a^8*b^4*d
^4 - 32*B*a^10*b^2*d^4) + (a + b*cot(c + d*x))^(1/2)*(16*B^2*b^10*d^3 + 32*
B^2*a^2*b^8*d^3 - 32*B^2*a^6*b^4*d^3 - 16*B^2*a^8*b^2*d^3)) + 24*B^3*a^3*b^

```
6*d^2 + 24*B^3*a^5*b^4*d^2 + 8*B^3*a^7*b^2*d^2 + 8*B^3*a*b^8*d^2)*(-((96*B^
4*a^2*b^4*d^4 - 16*B^4*b^6*d^4 - 144*B^4*a^4*b^2*d^4)^(1/2) - 4*B^2*a^3*d^2
 + 12*B^2*a*b^2*d^2)/(16*a^6*d^4 + 16*b^6*d^4 + 48*a^2*b^4*d^4 + 48*a^4*b^2
*d^4))^(1/2) + (2*A*b)/(d*(a^2 + b^2)*(a + b*cot(c + d*x))^(1/2)) - (2*B*a)
/(d*(a^2 + b^2)*(a + b*cot(c + d*x))^(1/2))
```

3.103 $\int \frac{A+B\cot(c+dx)}{(a+b\cot(c+dx))^{5/2}}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 185

$$\int \frac{A+B\cot(c+dx)}{(a+b\cot(c+dx))^{5/2}}\,dx=\frac{(iA+B)\operatorname{arctanh}\left(\frac{\sqrt{a+b\cot(c+dx)}}{\sqrt{a-ib}}\right)}{(a-ib)^{5/2}d}-\frac{(iA-B)\operatorname{arctanh}\left(\frac{\sqrt{a+b\cot(c+dx)}}{\sqrt{a+ib}}\right)}{(a+ib)^{5/2}d}+\frac{2(Ab-aB)}{3\left(a^2+b^2\right)d(a+b\cot(c+dx))^{3/2}}+\frac{2(2aAb-a^2B+b^2B)}{\left(a^2+b^2\right)^2d\sqrt{a+b\cot(c+dx)}}$$

```
[Out] (I*A+B)*arctanh((a+b*cot(d*x+c))^(1/2)/(a-I*b)^(1/2))/(a-I*b)^(5/2)/d-(I*A-
B)*arctanh((a+b*cot(d*x+c))^(1/2)/(a+I*b)^(1/2))/(a+I*b)^(5/2)/d+2/3*(A*b-B
*a)/(a^2+b^2)/d/(a+b*cot(d*x+c))^(3/2)+2*(2*A*a*b-B*a^2+B*b^2)/(a^2+b^2)^2/
d/(a+b*cot(d*x+c))^(1/2)
```

Rubi [A] (verified)

Time = 0.50 (sec) , antiderivative size = 185, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {3610, 3620, 3618, 65, 214}

$$\int \frac{A+B\cot(c+dx)}{(a+b\cot(c+dx))^{5/2}}\,dx=\frac{2(Ab-aB)}{3d\left(a^2+b^2\right)(a+b\cot(c+dx))^{3/2}}+\frac{2(a^2(-B)+2aAb+b^2B)}{d\left(a^2+b^2\right)^2\sqrt{a+b\cot(c+dx)}}+\frac{(B+iA)\operatorname{arctanh}\left(\frac{\sqrt{a+b\cot(c+dx)}}{\sqrt{a-ib}}\right)}{d(a-ib)^{5/2}}-\frac{(-B+iA)\operatorname{arctanh}\left(\frac{\sqrt{a+b\cot(c+dx)}}{\sqrt{a+ib}}\right)}{d(a+ib)^{5/2}}$$

```
[In] Int[(A + B*Cot[c + d*x])/(a + b*Cot[c + d*x])^(5/2),x]
```

```
[Out] ((I*A + B)*ArcTanh[Sqrt[a + b*Cot[c + d*x]]/Sqrt[a - I*b]])/((a - I*b)^(5/2
)*d) - ((I*A - B)*ArcTanh[Sqrt[a + b*Cot[c + d*x]]/Sqrt[a + I*b]])/((a + I*
b)^(5/2)*d) + (2*(A*b - a*B))/(3*(a^2 + b^2)*d*(a + b*Cot[c + d*x])^(3/2))
+ (2*(2*a*A*b - a^2*B + b^2*B))/((a^2 + b^2)^2*d*Sqrt[a + b*Cot[c + d*x]])
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 3610

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)]), x_Symbol] :> Simp[(b*c - a*d)*((a + b*Tan[e + f*x])^(m + 1)/
(f*(m + 1)*(a^2 + b^2))), x] + Dist[1/(a^2 + b^2), Int[(a + b*Tan[e + f*x])
^(m + 1)*Simp[a*c + b*d - (b*c - a*d)*Tan[e + f*x], x], x], x] /; FreeQ[{a,
 b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && LtQ[m, -1]
```

Rule 3618

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*tan[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Dist[c*(d/f), Subst[Int[(a + (b/d)*x)^m/(d^2 + c
*x), x], x, d*Tan[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b
*c - a*d, 0] && NeQ[a^2 + b^2, 0] && EqQ[c^2 + d^2, 0]
```

Rule 3620

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)]), x_Symbol] :> Dist[(c + I*d)/2, Int[(a + b*Tan[e + f*x])^m*(1
 - I*Tan[e + f*x]), x], x] + Dist[(c - I*d)/2, Int[(a + b*Tan[e + f*x])^m*(
1 + I*Tan[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b*c -
 a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] &&  !IntegerQ[m]
```

Rubi steps

$$\text{integral} = \frac{2(Ab-aB)}{3\left(a^2+b^2\right)d(a+b\cot(c+dx))^{3/2}}+\frac{\int \frac{aA+bB-(Ab-aB)\cot(c+dx)}{(a+b\cot(c+dx))^{3/2}}\,dx}{a^2+b^2}$$

$$= \frac{2(Ab-aB)}{3\left(a^2+b^2\right)d(a+b\cot(c+dx))^{3/2}} + \frac{2(2aAb-a^2B+b^2B)}{\left(a^2+b^2\right)^2 d\sqrt{a+b\cot(c+dx)}} + \frac{\int \frac{a^2A-Ab^2+2abB-(2aAb-a^2B+b^2B)\cot(c+dx)}{\sqrt{a+b\cot(c+dx)}}\,dx}{\left(a^2+b^2\right)^2}$$

$$= \frac{2(Ab-aB)}{3\left(a^2+b^2\right)d(a+b\cot(c+dx))^{3/2}} + \frac{2(2aAb-a^2B+b^2B)}{\left(a^2+b^2\right)^2 d\sqrt{a+b\cot(c+dx)}} + \frac{(A-iB)\int \frac{1+i\cot(c+dx)}{\sqrt{a+b\cot(c+dx)}}\,dx}{2(a-ib)^2} + \frac{(A+iB)\int \frac{1-i\cot(c+dx)}{\sqrt{a+b\cot(c+dx)}}\,dx}{2(a+ib)^2}$$

$$= \frac{2(Ab-aB)}{3\left(a^2+b^2\right)d(a+b\cot(c+dx))^{3/2}} + \frac{2(2aAb-a^2B+b^2B)}{\left(a^2+b^2\right)^2 d\sqrt{a+b\cot(c+dx)}} + \frac{(iA-B)\text{Subst}\left(\int \frac{1}{(-1+x)\sqrt{a+ibx}}\,dx, x, -i\cot(c+dx)\right)}{2(a+ib)^2d} - \frac{(iA+B)\text{Subst}\left(\int \frac{1}{(-1+x)\sqrt{a-ibx}}\,dx, x, i\cot(c+dx)\right)}{2(a-ib)^2d}$$

$$= \frac{2(Ab-aB)}{3\left(a^2+b^2\right)d(a+b\cot(c+dx))^{3/2}} + \frac{2(2aAb-a^2B+b^2B)}{\left(a^2+b^2\right)^2 d\sqrt{a+b\cot(c+dx)}} - \frac{(A-iB)\text{Subst}\left(\int \frac{1}{-1-\frac{ia}{b}+\frac{ix^2}{b}}\,dx, x, \sqrt{a+b\cot(c+dx)}\right)}{b(ia+b)^2d} - \frac{(A+iB)\text{Subst}\left(\int \frac{1}{-1+\frac{ia}{b}-\frac{ix^2}{b}}\,dx, x, \sqrt{a+b\cot(c+dx)}\right)}{(ia-b)^2bd}$$

$$= \frac{(iA+B)\text{arctanh}\left(\frac{\sqrt{a+b\cot(c+dx)}}{\sqrt{a-ib}}\right)}{(a-ib)^{5/2}d} - \frac{(iA-B)\text{arctanh}\left(\frac{\sqrt{a+b\cot(c+dx)}}{\sqrt{a+ib}}\right)}{(a+ib)^{5/2}d} + \frac{2(Ab-aB)}{3\left(a^2+b^2\right)d(a+b\cot(c+dx))^{3/2}} + \frac{2(2aAb-a^2B+b^2B)}{\left(a^2+b^2\right)^2 d\sqrt{a+b\cot(c+dx)}}$$

Mathematica [A] (verified)

Time = 3.76 (sec) , antiderivative size = 319, normalized size of antiderivative = 1.72

$$\int \frac{A+B\cot(c+dx)}{(a+b\cot(c+dx))^{5/2}}\,dx =$$

$$-\frac{\frac{3\left(2ab\left(A\sqrt{-b^2}+bB\right)+a^2\left(Ab-\sqrt{-b^2}B\right)+b^2\left(-Ab+\sqrt{-b^2}B\right)\right)\text{arctanh}\left(\frac{\sqrt{a+b\cot(c+dx)}}{\sqrt{a-\sqrt{-b^2}}}\right)}{\sqrt{-b^2}\sqrt{a-\sqrt{-b^2}}} + \frac{3\left(2ab\left(A\sqrt{-b^2}-bB\right)-a^2\left(Ab+\sqrt{-b^2}B\right)+b^2\left(\ldots\right.\right.}{\sqrt{-b^2}\sqrt{a\ldots}}}{3\left(a^2+b^2\right)^2d}$$

```
[In] Integrate[(A + B*Cot[c + d*x])/(a + b*Cot[c + d*x])^(5/2),x]
```

```
[Out] -1/3*((3*(2*a*b*(A*Sqrt[-b^2] + b*B) + a^2*(A*b - Sqrt[-b^2]*B) + b^2*(-(A*
b) + Sqrt[-b^2]*B))*ArcTanh[Sqrt[a + b*Cot[c + d*x]]/Sqrt[a - Sqrt[-b^2]]])
/(Sqrt[-b^2]*Sqrt[a - Sqrt[-b^2]]) + (3*(2*a*b*(A*Sqrt[-b^2] - b*B) - a^2*(
A*b + Sqrt[-b^2]*B) + b^2*(A*b + Sqrt[-b^2]*B))*ArcTanh[Sqrt[a + b*Cot[c +
d*x]]/Sqrt[a + Sqrt[-b^2]]])/(Sqrt[-b^2]*Sqrt[a + Sqrt[-b^2]]) + (2*(a^2 +
b^2)*(-(A*b) + a*B))/(a + b*Cot[c + d*x])^(3/2) + (6*(-2*a*A*b + a^2*B - b^
2*B))/Sqrt[a + b*Cot[c + d*x]])/((a^2 + b^2)^2*d)
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 4471 vs. $2(161) = 322$.

Time = 0.16 (sec) , antiderivative size = 4472, normalized size of antiderivative = 24.17

method	result	size
parts	Expression too large to display	4472
derivativedivides	Expression too large to display	12836
default	Expression too large to display	12836

```
[In] int((A+B*cot(d*x+c))/(a+b*cot(d*x+c))^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] A*(2/3/d*b/(a^2+b^2)/(a+b*cot(d*x+c))^(3/2)+2/d*b/(a^2+b^2)^3/(2*(a^2+b^2)^
(1/2)-2*a)^(1/2)*arctan((2*(a+b*cot(d*x+c))^(1/2)-(2*(a^2+b^2)^(1/2)+2*a)^(
1/2))/(2*(a^2+b^2)^(1/2)-2*a)^(1/2))*a^3-4/d*b/(a^2+b^2)^(7/2)/(2*(a^2+b^2)
^(1/2)-2*a)^(1/2)*arctan((2*(a+b*cot(d*x+c))^(1/2)-(2*(a^2+b^2)^(1/2)+2*a)^
(1/2))/(2*(a^2+b^2)^(1/2)-2*a)^(1/2))*a^4-4/d*b/(a^2+b^2)^(7/2)/(2*(a^2+b^2
)^(1/2)-2*a)^(1/2)*arctan((2*(a+b*cot(d*x+c))^(1/2)+(2*(a^2+b^2)^(1/2)+2*a)
^(1/2))/(2*(a^2+b^2)^(1/2)-2*a)^(1/2))*a^4+1/2/d*b/(a^2+b^2)^(7/2)*ln(b*cot
(d*x+c)+a-(a+b*cot(d*x+c))^(1/2)*(2*(a^2+b^2)^(1/2)+2*a)^(1/2)+(a^2+b^2)^(1
/2))*(2*(a^2+b^2)^(1/2)+2*a)^(1/2)*a^3-1/2/d*b/(a^2+b^2)^(7/2)*ln(b*cot(d*x
+c)+a+(a+b*cot(d*x+c))^(1/2)*(2*(a^2+b^2)^(1/2)+2*a)^(1/2)+(a^2+b^2)^(1/2))
*(2*(a^2+b^2)^(1/2)+2*a)^(1/2)*a^3-1/4/d/b/(a^2+b^2)^(7/2)*ln(b*cot(d*x+c)+
a-(a+b*cot(d*x+c))^(1/2)*(2*(a^2+b^2)^(1/2)+2*a)^(1/2)+(a^2+b^2)^(1/2))*(2*
(a^2+b^2)^(1/2)+2*a)^(1/2)*a^5+3/4/d*b^3/(a^2+b^2)^(7/2)*ln(b*cot(d*x+c)+a-
(a+b*cot(d*x+c))^(1/2)*(2*(a^2+b^2)^(1/2)+2*a)^(1/2)+(a^2+b^2)^(1/2))*(2*(a
^2+b^2)^(1/2)+2*a)^(1/2)*a+2/d*b^3/(a^2+b^2)^3/(2*(a^2+b^2)^(1/2)-2*a)^(1/2
)*arctan((2*(a+b*cot(d*x+c))^(1/2)-(2*(a^2+b^2)^(1/2)+2*a)^(1/2))/(2*(a^2+b
^2)^(1/2)-2*a)^(1/2))*a+1/4/d/b/(a^2+b^2)^3*ln(b*cot(d*x+c)+a-(a+b*cot(d*x+
c))^(1/2)*(2*(a^2+b^2)^(1/2)+2*a)^(1/2)+(a^2+b^2)^(1/2))*(2*(a^2+b^2)^(1/2)
+2*a)^(1/2)*a^4+1/d/b/(a^2+b^2)^(5/2)/(2*(a^2+b^2)^(1/2)-2*a)^(1/2)*arctan(
(2*(a+b*cot(d*x+c))^(1/2)-(2*(a^2+b^2)^(1/2)+2*a)^(1/2))/(2*(a^2+b^2)^(1/2)
-2*a)^(1/2))*a^4-1/d/b/(a^2+b^2)^(7/2)/(2*(a^2+b^2)^(1/2)-2*a)^(1/2)*arctan
((2*(a+b*cot(d*x+c))^(1/2)-(2*(a^2+b^2)^(1/2)+2*a)^(1/2))/(2*(a^2+b^2)^(1/2
)-2*a)^(1/2))*a^6+2/d*b^3/(a^2+b^2)^3/(2*(a^2+b^2)^(1/2)-2*a)^(1/2)*arctan(
```

(2*(a+b*cot(d*x+c))^(1/2)+(2*(a^2+b^2)^(1/2)+2*a)^(1/2))/(2*(a^2+b^2)^(1/2)
-2*a)^(1/2))*a-1/d/b/(a^2+b^2)^(7/2)/(2*(a^2+b^2)^(1/2)-2*a)^(1/2)*arctan((
2*(a+b*cot(d*x+c))^(1/2)+(2*(a^2+b^2)^(1/2)+2*a)^(1/2))/(2*(a^2+b^2)^(1/2)-
2*a)^(1/2))*a^6-1/d*b^3/(a^2+b^2)^(7/2)/(2*(a^2+b^2)^(1/2)-2*a)^(1/2)*arcta
n((2*(a+b*cot(d*x+c))^(1/2)+(2*(a^2+b^2)^(1/2)+2*a)^(1/2))/(2*(a^2+b^2)^(1/
2)-2*a)^(1/2))*a^2+1/4/d/b/(a^2+b^2)^(7/2)*ln(b*cot(d*x+c)+a+(a+b*cot(d*x+c
))^(1/2)*(2*(a^2+b^2)^(1/2)+2*a)^(1/2)+(a^2+b^2)^(1/2))*(2*(a^2+b^2)^(1/2)+
2*a)^(1/2)*a^5-1/d*b^3/(a^2+b^2)^(7/2)/(2*(a^2+b^2)^(1/2)-2*a)^(1/2)*arctan
((2*(a+b*cot(d*x+c))^(1/2)-(2*(a^2+b^2)^(1/2)+2*a)^(1/2))/(2*(a^2+b^2)^(1/2
)-2*a)^(1/2))*a^2-3/4/d*b^3/(a^2+b^2)^(7/2)*ln(b*cot(d*x+c)+a+(a+b*cot(d*x+
c))^(1/2)*(2*(a^2+b^2)^(1/2)+2*a)^(1/2)+(a^2+b^2)^(1/2))*(2*(a^2+b^2)^(1/2)
+2*a)^(1/2)*a+1/d/b/(a^2+b^2)^(5/2)/(2*(a^2+b^2)^(1/2)-2*a)^(1/2)*arctan((2
*(a+b*cot(d*x+c))^(1/2)+(2*(a^2+b^2)^(1/2)+2*a)^(1/2))/(2*(a^2+b^2)^(1/2)-2
*a)^(1/2))*a^4-1/4/d/b/(a^2+b^2)^3*ln(b*cot(d*x+c)+a+(a+b*cot(d*x+c))^(1/2)
(2(a^2+b^2)^(1/2)+2*a)^(1/2)+(a^2+b^2)^(1/2))*(2*(a^2+b^2)^(1/2)+2*a)^(1/
2)*a^4+2/d*b/(a^2+b^2)^3/(2*(a^2+b^2)^(1/2)-2*a)^(1/2)*arctan((2*(a+b*cot(d
x+c))^(1/2)+(2(a^2+b^2)^(1/2)+2*a)^(1/2))/(2*(a^2+b^2)^(1/2)-2*a)^(1/2))*
a^3+1/4/d*b^3/(a^2+b^2)^3*ln(b*cot(d*x+c)+a+(a+b*cot(d*x+c))^(1/2)*(2*(a^2+
b^2)^(1/2)+2*a)^(1/2)+(a^2+b^2)^(1/2))*(2*(a^2+b^2)^(1/2)+2*a)^(1/2)-1/4/d*
b^3/(a^2+b^2)^3*ln(b*cot(d*x+c)+a-(a+b*cot(d*x+c))^(1/2)*(2*(a^2+b^2)^(1/2)
+2*a)^(1/2)+(a^2+b^2)^(1/2))*(2*(a^2+b^2)^(1/2)+2*a)^(1/2)-1/d*b^3/(a^2+b^2
)^(5/2)/(2*(a^2+b^2)^(1/2)-2*a)^(1/2)*arctan((2*(a+b*cot(d*x+c))^(1/2)-(2*(
a^2+b^2)^(1/2)+2*a)^(1/2))/(2*(a^2+b^2)^(1/2)-2*a)^(1/2))+2/d*b^5/(a^2+b^2)
^(7/2)/(2*(a^2+b^2)^(1/2)-2*a)^(1/2)*arctan((2*(a+b*cot(d*x+c))^(1/2)-(2*(a
^2+b^2)^(1/2)+2*a)^(1/2))/(2*(a^2+b^2)^(1/2)-2*a)^(1/2))-1/d*b^3/(a^2+b^2)^
(5/2)/(2*(a^2+b^2)^(1/2)-2*a)^(1/2)*arctan((2*(a+b*cot(d*x+c))^(1/2)+(2*(a^
2+b^2)^(1/2)+2*a)^(1/2))/(2*(a^2+b^2)^(1/2)-2*a)^(1/2))+2/d*b^5/(a^2+b^2)^(
7/2)/(2*(a^2+b^2)^(1/2)-2*a)^(1/2)*arctan((2*(a+b*cot(d*x+c))^(1/2)+(2*(a^2
+b^2)^(1/2)+2*a)^(1/2))/(2*(a^2+b^2)^(1/2)-2*a)^(1/2))+4/d*b*a/(a^2+b^2)^2/
(a+b*cot(d*x+c))^(1/2))+B*(3/4/d/(a^2+b^2)^(7/2)*ln(b*cot(d*x+c)+a+(a+b*cot
(d*x+c))^(1/2)*(2*(a^2+b^2)^(1/2)+2*a)^(1/2)+(a^2+b^2)^(1/2))*(2*(a^2+b^2)^
(1/2)+2*a)^(1/2)*a^4-1/4/d/(a^2+b^2)^(7/2)*ln(b*cot(d*x+c)+a+(a+b*cot(d*x+c
))^(1/2)*(2*(a^2+b^2)^(1/2)+2*a)^(1/2)+(a^2+b^2)^(1/2))*(2*(a^2+b^2)^(1/2)+
2*a)^(1/2)*b^4+2/d/(a^2+b^2)^(5/2)/(2*(a^2+b^2)^(1/2)-2*a)^(1/2)*arctan((2*
(a+b*cot(d*x+c))^(1/2)+(2*(a^2+b^2)^(1/2)+2*a)^(1/2))/(2*(a^2+b^2)^(1/2)-2*
a)^(1/2))*a^3-1/d/(a^2+b^2)^(7/2)/(2*(a^2+b^2)^(1/2)-2*a)^(1/2)*arctan((2*(
a+b*cot(d*x+c))^(1/2)+(2*(a^2+b^2)^(1/2)+2*a)^(1/2))/(2*(a^2+b^2)^(1/2)-2*a
)^(1/2))*a^5-3/4/d/(a^2+b^2)^(7/2)*ln(b*cot(d*x+c)+a-(a+b*cot(d*x+c))^(1/2)
(2(a^2+b^2)^(1/2)+2*a)^(1/2)+(a^2+b^2)^(1/2))*(2*(a^2+b^2)^(1/2)+2*a)^(1/
2)*a^4+1/4/d/(a^2+b^2)^(7/2)*ln(b*cot(d*x+c)+a-(a+b*cot(d*x+c))^(1/2)*(2*(a
^2+b^2)^(1/2)+2*a)^(1/2)+(a^2+b^2)^(1/2))*(2*(a^2+b^2)^(1/2)+2*a)^(1/2)*b^4
+2/d/(a^2+b^2)^(5/2)/(2*(a^2+b^2)^(1/2)-2*a)^(1/2)*arctan((2*(a+b*cot(d*x+c
))^(1/2)-(2*(a^2+b^2)^(1/2)+2*a)^(1/2))/(2*(a^2+b^2)^(1/2)-2*a)^(1/2))*a^3-
1/d/(a^2+b^2)^(7/2)/(2*(a^2+b^2)^(1/2)-2*a)^(1/2)*arctan((2*(a+b*cot(d*x+c)
)^(1/2)-(2*(a^2+b^2)^(1/2)+2*a)^(1/2))/(2*(a^2+b^2)^(1/2)-2*a)^(1/2))*a^5-1

```
/2/d/(a^2+b^2)^3*ln(b*cot(d*x+c)+a+(a+b*cot(d*x+c))^(1/2)*(2*(a^2+b^2)^(1/2
)+2*a)^(1/2)+(a^2+b^2)^(1/2))*(2*(a^2+b^2)^(1/2)+2*a)^(1/2)*a^3-1/d/(a^2+b^
2)^3/(2*(a^2+b^2)^(1/2)-2*a)^(1/2)*arctan((2*(a+b*cot(d*x+c))^(1/2)+(2*(a^2
+b^2)^(1/2)+2*a)^(1/2))/(2*(a^2+b^2)^(1/2)-2*a)^(1/2))*a^4+1/d/(a^2+b^2)^3/
(2*(a^2+b^2)^(1/2)-2*a)^(1/2)*arctan((2*(a+b*cot(d*x+c))^(1/2)+(2*(a^2+b^2)
^(1/2)+2*a)^(1/2))/(2*(a^2+b^2)^(1/2)-2*a)^(1/2))*b^4+1/2/d/(a^2+b^2)^3*ln(
b*cot(d*x+c)+a-(a+b*cot(d*x+c))^(1/2)*(2*(a^2+b^2)^(1/2)+2*a)^(1/2)+(a^2+b^
2)^(1/2))*(2*(a^2+b^2)^(1/2)+2*a)^(1/2)*a^3-1/d/(a^2+b^2)^3/(2*(a^2+b^2)^(1
/2)-2*a)^(1/2)*arctan((2*(a+b*cot(d*x+c))^(1/2)-(2*(a^2+b^2)^(1/2)+2*a)^(1/
2))/(2*(a^2+b^2)^(1/2)-2*a)^(1/2))*a^4+1/d/(a^2+b^2)^3/(2*(a^2+b^2)^(1/2)-2
*a)^(1/2)*arctan((2*(a+b*cot(d*x+c))^(1/2)-(2*(a^2+b^2)^(1/2)+2*a)^(1/2))/(
2*(a^2+b^2)^(1/2)-2*a)^(1/2))*b^4+1/2/d/(a^2+b^2)^3*ln(b*cot(d*x+c)+a-(a+b*
cot(d*x+c))^(1/2)*(2*(a^2+b^2)^(1/2)+2*a)^(1/2)+(a^2+b^2)^(1/2))*(2*(a^2+b^
2)^(1/2)+2*a)^(1/2)*a*b^2-6/d/(a^2+b^2)^(7/2)/(2*(a^2+b^2)^(1/2)-2*a)^(1/2)
*arctan((2*(a+b*cot(d*x+c))^(1/2)+(2*(a^2+b^2)^(1/2)+2*a)^(1/2))/(2*(a^2+b^
2)^(1/2)-2*a)^(1/2))*a^3*b^2-5/d/(a^2+b^2)^(7/2)/(2*(a^2+b^2)^(1/2)-2*a)^(1
/2)*arctan((2*(a+b*cot(d*x+c))^(1/2)+(2*(a^2+b^2)^(1/2)+2*a)^(1/2))/(2*(a^2
+b^2)^(1/2)-2*a)^(1/2))*a*b^4-1/2/d/(a^2+b^2)^(7/2)*ln(b*cot(d*x+c)+a-(a+b*
cot(d*x+c))^(1/2)*(2*(a^2+b^2)^(1/2)+2*a)^(1/2)+(a^2+b^2)^(1/2))*(2*(a^2+b^
2)^(1/2)+2*a)^(1/2)*a^2*b^2+2/d/(a^2+b^2)^(5/2)/(2*(a^2+b^2)^(1/2)-2*a)^(1/
2)*arctan((2*(a+b*cot(d*x+c))^(1/2)-(2*(a^2+b^2)^(1/2)+2*a)^(1/2))/(2*(a^2+
b^2)^(1/2)-2*a)^(1/2))*a*b^2-6/d/(a^2+b^2)^(7/2)/(2*(a^2+b^2)^(1/2)-2*a)^(1
/2)*arctan((2*(a+b*cot(d*x+c))^(1/2)-(2*(a^2+b^2)^(1/2)+2*a)^(1/2))/(2*(a^2
+b^2)^(1/2)-2*a)^(1/2))*a^3*b^2-5/d/(a^2+b^2)^(7/2)/(2*(a^2+b^2)^(1/2)-2*a)
^(1/2)*arctan((2*(a+b*cot(d*x+c))^(1/2)-(2*(a^2+b^2)^(1/2)+2*a)^(1/2))/(2*(
a^2+b^2)^(1/2)-2*a)^(1/2))*a*b^4+1/2/d/(a^2+b^2)^(7/2)*ln(b*cot(d*x+c)+a+(a
+b*cot(d*x+c))^(1/2)*(2*(a^2+b^2)^(1/2)+2*a)^(1/2)+(a^2+b^2)^(1/2))*(2*(a^2
+b^2)^(1/2)+2*a)^(1/2)*a^2*b^2+2/d/(a^2+b^2)^(5/2)/(2*(a^2+b^2)^(1/2)-2*a)^
(1/2)*arctan((2*(a+b*cot(d*x+c))^(1/2)+(2*(a^2+b^2)^(1/2)+2*a)^(1/2))/(2*(a
^2+b^2)^(1/2)-2*a)^(1/2))*a*b^2-1/2/d/(a^2+b^2)^3*ln(b*cot(d*x+c)+a+(a+b*co
t(d*x+c))^(1/2)*(2*(a^2+b^2)^(1/2)+2*a)^(1/2)+(a^2+b^2)^(1/2))*(2*(a^2+b^2)
^(1/2)+2*a)^(1/2)*a*b^2-2/d/(a^2+b^2)^2/(a+b*cot(d*x+c))^(1/2)*a^2+2/d/(a^2
+b^2)^2/(a+b*cot(d*x+c))^(1/2)*b^2-2/3/d*a/(a^2+b^2)/(a+b*cot(d*x+c))^(3/2)
)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 7422 vs. $2(155) = 310$.

Time = 3.60 (sec) , antiderivative size = 7422, normalized size of antiderivative = 40.12

$$\int \frac{A + B\cot(c + dx)}{(a + b\cot(c + dx))^{5/2}}\, dx = \text{Too large to display}$$

```
[In] integrate((A+B*cot(d*x+c))/(a+b*cot(d*x+c))^(5/2),x, algorithm="fricas")
[Out] Too large to include
```

Sympy [F]

$$\int \frac{A+B\cot(c+dx)}{(a+b\cot(c+dx))^{5/2}}\,dx=\int \frac{A+B\cot(c+dx)}{(a+b\cot(c+dx))^{\frac{5}{2}}}\,dx$$

```
[In] integrate((A+B*cot(d*x+c))/(a+b*cot(d*x+c))**(5/2),x)
[Out] Integral((A + B*cot(c + d*x))/(a + b*cot(c + d*x))**(5/2), x)
```

Maxima [F]

$$\int \frac{A+B\cot(c+dx)}{(a+b\cot(c+dx))^{5/2}}\,dx=\int \frac{B\cot(dx+c)+A}{(b\cot(dx+c)+a)^{\frac{5}{2}}}\,dx$$

```
[In] integrate((A+B*cot(d*x+c))/(a+b*cot(d*x+c))^(5/2),x, algorithm="maxima")
[Out] integrate((B*cot(d*x + c) + A)/(b*cot(d*x + c) + a)^(5/2), x)
```

Giac [F]

$$\int \frac{A+B\cot(c+dx)}{(a+b\cot(c+dx))^{5/2}}\,dx=\int \frac{B\cot(dx+c)+A}{(b\cot(dx+c)+a)^{\frac{5}{2}}}\,dx$$

```
[In] integrate((A+B*cot(d*x+c))/(a+b*cot(d*x+c))^(5/2),x, algorithm="giac")
[Out] integrate((B*cot(d*x + c) + A)/(b*cot(d*x + c) + a)^(5/2), x)
```

Mupad [B] (verification not implemented)

Time = 31.32 (sec) , antiderivative size = 9453, normalized size of antiderivative = 51.10

$$\int \frac{A+B\cot(c+dx)}{(a+b\cot(c+dx))^{5/2}}\,dx=\text{Too large to display}$$

```
[In] int((A + B*cot(c + d*x))/(a + b*cot(c + d*x))^(5/2),x)
[Out] (log((((a + b*cot(c + d*x))^(1/2)*(320*A^2*a^4*b^14*d^3 - 16*A^2*b^18*d^3 +
 1024*A^2*a^6*b^12*d^3 + 1440*A^2*a^8*b^10*d^3 + 1024*A^2*a^10*b^8*d^3 + 32
0*A^2*a^12*b^6*d^3 - 16*A^2*a^16*b^2*d^3) + ((((320*A^4*a^2*b^8*d^4 - 16*A^
4*b^10*d^4 - 1760*A^4*a^4*b^6*d^4 + 1600*A^4*a^6*b^4*d^4 - 400*A^4*a^8*b^2*
d^4)^(1/2) - 4*A^2*a^5*d^2 + 40*A^2*a^3*b^2*d^2 - 20*A^2*a*b^4*d^2)/(a^10*d
```

```
^4 + b^10*d^4 + 5*a^2*b^8*d^4 + 10*a^4*b^6*d^4 + 10*a^6*b^4*d^4 + 5*a^8*b^2
*d^4))^(1/2)*(896*A*a^6*b^15*d^4 - ((((320*A^4*a^2*b^8*d^4 - 16*A^4*b^10*d^
4 - 1760*A^4*a^4*b^6*d^4 + 1600*A^4*a^6*b^4*d^4 - 400*A^4*a^8*b^2*d^4)^(1/2
) - 4*A^2*a^5*d^2 + 40*A^2*a^3*b^2*d^2 - 20*A^2*a*b^4*d^2)/(a^10*d^4 + b^10
*d^4 + 5*a^2*b^8*d^4 + 10*a^4*b^6*d^4 + 10*a^6*b^4*d^4 + 5*a^8*b^2*d^4))^(1
/2)*(a + b*cot(c + d*x))^(1/2)*(64*a*b^22*d^5 + 640*a^3*b^20*d^5 + 2880*a^5
*b^18*d^5 + 7680*a^7*b^16*d^5 + 13440*a^9*b^14*d^5 + 16128*a^11*b^12*d^5 +
13440*a^13*b^10*d^5 + 7680*a^15*b^8*d^5 + 2880*a^17*b^6*d^5 + 640*a^19*b^4*
d^5 + 64*a^21*b^2*d^5))/4 - 160*A*a^2*b^19*d^4 - 128*A*a^4*b^17*d^4 - 32*A*
b^21*d^4 + 3136*A*a^8*b^13*d^4 + 4928*A*a^10*b^11*d^4 + 4480*A*a^12*b^9*d^4
 + 2432*A*a^14*b^7*d^4 + 736*A*a^16*b^5*d^4 + 96*A*a^18*b^3*d^4))/4)*(((320
*A^4*a^2*b^8*d^4 - 16*A^4*b^10*d^4 - 1760*A^4*a^4*b^6*d^4 + 1600*A^4*a^6*b^
4*d^4 - 400*A^4*a^8*b^2*d^4)^(1/2) - 4*A^2*a^5*d^2 + 40*A^2*a^3*b^2*d^2 - 2
0*A^2*a*b^4*d^2)/(a^10*d^4 + b^10*d^4 + 5*a^2*b^8*d^4 + 10*a^4*b^6*d^4 + 10
*a^6*b^4*d^4 + 5*a^8*b^2*d^4))^(1/2))/4 - 96*A^3*a^3*b^13*d^2 - 240*A^3*a^5
*b^11*d^2 - 320*A^3*a^7*b^9*d^2 - 240*A^3*a^9*b^7*d^2 - 96*A^3*a^11*b^5*d^2
 - 16*A^3*a^13*b^3*d^2 - 16*A^3*a*b^15*d^2)*(((320*A^4*a^2*b^8*d^4 - 16*A^4
*b^10*d^4 - 1760*A^4*a^4*b^6*d^4 + 1600*A^4*a^6*b^4*d^4 - 400*A^4*a^8*b^2*d
^4)^(1/2) - 4*A^2*a^5*d^2 + 40*A^2*a^3*b^2*d^2 - 20*A^2*a*b^4*d^2)/(a^10*d^
4 + b^10*d^4 + 5*a^2*b^8*d^4 + 10*a^4*b^6*d^4 + 10*a^6*b^4*d^4 + 5*a^8*b^2*
d^4))^(1/2))/4 + (log((((a + b*cot(c + d*x))^(1/2)*(320*A^2*a^4*b^14*d^3 -
16*A^2*b^18*d^3 + 1024*A^2*a^6*b^12*d^3 + 1440*A^2*a^8*b^10*d^3 + 1024*A^2*
a^10*b^8*d^3 + 320*A^2*a^12*b^6*d^3 - 16*A^2*a^16*b^2*d^3) + ((-((320*A^4*a
^2*b^8*d^4 - 16*A^4*b^10*d^4 - 1760*A^4*a^4*b^6*d^4 + 1600*A^4*a^6*b^4*d^4
- 400*A^4*a^8*b^2*d^4)^(1/2) + 4*A^2*a^5*d^2 - 40*A^2*a^3*b^2*d^2 + 20*A^2*
a*b^4*d^2)/(a^10*d^4 + b^10*d^4 + 5*a^2*b^8*d^4 + 10*a^4*b^6*d^4 + 10*a^6*b
^4*d^4 + 5*a^8*b^2*d^4))^(1/2)*(896*A*a^6*b^15*d^4 - ((-((320*A^4*a^2*b^8*d
^4 - 16*A^4*b^10*d^4 - 1760*A^4*a^4*b^6*d^4 + 1600*A^4*a^6*b^4*d^4 - 400*A^
4*a^8*b^2*d^4)^(1/2) + 4*A^2*a^5*d^2 - 40*A^2*a^3*b^2*d^2 + 20*A^2*a*b^4*d^
2)/(a^10*d^4 + b^10*d^4 + 5*a^2*b^8*d^4 + 10*a^4*b^6*d^4 + 10*a^6*b^4*d^4 +
 5*a^8*b^2*d^4))^(1/2)*(a + b*cot(c + d*x))^(1/2)*(64*a*b^22*d^5 + 640*a^3*
b^20*d^5 + 2880*a^5*b^18*d^5 + 7680*a^7*b^16*d^5 + 13440*a^9*b^14*d^5 + 161
28*a^11*b^12*d^5 + 13440*a^13*b^10*d^5 + 7680*a^15*b^8*d^5 + 2880*a^17*b^6*
d^5 + 640*a^19*b^4*d^5 + 64*a^21*b^2*d^5))/4 - 160*A*a^2*b^19*d^4 - 128*A*a
^4*b^17*d^4 - 32*A*b^21*d^4 + 3136*A*a^8*b^13*d^4 + 4928*A*a^10*b^11*d^4 +
4480*A*a^12*b^9*d^4 + 2432*A*a^14*b^7*d^4 + 736*A*a^16*b^5*d^4 + 96*A*a^18*
b^3*d^4))/4)*(-((320*A^4*a^2*b^8*d^4 - 16*A^4*b^10*d^4 - 1760*A^4*a^4*b^6*d
^4 + 1600*A^4*a^6*b^4*d^4 - 400*A^4*a^8*b^2*d^4)^(1/2) + 4*A^2*a^5*d^2 - 40
*A^2*a^3*b^2*d^2 + 20*A^2*a*b^4*d^2)/(a^10*d^4 + b^10*d^4 + 5*a^2*b^8*d^4 +
 10*a^4*b^6*d^4 + 10*a^6*b^4*d^4 + 5*a^8*b^2*d^4))^(1/2))/4 - 96*A^3*a^3*b^
13*d^2 - 240*A^3*a^5*b^11*d^2 - 320*A^3*a^7*b^9*d^2 - 240*A^3*a^9*b^7*d^2 -
 96*A^3*a^11*b^5*d^2 - 16*A^3*a^13*b^3*d^2 - 16*A^3*a*b^15*d^2)*(-((320*A^4
*a^2*b^8*d^4 - 16*A^4*b^10*d^4 - 1760*A^4*a^4*b^6*d^4 + 1600*A^4*a^6*b^4*d^
4 - 400*A^4*a^8*b^2*d^4)^(1/2) + 4*A^2*a^5*d^2 - 40*A^2*a^3*b^2*d^2 + 20*A^
2*a*b^4*d^2)/(a^10*d^4 + b^10*d^4 + 5*a^2*b^8*d^4 + 10*a^4*b^6*d^4 + 10*a^6
```

*b^4*d^4 + 5*a^8*b^2*d^4))^(1/2))/4 - log(- ((a + b*cot(c + d*x))^(1/2)*(32
0*A^2*a^4*b^14*d^3 - 16*A^2*b^18*d^3 + 1024*A^2*a^6*b^12*d^3 + 1440*A^2*a^8
*b^10*d^3 + 1024*A^2*a^10*b^8*d^3 + 320*A^2*a^12*b^6*d^3 - 16*A^2*a^16*b^2*
d^3) - (((320*A^4*a^2*b^8*d^4 - 16*A^4*b^10*d^4 - 1760*A^4*a^4*b^6*d^4 + 16
00*A^4*a^6*b^4*d^4 - 400*A^4*a^8*b^2*d^4)^(1/2) - 4*A^2*a^5*d^2 + 40*A^2*a^
3*b^2*d^2 - 20*A^2*a*b^4*d^2)/(16*a^10*d^4 + 16*b^10*d^4 + 80*a^2*b^8*d^4 +
160*a^4*b^6*d^4 + 160*a^6*b^4*d^4 + 80*a^8*b^2*d^4))^(1/2)*((((320*A^4*a^2
*b^8*d^4 - 16*A^4*b^10*d^4 - 1760*A^4*a^4*b^6*d^4 + 1600*A^4*a^6*b^4*d^4 -
400*A^4*a^8*b^2*d^4)^(1/2) - 4*A^2*a^5*d^2 + 40*A^2*a^3*b^2*d^2 - 20*A^2*a*
b^4*d^2)/(16*a^10*d^4 + 16*b^10*d^4 + 80*a^2*b^8*d^4 + 160*a^4*b^6*d^4 + 16
0*a^6*b^4*d^4 + 80*a^8*b^2*d^4))^(1/2)*(a + b*cot(c + d*x))^(1/2)*(64*a*b^2
2*d^5 + 640*a^3*b^20*d^5 + 2880*a^5*b^18*d^5 + 7680*a^7*b^16*d^5 + 13440*a^
9*b^14*d^5 + 16128*a^11*b^12*d^5 + 13440*a^13*b^10*d^5 + 7680*a^15*b^8*d^5
+ 2880*a^17*b^6*d^5 + 640*a^19*b^4*d^5 + 64*a^21*b^2*d^5) - 32*A*b^21*d^4 -
160*A*a^2*b^19*d^4 - 128*A*a^4*b^17*d^4 + 896*A*a^6*b^15*d^4 + 3136*A*a^8*
b^13*d^4 + 4928*A*a^10*b^11*d^4 + 4480*A*a^12*b^9*d^4 + 2432*A*a^14*b^7*d^4
+ 736*A*a^16*b^5*d^4 + 96*A*a^18*b^3*d^4))*(((320*A^4*a^2*b^8*d^4 - 16*A^4
*b^10*d^4 - 1760*A^4*a^4*b^6*d^4 + 1600*A^4*a^6*b^4*d^4 - 400*A^4*a^8*b^2*d
^4)^(1/2) - 4*A^2*a^5*d^2 + 40*A^2*a^3*b^2*d^2 - 20*A^2*a*b^4*d^2)/(16*a^10
*d^4 + 16*b^10*d^4 + 80*a^2*b^8*d^4 + 160*a^4*b^6*d^4 + 160*a^6*b^4*d^4 + 8
0*a^8*b^2*d^4))^(1/2) - 96*A^3*a^3*b^13*d^2 - 240*A^3*a^5*b^11*d^2 - 320*A^
3*a^7*b^9*d^2 - 240*A^3*a^9*b^7*d^2 - 96*A^3*a^11*b^5*d^2 - 16*A^3*a^13*b^3
*d^2 - 16*A^3*a*b^15*d^2)*(((320*A^4*a^2*b^8*d^4 - 16*A^4*b^10*d^4 - 1760*A
^4*a^4*b^6*d^4 + 1600*A^4*a^6*b^4*d^4 - 400*A^4*a^8*b^2*d^4)^(1/2) - 4*A^2*
a^5*d^2 + 40*A^2*a^3*b^2*d^2 - 20*A^2*a*b^4*d^2)/(16*a^10*d^4 + 16*b^10*d^4
+ 80*a^2*b^8*d^4 + 160*a^4*b^6*d^4 + 160*a^6*b^4*d^4 + 80*a^8*b^2*d^4))^(1
/2) - log(- ((a + b*cot(c + d*x))^(1/2)*(320*A^2*a^4*b^14*d^3 - 16*A^2*b^18
*d^3 + 1024*A^2*a^6*b^12*d^3 + 1440*A^2*a^8*b^10*d^3 + 1024*A^2*a^10*b^8*d^
3 + 320*A^2*a^12*b^6*d^3 - 16*A^2*a^16*b^2*d^3) - (-((320*A^4*a^2*b^8*d^4 -
16*A^4*b^10*d^4 - 1760*A^4*a^4*b^6*d^4 + 1600*A^4*a^6*b^4*d^4 - 400*A^4*a^
8*b^2*d^4)^(1/2) + 4*A^2*a^5*d^2 - 40*A^2*a^3*b^2*d^2 + 20*A^2*a*b^4*d^2)/(
16*a^10*d^4 + 16*b^10*d^4 + 80*a^2*b^8*d^4 + 160*a^4*b^6*d^4 + 160*a^6*b^4*
d^4 + 80*a^8*b^2*d^4))^(1/2)*((-((320*A^4*a^2*b^8*d^4 - 16*A^4*b^10*d^4 - 1
760*A^4*a^4*b^6*d^4 + 1600*A^4*a^6*b^4*d^4 - 400*A^4*a^8*b^2*d^4)^(1/2) + 4
*A^2*a^5*d^2 - 40*A^2*a^3*b^2*d^2 + 20*A^2*a*b^4*d^2)/(16*a^10*d^4 + 16*b^1
0*d^4 + 80*a^2*b^8*d^4 + 160*a^4*b^6*d^4 + 160*a^6*b^4*d^4 + 80*a^8*b^2*d^4
))^(1/2)*(a + b*cot(c + d*x))^(1/2)*(64*a*b^22*d^5 + 640*a^3*b^20*d^5 + 288
0*a^5*b^18*d^5 + 7680*a^7*b^16*d^5 + 13440*a^9*b^14*d^5 + 16128*a^11*b^12*d
^5 + 13440*a^13*b^10*d^5 + 7680*a^15*b^8*d^5 + 2880*a^17*b^6*d^5 + 640*a^19
*b^4*d^5 + 64*a^21*b^2*d^5) - 32*A*b^21*d^4 - 160*A*a^2*b^19*d^4 - 128*A*a^
4*b^17*d^4 + 896*A*a^6*b^15*d^4 + 3136*A*a^8*b^13*d^4 + 4928*A*a^10*b^11*d^
4 + 4480*A*a^12*b^9*d^4 + 2432*A*a^14*b^7*d^4 + 736*A*a^16*b^5*d^4 + 96*A*a
^18*b^3*d^4))*(-((320*A^4*a^2*b^8*d^4 - 16*A^4*b^10*d^4 - 1760*A^4*a^4*b^6*
d^4 + 1600*A^4*a^6*b^4*d^4 - 400*A^4*a^8*b^2*d^4)^(1/2) + 4*A^2*a^5*d^2 - 4
0*A^2*a^3*b^2*d^2 + 20*A^2*a*b^4*d^2)/(16*a^10*d^4 + 16*b^10*d^4 + 80*a^2*b

^8*d^4 + 160*a^4*b^6*d^4 + 160*a^6*b^4*d^4 + 80*a^8*b^2*d^4))^(1/2) - 96*A^
3*a^3*b^13*d^2 - 240*A^3*a^5*b^11*d^2 - 320*A^3*a^7*b^9*d^2 - 240*A^3*a^9*b
^7*d^2 - 96*A^3*a^11*b^5*d^2 - 16*A^3*a^13*b^3*d^2 - 16*A^3*a*b^15*d^2)*(-(
(320*A^4*a^2*b^8*d^4 - 16*A^4*b^10*d^4 - 1760*A^4*a^4*b^6*d^4 + 1600*A^4*a^
6*b^4*d^4 - 400*A^4*a^8*b^2*d^4)^(1/2) + 4*A^2*a^5*d^2 - 40*A^2*a^3*b^2*d^2
 + 20*A^2*a*b^4*d^2)/(16*a^10*d^4 + 16*b^10*d^4 + 80*a^2*b^8*d^4 + 160*a^4*
b^6*d^4 + 160*a^6*b^4*d^4 + 80*a^8*b^2*d^4))^(1/2) + (log(40*B^3*a^8*b^8*d^
2 - 8*B^3*b^16*d^2 - 40*B^3*a^2*b^14*d^2 - 72*B^3*a^4*b^12*d^2 - 40*B^3*a^6
*b^10*d^2 - (((a + b*cot(c + d*x))^(1/2)*(320*B^2*a^4*b^14*d^3 - 16*B^2*b^1
8*d^3 + 1024*B^2*a^6*b^12*d^3 + 1440*B^2*a^8*b^10*d^3 + 1024*B^2*a^10*b^8*d
^3 + 320*B^2*a^12*b^6*d^3 - 16*B^2*a^16*b^2*d^3) - ((((320*B^4*a^2*b^8*d^4
- 16*B^4*b^10*d^4 - 1760*B^4*a^4*b^6*d^4 + 1600*B^4*a^6*b^4*d^4 - 400*B^4*a
^8*b^2*d^4)^(1/2) + 4*B^2*a^5*d^2 - 40*B^2*a^3*b^2*d^2 + 20*B^2*a*b^4*d^2)/
(a^10*d^4 + b^10*d^4 + 5*a^2*b^8*d^4 + 10*a^4*b^6*d^4 + 10*a^6*b^4*d^4 + 5*
a^8*b^2*d^4))^(1/2)*(96*B*a*b^20*d^4 - ((a + b*cot(c + d*x))^(1/2)*(((320*B
^4*a^2*b^8*d^4 - 16*B^4*b^10*d^4 - 1760*B^4*a^4*b^6*d^4 + 1600*B^4*a^6*b^4*
d^4 - 400*B^4*a^8*b^2*d^4)^(1/2) + 4*B^2*a^5*d^2 - 40*B^2*a^3*b^2*d^2 + 20*
B^2*a*b^4*d^2)/(a^10*d^4 + b^10*d^4 + 5*a^2*b^8*d^4 + 10*a^4*b^6*d^4 + 10*a
^6*b^4*d^4 + 5*a^8*b^2*d^4))^(1/2)*(64*a*b^22*d^5 + 640*a^3*b^20*d^5 + 2880
*a^5*b^18*d^5 + 7680*a^7*b^16*d^5 + 13440*a^9*b^14*d^5 + 16128*a^11*b^12*d^
5 + 13440*a^13*b^10*d^5 + 7680*a^15*b^8*d^5 + 2880*a^17*b^6*d^5 + 640*a^19*
b^4*d^5 + 64*a^21*b^2*d^5))/4 + 736*B*a^3*b^18*d^4 + 2432*B*a^5*b^16*d^4 +
4480*B*a^7*b^14*d^4 + 4928*B*a^9*b^12*d^4 + 3136*B*a^11*b^10*d^4 + 896*B*a^
13*b^8*d^4 - 128*B*a^15*b^6*d^4 - 160*B*a^17*b^4*d^4 - 32*B*a^19*b^2*d^4))/
4)*(((320*B^4*a^2*b^8*d^4 - 16*B^4*b^10*d^4 - 1760*B^4*a^4*b^6*d^4 + 1600*B
^4*a^6*b^4*d^4 - 400*B^4*a^8*b^2*d^4)^(1/2) + 4*B^2*a^5*d^2 - 40*B^2*a^3*b^
2*d^2 + 20*B^2*a*b^4*d^2)/(a^10*d^4 + b^10*d^4 + 5*a^2*b^8*d^4 + 10*a^4*b^6
*d^4 + 10*a^6*b^4*d^4 + 5*a^8*b^2*d^4))^(1/2))/4 + 72*B^3*a^10*b^6*d^2 + 40
*B^3*a^12*b^4*d^2 + 8*B^3*a^14*b^2*d^2)*(((320*B^4*a^2*b^8*d^4 - 16*B^4*b^1
0*d^4 - 1760*B^4*a^4*b^6*d^4 + 1600*B^4*a^6*b^4*d^4 - 400*B^4*a^8*b^2*d^4)^
(1/2) + 4*B^2*a^5*d^2 - 40*B^2*a^3*b^2*d^2 + 20*B^2*a*b^4*d^2)/(a^10*d^4 +
b^10*d^4 + 5*a^2*b^8*d^4 + 10*a^4*b^6*d^4 + 10*a^6*b^4*d^4 + 5*a^8*b^2*d^4)
)^(1/2))/4 + (log(40*B^3*a^8*b^8*d^2 - 8*B^3*b^16*d^2 - 40*B^3*a^2*b^14*d^2
 - 72*B^3*a^4*b^12*d^2 - 40*B^3*a^6*b^10*d^2 - (((a + b*cot(c + d*x))^(1/2)
*(320*B^2*a^4*b^14*d^3 - 16*B^2*b^18*d^3 + 1024*B^2*a^6*b^12*d^3 + 1440*B^2
*a^8*b^10*d^3 + 1024*B^2*a^10*b^8*d^3 + 320*B^2*a^12*b^6*d^3 - 16*B^2*a^16*
b^2*d^3) - ((-((320*B^4*a^2*b^8*d^4 - 16*B^4*b^10*d^4 - 1760*B^4*a^4*b^6*d^
4 + 1600*B^4*a^6*b^4*d^4 - 400*B^4*a^8*b^2*d^4)^(1/2) - 4*B^2*a^5*d^2 + 40*
B^2*a^3*b^2*d^2 - 20*B^2*a*b^4*d^2)/(a^10*d^4 + b^10*d^4 + 5*a^2*b^8*d^4 +
10*a^4*b^6*d^4 + 10*a^6*b^4*d^4 + 5*a^8*b^2*d^4))^(1/2)*(96*B*a*b^20*d^4 -
((a + b*cot(c + d*x))^(1/2)*(-((320*B^4*a^2*b^8*d^4 - 16*B^4*b^10*d^4 - 176
0*B^4*a^4*b^6*d^4 + 1600*B^4*a^6*b^4*d^4 - 400*B^4*a^8*b^2*d^4)^(1/2) - 4*B
^2*a^5*d^2 + 40*B^2*a^3*b^2*d^2 - 20*B^2*a*b^4*d^2)/(a^10*d^4 + b^10*d^4 +
5*a^2*b^8*d^4 + 10*a^4*b^6*d^4 + 10*a^6*b^4*d^4 + 5*a^8*b^2*d^4))^(1/2)*(64
*a*b^22*d^5 + 640*a^3*b^20*d^5 + 2880*a^5*b^18*d^5 + 7680*a^7*b^16*d^5 + 13

440*a^9*b^14*d^5 + 16128*a^11*b^12*d^5 + 13440*a^13*b^10*d^5 + 7680*a^15*b^8*d^5 + 2880*a^17*b^6*d^5 + 640*a^19*b^4*d^5 + 64*a^21*b^2*d^5))/4 + 736*B*a^3*b^18*d^4 + 2432*B*a^5*b^16*d^4 + 4480*B*a^7*b^14*d^4 + 4928*B*a^9*b^12*d^4 + 3136*B*a^11*b^10*d^4 + 896*B*a^13*b^8*d^4 - 128*B*a^15*b^6*d^4 - 160*B*a^17*b^4*d^4 - 32*B*a^19*b^2*d^4))/4)*(-((320*B^4*a^2*b^8*d^4 - 16*B^4*b^10*d^4 - 1760*B^4*a^4*b^6*d^4 + 1600*B^4*a^6*b^4*d^4 - 400*B^4*a^8*b^2*d^4)^(1/2) - 4*B^2*a^5*d^2 + 40*B^2*a^3*b^2*d^2 - 20*B^2*a*b^4*d^2)/(a^10*d^4 + b^10*d^4 + 5*a^2*b^8*d^4 + 10*a^4*b^6*d^4 + 10*a^6*b^4*d^4 + 5*a^8*b^2*d^4))^(1/2))/4 + 72*B^3*a^10*b^6*d^2 + 40*B^3*a^12*b^4*d^2 + 8*B^3*a^14*b^2*d^2)*(-((320*B^4*a^2*b^8*d^4 - 16*B^4*b^10*d^4 - 1760*B^4*a^4*b^6*d^4 + 1600*B^4*a^6*b^4*d^4 - 400*B^4*a^8*b^2*d^4)^(1/2) - 4*B^2*a^5*d^2 + 40*B^2*a^3*b^2*d^2 - 20*B^2*a*b^4*d^2)/(a^10*d^4 + b^10*d^4 + 5*a^2*b^8*d^4 + 10*a^4*b^6*d^4 + 10*a^6*b^4*d^4 + 5*a^8*b^2*d^4))^(1/2))/4 - log(((a + b*cot(c + d*x))^(1/2)*(320*B^2*a^4*b^14*d^3 - 16*B^2*b^18*d^3 + 1024*B^2*a^6*b^12*d^3 + 1440*B^2*a^8*b^10*d^3 + 1024*B^2*a^10*b^8*d^3 + 320*B^2*a^12*b^6*d^3 - 16*B^2*a^16*b^2*d^3) + (((320*B^4*a^2*b^8*d^4 - 16*B^4*b^10*d^4 - 1760*B^4*a^4*b^6*d^4 + 1600*B^4*a^6*b^4*d^4 - 400*B^4*a^8*b^2*d^4)^(1/2) + 4*B^2*a^5*d^2 - 40*B^2*a^3*b^2*d^2 + 20*B^2*a*b^4*d^2)/(16*a^10*d^4 + 16*b^10*d^4 + 80*a^2*b^8*d^4 + 160*a^4*b^6*d^4 + 160*a^6*b^4*d^4 + 80*a^8*b^2*d^4))^(1/2)*((a + b*cot(c + d*x))^(1/2)*(((320*B^4*a^2*b^8*d^4 - 16*B^4*b^10*d^4 - 1760*B^4*a^4*b^6*d^4 + 1600*B^4*a^6*b^4*d^4 - 400*B^4*a^8*b^2*d^4)^(1/2) + 4*B^2*a^5*d^2 - 40*B^2*a^3*b^2*d^2 + 20*B^2*a*b^4*d^2)/(16*a^10*d^4 + 16*b^10*d^4 + 80*a^2*b^8*d^4 + 160*a^4*b^6*d^4 + 160*a^6*b^4*d^4 + 80*a^8*b^2*d^4))^(1/2)*(64*a*b^22*d^5 + 640*a^3*b^20*d^5 + 2880*a^5*b^18*d^5 + 7680*a^7*b^16*d^5 + 13440*a^9*b^14*d^5 + 16128*a^11*b^12*d^5 + 13440*a^13*b^10*d^5 + 7680*a^15*b^8*d^5 + 2880*a^17*b^6*d^5 + 640*a^19*b^4*d^5 + 64*a^21*b^2*d^5) + 96*B*a*b^20*d^4 + 736*B*a^3*b^18*d^4 + 2432*B*a^5*b^16*d^4 + 4480*B*a^7*b^14*d^4 + 4928*B*a^9*b^12*d^4 + 3136*B*a^11*b^10*d^4 + 896*B*a^13*b^8*d^4 - 128*B*a^15*b^6*d^4 - 160*B*a^17*b^4*d^4 - 32*B*a^19*b^2*d^4))*(((320*B^4*a^2*b^8*d^4 - 16*B^4*b^10*d^4 - 1760*B^4*a^4*b^6*d^4 + 1600*B^4*a^6*b^4*d^4 - 400*B^4*a^8*b^2*d^4)^(1/2) + 4*B^2*a^5*d^2 - 40*B^2*a^3*b^2*d^2 + 20*B^2*a*b^4*d^2)/(16*a^10*d^4 + 16*b^10*d^4 + 80*a^2*b^8*d^4 + 160*a^4*b^6*d^4 + 160*a^6*b^4*d^4 + 80*a^8*b^2*d^4))^(1/2) - 8*B^3*b^16*d^2 - 40*B^3*a^2*b^14*d^2 - 72*B^3*a^4*b^12*d^2 - 40*B^3*a^6*b^10*d^2 + 40*B^3*a^8*b^8*d^2 + 72*B^3*a^10*b^6*d^2 + 40*B^3*a^12*b^4*d^2 + 8*B^3*a^14*b^2*d^2)*(((320*B^4*a^2*b^8*d^4 - 16*B^4*b^10*d^4 - 1760*B^4*a^4*b^6*d^4 + 1600*B^4*a^6*b^4*d^4 - 400*B^4*a^8*b^2*d^4)^(1/2) + 4*B^2*a^5*d^2 - 40*B^2*a^3*b^2*d^2 + 20*B^2*a*b^4*d^2)/(16*a^10*d^4 + 16*b^10*d^4 + 80*a^2*b^8*d^4 + 160*a^4*b^6*d^4 + 160*a^6*b^4*d^4 + 80*a^8*b^2*d^4))^(1/2) - log(((a + b*cot(c + d*x))^(1/2)*(320*B^2*a^4*b^14*d^3 - 16*B^2*b^18*d^3 + 1024*B^2*a^6*b^12*d^3 + 1440*B^2*a^8*b^10*d^3 + 1024*B^2*a^10*b^8*d^3 + 320*B^2*a^12*b^6*d^3 - 16*B^2*a^16*b^2*d^3) + (-((320*B^4*a^2*b^8*d^4 - 16*B^4*b^10*d^4 - 1760*B^4*a^4*b^6*d^4 + 1600*B^4*a^6*b^4*d^4 - 400*B^4*a^8*b^2*d^4)^(1/2) - 4*B^2*a^5*d^2 + 40*B^2*a^3*b^2*d^2 - 20*B^2*a*b^4*d^2)/(16*a^10*d^4 + 16*b^10*d^4 + 80*a^2*b^8*d^4 + 160*a^4*b^6*d^4 + 160*a^6*b^4*d^4 + 80*a^8*b^2*d^4))^(1/2)*((a + b*cot(c + d*

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x))^(1/2)*(-((320*B^4*a^2*b^8*d^4 - 16*B^4*b^10*d^4 - 1760*B^4*a^4*b^6*d^4
+ 1600*B^4*a^6*b^4*d^4 - 400*B^4*a^8*b^2*d^4)^(1/2) - 4*B^2*a^5*d^2 + 40*B^
2*a^3*b^2*d^2 - 20*B^2*a*b^4*d^2)/(16*a^10*d^4 + 16*b^10*d^4 + 80*a^2*b^8*d
^4 + 160*a^4*b^6*d^4 + 160*a^6*b^4*d^4 + 80*a^8*b^2*d^4))^(1/2)*(64*a*b^22*
d^5 + 640*a^3*b^20*d^5 + 2880*a^5*b^18*d^5 + 7680*a^7*b^16*d^5 + 13440*a^9*
b^14*d^5 + 16128*a^11*b^12*d^5 + 13440*a^13*b^10*d^5 + 7680*a^15*b^8*d^5 +
2880*a^17*b^6*d^5 + 640*a^19*b^4*d^5 + 64*a^21*b^2*d^5) + 96*B*a*b^20*d^4 +
 736*B*a^3*b^18*d^4 + 2432*B*a^5*b^16*d^4 + 4480*B*a^7*b^14*d^4 + 4928*B*a^
9*b^12*d^4 + 3136*B*a^11*b^10*d^4 + 896*B*a^13*b^8*d^4 - 128*B*a^15*b^6*d^4
 - 160*B*a^17*b^4*d^4 - 32*B*a^19*b^2*d^4))*(-((320*B^4*a^2*b^8*d^4 - 16*B^
4*b^10*d^4 - 1760*B^4*a^4*b^6*d^4 + 1600*B^4*a^6*b^4*d^4 - 400*B^4*a^8*b^2*
d^4)^(1/2) - 4*B^2*a^5*d^2 + 40*B^2*a^3*b^2*d^2 - 20*B^2*a*b^4*d^2)/(16*a^1
0*d^4 + 16*b^10*d^4 + 80*a^2*b^8*d^4 + 160*a^4*b^6*d^4 + 160*a^6*b^4*d^4 +
80*a^8*b^2*d^4))^(1/2) - 8*B^3*b^16*d^2 - 40*B^3*a^2*b^14*d^2 - 72*B^3*a^4*
b^12*d^2 - 40*B^3*a^6*b^10*d^2 + 40*B^3*a^8*b^8*d^2 + 72*B^3*a^10*b^6*d^2 +
 40*B^3*a^12*b^4*d^2 + 8*B^3*a^14*b^2*d^2)*(-((320*B^4*a^2*b^8*d^4 - 16*B^4
*b^10*d^4 - 1760*B^4*a^4*b^6*d^4 + 1600*B^4*a^6*b^4*d^4 - 400*B^4*a^8*b^2*d
^4)^(1/2) - 4*B^2*a^5*d^2 + 40*B^2*a^3*b^2*d^2 - 20*B^2*a*b^4*d^2)/(16*a^10
*d^4 + 16*b^10*d^4 + 80*a^2*b^8*d^4 + 160*a^4*b^6*d^4 + 160*a^6*b^4*d^4 + 8
0*a^8*b^2*d^4))^(1/2) - ((2*B*a)/(3*(a^2 + b^2)) + (2*B*(a^2 - b^2)*(a + b*
cot(c + d*x)))/(a^2 + b^2)^2)/(d*(a + b*cot(c + d*x))^(3/2)) + ((2*A*b)/(3*
(a^2 + b^2)) + (4*A*a*b*(a + b*cot(c + d*x)))/(a^2 + b^2)^2)/(d*(a + b*cot(
c + d*x))^(3/2))
```

3.104 $\int \frac{-a+b\cot(c+dx)}{\sqrt{a+b\cot(c+dx)}}\,dx$

Optimal result

Integrand size = 27, antiderivative size = 102

$$\int \frac{-a+b\cot(c+dx)}{\sqrt{a+b\cot(c+dx)}}\,dx = -\frac{(ia-b)\text{arctanh}\left(\frac{\sqrt{a+b\cot(c+dx)}}{\sqrt{a-ib}}\right)}{\sqrt{a-ib}d} + \frac{(ia+b)\text{arctanh}\left(\frac{\sqrt{a+b\cot(c+dx)}}{\sqrt{a+ib}}\right)}{\sqrt{a+ib}d}$$

```
[Out] -(I*a-b)*arctanh((a+b*cot(d*x+c))^(1/2)/(a-I*b)^(1/2))/d/(a-I*b)^(1/2)+(I*a
+b)*arctanh((a+b*cot(d*x+c))^(1/2)/(a+I*b)^(1/2))/d/(a+I*b)^(1/2)
```

Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 102, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.148, Rules used = {3620, 3618, 65, 214}

$$\int \frac{-a+b\cot(c+dx)}{\sqrt{a+b\cot(c+dx)}}\,dx = \frac{(b+ia)\text{arctanh}\left(\frac{\sqrt{a+b\cot(c+dx)}}{\sqrt{a+ib}}\right)}{d\sqrt{a+ib}} - \frac{(-b+ia)\text{arctanh}\left(\frac{\sqrt{a+b\cot(c+dx)}}{\sqrt{a-ib}}\right)}{d\sqrt{a-ib}}$$

```
[In] Int[(-a + b*Cot[c + d*x])/Sqrt[a + b*Cot[c + d*x]],x]
```

```
[Out] -(((I*a - b)*ArcTanh[Sqrt[a + b*Cot[c + d*x]]/Sqrt[a - I*b]])/(Sqrt[a - I*b
]*d)) + ((I*a + b)*ArcTanh[Sqrt[a + b*Cot[c + d*x]]/Sqrt[a + I*b]])/(Sqrt[a
 + I*b]*d)
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 3618

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*tan[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Dist[c*(d/f), Subst[Int[(a + (b/d)*x)^m/(d^2 + c
*x), x], x, d*Tan[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b
*c - a*d, 0] && NeQ[a^2 + b^2, 0] && EqQ[c^2 + d^2, 0]
```

Rule 3620

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)]), x_Symbol] :> Dist[(c + I*d)/2, Int[(a + b*Tan[e + f*x])^m*(1
 - I*Tan[e + f*x]), x], x] + Dist[(c - I*d)/2, Int[(a + b*Tan[e + f*x])^m*(
1 + I*Tan[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b*c -
 a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] &&  !IntegerQ[m]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{1}{2}(-a-ib)\int \frac{1+i\cot(c+dx)}{\sqrt{a+b\cot(c+dx)}}\,dx+\frac{1}{2}(-a+ib)\int \frac{1-i\cot(c+dx)}{\sqrt{a+b\cot(c+dx)}}\,dx\\
&= \frac{(ia-b)\text{Subst}\left(\int \frac{1}{(-1+x)\sqrt{a-ibx}}\,dx,x,i\cot(c+dx)\right)}{2d}\\
&\quad-\frac{(ia+b)\text{Subst}\left(\int \frac{1}{(-1+x)\sqrt{a+ibx}}\,dx,x,-i\cot(c+dx)\right)}{2d}\\
&= -\frac{(a-ib)\text{Subst}\left(\int \frac{1}{-1+\frac{ia}{b}-\frac{ix^2}{b}}\,dx,x,\sqrt{a+b\cot(c+dx)}\right)}{bd}\\
&\quad-\frac{(a+ib)\text{Subst}\left(\int \frac{1}{-1-\frac{ia}{b}+\frac{ix^2}{b}}\,dx,x,\sqrt{a+b\cot(c+dx)}\right)}{bd}\\
&= -\frac{(ia-b)\text{arctanh}\left(\frac{\sqrt{a+b\cot(c+dx)}}{\sqrt{a-ib}}\right)}{\sqrt{a-ib}d}+\frac{(ia+b)\text{arctanh}\left(\frac{\sqrt{a+b\cot(c+dx)}}{\sqrt{a+ib}}\right)}{\sqrt{a+ib}d}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.48 (sec) , antiderivative size = 137, normalized size of antiderivative = 1.34

$$\int \frac{-a+b\cot(c+dx)}{\sqrt{a+b\cot(c+dx)}}\,dx = \frac{\frac{\left(b^2-a\sqrt{-b^2}\right)\text{arctanh}\left(\frac{\sqrt{a+b\cot(c+dx)}}{\sqrt{a-\sqrt{-b^2}}}\right)}{\sqrt{a-\sqrt{-b^2}}}+\frac{\left(b^2+a\sqrt{-b^2}\right)\text{arctanh}\left(\frac{\sqrt{a+b\cot(c+dx)}}{\sqrt{a+\sqrt{-b^2}}}\right)}{\sqrt{a+\sqrt{-b^2}}}}{bd}$$

```
[In] Integrate[(-a + b*Cot[c + d*x])/Sqrt[a + b*Cot[c + d*x]],x]
```

```
[Out] (((b^2 - a*Sqrt[-b^2])*ArcTanh[Sqrt[a + b*Cot[c + d*x]]/Sqrt[a - Sqrt[-b^2]
]])/Sqrt[a - Sqrt[-b^2]] + ((b^2 + a*Sqrt[-b^2])*ArcTanh[Sqrt[a + b*Cot[c +
 d*x]]/Sqrt[a + Sqrt[-b^2]]])/Sqrt[a + Sqrt[-b^2]])/(b*d)
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 1890 vs. 2(84) = 168.

Time = 0.08 (sec) , antiderivative size = 1891, normalized size of antiderivative = 18.54

method	result	size
parts	Expression too large to display	1891
derivativedivides	Expression too large to display	1905
default	Expression too large to display	1905

```
[In] int((-a+b*cot(d*x+c))/(a+b*cot(d*x+c))^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] b/d*(-1/2/(a^2+b^2)^(1/2)*(-1/2*(2*(a^2+b^2)^(1/2)+2*a)^(1/2)*ln(b*cot(d*x+
c)+a+(a+b*cot(d*x+c))^(1/2)*(2*(a^2+b^2)^(1/2)+2*a)^(1/2)+(a^2+b^2)^(1/2))+
2*((a^2+b^2)^(1/2)-a)/(2*(a^2+b^2)^(1/2)-2*a)^(1/2)*arctan((2*(a+b*cot(d*x+
c))^(1/2)+(2*(a^2+b^2)^(1/2)+2*a)^(1/2))/(2*(a^2+b^2)^(1/2)-2*a)^(1/2)))-1/
2/(a^2+b^2)^(1/2)*(1/2*(2*(a^2+b^2)^(1/2)+2*a)^(1/2)*ln(b*cot(d*x+c)+a-(a+b
*cot(d*x+c))^(1/2)*(2*(a^2+b^2)^(1/2)+2*a)^(1/2)+(a^2+b^2)^(1/2))+2*((a^2+b
^2)^(1/2)-a)/(2*(a^2+b^2)^(1/2)-2*a)^(1/2)*arctan((2*(a+b*cot(d*x+c))^(1/2)
-(2*(a^2+b^2)^(1/2)+2*a)^(1/2))/(2*(a^2+b^2)^(1/2)-2*a)^(1/2))))-a*(-1/4/d/
b/(a^2+b^2)*ln(b*cot(d*x+c)+a+(a+b*cot(d*x+c))^(1/2)*(2*(a^2+b^2)^(1/2)+2*a
)^(1/2)+(a^2+b^2)^(1/2))*(2*(a^2+b^2)^(1/2)+2*a)^(1/2)*a^2-1/4/d*b/(a^2+b^2
)*ln(b*cot(d*x+c)+a+(a+b*cot(d*x+c))^(1/2)*(2*(a^2+b^2)^(1/2)+2*a)^(1/2)+(a
^2+b^2)^(1/2))*(2*(a^2+b^2)^(1/2)+2*a)^(1/2)+1/4/d/b/(a^2+b^2)^(3/2)*ln(b*c
ot(d*x+c)+a+(a+b*cot(d*x+c))^(1/2)*(2*(a^2+b^2)^(1/2)+2*a)^(1/2)+(a^2+b^2)^
(1/2))*(2*(a^2+b^2)^(1/2)+2*a)^(1/2)*a^3+1/4/d*b/(a^2+b^2)^(3/2)*ln(b*cot(d
*x+c)+a+(a+b*cot(d*x+c))^(1/2)*(2*(a^2+b^2)^(1/2)+2*a)^(1/2)+(a^2+b^2)^(1/2
))*(2*(a^2+b^2)^(1/2)+2*a)^(1/2)*a+1/d/b/(a^2+b^2)^(1/2)/(2*(a^2+b^2)^(1/2)
```

```
-2*a)^(1/2)*arctan((2*(a+b*cot(d*x+c))^(1/2)+(2*(a^2+b^2)^(1/2)+2*a)^(1/2))
/(2*(a^2+b^2)^(1/2)-2*a)^(1/2))*a^2+1/d*b/(a^2+b^2)^(1/2)/(2*(a^2+b^2)^(1/2
)-2*a)^(1/2)*arctan((2*(a+b*cot(d*x+c))^(1/2)+(2*(a^2+b^2)^(1/2)+2*a)^(1/2)
)/(2*(a^2+b^2)^(1/2)-2*a)^(1/2))-1/d/b/(a^2+b^2)^(3/2)/(2*(a^2+b^2)^(1/2)-2
*a)^(1/2)*arctan((2*(a+b*cot(d*x+c))^(1/2)+(2*(a^2+b^2)^(1/2)+2*a)^(1/2))/(
2*(a^2+b^2)^(1/2)-2*a)^(1/2))*a^4-3/d*b/(a^2+b^2)^(3/2)/(2*(a^2+b^2)^(1/2)-
2*a)^(1/2)*arctan((2*(a+b*cot(d*x+c))^(1/2)+(2*(a^2+b^2)^(1/2)+2*a)^(1/2))/
(2*(a^2+b^2)^(1/2)-2*a)^(1/2))*a^2-2/d*b^3/(a^2+b^2)^(3/2)/(2*(a^2+b^2)^(1/
2)-2*a)^(1/2)*arctan((2*(a+b*cot(d*x+c))^(1/2)+(2*(a^2+b^2)^(1/2)+2*a)^(1/2
))/(2*(a^2+b^2)^(1/2)-2*a)^(1/2))+1/4/d/b/(a^2+b^2)*ln(b*cot(d*x+c)+a-(a+b*
cot(d*x+c))^(1/2)*(2*(a^2+b^2)^(1/2)+2*a)^(1/2)+(a^2+b^2)^(1/2))*(2*(a^2+b^
2)^(1/2)+2*a)^(1/2)*a^2+1/4/d*b/(a^2+b^2)*ln(b*cot(d*x+c)+a-(a+b*cot(d*x+c)
)^(1/2)*(2*(a^2+b^2)^(1/2)+2*a)^(1/2)+(a^2+b^2)^(1/2))*(2*(a^2+b^2)^(1/2)+2
*a)^(1/2)-1/4/d/b/(a^2+b^2)^(3/2)*ln(b*cot(d*x+c)+a-(a+b*cot(d*x+c))^(1/2)*
(2*(a^2+b^2)^(1/2)+2*a)^(1/2)+(a^2+b^2)^(1/2))*(2*(a^2+b^2)^(1/2)+2*a)^(1/2
)*a^3-1/4/d*b/(a^2+b^2)^(3/2)*ln(b*cot(d*x+c)+a-(a+b*cot(d*x+c))^(1/2)*(2*(
a^2+b^2)^(1/2)+2*a)^(1/2)+(a^2+b^2)^(1/2))*(2*(a^2+b^2)^(1/2)+2*a)^(1/2)*a+
1/d/b/(a^2+b^2)^(1/2)/(2*(a^2+b^2)^(1/2)-2*a)^(1/2)*arctan((2*(a+b*cot(d*x+
c))^(1/2)-(2*(a^2+b^2)^(1/2)+2*a)^(1/2))/(2*(a^2+b^2)^(1/2)-2*a)^(1/2))*a^2
+1/d*b/(a^2+b^2)^(1/2)/(2*(a^2+b^2)^(1/2)-2*a)^(1/2)*arctan((2*(a+b*cot(d*x
+c))^(1/2)-(2*(a^2+b^2)^(1/2)+2*a)^(1/2))/(2*(a^2+b^2)^(1/2)-2*a)^(1/2))-1/
d/b/(a^2+b^2)^(3/2)/(2*(a^2+b^2)^(1/2)-2*a)^(1/2)*arctan((2*(a+b*cot(d*x+c)
)^(1/2)-(2*(a^2+b^2)^(1/2)+2*a)^(1/2))/(2*(a^2+b^2)^(1/2)-2*a)^(1/2))*a^4-3
/d*b/(a^2+b^2)^(3/2)/(2*(a^2+b^2)^(1/2)-2*a)^(1/2)*arctan((2*(a+b*cot(d*x+c
))^(1/2)-(2*(a^2+b^2)^(1/2)+2*a)^(1/2))/(2*(a^2+b^2)^(1/2)-2*a)^(1/2))*a^2-
2/d*b^3/(a^2+b^2)^(3/2)/(2*(a^2+b^2)^(1/2)-2*a)^(1/2)*arctan((2*(a+b*cot(d*
x+c))^(1/2)-(2*(a^2+b^2)^(1/2)+2*a)^(1/2))/(2*(a^2+b^2)^(1/2)-2*a)^(1/2)))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1219 vs. $2(75) = 150$.

Time = 0.31 (sec) , antiderivative size = 1219, normalized size of antiderivative = 11.95

$$\int \frac{-a + b\cot(c + dx)}{\sqrt{a + b\cot(c + dx)}}\, dx = \text{Too large to display}$$

```
[In] integrate((-a+b*cot(d*x+c))/(a+b*cot(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] -1/2*sqrt(-((a^2 + b^2)*d^2*sqrt(-(9*a^4*b^2 - 6*a^2*b^4 + b^6)/((a^4 + 2*a
^2*b^2 + b^4)*d^4)) + a^3 - 3*a*b^2)/((a^2 + b^2)*d^2))*log(-(3*a^4*b + 2*a
^2*b^3 - b^5)*sqrt((b*cos(2*d*x + 2*c) + a*sin(2*d*x + 2*c) + b)/sin(2*d*x
+ 2*c)) + ((a^4 - b^4)*d^3*sqrt(-(9*a^4*b^2 - 6*a^2*b^4 + b^6)/((a^4 + 2*a^
2*b^2 + b^4)*d^4)) + 2*(3*a^3*b^2 - a*b^4)*d)*sqrt(-((a^2 + b^2)*d^2*sqrt(-
(9*a^4*b^2 - 6*a^2*b^4 + b^6)/((a^4 + 2*a^2*b^2 + b^4)*d^4)) + a^3 - 3*a*b^
```

```
2)/((a^2 + b^2)*d^2))) + 1/2*sqrt(-((a^2 + b^2)*d^2*sqrt(-(9*a^4*b^2 - 6*a^
2*b^4 + b^6)/((a^4 + 2*a^2*b^2 + b^4)*d^4)) + a^3 - 3*a*b^2)/((a^2 + b^2)*d
^2))*log(-(3*a^4*b + 2*a^2*b^3 - b^5)*sqrt((b*cos(2*d*x + 2*c) + a*sin(2*d*
x + 2*c) + b)/sin(2*d*x + 2*c)) - ((a^4 - b^4)*d^3*sqrt(-(9*a^4*b^2 - 6*a^2
*b^4 + b^6)/((a^4 + 2*a^2*b^2 + b^4)*d^4)) + 2*(3*a^3*b^2 - a*b^4)*d)*sqrt(
-((a^2 + b^2)*d^2*sqrt(-(9*a^4*b^2 - 6*a^2*b^4 + b^6)/((a^4 + 2*a^2*b^2 + b
^4)*d^4)) + a^3 - 3*a*b^2)/((a^2 + b^2)*d^2))) + 1/2*sqrt(((a^2 + b^2)*d^2*
sqrt(-(9*a^4*b^2 - 6*a^2*b^4 + b^6)/((a^4 + 2*a^2*b^2 + b^4)*d^4)) - a^3 +
3*a*b^2)/((a^2 + b^2)*d^2))*log(-(3*a^4*b + 2*a^2*b^3 - b^5)*sqrt((b*cos(2*
d*x + 2*c) + a*sin(2*d*x + 2*c) + b)/sin(2*d*x + 2*c)) + ((a^4 - b^4)*d^3*s
qrt(-(9*a^4*b^2 - 6*a^2*b^4 + b^6)/((a^4 + 2*a^2*b^2 + b^4)*d^4)) - 2*(3*a^
3*b^2 - a*b^4)*d)*sqrt(((a^2 + b^2)*d^2*sqrt(-(9*a^4*b^2 - 6*a^2*b^4 + b^6)
/((a^4 + 2*a^2*b^2 + b^4)*d^4)) - a^3 + 3*a*b^2)/((a^2 + b^2)*d^2))) - 1/2*
sqrt(((a^2 + b^2)*d^2*sqrt(-(9*a^4*b^2 - 6*a^2*b^4 + b^6)/((a^4 + 2*a^2*b^2
 + b^4)*d^4)) - a^3 + 3*a*b^2)/((a^2 + b^2)*d^2))*log(-(3*a^4*b + 2*a^2*b^3
 - b^5)*sqrt((b*cos(2*d*x + 2*c) + a*sin(2*d*x + 2*c) + b)/sin(2*d*x + 2*c)
) - ((a^4 - b^4)*d^3*sqrt(-(9*a^4*b^2 - 6*a^2*b^4 + b^6)/((a^4 + 2*a^2*b^2
+ b^4)*d^4)) - 2*(3*a^3*b^2 - a*b^4)*d)*sqrt(((a^2 + b^2)*d^2*sqrt(-(9*a^4*
b^2 - 6*a^2*b^4 + b^6)/((a^4 + 2*a^2*b^2 + b^4)*d^4)) - a^3 + 3*a*b^2)/((a^
2 + b^2)*d^2)))
```

Sympy [F]

$$\int \frac{-a+b\cot(c+dx)}{\sqrt{a+b\cot(c+dx)}}\,dx=-\int \frac{a}{\sqrt{a+b\cot{(c+dx)}}}\,dx-\int \left(-\frac{b\cot{(c+dx)}}{\sqrt{a+b\cot{(c+dx)}}}\right)dx$$

```
[In] integrate((-a+b*cot(d*x+c))/(a+b*cot(d*x+c))**(1/2),x)
[Out] -Integral(a/sqrt(a + b*cot(c + d*x)), x) - Integral(-b*cot(c + d*x)/sqrt(a
+ b*cot(c + d*x)), x)
```

Maxima [F]

$$\int \frac{-a+b\cot(c+dx)}{\sqrt{a+b\cot(c+dx)}}\,dx=\int \frac{b\cot{(dx+c)}-a}{\sqrt{b\cot{(dx+c)}+a}}\,dx$$

```
[In] integrate((-a+b*cot(d*x+c))/(a+b*cot(d*x+c))^(1/2),x, algorithm="maxima")
[Out] integrate((b*cot(d*x + c) - a)/sqrt(b*cot(d*x + c) + a), x)
```

Giac [F]

$$\int \frac{-a+b\cot(c+dx)}{\sqrt{a+b\cot(c+dx)}}\,dx=\int \frac{b\cot(dx+c)-a}{\sqrt{b\cot(dx+c)+a}}\,dx$$

```
[In] integrate((-a+b*cot(d*x+c))/(a+b*cot(d*x+c))^(1/2),x, algorithm="giac")
[Out] integrate((b*cot(d*x + c) - a)/sqrt(b*cot(d*x + c) + a), x)
```

Mupad [B] (verification not implemented)

Time = 14.76 (sec) , antiderivative size = 2731, normalized size of antiderivative = 26.77

$$\int \frac{-a+b\cot(c+dx)}{\sqrt{a+b\cot(c+dx)}}\,dx=\text{Too large to display}$$

```
[In] int(-(a - b*cot(c + d*x))/(a + b*cot(c + d*x))^(1/2),x)
[Out] 2*atanh((32*a^4*b^2*d^2*(- (-16*a^4*b^2*d^4)^(1/2)/(16*(a^2*d^4 + b^2*d^4))
 - (a^3*d^2)/(4*(a^2*d^4 + b^2*d^4)))^(1/2)*(a + b*cot(c + d*x))^(1/2))/((1
6*a^4*b^5*d^5)/(a^2*d^4 + b^2*d^4) + (16*a^6*b^3*d^5)/(a^2*d^4 + b^2*d^4) +
 (4*a^3*b^3*d^4*(-16*a^4*b^2*d^4)^(1/2))/(a^2*d^5 + b^2*d^5) + (4*a*b^5*d^4
*(-16*a^4*b^2*d^4)^(1/2))/(a^2*d^5 + b^2*d^5)) - (32*a^2*b^2*(- (-16*a^4*b^
2*d^4)^(1/2)/(16*(a^2*d^4 + b^2*d^4)) - (a^3*d^2)/(4*(a^2*d^4 + b^2*d^4)))^
(1/2)*(a + b*cot(c + d*x))^(1/2))/((16*a^4*b^3*d^3)/(a^2*d^4 + b^2*d^4) + (
4*a*b^3*d^2*(-16*a^4*b^2*d^4)^(1/2))/(a^2*d^5 + b^2*d^5)) + (8*a*b^2*(- (-1
6*a^4*b^2*d^4)^(1/2)/(16*(a^2*d^4 + b^2*d^4)) - (a^3*d^2)/(4*(a^2*d^4 + b^2
*d^4)))^(1/2)*(a + b*cot(c + d*x))^(1/2)*(-16*a^4*b^2*d^4)^(1/2))/((16*a^4*
b^5*d^5)/(a^2*d^4 + b^2*d^4) + (16*a^6*b^3*d^5)/(a^2*d^4 + b^2*d^4) + (4*a^
3*b^3*d^4*(-16*a^4*b^2*d^4)^(1/2))/(a^2*d^5 + b^2*d^5) + (4*a*b^5*d^4*(-16*
a^4*b^2*d^4)^(1/2))/(a^2*d^5 + b^2*d^5)))*(- (-16*a^4*b^2*d^4)^(1/2)/(16*(a
^2*d^4 + b^2*d^4)) - (a^3*d^2)/(4*(a^2*d^4 + b^2*d^4)))^(1/2) - 2*atanh((32
*a^2*b^2*((-16*a^4*b^2*d^4)^(1/2)/(16*(a^2*d^4 + b^2*d^4)) - (a^3*d^2)/(4*(
a^2*d^4 + b^2*d^4)))^(1/2)*(a + b*cot(c + d*x))^(1/2))/((16*a^4*b^3*d^3)/(a
^2*d^4 + b^2*d^4) - (4*a*b^3*d^2*(-16*a^4*b^2*d^4)^(1/2))/(a^2*d^5 + b^2*d^
5)) - (32*a^4*b^2*d^2*((-16*a^4*b^2*d^4)^(1/2)/(16*(a^2*d^4 + b^2*d^4)) - (
a^3*d^2)/(4*(a^2*d^4 + b^2*d^4)))^(1/2)*(a + b*cot(c + d*x))^(1/2))/((16*a^
4*b^5*d^5)/(a^2*d^4 + b^2*d^4) + (16*a^6*b^3*d^5)/(a^2*d^4 + b^2*d^4) - (4*
a^3*b^3*d^4*(-16*a^4*b^2*d^4)^(1/2))/(a^2*d^5 + b^2*d^5) - (4*a*b^5*d^4*(-1
6*a^4*b^2*d^4)^(1/2))/(a^2*d^5 + b^2*d^5)) + (8*a*b^2*((-16*a^4*b^2*d^4)^(1
/2)/(16*(a^2*d^4 + b^2*d^4)) - (a^3*d^2)/(4*(a^2*d^4 + b^2*d^4)))^(1/2)*(a
+ b*cot(c + d*x))^(1/2)*(-16*a^4*b^2*d^4)^(1/2))/((16*a^4*b^5*d^5)/(a^2*d^4
 + b^2*d^4) + (16*a^6*b^3*d^5)/(a^2*d^4 + b^2*d^4) - (4*a^3*b^3*d^4*(-16*a^
4*b^2*d^4)^(1/2))/(a^2*d^5 + b^2*d^5) - (4*a*b^5*d^4*(-16*a^4*b^2*d^4)^(1/2
```

```
))/(a^2*d^5 + b^2*d^5)))*((-16*a^4*b^2*d^4)^(1/2)/(16*(a^2*d^4 + b^2*d^4))
 - (a^3*d^2)/(4*(a^2*d^4 + b^2*d^4)))^(1/2) + 2*atanh((32*b^4*((a*b^2*d^2)/(
4*(a^2*d^4 + b^2*d^4)) - (-16*b^6*d^4)^(1/2)/(16*(a^2*d^4 + b^2*d^4)))^(1/2
)*(a + b*cot(c + d*x))^(1/2))/((16*b^5)/d - (16*a^2*b^5*d^3)/(a^2*d^4 + b^2
*d^4) + (4*a*b^3*d^2*(-16*b^6*d^4)^(1/2))/(a^2*d^5 + b^2*d^5)) + (8*a*b^2*(
(a*b^2*d^2)/(4*(a^2*d^4 + b^2*d^4)) - (-16*b^6*d^4)^(1/2)/(16*(a^2*d^4 + b^
2*d^4)))^(1/2)*(a + b*cot(c + d*x))^(1/2)*(-16*b^6*d^4)^(1/2))/(16*b^7*d +
16*a^2*b^5*d - (16*a^2*b^7*d^5)/(a^2*d^4 + b^2*d^4) - (16*a^4*b^5*d^5)/(a^2
*d^4 + b^2*d^4) + (4*a*b^5*d^4*(-16*b^6*d^4)^(1/2))/(a^2*d^5 + b^2*d^5) + (
4*a^3*b^3*d^4*(-16*b^6*d^4)^(1/2))/(a^2*d^5 + b^2*d^5)) - (32*a^2*b^4*d^2*(
(a*b^2*d^2)/(4*(a^2*d^4 + b^2*d^4)) - (-16*b^6*d^4)^(1/2)/(16*(a^2*d^4 + b^
2*d^4)))^(1/2)*(a + b*cot(c + d*x))^(1/2))/(16*b^7*d + 16*a^2*b^5*d - (16*a
^2*b^7*d^5)/(a^2*d^4 + b^2*d^4) - (16*a^4*b^5*d^5)/(a^2*d^4 + b^2*d^4) + (4
*a*b^5*d^4*(-16*b^6*d^4)^(1/2))/(a^2*d^5 + b^2*d^5) + (4*a^3*b^3*d^4*(-16*b
^6*d^4)^(1/2))/(a^2*d^5 + b^2*d^5)))*((a*b^2*d^2)/(4*(a^2*d^4 + b^2*d^4)) -
 (-16*b^6*d^4)^(1/2)/(16*(a^2*d^4 + b^2*d^4)))^(1/2) + 2*atanh((8*a*b^2*((-
16*b^6*d^4)^(1/2)/(16*(a^2*d^4 + b^2*d^4)) + (a*b^2*d^2)/(4*(a^2*d^4 + b^2*
d^4)))^(1/2)*(a + b*cot(c + d*x))^(1/2)*(-16*b^6*d^4)^(1/2))/((16*a^2*b^7*d
^5)/(a^2*d^4 + b^2*d^4) - 16*a^2*b^5*d - 16*b^7*d + (16*a^4*b^5*d^5)/(a^2*d
^4 + b^2*d^4) + (4*a*b^5*d^4*(-16*b^6*d^4)^(1/2))/(a^2*d^5 + b^2*d^5) + (4*
a^3*b^3*d^4*(-16*b^6*d^4)^(1/2))/(a^2*d^5 + b^2*d^5)) - (32*b^4*((-16*b^6*d
^4)^(1/2)/(16*(a^2*d^4 + b^2*d^4)) + (a*b^2*d^2)/(4*(a^2*d^4 + b^2*d^4)))^(
1/2)*(a + b*cot(c + d*x))^(1/2))/((16*a^2*b^5*d^3)/(a^2*d^4 + b^2*d^4) - (1
6*b^5)/d + (4*a*b^3*d^2*(-16*b^6*d^4)^(1/2))/(a^2*d^5 + b^2*d^5)) + (32*a^2
*b^4*d^2*((-16*b^6*d^4)^(1/2)/(16*(a^2*d^4 + b^2*d^4)) + (a*b^2*d^2)/(4*(a^
2*d^4 + b^2*d^4)))^(1/2)*(a + b*cot(c + d*x))^(1/2))/((16*a^2*b^7*d^5)/(a^2
*d^4 + b^2*d^4) - 16*a^2*b^5*d - 16*b^7*d + (16*a^4*b^5*d^5)/(a^2*d^4 + b^2
*d^4) + (4*a*b^5*d^4*(-16*b^6*d^4)^(1/2))/(a^2*d^5 + b^2*d^5) + (4*a^3*b^3*
d^4*(-16*b^6*d^4)^(1/2))/(a^2*d^5 + b^2*d^5)))*((-16*b^6*d^4)^(1/2)/(16*(a^
2*d^4 + b^2*d^4)) + (a*b^2*d^2)/(4*(a^2*d^4 + b^2*d^4)))^(1/2)
```

3.105 $\int \frac{-a+b\cot(c+dx)}{(a+b\cot(c+dx))^{3/2}}\,dx$

Optimal result

Integrand size = 27, antiderivative size = 132

$$\int \frac{-a+b\cot(c+dx)}{(a+b\cot(c+dx))^{3/2}}\,dx = -\frac{(ia-b)\text{arctanh}\left(\frac{\sqrt{a+b\cot(c+dx)}}{\sqrt{a-ib}}\right)}{(a-ib)^{3/2}d} + \frac{(ia+b)\text{arctanh}\left(\frac{\sqrt{a+b\cot(c+dx)}}{\sqrt{a+ib}}\right)}{(a+ib)^{3/2}d} - \frac{4ab}{(a^2+b^2)\,d\sqrt{a+b\cot(c+dx)}}$$

```
[Out] -(I*a-b)*arctanh((a+b*cot(d*x+c))^(1/2)/(a-I*b)^(1/2))/(a-I*b)^(3/2)/d+(I*a
+b)*arctanh((a+b*cot(d*x+c))^(1/2)/(a+I*b)^(1/2))/(a+I*b)^(3/2)/d-4*a*b/(a^
2+b^2)/d/(a+b*cot(d*x+c))^(1/2)
```

Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 132, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.185, Rules used = {3610, 3620, 3618, 65, 214}

$$\int \frac{-a+b\cot(c+dx)}{(a+b\cot(c+dx))^{3/2}}\,dx = -\frac{4ab}{d(a^2+b^2)\sqrt{a+b\cot(c+dx)}} - \frac{(-b+ia)\text{arctanh}\left(\frac{\sqrt{a+b\cot(c+dx)}}{\sqrt{a-ib}}\right)}{d(a-ib)^{3/2}} + \frac{(b+ia)\text{arctanh}\left(\frac{\sqrt{a+b\cot(c+dx)}}{\sqrt{a+ib}}\right)}{d(a+ib)^{3/2}}$$

```
[In] Int[(-a + b*Cot[c + d*x])/(a + b*Cot[c + d*x])^(3/2),x]
```

```
[Out] -(((I*a - b)*ArcTanh[Sqrt[a + b*Cot[c + d*x]]/Sqrt[a - I*b]])/((a - I*b)^(3
/2)*d)) + ((I*a + b)*ArcTanh[Sqrt[a + b*Cot[c + d*x]]/Sqrt[a + I*b]])/((a +
 I*b)^(3/2)*d) - (4*a*b)/((a^2 + b^2)*d*Sqrt[a + b*Cot[c + d*x]])
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 3610

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)]), x_Symbol] :> Simp[(b*c - a*d)*((a + b*Tan[e + f*x])^(m + 1)/
(f*(m + 1)*(a^2 + b^2))), x] + Dist[1/(a^2 + b^2), Int[(a + b*Tan[e + f*x])
^(m + 1)*Simp[a*c + b*d - (b*c - a*d)*Tan[e + f*x], x], x], x] /; FreeQ[{a,
 b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && LtQ[m, -1]
```

Rule 3618

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*tan[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Dist[c*(d/f), Subst[Int[(a + (b/d)*x)^m/(d^2 + c
*x), x], x, d*Tan[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b
*c - a*d, 0] && NeQ[a^2 + b^2, 0] && EqQ[c^2 + d^2, 0]
```

Rule 3620

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)]), x_Symbol] :> Dist[(c + I*d)/2, Int[(a + b*Tan[e + f*x])^m*(1
 - I*Tan[e + f*x]), x], x] + Dist[(c - I*d)/2, Int[(a + b*Tan[e + f*x])^m*(
1 + I*Tan[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b*c -
 a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] &&  !IntegerQ[m]
```

Rubi steps

$$\text{integral} = -\frac{4ab}{(a^2+b^2)\,d\sqrt{a+b\cot(c+dx)}} + \frac{\int \frac{-a^2+b^2+2ab\cot(c+dx)}{\sqrt{a+b\cot(c+dx)}}\,dx}{a^2+b^2}$$

$$= -\frac{4ab}{(a^2+b^2)\,d\sqrt{a+b\cot(c+dx)}} - \frac{(a-ib)\int \frac{1-i\cot(c+dx)}{\sqrt{a+b\cot(c+dx)}}\,dx}{2(a+ib)} - \frac{(a+ib)\int \frac{1+i\cot(c+dx)}{\sqrt{a+b\cot(c+dx)}}\,dx}{2(a-ib)}$$

$$= -\frac{4ab}{(a^2+b^2)\,d\sqrt{a+b\cot(c+dx)}} - \frac{(a+ib)\text{Subst}\left(\int \frac{1}{(-1+x)\sqrt{a-ibx}}\,dx, x, i\cot(c+dx)\right)}{2(ia+b)d} - \frac{(ia+b)\text{Subst}\left(\int \frac{1}{(-1+x)\sqrt{a+ibx}}\,dx, x, -i\cot(c+dx)\right)}{2(a+ib)d}$$

$$= -\frac{4ab}{(a^2+b^2)\,d\sqrt{a+b\cot(c+dx)}} - \frac{(a-ib)\text{Subst}\left(\int \frac{1}{-1+\frac{ia}{b}-\frac{ix^2}{b}}\,dx, x, \sqrt{a+b\cot(c+dx)}\right)}{(a+ib)bd} - \frac{(a+ib)\text{Subst}\left(\int \frac{1}{-1-\frac{ia}{b}+\frac{ix^2}{b}}\,dx, x, \sqrt{a+b\cot(c+dx)}\right)}{(a-ib)bd}$$

$$= -\frac{(ia-b)\operatorname{arctanh}\left(\frac{\sqrt{a+b\cot(c+dx)}}{\sqrt{a-ib}}\right)}{(a-ib)^{3/2}d} + \frac{(ia+b)\operatorname{arctanh}\left(\frac{\sqrt{a+b\cot(c+dx)}}{\sqrt{a+ib}}\right)}{(a+ib)^{3/2}d} - \frac{4ab}{(a^2+b^2)\,d\sqrt{a+b\cot(c+dx)}}$$

Mathematica [A] (verified)

Time = 1.55 (sec) , antiderivative size = 190, normalized size of antiderivative = 1.44

$$\int \frac{-a+b\cot(c+dx)}{(a+b\cot(c+dx))^{3/2}}\,dx = \frac{b\left(\frac{\left(a^2-b^2+2a\sqrt{-b^2}\right)\operatorname{arctanh}\left(\frac{\sqrt{a+b\cot(c+dx)}}{\sqrt{a-\sqrt{-b^2}}}\right)}{\sqrt{-b^2}\sqrt{a-\sqrt{-b^2}}} + \frac{\left(-a^2+b^2+2a\sqrt{-b^2}\right)\operatorname{arctanh}\left(\frac{\sqrt{a+b\cot(c+dx)}}{\sqrt{a+\sqrt{-b^2}}}\right.}{\sqrt{-b^2}\sqrt{a+\sqrt{-b^2}}}\right.}{(a^2+b^2)\,d}$$

```
[In] Integrate[(-a + b*Cot[c + d*x])/(a + b*Cot[c + d*x])^(3/2),x]
```

```
[Out] (b*(((a^2 - b^2 + 2*a*Sqrt[-b^2])*ArcTanh[Sqrt[a + b*Cot[c + d*x]]/Sqrt[a -
 Sqrt[-b^2]]])/(Sqrt[-b^2]*Sqrt[a - Sqrt[-b^2]]) + ((-a^2 + b^2 + 2*a*Sqrt[
-b^2])*ArcTanh[Sqrt[a + b*Cot[c + d*x]]/Sqrt[a + Sqrt[-b^2]]])/(Sqrt[-b^2]*
Sqrt[a + Sqrt[-b^2]]) - (4*a)/Sqrt[a + b*Cot[c + d*x]]))/((a^2 + b^2)*d)
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 2290 vs. 2(112) = 224.

Time = 0.07 (sec) , antiderivative size = 2291, normalized size of antiderivative = 17.36

method	result	size
derivativedivides	Expression too large to display	2291
default	Expression too large to display	2291
parts	Expression too large to display	3684

```
[In] int((-a+b*cot(d*x+c))/(a+b*cot(d*x+c))^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] -2/d*b/(a^2+b^2)^2/(2*(a^2+b^2)^(1/2)-2*a)^(1/2)*arctan((2*(a+b*cot(d*x+c))
^(1/2)+(2*(a^2+b^2)^(1/2)+2*a)^(1/2))/(2*(a^2+b^2)^(1/2)-2*a)^(1/2))*a^3+1/
4/d/b/(a^2+b^2)^(5/2)*ln((a+b*cot(d*x+c))^(1/2)*(2*(a^2+b^2)^(1/2)+2*a)^(1/
2)-b*cot(d*x+c)-(a^2+b^2)^(1/2)-a)*(2*(a^2+b^2)^(1/2)+2*a)^(1/2)*a^5+1/d/b/
(a^2+b^2)^(5/2)/(2*(a^2+b^2)^(1/2)-2*a)^(1/2)*arctan((2*(a+b*cot(d*x+c))^(1
/2)+(2*(a^2+b^2)^(1/2)+2*a)^(1/2))/(2*(a^2+b^2)^(1/2)-2*a)^(1/2))*a^6+1/4/d
/b/(a^2+b^2)^2*ln(b*cot(d*x+c)+a+(a+b*cot(d*x+c))^(1/2)*(2*(a^2+b^2)^(1/2)+
2*a)^(1/2)+(a^2+b^2)^(1/2))*(2*(a^2+b^2)^(1/2)+2*a)^(1/2)*a^4-2/d*b^3/(a^2+
b^2)^2/(2*(a^2+b^2)^(1/2)-2*a)^(1/2)*arctan((2*(a+b*cot(d*x+c))^(1/2)+(2*(a
^2+b^2)^(1/2)+2*a)^(1/2))/(2*(a^2+b^2)^(1/2)-2*a)^(1/2))*a-3/4/d*b^3/(a^2+b
^2)^(5/2)*ln((a+b*cot(d*x+c))^(1/2)*(2*(a^2+b^2)^(1/2)+2*a)^(1/2)-b*cot(d*x
+c)-(a^2+b^2)^(1/2)-a)*(2*(a^2+b^2)^(1/2)+2*a)^(1/2)*a-1/d*b^3/(a^2+b^2)^(5
/2)/(2*(a^2+b^2)^(1/2)-2*a)^(1/2)*arctan((-2*(a+b*cot(d*x+c))^(1/2)+(2*(a^2
+b^2)^(1/2)+2*a)^(1/2))/(2*(a^2+b^2)^(1/2)-2*a)^(1/2))*a^2+1/d/b/(a^2+b^2)^
(3/2)/(2*(a^2+b^2)^(1/2)-2*a)^(1/2)*arctan((-2*(a+b*cot(d*x+c))^(1/2)+(2*(a
^2+b^2)^(1/2)+2*a)^(1/2))/(2*(a^2+b^2)^(1/2)-2*a)^(1/2))*a^4-1/d/b/(a^2+b^2
)^(5/2)/(2*(a^2+b^2)^(1/2)-2*a)^(1/2)*arctan((-2*(a+b*cot(d*x+c))^(1/2)+(2*
(a^2+b^2)^(1/2)+2*a)^(1/2))/(2*(a^2+b^2)^(1/2)-2*a)^(1/2))*a^6+3/4/d*b^3/(a
^2+b^2)^(5/2)*ln(b*cot(d*x+c)+a+(a+b*cot(d*x+c))^(1/2)*(2*(a^2+b^2)^(1/2)+2
*a)^(1/2)+(a^2+b^2)^(1/2))*(2*(a^2+b^2)^(1/2)+2*a)^(1/2)*a+1/d*b^3/(a^2+b^2
)^(5/2)/(2*(a^2+b^2)^(1/2)-2*a)^(1/2)*arctan((2*(a+b*cot(d*x+c))^(1/2)+(2*(
a^2+b^2)^(1/2)+2*a)^(1/2))/(2*(a^2+b^2)^(1/2)-2*a)^(1/2))*a^2-1/4/d/b/(a^2+
b^2)^(5/2)*ln(b*cot(d*x+c)+a+(a+b*cot(d*x+c))^(1/2)*(2*(a^2+b^2)^(1/2)+2*a)
^(1/2)+(a^2+b^2)^(1/2))*(2*(a^2+b^2)^(1/2)+2*a)^(1/2)*a^5-1/4/d/b/(a^2+b^2)
^2*ln((a+b*cot(d*x+c))^(1/2)*(2*(a^2+b^2)^(1/2)+2*a)^(1/2)-b*cot(d*x+c)-(a^
2+b^2)^(1/2)-a)*(2*(a^2+b^2)^(1/2)+2*a)^(1/2)*a^4+2/d*b^3/(a^2+b^2)^2/(2*(a
^2+b^2)^(1/2)-2*a)^(1/2)*arctan((-2*(a+b*cot(d*x+c))^(1/2)+(2*(a^2+b^2)^(1/
2)+2*a)^(1/2))/(2*(a^2+b^2)^(1/2)-2*a)^(1/2))*a-4/d*b/(a^2+b^2)^(5/2)/(2*(a
^2+b^2)^(1/2)-2*a)^(1/2)*arctan((-2*(a+b*cot(d*x+c))^(1/2)+(2*(a^2+b^2)^(1/
2)+2*a)^(1/2))/(2*(a^2+b^2)^(1/2)-2*a)^(1/2))*a^4+1/2/d*b/(a^2+b^2)^(5/2)*l
n(b*cot(d*x+c)+a+(a+b*cot(d*x+c))^(1/2)*(2*(a^2+b^2)^(1/2)+2*a)^(1/2)+(a^2+
b^2)^(1/2))*(2*(a^2+b^2)^(1/2)+2*a)^(1/2)*a^3+4/d*b/(a^2+b^2)^(5/2)/(2*(a^2
+b^2)^(1/2)-2*a)^(1/2)*arctan((2*(a+b*cot(d*x+c))^(1/2)+(2*(a^2+b^2)^(1/2)+
```

```
2*a)^(1/2))/(2*(a^2+b^2)^(1/2)-2*a)^(1/2))*a^4-1/2/d*b/(a^2+b^2)^(5/2)*ln((
a+b*cot(d*x+c))^(1/2)*(2*(a^2+b^2)^(1/2)+2*a)^(1/2)-b*cot(d*x+c)-(a^2+b^2)^
(1/2)-a)*(2*(a^2+b^2)^(1/2)+2*a)^(1/2)*a^3+2/d*b/(a^2+b^2)^2/(2*(a^2+b^2)^(
1/2)-2*a)^(1/2)*arctan((-2*(a+b*cot(d*x+c))^(1/2)+(2*(a^2+b^2)^(1/2)+2*a)^(
1/2))/(2*(a^2+b^2)^(1/2)-2*a)^(1/2))*a^3-1/d/b/(a^2+b^2)^(3/2)/(2*(a^2+b^2)
^(1/2)-2*a)^(1/2)*arctan((2*(a+b*cot(d*x+c))^(1/2)+(2*(a^2+b^2)^(1/2)+2*a)^
(1/2))/(2*(a^2+b^2)^(1/2)-2*a)^(1/2))*a^4-4*a*b/(a^2+b^2)/d/(a+b*cot(d*x+c)
)^(1/2)-1/4/d*b^3/(a^2+b^2)^2*ln(b*cot(d*x+c)+a+(a+b*cot(d*x+c))^(1/2)*(2*(
a^2+b^2)^(1/2)+2*a)^(1/2)+(a^2+b^2)^(1/2))*(2*(a^2+b^2)^(1/2)+2*a)^(1/2)+1/
4/d*b^3/(a^2+b^2)^2*ln((a+b*cot(d*x+c))^(1/2)*(2*(a^2+b^2)^(1/2)+2*a)^(1/2)
-b*cot(d*x+c)-(a^2+b^2)^(1/2)-a)*(2*(a^2+b^2)^(1/2)+2*a)^(1/2)+2/d*b^5/(a^2
+b^2)^(5/2)/(2*(a^2+b^2)^(1/2)-2*a)^(1/2)*arctan((-2*(a+b*cot(d*x+c))^(1/2)
+(2*(a^2+b^2)^(1/2)+2*a)^(1/2))/(2*(a^2+b^2)^(1/2)-2*a)^(1/2))-1/d*b^3/(a^2
+b^2)^(3/2)/(2*(a^2+b^2)^(1/2)-2*a)^(1/2)*arctan((-2*(a+b*cot(d*x+c))^(1/2)
+(2*(a^2+b^2)^(1/2)+2*a)^(1/2))/(2*(a^2+b^2)^(1/2)-2*a)^(1/2))-2/d*b^5/(a^2
+b^2)^(5/2)/(2*(a^2+b^2)^(1/2)-2*a)^(1/2)*arctan((2*(a+b*cot(d*x+c))^(1/2)+
(2*(a^2+b^2)^(1/2)+2*a)^(1/2))/(2*(a^2+b^2)^(1/2)-2*a)^(1/2))+1/d*b^3/(a^2+
b^2)^(3/2)/(2*(a^2+b^2)^(1/2)-2*a)^(1/2)*arctan((2*(a+b*cot(d*x+c))^(1/2)+(
2*(a^2+b^2)^(1/2)+2*a)^(1/2))/(2*(a^2+b^2)^(1/2)-2*a)^(1/2))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2574 vs. $2(103) = 206$.

Time = 0.36 (sec) , antiderivative size = 2574, normalized size of antiderivative = 19.50

$$\int \frac{-a+b\cot(c+dx)}{(a+b\cot(c+dx))^{3/2}}\,dx = \text{Too large to display}$$

```
[In] integrate((-a+b*cot(d*x+c))/(a+b*cot(d*x+c))^(3/2),x, algorithm="fricas")
```

```
[Out] -1/2*(8*a*b*sqrt((b*cos(2*d*x + 2*c) + a*sin(2*d*x + 2*c) + b)/sin(2*d*x +
2*c))*sin(2*d*x + 2*c) - ((a^2*b + b^3)*d*cos(2*d*x + 2*c) + (a^3 + a*b^2)*
d*sin(2*d*x + 2*c) + (a^2*b + b^3)*d)*sqrt(-(a^5 - 10*a^3*b^2 + 5*a*b^4 + (
a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6)*d^2*sqrt(-(25*a^8*b^2 - 100*a^6*b^4 + 11
0*a^4*b^6 - 20*a^2*b^8 + b^10)/((a^12 + 6*a^10*b^2 + 15*a^8*b^4 + 20*a^6*b^
6 + 15*a^4*b^8 + 6*a^2*b^10 + b^12)*d^4)))/((a^6 + 3*a^4*b^2 + 3*a^2*b^4 +
b^6)*d^2))*log((5*a^6*b - 5*a^4*b^3 - 9*a^2*b^5 + b^7)*sqrt((b*cos(2*d*x +
2*c) + a*sin(2*d*x + 2*c) + b)/sin(2*d*x + 2*c)) + ((a^9 - 6*a^5*b^4 - 8*a^
3*b^6 - 3*a*b^8)*d^3*sqrt(-(25*a^8*b^2 - 100*a^6*b^4 + 110*a^4*b^6 - 20*a^2
*b^8 + b^10)/((a^12 + 6*a^10*b^2 + 15*a^8*b^4 + 20*a^6*b^6 + 15*a^4*b^8 + 6
*a^2*b^10 + b^12)*d^4)) + (15*a^6*b^2 - 35*a^4*b^4 + 13*a^2*b^6 - b^8)*d)*s
qrt(-(a^5 - 10*a^3*b^2 + 5*a*b^4 + (a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6)*d^2*
sqrt(-(25*a^8*b^2 - 100*a^6*b^4 + 110*a^4*b^6 - 20*a^2*b^8 + b^10)/((a^12 +
 6*a^10*b^2 + 15*a^8*b^4 + 20*a^6*b^6 + 15*a^4*b^8 + 6*a^2*b^10 + b^12)*d^4
)))/((a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6)*d^2))) + ((a^2*b + b^3)*d*cos(2*d*
```

```
x + 2*c) + (a^3 + a*b^2)*d*sin(2*d*x + 2*c) + (a^2*b + b^3)*d)*sqrt(-(a^5 -
 10*a^3*b^2 + 5*a*b^4 + (a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6)*d^2*sqrt(-(25*a
^8*b^2 - 100*a^6*b^4 + 110*a^4*b^6 - 20*a^2*b^8 + b^10)/((a^12 + 6*a^10*b^2
 + 15*a^8*b^4 + 20*a^6*b^6 + 15*a^4*b^8 + 6*a^2*b^10 + b^12)*d^4)))/((a^6 +
 3*a^4*b^2 + 3*a^2*b^4 + b^6)*d^2))*log((5*a^6*b - 5*a^4*b^3 - 9*a^2*b^5 +
b^7)*sqrt((b*cos(2*d*x + 2*c) + a*sin(2*d*x + 2*c) + b)/sin(2*d*x + 2*c)) -
 ((a^9 - 6*a^5*b^4 - 8*a^3*b^6 - 3*a*b^8)*d^3*sqrt(-(25*a^8*b^2 - 100*a^6*b
^4 + 110*a^4*b^6 - 20*a^2*b^8 + b^10)/((a^12 + 6*a^10*b^2 + 15*a^8*b^4 + 20
*a^6*b^6 + 15*a^4*b^8 + 6*a^2*b^10 + b^12)*d^4)) + (15*a^6*b^2 - 35*a^4*b^4
 + 13*a^2*b^6 - b^8)*d)*sqrt(-(a^5 - 10*a^3*b^2 + 5*a*b^4 + (a^6 + 3*a^4*b^
2 + 3*a^2*b^4 + b^6)*d^2*sqrt(-(25*a^8*b^2 - 100*a^6*b^4 + 110*a^4*b^6 - 20
*a^2*b^8 + b^10)/((a^12 + 6*a^10*b^2 + 15*a^8*b^4 + 20*a^6*b^6 + 15*a^4*b^8
 + 6*a^2*b^10 + b^12)*d^4)))/((a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6)*d^2))) +
((a^2*b + b^3)*d*cos(2*d*x + 2*c) + (a^3 + a*b^2)*d*sin(2*d*x + 2*c) + (a^2
*b + b^3)*d)*sqrt(-(a^5 - 10*a^3*b^2 + 5*a*b^4 - (a^6 + 3*a^4*b^2 + 3*a^2*b
^4 + b^6)*d^2*sqrt(-(25*a^8*b^2 - 100*a^6*b^4 + 110*a^4*b^6 - 20*a^2*b^8 +
b^10)/((a^12 + 6*a^10*b^2 + 15*a^8*b^4 + 20*a^6*b^6 + 15*a^4*b^8 + 6*a^2*b^
10 + b^12)*d^4)))/((a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6)*d^2))*log((5*a^6*b -
 5*a^4*b^3 - 9*a^2*b^5 + b^7)*sqrt((b*cos(2*d*x + 2*c) + a*sin(2*d*x + 2*c)
 + b)/sin(2*d*x + 2*c)) + ((a^9 - 6*a^5*b^4 - 8*a^3*b^6 - 3*a*b^8)*d^3*sqrt
(-(25*a^8*b^2 - 100*a^6*b^4 + 110*a^4*b^6 - 20*a^2*b^8 + b^10)/((a^12 + 6*a
^10*b^2 + 15*a^8*b^4 + 20*a^6*b^6 + 15*a^4*b^8 + 6*a^2*b^10 + b^12)*d^4)) -
 (15*a^6*b^2 - 35*a^4*b^4 + 13*a^2*b^6 - b^8)*d)*sqrt(-(a^5 - 10*a^3*b^2 +
5*a*b^4 - (a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6)*d^2*sqrt(-(25*a^8*b^2 - 100*a
^6*b^4 + 110*a^4*b^6 - 20*a^2*b^8 + b^10)/((a^12 + 6*a^10*b^2 + 15*a^8*b^4
+ 20*a^6*b^6 + 15*a^4*b^8 + 6*a^2*b^10 + b^12)*d^4)))/((a^6 + 3*a^4*b^2 + 3
*a^2*b^4 + b^6)*d^2))) - ((a^2*b + b^3)*d*cos(2*d*x + 2*c) + (a^3 + a*b^2)*
d*sin(2*d*x + 2*c) + (a^2*b + b^3)*d)*sqrt(-(a^5 - 10*a^3*b^2 + 5*a*b^4 - (
a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6)*d^2*sqrt(-(25*a^8*b^2 - 100*a^6*b^4 + 11
0*a^4*b^6 - 20*a^2*b^8 + b^10)/((a^12 + 6*a^10*b^2 + 15*a^8*b^4 + 20*a^6*b^
6 + 15*a^4*b^8 + 6*a^2*b^10 + b^12)*d^4)))/((a^6 + 3*a^4*b^2 + 3*a^2*b^4 +
b^6)*d^2))*log((5*a^6*b - 5*a^4*b^3 - 9*a^2*b^5 + b^7)*sqrt((b*cos(2*d*x +
2*c) + a*sin(2*d*x + 2*c) + b)/sin(2*d*x + 2*c)) - ((a^9 - 6*a^5*b^4 - 8*a^
3*b^6 - 3*a*b^8)*d^3*sqrt(-(25*a^8*b^2 - 100*a^6*b^4 + 110*a^4*b^6 - 20*a^2
*b^8 + b^10)/((a^12 + 6*a^10*b^2 + 15*a^8*b^4 + 20*a^6*b^6 + 15*a^4*b^8 + 6
*a^2*b^10 + b^12)*d^4)) - (15*a^6*b^2 - 35*a^4*b^4 + 13*a^2*b^6 - b^8)*d)*s
qrt(-(a^5 - 10*a^3*b^2 + 5*a*b^4 - (a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6)*d^2*
sqrt(-(25*a^8*b^2 - 100*a^6*b^4 + 110*a^4*b^6 - 20*a^2*b^8 + b^10)/((a^12 +
 6*a^10*b^2 + 15*a^8*b^4 + 20*a^6*b^6 + 15*a^4*b^8 + 6*a^2*b^10 + b^12)*d^4
)))/((a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6)*d^2))))/((a^2*b + b^3)*d*cos(2*d*x
 + 2*c) + (a^3 + a*b^2)*d*sin(2*d*x + 2*c) + (a^2*b + b^3)*d)
```

Sympy [F]

$$\int \frac{-a+b\cot(c+dx)}{(a+b\cot(c+dx))^{3/2}}\,dx = -\int \frac{a}{a\sqrt{a+b\cot(c+dx)}+b\sqrt{a+b\cot(c+dx)}\cot(c+dx)}\,dx - \int \left(-\frac{b\cot(c+dx)}{a\sqrt{a+b\cot(c+dx)}+b\sqrt{a+b\cot(c+dx)}\cot(c+dx)}\right)dx$$

[In] integrate((-a+b*cot(d*x+c))/(a+b*cot(d*x+c))**(3/2),x)

[Out] -Integral(a/(a*sqrt(a + b*cot(c + d*x)) + b*sqrt(a + b*cot(c + d*x))*cot(c + d*x)), x) - Integral(-b*cot(c + d*x)/(a*sqrt(a + b*cot(c + d*x)) + b*sqrt(a + b*cot(c + d*x))*cot(c + d*x)), x)

Maxima [F]

$$\int \frac{-a+b\cot(c+dx)}{(a+b\cot(c+dx))^{3/2}}\,dx = \int \frac{b\cot(dx+c)-a}{(b\cot(dx+c)+a)^{\frac{3}{2}}}\,dx$$

[In] integrate((-a+b*cot(d*x+c))/(a+b*cot(d*x+c))^(3/2),x, algorithm="maxima")

[Out] integrate((b*cot(d*x + c) - a)/(b*cot(d*x + c) + a)^(3/2), x)

Giac [F]

$$\int \frac{-a+b\cot(c+dx)}{(a+b\cot(c+dx))^{3/2}}\,dx = \int \frac{b\cot(dx+c)-a}{(b\cot(dx+c)+a)^{\frac{3}{2}}}\,dx$$

[In] integrate((-a+b*cot(d*x+c))/(a+b*cot(d*x+c))^(3/2),x, algorithm="giac")

[Out] integrate((b*cot(d*x + c) - a)/(b*cot(d*x + c) + a)^(3/2), x)

Mupad [B] (verification not implemented)

Time = 19.01 (sec) , antiderivative size = 5475, normalized size of antiderivative = 41.48

$$\int \frac{-a+b\cot(c+dx)}{(a+b\cot(c+dx))^{3/2}}\,dx = \text{Too large to display}$$

```
[In] int(-(a - b*cot(c + d*x))/(a + b*cot(c + d*x))^(3/2),x)
```

```
[Out] log(8*a*b^11*d^2 - (((((24*a*b^4*d^2 - 8*a^3*b^2*d^2)^2/4 - b^4*(16*a^6*d^4
 + 16*b^6*d^4 + 48*a^2*b^4*d^4 + 48*a^4*b^2*d^4))^(1/2) - 12*a*b^4*d^2 + 4*
a^3*b^2*d^2)/(16*(a^6*d^4 + b^6*d^4 + 3*a^2*b^4*d^4 + 3*a^4*b^2*d^4)))^(1/2
)*(64*a^6*b^7*d^4 - 96*a^2*b^11*d^4 - 64*a^4*b^9*d^4 - 32*b^13*d^4 + 96*a^8
*b^5*d^4 + 32*a^10*b^3*d^4 + (a + b*cot(c + d*x))^(1/2)*((((24*a*b^4*d^2 -
8*a^3*b^2*d^2)^2/4 - b^4*(16*a^6*d^4 + 16*b^6*d^4 + 48*a^2*b^4*d^4 + 48*a^4
*b^2*d^4))^(1/2) - 12*a*b^4*d^2 + 4*a^3*b^2*d^2)/(16*(a^6*d^4 + b^6*d^4 + 3
*a^2*b^4*d^4 + 3*a^4*b^2*d^4)))^(1/2)*(64*a*b^12*d^5 + 320*a^3*b^10*d^5 + 6
40*a^5*b^8*d^5 + 640*a^7*b^6*d^5 + 320*a^9*b^4*d^5 + 64*a^11*b^2*d^5)) + (a
 + b*cot(c + d*x))^(1/2)*(16*b^12*d^3 + 32*a^2*b^10*d^3 - 32*a^6*b^6*d^3 -
16*a^8*b^4*d^3))*((((24*a*b^4*d^2 - 8*a^3*b^2*d^2)^2/4 - b^4*(16*a^6*d^4 +
16*b^6*d^4 + 48*a^2*b^4*d^4 + 48*a^4*b^2*d^4))^(1/2) - 12*a*b^4*d^2 + 4*a^3
*b^2*d^2)/(16*(a^6*d^4 + b^6*d^4 + 3*a^2*b^4*d^4 + 3*a^4*b^2*d^4)))^(1/2) +
 24*a^3*b^9*d^2 + 24*a^5*b^7*d^2 + 8*a^7*b^5*d^2)*((((24*a*b^4*d^2 - 8*a^3*
b^2*d^2)^2/4 - b^4*(16*a^6*d^4 + 16*b^6*d^4 + 48*a^2*b^4*d^4 + 48*a^4*b^2*d
^4))^(1/2) - 12*a*b^4*d^2 + 4*a^3*b^2*d^2)/(16*(a^6*d^4 + b^6*d^4 + 3*a^2*b
^4*d^4 + 3*a^4*b^2*d^4)))^(1/2) + log(8*a*b^11*d^2 - ((-(((24*a*b^4*d^2 - 8
*a^3*b^2*d^2)^2/4 - b^4*(16*a^6*d^4 + 16*b^6*d^4 + 48*a^2*b^4*d^4 + 48*a^4*
b^2*d^4))^(1/2) + 12*a*b^4*d^2 - 4*a^3*b^2*d^2)/(16*(a^6*d^4 + b^6*d^4 + 3*
a^2*b^4*d^4 + 3*a^4*b^2*d^4)))^(1/2)*(64*a^6*b^7*d^4 - 96*a^2*b^11*d^4 - 64
*a^4*b^9*d^4 - 32*b^13*d^4 + 96*a^8*b^5*d^4 + 32*a^10*b^3*d^4 + (a + b*cot(
c + d*x))^(1/2)*(-(((24*a*b^4*d^2 - 8*a^3*b^2*d^2)^2/4 - b^4*(16*a^6*d^4 +
16*b^6*d^4 + 48*a^2*b^4*d^4 + 48*a^4*b^2*d^4))^(1/2) + 12*a*b^4*d^2 - 4*a^3
*b^2*d^2)/(16*(a^6*d^4 + b^6*d^4 + 3*a^2*b^4*d^4 + 3*a^4*b^2*d^4)))^(1/2)*(
64*a*b^12*d^5 + 320*a^3*b^10*d^5 + 640*a^5*b^8*d^5 + 640*a^7*b^6*d^5 + 320*
a^9*b^4*d^5 + 64*a^11*b^2*d^5)) + (a + b*cot(c + d*x))^(1/2)*(16*b^12*d^3 +
 32*a^2*b^10*d^3 - 32*a^6*b^6*d^3 - 16*a^8*b^4*d^3))*(-(((24*a*b^4*d^2 - 8*
a^3*b^2*d^2)^2/4 - b^4*(16*a^6*d^4 + 16*b^6*d^4 + 48*a^2*b^4*d^4 + 48*a^4*b
^2*d^4))^(1/2) + 12*a*b^4*d^2 - 4*a^3*b^2*d^2)/(16*(a^6*d^4 + b^6*d^4 + 3*a
^2*b^4*d^4 + 3*a^4*b^2*d^4)))^(1/2) + 24*a^3*b^9*d^2 + 24*a^5*b^7*d^2 + 8*a
^7*b^5*d^2)*(-(((24*a*b^4*d^2 - 8*a^3*b^2*d^2)^2/4 - b^4*(16*a^6*d^4 + 16*b
^6*d^4 + 48*a^2*b^4*d^4 + 48*a^4*b^2*d^4))^(1/2) + 12*a*b^4*d^2 - 4*a^3*b^2
*d^2)/(16*(a^6*d^4 + b^6*d^4 + 3*a^2*b^4*d^4 + 3*a^4*b^2*d^4)))^(1/2) + (lo
g((((a + b*cot(c + d*x))^(1/2)*(16*a^2*b^10*d^3 + 32*a^4*b^8*d^3 - 32*a^8*b
^4*d^3 - 16*a^10*b^2*d^3) - ((((96*a^6*b^4*d^4 - 16*a^4*b^6*d^4 - 144*a^8*b
^2*d^4)^(1/2) - 4*a^5*d^2 + 12*a^3*b^2*d^2)/(a^6*d^4 + b^6*d^4 + 3*a^2*b^4*
```

d^4 + 3*a^4*b^2*d^4))^(1/2)*(((((96*a^6*b^4*d^4 - 16*a^4*b^6*d^4 - 144*a^8*
b^2*d^4)^(1/2) - 4*a^5*d^2 + 12*a^3*b^2*d^2)/(a^6*d^4 + b^6*d^4 + 3*a^2*b^4
*d^4 + 3*a^4*b^2*d^4))^(1/2)*(a + b*cot(c + d*x))^(1/2)*(64*a*b^12*d^5 + 32
0*a^3*b^10*d^5 + 640*a^5*b^8*d^5 + 640*a^7*b^6*d^5 + 320*a^9*b^4*d^5 + 64*a
^11*b^2*d^5))/4 + 64*a^2*b^11*d^4 + 256*a^4*b^9*d^4 + 384*a^6*b^7*d^4 + 256
*a^8*b^5*d^4 + 64*a^10*b^3*d^4))/4)*(((96*a^6*b^4*d^4 - 16*a^4*b^6*d^4 - 14
4*a^8*b^2*d^4)^(1/2) - 4*a^5*d^2 + 12*a^3*b^2*d^2)/(a^6*d^4 + b^6*d^4 + 3*a
^2*b^4*d^4 + 3*a^4*b^2*d^4))^(1/2))/4 + 8*a^3*b^9*d^2 + 24*a^5*b^7*d^2 + 24
*a^7*b^5*d^2 + 8*a^9*b^3*d^2)*(((96*a^6*b^4*d^4 - 16*a^4*b^6*d^4 - 144*a^8*
b^2*d^4)^(1/2) - 4*a^5*d^2 + 12*a^3*b^2*d^2)/(a^6*d^4 + b^6*d^4 + 3*a^2*b^4
*d^4 + 3*a^4*b^2*d^4))^(1/2))/4 + (log((((a + b*cot(c + d*x))^(1/2)*(16*a^2
*b^10*d^3 + 32*a^4*b^8*d^3 - 32*a^8*b^4*d^3 - 16*a^10*b^2*d^3) - ((-((96*a^
6*b^4*d^4 - 16*a^4*b^6*d^4 - 144*a^8*b^2*d^4)^(1/2) + 4*a^5*d^2 - 12*a^3*b^
2*d^2)/(a^6*d^4 + b^6*d^4 + 3*a^2*b^4*d^4 + 3*a^4*b^2*d^4))^(1/2)*(((-((96*
a^6*b^4*d^4 - 16*a^4*b^6*d^4 - 144*a^8*b^2*d^4)^(1/2) + 4*a^5*d^2 - 12*a^3*
b^2*d^2)/(a^6*d^4 + b^6*d^4 + 3*a^2*b^4*d^4 + 3*a^4*b^2*d^4))^(1/2)*(a + b*
cot(c + d*x))^(1/2)*(64*a*b^12*d^5 + 320*a^3*b^10*d^5 + 640*a^5*b^8*d^5 + 6
40*a^7*b^6*d^5 + 320*a^9*b^4*d^5 + 64*a^11*b^2*d^5))/4 + 64*a^2*b^11*d^4 +
256*a^4*b^9*d^4 + 384*a^6*b^7*d^4 + 256*a^8*b^5*d^4 + 64*a^10*b^3*d^4))/4)*
(-((96*a^6*b^4*d^4 - 16*a^4*b^6*d^4 - 144*a^8*b^2*d^4)^(1/2) + 4*a^5*d^2 -
12*a^3*b^2*d^2)/(a^6*d^4 + b^6*d^4 + 3*a^2*b^4*d^4 + 3*a^4*b^2*d^4))^(1/2))
/4 + 8*a^3*b^9*d^2 + 24*a^5*b^7*d^2 + 24*a^7*b^5*d^2 + 8*a^9*b^3*d^2)*(-((9
6*a^6*b^4*d^4 - 16*a^4*b^6*d^4 - 144*a^8*b^2*d^4)^(1/2) + 4*a^5*d^2 - 12*a^
3*b^2*d^2)/(a^6*d^4 + b^6*d^4 + 3*a^2*b^4*d^4 + 3*a^4*b^2*d^4))^(1/2))/4 -
log(8*a^3*b^9*d^2 - ((a + b*cot(c + d*x))^(1/2)*(16*a^2*b^10*d^3 + 32*a^4*b
^8*d^3 - 32*a^8*b^4*d^3 - 16*a^10*b^2*d^3) + (((96*a^6*b^4*d^4 - 16*a^4*b^6
*d^4 - 144*a^8*b^2*d^4)^(1/2) - 4*a^5*d^2 + 12*a^3*b^2*d^2)/(16*a^6*d^4 + 1
6*b^6*d^4 + 48*a^2*b^4*d^4 + 48*a^4*b^2*d^4))^(1/2)*(64*a^2*b^11*d^4 - (((9
6*a^6*b^4*d^4 - 16*a^4*b^6*d^4 - 144*a^8*b^2*d^4)^(1/2) - 4*a^5*d^2 + 12*a^
3*b^2*d^2)/(16*a^6*d^4 + 16*b^6*d^4 + 48*a^2*b^4*d^4 + 48*a^4*b^2*d^4))^(1/
2)*(a + b*cot(c + d*x))^(1/2)*(64*a*b^12*d^5 + 320*a^3*b^10*d^5 + 640*a^5*b
^8*d^5 + 640*a^7*b^6*d^5 + 320*a^9*b^4*d^5 + 64*a^11*b^2*d^5) + 256*a^4*b^9
*d^4 + 384*a^6*b^7*d^4 + 256*a^8*b^5*d^4 + 64*a^10*b^3*d^4))*(((96*a^6*b^4*
d^4 - 16*a^4*b^6*d^4 - 144*a^8*b^2*d^4)^(1/2) - 4*a^5*d^2 + 12*a^3*b^2*d^2)
/(16*a^6*d^4 + 16*b^6*d^4 + 48*a^2*b^4*d^4 + 48*a^4*b^2*d^4))^(1/2) + 24*a^
5*b^7*d^2 + 24*a^7*b^5*d^2 + 8*a^9*b^3*d^2)*(((96*a^6*b^4*d^4 - 16*a^4*b^6*
d^4 - 144*a^8*b^2*d^4)^(1/2) - 4*a^5*d^2 + 12*a^3*b^2*d^2)/(16*a^6*d^4 + 16
*b^6*d^4 + 48*a^2*b^4*d^4 + 48*a^4*b^2*d^4))^(1/2) - log(8*a^3*b^9*d^2 - ((
a + b*cot(c + d*x))^(1/2)*(16*a^2*b^10*d^3 + 32*a^4*b^8*d^3 - 32*a^8*b^4*d^
3 - 16*a^10*b^2*d^3) + (-((96*a^6*b^4*d^4 - 16*a^4*b^6*d^4 - 144*a^8*b^2*d^
4)^(1/2) + 4*a^5*d^2 - 12*a^3*b^2*d^2)/(16*a^6*d^4 + 16*b^6*d^4 + 48*a^2*b^
4*d^4 + 48*a^4*b^2*d^4))^(1/2)*(64*a^2*b^11*d^4 - (-((96*a^6*b^4*d^4 - 16*a
^4*b^6*d^4 - 144*a^8*b^2*d^4)^(1/2) + 4*a^5*d^2 - 12*a^3*b^2*d^2)/(16*a^6*d
^4 + 16*b^6*d^4 + 48*a^2*b^4*d^4 + 48*a^4*b^2*d^4))^(1/2)*(a + b*cot(c + d*
x))^(1/2)*(64*a*b^12*d^5 + 320*a^3*b^10*d^5 + 640*a^5*b^8*d^5 + 640*a^7*b^6

*d^5 + 320*a^9*b^4*d^5 + 64*a^11*b^2*d^5) + 256*a^4*b^9*d^4 + 384*a^6*b^7*d
^4 + 256*a^8*b^5*d^4 + 64*a^10*b^3*d^4))*(-((96*a^6*b^4*d^4 - 16*a^4*b^6*d^
4 - 144*a^8*b^2*d^4)^(1/2) + 4*a^5*d^2 - 12*a^3*b^2*d^2)/(16*a^6*d^4 + 16*b
^6*d^4 + 48*a^2*b^4*d^4 + 48*a^4*b^2*d^4))^(1/2) + 24*a^5*b^7*d^2 + 24*a^7*
b^5*d^2 + 8*a^9*b^3*d^2)*(-((96*a^6*b^4*d^4 - 16*a^4*b^6*d^4 - 144*a^8*b^2*
d^4)^(1/2) + 4*a^5*d^2 - 12*a^3*b^2*d^2)/(16*a^6*d^4 + 16*b^6*d^4 + 48*a^2*
b^4*d^4 + 48*a^4*b^2*d^4))^(1/2) - log(((a + b*cot(c + d*x))^(1/2)*(16*b^12
*d^3 + 32*a^2*b^10*d^3 - 32*a^6*b^6*d^3 - 16*a^8*b^4*d^3) + (((96*a^2*b^8*d
^4 - 16*b^10*d^4 - 144*a^4*b^6*d^4)^(1/2) - 12*a*b^4*d^2 + 4*a^3*b^2*d^2)/(
16*a^6*d^4 + 16*b^6*d^4 + 48*a^2*b^4*d^4 + 48*a^4*b^2*d^4))^(1/2)*(32*b^13*
d^4 + (((96*a^2*b^8*d^4 - 16*b^10*d^4 - 144*a^4*b^6*d^4)^(1/2) - 12*a*b^4*d
^2 + 4*a^3*b^2*d^2)/(16*a^6*d^4 + 16*b^6*d^4 + 48*a^2*b^4*d^4 + 48*a^4*b^2*
d^4))^(1/2)*(a + b*cot(c + d*x))^(1/2)*(64*a*b^12*d^5 + 320*a^3*b^10*d^5 +
640*a^5*b^8*d^5 + 640*a^7*b^6*d^5 + 320*a^9*b^4*d^5 + 64*a^11*b^2*d^5) + 96
*a^2*b^11*d^4 + 64*a^4*b^9*d^4 - 64*a^6*b^7*d^4 - 96*a^8*b^5*d^4 - 32*a^10*
b^3*d^4))*(((96*a^2*b^8*d^4 - 16*b^10*d^4 - 144*a^4*b^6*d^4)^(1/2) - 12*a*b
^4*d^2 + 4*a^3*b^2*d^2)/(16*a^6*d^4 + 16*b^6*d^4 + 48*a^2*b^4*d^4 + 48*a^4*
b^2*d^4))^(1/2) + 8*a*b^11*d^2 + 24*a^3*b^9*d^2 + 24*a^5*b^7*d^2 + 8*a^7*b^
5*d^2)*(((96*a^2*b^8*d^4 - 16*b^10*d^4 - 144*a^4*b^6*d^4)^(1/2) - 12*a*b^4*
d^2 + 4*a^3*b^2*d^2)/(16*a^6*d^4 + 16*b^6*d^4 + 48*a^2*b^4*d^4 + 48*a^4*b^2
*d^4))^(1/2) - log(((a + b*cot(c + d*x))^(1/2)*(16*b^12*d^3 + 32*a^2*b^10*d
^3 - 32*a^6*b^6*d^3 - 16*a^8*b^4*d^3) + (-((96*a^2*b^8*d^4 - 16*b^10*d^4 -
144*a^4*b^6*d^4)^(1/2) + 12*a*b^4*d^2 - 4*a^3*b^2*d^2)/(16*a^6*d^4 + 16*b^6
*d^4 + 48*a^2*b^4*d^4 + 48*a^4*b^2*d^4))^(1/2)*(32*b^13*d^4 + (-((96*a^2*b^
8*d^4 - 16*b^10*d^4 - 144*a^4*b^6*d^4)^(1/2) + 12*a*b^4*d^2 - 4*a^3*b^2*d^2
)/(16*a^6*d^4 + 16*b^6*d^4 + 48*a^2*b^4*d^4 + 48*a^4*b^2*d^4))^(1/2)*(a + b
*cot(c + d*x))^(1/2)*(64*a*b^12*d^5 + 320*a^3*b^10*d^5 + 640*a^5*b^8*d^5 +
640*a^7*b^6*d^5 + 320*a^9*b^4*d^5 + 64*a^11*b^2*d^5) + 96*a^2*b^11*d^4 + 64
*a^4*b^9*d^4 - 64*a^6*b^7*d^4 - 96*a^8*b^5*d^4 - 32*a^10*b^3*d^4))*(-((96*a
^2*b^8*d^4 - 16*b^10*d^4 - 144*a^4*b^6*d^4)^(1/2) + 12*a*b^4*d^2 - 4*a^3*b^
2*d^2)/(16*a^6*d^4 + 16*b^6*d^4 + 48*a^2*b^4*d^4 + 48*a^4*b^2*d^4))^(1/2) +
 8*a*b^11*d^2 + 24*a^3*b^9*d^2 + 24*a^5*b^7*d^2 + 8*a^7*b^5*d^2)*(-((96*a^2
*b^8*d^4 - 16*b^10*d^4 - 144*a^4*b^6*d^4)^(1/2) + 12*a*b^4*d^2 - 4*a^3*b^2*
d^2)/(16*a^6*d^4 + 16*b^6*d^4 + 48*a^2*b^4*d^4 + 48*a^4*b^2*d^4))^(1/2) - (
4*a*b)/(d*(a^2 + b^2)*(a + b*cot(c + d*x))^(1/2))

3.106 $\int \frac{-a+b\cot(c+dx)}{(a+b\cot(c+dx))^{5/2}}\,dx$

Optimal result

Integrand size = 27, antiderivative size = 174

$$\begin{aligned}\int \frac{-a+b\cot(c+dx)}{(a+b\cot(c+dx))^{5/2}}\,dx &= -\frac{(ia-b)\operatorname{arctanh}\left(\frac{\sqrt{a+b\cot(c+dx)}}{\sqrt{a-ib}}\right)}{(a-ib)^{5/2}d} \\ &\quad+\frac{(ia+b)\operatorname{arctanh}\left(\frac{\sqrt{a+b\cot(c+dx)}}{\sqrt{a+ib}}\right)}{(a+ib)^{5/2}d} \\ &\quad-\frac{4ab}{3(a^2+b^2)\,d(a+b\cot(c+dx))^{3/2}}-\frac{2b(3a^2-b^2)}{(a^2+b^2)^2\,d\sqrt{a+b\cot(c+dx)}}\end{aligned}$$

```
[Out] -(I*a-b)*arctanh((a+b*cot(d*x+c))^(1/2)/(a-I*b)^(1/2))/(a-I*b)^(5/2)/d+(I*a
+b)*arctanh((a+b*cot(d*x+c))^(1/2)/(a+I*b)^(1/2))/(a+I*b)^(5/2)/d-4/3*a*b/(
a^2+b^2)/d/(a+b*cot(d*x+c))^(3/2)-2*b*(3*a^2-b^2)/(a^2+b^2)^2/d/(a+b*cot(d*
x+c))^(1/2)
```

Rubi [A] (verified)

Time = 0.48 (sec) , antiderivative size = 174, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.185, Rules used = {3610, 3620, 3618, 65, 214}

$$\begin{aligned}\int \frac{-a+b\cot(c+dx)}{(a+b\cot(c+dx))^{5/2}}\,dx &= -\frac{2b(3a^2-b^2)}{d(a^2+b^2)^2\sqrt{a+b\cot(c+dx)}} \\ &\quad-\frac{4ab}{3d(a^2+b^2)(a+b\cot(c+dx))^{3/2}} \\ &\quad-\frac{(-b+ia)\operatorname{arctanh}\left(\frac{\sqrt{a+b\cot(c+dx)}}{\sqrt{a-ib}}\right)}{d(a-ib)^{5/2}}+\frac{(b+ia)\operatorname{arctanh}\left(\frac{\sqrt{a+b\cot(c+dx)}}{\sqrt{a+ib}}\right)}{d(a+ib)^{5/2}}\end{aligned}$$

```
[In] Int[(-a + b*Cot[c + d*x])/(a + b*Cot[c + d*x])^(5/2),x]
```

```
[Out] -(((I*a - b)*ArcTanh[Sqrt[a + b*Cot[c + d*x]]/Sqrt[a - I*b]])/((a - I*b)^(5
/2)*d)) + ((I*a + b)*ArcTanh[Sqrt[a + b*Cot[c + d*x]]/Sqrt[a + I*b]])/((a +
 I*b)^(5/2)*d) - (4*a*b)/(3*(a^2 + b^2)*d*(a + b*Cot[c + d*x])^(3/2)) - (2*
b*(3*a^2 - b^2))/((a^2 + b^2)^2*d*Sqrt[a + b*Cot[c + d*x]])
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 3610

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)]), x_Symbol] :> Simp[(b*c - a*d)*((a + b*Tan[e + f*x])^(m + 1)/
(f*(m + 1)*(a^2 + b^2))), x] + Dist[1/(a^2 + b^2), Int[(a + b*Tan[e + f*x])
^(m + 1)*Simp[a*c + b*d - (b*c - a*d)*Tan[e + f*x], x], x], x] /; FreeQ[{a,
 b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && LtQ[m, -1]
```

Rule 3618

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*tan[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Dist[c*(d/f), Subst[Int[(a + (b/d)*x)^m/(d^2 + c
*x), x], x, d*Tan[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b
*c - a*d, 0] && NeQ[a^2 + b^2, 0] && EqQ[c^2 + d^2, 0]
```

Rule 3620

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)]), x_Symbol] :> Dist[(c + I*d)/2, Int[(a + b*Tan[e + f*x])^m*(1
 - I*Tan[e + f*x]), x], x] + Dist[(c - I*d)/2, Int[(a + b*Tan[e + f*x])^m*(
1 + I*Tan[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b*c -
 a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] &&  !IntegerQ[m]
```

Rubi steps

$$\text{integral} = -\frac{4ab}{3\left(a^2+b^2\right)d(a+b\cot(c+dx))^{3/2}}+\frac{\int \frac{-a^2+b^2+2ab\cot(c+dx)}{(a+b\cot(c+dx))^{3/2}}\,dx}{a^2+b^2}$$

$$= -\frac{4ab}{3(a^2+b^2)d(a+b\cot(c+dx))^{3/2}} - \frac{2b(3a^2-b^2)}{(a^2+b^2)^2 d\sqrt{a+b\cot(c+dx)}} + \frac{\int \frac{-a(a^2-3b^2)+b(3a^2-b^2)\cot(c+dx)}{\sqrt{a+b\cot(c+dx)}}dx}{(a^2+b^2)^2}$$

$$= -\frac{4ab}{3(a^2+b^2)d(a+b\cot(c+dx))^{3/2}} - \frac{2b(3a^2-b^2)}{(a^2+b^2)^2 d\sqrt{a+b\cot(c+dx)}} - \frac{(a-ib)\int \frac{1-i\cot(c+dx)}{\sqrt{a+b\cot(c+dx)}}dx}{2(a+ib)^2} - \frac{(a+ib)\int \frac{1+i\cot(c+dx)}{\sqrt{a+b\cot(c+dx)}}dx}{2(a-ib)^2}$$

$$= -\frac{4ab}{3(a^2+b^2)d(a+b\cot(c+dx))^{3/2}} - \frac{2b(3a^2-b^2)}{(a^2+b^2)^2 d\sqrt{a+b\cot(c+dx)}} + \frac{(ia-b)\text{Subst}\left(\int \frac{1}{(-1+x)\sqrt{a-ibx}}dx, x, i\cot(c+dx)\right)}{2(a-ib)^2 d} - \frac{(ia+b)\text{Subst}\left(\int \frac{1}{(-1+x)\sqrt{a+ibx}}dx, x, -i\cot(c+dx)\right)}{2(a+ib)^2 d}$$

$$= -\frac{4ab}{3(a^2+b^2)d(a+b\cot(c+dx))^{3/2}} - \frac{2b(3a^2-b^2)}{(a^2+b^2)^2 d\sqrt{a+b\cot(c+dx)}} + \frac{(a+ib)\text{Subst}\left(\int \frac{1}{-1-\frac{ia}{b}+\frac{ix^2}{b}}dx, x, \sqrt{a+b\cot(c+dx)}\right)}{b(ia+b)^2 d} + \frac{(i(ia+b))\text{Subst}\left(\int \frac{1}{-1+\frac{ia}{b}-\frac{ix^2}{b}}dx, x, \sqrt{a+b\cot(c+dx)}\right)}{(a+ib)^2 bd}$$

$$= -\frac{(ia-b)\text{arctanh}\left(\frac{\sqrt{a+b\cot(c+dx)}}{\sqrt{a-ib}}\right)}{(a-ib)^{5/2}d} + \frac{(ia+b)\text{arctanh}\left(\frac{\sqrt{a+b\cot(c+dx)}}{\sqrt{a+ib}}\right)}{(a+ib)^{5/2}d} - \frac{4ab}{3(a^2+b^2)d(a+b\cot(c+dx))^{3/2}} - \frac{2b(3a^2-b^2)}{(a^2+b^2)^2 d\sqrt{a+b\cot(c+dx)}}$$

Mathematica [A] (verified)

Time = 3.16 (sec) , antiderivative size = 253, normalized size of antiderivative = 1.45

$$\int \frac{-a+b\cot(c+dx)}{(a+b\cot(c+dx))^{5/2}}dx = \frac{b\left(\frac{3\left(a^3-3ab^2+3a^2\sqrt{-b^2}+(-b^2)^{3/2}\right)\text{arctanh}\left(\frac{\sqrt{a+b\cot(c+dx)}}{\sqrt{a-\sqrt{-b^2}}}\right)}{\sqrt{-b^2}\sqrt{a-\sqrt{-b^2}}} + \frac{3\left(-a^3+3ab^2+3a^2\sqrt{-b^2}+(-b^2)^{3/2}\right.}{\sqrt{-b^2}\sqrt{}}\right.}{3(a^2+b^2)^2 d}$$

```
[In] Integrate[(-a + b*Cot[c + d*x])/(a + b*Cot[c + d*x])^(5/2),x]
```

```
[Out] (b*((3*(a^3 - 3*a*b^2 + 3*a^2*Sqrt[-b^2] + (-b^2)^(3/2))*ArcTanh[Sqrt[a + b
*Cot[c + d*x]]/Sqrt[a - Sqrt[-b^2]]])/(Sqrt[-b^2]*Sqrt[a - Sqrt[-b^2]]) + (
3*(-a^3 + 3*a*b^2 + 3*a^2*Sqrt[-b^2] + (-b^2)^(3/2))*ArcTanh[Sqrt[a + b*Cot
[c + d*x]]/Sqrt[a + Sqrt[-b^2]]])/(Sqrt[-b^2]*Sqrt[a + Sqrt[-b^2]]) - (4*a*
(a^2 + b^2))/(a + b*Cot[c + d*x])^(3/2) + (6*(-3*a^2 + b^2))/Sqrt[a + b*Cot
[c + d*x]]))/(3*(a^2 + b^2)^2*d)
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 3054 vs. $2(150) = 300$.

Time = 0.12 (sec) , antiderivative size = 3055, normalized size of antiderivative = 17.56

method	result	size
derivativedivides	Expression too large to display	3055
default	Expression too large to display	3055
parts	Expression too large to display	4473

```
[In] int((-a+b*cot(d*x+c))/(a+b*cot(d*x+c))^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] -6/d*b/(a^2+b^2)^2/(a+b*cot(d*x+c))^(1/2)*a^2+1/4/d*b^5/(a^2+b^2)^(7/2)*ln(
(a+b*cot(d*x+c))^(1/2)*(2*(a^2+b^2)^(1/2)+2*a)^(1/2)-b*cot(d*x+c)-(a^2+b^2)
^(1/2)-a)*(2*(a^2+b^2)^(1/2)+2*a)^(1/2)-1/4/d*b^5/(a^2+b^2)^(7/2)*ln(b*cot(
d*x+c)+a+(a+b*cot(d*x+c))^(1/2)*(2*(a^2+b^2)^(1/2)+2*a)^(1/2)+(a^2+b^2)^(1/
2))*(2*(a^2+b^2)^(1/2)+2*a)^(1/2)+1/d*b^5/(a^2+b^2)^3/(2*(a^2+b^2)^(1/2)-2*
a)^(1/2)*arctan((2*(a+b*cot(d*x+c))^(1/2)+(2*(a^2+b^2)^(1/2)+2*a)^(1/2))/(2
*(a^2+b^2)^(1/2)-2*a)^(1/2))-1/d*b^5/(a^2+b^2)^3/(2*(a^2+b^2)^(1/2)-2*a)^(1
/2)*arctan((-2*(a+b*cot(d*x+c))^(1/2)+(2*(a^2+b^2)^(1/2)+2*a)^(1/2))/(2*(a^
2+b^2)^(1/2)-2*a)^(1/2))+1/d/b/(a^2+b^2)^(7/2)/(2*(a^2+b^2)^(1/2)-2*a)^(1/2
)*arctan((2*(a+b*cot(d*x+c))^(1/2)+(2*(a^2+b^2)^(1/2)+2*a)^(1/2))/(2*(a^2+b
^2)^(1/2)-2*a)^(1/2))*a^7-5/d*b^3/(a^2+b^2)^(7/2)/(2*(a^2+b^2)^(1/2)-2*a)^(
1/2)*arctan((2*(a+b*cot(d*x+c))^(1/2)+(2*(a^2+b^2)^(1/2)+2*a)^(1/2))/(2*(a^
2+b^2)^(1/2)-2*a)^(1/2))*a^3+1/4/d/b/(a^2+b^2)^(7/2)*ln((a+b*cot(d*x+c))^(1
/2)*(2*(a^2+b^2)^(1/2)+2*a)^(1/2)-b*cot(d*x+c)-(a^2+b^2)^(1/2)-a)*(2*(a^2+b
^2)^(1/2)+2*a)^(1/2)*a^6-5/4/d*b^3/(a^2+b^2)^(7/2)*ln((a+b*cot(d*x+c))^(1/2
)*(2*(a^2+b^2)^(1/2)+2*a)^(1/2)-b*cot(d*x+c)-(a^2+b^2)^(1/2)-a)*(2*(a^2+b^2
)^(1/2)+2*a)^(1/2)*a^2+7/d*b^5/(a^2+b^2)^(7/2)/(2*(a^2+b^2)^(1/2)-2*a)^(1/2
)*arctan((-2*(a+b*cot(d*x+c))^(1/2)+(2*(a^2+b^2)^(1/2)+2*a)^(1/2))/(2*(a^2+
b^2)^(1/2)-2*a)^(1/2))*a-1/d/b/(a^2+b^2)^(7/2)/(2*(a^2+b^2)^(1/2)-2*a)^(1/2
)*arctan((-2*(a+b*cot(d*x+c))^(1/2)+(2*(a^2+b^2)^(1/2)+2*a)^(1/2))/(2*(a^2+
b^2)^(1/2)-2*a)^(1/2))*a^7+5/d*b^3/(a^2+b^2)^(7/2)/(2*(a^2+b^2)^(1/2)-2*a)^
(1/2)*arctan((-2*(a+b*cot(d*x+c))^(1/2)+(2*(a^2+b^2)^(1/2)+2*a)^(1/2))/(2*(
a^2+b^2)^(1/2)-2*a)^(1/2))*a^3+1/4/d/b/(a^2+b^2)^3*ln(b*cot(d*x+c)+a+(a+b*c
ot(d*x+c))^(1/2)*(2*(a^2+b^2)^(1/2)+2*a)^(1/2)+(a^2+b^2)^(1/2))*(2*(a^2+b^2
)^(1/2)+2*a)^(1/2)*a^5+2/d*b^3/(a^2+b^2)^3/(2*(a^2+b^2)^(1/2)-2*a)^(1/2)*ar
```

```
ctan((-2*(a+b*cot(d*x+c))^(1/2)+(2*(a^2+b^2)^(1/2)+2*a)^(1/2))/(2*(a^2+b^2)
^(1/2)-2*a)^(1/2))*a^2-1/4/d/b/(a^2+b^2)^3*ln((a+b*cot(d*x+c))^(1/2)*(2*(a^
2+b^2)^(1/2)+2*a)^(1/2)-b*cot(d*x+c)-(a^2+b^2)^(1/2)-a)*(2*(a^2+b^2)^(1/2)+
2*a)^(1/2)*a^5+3/4/d*b^3/(a^2+b^2)^3*ln((a+b*cot(d*x+c))^(1/2)*(2*(a^2+b^2)
^(1/2)+2*a)^(1/2)-b*cot(d*x+c)-(a^2+b^2)^(1/2)-a)*(2*(a^2+b^2)^(1/2)+2*a)^(
1/2)*a-3/4/d*b^3/(a^2+b^2)^3*ln(b*cot(d*x+c)+a+(a+b*cot(d*x+c))^(1/2)*(2*(a
^2+b^2)^(1/2)+2*a)^(1/2)+(a^2+b^2)^(1/2))*(2*(a^2+b^2)^(1/2)+2*a)^(1/2)*a-2
/d*b^3/(a^2+b^2)^3/(2*(a^2+b^2)^(1/2)-2*a)^(1/2)*arctan((2*(a+b*cot(d*x+c))
^(1/2)+(2*(a^2+b^2)^(1/2)+2*a)^(1/2))/(2*(a^2+b^2)^(1/2)-2*a)^(1/2))*a^2+5/
4/d*b/(a^2+b^2)^(7/2)*ln(b*cot(d*x+c)+a+(a+b*cot(d*x+c))^(1/2)*(2*(a^2+b^2)
^(1/2)+2*a)^(1/2)+(a^2+b^2)^(1/2))*(2*(a^2+b^2)^(1/2)+2*a)^(1/2)*a^4+3/d*b/
(a^2+b^2)^(7/2)/(2*(a^2+b^2)^(1/2)-2*a)^(1/2)*arctan((2*(a+b*cot(d*x+c))^(1
/2)+(2*(a^2+b^2)^(1/2)+2*a)^(1/2))/(2*(a^2+b^2)^(1/2)-2*a)^(1/2))*a^5+1/2/d
*b/(a^2+b^2)^3*ln((a+b*cot(d*x+c))^(1/2)*(2*(a^2+b^2)^(1/2)+2*a)^(1/2)-b*co
t(d*x+c)-(a^2+b^2)^(1/2)-a)*(2*(a^2+b^2)^(1/2)+2*a)^(1/2)*a^3-3/d*b/(a^2+b^
2)^3/(2*(a^2+b^2)^(1/2)-2*a)^(1/2)*arctan((2*(a+b*cot(d*x+c))^(1/2)+(2*(a^2
+b^2)^(1/2)+2*a)^(1/2))/(2*(a^2+b^2)^(1/2)-2*a)^(1/2))*a^4+3/d*b/(a^2+b^2)^
3/(2*(a^2+b^2)^(1/2)-2*a)^(1/2)*arctan((-2*(a+b*cot(d*x+c))^(1/2)+(2*(a^2+b
^2)^(1/2)+2*a)^(1/2))/(2*(a^2+b^2)^(1/2)-2*a)^(1/2))*a^4-1/2/d*b/(a^2+b^2)^
3*ln(b*cot(d*x+c)+a+(a+b*cot(d*x+c))^(1/2)*(2*(a^2+b^2)^(1/2)+2*a)^(1/2)+(a
^2+b^2)^(1/2))*(2*(a^2+b^2)^(1/2)+2*a)^(1/2)*a^3-5/4/d*b/(a^2+b^2)^(7/2)*ln
((a+b*cot(d*x+c))^(1/2)*(2*(a^2+b^2)^(1/2)+2*a)^(1/2)-b*cot(d*x+c)-(a^2+b^2
)^(1/2)-a)*(2*(a^2+b^2)^(1/2)+2*a)^(1/2)*a^4-4/3*a*b/(a^2+b^2)/d/(a+b*cot(d
*x+c))^(3/2)+2/d*b^3/(a^2+b^2)^2/(a+b*cot(d*x+c))^(1/2)+1/d/b/(a^2+b^2)^(5/
2)/(2*(a^2+b^2)^(1/2)-2*a)^(1/2)*arctan((-2*(a+b*cot(d*x+c))^(1/2)+(2*(a^2+
b^2)^(1/2)+2*a)^(1/2))/(2*(a^2+b^2)^(1/2)-2*a)^(1/2))*a^5-3/d*b^3/(a^2+b^2)
^(5/2)/(2*(a^2+b^2)^(1/2)-2*a)^(1/2)*arctan((-2*(a+b*cot(d*x+c))^(1/2)+(2*(
a^2+b^2)^(1/2)+2*a)^(1/2))/(2*(a^2+b^2)^(1/2)-2*a)^(1/2))*a-2/d*b/(a^2+b^2)
^(5/2)/(2*(a^2+b^2)^(1/2)-2*a)^(1/2)*arctan((-2*(a+b*cot(d*x+c))^(1/2)+(2*(
a^2+b^2)^(1/2)+2*a)^(1/2))/(2*(a^2+b^2)^(1/2)-2*a)^(1/2))*a^3-1/d/b/(a^2+b^
2)^(5/2)/(2*(a^2+b^2)^(1/2)-2*a)^(1/2)*arctan((2*(a+b*cot(d*x+c))^(1/2)+(2*
(a^2+b^2)^(1/2)+2*a)^(1/2))/(2*(a^2+b^2)^(1/2)-2*a)^(1/2))*a^5+3/d*b^3/(a^2
+b^2)^(5/2)/(2*(a^2+b^2)^(1/2)-2*a)^(1/2)*arctan((2*(a+b*cot(d*x+c))^(1/2)+
(2*(a^2+b^2)^(1/2)+2*a)^(1/2))/(2*(a^2+b^2)^(1/2)-2*a)^(1/2))*a+2/d*b/(a^2+
b^2)^(5/2)/(2*(a^2+b^2)^(1/2)-2*a)^(1/2)*arctan((2*(a+b*cot(d*x+c))^(1/2)+(
2*(a^2+b^2)^(1/2)+2*a)^(1/2))/(2*(a^2+b^2)^(1/2)-2*a)^(1/2))*a^3-3/d*b/(a^2
+b^2)^(7/2)/(2*(a^2+b^2)^(1/2)-2*a)^(1/2)*arctan((-2*(a+b*cot(d*x+c))^(1/2)
+(2*(a^2+b^2)^(1/2)+2*a)^(1/2))/(2*(a^2+b^2)^(1/2)-2*a)^(1/2))*a^5-1/4/d/b/
(a^2+b^2)^(7/2)*ln(b*cot(d*x+c)+a+(a+b*cot(d*x+c))^(1/2)*(2*(a^2+b^2)^(1/2)
+2*a)^(1/2)+(a^2+b^2)^(1/2))*(2*(a^2+b^2)^(1/2)+2*a)^(1/2)*a^6+5/4/d*b^3/(a
^2+b^2)^(7/2)*ln(b*cot(d*x+c)+a+(a+b*cot(d*x+c))^(1/2)*(2*(a^2+b^2)^(1/2)+2
*a)^(1/2)+(a^2+b^2)^(1/2))*(2*(a^2+b^2)^(1/2)+2*a)^(1/2)*a^2-7/d*b^5/(a^2+b
^2)^(7/2)/(2*(a^2+b^2)^(1/2)-2*a)^(1/2)*arctan((2*(a+b*cot(d*x+c))^(1/2)+(2
*(a^2+b^2)^(1/2)+2*a)^(1/2))/(2*(a^2+b^2)^(1/2)-2*a)^(1/2))*a
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 3922 vs. $2(141) = 282$.

Time = 0.42 (sec) , antiderivative size = 3922, normalized size of antiderivative = 22.54

$$\int \frac{-a+b\cot(c+dx)}{(a+b\cot(c+dx))^{5/2}}\,dx = \text{Too large to display}$$

```
[In] integrate((-a+b*cot(d*x+c))/(a+b*cot(d*x+c))^(5/2),x, algorithm="fricas")
```

```
[Out] -1/6*(3*((a^6 + a^4*b^2 - a^2*b^4 - b^6)*d*cos(2*d*x + 2*c) - 2*(a^5*b + 2*
a^3*b^3 + a*b^5)*d*sin(2*d*x + 2*c) - (a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6)*d
)*sqrt(-(a^7 - 21*a^5*b^2 + 35*a^3*b^4 - 7*a*b^6 + (a^10 + 5*a^8*b^2 + 10*a
^6*b^4 + 10*a^4*b^6 + 5*a^2*b^8 + b^10)*d^2*sqrt(-(49*a^12*b^2 - 490*a^10*b
^4 + 1519*a^8*b^6 - 1484*a^6*b^8 + 511*a^4*b^10 - 42*a^2*b^12 + b^14)/((a^2
0 + 10*a^18*b^2 + 45*a^16*b^4 + 120*a^14*b^6 + 210*a^12*b^8 + 252*a^10*b^10
 + 210*a^8*b^12 + 120*a^6*b^14 + 45*a^4*b^16 + 10*a^2*b^18 + b^20)*d^4)))/(
(a^10 + 5*a^8*b^2 + 10*a^6*b^4 + 10*a^4*b^6 + 5*a^2*b^8 + b^10)*d^2))*log(-
(7*a^8*b - 28*a^6*b^3 - 14*a^4*b^5 + 20*a^2*b^7 - b^9)*sqrt((b*cos(2*d*x +
2*c) + a*sin(2*d*x + 2*c) + b)/sin(2*d*x + 2*c)) + ((a^14 - a^12*b^2 - 19*a
^10*b^4 - 45*a^8*b^6 - 45*a^6*b^8 - 19*a^4*b^10 - a^2*b^12 + b^14)*d^3*sqrt
(-(49*a^12*b^2 - 490*a^10*b^4 + 1519*a^8*b^6 - 1484*a^6*b^8 + 511*a^4*b^10
- 42*a^2*b^12 + b^14)/((a^20 + 10*a^18*b^2 + 45*a^16*b^4 + 120*a^14*b^6 + 2
10*a^12*b^8 + 252*a^10*b^10 + 210*a^8*b^12 + 120*a^6*b^14 + 45*a^4*b^16 + 1
0*a^2*b^18 + b^20)*d^4)) + 4*(7*a^9*b^2 - 42*a^7*b^4 + 56*a^5*b^6 - 22*a^3*
b^8 + a*b^10)*d)*sqrt(-(a^7 - 21*a^5*b^2 + 35*a^3*b^4 - 7*a*b^6 + (a^10 + 5
*a^8*b^2 + 10*a^6*b^4 + 10*a^4*b^6 + 5*a^2*b^8 + b^10)*d^2*sqrt(-(49*a^12*b
^2 - 490*a^10*b^4 + 1519*a^8*b^6 - 1484*a^6*b^8 + 511*a^4*b^10 - 42*a^2*b^1
2 + b^14)/((a^20 + 10*a^18*b^2 + 45*a^16*b^4 + 120*a^14*b^6 + 210*a^12*b^8
+ 252*a^10*b^10 + 210*a^8*b^12 + 120*a^6*b^14 + 45*a^4*b^16 + 10*a^2*b^18 +
 b^20)*d^4)))/((a^10 + 5*a^8*b^2 + 10*a^6*b^4 + 10*a^4*b^6 + 5*a^2*b^8 + b^
10)*d^2))) - 3*((a^6 + a^4*b^2 - a^2*b^4 - b^6)*d*cos(2*d*x + 2*c) - 2*(a^5
*b + 2*a^3*b^3 + a*b^5)*d*sin(2*d*x + 2*c) - (a^6 + 3*a^4*b^2 + 3*a^2*b^4 +
 b^6)*d)*sqrt(-(a^7 - 21*a^5*b^2 + 35*a^3*b^4 - 7*a*b^6 + (a^10 + 5*a^8*b^2
 + 10*a^6*b^4 + 10*a^4*b^6 + 5*a^2*b^8 + b^10)*d^2*sqrt(-(49*a^12*b^2 - 490
*a^10*b^4 + 1519*a^8*b^6 - 1484*a^6*b^8 + 511*a^4*b^10 - 42*a^2*b^12 + b^14
)/((a^20 + 10*a^18*b^2 + 45*a^16*b^4 + 120*a^14*b^6 + 210*a^12*b^8 + 252*a^
10*b^10 + 210*a^8*b^12 + 120*a^6*b^14 + 45*a^4*b^16 + 10*a^2*b^18 + b^20)*d
^4)))/((a^10 + 5*a^8*b^2 + 10*a^6*b^4 + 10*a^4*b^6 + 5*a^2*b^8 + b^10)*d^2)
)*log(-(7*a^8*b - 28*a^6*b^3 - 14*a^4*b^5 + 20*a^2*b^7 - b^9)*sqrt((b*cos(2
*d*x + 2*c) + a*sin(2*d*x + 2*c) + b)/sin(2*d*x + 2*c)) - ((a^14 - a^12*b^2
 - 19*a^10*b^4 - 45*a^8*b^6 - 45*a^6*b^8 - 19*a^4*b^10 - a^2*b^12 + b^14)*d
^3*sqrt(-(49*a^12*b^2 - 490*a^10*b^4 + 1519*a^8*b^6 - 1484*a^6*b^8 + 511*a^
4*b^10 - 42*a^2*b^12 + b^14)/((a^20 + 10*a^18*b^2 + 45*a^16*b^4 + 120*a^14*
b^6 + 210*a^12*b^8 + 252*a^10*b^10 + 210*a^8*b^12 + 120*a^6*b^14 + 45*a^4*b
```

^16 + 10*a^2*b^18 + b^20)*d^4)) + 4*(7*a^9*b^2 - 42*a^7*b^4 + 56*a^5*b^6 -
22*a^3*b^8 + a*b^10)*d)*sqrt(-(a^7 - 21*a^5*b^2 + 35*a^3*b^4 - 7*a*b^6 + (a
^10 + 5*a^8*b^2 + 10*a^6*b^4 + 10*a^4*b^6 + 5*a^2*b^8 + b^10)*d^2*sqrt(-(49
*a^12*b^2 - 490*a^10*b^4 + 1519*a^8*b^6 - 1484*a^6*b^8 + 511*a^4*b^10 - 42*
a^2*b^12 + b^14)/((a^20 + 10*a^18*b^2 + 45*a^16*b^4 + 120*a^14*b^6 + 210*a^
12*b^8 + 252*a^10*b^10 + 210*a^8*b^12 + 120*a^6*b^14 + 45*a^4*b^16 + 10*a^2
*b^18 + b^20)*d^4)))/((a^10 + 5*a^8*b^2 + 10*a^6*b^4 + 10*a^4*b^6 + 5*a^2*b
^8 + b^10)*d^2))) - 3*((a^6 + a^4*b^2 - a^2*b^4 - b^6)*d*cos(2*d*x + 2*c) -
2*(a^5*b + 2*a^3*b^3 + a*b^5)*d*sin(2*d*x + 2*c) - (a^6 + 3*a^4*b^2 + 3*a^
2*b^4 + b^6)*d)*sqrt(-(a^7 - 21*a^5*b^2 + 35*a^3*b^4 - 7*a*b^6 - (a^10 + 5*
a^8*b^2 + 10*a^6*b^4 + 10*a^4*b^6 + 5*a^2*b^8 + b^10)*d^2*sqrt(-(49*a^12*b^
2 - 490*a^10*b^4 + 1519*a^8*b^6 - 1484*a^6*b^8 + 511*a^4*b^10 - 42*a^2*b^12
+ b^14)/((a^20 + 10*a^18*b^2 + 45*a^16*b^4 + 120*a^14*b^6 + 210*a^12*b^8 +
252*a^10*b^10 + 210*a^8*b^12 + 120*a^6*b^14 + 45*a^4*b^16 + 10*a^2*b^18 +
b^20)*d^4)))/((a^10 + 5*a^8*b^2 + 10*a^6*b^4 + 10*a^4*b^6 + 5*a^2*b^8 + b^1
0)*d^2))*log(-(7*a^8*b - 28*a^6*b^3 - 14*a^4*b^5 + 20*a^2*b^7 - b^9)*sqrt((
b*cos(2*d*x + 2*c) + a*sin(2*d*x + 2*c) + b)/sin(2*d*x + 2*c)) + ((a^14 - a
^12*b^2 - 19*a^10*b^4 - 45*a^8*b^6 - 45*a^6*b^8 - 19*a^4*b^10 - a^2*b^12 +
b^14)*d^3*sqrt(-(49*a^12*b^2 - 490*a^10*b^4 + 1519*a^8*b^6 - 1484*a^6*b^8 +
511*a^4*b^10 - 42*a^2*b^12 + b^14)/((a^20 + 10*a^18*b^2 + 45*a^16*b^4 + 12
0*a^14*b^6 + 210*a^12*b^8 + 252*a^10*b^10 + 210*a^8*b^12 + 120*a^6*b^14 + 4
5*a^4*b^16 + 10*a^2*b^18 + b^20)*d^4)) - 4*(7*a^9*b^2 - 42*a^7*b^4 + 56*a^5
*b^6 - 22*a^3*b^8 + a*b^10)*d)*sqrt(-(a^7 - 21*a^5*b^2 + 35*a^3*b^4 - 7*a*b
^6 - (a^10 + 5*a^8*b^2 + 10*a^6*b^4 + 10*a^4*b^6 + 5*a^2*b^8 + b^10)*d^2*sq
rt(-(49*a^12*b^2 - 490*a^10*b^4 + 1519*a^8*b^6 - 1484*a^6*b^8 + 511*a^4*b^1
0 - 42*a^2*b^12 + b^14)/((a^20 + 10*a^18*b^2 + 45*a^16*b^4 + 120*a^14*b^6 +
210*a^12*b^8 + 252*a^10*b^10 + 210*a^8*b^12 + 120*a^6*b^14 + 45*a^4*b^16 +
10*a^2*b^18 + b^20)*d^4)))/((a^10 + 5*a^8*b^2 + 10*a^6*b^4 + 10*a^4*b^6 +
5*a^2*b^8 + b^10)*d^2))) + 3*((a^6 + a^4*b^2 - a^2*b^4 - b^6)*d*cos(2*d*x +
2*c) - 2*(a^5*b + 2*a^3*b^3 + a*b^5)*d*sin(2*d*x + 2*c) - (a^6 + 3*a^4*b^2
+ 3*a^2*b^4 + b^6)*d)*sqrt(-(a^7 - 21*a^5*b^2 + 35*a^3*b^4 - 7*a*b^6 - (a^
10 + 5*a^8*b^2 + 10*a^6*b^4 + 10*a^4*b^6 + 5*a^2*b^8 + b^10)*d^2*sqrt(-(49*
a^12*b^2 - 490*a^10*b^4 + 1519*a^8*b^6 - 1484*a^6*b^8 + 511*a^4*b^10 - 42*a
^2*b^12 + b^14)/((a^20 + 10*a^18*b^2 + 45*a^16*b^4 + 120*a^14*b^6 + 210*a^1
2*b^8 + 252*a^10*b^10 + 210*a^8*b^12 + 120*a^6*b^14 + 45*a^4*b^16 + 10*a^2*
b^18 + b^20)*d^4)))/((a^10 + 5*a^8*b^2 + 10*a^6*b^4 + 10*a^4*b^6 + 5*a^2*b^
8 + b^10)*d^2))*log(-(7*a^8*b - 28*a^6*b^3 - 14*a^4*b^5 + 20*a^2*b^7 - b^9)
*sqrt((b*cos(2*d*x + 2*c) + a*sin(2*d*x + 2*c) + b)/sin(2*d*x + 2*c)) - ((a
^14 - a^12*b^2 - 19*a^10*b^4 - 45*a^8*b^6 - 45*a^6*b^8 - 19*a^4*b^10 - a^2*
b^12 + b^14)*d^3*sqrt(-(49*a^12*b^2 - 490*a^10*b^4 + 1519*a^8*b^6 - 1484*a^
6*b^8 + 511*a^4*b^10 - 42*a^2*b^12 + b^14)/((a^20 + 10*a^18*b^2 + 45*a^16*b
^4 + 120*a^14*b^6 + 210*a^12*b^8 + 252*a^10*b^10 + 210*a^8*b^12 + 120*a^6*b
^14 + 45*a^4*b^16 + 10*a^2*b^18 + b^20)*d^4)) - 4*(7*a^9*b^2 - 42*a^7*b^4 +
56*a^5*b^6 - 22*a^3*b^8 + a*b^10)*d)*sqrt(-(a^7 - 21*a^5*b^2 + 35*a^3*b^4
- 7*a*b^6 - (a^10 + 5*a^8*b^2 + 10*a^6*b^4 + 10*a^4*b^6 + 5*a^2*b^8 + b^10)

```
*d^2*sqrt(-(49*a^12*b^2 - 490*a^10*b^4 + 1519*a^8*b^6 - 1484*a^6*b^8 + 511*
a^4*b^10 - 42*a^2*b^12 + b^14)/((a^20 + 10*a^18*b^2 + 45*a^16*b^4 + 120*a^1
4*b^6 + 210*a^12*b^8 + 252*a^10*b^10 + 210*a^8*b^12 + 120*a^6*b^14 + 45*a^4
*b^16 + 10*a^2*b^18 + b^20)*d^4)))/((a^10 + 5*a^8*b^2 + 10*a^6*b^4 + 10*a^4
*b^6 + 5*a^2*b^8 + b^10)*d^2))) - 4*(11*a^3*b - a*b^3 - (11*a^3*b - a*b^3)*
cos(2*d*x + 2*c) + 3*(3*a^2*b^2 - b^4)*sin(2*d*x + 2*c))*sqrt((b*cos(2*d*x
+ 2*c) + a*sin(2*d*x + 2*c) + b)/sin(2*d*x + 2*c)))/((a^6 + a^4*b^2 - a^2*b
^4 - b^6)*d*cos(2*d*x + 2*c) - 2*(a^5*b + 2*a^3*b^3 + a*b^5)*d*sin(2*d*x +
2*c) - (a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6)*d)
```

Sympy [F]

$$\int \frac{-a+b\cot(c+dx)}{(a+b\cot(c+dx))^{5/2}}\,dx =$$
$$-\int \frac{a}{a^2\sqrt{a+b\cot(c+dx)}+2ab\sqrt{a+b\cot(c+dx)}\cot(c+dx)+b^2\sqrt{a+b\cot(c+dx)}\cot^2(c+dx)}\,dx$$
$$-\int \left(-\frac{b\cot(c+dx)}{a^2\sqrt{a+b\cot(c+dx)}+2ab\sqrt{a+b\cot(c+dx)}\cot(c+dx)+b^2\sqrt{a+b\cot(c+dx)}\cot^2(c+dx)}\right.$$

```
[In] integrate((-a+b*cot(d*x+c))/(a+b*cot(d*x+c))**(5/2),x)
```

```
[Out] -Integral(a/(a**2*sqrt(a + b*cot(c + d*x)) + 2*a*b*sqrt(a + b*cot(c + d*x))
*cot(c + d*x) + b**2*sqrt(a + b*cot(c + d*x))*cot(c + d*x)**2), x) - Integr
al(-b*cot(c + d*x)/(a**2*sqrt(a + b*cot(c + d*x)) + 2*a*b*sqrt(a + b*cot(c
+ d*x))*cot(c + d*x) + b**2*sqrt(a + b*cot(c + d*x))*cot(c + d*x)**2), x)
```

Maxima [F]

$$\int \frac{-a+b\cot(c+dx)}{(a+b\cot(c+dx))^{5/2}}\,dx = \int \frac{b\cot(dx+c)-a}{(b\cot(dx+c)+a)^{\frac{5}{2}}}\,dx$$

```
[In] integrate((-a+b*cot(d*x+c))/(a+b*cot(d*x+c))^(5/2),x, algorithm="maxima")
```

```
[Out] integrate((b*cot(d*x + c) - a)/(b*cot(d*x + c) + a)^(5/2), x)
```

Giac [F]

$$\int \frac{-a+b\cot(c+dx)}{(a+b\cot(c+dx))^{5/2}}\,dx = \int \frac{b\cot(dx+c)-a}{(b\cot(dx+c)+a)^{\frac{5}{2}}}\,dx$$

```
[In] integrate((-a+b*cot(d*x+c))/(a+b*cot(d*x+c))^(5/2),x, algorithm="giac")
[Out] integrate((b*cot(d*x + c) - a)/(b*cot(d*x + c) + a)^(5/2), x)
```

Mupad [B] (verification not implemented)

Time = 29.37 (sec) , antiderivative size = 8438, normalized size of antiderivative = 48.49

$$\int \frac{-a+b\cot(c+dx)}{(a+b\cot(c+dx))^{5/2}}\,dx = \text{Too large to display}$$

```
[In] int(-(a - b*cot(c + d*x))/(a + b*cot(c + d*x))^(5/2),x)
[Out] (log(((-(4*a^7*d^2 - (320*a^6*b^8*d^4 - 16*a^4*b^10*d^4 - 1760*a^8*b^6*d^4
+ 1600*a^10*b^4*d^4 - 400*a^12*b^2*d^4)^(1/2) + 20*a^3*b^4*d^2 - 40*a^5*b^2
*d^2)/(a^10*d^4 + b^10*d^4 + 5*a^2*b^8*d^4 + 10*a^4*b^6*d^4 + 10*a^6*b^4*d^
4 + 5*a^8*b^2*d^4))^(1/2)*((a + b*cot(c + d*x))^(1/2)*(320*a^6*b^14*d^3 - 1
6*a^2*b^18*d^3 + 1024*a^8*b^12*d^3 + 1440*a^10*b^10*d^3 + 1024*a^12*b^8*d^3
 + 320*a^14*b^6*d^3 - 16*a^18*b^2*d^3) - ((-(4*a^7*d^2 - (320*a^6*b^8*d^4 -
 16*a^4*b^10*d^4 - 1760*a^8*b^6*d^4 + 1600*a^10*b^4*d^4 - 400*a^12*b^2*d^4)
^(1/2) + 20*a^3*b^4*d^2 - 40*a^5*b^2*d^2)/(a^10*d^4 + b^10*d^4 + 5*a^2*b^8*
d^4 + 10*a^4*b^6*d^4 + 10*a^6*b^4*d^4 + 5*a^8*b^2*d^4))^(1/2)*(((-(4*a^7*d^
2 - (320*a^6*b^8*d^4 - 16*a^4*b^10*d^4 - 1760*a^8*b^6*d^4 + 1600*a^10*b^4*d
^4 - 400*a^12*b^2*d^4)^(1/2) + 20*a^3*b^4*d^2 - 40*a^5*b^2*d^2)/(a^10*d^4 +
 b^10*d^4 + 5*a^2*b^8*d^4 + 10*a^4*b^6*d^4 + 10*a^6*b^4*d^4 + 5*a^8*b^2*d^4
))^(1/2)*(a + b*cot(c + d*x))^(1/2)*(64*a*b^22*d^5 + 640*a^3*b^20*d^5 + 288
0*a^5*b^18*d^5 + 7680*a^7*b^16*d^5 + 13440*a^9*b^14*d^5 + 16128*a^11*b^12*d
^5 + 13440*a^13*b^10*d^5 + 7680*a^15*b^8*d^5 + 2880*a^17*b^6*d^5 + 640*a^19
*b^4*d^5 + 64*a^21*b^2*d^5))/4 - 32*a*b^21*d^4 - 160*a^3*b^19*d^4 - 128*a^5
*b^17*d^4 + 896*a^7*b^15*d^4 + 3136*a^9*b^13*d^4 + 4928*a^11*b^11*d^4 + 448
0*a^13*b^9*d^4 + 2432*a^15*b^7*d^4 + 736*a^17*b^5*d^4 + 96*a^19*b^3*d^4))/4
))/4 + 16*a^4*b^15*d^2 + 96*a^6*b^13*d^2 + 240*a^8*b^11*d^2 + 320*a^10*b^9*
d^2 + 240*a^12*b^7*d^2 + 96*a^14*b^5*d^2 + 16*a^16*b^3*d^2)*(-(4*a^7*d^2 -
(320*a^6*b^8*d^4 - 16*a^4*b^10*d^4 - 1760*a^8*b^6*d^4 + 1600*a^10*b^4*d^4 -
 400*a^12*b^2*d^4)^(1/2) + 20*a^3*b^4*d^2 - 40*a^5*b^2*d^2)/(a^10*d^4 + b^1
0*d^4 + 5*a^2*b^8*d^4 + 10*a^4*b^6*d^4 + 10*a^6*b^4*d^4 + 5*a^8*b^2*d^4))^(
1/2))/4 - log(16*a^4*b^15*d^2 - (-(4*a^7*d^2 - (320*a^6*b^8*d^4 - 16*a^4*b^
10*d^4 - 1760*a^8*b^6*d^4 + 1600*a^10*b^4*d^4 - 400*a^12*b^2*d^4)^(1/2) + 2
0*a^3*b^4*d^2 - 40*a^5*b^2*d^2)/(16*a^10*d^4 + 16*b^10*d^4 + 80*a^2*b^8*d^4
```

+ 160*a^4*b^6*d^4 + 160*a^6*b^4*d^4 + 80*a^8*b^2*d^4))^(1/2)*((a + b*cot(c + d*x))^(1/2)*(320*a^6*b^14*d^3 - 16*a^2*b^18*d^3 + 1024*a^8*b^12*d^3 + 1440*a^10*b^10*d^3 + 1024*a^12*b^8*d^3 + 320*a^14*b^6*d^3 - 16*a^18*b^2*d^3) + (-(4*a^7*d^2 - (320*a^6*b^8*d^4 - 16*a^4*b^10*d^4 - 1760*a^8*b^6*d^4 + 1600*a^10*b^4*d^4 - 400*a^12*b^2*d^4)^(1/2) + 20*a^3*b^4*d^2 - 40*a^5*b^2*d^2)/(16*a^10*d^4 + 16*b^10*d^4 + 80*a^2*b^8*d^4 + 160*a^4*b^6*d^4 + 160*a^6*b^4*d^4 + 80*a^8*b^2*d^4))^(1/2)*(896*a^7*b^15*d^4 - (-(4*a^7*d^2 - (320*a^6*b^8*d^4 - 16*a^4*b^10*d^4 - 1760*a^8*b^6*d^4 + 1600*a^10*b^4*d^4 - 400*a^12*b^2*d^4)^(1/2) + 20*a^3*b^4*d^2 - 40*a^5*b^2*d^2)/(16*a^10*d^4 + 16*b^10*d^4 + 80*a^2*b^8*d^4 + 160*a^4*b^6*d^4 + 160*a^6*b^4*d^4 + 80*a^8*b^2*d^4))^(1/2)*(a + b*cot(c + d*x))^(1/2)*(64*a*b^22*d^5 + 640*a^3*b^20*d^5 + 2880*a^5*b^18*d^5 + 7680*a^7*b^16*d^5 + 13440*a^9*b^14*d^5 + 16128*a^11*b^12*d^5 + 13440*a^13*b^10*d^5 + 7680*a^15*b^8*d^5 + 2880*a^17*b^6*d^5 + 640*a^19*b^4*d^5 + 64*a^21*b^2*d^5) - 160*a^3*b^19*d^4 - 128*a^5*b^17*d^4 - 32*a*b^21*d^4 + 3136*a^9*b^13*d^4 + 4928*a^11*b^11*d^4 + 4480*a^13*b^9*d^4 + 2432*a^15*b^7*d^4 + 736*a^17*b^5*d^4 + 96*a^19*b^3*d^4)) + 96*a^6*b^13*d^2 + 240*a^8*b^11*d^2 + 320*a^10*b^9*d^2 + 240*a^12*b^7*d^2 + 96*a^14*b^5*d^2 + 16*a^16*b^3*d^2)*(-(4*a^7*d^2 - (320*a^6*b^8*d^4 - 16*a^4*b^10*d^4 - 1760*a^8*b^6*d^4 + 1600*a^10*b^4*d^4 - 400*a^12*b^2*d^4)^(1/2) + 20*a^3*b^4*d^2 - 40*a^5*b^2*d^2)/(16*a^10*d^4 + 16*b^10*d^4 + 80*a^2*b^8*d^4 + 160*a^4*b^6*d^4 + 160*a^6*b^4*d^4 + 80*a^8*b^2*d^4))^(1/2) + (log(((((320*a^2*b^12*d^4 - 16*b^14*d^4 - 1760*a^4*b^10*d^4 + 1600*a^6*b^8*d^4 - 400*a^8*b^6*d^4)^(1/2) + 20*a*b^6*d^2 - 40*a^3*b^4*d^2 + 4*a^5*b^2*d^2)/(a^10*d^4 + b^10*d^4 + 5*a^2*b^8*d^4 + 10*a^4*b^6*d^4 + 10*a^6*b^4*d^4 + 5*a^8*b^2*d^4))^(1/2)*(((((320*a^2*b^12*d^4 - 16*b^14*d^4 - 1760*a^4*b^10*d^4 + 1600*a^6*b^8*d^4 - 400*a^8*b^6*d^4)^(1/2) + 20*a*b^6*d^2 - 40*a^3*b^4*d^2 + 4*a^5*b^2*d^2)/(a^10*d^4 + b^10*d^4 + 5*a^2*b^8*d^4 + 10*a^4*b^6*d^4 + 10*a^6*b^4*d^4 + 5*a^8*b^2*d^4))^(1/2)*(96*a*b^21*d^4 + 736*a^3*b^19*d^4 + 2432*a^5*b^17*d^4 + 4480*a^7*b^15*d^4 + 4928*a^9*b^13*d^4 + 3136*a^11*b^11*d^4 + 896*a^13*b^9*d^4 - 128*a^15*b^7*d^4 - 160*a^17*b^5*d^4 - 32*a^19*b^3*d^4 - ((((320*a^2*b^12*d^4 - 16*b^14*d^4 - 1760*a^4*b^10*d^4 + 1600*a^6*b^8*d^4 - 400*a^8*b^6*d^4)^(1/2) + 20*a*b^6*d^2 - 40*a^3*b^4*d^2 + 4*a^5*b^2*d^2)/(a^10*d^4 + b^10*d^4 + 5*a^2*b^8*d^4 + 10*a^4*b^6*d^4 + 10*a^6*b^4*d^4 + 5*a^8*b^2*d^4))^(1/2)*(a + b*cot(c + d*x))^(1/2)*(64*a*b^22*d^5 + 640*a^3*b^20*d^5 + 2880*a^5*b^18*d^5 + 7680*a^7*b^16*d^5 + 13440*a^9*b^14*d^5 + 16128*a^11*b^12*d^5 + 13440*a^13*b^10*d^5 + 7680*a^15*b^8*d^5 + 2880*a^17*b^6*d^5 + 640*a^19*b^4*d^5 + 64*a^21*b^2*d^5))/4))/4 - (a + b*cot(c + d*x))^(1/2)*(320*a^4*b^16*d^3 - 16*b^20*d^3 + 1024*a^6*b^14*d^3 + 1440*a^8*b^12*d^3 + 1024*a^10*b^10*d^3 + 320*a^12*b^8*d^3 - 16*a^16*b^4*d^3)))/4 - 8*b^19*d^2 - 40*a^2*b^17*d^2 - 72*a^4*b^15*d^2 - 40*a^6*b^13*d^2 + 40*a^8*b^11*d^2 + 72*a^10*b^9*d^2 + 40*a^12*b^7*d^2 + 8*a^14*b^5*d^2)*(((320*a^2*b^12*d^4 - 16*b^14*d^4 - 1760*a^4*b^10*d^4 + 1600*a^6*b^8*d^4 - 400*a^8*b^6*d^4)^(1/2) + 20*a*b^6*d^2 - 40*a^3*b^4*d^2 + 4*a^5*b^2*d^2)/(a^10*d^4 + b^10*d^4 + 5*a^2*b^8*d^4 + 10*a^4*b^6*d^4 + 10*a^6*b^4*d^4 + 5*a^8*b^2*d^4))^(1/2))/4 + (log(((-((320*a^2*b^12*d^4 - 16*b^14*d^4 - 1760*a^4*b^10*d^4 + 1600*a^6*b^8*d^4 - 400*a^8*b^6*d^4)^(1/2

) - 20*a*b^6*d^2 + 40*a^3*b^4*d^2 - 4*a^5*b^2*d^2)/(a^10*d^4 + b^10*d^4 + 5
*a^2*b^8*d^4 + 10*a^4*b^6*d^4 + 10*a^6*b^4*d^4 + 5*a^8*b^2*d^4))^(1/2)*(((-
((320*a^2*b^12*d^4 - 16*b^14*d^4 - 1760*a^4*b^10*d^4 + 1600*a^6*b^8*d^4 - 4
00*a^8*b^6*d^4)^(1/2) - 20*a*b^6*d^2 + 40*a^3*b^4*d^2 - 4*a^5*b^2*d^2)/(a^1
0*d^4 + b^10*d^4 + 5*a^2*b^8*d^4 + 10*a^4*b^6*d^4 + 10*a^6*b^4*d^4 + 5*a^8*
b^2*d^4))^(1/2)*(96*a*b^21*d^4 + 736*a^3*b^19*d^4 + 2432*a^5*b^17*d^4 + 448
0*a^7*b^15*d^4 + 4928*a^9*b^13*d^4 + 3136*a^11*b^11*d^4 + 896*a^13*b^9*d^4
- 128*a^15*b^7*d^4 - 160*a^17*b^5*d^4 - 32*a^19*b^3*d^4 - ((-((320*a^2*b^12
*d^4 - 16*b^14*d^4 - 1760*a^4*b^10*d^4 + 1600*a^6*b^8*d^4 - 400*a^8*b^6*d^4
)^(1/2) - 20*a*b^6*d^2 + 40*a^3*b^4*d^2 - 4*a^5*b^2*d^2)/(a^10*d^4 + b^10*d
^4 + 5*a^2*b^8*d^4 + 10*a^4*b^6*d^4 + 10*a^6*b^4*d^4 + 5*a^8*b^2*d^4))^(1/2
)*(a + b*cot(c + d*x))^(1/2)*(64*a*b^22*d^5 + 640*a^3*b^20*d^5 + 2880*a^5*b
^18*d^5 + 7680*a^7*b^16*d^5 + 13440*a^9*b^14*d^5 + 16128*a^11*b^12*d^5 + 13
440*a^13*b^10*d^5 + 7680*a^15*b^8*d^5 + 2880*a^17*b^6*d^5 + 640*a^19*b^4*d^
5 + 64*a^21*b^2*d^5))/4))/4 - (a + b*cot(c + d*x))^(1/2)*(320*a^4*b^16*d^3
- 16*b^20*d^3 + 1024*a^6*b^14*d^3 + 1440*a^8*b^12*d^3 + 1024*a^10*b^10*d^3
+ 320*a^12*b^8*d^3 - 16*a^16*b^4*d^3)))/4 - 8*b^19*d^2 - 40*a^2*b^17*d^2 -
72*a^4*b^15*d^2 - 40*a^6*b^13*d^2 + 40*a^8*b^11*d^2 + 72*a^10*b^9*d^2 + 40*
a^12*b^7*d^2 + 8*a^14*b^5*d^2)*(-((320*a^2*b^12*d^4 - 16*b^14*d^4 - 1760*a^
4*b^10*d^4 + 1600*a^6*b^8*d^4 - 400*a^8*b^6*d^4)^(1/2) - 20*a*b^6*d^2 + 40*
a^3*b^4*d^2 - 4*a^5*b^2*d^2)/(a^10*d^4 + b^10*d^4 + 5*a^2*b^8*d^4 + 10*a^4*
b^6*d^4 + 10*a^6*b^4*d^4 + 5*a^8*b^2*d^4))^(1/2))/4 - log((((320*a^2*b^12*d
^4 - 16*b^14*d^4 - 1760*a^4*b^10*d^4 + 1600*a^6*b^8*d^4 - 400*a^8*b^6*d^4)^
(1/2) + 20*a*b^6*d^2 - 40*a^3*b^4*d^2 + 4*a^5*b^2*d^2)/(16*a^10*d^4 + 16*b^
10*d^4 + 80*a^2*b^8*d^4 + 160*a^4*b^6*d^4 + 160*a^6*b^4*d^4 + 80*a^8*b^2*d^
4))^(1/2)*((((320*a^2*b^12*d^4 - 16*b^14*d^4 - 1760*a^4*b^10*d^4 + 1600*a^6
*b^8*d^4 - 400*a^8*b^6*d^4)^(1/2) + 20*a*b^6*d^2 - 40*a^3*b^4*d^2 + 4*a^5*b
^2*d^2)/(16*a^10*d^4 + 16*b^10*d^4 + 80*a^2*b^8*d^4 + 160*a^4*b^6*d^4 + 160
*a^6*b^4*d^4 + 80*a^8*b^2*d^4))^(1/2)*(96*a*b^21*d^4 + 736*a^3*b^19*d^4 + 2
432*a^5*b^17*d^4 + 4480*a^7*b^15*d^4 + 4928*a^9*b^13*d^4 + 3136*a^11*b^11*d
^4 + 896*a^13*b^9*d^4 - 128*a^15*b^7*d^4 - 160*a^17*b^5*d^4 - 32*a^19*b^3*d
^4 + (((320*a^2*b^12*d^4 - 16*b^14*d^4 - 1760*a^4*b^10*d^4 + 1600*a^6*b^8*d
^4 - 400*a^8*b^6*d^4)^(1/2) + 20*a*b^6*d^2 - 40*a^3*b^4*d^2 + 4*a^5*b^2*d^2
)/(16*a^10*d^4 + 16*b^10*d^4 + 80*a^2*b^8*d^4 + 160*a^4*b^6*d^4 + 160*a^6*b
^4*d^4 + 80*a^8*b^2*d^4))^(1/2)*(a + b*cot(c + d*x))^(1/2)*(64*a*b^22*d^5 +
640*a^3*b^20*d^5 + 2880*a^5*b^18*d^5 + 7680*a^7*b^16*d^5 + 13440*a^9*b^14*
d^5 + 16128*a^11*b^12*d^5 + 13440*a^13*b^10*d^5 + 7680*a^15*b^8*d^5 + 2880*
a^17*b^6*d^5 + 640*a^19*b^4*d^5 + 64*a^21*b^2*d^5)) + (a + b*cot(c + d*x))^
(1/2)*(320*a^4*b^16*d^3 - 16*b^20*d^3 + 1024*a^6*b^14*d^3 + 1440*a^8*b^12*d
^3 + 1024*a^10*b^10*d^3 + 320*a^12*b^8*d^3 - 16*a^16*b^4*d^3)) - 8*b^19*d^2
- 40*a^2*b^17*d^2 - 72*a^4*b^15*d^2 - 40*a^6*b^13*d^2 + 40*a^8*b^11*d^2 +
72*a^10*b^9*d^2 + 40*a^12*b^7*d^2 + 8*a^14*b^5*d^2)*(((320*a^2*b^12*d^4 - 1
6*b^14*d^4 - 1760*a^4*b^10*d^4 + 1600*a^6*b^8*d^4 - 400*a^8*b^6*d^4)^(1/2)
+ 20*a*b^6*d^2 - 40*a^3*b^4*d^2 + 4*a^5*b^2*d^2)/(16*a^10*d^4 + 16*b^10*d^4
+ 80*a^2*b^8*d^4 + 160*a^4*b^6*d^4 + 160*a^6*b^4*d^4 + 80*a^8*b^2*d^4))^(1

/2) - log((-((320*a^2*b^12*d^4 - 16*b^14*d^4 - 1760*a^4*b^10*d^4 + 1600*a^6
*b^8*d^4 - 400*a^8*b^6*d^4)^(1/2) - 20*a*b^6*d^2 + 40*a^3*b^4*d^2 - 4*a^5*b
^2*d^2)/(16*a^10*d^4 + 16*b^10*d^4 + 80*a^2*b^8*d^4 + 160*a^4*b^6*d^4 + 160
*a^6*b^4*d^4 + 80*a^8*b^2*d^4))^(1/2)*((-((320*a^2*b^12*d^4 - 16*b^14*d^4 -
 1760*a^4*b^10*d^4 + 1600*a^6*b^8*d^4 - 400*a^8*b^6*d^4)^(1/2) - 20*a*b^6*d
^2 + 40*a^3*b^4*d^2 - 4*a^5*b^2*d^2)/(16*a^10*d^4 + 16*b^10*d^4 + 80*a^2*b^
8*d^4 + 160*a^4*b^6*d^4 + 160*a^6*b^4*d^4 + 80*a^8*b^2*d^4))^(1/2)*(96*a*b^
21*d^4 + 736*a^3*b^19*d^4 + 2432*a^5*b^17*d^4 + 4480*a^7*b^15*d^4 + 4928*a^
9*b^13*d^4 + 3136*a^11*b^11*d^4 + 896*a^13*b^9*d^4 - 128*a^15*b^7*d^4 - 160
*a^17*b^5*d^4 - 32*a^19*b^3*d^4 + (-((320*a^2*b^12*d^4 - 16*b^14*d^4 - 1760
*a^4*b^10*d^4 + 1600*a^6*b^8*d^4 - 400*a^8*b^6*d^4)^(1/2) - 20*a*b^6*d^2 +
40*a^3*b^4*d^2 - 4*a^5*b^2*d^2)/(16*a^10*d^4 + 16*b^10*d^4 + 80*a^2*b^8*d^4
 + 160*a^4*b^6*d^4 + 160*a^6*b^4*d^4 + 80*a^8*b^2*d^4))^(1/2)*(a + b*cot(c
+ d*x))^(1/2)*(64*a*b^22*d^5 + 640*a^3*b^20*d^5 + 2880*a^5*b^18*d^5 + 7680*
a^7*b^16*d^5 + 13440*a^9*b^14*d^5 + 16128*a^11*b^12*d^5 + 13440*a^13*b^10*d
^5 + 7680*a^15*b^8*d^5 + 2880*a^17*b^6*d^5 + 640*a^19*b^4*d^5 + 64*a^21*b^2
*d^5)) + (a + b*cot(c + d*x))^(1/2)*(320*a^4*b^16*d^3 - 16*b^20*d^3 + 1024*
a^6*b^14*d^3 + 1440*a^8*b^12*d^3 + 1024*a^10*b^10*d^3 + 320*a^12*b^8*d^3 -
16*a^16*b^4*d^3)) - 8*b^19*d^2 - 40*a^2*b^17*d^2 - 72*a^4*b^15*d^2 - 40*a^6
*b^13*d^2 + 40*a^8*b^11*d^2 + 72*a^10*b^9*d^2 + 40*a^12*b^7*d^2 + 8*a^14*b^
5*d^2)*(-((320*a^2*b^12*d^4 - 16*b^14*d^4 - 1760*a^4*b^10*d^4 + 1600*a^6*b^
8*d^4 - 400*a^8*b^6*d^4)^(1/2) - 20*a*b^6*d^2 + 40*a^3*b^4*d^2 - 4*a^5*b^2*
d^2)/(16*a^10*d^4 + 16*b^10*d^4 + 80*a^2*b^8*d^4 + 160*a^4*b^6*d^4 + 160*a^
6*b^4*d^4 + 80*a^8*b^2*d^4))^(1/2) + (log(16*a^4*b^15*d^2 - ((((-(4*a^7*d^2
 + (320*a^6*b^8*d^4 - 16*a^4*b^10*d^4 - 1760*a^8*b^6*d^4 + 1600*a^10*b^4*d^
4 - 400*a^12*b^2*d^4)^(1/2) + 20*a^3*b^4*d^2 - 40*a^5*b^2*d^2)/(a^10*d^4 +
b^10*d^4 + 5*a^2*b^8*d^4 + 10*a^4*b^6*d^4 + 10*a^6*b^4*d^4 + 5*a^8*b^2*d^4)
)^(1/2)*(((a + b*cot(c + d*x))^(1/2)*(-(4*a^7*d^2 + (320*a^6*b^8*d^4 - 16*a
^4*b^10*d^4 - 1760*a^8*b^6*d^4 + 1600*a^10*b^4*d^4 - 400*a^12*b^2*d^4)^(1/2
) + 20*a^3*b^4*d^2 - 40*a^5*b^2*d^2)/(a^10*d^4 + b^10*d^4 + 5*a^2*b^8*d^4 +
 10*a^4*b^6*d^4 + 10*a^6*b^4*d^4 + 5*a^8*b^2*d^4))^(1/2)*(64*a*b^22*d^5 + 6
40*a^3*b^20*d^5 + 2880*a^5*b^18*d^5 + 7680*a^7*b^16*d^5 + 13440*a^9*b^14*d^
5 + 16128*a^11*b^12*d^5 + 13440*a^13*b^10*d^5 + 7680*a^15*b^8*d^5 + 2880*a^
17*b^6*d^5 + 640*a^19*b^4*d^5 + 64*a^21*b^2*d^5))/4 - 32*a*b^21*d^4 - 160*a
^3*b^19*d^4 - 128*a^5*b^17*d^4 + 896*a^7*b^15*d^4 + 3136*a^9*b^13*d^4 + 492
8*a^11*b^11*d^4 + 4480*a^13*b^9*d^4 + 2432*a^15*b^7*d^4 + 736*a^17*b^5*d^4
+ 96*a^19*b^3*d^4))/4 - (a + b*cot(c + d*x))^(1/2)*(320*a^6*b^14*d^3 - 16*a
^2*b^18*d^3 + 1024*a^8*b^12*d^3 + 1440*a^10*b^10*d^3 + 1024*a^12*b^8*d^3 +
320*a^14*b^6*d^3 - 16*a^18*b^2*d^3))*(-(4*a^7*d^2 + (320*a^6*b^8*d^4 - 16*a
^4*b^10*d^4 - 1760*a^8*b^6*d^4 + 1600*a^10*b^4*d^4 - 400*a^12*b^2*d^4)^(1/2
) + 20*a^3*b^4*d^2 - 40*a^5*b^2*d^2)/(a^10*d^4 + b^10*d^4 + 5*a^2*b^8*d^4 +
 10*a^4*b^6*d^4 + 10*a^6*b^4*d^4 + 5*a^8*b^2*d^4))^(1/2))/4 + 96*a^6*b^13*d
^2 + 240*a^8*b^11*d^2 + 320*a^10*b^9*d^2 + 240*a^12*b^7*d^2 + 96*a^14*b^5*d
^2 + 16*a^16*b^3*d^2)*(-(4*a^7*d^2 + (320*a^6*b^8*d^4 - 16*a^4*b^10*d^4 - 1
760*a^8*b^6*d^4 + 1600*a^10*b^4*d^4 - 400*a^12*b^2*d^4)^(1/2) + 20*a^3*b^4*

```
d^2 - 40*a^5*b^2*d^2)/(a^10*d^4 + b^10*d^4 + 5*a^2*b^8*d^4 + 10*a^4*b^6*d^4
 + 10*a^6*b^4*d^4 + 5*a^8*b^2*d^4))^(1/2))/4 - log(16*a^4*b^15*d^2 - ((-(4*
a^7*d^2 + (320*a^6*b^8*d^4 - 16*a^4*b^10*d^4 - 1760*a^8*b^6*d^4 + 1600*a^10
*b^4*d^4 - 400*a^12*b^2*d^4)^(1/2) + 20*a^3*b^4*d^2 - 40*a^5*b^2*d^2)/(16*a
^10*d^4 + 16*b^10*d^4 + 80*a^2*b^8*d^4 + 160*a^4*b^6*d^4 + 160*a^6*b^4*d^4
+ 80*a^8*b^2*d^4))^(1/2)*(896*a^7*b^15*d^4 - 32*a*b^21*d^4 - 160*a^3*b^19*d
^4 - 128*a^5*b^17*d^4 - (a + b*cot(c + d*x))^(1/2)*(-(4*a^7*d^2 + (320*a^6*
b^8*d^4 - 16*a^4*b^10*d^4 - 1760*a^8*b^6*d^4 + 1600*a^10*b^4*d^4 - 400*a^12
*b^2*d^4)^(1/2) + 20*a^3*b^4*d^2 - 40*a^5*b^2*d^2)/(16*a^10*d^4 + 16*b^10*d
^4 + 80*a^2*b^8*d^4 + 160*a^4*b^6*d^4 + 160*a^6*b^4*d^4 + 80*a^8*b^2*d^4))^
(1/2)*(64*a*b^22*d^5 + 640*a^3*b^20*d^5 + 2880*a^5*b^18*d^5 + 7680*a^7*b^16
*d^5 + 13440*a^9*b^14*d^5 + 16128*a^11*b^12*d^5 + 13440*a^13*b^10*d^5 + 768
0*a^15*b^8*d^5 + 2880*a^17*b^6*d^5 + 640*a^19*b^4*d^5 + 64*a^21*b^2*d^5) +
3136*a^9*b^13*d^4 + 4928*a^11*b^11*d^4 + 4480*a^13*b^9*d^4 + 2432*a^15*b^7*
d^4 + 736*a^17*b^5*d^4 + 96*a^19*b^3*d^4) + (a + b*cot(c + d*x))^(1/2)*(320
*a^6*b^14*d^3 - 16*a^2*b^18*d^3 + 1024*a^8*b^12*d^3 + 1440*a^10*b^10*d^3 +
1024*a^12*b^8*d^3 + 320*a^14*b^6*d^3 - 16*a^18*b^2*d^3))*(-(4*a^7*d^2 + (32
0*a^6*b^8*d^4 - 16*a^4*b^10*d^4 - 1760*a^8*b^6*d^4 + 1600*a^10*b^4*d^4 - 40
0*a^12*b^2*d^4)^(1/2) + 20*a^3*b^4*d^2 - 40*a^5*b^2*d^2)/(16*a^10*d^4 + 16*
b^10*d^4 + 80*a^2*b^8*d^4 + 160*a^4*b^6*d^4 + 160*a^6*b^4*d^4 + 80*a^8*b^2*
d^4))^(1/2) + 96*a^6*b^13*d^2 + 240*a^8*b^11*d^2 + 320*a^10*b^9*d^2 + 240*a
^12*b^7*d^2 + 96*a^14*b^5*d^2 + 16*a^16*b^3*d^2)*(-(4*a^7*d^2 + (320*a^6*b^
8*d^4 - 16*a^4*b^10*d^4 - 1760*a^8*b^6*d^4 + 1600*a^10*b^4*d^4 - 400*a^12*b
^2*d^4)^(1/2) + 20*a^3*b^4*d^2 - 40*a^5*b^2*d^2)/(16*a^10*d^4 + 16*b^10*d^4
 + 80*a^2*b^8*d^4 + 160*a^4*b^6*d^4 + 160*a^6*b^4*d^4 + 80*a^8*b^2*d^4))^(1
/2) - ((2*a*b)/(3*(a^2 + b^2)) + (2*b*(a^2 - b^2)*(a + b*cot(c + d*x)))/(a^
2 + b^2)^2)/(d*(a + b*cot(c + d*x))^(3/2)) - ((2*a*b)/(3*(a^2 + b^2)) + (4*
a^2*b*(a + b*cot(c + d*x)))/(a^2 + b^2)^2)/(d*(a + b*cot(c + d*x))^(3/2))
```

CHAPTER 4

APPENDIX

4.1 Listing of Grading functions

The following are the current version of the grading functions used for grading the quality of the antiderivative with reference to the optimal antiderivative included in the test suite.

There is a version for Maple and for Mathematica/Rubi. There is a version for grading Sympy and version for use with Sagemath.

The following are links to the current source code.

The following are the listings of source code of the grading functions.

Mathematica and Rubi grading function

```
(* Original version thanks to Albert Rich emailed on 03/21/2017 *)
(* ::Package:: *)

(* Nasser: April 7,2022. add second output which gives reason for the grade *)
(*                       Small rewrite of logic in main function to make it*)
(*                       match Maple's logic. No change in functionality otherwise*)

(* ::Subsection:: *)
(*GradeAntiderivative[result,optimal]*)


(* ::Text:: *)
(*If result and optimal are mathematical expressions, *)
```

```
(*          GradeAntiderivative[result,optimal] returns*)
(*  "F" if the result fails to integrate an expression that*)
(*      is integrable*)
(*  "C" if result involves higher level functions than necessary*)
(*  "B" if result is more than twice the size of the optimal*)
(*      antiderivative*)
(*  "A" if result can be considered optimal*)


GradeAntiderivative[result_,optimal_] := Module[{expnResult,expnOptimal,leafCountResult,leafCou
    expnResult  = ExpnType[result];
    expnOptimal = ExpnType[optimal];
    leafCountResult = LeafCount[result];
    leafCountOptimal = LeafCount[optimal];

    (*Print["expnResult=",expnResult," expnOptimal=",expnOptimal];*)
    If[expnResult<=expnOptimal,
        If[Not[FreeQ[result,Complex]], (*result contains complex*)
            If[Not[FreeQ[optimal,Complex]], (*optimal contains complex*)
                If[leafCountResult<=2*leafCountOptimal,
                    finalresult={"A",""}
                    ,(*ELSE*)
                    finalresult={"B","Both result and optimal contain complex but leaf count is
                  ]
            ,(*ELSE*)
                finalresult={"C","Result contains complex when optimal does not."}
            ]
        ,(*ELSE*)(*result does not contains complex*)
            If[leafCountResult<=2*leafCountOptimal,
               finalresult={"A",""}
            ,(*ELSE*)
               finalresult={"B","Leaf count is larger than twice the leaf count of optimal. $"<
              ]
        ]
    ,(*ELSE*) (*expnResult>expnOptimal*)
        If[FreeQ[result,Integrate] && FreeQ[result,Int],
            finalresult={"C","Result contains higher order function than in optimal. Order "<>T
            ,
            finalresult={"F","Contains unresolved integral."}
        ]
    ];

    finalresult
]

(* ::Text:: *)
(*The following summarizes the type number assigned an *)
```

```
(*expression based on the functions it involves*)
(*1 = rational function*)
(*2 = algebraic function*)
(*3 = elementary function*)
(*4 = special function*)
(*5 = hyperpergeometric function*)
(*6 = appell function*)
(*7 = rootsum function*)
(*8 = integrate function*)
(*9 = unknown function*)


ExpnType[expn_] :=
  If[AtomQ[expn],
    1,
  If[ListQ[expn],
    Max[Map[ExpnType,expn]],
  If[Head[expn]===Power,
    If[IntegerQ[expn[[2]]],
      ExpnType[expn[[1]]],
    If[Head[expn[[2]]]===Rational,
      If[IntegerQ[expn[[1]]] || Head[expn[[1]]]===Rational,
        1,
      Max[ExpnType[expn[[1]]],2]],
    Max[ExpnType[expn[[1]]],ExpnType[expn[[2]]],3]]],
  If[Head[expn]===Plus || Head[expn]===Times,
    Max[ExpnType[First[expn]],ExpnType[Rest[expn]]],
  If[ElementaryFunctionQ[Head[expn]],
    Max[3,ExpnType[expn[[1]]]],
  If[SpecialFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],4]],
  If[HypergeometricFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],5]],
  If[AppellFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],6]],
  If[Head[expn]===RootSum,
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],7]],
  If[Head[expn]===Integrate || Head[expn]===Int,
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],8]],
  9]]]]]]]]]]


ElementaryFunctionQ[func_] :=
  MemberQ[{
  Exp,Log,
  Sin,Cos,Tan,Cot,Sec,Csc,
  ArcSin,ArcCos,ArcTan,ArcCot,ArcSec,ArcCsc,
```

```
  Sinh,Cosh,Tanh,Coth,Sech,Csch,
  ArcSinh,ArcCosh,ArcTanh,ArcCoth,ArcSech,ArcCsch
},func]


SpecialFunctionQ[func_] :=
  MemberQ[{
  Erf, Erfc, Erfi,
  FresnelS, FresnelC,
  ExpIntegralE, ExpIntegralEi, LogIntegral,
  SinIntegral, CosIntegral, SinhIntegral, CoshIntegral,
  Gamma, LogGamma, PolyGamma,
  Zeta, PolyLog, ProductLog,
  EllipticF, EllipticE, EllipticPi
},func]


HypergeometricFunctionQ[func_] :=
  MemberQ[{Hypergeometric1F1,Hypergeometric2F1,HypergeometricPFQ},func]


AppellFunctionQ[func_] :=
  MemberQ[{AppellF1},func]
```

Maple grading function

```
# File: GradeAntiderivative.mpl
# Original version thanks to Albert Rich emailed on 03/21/2017

#Nasser 03/22/2017  Use Maple leaf count instead since buildin
#Nasser 03/23/2017  missing 'ln' for ElementaryFunctionQ added
#Nasser 03/24/2017  corrected the check for complex result
#Nasser 10/27/2017  check for leafsize and do not call ExpnType()
#                   if leaf size is "too large". Set at 500,000
#Nasser 12/22/2019  Added debug flag, added 'dilog' to special functions
#                   see problem 156, file Apostol_Problems
#Nasser 4/07/2022   add second output which gives reason for the grade

GradeAntiderivative := proc(result,optimal)
local leaf_count_result,
        leaf_count_optimal,
        ExpnType_result,
        ExpnType_optimal,
        debug:=false;

        leaf_count_result:=leafcount(result);
```

```
        #do NOT call ExpnType() if leaf size is too large. Recursion problem
        if leaf_count_result > 500000 then
            return "B","result has leaf size over 500,000. Avoiding possible recursion issues
        fi;

        leaf_count_optimal := leafcount(optimal);
        ExpnType_result  := ExpnType(result);
        ExpnType_optimal := ExpnType(optimal);

        if debug then
            print("ExpnType_result",ExpnType_result," ExpnType_optimal=",ExpnType_optimal);
        fi;

# If result and optimal are mathematical expressions,
#  GradeAntiderivative[result,optimal] returns
#   "F" if the result fails to integrate an expression that
#       is integrable
#   "C" if result involves higher level functions than necessary
#   "B" if result is more than twice the size of the optimal
#       antiderivative
#   "A" if result can be considered optimal

    #This check below actually is not needed, since I only
    #call this grading only for passed integrals. i.e. I check
    #for "F" before calling this. But no harm of keeping it here.
    #just in case.


    if not type(result,freeof('int')) then
        return "F","Result contains unresolved integral";
    fi;


    if ExpnType_result<=ExpnType_optimal then
        if debug then
             print("ExpnType_result<=ExpnType_optimal");
        fi;
        if is_contains_complex(result) then
            if is_contains_complex(optimal) then
                if debug then
                        print("both result and optimal complex");
                fi;
                if leaf_count_result<=2*leaf_count_optimal then
                    return "A"," ";
                else
                   return "B",cat("Both result and optimal contain complex but leaf count of
                                convert(leaf_count_result,string)," vs. $2 (",
```

```
                                   convert(leaf_count_optimal,string)," ) = ",convert(2*leaf_c
                end if
            else #result contains complex but optimal is not
                if debug then
                        print("result contains complex but optimal is not");
                fi;
                return "C","Result contains complex when optimal does not.";
            fi;
        else # result do not contain complex
             # this assumes optimal do not as well. No check is needed here.
            if debug then
                   print("result do not contain complex, this assumes optimal do not as well")
            fi;
            if leaf_count_result<=2*leaf_count_optimal then
                if debug then
                    print("leaf_count_result<=2*leaf_count_optimal");
                fi;
                return "A"," ";
            else
                if debug then
                    print("leaf_count_result>2*leaf_count_optimal");
                fi;
                return "B",cat("Leaf count of result is larger than twice the leaf count of opt
                                   convert(leaf_count_result,string),"$ vs. $2(",
                                   convert(leaf_count_optimal,string),")=",convert(2*leaf_count
            fi;
        fi;
    else #ExpnType(result) > ExpnType(optimal)
        if debug then
            print("ExpnType(result) > ExpnType(optimal)");
        fi;
        return "C",cat("Result contains higher order function than in optimal. Order ",
                       convert(ExpnType_result,string)," vs. order ",
                       convert(ExpnType_optimal,string),".");
    fi;

end proc:

#
# is_contains_complex(result)
# takes expressions and returns true if it contains "I" else false
#
#Nasser 032417
is_contains_complex:= proc(expression)
  return (has(expression,I));
end proc:
```

```
# The following summarizes the type number assigned an expression
# based on the functions it involves
# 1 = rational function
# 2 = algebraic function
# 3 = elementary function
# 4 = special function
# 5 = hyperpergeometric function
# 6 = appell function
# 7 = rootsum function
# 8 = integrate function
# 9 = unknown function

ExpnType := proc(expn)
  if type(expn,'atomic') then
    1
  elif type(expn,'list') then
    apply(max,map(ExpnType,expn))
  elif type(expn,'sqrt') then
    if type(op(1,expn),'rational') then
       1
    else
       max(2,ExpnType(op(1,expn)))
    end if
  elif type(expn,'`^`') then
    if type(op(2,expn),'integer') then
      ExpnType(op(1,expn))
    elif type(op(2,expn),'rational') then
      if type(op(1,expn),'rational') then
         1
      else
         max(2,ExpnType(op(1,expn)))
      end if
    else
         max(3,ExpnType(op(1,expn)),ExpnType(op(2,expn)))
    end if
  elif type(expn,'`+`') or type(expn,'`*`') then
    max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
  elif ElementaryFunctionQ(op(0,expn)) then
    max(3,ExpnType(op(1,expn)))
  elif SpecialFunctionQ(op(0,expn)) then
    max(4,apply(max,map(ExpnType,[op(expn)])))
  elif HypergeometricFunctionQ(op(0,expn)) then
    max(5,apply(max,map(ExpnType,[op(expn)])))
  elif AppellFunctionQ(op(0,expn)) then
    max(6,apply(max,map(ExpnType,[op(expn)])))
  elif op(0,expn)='int' then
    max(8,apply(max,map(ExpnType,[op(expn)]))) else
```

```
  9
  end if
end proc:


ElementaryFunctionQ := proc(func)
  member(func,[
        exp,log,ln,
        sin,cos,tan,cot,sec,csc,
        arcsin,arccos,arctan,arccot,arcsec,arccsc,
        sinh,cosh,tanh,coth,sech,csch,
        arcsinh,arccosh,arctanh,arccoth,arcsech,arccsch])
end proc:

SpecialFunctionQ := proc(func)
  member(func,[
        erf,erfc,erfi,
        FresnelS,FresnelC,
        Ei,Ei,Li,Si,Ci,Shi,Chi,
        GAMMA,lnGAMMA,Psi,Zeta,polylog,dilog,LambertW,
        EllipticF,EllipticE,EllipticPi])
end proc:

HypergeometricFunctionQ := proc(func)
  member(func,[Hypergeometric1F1,hypergeom,HypergeometricPFQ])
end proc:

AppellFunctionQ := proc(func)
  member(func,[AppellF1])
end proc:

# u is a sum or product.  rest(u) returns all but the
# first term or factor of u.
rest := proc(u) local v;
  if nops(u)=2 then
     op(2,u)
  else
     apply(op(0,u),op(2..nops(u),u))
  end if
end proc:

#leafcount(u) returns the number of nodes in u.
#Nasser 3/23/17  Replaced by build-in leafCount from package in Maple
leafcount := proc(u)
   MmaTranslator[Mma][LeafCount](u);
end proc:
```

Sympy grading function

```
#Dec 24, 2019. Nasser M. Abbasi:
#             Port of original Maple grading function by
#             Albert Rich to use with Sympy/Python
#Dec 27, 2019  Nasser. Added `RootSum`. See problem 177, Timofeev file
#             added 'exp_polar'
from sympy import *

def leaf_count(expr):
   #sympy do not have leaf count function. This is approximation
   return round(1.7*count_ops(expr))

def is_sqrt(expr):
   if isinstance(expr,Pow):
      if expr.args[1] == Rational(1,2):
         return True
      else:
         return False
   else:
      return False

def is_elementary_function(func):
   return func in [exp,log,ln,sin,cos,tan,cot,sec,csc,
         asin,acos,atan,acot,asec,acsc,sinh,cosh,tanh,coth,sech,csch,
         asinh,acosh,atanh,acoth,asech,acsch
      ]

def is_special_function(func):
   return func in [ erf,erfc,erfi,
          fresnels,fresnelc,Ei,Ei,Li,Si,Ci,Shi,Chi,
          gamma,loggamma,digamma,zeta,polylog,LambertW,
          elliptic_f,elliptic_e,elliptic_pi,exp_polar
       ]

def is_hypergeometric_function(func):
   return func in [hyper]

def is_appell_function(func):
   return func in [appellf1]

def is_atom(expn):
   try:
      if expn.isAtom or isinstance(expn,int) or isinstance(expn,float):
        return True
      else:
        return False
```

```
    except AttributeError as error:
        return False

def expnType(expn):
    debug=False
    if debug:
        print("expn=",expn,"type(expn)=",type(expn))

    if is_atom(expn):
       return 1
    elif isinstance(expn,list):
        return max(map(expnType, expn))    #apply(max,map(ExpnType,expn))
    elif  is_sqrt(expn):
        if isinstance(expn.args[0],Rational): #type(op(1,expn),'rational')
            return 1
        else:
            return max(2,expnType(expn.args[0]))   #max(2,ExpnType(op(1,expn)))
    elif isinstance(expn,Pow):    #type(expn,'`^`')
        if isinstance(expn.args[1],Integer):   #type(op(2,expn),'integer')
            return expnType(expn.args[0])    #ExpnType(op(1,expn))
        elif isinstance(expn.args[1],Rational):   #type(op(2,expn),'rational')
            if isinstance(expn.args[0],Rational): #type(op(1,expn),'rational')
                return 1
            else:
                return max(2,expnType(expn.args[0]))   #max(2,ExpnType(op(1,expn)))
        else:
            return max(3,expnType(expn.args[0]),expnType(expn.args[1]))  #max(3,ExpnType(op(1,expn)),ExpnTy
    elif isinstance(expn,Add) or isinstance(expn,Mul): #type(expn,'`+`') or type(expn,'`*`')
        m1 = expnType(expn.args[0])
        m2 = expnType(list(expn.args[1:]))
        return max(m1,m2)   #max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
    elif is_elementary_function(expn.func):   #ElementaryFunctionQ(op(0,expn))
        return max(3,expnType(expn.args[0]))   #max(3,ExpnType(op(1,expn)))
    elif is_special_function(expn.func): #SpecialFunctionQ(op(0,expn))
        m1 = max(map(expnType, list(expn.args)))
        return max(4,m1)    #max(4,apply(max,map(ExpnType,[op(expn)])))
    elif is_hypergeometric_function(expn.func): #HypergeometricFunctionQ(op(0,expn))
        m1 = max(map(expnType, list(expn.args)))
        return max(5,m1)    #max(5,apply(max,map(ExpnType,[op(expn)])))
    elif is_appell_function(expn.func):
        m1 = max(map(expnType, list(expn.args)))
        return max(6,m1)    #max(5,apply(max,map(ExpnType,[op(expn)])))
    elif isinstance(expn,RootSum):
        m1 = max(map(expnType, list(expn.args))) #Apply[Max,Append[Map[ExpnType,Apply[List,expn]],7]],
        return max(7,m1)
    elif str(expn).find("Integral") != -1:
```

```
        m1 = max(map(expnType, list(expn.args)))
        return max(8,m1)    #max(5,apply(max,map(ExpnType,[op(expn)])))
    else:
        return 9

#main function
def grade_antiderivative(result,optimal):

    #print ("Enter grade_antiderivative for sagemath")
    #print("Enter grade_antiderivative, result=",result," optimal=",optimal)

    leaf_count_result  = leaf_count(result)
    leaf_count_optimal = leaf_count(optimal)

    #print("leaf_count_result=",leaf_count_result)
    #print("leaf_count_optimal=",leaf_count_optimal)

    expnType_result  = expnType(result)
    expnType_optimal = expnType(optimal)

    if str(result).find("Integral") != -1:
        grade = "F"
        grade_annotation =""
    else:
        if expnType_result <= expnType_optimal:
            if result.has(I):
                if optimal.has(I): #both result and optimal complex
                    if leaf_count_result <= 2*leaf_count_optimal:
                        grade = "A"
                        grade_annotation =""
                    else:
                        grade = "B"
                        grade_annotation ="Both result and optimal contain complex but leaf count of result is large
                else: #result contains complex but optimal is not
                    grade = "C"
                    grade_annotation ="Result contains complex when optimal does not."
            else: # result do not contain complex, this assumes optimal do not as well
                if leaf_count_result <= 2*leaf_count_optimal:
                    grade = "A"
                    grade_annotation =""
                else:
                    grade = "B"
                    grade_annotation ="Leaf count of result is larger than twice the leaf count of optimal. "+str(le
        else:
            grade = "C"
            grade_annotation ="Result contains higher order function than in optimal. Order "+str(ExpnType_
```

```
    #print("Before returning. grade=",grade, " grade_annotation=",grade_annotation)

    return grade, grade_annotation
```

SageMath grading function

```
#Dec 24, 2019. Nasser: Ported original Maple grading function by
#          Albert Rich to use with Sagemath. This is used to
#          grade Fricas, Giac and Maxima results.
#Dec 24, 2019. Nasser: Added 'exp_integral_e' and 'sng', 'sin_integral'
#          'arctan2','floor','abs','log_integral'
#June 4, 2022  Made default grade_annotation "none" instead of "" due
#          issue later when reading the file.
#July 14, 2022. Added ellipticF. This is until they fix sagemath, then remove it.

from sage.all import *
from sage.symbolic.operators import add_vararg, mul_vararg

debug=False;

def tree_size(expr):
    r"""
    Return the tree size of this expression.
    """
    #print("Enter tree_size, expr is ",expr)

    if expr not in SR:
        # deal with lists, tuples, vectors
        return 1 + sum(tree_size(a) for a in expr)
    expr = SR(expr)
    x, aa = expr.operator(), expr.operands()
    if x is None:
        return 1
    else:
        return 1 + sum(tree_size(a) for a in aa)

def is_sqrt(expr):
    if expr.operator() == operator.pow:   #isinstance(expr,Pow):
        if expr.operands()[1]==1/2: #expr.args[1] == Rational(1,2):
            if debug: print ("expr is sqrt")
            return True
        else:
            return False
    else:
        return False
```

```
def is_elementary_function(func):
    #debug=False
    m = func.name() in ['exp','log','ln',
            'sin','cos','tan','cot','sec','csc',
            'arcsin','arccos','arctan','arccot','arcsec','arccsc',
            'sinh','cosh','tanh','coth','sech','csch',
            'arcsinh','arccosh','arctanh','arccoth','arcsech','arccsch','sgn',
        'arctan2','floor','abs'
        ]
    if debug:
        if m:
            print ("func ", func , " is elementary_function")
        else:
            print ("func ", func , " is NOT elementary_function")


    return m

def is_special_function(func):
    #debug=False
    if debug:
        print ("type(func)=", type(func))

    m= func.name() in ['erf','erfc','erfi','fresnel_sin','fresnel_cos','Ei',
          'Ei','Li','Si','sin_integral','Ci','cos_integral','Shi','sinh_integral'
          'Chi','cosh_integral','gamma','log_gamma','psi,zeta',
          'polylog','lambert_w','elliptic_f','elliptic_e','ellipticF',
          'elliptic_pi','exp_integral_e','log_integral']

    if debug:
        print ("m=",m)
        if m:
            print ("func ", func ," is special_function")
        else:
            print ("func ", func ," is NOT special_function")


    return m


def is_hypergeometric_function(func):
    return func.name() in ['hypergeometric','hypergeometric_M','hypergeometric_U']

def is_appell_function(func):
    return func.name() in ['hypergeometric']   #[appellf1] can't find this in sagemath
```

```
def is_atom(expn):

    #debug=False
    if debug:
        print ("Enter is_atom, expn=",expn)

    if not hasattr(expn, 'parent'):
        return False


    #thanks to answer at https://ask.sagemath.org/question/49179/what-is-sagemath-equivalent-to-atomic-t
    try:
        if expn.parent() is SR:
            return expn.operator() is None
        if expn.parent() in (ZZ, QQ, AA, QQbar):
            return expn in expn.parent() # Should always return True
        if hasattr(expn.parent(),"base_ring") and hasattr(expn.parent(),"gens"):
            return expn in expn.parent().base_ring() or expn in expn.parent().gens()

        return False

    except AttributeError as error:
        print("Exception,AttributeError in  is_atom")
        print ("cought exception" , type(error).__name__ )
        return False


def expnType(expn):

    if debug:
        print (">>>>>Enter expnType, expn=", expn)
        print (">>>>>is_atom(expn)=", is_atom(expn))

    if is_atom(expn):
        return 1
    elif type(expn)==list:   #isinstance(expn,list):
        return max(map(expnType, expn))    #apply(max,map(ExpnType,expn))
    elif  is_sqrt(expn):
        if  type(expn.operands()[0])==Rational:  #type(isinstance(expn.args[0],Rational):
            return 1
        else:
            return max(2,expnType(expn.operands()[0]))   #max(2,expnType(expn.args[0]))
    elif expn.operator() == operator.pow:    #isinstance(expn,Pow)
        if type(expn.operands()[1])==Integer:   #isinstance(expn.args[1],Integer)
            return expnType(expn.operands()[0])    #expnType(expn.args[0])
        elif type(expn.operands()[1])==Rational:   #isinstance(expn.args[1],Rational)
            if type(expn.operands()[0])==Rational:  #isinstance(expn.args[0],Rational)
```

```
            return 1
        else:
            return max(2,expnType(expn.operands()[0]))  #max(2,expnType(expn.args[0]))
    else:
        return max(3,expnType(expn.operands()[0]),expnType(expn.operands()[1])) #max(3,expnType(expn.
elif expn.operator() == add_vararg or expn.operator() == mul_vararg: #isinstance(expn,Add) or isinsta
    m1 = expnType(expn.operands()[0]) #expnType(expn.args[0])
    m2 = expnType(expn.operands()[1:]) #expnType(list(expn.args[1:]))
    return max(m1,m2)  #max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
elif is_elementary_function(expn.operator()):  #is_elementary_function(expn.func)
    return max(3,expnType(expn.operands()[0]))
elif is_special_function(expn.operator()): #is_special_function(expn.func)
    m1 = max(map(expnType, expn.operands()))      #max(map(expnType, list(expn.args)))
    return max(4,m1)   #max(4,m1)
elif is_hypergeometric_function(expn.operator()): #is_hypergeometric_function(expn.func)
    m1 = max(map(expnType, expn.operands()))      #max(map(expnType, list(expn.args)))
    return max(5,m1)   #max(5,m1)
elif is_appell_function(expn.operator()):
    m1 = max(map(expnType, expn.operands()))      #max(map(expnType, list(expn.args)))
    return max(6,m1)   #max(6,m1)
elif str(expn).find("Integral") != -1: #this will never happen, since it
        #is checked before calling the grading function that is passed.
        #but kept it here.
    m1 = max(map(expnType, expn.operands()))      #max(map(expnType, list(expn.args)))
    return max(8,m1)   #max(5,apply(max,map(ExpnType,[op(expn)])))
else:
    return 9

#main function
def grade_antiderivative(result,optimal):


    if debug:
        print ("Enter grade_antiderivative for sagemath")
        print("Enter grade_antiderivative, result=",result)
        print("Enter grade_antiderivative, optimal=",optimal)
        print("type(anti)=",type(result))
        print("type(optimal)=",type(optimal))

    leaf_count_result  = tree_size(result) #leaf_count(result)
    leaf_count_optimal = tree_size(optimal) #leaf_count(optimal)

    #if debug: print ("leaf_count_result=", leaf_count_result, "leaf_count_optimal=",leaf_count_optimal)


    expnType_result  = expnType(result)
    expnType_optimal = expnType(optimal)
```

```
if debug: print ("expnType_result=", expnType_result, "expnType_optimal=",expnType_optimal)

if expnType_result <= expnType_optimal:
    if result.has(I):
        if optimal.has(I): #both result and optimal complex
            if leaf_count_result <= 2*leaf_count_optimal:
                grade = "A"
                grade_annotation ="none"
            else:
                grade = "B"
                grade_annotation ="Both result and optimal contain complex but leaf count of result is larger th
        else: #result contains complex but optimal is not
            grade = "C"
            grade_annotation ="Result contains complex when optimal does not."
    else: # result do not contain complex, this assumes optimal do not as well
        if leaf_count_result <= 2*leaf_count_optimal:
            grade = "A"
            grade_annotation ="none"
        else:
            grade = "B"
            grade_annotation ="Leaf count of result is larger than twice the leaf count of optimal. "+str(leaf_c
else:
    grade = "C"
    grade_annotation ="Result contains higher order function than in optimal. Order "+str(expnType_resul


print("Before returning. grade=",grade, " grade_annotation=",grade_annotation)

return grade, grade_annotation
```